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## Identifying the causes of unemployment: Estimation and control of a small macroeconomic model

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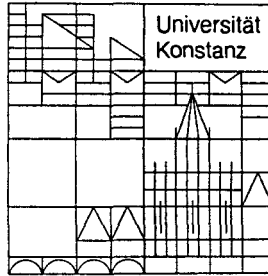
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**Fakultät für  
Wirtschaftswissenschaften  
und Statistik**

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Joachim Möller

**Identifying the Causes  
of Unemployment**

Estimation and Control  
of a Small Macroeconomic Model

Diskussionsbeiträge

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# IDENTIFYING THE CAUSES OF UNEMPLOYMENT

Estimation and Control of a Small Macroeconomic Model \*

JOACHIM MÖLLER

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*The European unemployment problem is becoming so severe and prolonged that the influence of that event will almost certainly change theories of unemployment in the next few years.*

*Dornbusch, Fischer [1987]*

## 1. Introduction

Persistent unemployment in Western Europe remains a challenge for theory and policy analysis. One of the central questions is, whether the chronic slackness of labour markets after 1974 was inevitable or not given the political preference for a low level of the inflation rate. At first glance, there seems to be little scope for policy measures. A broad consensus among economists exists that the trade-off between the major macroeconomic targets has considerably deteriorated as compared with the sixties or early seventies. Strong empirical evidence supports the hypothesis that the non-accelerating rate of inflation (NAIRU) has increased markedly in most of the major European countries (See Coe, Gagliardi [1985]). But this is merely an empirical description rather than an explanation of the puzzling fact that the NAIRU more or less mimics the actual unemployment rate. The NAIRU could be a misleading guidance for policy and Alan Blinders warning should be born in mind: "... (an approach) that allows the natural rate to trundle along after the actual rate is not a natural rate theory at all." <sup>1</sup>

Some authors ascribe the increase in the NAIRU to changes in the labour market structure and in institutional conditions entailing a different behaviour of market participants (for example, Minford [1983]). This approach neglects the fact that employment, output, wages and prices are jointly determined in an interdependent system of markets. For instance, productivity (one of the most important variables affecting the NAIRU) may not be taken as an exogenous variable but is implicitly given by the production and the labour demand decisions. Rather than restricting the analysis to the labour market, we interpret the NAIRU as a part of the dynamic equilibrium solution of a complete macroeconomic model.

The present paper provides an empirical investigation of the West German economy. A small macroeconomic model focusing on the wage/price behaviour and the production/employment decisions is developed. The approach is inspired by a model of Layard and Nickell [1986]. In contrast to the literature of disequilibrium macroeconomics (Barro and Grossman [1976], Malinvaud [1977,1980]) and the wage-gap analysis of Bruno and Sachs [1985] we adopt a framework of imperfect competition which leads to mark-up pricing and to labour demand functions depending on output as well as on factor prices. Coen and Hickman [1987, p.124] emphasize that such an approach "...allows for coexistence of classical and Keynesian unemployment instead of explaining them as separate regimes in markets with price-taking firms and rationed buyers and sellers." Thus, in the framework of imperfect competition some of the features of disequilibrium macroeconomics are preserved while the severe theoretical and empirical problems arising in that field (see Laffont [1985] for an overview) are avoided.

As compared to the approach of Layard and Nickell, we start with a more general framework in order to investigate some of the underlying assumptions concerning the production technique and the pricing behavior. Since the underlying model is highly interdependent, simultaneous equation techniques are used for the estimation of the so-specified equations. Since it can be argued that the model structure is not constant in the sample period, we test whether the coefficients of our model are time-varying by applying Kalman filtering methods.

In the interpretation of the estimation results the focus is on the dynamic equilibrium

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<sup>1</sup> Blinder [1987], p.132.

implications of our model. We calculate the non-accelerating inflation rate of unemployment and identify the underlying factors which are responsible for the rise in this variable. It is shown that our model exhibits the hysteresis property: the NAIRU depends on the history of shocks affecting employment (including policy measures). Thus the empirical evidence that NAIRU follows the actual unemployment rate rather closely is no longer surprising.

In order to demonstrate the inherited effects and to present alternatives to the policy actions in the past we apply an optimal control algorithm. It can be shown that the rise in the NAIRU could have been avoided by means of appropriate monetary and fiscal measures.

## 2. The theoretical model

We first discuss the supply side of the model focusing on the production decision and the determination of output prices. According to the hypothesis of imperfect competition, prices are set by the firms and not by impersonal forces. Pricing behaviour of a firm can be described by a mark-up over short run marginal costs, evaluated at the level of expected demand. For an individual firm, expected demand - which is a function of the output price relative to that of its competitors and of aggregate demand - determines its output activity. For a given output level, the firm can be optimizing in its employment decision. This leads to 'hybrid' labour demand functions, depending on the level of demand as well as on the level of real factor costs. Expressed in the terms of the quantity rationing theory, firms always operate on their effective demand curves.

### *Price setting and the determination of output*

Output ( $V$ ) is produced by means of (effectively used) labour ( $L$ ) and of non-labour factors ( $K$ ), which include the capital stock, state of technology, availability of resources etc.:

$$V = f(L, K), \quad f_i > 0, f_{ii} < 0 \quad (i = L, K). \quad (1)$$

Since we do not intend to analyse the process of capital formation explicitly, non-labour factors  $K$  are taken as exogenous in the following. Actual production is a fraction  $\Phi$  of potential output  $V^*$  which is in general determined by labour supply and the non-labour factors  $K^*$ . Under certain special conditions,  $\Phi$  can be defined as the rate of capacity utilization.<sup>2</sup> So we assume that firms determine output by choosing the rate of capacity utilization. This production decision of a representative firm depends on output prices of this firm  $P_i$  in relation to the expected global price level  $P^e$  and expected demand factors  $V_i^d$  (which we discuss later). So in the aggregate we have:

$$V = \Phi(P/P^e, V^d) \cdot V^*(K^*), \quad \Phi(\cdot) \leq 1. \quad (2)$$

The labour requirement for producing an additional unit of output (the marginal labour coefficient) is  $1/f_L$ . Short-run marginal costs ( $SMC$ ) are obtained by multiplying the marginal labour coefficient and the wage rate  $W$ :

$$SMC = 1/f_L W. \quad (3)$$

<sup>2</sup> Since most of the time period which we consider has substantial unemployment, we exclude the possibility that available labour is a limiting factor in the production process. This assumption is problematic only under the economic conditions of the early seventies.

The mark-up hypothesis leads to the desired price level:

$$P^* = \mu(\cdot) SMC, \quad (4)$$

where the arguments in the function  $\mu(\cdot)$  may be trend factors or cyclical influences, reflecting, for instance, changes in the effective capital requirement of labour or fluctuations of the elasticity of demand  $\nu$ . If a profit maximizing firm takes into account only labour costs, then the mark-up rate will be  $(1 - 1/\nu)^{-1}$ . Given that the elasticity of demand follows a countercyclical pattern (demand becoming more price sensitive in an upswing) this implies the mark-up rate is procyclical.

### *Wage formation*

As discussed by Layard and Nickell [1986] or Johnson and Layard [1986] there are essentially four different approaches for modelling the wage process. The first stresses the role of the labour unions for wage setting. The unions maximize the expected utility of a member which takes into account the risk of becoming unemployed<sup>3</sup>. The second is a market approach which combines demand and supply to an equilibrium real wage condition. The third concentrates on the firm's wage offer assuming that a wage differential pays out for various reasons. This approach is typically presented by the efficiency wage type models<sup>4</sup>. The fourth approach focuses on a bargaining process between firms and employees or between their representative organisations.

It is important to point out, that all these different theories of the wage process lead, in the aggregate, to very similar empirical formulations. In the market approach all variables determining profitability come into play together with all variables that constitute a wedge between labour costs and take home pay. The same may be true for the models of union wage setting, if the interdependence of the central variables is included correctly. As well as in the efficiency wage type models the influences affecting the opportunity costs of being employed play an important role. Finally, in the bargaining approach models we have a sort of joint maximisation of the firms' profit function and the unions' utility function under the condition that both sides of the market know that the outcome of this process determines not only wages but also employment. The utility functions of the unions typically depend positively on the real net take-home pay and negatively on the threat of unemployment. Unions will try not only to demand a compensation for inflation but also for increasing social security burdens, labour income taxes, and import price shocks. The bargaining power is negatively related to the opportunity costs of unemployment which in turn depend on the level of unemployment compensation and similar factors. The willingness to make concessions on the part of the employers is influenced by all factors determining profitability (demand forces, productivity gains, taxes, non labour costs etc.).

Following these theoretical considerations, the wage process may be formulated as follows

$$W = W(P, U, \frac{V}{L}, IPS, \tau), \quad (5)$$

where  $W$  are nominal wages,  $P$  are consumer prices excluding import price shocks ( $IPS$ ),  $U$  is the unemployment rate,  $V/L$  stands for productivity and  $\tau$  for the so-called wedge variable, representing the difference between gross labour costs and the take-home pay

<sup>3</sup> A model of this kind is provided by McDonald and Solow [1981].

<sup>4</sup> See, for instance, Shapiro, Stiglitz [1984].

(social security burdens as well as labour income taxes). An important implication of eq.(5) is that the coefficient of prices is not restricted to one, so we do not a priori impose the so-called homogeneity assumption (see Blanchard [1987]).

#### *Internal and external demand for output*

Total demand for output is divided into an internal and an external component:  $V^d = V_{int}^d + V_{ext}^d$ . According to standard macroeconomic textbook versions, internal demand is modelled as a function of autonomous spending  $AS$  and real balances  $M/P$ :

$$V_{int}^d = V_{int}^d \left( AS, \frac{M}{P} \right), \quad (6)$$

where eq.(6) is a reduced form of the well-known IS-LM type models. External demand, on the other hand, is a function of the level of world trade activity ( $WT$ ) and an indicator of competitiveness. The latter may be constructed as the product of the exchange rate  $E$  and the ratio of external to internal prices  $P^x/P$

$$V_{ext}^d = V_{ext}^d \left( WT, \frac{EP^x}{P} \right). \quad (7)$$

#### *Labour Demand*

If labour can be substituted by non-labour factors, the desired demand for labour conditional to the expected demand on the goods market (which depends on cost-based pricing) is obtained in a very similar way as in the traditional approach of short-run employment functions (see, for example, Black, Kelejian [1970]). Labour demand, thus, is a function of the real product wage (including taxes and social contributions) and the level of demand:

$$L^* = L^* \left( V, \frac{W}{P}, K \right). \quad (8)$$

According to Layard, Nickell [1986, p.121] "... (this approach) conforms both to common sense and to the data." An alternative derivation is given by Hickman [1987] on the basis of cost-minimization. Hickman [p.1535] writes: "Labour demand can therefore fall short of full employment either because effective demand is too low or because the real wage is too high, or both. Keynesian and classical unemployment may coexist rather than occurring in separate regimes as in the standard fixprice disequilibrium model."

