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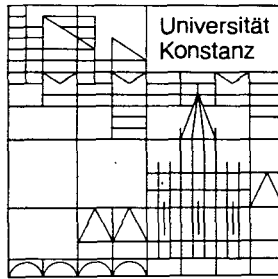
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**Monopolistic Competition with
Pro- and Countercyclical Pricing**

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MONOPOLISTIC COMPETITION WITH PRO- AND COUNTERCYCLICAL PRICING

by

Helmut Zink

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ABSTRACT:

We develop a market model which explains how prices react to short-run demand variations when the number of active price-setting firms is held fixed on its long-run level. We assume that for each firm the average production cost function is U-shaped, that customers are imperfectly informed about the quality of offers, and that customers may search for better offers.

For low degrees of market transparency the long-run market outcome exhibits price dispersion with an endogenous finite number of firms. In this case, in the short-run, price mark-ups respond countercyclically to demand variations and productivity is procyclical. In the complementary case of higher degrees of market transparency, in the long-run we have a single-price equilibrium. In that case, in the short-run price mark-ups fall with decreasing demand while productivity diminishes with any deviation of demand from its long-run level.

JEL Classification new No.: D40, D83, L11

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1. INTRODUCTION

According to both classical economics and the "General Theory" of Keynes, the relation of output prices to input prices should move procyclically over business cycles and productivity should be countercyclical. However, so far, no such hypothesis has found unanimous empirical support. On the contrary, there is strong evidence for procyclical behavior of productivity (compare e.g. R.E. Hall (1987)). Hall (1986) shows that in most U.S. manufacturing industries (from 1949 to 1978), prices are well above marginal costs while profits are nonexcessive. He suggests that in these industries firms are frequently operating on the decreasing portions of their average cost curves. M. Bils (1987) supports Hall's findings. He further shows that in most U.S. manufacturing sectors since 1956, price-marginal cost mark-ups are strongly countercyclical, and even prices are countercyclical.

However, Bils also indicates that cyclicity varies strongly over industries. In some sectors price mark-ups are procyclical while productivity is countercyclical. In Zink (1989, 1991) we provide for an explanation of countercyclical, positive price mark-ups and procyclical productivity. That explanation is based on a market structure in which firms produce with positive overhead costs and constant marginal costs while customers are imperfectly informed about the quality of offers.¹ In the present paper we develop a more general model for U-shaped average cost curves. Depending on its parameters, this model exhibits either countercyclical or (quasi)-procyclical price-marginal cost mark-ups. Correspondingly two types of productivity behavior may occur. For simplicity, we confine ourselves here to the analysis of a partial model in which input prices are exogenous and constant over time, and in which demand variations occur exogenously.

We construct a market model in which price setting firms produce a good at different quality levels. Average production cost per quality unit form a U-shaped curve. The market is intransparent with respect to the quality of the goods. That is, each customer knows the quality-unit price of a given firm with a certain probability only. This probability is denoted as publicity degree or degree of market transparency. Each customer buys a fixed number of quality units from the cheapest firm among those whose offer quality is known to him. Customers can search for cheaper offers trading-off search costs with expected reductions of expenditure. In an alternative interpretation of the model, customers have perfect information but products and preferences are

strongly heterogeneous and each customer can adapt to offers which are originally incompatible with his preferences.²

We find that under continuity assumptions there always exists a unique, so called canonical equilibrium under which each firm makes expected profits of zero and no single firm can increase its expected profits. This equilibrium is calculated explicitly. Depending on the parameters of the model, this long-run equilibrium either corresponds with classical views and then has a single market price equal to minimal average cost or it exhibits price dispersion and firms produce at the downward sloping part of the average cost curve. The relevant parameters refer to the degree of market transparency (respectively the degree of product differentiation) and to the relative size of aggregate demand with respect to the capacity of single firms. With high degrees of market transparency and small capacities we get the classical outcome. In the complementary case of small degrees of market transparency we get the equilibrium with price dispersion.

To analyze how firms respond in the short-run to demand variations we hold the number of firms fixed on its long-run level and determine their profit and prices endogenously. We find that prices and mark-ups behave (quasi)-procyclically if and only if the degree of market transparency is so high that the corresponding long-run equilibrium is classical. In the complementary case of low degrees of transparency and high product differentiation, however, we get countercyclical movements of prices and mark-ups. In the classical case any deviation of demand from its long-run level reduces (total factor) productivity, in the complementary case productivity is procyclical.

For related models on market intransparency and product differentiation compare G. Butters (1977) and O. Hart (1985).³

This paper is organized according to the following plan. In subsection 2.1 we introduce the assumptions of the model. A continuous version of it, which is more accessible to analysis, is elaborated in subsection 2.2. In subsection 2.3 we present our equilibrium concepts for the long-run and for the short-run. Section 3 gives an explicit characterization of long- and short-run market solutions. Some conclusions are derived in Section 4 while Section 5 gives a summary. Proofs are contained in the Appendix.

2. THE MODEL

2.1 Assumptions

Let us consider a market for a good which can be produced at different (one-dimensional) quality levels. Each firm $i=1,2,\dots$ can produce this good at constant marginal cost, a , per quality unit as long as its production level x does not exceed its capacity k . However, positive overhead cost, b , occur per unit of time and capacity overutilization (i.e. $x>k$) involves increasing average cost. Hence, each firm can produce x quality units at average cost (per quality unit) of

$$(1) \quad f(x) = a + \frac{b}{x} + u(\max(x-k, 0)), \quad x \geq 0,$$

where $a \in (0, \infty)$, $b \in (0, \infty)$, $k \in (0, \infty)$, and u is a real-valued differentiable function with $u(0)=0$, $\dot{u}(0) \geq b/k^2$ and $[u(y)-u(0)]\ddot{u}(y)/[\dot{u}(y)]^2 \geq 1$ for all $y \geq 0$ so that u is growing with increasing rates and f has a unique minimum at $x=k$. Let

$$(2) \quad p^\circ := f(k) = a + \frac{b}{k}.$$

FIGURE 1: Average production cost as a function of output

Each firm i charges a nominal price. However, its price per quality unit, p_i , may be unknown to customers. Henceforth, the term "price" refers to price per quality unit. Suppose in a period there is an expected number of q_i customers each of whom is going to buy θ quality units of the good from firm i during this period, $\theta > 0$. Then, for simplicity, the expected profit of this firm is assumed to be⁴

$$(3) \quad g_i = \theta q_i (p_i - f(\theta q_i)).$$

Let there be $c \in (1, \infty)$ new customers at the market in each period. Each customer stays at the market for one period. He buys θ quality units of the good from the cheapest firm he is informed about. (If there are several firms known to a customer with the same minimal price then this customer chooses randomly between them with equal probabilities.) We assume that any given customer - prior to his search - is informed about the price (per quality unit) of any specific firm i with a probability $\pi \in (0, 1)$, stochastically independent of all other relations. This probability is denoted as publicity degree of firms. We refer to this sort of imperfect information as market intransparency.

Each customer is informed about the distribution of prices and can search for cheaper offers. Each search step costs $\delta > 0$ and makes known to him the offer of exactly one firm. The probability that a search step informs about a specific firm i is proportional to the size of this firm as measured by the expected number of customers it receives prior to search. The gain to the customer from a single search step is measured by the reduction of his buying expenditures. It is equal to $\max(\theta(p' - p), 0)$ if p' is the smallest price of all offers known to him prior to the search step (p' is equal to ∞ if the customer does not know any offer) and p is the (random) price revealed to him by the considered search step. A customer searches as long as his expected gain from search is greater than his search cost δ . Search is assumed to take no time.

Each firm i may reject customers if it prefers to produce at a smaller level. Let $\omega_i \in (0, 1]$ describe the proportion of customers accepted by firm i . Then its expected profit is $g_i = \omega_i \theta q_i (p_i - f(\omega_i \theta q_i))$. Denote the firm as nonrejecting if it chooses $\omega_i = 1$. Since we will only consider market solutions where no active firm has any incentive to reject customers, we assume for simplicity that rejected customers do not buy until the next period.

In each period each firm i chooses its control variables $(p_i, \omega_i) \in [0, \infty] \times [0, 1]$ such as to maximize its expected profit g_i which also depends on the control variables of all other firms. Firms with an offer price $p_i = \infty$ do not get any customers. They are assumed to have a publicity degree of zero, make zero expected profits and are called inactive. Customers are not considered as players. We are looking for an assignment of strategies $(p_i, \omega_i)_{i=1,2,\dots}$ such that no firm, active or inactive, can increase its expected profit by changing its strategy (given that competing firms stick to their strategies).

Finally, we note that there are alternative interpretations of this model where market intransparency is replaced by product differentiation, that is, customers have perfect information but products and preferences are heterogeneous (see Zink (1989, 1991)).

2.2 Continuous Price Distributions

In this section we introduce the concept of a general price distribution. Let $H(p)$ denote the number of those active firms each of which offers a price below p , $p \in [0, \infty]$. Within the discrete model the values of any distribution H

are restricted to natural numbers. Now we transfer to a continuous version of the model. We allow H to be any real-valued, monotonously increasing and left-continuous function on $[0, \infty)$. We note that H may have both continuous parts and atoms (as well as singular continuous parts). We note also that even though the number of firms $H(p)$ may vary continuously in p , the total number of active firms $H(\infty)$ will be finite.⁵

Before analyzing the continuous version of the model we make precise how (i) a customer's expected gain from a search step and (ii) the expected number of a firm's customers are defined within the continuous model, that is in a market where firms are distributed according to a general distribution H .

Suppose firms' prices are distributed according to H and no active firm rejects customers. Let p' be the lowest price among all offers known to a given customer (respectively, $p' = \infty$ if he does not know any firm). Then this customer's expected gain from one search step is defined as⁶

$$(4) \quad R(p' | H) = \theta \int_0^{p'} (p' - p) \frac{q(p)}{\int_0^\infty q(x) dH(x)} dH(p).$$

Here $q(p)dH(p)$ denotes the distribution of customers over prices since firms are distributed according to H and any firm with price p expects to receive $q(p)$ customers. Thus, $q(p)dH(p)/\int_0^\infty qdH$ denotes the probability distribution over prices made available by one search step. The whole expression (4) describes the expected reduction of that customer's expenditure.

To define the expected number of customers of a firm we first define the reservation price of customers, $\hat{p}(H)$, as a solution of

$$(5) \quad R(\hat{p}(H) | H) = \delta.$$

Price \hat{p} is well-defined since $R(p' | H)$ increases continuously in p' from zero to infinity. Thus, if p' is the cheapest offer known to a customer then this customer searches if $p' > \hat{p}$ and he buys at p' if $p' \leq \hat{p}$.

The expected number of customers of a firm i with offer price $p \leq \hat{p}$ - before any customer may get rejected - is defined as

$$(6) \quad q(p | H) := \kappa(p) c \pi (1 - \pi)^{H(p)} + c (1 - \pi)^{H(\infty)} \frac{\kappa(p) c \pi (1 - \pi)^{H(p)}}{c - c (1 - \pi)^{H(\infty)}}$$

where $\kappa(p) := 1$ if H is continuous at p .⁷

The interpretation of this equation follows from the case in which $H(p)$ and $H(\infty)$ are natural numbers. Then the term $\pi(1-\pi)^{H(p)}$ describes the probability that - prior to search - any given customer is informed about firm i but not about any cheaper firm the number of which is $H(p)$. In case of $\kappa(p)=1$ such customers buy from firm i . The term $c(1-\pi)^{H(\infty)}$ describes the expected number of customers who - prior to search - have not found any firm they are content with. All these customers search until they find a firm cheaper than or equally expensive as \hat{p} . Any given customer among them finds firm i with a probability equal to the proportion of those customers which firm i attracts from all customers buying prior to search. For later use we note that (6) simplifies to

$$(7) \quad q(p|H) = \kappa(p)c\pi(1-\pi)^{H(p)} \frac{1}{1-(1-\pi)^{H(\infty)}} \quad \text{if } p \leq \hat{p}.$$

If firm i charges a price $p > \hat{p}$ then all customers prefer searching over buying at that price and $q(p|H)=0$.

