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Demographic changes and the implicit tax rate in a pay-as-you-go pension system

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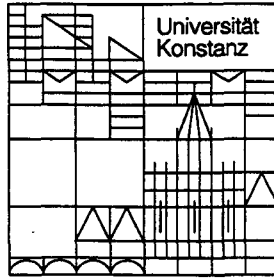
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Diskussionsbeiträge

**Demographic Changes and the Implicit Tax Rate
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**Mathias Kifmann
Dirk Schindler**

Serie I – Nr. 308

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Demographic Changes and the Implicit Tax Rate in a Pay-as-you-go Pension System*

Mathias Kifmann Dirk Schindler[†]

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Abstract

We analyze the effects of a permanent increase in life expectancy and of a permanent decline in the rate of population growth on intergenerational redistribution in a pay-as-you-go pension system with a constant contribution or constant replacement rate. We show that under these policies both demographic changes increase the implicit tax rate of later-born generations to the pension system. In addition, we characterize policies which smooth the implicit tax rate. To reach this objective, generation-specific contribution or replacement rates are necessary. Partial funding of the pension system is indispensable if the rate of population growth falls. If life expectancy increases, the contribution rate fluctuates and may not converge to a new steady state value unless funded elements are introduced.

JEL-classification: H55, J18.

Keywords: Pay-as-You-Go, Implicit Tax Rate,
Life Expectancy, Population Growth.

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1 Introduction

Pay-as-you-go (PAYG) pension systems redistribute between generations. If an economy is dynamically efficient, generations who receive pensions without having contributed to the pension system are better off at the expense of later generations whose return in the PAYG system is lower than on the capital market.¹ As Breyer (1989) and Fenge (1995) have shown, it is generally not possible to switch back to a funded system without making at least one generation worse off. An empirical study by Abel *et al.* (1989) supports the hypothesis that the condition for dynamic efficiency has been satisfied in major industrialized countries in the last decades. It is therefore plausible to regard PAYG pensions as a transfer system which redistributes from younger to older generations.

Two major demographic changes in the more developed countries are having an impact on this intergenerational redistribution. On the one hand, life expectancy is rising as shown for six major industrialized countries in Table 1.² According to these estimates and projections by the United Nations Population Division, life expectancy increases considerably over the time period from 1950 to 2040 in all countries. On the other hand, the rate of population growth has significantly fallen in the industrialized countries. Table 2 shows the estimates and projections for the net reproduction rate. Until 1970, this rate is close to one in most countries. Then the net reproduction rate drops considerably in all countries. For future decades, the United Nations projects a small increase in the net reproduction rate. However, the net reproduction rate is to remain well below its pre-decline level. The projections of the United Nations therefore point to a *permanent* increase in life expectancy and a *permanent* decline in the rate of population growth. In addition, the United Nations projects that immigration from countries with higher rates of population growth will be far from sufficient to counteract the decline in the rate of population growth. Of the six countries presented, only the United States is projected to have a growing population until 2040.

¹The redistribution under dynamic efficiency has been justified by *intergenerational risk sharing*. Smith (1982) and Gordon and Varian (1988) adopt the perspective of an individual who does not know in which generation he will be born. In addition, they assume that the return on labor income of the generations is stochastic. Then a PAYG system can provide insurance against the risk of being born in a generation whose productivity is low during their working age as lucky generations transfers a part of their income to unlucky generations.

²The numbers for the 1950 - 1990 period are demographic estimates of the United Nations while the numbers for the 2000 - 2040 period are from the United Nations Population Division's medium variant population projection.

	1950	1960	1970	1980	1990	2000	2010	2020	2030	2040
France	66.5	71.0	72.3	74.7	77.2	78.8	79.7	80.6	81.5	82.2
Germany	67.5	70.3	71.0	73.8	76.0	77.8	78.9	79.8	80.7	81.5
Italy	66.0	69.9	72.1	74.5	77.2	78.8	79.8	80.7	81.5	82.2
Japan	63.9	69.0	73.3	76.9	79.5	80.3	81.1	81.9	82.7	83.4
UK	69.2	70.8	72.0	74.0	76.2	78.0	79.0	80.0	80.8	81.6
USA	69.0	70.0	71.3	74.5	75.7	77.4	78.7	79.7	80.6	81.4

Table 1: Estimated and projected life expectancy at birth in years
Source: United Nations (1998).