#### *External balance*

With the internal balance being given by the level of employment and the rate of capacity utilization, we have to complete the model by adding an equation for the external balance. The current account in relation to potential output is modelled as a function of domestic demand, the level of external trade activity and the degree of competitiveness:

$$\frac{B}{V^*} = \frac{B}{V^*} \left( V_{int}^d, WT, \frac{EP^x}{P} \right). \quad (9)$$



### 3. The estimated simultaneous equation model

#### *Choosing the empirical framework*

In the following we assume that all equations are linear in logarithms and small letters denote logarithms of the corresponding variable. Before estimating the complete model we have to select between alternative explanatory or endogenous variables and to carry out a number of specification tests on the dynamic structure of the model and on possible parameter restrictions. Partially, our preliminary empirical results contradict some assumptions made in other studies.

For the price equation, we get the following results: The hypothesis of instantaneous adjustment ( $\lambda = 1$ ) is not supported by the data. A likelihood-ratio test rejects the restriction  $\lambda = 1$  at the 1%-level. This finding contradicts the use of  $(p - w)$  on the left side of an aggregate price equation (as in Layard, Nickell [1986]), at least if the periodicity of the data is less than one year. Beside this, restrictive assumptions concerning the production technology are not justified on empirical reasons. Imposing a scale parameter of 1 (constant returns to scale), implies equality of the coefficients of output and employment in absolute values. This assumption is rejected by a likelihood ratio test at the 1%-level. The same applies to an unit elasticity of substitution. We conclude that, in combination with mark-up pricing, the assumption of a Cobb-Douglas technology is not appropriate and a more general formulation should be used.

For the nominal wage equation, we decided to take *per capita* wage income as the left hand variable instead of an index of hourly wages, since there are some serious data problems with the quarterly index of aggregate working hours. Working hours fluctuate around a declining trend, and it should be kept in mind that our model does not accommodate this fluctuation. Apart from the variables of the theoretical approach presented above, the lagged endogenous variable was included. The unemployment rate is proxied by the logarithmic difference between potential and actual employment  $l^* - l$ . Potential labour is determined as the sum of employed and unemployed persons, so we implicitly assume that labour supply is exogenous and all unemployed persons are officially registered. Labour productivity is given by the logarithmic difference between output and employment  $(v - l)$ , and the wedge variable  $\tau$  is defined as one minus the ratio between gross labour costs and take-home pay. The import price shocks, measured in logs as the difference of the final demand deflator and the BIP-deflator was found to be weakly significant in some versions of single equation specifications, but in the estimation of the simultaneous system it was not even significant at the 10 percent level. Therefore, the variable was excluded from the model.

As derived from theoretical considerations, employment depends on a real wage income variable and actual output. According to tests of the dynamic structure of the model, lags of the endogenous variables up to at least two quarters are required on the right hand side of the equation. The same is the case for the rate of capacity utilisation. As indicators of demand forces, affecting the rate of capacity utilisation, are chosen: a detrended volume index of world exports  $WT$ , an indicator of competitiveness, and real balances  $(M/P)$  as the ratio of  $M1$  and the BIP-deflator. A cyclically adjusted fiscal indicator, which should proxy autonomous spending, was tried, but was not found to be significant. In spite of numerous attempts, an indicator of competitiveness as the product of the exchange rate  $E$  and the relation between internal and external prices  $P^x/P$  was not successful as a regressor in the equation of internal demand. Including the variables from which the indicator is constructed separately, gives the following result: In the demand equation the effective exchange rate appears statistically significant with the theoretically expected positive sign, whereas world prices (when used instead) have a robust negative sign and internal prices turn out to be insignificant. Obviously, the problem is caused by the price level variables, reflecting not only information about competitiveness but also indicating important macroeconomic effects (as the real balance effect on world level, for instance). So, our decision was to use

simply the effective exchange rate as a proxy for price competitiveness.

It need hardly be mentioned that the determination of the price, wage, employment and capacity utilisation (or output) is highly interdependent<sup>5</sup>. In addition to this interdependent block, the equation for the real current account surplus (relative to potential nominal GNP) is modelled as dependent on the level of internal and external demand, and an indicator of competitiveness, where the endogenous variable is defined as  $\ln(1 + CA/V^*)$  which is approximately equal to  $CA/V^*$ . It turns out that world prices have a robust negative sign in this equation, too. So, the argument made before is reinforced. By contrast to the equation for capacity utilisation, however, an exchange rate variable is not successful.

Apart from some modifications concerning the lag structure of exogenous variables, the complete model used for simultaneous equation estimation is given as follows:<sup>6</sup>

### *The complete model*

wage equation

$$w_t = a_0 + a_1 w_{t-1} + a_2 p_t + a_3 (v_t - l_t) + a_4 (\ell_t^s - l_t) + a_5 \tau_t + a_6 t \quad (10)$$

price equation

$$p_t = b_0 + b_1 p_{t-1} + b_2 w_t + b_3 v_t + b_4 l_t + b_5 t \quad (11)$$

employment equation

$$l_t = c_0 + c_1 l_{t-1} + c_2 l_{t-2} + c_3 (w_t - p_t) + c_4 v_t + c_5 t \quad (12)$$

capacity utilisation

$$\phi_t = d_0 + d_1 \phi_{t-1} + d_2 \phi_{t-2} + d_3 (m_t - p_t) + d_4 WT_t + d_5 \varepsilon_t \quad (13)$$

current account surplus

$$\psi_t = e_0 + e_1 \psi_{t-1} + e_2 (m_t - p_t) + e_3 p_t^x + e_4 WT_t + e_5 t \quad (14)$$

identity for output determination

$$v_t = \phi_t + v_t^* \quad (15)$$

The explanatory variables of the system are divided in those which are open to possible control action (i.e. the money supply  $m$  and the fiscal variable  $\tau$ ) and those which are not. We denote the former as control variables  $z_t$ , the latter as the exogenous variables  $x_t$ . With the vector of endogenous variables  $y_t' = (w_t, p_t, l_t, \phi_t, \psi_t)$ , the vector of control variables  $z_t' = (m_t, \tau_t)$  and the vector of exogenous variables  $x_t' = (1, v_t^*, \ell_t^s, WT_t, \varepsilon_t, p_t^x, t)$  our specific simultaneous model is

$$G_0 y_t = G_1 y_{t-1} + G_2 y_{t-2} + H_1 z_t + H_2 x_t, \quad (16)$$

where  $G_i, H_i$  are properly dimensioned matrices of coefficients.

<sup>5</sup> The product of the rate of capacity utilisation and potential output gives the actual output.

<sup>6</sup> The identification of the system was checked using a method proposed by Hsiao [1986]. All equations are identified.

#### 4. Empirical results

##### *Data sources and estimation method*

The following estimation results are based on quarterly, seasonally adjusted data. The estimation period was 1970,3 to 1987,2. Most of the time series are taken from publications of the Deutsche Bundesbank. The world trade (export) index and the world price index (unit value of exports) are from the Monthly Bulletin of Statistics of the United Nations. The effective exchange rate is from publications of the International Monetary Fund.

We used iterated 3SLS to estimate the coefficients of the above model. The iteration procedure was stopped when the quadratic difference of the coefficient vector with respect to the last iteration was less than 0.01. In all cases convergence was achieved in less than 12 steps.

##### *Estimation results and stability of parameters*

Generally, the overall statistics of *table 1* appear to be favourable. There is no hint for autocorrelation in any of these equations. With respect to the possibility that the estimated coefficients might change in time, we carried through a corresponding test. For this purpose a state space model of the following form is used

$$y_t = X_t \beta_t + \zeta_t \quad (17)$$

$$\beta_t = \beta_{t-1} + \eta_t, \quad (18)$$

where  $y_t$  is the vector of endogenous variables,  $X_t$  is the matrix of (in wide sense) explanatory variables containing lagged endogenous, exogenous and control variables,  $\beta_t$  is the vector of coefficients in time period  $t$ . The disturbances  $\zeta_t$  and  $\eta_t$  are normally independently distributed with covariance matrix  $\Sigma_\zeta$  and  $\Sigma_\eta$ , where it is assumed that  $\Sigma_\eta$  is a diagonal matrix. An element of the coefficient vector is invariant in time, if the corresponding diagonal element of  $\Sigma_\eta$  equals zero.

Applying a method described elsewhere <sup>7</sup>, we tried to find out a time varying pattern in the parameters using ML-estimation of the variances in the corresponding state space model. The general finding is: the hypothesis of time invariant parameters cannot be rejected on the basis of a likelihood ratio test.

This result is of special importance because it shows that in spite of severe changes in economic policy, in spite of changing expectations, changes in the structure of world trade, changes in technical progress - only to mention a few - some of the basic relations of the economy have remained more or less the same. In our view, the stability of estimation results is an central argument against the famous Lucas critique of macroeconomic modelling, which asserts structural shifts in economic relations as a response to changing policy rules that are perceived by individuals with rational expectations. If the Lucas critique does not apply, the estimated relationships may be used for policy simulations, at least, if the range of the control variables does not exceed the range of the observed data in our sample.

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<sup>7</sup> See Möller, Wais [1987] and Möller [1987].

TABLE 1

Estimation Results for the Simultaneous Equation Model (1970,3 - 1987,2)

<i>wages w</i>		<i>prices p</i>	
constant	-7.13 (5.74)	constant	3.90 (4.41)
$w_{t-1}$	0.18 (1.56)	$p_{t-1}$	0.43 (3.69)
$p_t$	1.44 (6.56)	$w_t$	0.31 (4.90)
$(\ell_t^s - \ell_t)$	-0.63 (5.94)	$v_t$	-0.25 (3.96)
$(v_t - \ell_t)$	0.30 (3.28)	$\ell_t$	0.17 (3.24)
$\tau_t$	0.55 (3.78)	$t(\times 10)$	0.10 (4.45)
$t(\times 10)$	-0.17 (5.15)		
<i>statistics</i>		<i>statistics</i>	
DW	2.07	DW	2.03
Durbin's h	-0.91	Durbin's h	-0.47
$R^2$ (2SLS)	0.99	$R^2$ (2SLS)	0.99
ln likelih. (2SLS)	240.67	ln likelih. (2SLS)	269.05

<i>employment <math>\ell</math></i>		<i>capacity utilisation <math>\phi</math></i>	
constant	-1.30 (4.10)	constant	1.51 (4.17)
$\ell_{t-1}$	1.30(10.51)	$\phi_{t-1}$	0.53 (4.76)
$\ell_{t-2}$	-0.43 (3.95)	$\phi_{t-2}$	0.32 (2.89)
$(w_t - p_t)$	-0.07 (4.46)	$WT_{t-1}$	0.10 (1.86)
$v_t$	0.15 (4.68)	$\varepsilon_{t-2}$	-0.23 (4.16)
$t(\times 100)$	-0.18 (4.37)	$(m_t - p_t)$	0.28 (4.48)
<i>statistics</i>		<i>statistics</i>	
DW	1.82	DW	1.95
Durbin's h	-	Durbin's h	-0.52
$R^2$ (2SLS)	0.99	$R^2$ (2SLS)	0.81
ln likelih. (2SLS)	349.94	ln likelih. (2SLS)	172.07

<i>current account <math>\psi</math></i>		<i>statistics</i>	
constant	1.15 (3.57)	DW	2.27
$\psi_{t-1}$	0.76(11.31)	Durbin's h	-1.38
$p_{t-2}^x$	-1.39 (2.71)	$R^2$ (2SLS)	0.83
$(m_t - p_t)$	-3.45 (2.05)	ln likelih. (2SLS)	260.58
$WT_{t-1}$	2.22 (1.36)		
$t(\times 10)$	2.40 (3.46)		

## Notes:

- results based on iterated 3SLS estimation
- asymptotic absolute t-ratios in parentheses
- Durbin's h-statistic in the employment equation is not calculable.