For discontinuous distributions H we define^{8 9 10}

$$(8) \quad \kappa(p) := \begin{cases} \frac{1}{\rho} \frac{1}{m} [1-(1-\pi)^m] & \text{if } \lim_{x \downarrow p} H(x) - H(p) = m > 0 \\ 1 & \text{if } \lim_{x \downarrow p} H(x) - H(p) = m = 0 \end{cases}$$

with the abbreviation

$$(9) \quad \rho := -\ln(1-\pi).$$

Now, the expected profit of a firm with price p and acceptance rate ω (under a general distribution H) is given by

$$(10) \quad g(p, \omega|H) = \omega \theta q(p|H) [p - f(\omega \theta q(p|H))].$$

As abbreviation we set $g(p|H) := g(p, 1|H)$.

2.3 Market Equilibria

Now we are ready to define under what condition we consider a market to be in equilibrium. We note that the following definitions refer to the continuous version of the model, where for any distribution H , $H(p)$ describes the number of active firms offering a price below p .

DEFINITION 1: A distribution H of active nonrejecting firms is called long-run canonical (with respect to $(\theta, \pi, k, b, a, \delta)$) if expected profits fulfill

$$(11) \quad g(p, \omega | H) \quad \begin{cases} = 0 & \text{for } p \in S(H), \omega = 1 \\ \leq 0 & \text{else} \end{cases}$$

where $S(H) := \{p: H(p-\varepsilon) < H(p+\varepsilon) \text{ for all } \varepsilon > 0\}$ is the support of H . \square

We will see that under canonical distributions each firm forms a negligible part of the whole market if the market size c increases to infinity. Then, for finite c , $g(p, \omega | H)$ can be interpreted to approximately describe the profit possibilities of both (i) an additional firm previously being inactive and not considered by H and (ii) any firm previously being active and considered by H which evaluates a variation of its control variables (p, ω) . Hence, under a canonical distribution no firm - old or new - can expect positive profit possibilities from switching its price or from rejecting customers.¹¹

Long-run canonical distributions treat the number of active firms as endogenous while the level of expected profits is exogenously given equal to zero as it is motivated by exit and entry possibilities of firms in the long-run. To analyze the effects of a short-run variation of parameters, however, we take the number of active firms as exogenous while the level of expected profits is left endogenous.¹²

DEFINITION 2: A distribution H of active nonrejecting firms is called short-run canonical with respect to an exogenous number of active firms, N , if $H(\infty) = N$ and there exists a profit level $\gamma > -b$ such that expected profits fulfill

$$(12) \quad g(p, \omega | H) \quad \begin{cases} = \gamma & \text{for } p \in S(H), \omega = 1 \\ \leq \gamma & \text{else } \square \end{cases}$$

Finding short-run canonical distributions amounts to solving for long-run canonical distributions while treating the number of active firms N as exogenous and overhead costs $\tilde{b} = b + \gamma$ as endogenous.

To evaluate a distribution of prices in the form of a one-dimensional magnitude we introduce the concept of an average price as the expected price to be paid by a randomly chosen customer. As we will see, it suffices to calculate the average price for distributions under which all active firms (i)

produce at a level smaller or equal to the capacity level k and (ii) expect the same level of profits γ . In these cases active firms have overhead costs of b and constant variable costs of a . This allows us to define the average price in a simple way as that price p^* which equates aggregate expenditures of customers $\theta c p^*$ with the sum of aggregate production cost $Nb + \theta c a$ and aggregate profits $N\gamma$. Thus we get

DEFINITION 3: For a distribution of N active firms under which each active firm produces below or at the capacity level k and expects the same profit level γ , the average price is defined as

$$(13) \quad p^* = a + \frac{N(b+\gamma)}{\theta c}. \quad \square$$

3. LONG-RUN AND SHORT-RUN CANONICAL DISTRIBUTIONS

3.1 First we characterize long-run canonical distributions.

In this Section we will show that there always exists a unique long-run canonical distribution. The specific form of it depends on the intensity of demand θ , the degree of market transparency π , and the cost and capacity parameters b , a , δ , k . To describe canonical distributions we first define three domains of demand intensities.

Let N° be the unique solution of

$$(14) \quad \delta \frac{c\pi}{b} = e^{\rho N^\circ} - (1 + \rho N^\circ), \quad \text{where } \rho = -\ln(1-\pi),$$

and define $\theta_1 := [k/c\pi][1-(1-\pi)^{N^\circ}]$, $\theta_2 := [k/c\pi]\rho N^\circ$. Then $0 < \theta_1 < \theta_2 < \infty$. Now long-run canonical distributions are characterized by the following three theorems.

THEOREM 1.1:

If $\theta \in (0, \theta_1]$ then the long-run canonical distribution, $H_{\theta, b}$, is unique and absolutely continuous with the density

$$(15) \quad h(p) = \begin{cases} \frac{1}{\rho} \frac{1}{p-a} & \text{for } p \in [p^1, p^2] \\ 0 & \text{else,} \end{cases}$$

where

$$(16) \quad p^1 := a + \frac{b}{\theta c \pi} [1-(1-\pi)^{N^\circ}],$$

$$(17) \quad p^2 := a + \frac{b}{\theta c \pi} \frac{1 - (1 - \pi)^{N^\circ}}{(1 - \pi)^{N^\circ}}$$

and $p^\circ \leq p^1 < p^2 < \infty$. The number of active firms, $N_\theta := H_{\theta, b}(\infty)$, is equal to N° and $N^\circ \in (0, \infty)$. If θ increases to θ_1 then p^1 decreases to p° . The average price p^* of H is given by

$$(18) \quad p^* = a + \frac{N^\circ b}{\theta c}.$$

which decreases in θ . \square

THEOREM 1.2:

If $\theta \in (\theta_1, \theta_2)$ then the long-run canonical distribution, $H_{\theta, b}$, is unique. It has an atom of mass $m > 0$ at p° while beside p° it is absolutely continuous with the density

$$(19) \quad h(p) = \begin{cases} \frac{1}{\rho} \frac{1}{p - a} & \text{for } p \in [p^1, p^2] \\ 0 & \text{else,} \end{cases}$$

where

$$(20) \quad p^1 = a + \frac{b}{\theta c \pi} \frac{1 - (1 - \pi)^{N^\circ}}{(1 - \pi)^m},$$

$$(21) \quad p^2 := a + \frac{b}{\theta c \pi} \frac{1 - (1 - \pi)^{N^\circ}}{(1 - \pi)^{N^\circ}},$$

and m is the unique solution of

$$(22) \quad \frac{1}{m} [1 - (1 - \pi)^m] = \frac{\rho}{\pi} \frac{k}{\theta c} [1 - (1 - \pi)^{N^\circ}].$$

The number of active firms, N_θ , is again equal to N° and $N^\circ \in (0, \infty)$. Further $m \in (0, N^\circ)$. If θ decreases to θ_1 then m decreases to zero and p^1 decreases to p° . If θ increases to θ_2 then m increases to N° , p^2 decreases to $p^\circ + (\delta/\theta)$, and p^1 increases to $p^\circ + (\delta/\theta)$. The average price is again given by (18) and thus it decreases in θ . \square

THEOREM 1.3:

If $\theta \in [\theta_2, \infty)$ then the long-run canonical distribution, $H_{\theta, b}$, is unique. It consists of a single atom at p° and the number of active firms, N_θ , is given by

$$(23) \quad N_\theta = \frac{\pi}{\rho} \frac{\theta c}{k}.$$

In case of $\theta = \theta_2$ we have $N_\theta = N^\circ$. \square

For a proof see Appendix A. The main results of the above theorems can be described by Figure 2 and Figure 3.

FIGURE 2: Shape of the long-run canonical density of prices

FIGURE 3: Long-run canonical prices as a function of demand

For each demand intensity $\theta \in (0, \theta_2)$ the continuous part of the canonical distribution $H_{\theta,b}$ is given by the density $h = h_{\theta,b}$ which is depicted in Figure 2. While the support of $h_{\theta,b}$ depends on θ , its shape is independent thereof. Under $H_{\theta,b}$ each active firm is indifferent with respect to any variation of its own price within the support of $H_{\theta,b}$, $S(H_{\theta,b})$. Any price increase within $S(H_{\theta,b})$ would increase its revenue received per unit sold, but it would reduce its expected number of customers since the number of firms with lower prices would increase. The canonical density is such that both these effects just balance. Further, at prices below $\inf S(H_{\theta,b})$ or within (p°, p^1) firms could not cover costs. At prices above p^2 customers would prefer searching.

The graphs in Figure 3 describe for each demand intensity θ the boundaries of the support of $H_{\theta,b}$. For $\theta < \theta_1$ the canonical distribution is absolutely continuous with support $[p^1, p^2]$. As θ increases to θ_1 firms produce at higher levels and prices fall. The number of active firms, N° , remains constant throughout. The lower bound of the canonical support, p^1 , converges to the minimum of average costs, p° . Further increases of demand cannot result in still lower prices since some firms start producing at their capacity level k . Now the number of firms offering at p° increases. For $\theta > \theta_1$ the canonical distribution has an atom in p° . With increasing mass in p° the number of customers left over for firms with prices above p° shrinks. Thus, above p° the next cheapest price p^1 has to increase in θ to cover production cost in spite of increasing demand per customer. The number of active firms offering above p° decreases correspondingly. As θ has increased to θ_2 , p^1 has risen so high that customers prefer searching over buying at p^1 . For $\theta \leq \theta_2$ the number of active firms has remained constant, N° . Further growth of demand results in a growing number of active firms all of which produce at capacity level k and charge minimal price p° . Figure 3 also shows the average price p^* which decreases monotonously in θ while remaining constant for $\theta \geq \theta_2$.

The property that the canonical number of active firms remains constant for all $\theta < \theta_1$ is due to the simple form of the average production cost

function. We give an intuition for this constancy. Suppose firms are distributed according to the canonical distribution and the demand intensity reduces to half its value. How would prices and the number of active firms respond in order to maintain zero expected profits? We will argue that profits and the number of active firms remain invariant according to equation (3) if all firms double their price mark-up $p-a$ in response. By this price increase each active firm maintains the size of its customer stock q provided the mark-up of the reservation price of customers $\hat{p}-a$ doubles too. At any demand intensity the reservation price is determined by the condition that a customer (who does not know any firm prior to search) faces equal expected expenditure either (i) from buying at the reservation price which costs $\theta\hat{p}$ or (ii) from searching and buying at the expected price of the firm found by search, which costs $\delta+\theta p_{\text{search}}$. Since the price mark-up of each active firm doubles, so will the mark-up of the expected price found by search, and hence the mark-up of the reservation price. Thus, it is plausible that the canonical number of active firms and expected profits remain invariant if price mark-ups double in response to a halving of demand.