	1950	1960	1970	1980	1990	2000	2010	2020	2030	2040
France	1.26	1.34	1.10	0.90	0.83	0.88	0.91	0.95	0.95	0.95
Germany	0.85	1.06	0.77	0.70	0.62	0.64	0.69	0.76	0.79	0.79
Italy	1.09	1.16	1.05	0.74	0.61	0.58	0.64	0.71	0.77	0.80
Japan	1.19	0.94	0.98	0.85	0.72	0.71	0.77	0.84	0.85	0.85
UK	1.02	1.34	0.97	0.87	0.86	0.83	0.88	0.92	0.92	0.92
USA	1.60	1.56	0.96	0.87	0.96	0.93	0.92	0.92	0.92	0.92

Table 2: Estimated and projected net reproduction rates
Source: United Nations (1998).

Both demographic changes make future generations worse off. For example, Keyfitz (1985) shows for the United States that under a defined benefit as well as under a defined contribution PAYG system, the implicit rate of return of the PAYG system turns negative for later-born generations. The same result is obtained by Schnabel (1998) for the German PAYG pension system.

A number of proposals have been put forward to reduce this burden for younger generations. Keyfitz (1988) proposes a partial funding scheme which would permit all generations born within 100 years to pay the same contributions and receive the same benefits. Hagemann and Nicoletti (1989) show for four countries how the average contribution rate to the public pension would increase over time and could be smoothed by a temporary fund. Similarly, the Council of Scientific Advisors of the German Ministry of Economics (1998) has proposed to build up a capital fund to stabilize the contribution rate in Germany.³ For the US Social Security Program, Feldstein and Samwick (1998) have suggested a partially funded scheme in order to lower the future contribution rate. According to their proposal, the federal budget surplus should be used to lower future pension obligations un-

³See Bundesministerium für Wirtschaft (1998).

der the PAYG system. Fehr (2000) analyzes several reform proposals for the German pension system in a computable general equilibrium model. He concludes that an option using partial funding is most attractive because it yields the highest efficiency gains and distributes the burden of reform most evenly among the currently old generations. Using a intergenerational welfare function, Blanchet and Kessler (1991) solve numerically for the optimal mixture between a PAYG and a funded system in a closed economy with endogenous return on investment and demographic instability. Considering different intergenerational welfare functions, they establish that partial funding is generally part of an optimal solution.

These studies show that partial funding is essential to share the burden between generations evenly. With the exception of Fehr and Blanchet and Kessler, however, they focus on the contribution rate as a measure for intergenerational redistribution. We think this measure is not satisfactory because it does not take into account that individuals also receive benefits from the PAYG system. These benefits are also affected by demographic changes. The burden that the PAYG pension system puts on a generation is better captured by the *implicit taxes* which generations pay due to their participation in the PAYG system. These are defined as the difference between the present value of PAYG contributions and the present value of PAYG benefits. In particular, the average *implicit tax rate*, i.e. the ratio of implicit taxes to the present value of life time income, captures the burden the PAYG system puts on a generation. This concept is used by Thum und von Weizsäcker (2000) who determine the evolution of the implicit tax rate under the current reform proposal for the German PAYG system. In a similar study, Bonin und Feist (1999) apply a related concept in a generational accounting framework.

Our paper is complementary to the analysis of Thum and von Weizsäcker. While they analyze the implications of particular reform proposals on the implicit tax rate, our emphasis is on the instruments which are necessary to smooth implicit tax rates after both demographic changes. In a steady state analysis we show that *generation-specific* contribution and replacement rates are necessary to smooth the implicit tax rate. This is an important result since these instruments are not part of current German reform proposals. Furthermore, we examine how the implicit tax rate can be smoothed in a transition phase. We demonstrate that generation-specific contribution and replacement rates may allow to smooth the implicit tax rate even without using funded elements. This is possible if only life expectancy increases and one is willing to tolerate fluctuations of the contribution rate. However, partial funding is necessary if a stable contribution rate is preferred. If the rate of population growth falls, then partial funding combined with generation-specific contribution and replacement rates is needed to lower the burden for later-born generations.

Our results are therefore highly relevant if the smoothing of the implicit tax rate is regarded as desirable. We think that this is an attractive policy goal for several reasons.⁴

1. From the theory of taxation it is well-known that the excess burden of a tax grows at an increasing rate with the tax rate. If one applies this result in an intergenerational context and assumes that all generations have the same elasticity of labor supply, then a constant implicit tax rate produces a lower aggregate excess burden than a fluctuating implicit tax rate.
2. Lowering the implicit tax burden for later-born generations may be in the self-interest of the generations currently alive as Breyer and Stolte (2000) have shown. If the implicit tax rate increases by too much, then future generations may evade their implicit tax payments by working less or in the informal sector or by migrating to a different country. This would lead to a cut in the pensions of the currently living generations.
3. Last but not least, the smoothing of the implicit tax rate is desirable for reasons of intergenerational justice. Under a constant implicit tax rate, each generation contributes the same proportion of life time income in the face of the inevitable burden created by earlier transfers in the PAYG system. If productivity is constant, this implies that all generations pay the same implicit taxes. If productivity grows, then later-born generations pay higher implicit taxes, which may be regarded as fair because it is also easier for these generations to earn income.