*Estimation results*

The comparison of the wage and price equation yields the following result: in the price equation rigidities are clearly present as indicated by the coefficient of the lagged dependent variable which is significantly greater than zero. The case is not evident for the wage equation. The coefficient of the lagged endogenous is statistically significant in OLS-regressions on the 5% level but in 3SLS only at the 10% level. In any case, the speed of adjustment appears to be greater in the wage equation (about 0.8) than in the price

equation (about 0.6). The coefficient of prices in the wage equation exceeds one, while the coefficient of wages in the price equation is much lower (0.3). These estimates indicate an over-proportional reaction of wages to prices, in contrast to a rather weak (short-run) reaction of prices to wages.

Furthermore, a strong negative effect of unemployment on *per capita* wage income of the employed (-0.6) is evident, together with a statistically well established positive influence of productivity. Institutional influences on the wage process are expressed by the strong positive coefficient of the tax and social security contribution variable. According to this result, labour obviously managed to gain a certain compensation for these considerably increased burdens in the bargaining process.

Output and employment affect prices in the expected way. It has to be mentioned that the estimated speed of adjustment does not differ significantly at the 5% level from the wage coefficient, so our empirical results are not contradictory to the theoretical model.

A look at the next two equations shows that rigidities in attaining the desired levels are much higher in the employment and capacity utilisation equations than in the price and wage equation. Evidently the lowest speed of adjustment is found with respect to the employment decision (about 0.13), while the corresponding value for capacity utilisation is about 0.15.

As could be expected from theory, employment responds negatively to real wage income and positively to output. The negative coefficient of the trend term in the employment equation reflects (labour augmenting) technical progress.

In the equation for capacity utilisation the results indicate a positive influence of the detrended world trade index in combination with strong effects of the effective exchange rate (as a proxy for price competitiveness), and of real balances as a domestic demand indicator.

According to our theoretic expectations, external demand exhibits a positive, but rather weak influence (significant only at the 10% percent level) on the current account, while the negative influence of internal demand is well established. Apart from the statistically significant negative sign of world export prices, indicating a real balance effect on world level, we find a positive trend for the estimation period.

### *Long-run effects*

Table 2 gives the long-run multipliers of the exogenous variables which are of special interest. Since most of the variables are in logarithms, the reported multipliers (with exception of the current account ratio and the wedge variable) are to be interpreted as elasticities.<sup>8</sup>

A first look on the results shows that all long-run multipliers with respect to *per capita* wage income are greater in absolute value than the corresponding multipliers with respect to prices. This finding implies real wage effects in the long run, which are positive for all exogenous variables but for labour supply. The effects on the nominal variables are always working in the same direction, whereas this is not the case in the real part of the model.

Turning now to the details, we make the following observations: Internal and external demand (proxied by the money stock and the world trade activity) drive the nominal as well as the real variables of the model. Working through the channel of higher demand, higher prices and higher productivity, the long-run elasticity of the money stock with respect to nominal *per capita* wage income is about 1.5. The corresponding multipliers with respect

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<sup>8</sup> The reported multipliers of the current account variable have to be divided by the variable itself to obtain the corresponding elasticity. The same applies to the wedge variable.

TABLE 2  
Long-run Multipliers

endogenous variables	exogenous variables						
	$m$	$\tau$	$v^*$	$\ell^*$	$WT$	$\epsilon$	$p^x$
$w$	1.47	0.74	0.84	-0.84	0.53	1.24	-
$p$	0.62	0.42	0.35	-0.47	0.23	0.52	-
$\ell$	0.32	-1.02	0.18	1.16	0.12	0.27	-
$\phi$	0.67	-0.74	-0.62	0.84	0.24	0.56	-
$\psi$	-0.06	0.06	0.05	-0.07	0.13	-	-0.06

to prices and capacity utilisation have values of about 2/3, whereas the effect on employment seems to be lower (0.3). Since the money stock leads to internal expansion, which induces higher imports, the effect on the current account is clearly negative. As should be expected, the effects of an external demand expansion are all smaller in magnitude than those of an internal expansion but the effects on the endogenous variables are very similar in proportion. The main difference between an internal and an external expansion lies, of course, in the effect on the current account.

The second column of *table 2* shows the importance of the wedge variable. Higher taxes on wages or higher contributions to social security lead to higher price and especially wage levels, thereby depressing capacity utilisation and employment. According to this result, a rise in the average tax rate especially has a detrimental effect on employment. As it is the case with all variables depressing the real part of the model, an increasing wedge variable improves the current account.

Higher potential output involves a simultaneous rise in wages and employment in the long run, although the effect on employment appears to be much weaker. Furthermore, growth of potential output *cet.par.* lowers capacity utilisation. Due to the interdependence of the model, however, the long-run elasticity is considerably smaller than 1. Taking into account the negative influence of output in the price equation, it seems surprising at first glance, that higher production possibilities affect prices positively after all adjustments are made. The explanation is that the stimulation of wage-costs more than compensates for the productivity effect.

Growing labour supply depresses wages to a higher extend than prices and augments employment and capacity utilisation in almost a one to one proportion.

Column 6 of *table 2* gives the long-run elasticities of a depreciation of the German Mark. Increasing competitiveness stimulates prices and (especially) wages as well as employment and production. Finally, as shown in column 7 the price index of world exports only affects the current account negatively, since the last equation is exogenous with respect to the first four equations of our model.

### Calculating the NAIRU

As is well known in state space modelling, eq.(16) can be reformulated as:

$$\Gamma_0 y_t^+ = \Gamma_1 y_{t-1}^+ + B_1 z_t^+ + B_2 x_t^+ \quad (19)$$

where

$$\begin{aligned} \Gamma_0 &= \begin{pmatrix} G_0 & 0 \\ 0 & I \end{pmatrix} & \Gamma_1 &= \begin{pmatrix} G_1 & G_2 \\ I & 0 \end{pmatrix} \\ B'_1 &= (H'_1 \quad 0) & B'_2 &= (H'_2 \quad 0), \end{aligned}$$

and  $y_t^{+'} = (y_t' \ y_{t-1}')$ ,  $x_t^{+'} = (x_t' \ 0)$ ,  $z_t^{+'} = (z_t' \ 0)$ . Omitting the superscripts, we suppose in the following that the system is in the form of eq.(19) .

The reduced form of the system is obtained by premultiplying eq.(19) with the inverse of  $\Gamma_0$ :

$$y_t = \Gamma_0^{-1} \Gamma_1 y_{t-1} + \Gamma_0^{-1} B_1 z_t + \Gamma_0^{-1} B_2 x_t$$

or

$$y_t = \Pi_1 y_{t-1} + \Pi_2 z_t + \Pi_3 x_t. \quad (20)$$

Eq.(20) can be used to examine the stability of the system. A sufficient condition for stability is that the real parts of the eigenvalues of  $\Pi_1$  are less than 1 in absolute value. The largest eigenvalue of  $\Pi_1$  is 0.95, so the system is stable.

Setting  $y_t = y_{t-1}$  yields the steady state solution of the system:

$$y_t^* = (I - \Pi_1)^{-1} \Pi_2 z_t + (I - \Pi_1)^{-1} \Pi_3 x_t. \quad (21)$$

Since the system is formulated in logarithms, first differences of eq.(21) approximately give the long-run dynamic solution which satisfies the condition ( $\hat{y}_t = \hat{y}_{t-1}$ ) (see Currie [1981]), where a hat over a variable denotes the growth rate of that variable:

$$\hat{y}_t^* \approx (I - \Pi_1)^{-1} \Pi_2 \hat{z}_t + (I - \Pi_1)^{-1} \Pi_3 \hat{x}_t. \quad (22)$$

One part of the dynamic solution of the system has an important economic interpretation: the third component of the vector  $\hat{y}_t^*$  gives the equilibrium employment path consistent with constant inflation rates. Together with data for potential labour, changes in the non-accelerating inflation rate of unemployment (NAIRU), which we denote by  $u^*$ , can be calculated

$$\Delta u_t^* \approx \hat{l}_t^* - \gamma \hat{z}_t - \delta \hat{x}_t, \quad (23)$$

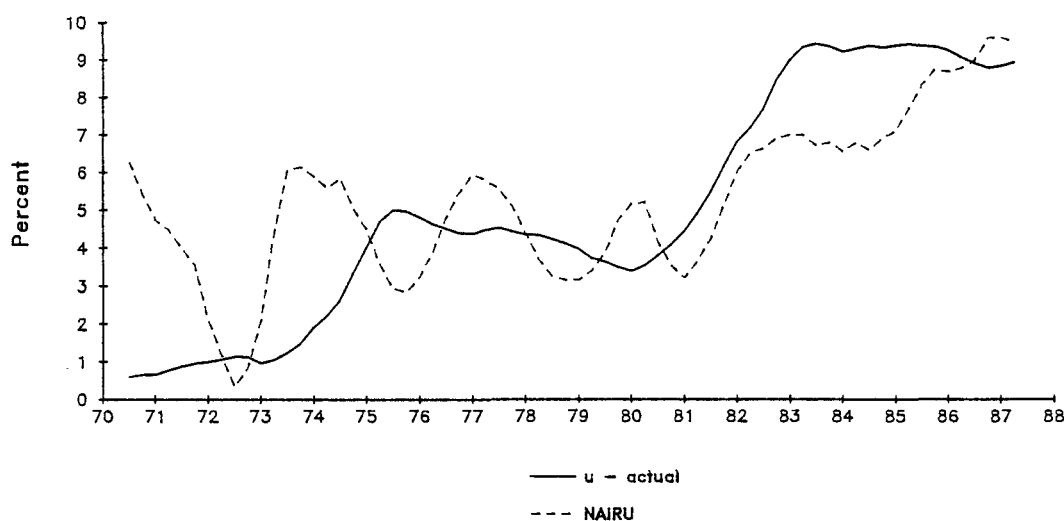


FIGURE 1:  
Actual and non-accelerating-inflation rate of unemployment

where  $\gamma$  and  $\delta$  are the corresponding rows of  $(I - \Pi_1)^{-1}\Pi_2$  and  $(I - \Pi_1)^{-1}\Pi_3$ , respectively. It is evident from eq.(23), that the NAIRU is influenced by policy actions in the past as well as by changes in the supply of labour and other exogenous variables of the system. Adding  $u_{t-1}^*$  on both sides of eq.(23) and substituting subsequently for the lagged term on the right hand side gives the final form

$$u_t^* \approx \sum_{i=0}^{t-1} (\hat{i}_{t-i}^s - \gamma \hat{z}_{t-i} - \delta \hat{x}_{t-i}) + u_0^* \quad (24)$$

On the basis of eq.(24), a time path for the NAIRU can be calculated up to a constant determined by the starting value  $u_0^*$ , which is unknown.