3.2 Next we characterize short-run canonical distributions.

Let $\bar{\theta} \in (0, \infty)$ be the long-run demand intensity and $N_{\bar{\theta}}$ the number of active firms under the long-run canonical distribution $H_{\bar{\theta}, b}$ with respect to the demand intensity $\bar{\theta}$ and overhead cost b . We distinguish two cases with respect to $\bar{\theta}$.

THEOREM 2.1:

1. Let $\bar{\theta} < \theta_2$ and $\theta < \theta_2$. Then the short-run canonical distribution with respect to the demand intensity θ and $N_{\bar{\theta}}$ active firms, $H_{\theta, \bar{\theta}}$, is unique and identical to the long-run canonical distribution with respect to (θ, b) , $H_{\theta, b}$. Short-run expected profits are zero and the average price $p^*(\theta, \bar{\theta})$ decreases in θ ,

$$(24) \quad p^*(\theta, \bar{\theta}) = a + \frac{N_{\bar{\theta}} b}{\theta c}.$$

2. Let $\bar{\theta} < \theta_2$ and $\theta > \theta_2$. Then a short-run canonical distribution with respect to the demand intensity θ and $N_{\bar{\theta}}$ active firms, $H_{\theta, \bar{\theta}}$, is given by the single-price equilibrium $p^+(\theta, \bar{\theta})$ where all firms choose the same price equal to marginal production cost, that is

$$(25) \quad p^+(\theta, \bar{\theta}) := f(x_{\theta}) + x_{\theta} \dot{f}(x_{\theta}) > p^{\circ}$$

and $x_{\theta} := x(\theta, \bar{\theta}) := (\pi/\rho)\theta c/N_{\bar{\theta}} = (\theta/\theta_2)k > k$.¹³ Then each active firm makes positive

expected profits

$$(26) \quad g(p^+ | H_{\theta, \bar{\theta}}) = x_{\theta}^2 \dot{f}(x_{\theta}) > 0.$$

Both the common price p^+ and expected profits increase in θ .¹⁴ There is no other single-price canonical distribution than that described in (25). \square

For a proof of Theorem 2.1 see Appendix C. The main results of the theorem can again be described graphically. In Figure 4 we describe the support of short-run canonical distributions for a long-run demand intensity $\bar{\theta} < \theta_2$ and variable short-run demand intensities θ .

FIGURE 4: Short-run canonical prices as a function of demand,
inverse price reaction case

If $\bar{\theta} < \theta_2$ and $\theta < \theta_2$ then the short-run canonical distribution of $H_{\theta, \bar{\theta}}$ coincides with the long-run canonical distribution of $H_{\theta, b}$. The reason is that the number of active firms under the long-run canonical distribution $H_{\theta, b}$ is independent of θ as can be seen from Theorem 1.1, Theorem 1.2, and (14). Hence, according to the definition of short-run canonical distributions, short-run expected profits remain zero for any $\theta \in (0, \theta_2)$. This invariance of short-run expected profits can be made plausible in analogy to the intuition given in connection with Theorem 1.1.

Suppose again, firms are distributed according to the canonical distribution and the demand intensity reduces to half its value. How would prices and profits react if the number of active firms remains fixed? Again, profits remain invariant if all firms double their price mark-up p -a in response. By this price increase each active firm maintains the size of its customer stock since the mark-up of the reservation price of customers doubles too as we saw above, and, due to the price increase the revenue received per customer remains invariant. Thus the resulting distribution is again canonical with a profit level of zero.

The identity of short- and long-run canonical distributions implies that any decrease in demand pushes up the average price mark-up $p^*(\theta, \bar{\theta})$ -a by the same proportion. On the other hand, an increase in demand pushes down the average price analogously as long as $\theta \leq \theta_2$. The left branch of the graphs in Figure 4 is equal to that of Figure 3.

For $\bar{\theta} < \theta_2$ and $\theta > \theta_2$, however, the long-run canonical distribution $H_{\theta, b}$ would require a larger number of active firms than under $\bar{\theta}$, $N_{\theta, b} > N_{\bar{\theta}, b} = N^*$. Hence, with all N^* active firms charging the same price p^+ , each firm produces at a level above capacity k . Then each firm could lower its average production cost by rejecting customers and it could increase sales by slightly reducing its price. Therefore, $p^+(\theta, \bar{\theta})$ is the unique price at which the level of expected profits is so high that no firm can increase its expected profits, neither by any of these two strategies nor by any price increase. Thus, in the short-run an increase of demand θ above θ_2 pushes up average production cost, prices and profits.

THEOREM 2.2:

1. Let $\bar{\theta} > \theta_2$ and $\theta > \bar{\theta}$. Then a short-run canonical distribution with respect to the demand intensity θ and $N_{\bar{\theta}}$ active firms, $H_{\theta, \bar{\theta}}$, is again given by the single-price equilibrium $p^+(\theta, \bar{\theta})$ where all firms choose the same price equal to marginal production cost as it is described by (25) where the production level is now $x(\theta, \bar{\theta}) = (\pi/\rho)\theta c/N_{\bar{\theta}} = (\theta/\bar{\theta})k$.

Each active firm makes positive expected profits as described by (26) and both, the common price p^+ and expected profits increase in θ . There is no other single-price canonical distribution.

2. Let $\bar{\theta} > \theta_2$ and $\theta < \bar{\theta}$. Then a short-run canonical distribution with respect to the demand intensity θ and $N_{\bar{\theta}}$ active firms, $H_{\theta, \bar{\theta}}$, is uniquely given by the long-run canonical distribution with respect to θ and the fixed cost level $\tilde{b}(N_{\bar{\theta}})$, $H_{\theta, \tilde{b}(N_{\bar{\theta}})}$, where

$$(27) \quad \tilde{b}(N_{\bar{\theta}}) = \delta c \pi / [e^{\rho N_{\bar{\theta}}} - (1 + \rho N_{\bar{\theta}})] > 0.$$

Now each active firm incurs negative expected profits which are independent of θ ,

$$(28) \quad g(p | H_{\theta, \tilde{b}(N_{\bar{\theta}})}) = \tilde{b}(N_{\bar{\theta}}) - b \in (-b, 0) \quad \text{for } p \in S(H_{\theta, \tilde{b}}).$$

The average price $p^*(\theta, \bar{\theta})$ decreases in θ ,

$$(29) \quad p^*(\theta, \bar{\theta}) = a + \frac{N_{\bar{\theta}} \tilde{b}(N_{\bar{\theta}})}{\theta c},$$

with

$$(30) \quad \lim_{\theta \rightarrow \bar{\theta}} p^*(\theta, \bar{\theta}) = a + \frac{\tilde{b}(N_{\bar{\theta}})}{b} [p^0 - a] \in (a, p^0). \quad \square^{15}$$

In Figure 5 we describe the support of short-run canonical distributions for a long-run demand intensity $\bar{\theta} > \theta_2$ and variable short-run demand intensities θ .

FIGURE 5: Short-run canonical prices as a function of demand,
normal price reaction case

If $\bar{\theta} > \theta_2$ and $\theta > \bar{\theta}$ then, again, a single price $p^+(\theta, \bar{\theta})$ forms a short-run canonical distribution as in case of $\bar{\theta} < \theta_2 < \theta$. The only difference is that the number of active firms is higher now, $N_{\bar{\theta}} > N^0$. Again, in the short-run any increase of demand θ pushes up average production cost, prices and profits.

However, if $\bar{\theta} > \theta_2$ and $\theta < \bar{\theta}$ then the number of active firms, $N_{\bar{\theta}}$, is too large to allow for a competitive outcome with zero profit level. Competition will push down the average price. In the short-run, any active firm would rather sell at a price just above a , but below minimal average production cost p^0 , than being left without customers. A canonical distribution with a number of firms higher than N^0 requires a sum of overhead cost and expected profit smaller than b . Hence, for $\theta < \bar{\theta}$ the short-run canonical distributions can be described as in the above case of $\bar{\theta} < \theta_2 < \theta$, but with a negative level of expected profit. This level depends on $N_{\bar{\theta}}/N^0$ but is independent of θ . Thus, in our simple model, the average price will jump down discontinuously to $\tilde{p}^0 := a + [\tilde{b}(N_{\bar{\theta}})/b][p^0 - a]$ when θ decreases from its long-run level $\bar{\theta}$. For $\theta < \bar{\theta}$, though, further decreases of demand push up average production cost and the average price p^* while expected profits remain constant on their negative level. For small variations of θ around $\bar{\theta}$, thus, both the average price and the average mark-up are (quasi-) procyclical.

3.3 Next, we calculate how the canonical distribution evolves if the market size c increases to infinity. With increasing market size we do not want to hold the publicity degree of firms, π , exogenously fixed. In Zink (1989, 1991) we endogenized π by allowing each firm i to increase its own publicity degree π_i at the expense of increasing marketing cost, $b = b(\pi, c)$. There we found that for a overhead cost functions like $b(\pi, c) = \beta e^{\nu c \pi}$ with $\nu \geq 1$ and $\beta > 0$ each firm chooses the publicity degree $\pi = 1/(\nu c)$ which minimizes its average overhead cost per unit of sale. In the present model, too, with this marketing cost function each firm would choose $\pi = 1/(\nu c)$ since each firm's customer stock is linear in its own publicity degree. We get the following asymptotic properties.

THEOREM 3: If overhead cost are given by $b(\pi, c) = \beta(c) e^{\nu c \pi}$ with $\lim_{c \rightarrow \infty} \beta(c) \in (0, \infty)$ and each firm chooses the publicity degree $\pi = 1/\nu c$, then the long-run canonical distribution exhibits the following properties:

$$(31) \quad \lim_{c \rightarrow \infty} N^\circ = \infty, \quad \lim_{c \rightarrow \infty} \rho N^\circ \in (0, \infty),$$

$$(32) \quad 0 < \lim_{c \rightarrow \infty} \theta_1 < \lim_{c \rightarrow \infty} \theta_2 < \infty,$$

$$(33) \quad \lim_{c \rightarrow \infty} p_\theta^* = a + \frac{\nu}{\theta} \lim_{c \rightarrow \infty} \rho N^\circ \quad \text{for } \theta < \lim_{c \rightarrow \infty} \theta_2,$$

and, in case of $\bar{\theta} > \theta_2$, the size of the price jump \tilde{p}°/p° is independent of c.o

Theorem 3 follows from straightforward calculation.¹⁶ It suggests that all features recognized in Theorems 1 and 2 remain valid if the market size is increased to infinity.