Thum and von Weizsäcker find that the current reform proposals do not smooth the implicit tax rate. Our paper shows that these proposals are *a priori* unable to do so. They generally rely on contribution and replacement rates which apply to all generations whereas generation-specific contribution or replacement rates are necessary to smooth the implicit tax rate.

Throughout the paper, we focus on a small open economy for which the interest rate and the rate of productivity growth are assumed to be constant. Although the demographic changes are projected in all major industrialized countries and are therefore likely to have an effect on these variables as well, we think this is a useful starting point for the analysis. Furthermore, not all countries are affected by the demographic changes to the same extent. Table 2 shows that in Germany and Italy the decline in the rate of population growth is particularly large. Finally,

⁴See also Thum and von Weizsäcker (2000).

we focus on permanent changes in the demographic variables as projected by the United Nations. With respect to life expectancy, we think this is a reasonable assumption. Concerning the rate of population growth, such projections are to be interpreted with more caution since future fertility is unknown.

The paper proceeds as follows. In the following Section 2, we present the model and characterize the steady states under pension policies with constant contribution and replacement rates. Section 3 analyzes the consequences of an increase in life expectancy. The changes in steady states are examined and policies are presented which stabilize the implicit tax rate. The same analysis is performed in Section 4 for a decline in the rate of population growth. Section 5 summarizes the results.

2 The model

In a continuous-time model, we examine a steady state economy in which the interest rate r , the rate of population growth m and the rate of wage growth g are all constant. All individuals are assumed to have a life expectancy of exactly T . They work during the first E periods and retire for the remaining time $T - E$. $N(s)$ denotes the number of individuals entering the labor force at time $t = s$. Therefore the size of the labor force at t is given by

$$L(t) = \int_{t-E}^t N(s) ds. \quad (1)$$

Likewise

$$P(t) = \int_{t-T}^{t-E} N(s) ds \quad (2)$$

will be retired. With m as the constant rate of population growth, the size of a cohort at time $t = s$ is

$$N(s) = N_0 e^{ms} \quad (3)$$

with $N_0 > 0$. Thus, in a steady state, the number of workers $L^*(t)$ and retirees $P^*(t)$ (steady state variables are denoted by a $*$) simplifies to

$$L^*(t) = \begin{cases} \frac{N_0}{m} (e^{mt} - e^{m(t-E)}) & \text{if } m \neq 0 \\ N_0 E & \text{if } m = 0 \end{cases}, \quad (4)$$

$$P^*(t) = \begin{cases} \frac{N_0}{m} (e^{m(t-E)} - e^{m(t-T)}) & \text{if } m \neq 0 \\ N_0 (T - E) & \text{if } m = 0 \end{cases}. \quad (5)$$

The dependency ratio q is defined as the number of retirees per worker. Hence, the steady state dependency ratio corresponds to

$$q^*(m, T, E) \equiv \frac{P^*(t)}{L^*(t)} = \begin{cases} \frac{e^{m(T-E)} - 1}{e^{mT} - e^{m(T-E)}} & \text{if } m \neq 0 \\ \frac{T-E}{E} & \text{if } m = 0 \end{cases}. \quad (6)$$

and is independent of t in a steady state. Since we are interested in the effects of changes in life expectancy and the rate of population growth, the following two

properties of the dependency ratio $q^*(m, T, E)$ will turn out to be important:

Lemma 1: *A higher life expectancy T leads to an increase in the steady state dependency ratio while an increase in the rate of population growth lowers the steady state dependency ratio.*

Proof: see Appendix.

Pay-as-you-go pension systems are characterized by a pension $x(t)$ which is assumed to be the same for all pensioners and a contribution rate $b(t)$. The balanced budget condition for the pension system therefore is

$$L(t)b(t)w(t) = P(t)x(t) \quad (7)$$

where $w(t)$ is the wage rate at time t which applies to all workers. On the basis of this balanced budget condition, we can discuss the properties of following two pay-as-you-go pension systems:

1. *Constant replacement rate (CRR)*

In a defined benefit PAYG system with a constant replacement rate \bar{n} , pensions are always a constant proportion of wages, i.e.

$$x(t) = \bar{n}w(t). \quad (8)$$

The balanced budget condition (7) and the definition of the dependency ratio imply

$$b(\bar{n}, t) = \bar{n}q, \quad (9)$$

Therefore, the contribution rate is constant in a steady state.