Figure 1 depicts the development of actual unemployment and a time path for NAIRU, where we assume that the mean of NAIRU in the time period 1976 to 1979,2 - a period of a rather stable inflation rate - was equal to the mean of the actual unemployment rate.

According to figure 1, NAIRU shows large cyclical swings in the seventies without a significant trend. By contrast, a marked upward shift is observed in the eighties, drifting up the average NAIRU more than 3 percent. Comparing the two major recessions included in our estimation period, it is evident that a strong increase of NAIRU preceded the actual unemployment in the mid-seventies, whereas NAIRU followed the actual development with a lag at the beginning of the eighties.

These results are roughly consistent with the estimations of NAIRU by Layard, Basevi et al. [1984] for the European Community as a group, which report a more or less stable NAIRU of about 5.3% in the seventies and a considerable increase at the begin of the eighties. Coe and Gagliardi [1985], as cited in Sachs [1987], give results for Germany which show a slight decrease in the seventies and a strong upward shift thereafter.

Although leading to comparable results, our approach differs from the method adopted by



other authors. Other studies <sup>9</sup> start with a reduced form of a simple wage-price model, where unemployment and productivity are taken as exogenous variables. Estimates for the NAIRU are obtained, if the model is solved for a constant unemployment and a stable inflation rate. By contrast, employment and productivity in our approach are modelled as *endogenous* variables, which gives some new insights into the concept of the NAIRU. The main point is, that NAIRU as a part of the dynamic solution of the system becomes dependent on the explanatory variables not only of the wage and price equations but also of the real part of the system <sup>10</sup>. Since some of these explanatory variables are at least partially under the control of political authorities, the observed time path of the NAIRU may reflect a 'wrong' policy in the past.

The question arises, which factors exhibit a dominant influence on NAIRU, especially, which factors have caused the dramatic change in the eighties? In *table 3*, we use our estimation results to decompose the changes in NAIRU. The calculation presupposes a reasonable division of the sample period. For this purpose we take the peaks of the NAIRU time path to split the estimation period up in five subperiods. The first subperiod, reaching from the beginning of the sample to 1973,4, is characterised by a more or less full employment situation. At the same time, the mean of NAIRU exceeds the mean of the actual unemployment rate, which seems plausible because the inflation process began to accelerate at the beginning of the seventies. The second subperiod from 1974,1 to 1977,1 contains the first jump in the actual unemployment rate from about 1.5 to nearly 5 percent caused by the first OPEC shock. According to *table 3* the increase in NAIRU was much less dramatic (one percent as compared with the first subperiod or about 0.3 on a yearly basis).

In the late seventies, NAIRU temporary stopped raising. The comparison of the third subperiod (1977,2 - 1980,1) with the second even shows a moderate decline of the average NAIRU (-0.25). Things become different in the subperiod 1980,2 to 1983,1. The second oil price shock occurred with the actual unemployment rate jumping from about 4 percent to more than 9 percent. The NAIRU has been almost monotonically increasing since then. The upward shift is especially evident in the last subperiod (from 1983,2 to the end of our estimation period). At the same time, the NAIRU lies below the actual unemployment rate with the exception of the very end of our sample period.

The decomposition of the NAIRU in *table 3* shows that the global change of this variable stems from a number of effects which partially neutralize each other. First, as already indicated by the long-run multipliers of the model, labour supply does not contribute substantially to changes of the NAIRU, although there have been considerable changes in the labour supply within the sample period. The negative labour market effect of a growing labour force is more than outweighed in long-run equilibrium by the indirect effects which arise from the pressure on wages in the case of excess supply on the labour market. To see this, we split up the total effect of an increase in labour supply - which is rather small in all subperiods - in the direct (pure supply) effect and the indirect effects. Second, the major factors which tend to reduce the NAIRU over all subperiods are the money supply as an indicator of demand and the growth of potential output. Forces which tend to increase non-inflationary unemployment are the tax rate, the exchange rate and the time trend.

In the second half of the seventies, internal demand as well as the growth of potential output (which reflects investment decisions in that time period) were strong factors in working against a rise in NAIRU more than neutralizing the detrimental effects of the exchange rate and the increasing wedge. In the eighties, we find relative weakness of demand as well as slow growth of potential output as the major causes for the rise in the NAIRU. The burden of labour taxes and social contribution and the slackness of world trade considerably

<sup>9</sup> A typical approach is described by Sachs, Wyplosz [1986].

<sup>10</sup> For example, NAIRU is influenced by demand factors. With the traditional link of NAIRU with frictional unemployment in mind, this might be surprising, but it merely is a consequence of model interdependence.

TABLE 3

## Changes of the NAIRU and Effects of the Explanatory Variables

	74,1 - 77,1 vs. 70,3 - 73,4		77,2 - 80,1 vs. 74,1 - 77,1	
	(1)	(2)	(1)	(2)
<i>NAIRU</i>	1.11	0.31	-0.25	-0.07
<i>labour supply</i>				
- direct effect	1.68	0.46	0.43	0.13
- indirect effect	-1.95	-0.54	-0.50	-0.15
- total	-0.27	-0.08	-0.07	-0.02
<i>other factors</i>				
<i>m</i>	-9.44	-2.60	-10.14	-3.00
$\tau$	4.09	1.13	2.03	0.60
$v^*$	-2.96	-0.82	-1.32	-0.39
<i>WT</i>	-0.99	-0.27	-0.27	-0.08
$\varepsilon$	5.29	1.46	4.54	1.34
<i>t</i>	5.38	1.48	4.98	1.48
	80,2 - 83,1 vs. 77,2 - 80,1		83,2 - 87,2 vs. 80,2 - 83,1	
	(1)	(2)	(1)	(2)
<i>NAIRU</i>	0.44	0.13	2.71	0.70
<i>labour supply</i>				
- direct effect	3.48	1.07	1.80	0.46
- indirect effect	-4.03	-1.24	-2.08	-0.54
- total	-0.55	-0.17	-0.28	-0.12
<i>other factors</i>				
<i>m</i>	-4.79	-1.47	-6.27	-1.62
$\tau$	0.78	0.24	2.48	0.64
$v^*$	-1.45	-0.45	-0.55	-0.14
<i>WT</i>	0.27	0.08	0.87	0.23
$\varepsilon$	1.39	0.43	0.69	0.18
<i>t</i>	4.78	1.48	5.78	1.48

## Notes:

- all numbers are in percent
- (1) total change, (2) change on a yearly basis

reinforce this tendency in the last subperiod. On the other hand, the effective exchange rate is not so important a factor as in the seventies.

## 5. Implications for policy

Slackness of labour markets in Western Europe lasts for more than 10 years. Having identified the factors which are responsible for the rise of the NAIRU, we may ask whether this development was inevitable or not. Before we try to answer this question, we first analyse the way our systems reacts to shocks.

### *The hysteresis property*

One important argument explaining the the persistence of slackness rests on the hysteresis property of non-inflationary unemployment. Hysteresis, in a technical sense, occurs if the transition matrix  $A$  in a linear inhomogenous differential equation system  $\dot{y} = Ay - x$  is singular.<sup>11</sup> As a consequence, the endogenous variables become path-dependent: shocks are not worked out through the system dynamics but have persistent effects. So, after a disturbance has disappeared, the system does not re-attain the initial equilibrium. The steady state solution of the system is no longer unique for a given set of exogenous variables, but depends on the history of deviations from equilibrium.

As a special case, a system exhibits the hysteresis property if  $A$  is the zero matrix or if the changes of the endogenous variables depend only on exogenous influences and not on the (past) level of the endogenous. Obviously, this is the case with the NAIRU as formulated in the last section. Consequently, the non-inflationary unemployment, in our view, is a variable with an infinite memory of shocks that occurred in the past. A deviation of an exogenous variable from its 'normal level' (for example, the growth rate of potential output) will be reflected by changes in the NAIRU. The NAIRU will only return to its initial state after a shock in the opposite direction of equal magnitude. With non-inflationary unemployment depending on the whole history of employment affecting variables, it is not surprising that empirical studies find a strong correlation between the NAIRU and (past) unemployment.

Following Hargraves Heap [1980], some authors argue, that the change of the NAIRU is affected by the slackness of the labour market itself. If policy makers always choose an unemployment rate close to the actual NAIRU, a positive feedback may occur. Neglecting all other explanatory variables and defining slackness as a difference between the actual and the equilibrium unemployment rate, this leads to a formulation of a linear differential equation used by Sachs [1987]

$$\dot{u}_t^* = f(u_t - u_t^*), \quad f > 0. \quad (25)$$

Sachs shows that if eq.(25) is combined with a Phillips-curve in a simple dynamic control approach, the model exhibits the hysteresis property.

As discussed by Franz [1987] in more detail, there are at least three justifications for a relationship like eq.(25) : First, during long periods of unemployment human capital depreciates. The argument is reinforced by the practice of employers of using the status of unemployment as a negative screening device. Second, the long-term unemployed are not in direct competition with the employed persons, so that there is little downward pressure on wages stemming from this group (the so-called insider-outsider-effect). Third, slack labour markets may be an indicator for slack demand in the economy, and slack demand could entail a reduction in capital formation which in turn increases NAIRU, acting like an adverse supply shock.

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<sup>11</sup> See Giavazzi, Wyplosz [1985].

The discussion of the first two questions, which extend the traditional view of combining the NAIRU with frictional unemployment, is not the aim of the present paper, although it seems a fruitful task to look behind some of the effects which, in our approach, are simply captured by trend terms. The link between the labour market and capital formation could be incorporated in our model in a straightforward way but has not been done yet. As mentioned above, the weakness of capital formation is a main factor in explaining the shift of the NAIRU which occurred in the eighties and it would be very important to know whether this phenomenon could be explained by the model.

### *NAIRU and the scope for macroeconomic policy*

In a myopic view NAIRU determines the scope for macroeconomic policy. Any attempt to reduce unemployment below this mark would unavoidably entail inflationary consequences which, in the medium run, would have to be counteracted. Looking at *figure 1* there seems to be little hope of finding a strategy to combat unemployment. But, taking the hysteresis property and eq.(25) into account, things might be somewhat different. As Sachs [1987] emphasizes, the policy implications of hysteresis have not yet been worked out. His own preliminary results in this area indicate that a simple 'going for growth' (to lower NAIRU sharply after several years of tight labour markets) might not be optimal: "Given the inflationary costs of expansion when the initial NAIRU is high, there is no general case for an expansion in the presence of hysteresis" (Sachs [1987,p.33]).