4. A TYPOLOGY: CLASSICAL AND NON-CLASSICAL INDUSTRIES

Depending on the relative size of long-run demand $\bar{\theta}$ and threshold θ_2 the behavior of the industry can be of two types. If $\bar{\theta} < \theta_2$ then average price and average price mark-up above marginal cost both respond countercyclically to short-run demand variations. If $\bar{\theta} > \theta_2$ then the average price responds quasi-procyclically: That is, if θ increases from $\bar{\theta} > \theta_2$ then the average price of the corresponding short-run canonical distribution increases while the average mark-up remains zero. If θ decreases from $\bar{\theta} > \theta_2$ then both average price and average mark-up jump downward. Let us denote a parameter combination with $\bar{\theta} > \theta_2$ as classical or normal price reaction case, and a combination with $\bar{\theta} < \theta_2$ as non-classical or inverse price reaction case. Using the definition $\theta_2 := [k/c\pi]\rho N^\circ$ and N° being the solution of $\delta c\pi/b = \Psi(\rho N^\circ)$ with $\Psi(x) := e^x - (1+x)$ we get

$$(34) \quad \theta_2 = \frac{k}{c\pi} \Psi^{-1}\left(\frac{\delta c\pi}{b}\right).$$

Since Ψ^{-1} increases from the origin with diminishing slope, θ_2 increases in k and δ/b and it decreases in $c\pi$. That is, the normal price reaction case emerges for small search cost δ , high publicity degree π , small capacity level k , and large overhead cost b . Vice versa relations hold true for the inverse price reaction case.¹⁷

We may also classify price reactions not with respect to parameters but with respect to the characteristics which the model exhibits in the long-run. The normal price reaction case emerges if there is no price dispersion in the long-run. The inverse price reaction case emerges if price dispersion does prevail in the long-run.¹⁸

Finally we consider the short-run behavior of productivity. Total factor productivity η is defined as the ratio of aggregate real output over aggregate real input. If $\theta \geq \max(\theta_2, \bar{\theta})$ then all N firms produce at the same level, denoted as y . Hence aggregate output is equal to Ny and aggregate input, which is aggregate cost, is equal to $Nyf(y)$. Thus $\eta = Ny/[Nyf(y)] = 1/f(y)$. Deviating from our definitions (7) and (8) for simplicity, we set $y = \theta c/N$ since there are c customers each of whom buys θ units and all these goods are produced by N firms. Then $\eta = 1/f(\theta c/N)$. If $\theta \leq \max(\theta_2, \bar{\theta})$ then all firms produce within capacity, that is all firms face the same constant marginal cost a . Again ignoring definitions (7) and (8) for simplicity, we set total output equal to θc . Then $\eta = \theta c/[\theta ca + Nb] = 1/[a + b/(\theta c/N)] = 1/f(\theta c/N)$. Thus, in each case total factor productivity η is the inverse of the average cost function evaluated at the average production level $\theta c/N$. Then, in the classical case total factor productivity declines with any deviation of demand from its long-run level, and in the complementary non-classical case productivity is procyclical.

5. SUMMARY

We have developed a market model which explains how prices react to short-run demand variations when the number of active firms is held fixed on its long-run level. We assumed that for each firm the average production cost function is U-shaped, that customers are imperfectly informed about the quality of offers and that customers may search for better offers.

We showed that for each intensity of demand there exists a unique canonical distribution describing the market outcome. For low degrees of market transparency this canonical distribution exhibits price dispersion, for high degrees of transparency it is degenerated to a single-price equilibrium. We noted that market intransparency can be alternatively interpreted as product-differentiation with perfect information.

While we treated the number of active firms as endogenous in the long-run with expected profits being exogenously equal to zero, in the short-run we took the number of active firms as given leaving expected profits and prices as endogenous. We distinguished two cases. First, with high enough market transparency, rising demand results in the short-run in increasing prices and increasing profits. Declining demand, on the other hand, results in the short-run in a downward jump of all the average price, the average mark-up,

and the profit level. Total factor productivity is reduced by any deviation of demand from its long-run level. We referred to this case as the normal price reaction case or classical case. Second, with low enough market transparency, the average price and the average mark-up respond countercyclically to short-run demand variations. In this case productivity is procyclical. We referred to the second case as inverse price reaction case. We showed that the normal price reaction case arises if there is no price dispersion in the long-run. The inverse price reaction case arises if there is price dispersion in the long-run.¹⁹

APPENDIX A

PROOF OF THEOREM 1:

1.1. In Section 1 of this appendix we prove the existence of a canonical distribution. In Section 1.1 we construct a distribution H^* which will be shown to be canonical in Section 1.2.

For each $n \in (0, \infty)$ let $H_n: [0, \infty) \rightarrow [0, \infty)$ be that function which is uniquely determined by (A1)-(A5):

$$(A1) \quad H_n(p) := \begin{cases} 0 & \text{for } p \in [0, p^\circ] \\ m_n & \text{for } p \in (p^\circ, p_n^1] \\ m_n + \int_{p_n^1}^p \frac{1}{\rho} \frac{1}{x-a} dx & \text{for } p \in [p_n^1, p_n^2] \\ m_n + \int_{p_n^1}^{p_n^2} \frac{1}{\rho} \frac{1}{x-a} dx & \text{for } p \in [p_n^2, \infty) \end{cases}$$

$$(A2) \quad m_n := \max(\mu^{-1}(\frac{\rho}{\pi} \frac{k}{\theta c} [1 - (1-\pi)^n]), 0)$$

$$(A3) \quad \mu(x) := \begin{cases} [1 - (1-\pi)^x]/x & \text{for } x \neq 0 \\ \rho & \text{for } x = 0 \end{cases}$$

$$(A4) \quad p_n^1 := a + \frac{b}{\theta c \pi} \frac{1 - (1-\pi)^n}{m_n (1-\pi)}$$

$$(A5) \quad p_n^2 := \inf \{p \geq p_n^1 : R(p|H_n) \geq \delta\}$$

First we show that H_n is a distribution with $p^\circ \leq p_n^1 \leq p_n^2$ and $m_n \geq 0$. The last two inequalities follow directly from (A2) and (A5). To show $p^\circ \leq p_n^1$ we

note that

$$(A6) \quad p_n^1 = a + \frac{b}{k} \left[\frac{k}{\theta c \pi} \frac{1 - (1 - \pi)^n}{(1 - \pi)^{m_n}} \right].$$

Hence we have to prove that the bracket term is not smaller than unity. From (A2) we know that

$$(A7) \quad \frac{k}{\theta c \pi} [1 - (1 - \pi)^n] \begin{cases} = \frac{1}{\rho} \frac{1}{m_n} [1 - (1 - \pi)^{m_n}] & \text{if } m_n > 0 \\ \geq 1 & \text{if } m_n = 0. \end{cases}$$

Inserting (A7) into (A6) we get

$$(A8) \quad \frac{k}{\theta c \pi} \frac{1 - (1 - \pi)^n}{(1 - \pi)^{m_n}} \begin{cases} = \frac{1}{\rho^{m_n}} [e^{\rho m_n} - 1] > 1 & \text{if } m_n > 0 \\ \geq 1 & \text{if } m_n = 0 \end{cases}$$

Thus, $p^0 \leq p_n^1$ and strict inequality holds in case of $m_n > 0$.

Now we show that there exists a $n^* > 0$ such that $H_n^*(\omega) = n^*$ and $m_n^* \in [0, n^*]$.

Let

$$(A9) \quad \hat{n} := \frac{\pi}{\rho} \frac{\theta c}{k}.$$

Then (A2) implies that $[m_n \leq n \text{ if and only if } n \geq \hat{n}]$. For each $n \geq \hat{n}$ let

$$(A10) \quad \varphi(n) := H_n(\omega).$$

Since $m_n \geq 0$ for all n , it remains to be shown that function φ has a fixpoint $n^* \in [\hat{n}, \infty)$. Since $p_n^1 \leq p_n^2$ and $m_{\hat{n}} = \hat{n}$, (A1) implies $\varphi(\hat{n}) \geq \hat{n}$. Since φ is continuous in n , it suffices for the existence of a fixpoint $n^* \in [\hat{n}, \infty)$ to show that $\varphi(n) < n$ for some finite $n \geq \hat{n}$.

Since the expected gain from one search step, $R(p|H_n)$, is continuous on $[p^0, \infty)$, (A5) implies that $p_n^2 = p_n^1$ or $R(p_n^2|H_n) = \delta$ (or both). On the other side, (A1) and (4) imply that $R(p_n^2|H_n) \geq \theta(p_n^2 - p_n^1)/2$, since (i) the probability density of finding a firm with a specific price p by one search step decreases in p for $p \in (p_n^1, p_n^2)$, and (ii) $p_n^2 = \sup S(H_n)$. Hence we get $p_n^2 - p_n^1 \leq 2\delta/\theta$. Now we get from (A1)

$$(A11) \quad \varphi(n) \leq m_n + 2 \frac{\delta}{\theta} \frac{1}{\rho} \frac{1}{p_n^{1-a}}.$$

As n increases to infinity, m_n converges to zero according to (A2) and $(p_n^1)^{1-a}$ converges to $b/\theta c \pi$ according to (A4). Hence, (A11) implies that $\varphi(n) < n$ for large n which proves the existence of a fixpoint n^* with $H_n^*(\omega) = n^*$ and $m_n^* \leq n^*$.

1.2. Now we prove that H_N^* is canonical, i.e. that (11) is fulfilled. We consider two cases with respect to $p_N^{2*} - p_N^{1*}$. For notational simplicity we set $N := n^*$.

First, suppose that $p_N^2 = p_N^1$. Then (A1) implies that $S(H_N) = \{p^\circ\}$ and $m_N = N > 0$. Hence (A2) implies $N = (\pi/\rho)\theta c/k$. Further, $p_N^2 = p_N^1$ and (A5) imply that $p^\circ < \hat{p}(H_N) \leq p_N^1$. Using these relations we can calculate the expected profit for alternative prices p according to (10), (1), and (7). For $p = p^\circ$ we get

$$(A12) \quad g(p^\circ | H_N) = \theta c \frac{\pi}{\rho m_N} [1 - (1 - \pi)^{m_N}] \frac{1}{1 - (1 - \pi)^N} (p^\circ - a) - b \\ = k (p^\circ - a) - b = 0.$$

For $p \in (p^\circ, \hat{p}(H_N)]$ we get

$$(A13) \quad g(p | H_N) = \theta \pi c \frac{(1 - \pi)^N}{1 - (1 - \pi)^N} (p - a) - b \\ \leq \theta \pi c \frac{(1 - \pi)^N}{1 - (1 - \pi)^N} (p_N^1 - a) - b = 0.$$

At prices below p° , expected profits are negative since average costs are above p° for any output level. At prices above $\hat{p}(H_N)$, sales are zero because customers prefer searching.

Now, suppose $p_N^1 < p_N^2$. Then (A5) implies $\hat{p}(H_N) = p_N^2$. From (A4) we get for $p \in (p^\circ, p_N^1]$

$$(A14) \quad g(p | H_N) = \theta \pi c \frac{(1 - \pi)^{m_N}}{1 - (1 - \pi)^N} (p - a) - b \\ \leq \theta \pi c \frac{(1 - \pi)^{m_N}}{1 - (1 - \pi)^N} (p_N^1 - a) - b = 0.$$

Since

$$(A15) \quad H_N(p) = m_N + \frac{1}{\rho} \ln \frac{p - a}{p_N^1 - a} \quad \text{for } p \in [p_N^1, p_N^2],$$

we get for $p \in [p_N^1, p_N^2]$

$$(A16) \quad g(p | H_N) = \theta \pi c (1 - \pi)^{H_N(p)} \frac{1}{1 - (1 - \pi)^N} (p - a) - b$$

$$= \theta \pi c (1-\pi)^{m_N} \frac{p_N^1 - a}{p - a} \frac{p - a}{1 - (1-\pi)^N} - b = 0.$$

In case of $m_N > 0$ we get for $p = p^\circ$ according to (A2)

$$(A17) \quad g(p|H_N) = \theta c \frac{\pi}{\rho m_N} [1 - (1-\pi)^{m_N}] \frac{p^\circ - a}{1 - (1-\pi)^N} - b$$

$$= k(p^\circ - a) - b = 0.$$

In case of $m_N = 0$ nonpositive expected profits in p° follow from (A16) and $p^\circ \leq p_N^1$. At prices below p° expected profits are negative again since average production costs are higher than p° . At $p > p_N^2$ each firm would receive no customer and thus it would make losses. Hence, H is proven to be canonical.

2.1. In Section 2 of this appendix we prove the uniqueness of the canonical distribution and, as a by-product, the characterizations described in Theorem 1.

For this, in Section 2.1 we characterize the support of canonical distributions. Let H be any canonical distribution. We show that there exist real numbers p^1, p^2 with $p^\circ \leq p^1 < p^2 = \hat{p}(H)$ such that $S(H)$ is equal to either $\{p^\circ\}$, $\{p^\circ\} \cup [p^1, p^2]$ or $[p^1, p^2]$, where $\hat{p}(H)$ is defined in (5).

First, we note that $S(H)$ is nonempty and $S(H) \subset [p^\circ, \hat{p}(H)]$: At prices below p° any firm would make losses since average costs remain above or at p° , and at prices above \hat{p} a firm would get no customers.

Second, to simplify the production cost function (1) for relevant prices, we show that

$$(A18) \quad \theta q(p|H) \leq k \quad \text{for } p \geq p^\circ.$$

This would imply

$$(A19) \quad \mu(\max(\theta q(p|H) - k), 0) = 0 \text{ for } p \geq p^\circ.$$

A further consequence of (A18) is that under H no active firm ever has an incentive to reject customers.

To show (A18), let $\acute{p} := \inf S(H)$. Since $S(H)$ is closed, $\acute{p} \in S(H)$. We consider two cases. First, suppose there were a $\tilde{p} \in [\acute{p}, \infty)$ with $\theta q(\tilde{p}|H) > k$. Then, according to (6), we also had $\theta q(\acute{p}|H) > 0$. But a firm at \acute{p} could increase its profits

simply by rejecting some of its customers thus producing at lower average costs: Choosing $\omega \in (0,1)$ such that $\omega \theta q(\hat{p}|H) = k$, we would get $g(\hat{p}, \omega|H) = k(\hat{p} - p^\circ) > 0$. This contradicts with H being canonical. Now, suppose we had $p^\circ < \hat{p}$ and there were a $\tilde{p} \in [p^\circ, \hat{p})$ with $\theta q(\tilde{p}|H) = \theta \tilde{q} > k$ for some \tilde{q} . Then $\theta q(\tilde{p}|H) = \theta \tilde{q}$ for all $p < \hat{p}$. Again, with $\omega \in (0,1)$ such that $\omega \theta \tilde{q} = k$, we would get

$$(A20) \quad \lim_{p \nearrow \hat{p}} g(p, \theta|H) = \lim_{p \nearrow \hat{p}} \omega \theta \tilde{q} [p - p^\circ] = k[\hat{p} - p^\circ] > 0.$$

Again, this is a contradiction proving (A18) and (A19).

Now we can estimate H . Equations (11), (10), (1) and (A19) imply

$$(A21) \quad \theta q(p|H) \begin{cases} = b/(p-a) & \text{for } p \in S(H) \\ \leq b/(p-a) & \text{for } p \in [p^\circ, \hat{p}]. \end{cases}$$

Inserting (7) into (A21) we get

$$(A22) \quad H(p) \begin{cases} = F(p) - \eta(p) & \text{for } p \in [p^\circ, \hat{p}] \cap S(H) \\ \geq F & \text{for } p \in [p^\circ, \hat{p}] \setminus S(H), \end{cases}$$

where

$$(A23) \quad F(p) := \frac{1}{\rho} [\ln(p-a) - \ln \frac{b}{\theta c \pi} [1 - (1-\pi)^{H(\omega)}]],$$

$$(A24) \quad \eta(p) := [-\ln(\kappa(p))]/\rho.$$

Then $\eta(p) = 0$ if H is continuous at p , otherwise $\eta(p) > 0$.

Now the application of Lemma 1 from Appendix B on (A21) yields that for any canonical distribution H there are real numbers p^1, p^2 with $p^\circ \leq p^1 < p^2 = \hat{p}(H)$ such that $S(H)$ is equal to either $\{p^\circ\}$, $\{p^\circ\} \cup [p^1, p^2]$ or $[p^1, p^2]$.

For later use we note that Lemma 1 further states that

$$(A25) \quad H(p) = F(p) \quad \text{for } p \in S(H) \setminus \{p^\circ\}.$$

Hence, H is absolutely continuous on $S(H) \setminus \{p^\circ\}$ with the density

$$(A26) \quad h(p) = \begin{cases} F'(p) = \frac{1}{\rho} \frac{1}{p-a} & \text{for } p \in S(H) \setminus \{p^\circ\} \\ 0 & \text{else,} \end{cases}$$

and

$$(A27) \quad (1-\pi)^{H(p)} = \frac{1}{p-a} \frac{b}{\theta c \pi} [1 - (1-\pi)^{H(\hat{p})}] \quad \text{for } p \in S(H) \setminus \{p^\circ\}.$$

Finally Lemma 1 states that if H has an atom in p° and $S(H) = \{p^\circ\} \cup [p^1, p^2]$

then $p^\circ < p^1$.

2.2. As further preparation for the uniqueness of canonical distributions we now show that for each $n \in (0, \infty)$ there exists at most one canonical distribution H with $H(\infty) = n$. Suppose H is a canonical distribution with $H(\infty) = n$. Due to the characterization of the support $S(H)$ and that of the canonical density h in (A26), it suffices to show that mass m of H in p° and the numbers p^1 and p^2 (in case of $S(H) = \{p^\circ\} \cup [p^1, p^2]$ or $S(H) = [p^1, p^2]$) are uniquely determined by n .

First, we determine the number $m(n)$ of firms at p° . The zero profit condition requires in case of $p^\circ \in S(H)$ that each firm at p° expects sales of k . Hence, (7) and (8) imply for $p^\circ \in S(H)$

$$(A28) \quad k = \theta q(p^\circ | H) = \theta c \frac{\pi}{\rho} \mu(m(n)) \frac{1}{1 - (1 - \pi)^n},$$

respectively

$$(A29) \quad m(n) = \max(\mu^{-1}(\frac{\rho}{\pi} \frac{k}{\theta c} [1 - (1 - \pi)^n]), 0)$$

where μ was defined in (A3). The solution of (A29) is unique since the derivative of μ is strictly negative. In case of $p^\circ \notin S(H)$ we have $m(n) = 0$.

Next we determine $p^1(n)$ and $p^2(n)$. Since H has no atom in $p^1(n)$ and $p^2(n)$, the expected numbers of customers are given by

$$(A30) \quad q(p^1(n) | H) = c\pi(1 - \pi)^{m(n)} \frac{1}{1 - (1 - \pi)^n},$$

$$(A31) \quad q(p^2(n) | H) = c\pi(1 - \pi)^n \frac{1}{1 - (1 - \pi)^n}.$$

Since active firms gain zero expected profits in $p^1(n)$ and $p^2(n)$ without rejecting customers we get

$$(A32) \quad p^1(n) = a + \frac{b}{\theta q(p^1 | H)} = a + \frac{b}{\theta c \pi} \frac{1 - (1 - \pi)^n}{(1 - \pi)^{m(n)}},$$

$$(A33) \quad p^2(n) = a + \frac{b}{\theta q(p^2 | H)} = a + \frac{b}{\theta c \pi} \frac{1 - (1 - \pi)^n}{(1 - \pi)^n}.$$

2.3. Now we complete the uniqueness proof. Let H be a canonical distribution with n active firms, $H(\infty) = n$. There are three possible cases for the support of $S(H)$: $\{p^\circ\}$, $\{p^\circ\} \cup [p^1(n), p^2(n)]$ with $p^\circ < p^1 < p^2$, and $[p^1(n), p^2(n)]$. We will show (i) that in each of the three cases n is uniquely determined by the parameters of the model and (ii) that each case requires a separate domain for θ :

$S(H)=\{p^\circ\}$ requires $\theta \geq \theta_2$, $S(H)=\{p^\circ\} \cup [p^1, p^2]$ with $p^\circ < p^1 < p^2$ requires $\theta \in (\theta_1, \theta_2)$, $S(H)=[p^1, p^2]$ requires $\theta \leq \theta_1$. Together, the last three relations imply the following: $\theta \geq \theta_2$ requires $S(H)=\{p^\circ\}$, $\theta \in (\theta_1, \theta_2)$ requires $S(H)=\{p^\circ\} \cup [p^1, p^2]$ with $p^\circ < p^1 < p^2$, and $\theta \leq \theta_1$ requires $S(H)=[p^1, p^2]$. Thus, in particular, for each θ the number of active firms is unique. All other properties stated in Theorem 1, too, are proven in the following subsections.

2.3.1. Suppose, $S(H)=[p^1(n), p^2(n)]$. Then $p^1(n)$, $p^2(n)$ fulfill (A32), (A33) respectively (16), (17) with $m(n)=0$. To determine n we make use of (5), $R(p^2|H)=\delta$. According to (4), (7), (A26), (A27), (A32) and (A33) we get

$$(A34) \quad \delta = R(p^2|H) = \theta \frac{\int_{p^1}^{p^2} (p^2-p) \frac{c\pi(1-\pi)^{H(p)}}{1-(1-\pi)^n} h(p) dp}{\int_{p^1}^{p^2} \frac{c\pi(1-\pi)^{H(p)}}{1-(1-\pi)^n} h(p) dp}$$

$$= \frac{b}{c\pi} [e^{\rho n} - (1+\rho n)].$$

Since the right-hand side increases continuously in n from zero to infinity, there exists a unique solution for n denoted by N . Comparing (A34) with (14) we get $N=N^\circ$.

Condition $m(N)=0$ imposes a condition on θ . According to (A29) and (A3) we get $m(N)=0$ if and only if $\theta \leq \theta_1$. We also note that, according to (A32), p^1 decreases to p° if θ increases to θ_1 .

2.3.2. Suppose $S(H)=\{p^\circ\} \cup [p^1(n), p^2(n)]$ with $p^\circ < p^1 < p^2$. Then m , p^1 , p^2 fulfill (A29), (A32), (A33), respectively (22), (20), (21). Again we can determine n from (5), $R(p^2|H)=\delta$. According to (4), (7), (A27), (A26), (2), (A29), (A32) and (A33) we get

$$(A35) \quad \delta = R(p^2|H) = \theta \frac{(p^2-p^\circ) c \frac{\pi}{\rho} \frac{1-(1-\pi)^m}{1-(1-\pi)^n} + \int_{p^1}^{p^2} (p^2-x) \frac{c\pi(1-\pi)^{H(x)}}{1-(1-\pi)^n} h(x) dx}{c \frac{\pi}{\rho} \frac{1-(1-\pi)^m}{1-(1-\pi)^n} + \int_{p^1}^{p^2} \frac{c\pi(1-\pi)^{H(x)}}{1-(1-\pi)^n} h(x) dx}$$

$$= \frac{b}{c\pi} [e^{\rho n} - (1+\rho n)].$$

Hence, the number of active firms, N is uniquely determined by the unique solution of (A35) in n , N° , as in case of $S(H)=[p^1, p^2]$.

To find the condition on θ we use $m(N) \in [0, N]$. This requires $\theta \in (\theta_1, \theta_2)$ as can be seen directly from (A29), (A3) and (A32). These equations also show that m decreases to zero and p^1 decreases to p° as θ decreases to θ_1 . As θ increases to θ_2 , m increases to N . Further, as θ increases to θ_2 , p^2 decreases to $p^\circ + (\delta/\theta)$ and p^1 increases to $p^\circ + (\delta/\theta)$. This follows since (i) $(p^1 - a)/(p^2 - a) = (1 - \pi)^{N-m}$ converges to unity for $m \rightarrow N$ and (ii) $R(p^\circ + (\delta/\theta) | H: S(H) = \{p^\circ\}) = \delta$.