Since the approach used by Sachs is purely theoretical, it is not able to answer the question when and to what extent policy intervention should have been undertaken in the past and what would have been their costs. A further argument has to be taken into account: Not only could a 'going for growth'-strategy drive up the inflation rate to a politically unacceptable amount, but also, as Modigliani et al. [1986] have pointed out for the case of Italy, a traditional policy of stimulating demand could quickly lead to external imbalance.

With regard to the external balance, the case of Germany might be different. The predominant situation of current account surpluses, perhaps leaves scope for demand management. But what about the inflation rate? There are no clear-cut answers possible. The problem has to be solved by optimal control techniques.

## 6. An optimal control approach

In the typically optimal control approach <sup>12</sup> a quadratic cost functional is defined which is minimised over time taking into account the macroeconomic system as a whole. Defining  $\tilde{y}_t$  and  $\tilde{z}_t$  as the vector of deviations of the endogenous variables and control variables from target values, the problem can be defined as follows:

$$\min_{\tilde{z}_t} C = \frac{1}{2} \sum_{t=1}^T \tilde{y}_t' Q \tilde{y}_t + \tilde{z}_t' R \tilde{z}_t \quad (26)$$

s.t.

$$\begin{aligned} y_t &= \Pi_1 y_{t-1} + \Pi_2 z_t + \Pi_3 x_t \\ y_0 &= \tilde{y}_0, \end{aligned}$$

where  $Q$  is a positive semi-definite and  $R$  a positive definite matrix.

<sup>12</sup> See, for instance, Chow [1975], Aoki [1976], and Rao [1987].

The Hamiltonian is

$$H(y_t, z_t, \lambda_{t+1}) = \frac{1}{2} [\tilde{y}'_t Q \tilde{y}_t + \tilde{z}'_t R \tilde{z}_t] + \lambda_{t+1} [\Pi_1 y_t + \Pi_2 z_{t+1} + \Pi_3 x_{t+1}]. \quad (27)$$

If  $z_t^c$  denotes the optimal level of the control variable, and  $y_{t-1}^c$  is the endogenous variable under control influence, the solution of the problem leads to an optimal feedback rule of the form

$$z_t^c = -G_t y_{t-1}^c - J_t \quad (28)$$

where

$$\begin{aligned} G_t &= M_t \Pi'_2 H_t \Pi_1 \\ J_t &= M_t (\Pi'_2 H_t \Pi_3 x_t - \Pi'_2 h_t - R z_t) \end{aligned}$$

with  $M_t = (R + \Pi'_2 H_t \Pi_2)^{-1}$ .  $H_t$  is given from the matrix Ricatti equation

$$H_{t-1} = Q + \Pi'_1 H_t \Pi_1 - \Pi'_1 H_t \Pi_2 G_t \quad (29)$$

and  $h_t$  from the tracking equation

$$h_{t-1} = Q y_{t-1} + \Pi'_1 H_t (\Pi_2 J_t - \Pi_3 x_t) + \Pi_1 h_t. \quad (30)$$

The matrix Ricatti equation is to be solved recursively with  $H_T = Q$  as initial condition. Similarly  $G_t$  and  $h_{t-1}$  can be calculated. Substitution in eq.(28) gives the solution of the feed-back problem.

### *Control experiment 1*

The period chosen for the control experiments is 1971,2 to 1985,4<sup>13</sup>. We focus on three-target variables: the inflation rate, the unemployment rate, and the current account ratio. To run a control experiment, first, the desired levels of this target variables have to be specified. This was simply done by defining a zero level of unemployment, inflation and the current account ratio<sup>14</sup>. In a second step, the weight matrices  $Q$  and  $R$  are to be set.

<sup>13</sup> Rao [1987,p. 185f.], among others, points out that the optimal path of the control variables towards the end of the planning period sometimes seems to be rather unrealistic, "regardless of the consequences stemming from such actions thereafter". The problem lies in the finiteness of the planning horizon and the dynamic structure of the model. To avoid complications, the period of our policy simulation ends in 1985, but the cost of deviation from the targets are calculated through 1987,2. As our model implies a rather quick response of price level variable, at least most of the inflation effects of policy actions towards the end of our policy simulation period are captured.

<sup>14</sup> It could be argued that the optimal unemployment rate should allow for frictional unemployment to some extent. As our experiments have shown, including a positive level of unemployment of 1 or 2 percent does not make much difference to the results referred to above.

TABLE 4  
Cost components - Control experiment 1

	total costs	cost components associated with deviations from targets				
		inflation rate	unemployment rate	current account	money supply	tax rate
historical data	100.00	70.00	20.00	10.00	—	—
optimal control	66.13	50.46	8.32	4.03	3.15	0.17

With the exception of the special structure of the matrix  $Q$  with regard to the variable  $p$  (which was necessary because we intended to control for the inflation rate and not for the price level) it was assumed that these matrices are diagonal and that the elements of  $Q$  corresponding to the non-target variables are set equal to zero.

We calculated the deviation of the historical values from the optimal path and used these values to determine the elements of the  $Q$  matrix according to a pre-chosen relative weight scheme.

In our first control experiment emphasis was laid on anti-inflationary policy, setting a relation of 70:20:10 for the weights of the target variables (inflation rate, unemployment rate, current account ratio). The relative cost of using the instrument variables (nominal money supply and the tax rate) was specified as 10:100<sup>15</sup>. The high costs imposed on variations of the tax rate are quasi prohibitive, so in this experiment we have only one instrument in effect.

We first have a look on the optimal time path of the money supply. *Figure 2* shows that the actual and the optimal money supply in the first half of the seventies fluctuates around the actual money supply. On the other hand, in the second half of the seventies, and to an even larger extent in the time period 1982-1985, the control algorithm recommends a more expansionary policy than actually chosen. This is the case, although inflation is 'expensive' in this setting. Further, the responses of the money supply to both oil price shocks are interesting: optimal control obviously does not support simultaneous accommodation policy. The optimal path of the money supply clearly implies a restrictive policy in both time periods.

In general we conclude, that in spite of the relatively high weight of the zero inflation target, the optimal control algorithm does not choose an extremely anti-inflationary cure. Instead, the controlled inflation rate seems to be more or less a smoothed version of the actual inflation rate, showing a declining trend over the sample (see *figure 3*)<sup>16</sup>. *Table 4* indicates that there are still some gains with respect to the inflation target (although these can essentially be attributed to the smoother course of the optimal path).

As can be seen from *figure 4*, the effects of the expansionary policy on unemployment are considerable. Optimal control of the money supply would have lowered unemployment up to 1.5 percent in the period 1978/79 and up to 3 percent in the years after 1983. Only at the beginning of the sample and in the period 1973/74 controlled unemployment exceeds actual

<sup>15</sup> It should be mentioned, that the detrimental effects of extensive money supply are also caught indirectly by punishing deviations of the inflation rate target.

<sup>16</sup> The figure depicts a smoothed time path of the actual inflation rate. The quarterly inflation rate (BIP-deflator) is still more volatile.

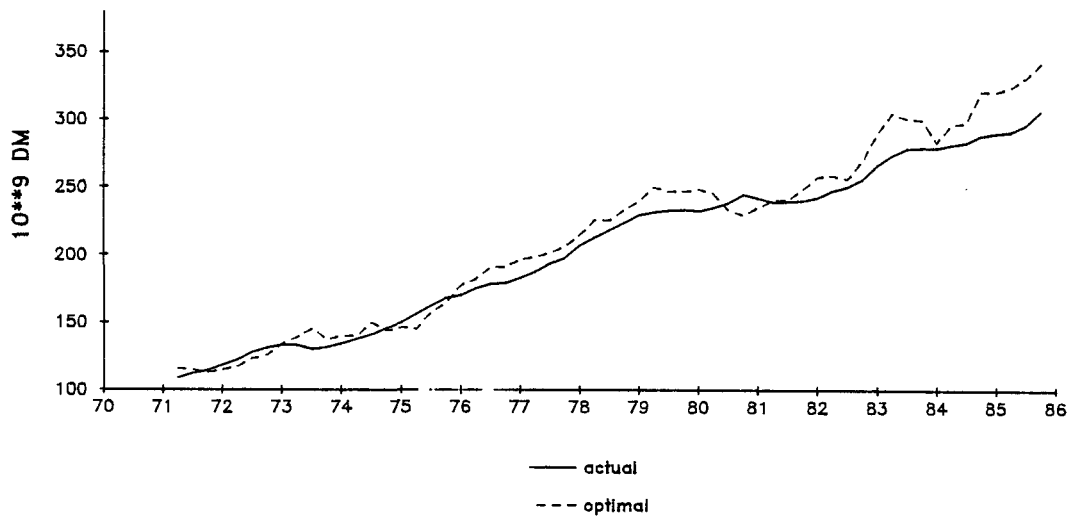


FIGURE 2:  
Actual and controlled money supply (experiment 1)

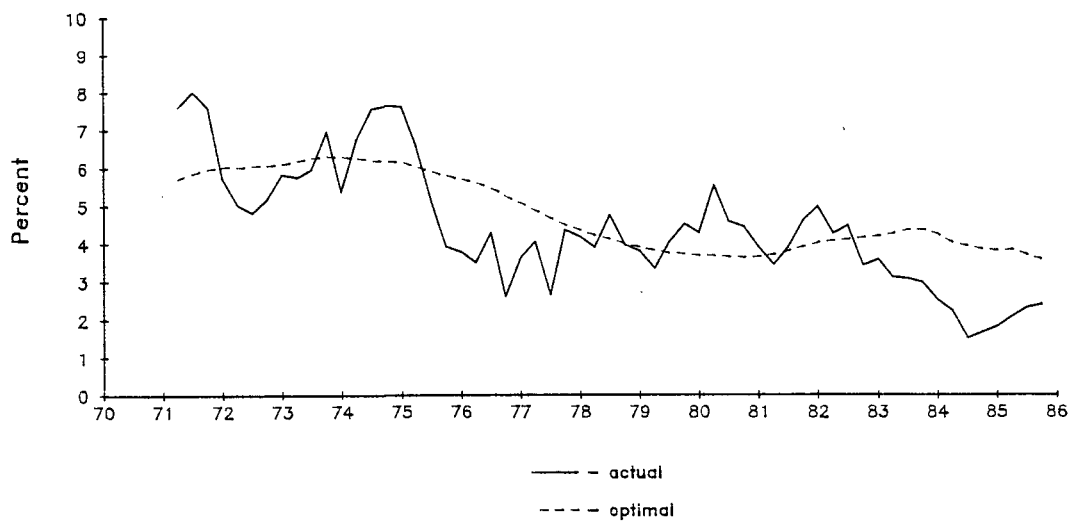


FIGURE 3:  
Actual and controlled inflation rate (experiment 1)