2.3.3. Now suppose $S(H) = \{p^\circ\}$. Then $m(n) = n$ and (A29) implies that the number of active firms, N , is uniquely given by $M_\theta := (\pi/\rho)\theta c/k$ as stated in (20). In particular we have $M(\theta_2) = N^\circ$.

Now we get a condition on θ by the requirement that there are no profit possibilities at prices above p° . Under H the expected number of customers of a firm with a price $p > p^\circ$ is given by (31) provided customers do not prefer searching at p . Hence, given that customers do not search at p , positive profits can be expected at p if $p > p^2$ where p^2 is determined in (A33). Thus, the no profit condition requires that customers do search at prices above p^2 , that is $R(p^2 | H) \geq \delta$. According to (4), (A33), (2) and $\rho N = \theta c \pi / k$ we have

$$(A36) \quad R(p^2 | H) = \theta(p^2 - p^\circ) = \frac{b}{c\pi} [e^{\rho N} - (1 + \rho N)].$$

Then we get from (14) that $R(p^2 | H) \geq \delta$ if and only if $N \geq N^\circ$. Using the definition of θ_2 we have $\rho N^\circ = \theta_2 c \pi / k$. Hence, our condition is $\theta \geq \theta_2$. \square

APPENDIX B

LEMMA 1: Let H be any (left-continuous) distribution function on $[p_0, \infty)$ with $p_0 \in [0, \infty)$ and $S(H)$ the (nonempty) support of H with $S(H) \subset [p_0, \hat{p}]$ for some $\hat{p} \in (p_0, \infty)$. Suppose there is a differentiable function F on $[p_0, \infty)$ with a positive derivative and a nonnegative function η on $[p_0, \infty)$ such that

$$(B1) \quad H(p) = F(p) - \eta(p) \quad \text{if } p \in S(H)$$

$$(B2) \quad H(p) \geq F(p) \quad \text{if } p \in [p_0, \hat{p}] \setminus S(H)$$

$$(B3) \quad \eta(p) = 0 \quad \text{if and only if } H \text{ is continuous at } p.$$

Then there is a $p_1 \in [p_0, \hat{p})$ such that $S(H)$ is equal to either $\{p_0\}$, $\{p_0\} \cup [p_1, \hat{p}]$ or $[p_1, \hat{p}]$. H is absolutely continuous on $S(H) \setminus \{p_0\}$ with the density F' . If $S(H) = \{p_0\} \cup [p_1, \hat{p}]$ and H has an atom in p_0 then $p_0 < p_1$. \square

PROOF OF LEMMA 1: First, we show that for any $p' \in (p_0, \hat{p}]$, H can have no atom in p' : On the one hand there exists a sequence of points $(x_n)_{n=1,2,\dots}$ which converge to p' from below such that for each n either $x_n \notin S(H)$ or $\{x_n \in S(H) \text{ and } \eta(x_n)=0\}$, since any distribution function has at most countably many points of discontinuity. Due to (B1), (B2) and (B3), for each n we have $H(x_n) \geq F(x_n)$ and, hence, $H(p') \geq F(p')$. But on the other hand, if H had an atom in p' then $p' \in S(H)$ and $\eta(p') > 0$. According to (B1), this would imply $H(p') < F(p')$ which contradicts the above argument.

Next, we show that $S(H) = [p_1, \hat{p}]$ in case of $p_0 \notin S(H)$, and that $S(H) = \{p_0\}$ or $S(H) = \{p_0\} \cup [p_1, \hat{p}]$ in case of $p_0 \in S(H)$, where $p_1 := \inf\{p > p_0 : p \in S(H) \text{ or } p = \infty\}$. First we note that $p_1 < \hat{p}$ since otherwise we had $S(H) \setminus \{p_0\} = \{\hat{p}\}$ which contradicts with H having no atom in $(p_0, \hat{p}]$. Since $S(H)$ is closed and $S(H) \subset [p_0, \hat{p}]$ it suffices to show that $(p_1, \hat{p}) \subset S(H)$. We give a proof by contradiction. If the assertion were not true then there would exist a $p' \in (p_1, \hat{p})$ with $p' \notin S(H)$. Let $p_2 := \sup\{p < p' : p \in S(H)\}$, $p_3 := \inf\{p > p' : p \in S(H) \text{ or } p = \hat{p}\}$. Since $S(H)$ is closed we get $p_2 < p_3$, $p_2 \in S(H)$, and $(p_2, p_3) \subset (p_0, \hat{p}) \setminus S(H)$. We show that this implies H having an atom in p_2 . From (B2) we know that $H(p) \geq F(p)$ for $p \in (p_2, p_3)$, and from (B1) that $H(p_2) \leq F(p_2)$. Hence, since H is constant on (p_2, p_3) but $F(p_2) < F(p_3)$, we get $H(p_2) < \lim_{p \nearrow p_2} H(p)$. In case of $p_2 > p_0$, this is a contradiction since we have already shown the nonexistence of atoms in $(p_0, \hat{p}]$. In case of $p_2 = p_0$ we have $p_0 = p_1 = p_2 \in S(H)$ and $(p_2, p_3) \subset (p_0, \hat{p}) \setminus S(H)$ which is in contradiction to the definition of p_1 .

We have shown that for each $p \in S(H) \setminus \{p_0\}$ H has no atom in p . Hence, $\eta = 0$ in these points due to (B3). Then (B1) implies that H is absolutely continuous on $S(H) \setminus \{p_0\}$ with F' as density.

Finally, we show that $p_0 < p_1$ if $S(H) = \{p_0\} \cup [p_1, \hat{p}]$ and H has an atom in p_0 . Suppose H had an atom in p_0 and a support equal to $\{p_0\} \cup [p_1, \hat{p}]$ with $p_0 = p_1$. Then, according to (B1) and (B3), $H(p_0) > F(p_0)$. But since H is continuous on $(p_1, \hat{p}]$, (B1) and (B3) also imply that for the tail of any sequence $(p_n)_{n=1}^{\infty}$ converging to p_0 from above, we have $H(p_n) = F(p_n)$. This contradicts with H being monotonously increasing and, thus, proves the lemma. \square

APPENDIX C

PROOF OF THEOREM 2.1 and THEOREM 2.2:

1. First we prove for $\theta < \max(\bar{\theta}, \theta_2)$ that the short-run canonical distribution

wrt $(\theta, N_{\bar{\theta}})$ is uniquely given by the long-run canonical distribution wrt $(\theta, \tilde{b}(N_{\bar{\theta}}))$. The term $\tilde{b}(N_{\bar{\theta}})$ is evaluated below. From Theorem 1.1 we know that the short-run canonical distribution must be equal to the long-run canonical distribution for some level of fixed costs, \tilde{b} , which includes endogenous expected profits, $\tilde{b}=b+\gamma$. Thus, it suffices to show that this \tilde{b} is uniquely determined by $N_{\bar{\theta}}$. Equation (14) implies that for each $\theta < \max(\bar{\theta}, \theta_2)$ and any positive N there is a unique fixed cost level $\tilde{b}(N)$ such that N is the number of firms active under the long-run canonical distribution with respect to $(\theta, \tilde{b}(N))$. In particular, (14) implies (27).

2. To evaluate $\tilde{b}(N_{\bar{\theta}})$ we distinguish three cases for $(\theta, \bar{\theta})$.

2.1. In case of $\bar{\theta} \leq \theta_2$ and $\theta < \theta_2$, Theorem 1.1 implies $N_{\bar{\theta}} = N_{\bar{\theta}}^{\circ}$. Hence equation (14) yields $\tilde{b}(N_{\bar{\theta}}) = b$ and, thus, zero expected profits for active firms.

2.2 In case of $\bar{\theta} > \theta_2$ and $\theta < \bar{\theta}$ we have $N_{\bar{\theta}} > N^{\circ}$ due to (23). Now (14) implies $\tilde{b}(N_{\bar{\theta}}) < b$. Under $H_{\theta, \tilde{b}(N_{\bar{\theta}})}$ expected profits would be zero if overhead costs were given by $\tilde{b}(N_{\bar{\theta}})$. Since overhead costs are actually given by b , short-run expected profits of active firms are given by $\tilde{b}(N_{\bar{\theta}}) - b$ as stated in (28). Hence, expected profits are negative and independent of θ . Finally (30) follows from (29), $p^{\circ} = a + N_{\bar{\theta}}b/(\bar{\theta}c)$, and $\tilde{b}(N_{\bar{\theta}}) < b$.

2.3. Finally, let $\theta \geq \max(\bar{\theta}, \theta_2)$. We show that there is a unique price $p^+(\theta, \bar{\theta})$ such that the distribution which consists of all $N_{\bar{\theta}}$ active firms offering at p^+ , forms a short-run canonical distribution with respect to $(\theta, N_{\bar{\theta}})$, denoted as $H_{\theta, \bar{\theta}}$.

2.3.1. First we determine $p^+(\theta, \bar{\theta})$. Suppose all $N_{\bar{\theta}}$ active firms offer at the common price p . Then, according to (7) and (8) for each of these firms the expected number of customers is

$$(C5) \quad q(p^+ | H_{\theta, \bar{\theta}}) = (\pi/\rho)c/N_{\bar{\theta}}$$

and, hence, the production level is $x(\theta, \bar{\theta}) := \theta(\pi/\rho)(c/N_{\bar{\theta}})$. If $\theta_2 \leq \bar{\theta} < \theta$ then $N_{\bar{\theta}} = (\pi/\rho)\bar{\theta}c/k$ due to (23), and hence $x(\theta, \bar{\theta}) > k$. If $\bar{\theta} \leq \theta_2 < \theta$ then $N_{\bar{\theta}} = N^{\circ} = (\pi/\rho)\theta_2c/k$, and hence $x(\theta, \bar{\theta}) > k$ again. Thus, in case of $\theta > \max(\bar{\theta}, \theta_2)$ each active firms produces at a level above k .

Any of the $N_{\bar{\theta}}$ firms can decrease its sales x by rejecting customers, and it can increase its sales x by marginally decreasing its price. Thus, by varying its sales x in this way any firms can get expected profits of

$$(C6) \quad \tilde{g}(x|p) = x[p-f(x)].$$

If p describes the short-run equilibrium $H_{\theta, \bar{\theta}}$, then (12) requires

$$(C7) \quad \tilde{g}(x_{\theta}|p) \geq \tilde{g}(x|p) \text{ for all } x \geq 0.$$

Otherwise each active firm could increase its expected profit by either rejecting customers or by slightly reducing its price. Maximizing \tilde{g} over x yields

$$(C8) \quad p = f(x) + x\dot{f}(x).$$

Hence the canonical price must be given by $p^+(\theta, \bar{\theta}) = f(x_{\theta}) + x_{\theta}\dot{f}(x_{\theta})$, as in (25).

2.3.2. We have already shown that under $H_{\theta, \bar{\theta}}$ no active firm can increase its expected profit by rejecting customers or decreasing its price. Now we check that no firm can increase its profit by increasing its price. It suffices to show that any deviating firm would expect a smaller level of profits at the reservation price \hat{p} , the maximal price at which customers do not start searching.

The reservation price is given by $\hat{p} = p^+(\theta, \bar{\theta}) + (\delta/\theta)$ since $R(p^+ + (\delta/\theta) | H_{\theta, \bar{\theta}}) = \delta$ according to (4). We show the negativity of the profit differential

$$(C9) \quad \tilde{D} := g(p^+ + (\delta/\theta) | H_{\theta, \bar{\theta}}) - g(p^+ | H_{\theta, \bar{\theta}}).$$

Under $H_{\theta, \bar{\theta}}$ the number of customers at \hat{p} is given by

$$(C10) \quad q(\hat{p} | H_{\theta, \bar{\theta}}) = c\pi(1-\pi)^{N_{\bar{\theta}}}/[1-(1-\pi)^{N_{\bar{\theta}}}].$$

Then the expected number of quality units sold at \hat{p} is equal to $y_{\theta} := \theta q(\hat{p} | H_{\theta, \bar{\theta}})$. Henceforth, for simplicity, we suppress the index θ in x_{θ} and y_{θ} . Then we get from (29), (30), and (1)

$$\begin{aligned} (C11) \quad \tilde{D} &= y[f(x) + x\dot{f}(x) + (\delta/\theta) - f(y)] - x^2\dot{f}(x) \\ &= y[f(x) - f(y)] - x\dot{f}(x)[x-y] + y\delta/\theta \\ &= y[u(x-k) - u(y-k)] - x(x-y)\dot{u}(x-k) + y\delta/\theta. \end{aligned}$$

To estimate \tilde{D} we make use of the following properties:

$$(C12) \quad y < x, \quad x > k,$$

$$(C13) \quad x/y = (e^{\rho N_{\bar{\theta}}} - 1)/(\rho N_{\bar{\theta}}),$$

$$(C14) \quad \frac{\delta}{\theta} \leq k^2 u(0) \left(\frac{1}{y} - \frac{1}{x} \right),$$

where (C13) follows from

$$(C15) \quad y = \theta c\pi / [e^{\rho N_{\theta}^-} - 1] \quad \text{and} \quad x = \theta c\pi / \rho N_{\theta}^- ,$$

and (C14) follows from

$$(C16) \quad \delta \frac{c\pi}{b} = e^{\rho N^{\circ}} - (1 + \rho N^{\circ}) \leq (e^{\rho N_{\theta}^-} - 1) - \rho N_{\theta}^- = \theta c\pi \left(\frac{1}{y} - \frac{1}{x} \right),$$

$$(C17) \quad \dot{u}(0) \geq b/k^2;$$

(C16) is due to (14), $N_{\theta}^- \geq N^{\circ}$ and (C15). Now we get from (C11) and (C14)

$$(C18) \quad \begin{aligned} \tilde{D} &= (1-y/x) k^2 \dot{u}(0) - (1-y/x) x^2 \dot{u}(x-k) + (y/x) x [u(x-k) - u(y-k)] \\ &\leq (1-y/x) k^2 \dot{u}(0) - (1-y/x) x \left\{ x \dot{u}(x-k) + \frac{y/x}{1-y/x} [u(x-k) - u(0)] \right\} =: \Psi(x). \end{aligned}$$

For $x=k$ the right side of (C18), $\Psi(k)$, is equal to zero. Thus, for the negativity of \tilde{D} , it suffices to show that Ψ is decreasing in x . Since y/x is independent of x and smaller than unity due to (C12) and (C13), it suffices to show that the curved bracket expression,

$$(C19) \quad \Phi(x) := x \dot{u}(x-k) - A[u(x-k) - u(0)],$$

is increasing in x where $A := (y/x) / [1-y/x]$. This property follows from

$$(C20) \quad \Phi(x) = \left\{ \frac{\dot{u}(x-k)}{u(x-k) - u(0)} - \frac{A}{x} \right\} x [u(x-k) - u(0)]$$

and our assumptions about u , $\dot{u} > 0$ and $[u(y) - u(0)]\ddot{u}(y) / [\dot{u}(y)]^2 \geq 1$.

The positive dependence of p^+ on θ follows directly from (22) and the definition of x_{θ} .

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NOTES

- 1 Compare Hall (1986) for a related proposal.
- 2 Whatever property of the imperfect information version of the model is valid for small (respectively high) degrees of market transparency, it holds true also in the product differentiation version for high (respectively small) degrees of product differentiation. See Zink (1989 and 1991) for a more detailed discussion of the product differentiation version of the model.
- 3 Butters constructs price distributions related to ours. However, he concentrates on an analysis of limit economies where the number of firms have been sent to infinity. Such an approach is not applicable for the analysis of short-run price responses for which the number of active firms has to be held constant over short-run demand variations. For a more thorough discussion of the literature compare Zink (1989).
- 4 Equation (3) measures the profit which occurs if sales are equal to its expected number. Hence, with random sales we implicitly assume that firms costlessly hold inventories to smooth out their day to day sales fluctuations. However inventories are not so large as to compensate for systematic demand variations.
- 5 In Zink (1989, 1991) we consider the special case of $k=\infty$. We find that within the discrete model there does not exist any Nash equilibrium. However we find Epsilon-Nash equilibria. That is, for each positive ϵ there is a market size c above which all profit possibilities remain below ϵ . With positive switching cost (which incur to a firm any time it varies its price) these Epsilon Nash equilibria become Nash equilibria. To analyze their properties we introduce a continuous version of the model, in analogy to the procedure of the present paper. Within the continuous version we find unique solutions, and we prove that discrete approximations of these solutions form Epsilon-Nash equilibria of the original discrete model. In the present paper it is hypothesized that for $k<\infty$ the solutions of the continuous version have the same property: discrete approximations of them form Epsilon Nash equilibria. For simplicity, however, we confine ourselves here to the analysis of the continuous version.
- 6 Here $dH(p)$ denotes Lebesgue-Stieltjes integration. That is, for example, if H has an absolutely continuous part with density h and atoms of mass $m(x_i)$ at x_i , $i=1,2,\dots$, then $\int_0^p f(x)dH(p) = \int_0^x f(x)h(x)dx + \sum_{x_i < p} f(x_i)m(x_i)$.

7 Definition (6) makes sense for $H(\infty) > 1$ only. Later we will see that the emerging market solutions fulfill this condition for large enough market sizes c .

8 For reasons given later, this definition differs from what we could infer for the discrete model. For example, suppose that all N firms offered at p^0 . Within the discrete model, we then could infer that for any given firm the expected number of customers is equal to c/N . Within the continuous version of the model, however, equation (7) and (8) define this expected number to be equal to $(c/N)(\pi/\rho)$. We introduce this correction factor since without it there would be a small θ -interval over which canonical distributions were not unique.

9 Hence, κ is continuous in the size of the atom, m , for $m \geq 0$. If H is continuous at p then $\kappa(p) = 1$, and otherwise $\kappa(p) \in (0, 1)$. The expression $\pi\kappa(p) = [\pi/\rho][1 - (1-\pi)^m]/m$ is assumed to describe the probability with which a given customer buys from a specific firm i at price p if under H there are m firms at price p (given that the customer does not know any cheaper firm). The correction factor π/ρ arises from our identifying the profit possibilities of an old active firm at p with those of an additional firm at p . It is uniquely determined by the m -continuity requirement and converges to unity as π decreases to zero.

10 To describe the profit possibilities of an old active firm at p (which previously offered at a higher price) we had to define

$$\kappa^{\text{old}}(p) = \frac{1}{\pi} \frac{1}{m} [1 - (1-\pi)]^m$$

while for an additional firm the adequate definition would be

$$\kappa^{\text{add}}(p) = \frac{1}{\pi} \frac{1}{m+1} [1 - (1-\pi)^{m+1}].$$

Our definition of κ ensures that for small π

$$\kappa^{\text{old}} \leq \kappa \leq \kappa^{\text{new}}.$$

The correction factor is uniquely determined by the requirement that for each m there is a $\tilde{\pi}(m)$ such that the above inequality holds for $\pi \leq \tilde{\pi}(m)$.

11 Compare Note 6.

12 For simplicity we assume that for each firm the short-run average production cost function coincides with the long-run average production cost function as it is introduced in (1). Then, in particular, each firm would incur overhead costs of b over the short-run even if it left the market. Hence over the short-run each firm stays in the market as long as its expected profit level is not below $-b$.

13 The correction factor (π/ρ) is due to our definition of q in (7) and (8).

14 We note that $\lim_{\theta \rightarrow \theta_2} p^+(\theta, \bar{\theta}) \geq p^\circ$ with strict equality for $\dot{u}(0) = b/k^2$.

15 The price jump at $\theta = \bar{\theta}$ is due to the average cost function f having a kink at the capacity level.

16 According to equation (24) we have,

$$(\tilde{p}^\circ - a)/(p^\circ - a) = \tilde{b}(N_{\bar{\theta}})/b = (\delta/\nu e)/[e^{\theta/(k\nu)} - (1 + \theta/(k\nu))]$$

which is independent of market size c .

17 The effect of b on the pricing type might seem to be counterintuitive. But, starting in the countercyclical pricing case, with increasing overhead cost the number of active firms decreases and each firm's production level increases. If all firms' production levels are rising above capacity then pricing becomes procyclical.

18 With price dispersion in the long-run at least some firms produce below capacity k . Then firms may have an incentive to merge production activities while keeping selling activities separate. However such incentives vanish if overhead cost accrue completely from marketing cost or if the model is interpreted within the product differentiation version mentioned above (see Zink (1991)).

19 We note that our model offers an alternative explanation of the stylized empirical findings associated with the "Administered Prices" Hypothesis. (Compare G. Means (1935 and 1972), G. Stigler and J. Kindahl (1970), and H. Ross and J. Krausz (1986).) According to this hypothesis, in concentrated industries, prices move countercyclically with respect to demand variations. Many proponents of the "Administered Prices" hypothesis referred to collusion as an explanation. Instead, we refer to the prevalence of increasing returns, product differentiation and market intransparency in these industries. In the present paper we show that these three characteristics of markets can result in countercyclical pricing. Further, we show that inverse pricing is associated with price dispersion. Price dispersion exhibits asymmetric distributions of firms sizes and thus, higher concentration ratios than without price dispersion where all firms have the same size.