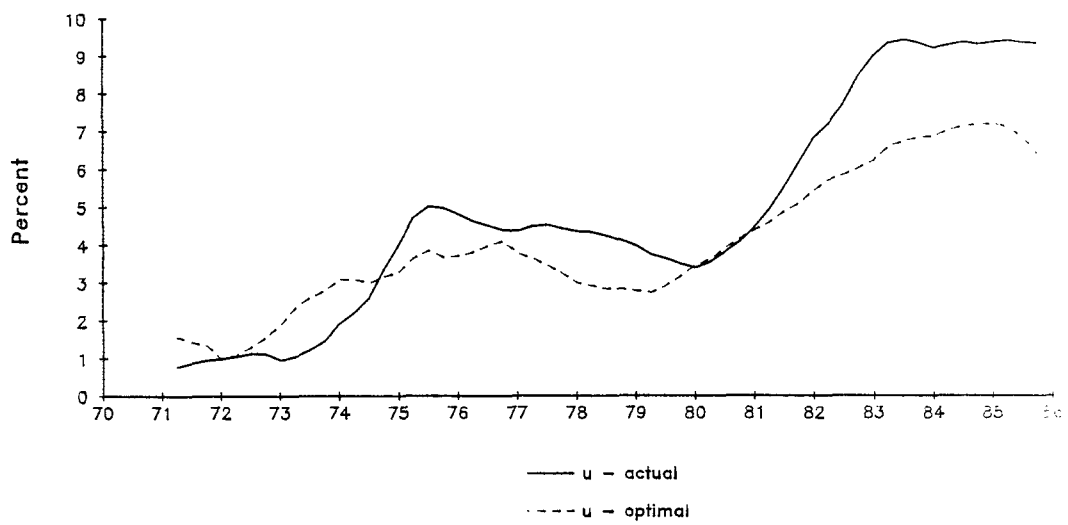


FIGURE 4:  
Actual and controlled unemployment rate (experiment 1)

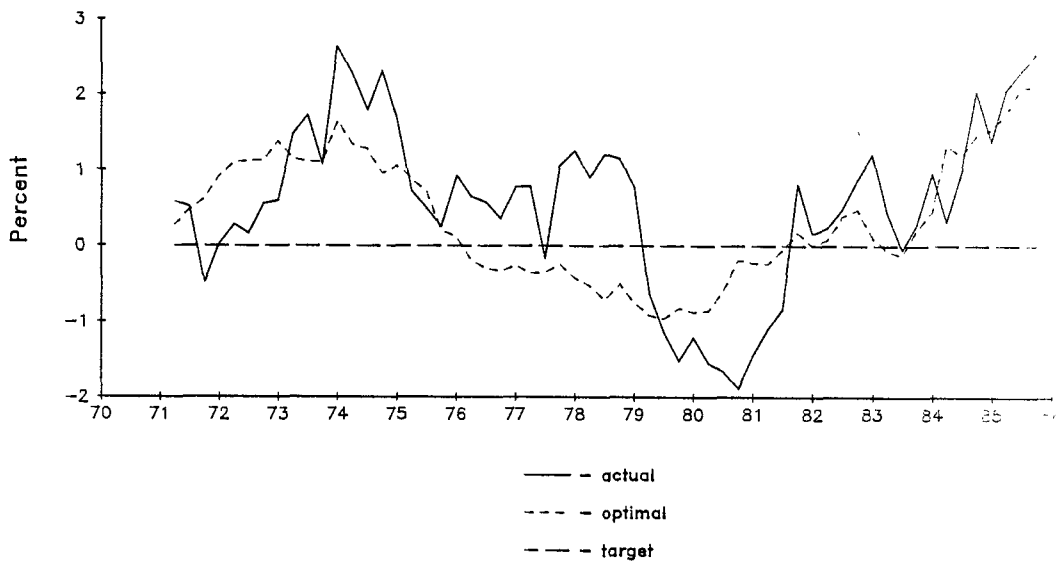


FIGURE 5:  
Actual and controlled current account surplus (experiment 1)



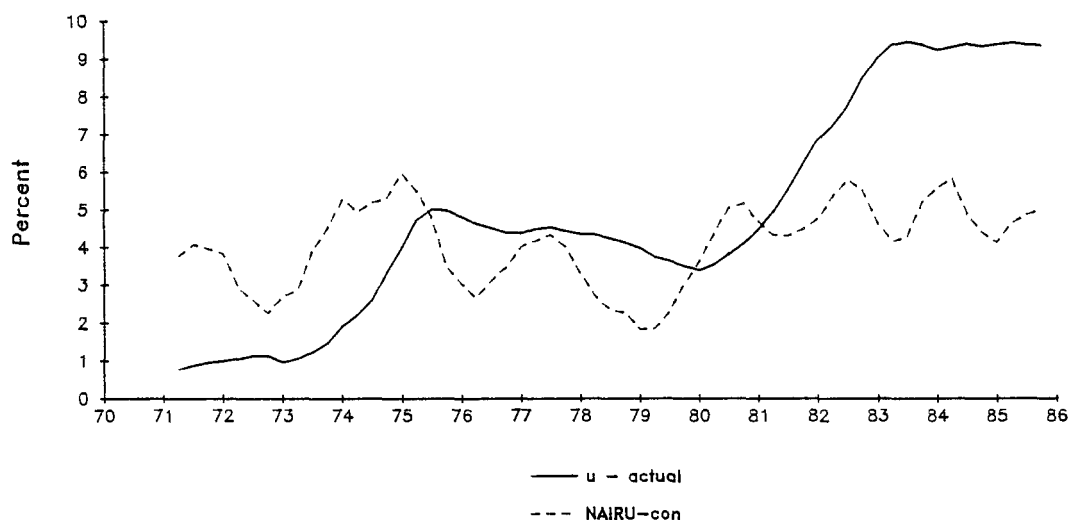


FIGURE 6:  
Actual and non-accelerating-inflation rate of unemployment (experiment 1)

unemployment. That is, the optimal control algorithm reveals a sizable scope for demand-managed labour market policy. On the other hand, a large increase in unemployment at the beginning of the eighties could not have been avoided by means of money supply policy alone, at least, if policy makers had a strong preference for fighting against inflation, which is assumed here <sup>17</sup>.

What are the effects of the optimal control strategy on the current account ratio? A look at *table 4* reveals that there are gains with respect to this target, too. *Figure 5* depicts the time path of the controlled current account ratio as compared to the actual time path. The result is, that under control conditions the large surpluses are reduced as well as the large deficit appearing in the actual series in the years 1980/81. The deficit period is longer, reaching from 1976 to 1981, but never exceeds 1% of potential output. On the other hand, the large current account surplus at the end of the simulation period is not substantially reduced.

An interesting question is, what the effects of such an optimal strategy on the NAIRU would have been. *Figure 6* depicts the time path of NAIRU under conditions of optimal control of the money supply process. As compared with *figure 1* the large swings of the NAIRU are somewhat dampened. Beside this, the mean of both variables is rather similar in the seventies. In the eighties, however, the time path of NAIRU under control conditions appears to be quite different: Especially the upward shift in the eighties is much smaller (about 1.5 percent as compared with the mean of the seventies). The remaining upward shift essentially stems from slack capital formation and increased tax burdens of labour.

<sup>17</sup> It should be mentioned that the results could be more favourable if the feedback-mechanisms on labour supply and capital formation - as discussed in the context of hysteresis - are taken into account.

*Control experiment 2*

In a second experiment we analyse the combined effect of monetary and fiscal policy by reducing the costs of deviations of the wedge variable from the actual path, so that these costs are no longer prohibitive. But still, the costs of using the wedge variable are chosen higher than the costs of using monetary policy (25:10). This can be justified by the argument, that the costs of an extension of the money supply are found in the induced inflation process (which is valued separately), but that a reduction in the tax rate (to lower the wedge) perhaps entails a higher public budget deficit, a mechanism which our model does not encompass.

TABLE 5  
Cost components - Control experiment 2

	total costs	cost components associated with				
		deviations from targets			use of controls	
		inflation rate	unemployment rate	current account	money supply	tax rate
historical data	100.00	40.00	40.00	20.00	—	—
optimal control	42.66	32.08	2.56	5.61	0.44	1.97

*Figure 7* shows the development of the money stock in control experiment 2. In general, the pattern is quite similar to that in experiment 1, although there seems to be an even more expansionary policy possible. *Figure 8* depicts the time path of  $\tau$ , the second control variable. The optimal control algorithm recommends a smaller increase in the wedge variable after the first oil price shock, the optimal path is about 1 to 2.0% lower than the actual in the second half of the seventies and after the year 1982, while it comes close to the actual path at the begin og the eighties.

The effect on the target variables is described by the following figures. We see that the optimal inflation rate with respect to the first half of the seventies has declined from about 6% to about 4% at the begining of the eighties. After a slight increase in 1983/84 it falls again at the end of the sample, yet somewhat exceeding the actual inflation rate in the last subperiod.

Although the variations of the control variables are moderate and the effect on the inflation rate in the last subperiod seems tolerable, the labour market consequences appear to be striking: unemployment under this control conditions has been only slightly increasing since the seventies, and does not exceed 3.0% in the eighties. A look at the current account under control conditions shows that the deficit period as compared to experiment 1 has been prolonged considerably (reaching from 1976 to mid 1984), but again does not substantially exceed 1% of potential output. Taking into account the large surplus in the first half of the seventies and again after mid 1984, the results of the control experiment may be regarded as acceptable. So, it can be established in general, that the model simulates a more or less full employment situation throughout the sample period without requiring unrealistic policy parameters.

*Figure 12* reveals that in the setting of this control experiment the NAIRU has no upward trend during the whole sample period. So we conclude that the increase in NAIRU has not been inevitable but could have been avoided by well-timed changes in the instruments of fiscal and monetary policy.