FIGURES

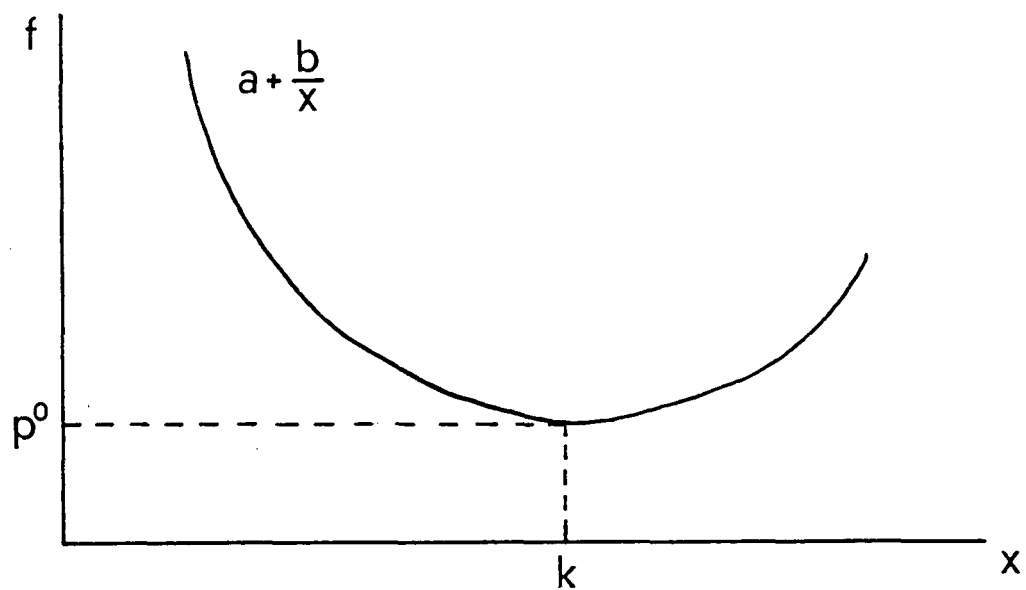


FIGURE 1: Average production cost as a function of output

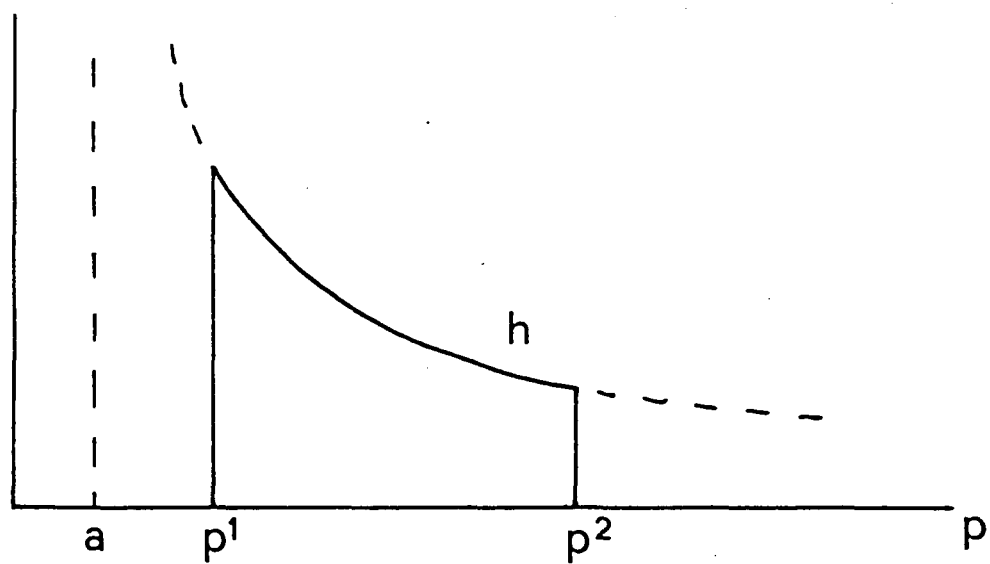


FIGURE 2: Shape of the canonical density of prices

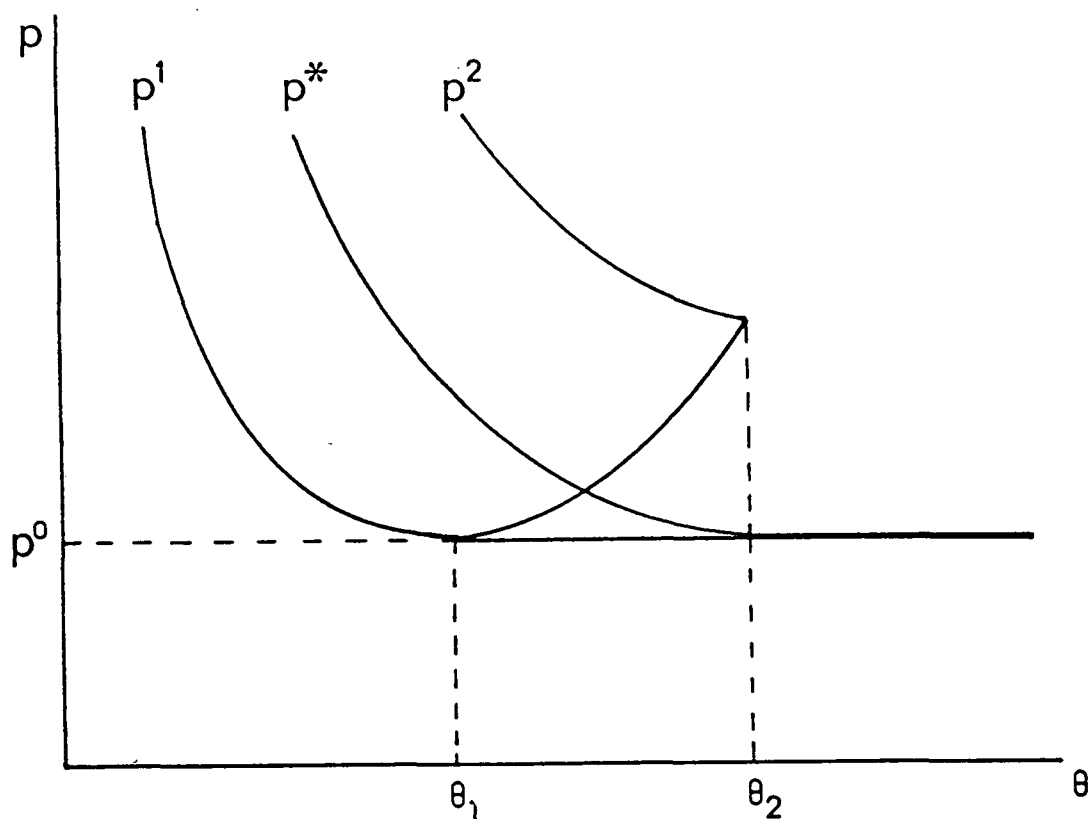


FIGURE 3: Canonical prices as a function of demand

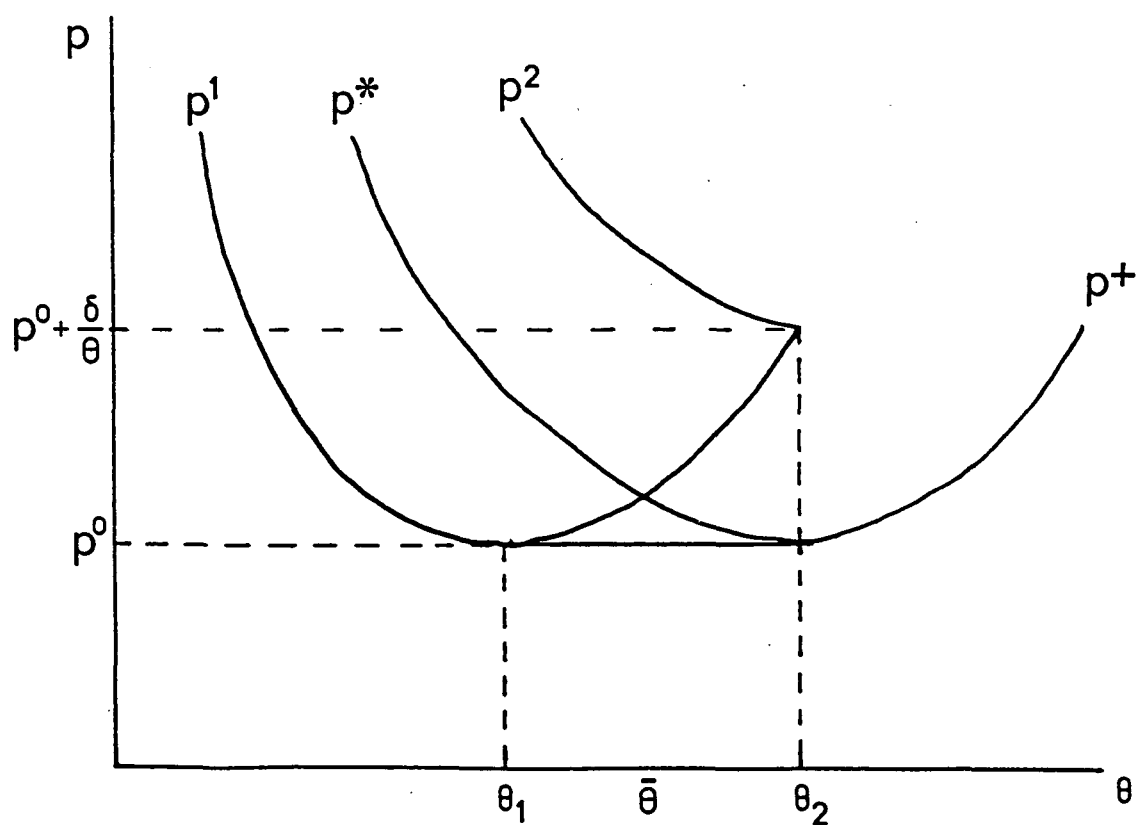


FIGURE 4: Short-run canonical prices as a function of demand, inverse price reaction case

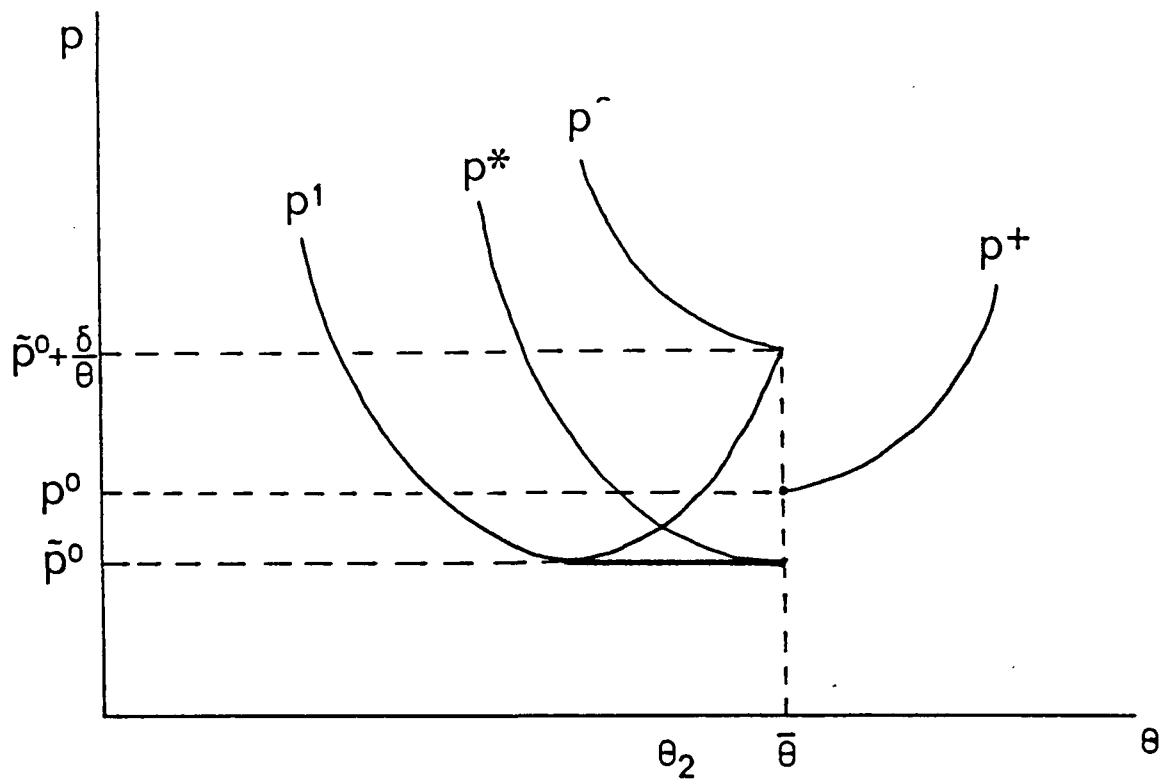


FIGURE 5: Short-run canonical prices as a function of demand, normal price reaction case