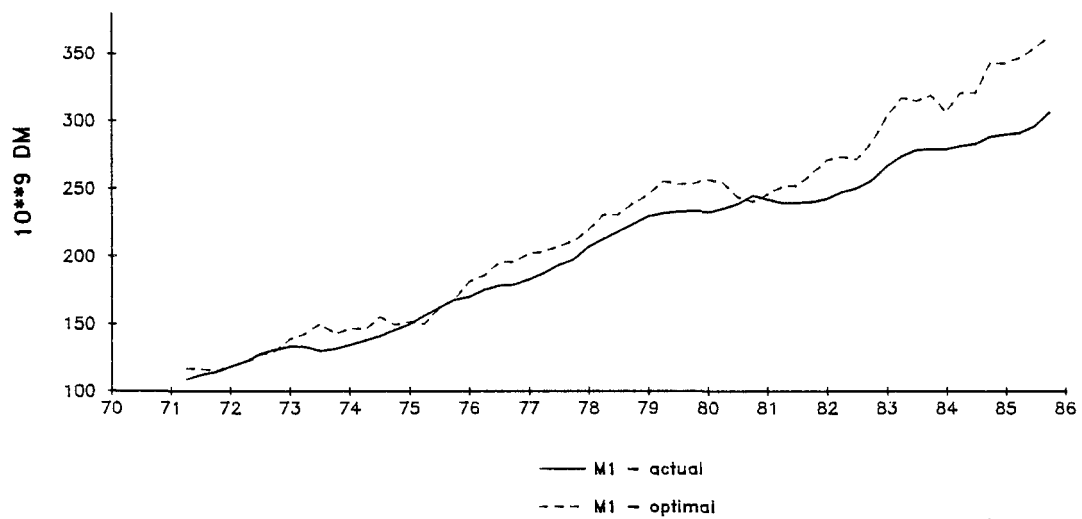


FIGURE 7:  
Actual and controlled money supply (experiment 2)

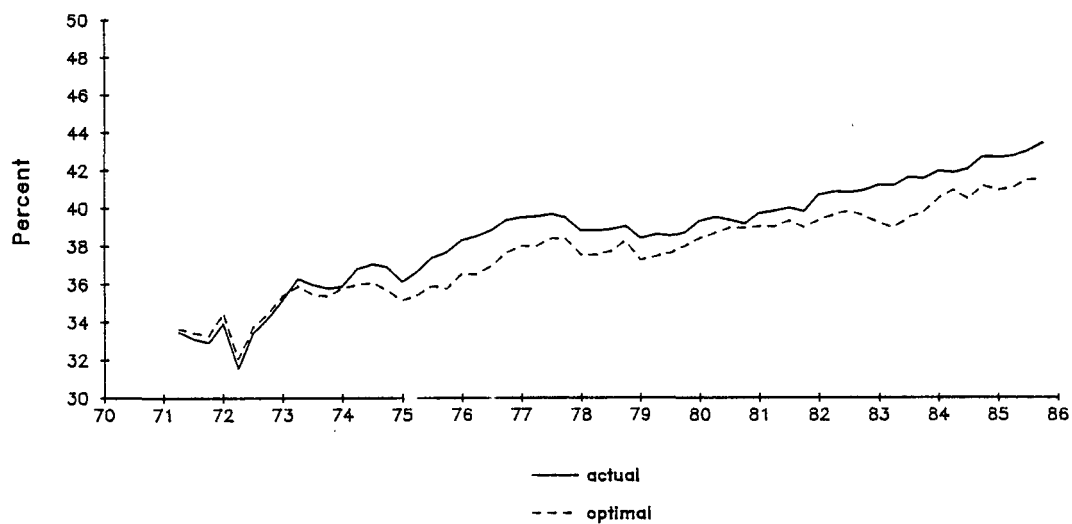


FIGURE 8:  
Actual and controlled wedge variable (experiment 2)

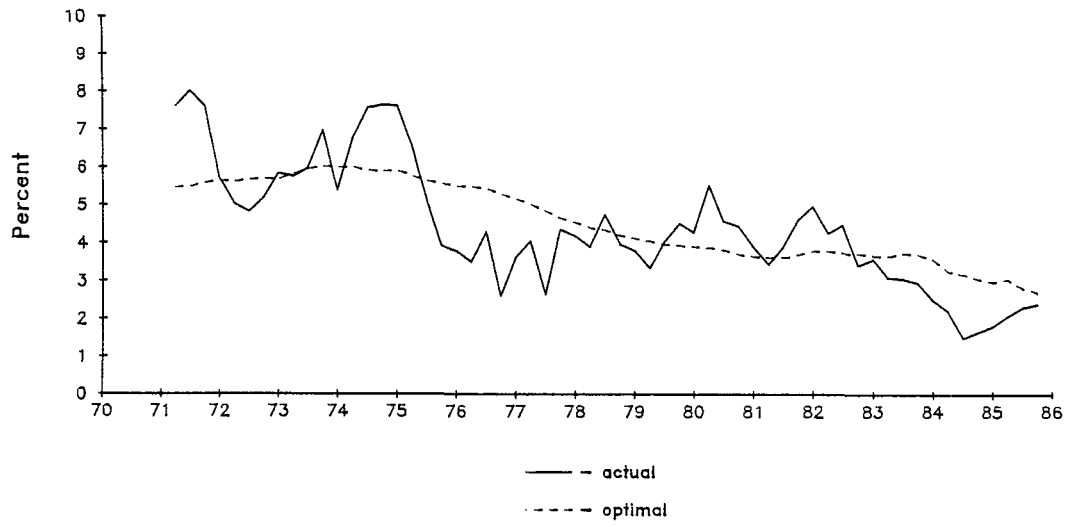


FIGURE 9:  
Actual and controlled inflation rate (experiment 2)

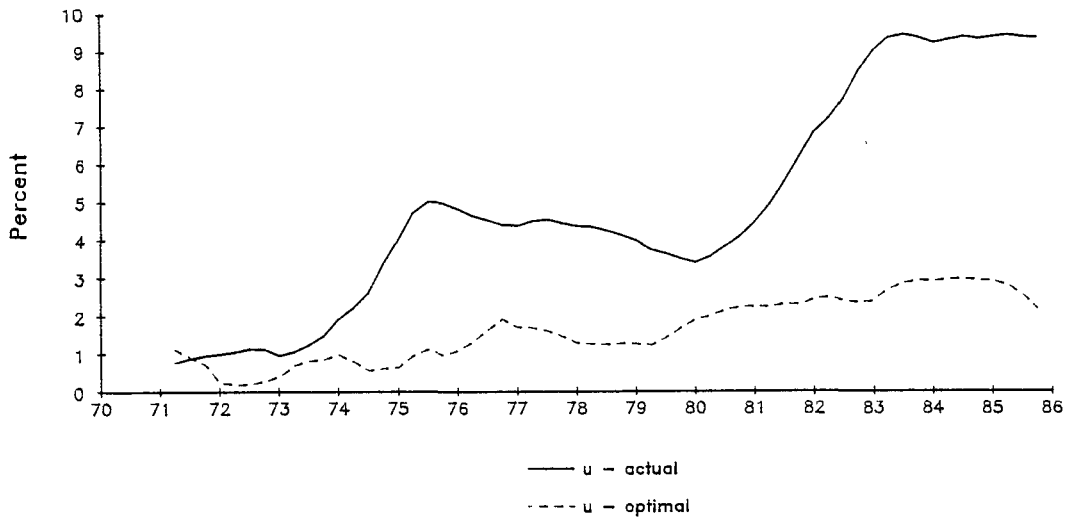


FIGURE 10:  
Actual and controlled unemployment rate (experiment 2)

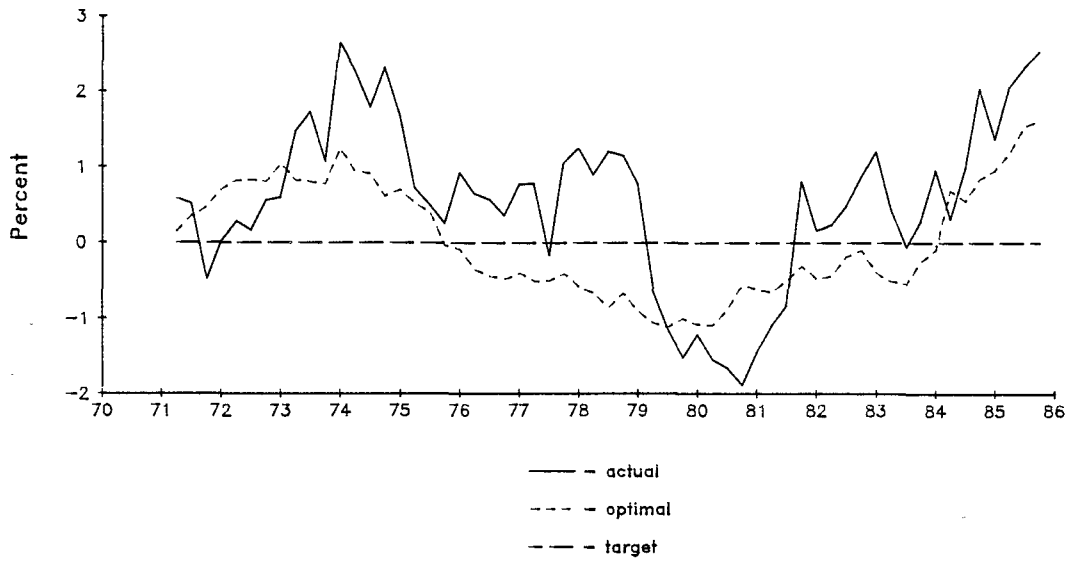


FIGURE 11:  
Actual and controlled current account surplus (experiment 2)

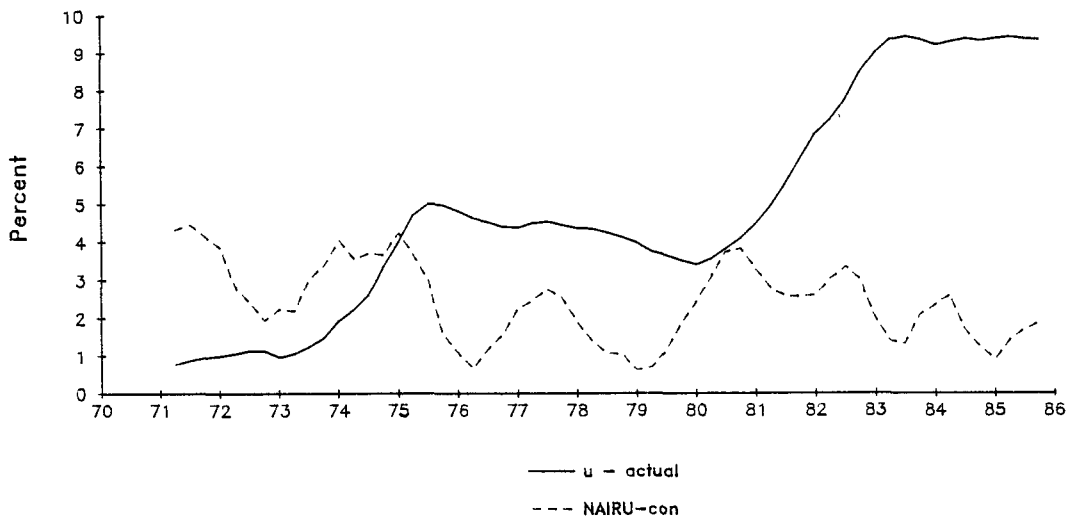


FIGURE 12:  
Actual and non-accelerating-inflation rate of unemployment (exp.2)

## 7. Changing labour market structure

The simulations of optimal policy actions reveal an optimal strategy for the past, but what about the future? What are the inherited effects of high unemployment on the scope of macroeconomic policy? Do the feedback effects discussed in the context of hysteresis prevent us from returning to a full-employment path?

The Beveridge curve gives an impression of the changing labour market structure. A first glance at the data shows that the relationship has not been stable during the last 15 years or so. This phenomenon is known as the (outward) shift of the Beveridge curve (Franz[1987]). Empirically this shift is usually described by trend variables.

The question remains whether the changing structure of the curve could also imply a time-varying pattern in the sensitivity of the unemployment rate with respect to the vacancy rate. Since this question cannot be answered exactly by the use of traditional econometric methods, the relationship, therefore, seems to be a natural field of application for time varying coefficient estimation. We first carried out a maximum likelihood estimation of the variances in a state space model and then applied the Kalman smoothing algorithm.

The model is

$$u_t = \alpha_{1t} + \alpha_{2t}u_{t-1} + \alpha_{3t}vr_t + \alpha_{4t}t + \zeta_t \quad (31)$$

with  $u_t$  as the log of the (official) unemployment rate and  $vr_t$  as the log of the vacancy rate as reported by the Bundesanstalt für Arbeit. The state equation is modelled as a random walk as described above.

TABLE 6  
Estimation of the Beveridge-Curve 1970,3 - 1987,2

explanatory variables	est.par. (1)	dependent variable $u$		est.var. (state eqtn.) (4)
		est.var. (state eqtn.) (2)	est. par. (3)	
constant term	<i>time varying</i>	$0.63 \times 10^{-3}$	<i>time varying</i>	$0.58 \times 10^{-3}$
$u_{t-1}$	0.50 (6.45)	0.00	0.54 (7.34)	0.00
$vr$	<i>time varying</i>	$1.89 \times 10^{-3}$	<i>time varying</i>	$1.90 \times 10^{-3}$
$t \times 10$	0.21 (1.16)	0.00	—	—
$\sigma_\zeta^2$	$3.72 \times 10^{-3}$		$4.38 \times 10^{-3}$	
ln likelih.(Kalman)	112.53		111.62	
ln likelih.(OLS)	85.64		85.50	
likelih.ratio	$53.78\tilde{\chi}^2(4)$		$52.24\tilde{\chi}^2(3)$	

Notes:

- asymptotic absolute t-statistics in parenthesis
- the likelihood ratio test refers to the OLS-regression
- columns (1) and (3) give the estimated parameter when not time-varying
- columns (2) and (4) give the estimated element of the covariance matrix of the state equation  $\Sigma_\eta$

Restricting the covariance matrix of the state equation  $\Sigma_n$  to be a diagonal matrix, we get the following results: As indicated by the likelihood ratio test in *table 6* the hypothesis of parameter stability is strongly rejected. Apart from the changing coefficient of the regression constant, which reflects exogenous factors, the variability of the model parameters primarily stems from the time varying coefficient of the vacancy rate. On the other hand, the coefficient of the lagged endogenous variable is about 0.5 in value and not time varying. Somewhat surprisingly, the trend variable turns out to be insignificant, and can be omitted from the regression.

*Figure 14* shows the time varying coefficient of the vacancy rate in the Beveridge curve. The results indicate that in absolute value the 'long-run' elasticity of the unemployment rate with respect to the vacancy rate has almost continuously fallen from about 1 in the early seventies to less than 0.5 in the mid-eighties. According to our results, the hypothesis of an outward shift is not supported by the data, but the curve has become much steeper.

Using the results of the time-varying parameter regression the equilibrium vacancy rates for alternative unemployment rates of 2%, 5% and 8% at the beginning and at the end of the sample period are as follows:

	1970			1986		
Unemployment Rate	2.0	5.0	8.0	2.0	5.0	8.0
Vacancy Rate	1.8	0.9	0.6	8.9	1.7	0.7

The reported values indicate that the vacancy rate associated with low unemployment rates has risen dramatically while there are no striking differences at higher rates of unemployment.<sup>18</sup>

Given a stable Beveridge curve with parameters as at the end of our sample period, more than 2 million vacancies are permanently required to reduce the unemployment rate to 2%. This number has to be compared with the maximum of 795 000 vacancies reported for the full employment year 1970.

The increasing slope of the Beveridge-curve is not consistent with the hypothesis of an overall decreasing intensity of job search. Rather, it affirms the feedback argument in the context of hysteresis: persistent unemployment tends to produce a hard core of problem groups in the labour market. Unemployment as a negative screening device and the depreciation of human capital during long-term unemployment are the basic mechanisms which lead to this result.

According to these calculations, the changing labour market structure implies a severe burden for full employment policy.

<sup>18</sup> It should be mentioned that there are some problems with the data, especially with the vacancy rate. The vacancies reported in the official statistics only include those new jobs which are registered by the labour office. Only a fraction of new hires is managed by the labour office. As Franz and König [1986] have pointed out, this fraction fluctuates around a decreasing trend. Although no outward shift of the Beveridge curve is suggested by the use of the official data, the bias in the vacancy rate would imply a certain outward shift of the 'corrected' curve. The changes in the slope of the curve are more likely to be underestimated if the official data are used.

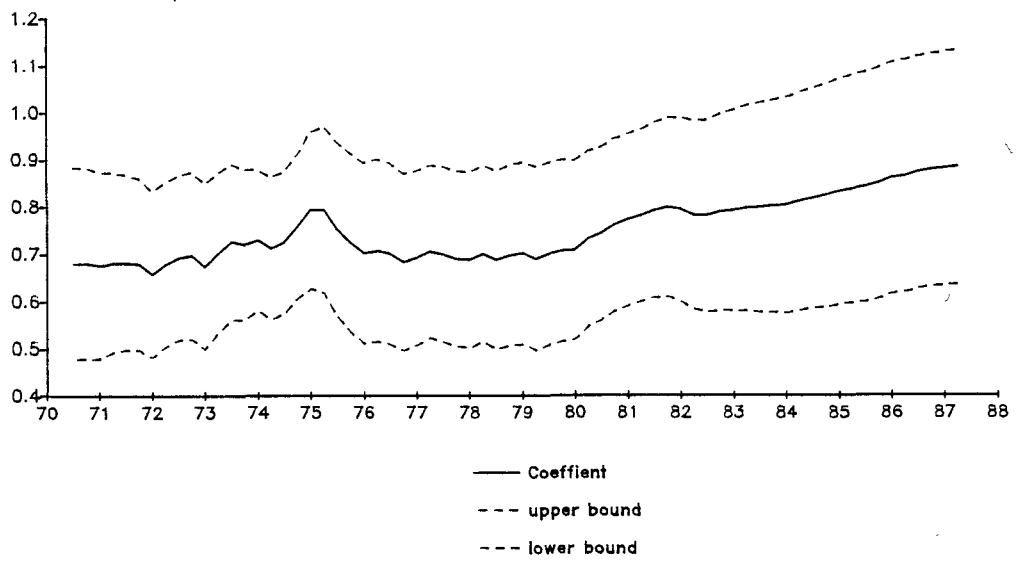


FIGURE 13:  
 Beveridge Curve: Estimated time path of coefficient  $\alpha_1$

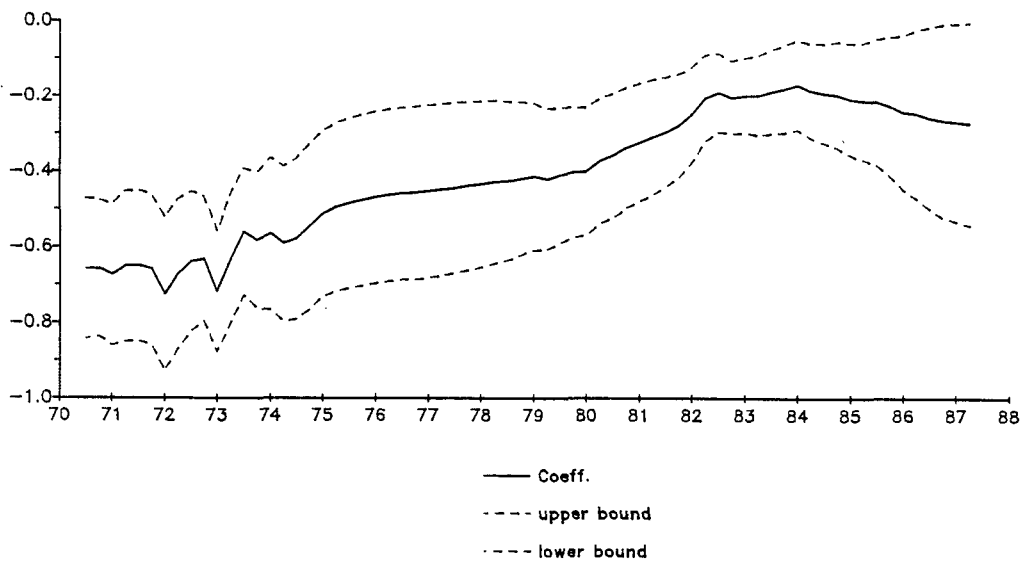


FIGURE 14:  
 Beveridge Curve: Estimated time path of coefficient  $\alpha_3$



## 8. Conclusions

Starting from a theoretical framework with incomplete competition, the present paper presents a small macroeconomic model which focuses on the interdependence of prices and wages, employment and capacity utilisation. External balance is incorporated by introducing a current account equation.

With respect to the wage/price dynamics, we find that rigidities are present but are smaller in magnitude than the rigidities in the quantity variables. Beside this, there is some evidence that prices respond less promptly to wages than wages respond to prices. Wages (measured as wage income per capita) react strongly to unemployment and to increased tax and social security burdens. On the other hand, import prices, which considerably affect purchasing power, do not seem to be reflected in nominal wages. According to our theoretical expectations, prices respond positively to employment and negatively to output. Employment is strongly affected by a real wage variable as well as by output. The adjustment process of employment is found to be the most rigid. Capacity utilisation is influenced by factors of internal and external demand, whereas the current account depends negatively on internal demand and (weakly significant) positively on an indicator of world demand.

From the long-run solution of the model a decomposition of the non-accelerating unemployment rate (NAIRU) is calculated. This decomposition reveals the strong influence of (internal and external) demand factors, of the slow increase of the production potential, and of the wedge variable (reflecting the increasing burden of taxes and social contributions) on changes of the non-inflationary unemployment rate. Especially in the 1983 to 1987 time period, demand factors and a slow growth rate of potential output were dominating for the shift in the NAIRU.

In general, we find that the hypothesis of incomplete competition when combined with partial adjustment processes works well in explaining the data. The stability of the estimation results is investigated by the use of a maximum-likelihood method based on Kalman-filtering. According to likelihood ratio tests, there are no hints of a time-varying pattern in our model structure. Consequently, the model may be used for application of optimal control techniques in order to find out ex-post a superior strategy for monetary and fiscal policy.

In the last part of the paper, we present the results of two control experiments for the time period 1971,1 to 1985,4. The control algorithm recommends a restrictive monetary policy at the time of the two oil price shocks. On the other hand, our results indicate that there has been a sizeable scope for non-inflationary demand expansion in the second half of the seventies and especially in the eighties after 1981. If this policy is supported by appropriate fiscal measures (to lower the wedge between labour costs and take-home pay), a near to full employment path without accelerating inflation can be reached. At the same time, the effects on the current account are found to be tolerable.

Although it can be shown that the optimal policy could have avoided most of the unemployment problem we are now facing, returning to low unemployment rates after several years of a slack labour market might be difficult. Estimation results for the Beveridge curve show an increasing slope of this curve. This means, that an additional burden for future employment policy exists and that the costs of suboptimal policy in the past (leading to sizable depreciation of human capital) are underestimated in general.

Nevertheless, it should be borne in mind that our model has several limitations. It remains for future research to incorporate the capital formation process and a better formulation of the expectations process.

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