2. *Constant contribution rate (CCR)*

In a defined contribution PAYG system with a constant contribution rate \bar{b} , we can solve (7) and (8) for the replacement rate which yields

$$n(\bar{b}, t) = \frac{\bar{b}}{q}. \quad (10)$$

Thus the replacement rate is also constant in a steady state.

We measure the intergenerational distribution effects of changes in demographic variables by the implicit taxes $\mathcal{T}(t)$ of the individuals born at time t . These correspond to the difference between the present value of contributions and the present value of pension payments. Given a steady state interest rate of r , the implicit taxes are therefore defined by

$$\mathcal{T}(t) \equiv \int_t^{t+E} e^{-r(s-t)} b(s) w(s) ds - \int_{E+t}^{T+t} e^{-r(s-t)} x(s) ds. \quad (11)$$

Under PAYG-pension systems with a constant replacement rate or a constant contribution rate, b and n are constant in a steady state and (11) simplifies to

$$\mathcal{T}(t) = b \int_t^{E+t} e^{-r(s-t)} w(s) ds - n \int_{E+t}^{T+t} e^{-r(s-t)} w(s) ds. \quad (12)$$

The implicit tax rate $\tau(t)$ is defined as the share of the present value of life time earnings paid as implicit taxes, i.e.

$$\tau(t) \equiv \frac{\mathcal{T}(t)}{\int_t^{E+t} e^{-r(s-t)} w(s) ds} = b - n \frac{\int_{E+t}^{T+t} e^{-r(s-t)} w(s) ds}{\int_t^{E+t} e^{-r(s-t)} w(s) ds}. \quad (13)$$

We assume that in our steady state economy, productivity and wages grow at a constant rate g . Equation (13) can thus be simplified to

$$\tau^* = b - nu(r, g, T, E) \quad (14)$$

where

$$u(r, g, T, E) \equiv \begin{cases} \frac{e^{(r-g)(T-E)} - 1}{e^{(r-g)T} - e^{(r-g)(T-E)}} & \text{if } r \neq g \\ \frac{T-E}{E} & \text{if } r = g \end{cases}. \quad (15)$$

Since the function $u(r, g, T, E)$ is constant in a steady state, we obtain:

Proposition 1: *The implicit tax rate under either CRR and CCR is constant in a steady state.*

Furthermore, we can show

Proposition 2: *In a steady state, the implicit tax rate is positive under a constant replacement rate and under a constant contribution rate PAYG system if and only if $r > m + g$.*

Proof: see Appendix.

Proposition 2 replicates a well-known result which was first shown by Aaron (1966). $m + g$ is the internal rate of return of a PAYG system with a constant replacement and contribution rate. If $r > m + g$, then this rate of return is lower than the rate of return on the capital market and individuals pay implicit taxes to the pension system. This condition corresponds to a situation of a dynamically efficient economy. The empirical study by Abel (1989) *et al.* supports the hypothesis that it was satisfied for major industrialized countries in the past decades.

Breyer (1989) has shown that a PAYG system is Pareto-efficient if $r > m + g$.⁵ His result has been extended by Sinn (2000) who demonstrates that the PAYG system is a zero-sum game between generations if $r > m + g$. In this model, this implies that the implicit tax rate of one generation cannot be lowered without increasing the implicit tax rate of another generation. A positive implicit tax rate is therefore due to the existence of earlier generations who participated in the PAYG system and obtained an implicit subsidy. These transfers to earlier generations created an implicit public debt and the implicit taxes finance the implicit interest payments on this debt.

Proposition 1 shows that CRR and CCR smooth the implicit tax rate in a steady state. However, the following sections show that this is not the case if life expectancy or the rate of population growth changes. Therefore, we also characterize policy measures which try to keep the implicit tax rate constant besides analyzing the effects on implicit taxes under CRR and CCR. We do so for the empirically relevant case $r > m + g$.

⁵Similar to this model, Breyer assumes that contributions have a lump-sum character. Brunner (1994, 1996) and Fenge (1995) extend his result to contributions financed by a distortionary wage tax.

3 An increase in life expectancy

In this section, we analyze the effects of changes in life expectancy T depending on the policy regime. First, we determine the steady state effects. Second, we use numerical simulations to show the time path of the endogenous variables until the new steady state is reached. In addition, we determine how b and n would have to change if τ is to remain constant in the new steady state and discuss three policies which keep the implicit tax rate constant in all periods.

3.1 Constant replacement rate

If the replacement rate \bar{n} is fixed, then an increase in life expectancy will increase both the contribution rate and the implicit tax rate in the steady state:

Proposition 3.1: *Under a constant replacement rate \bar{n} , an increase in life expectancy leads to an increase in the steady state contribution rate $b^*(\bar{n})$. If $r > m + g$, then the steady state implicit tax rate $\tau^*(\bar{n})$ rises.*

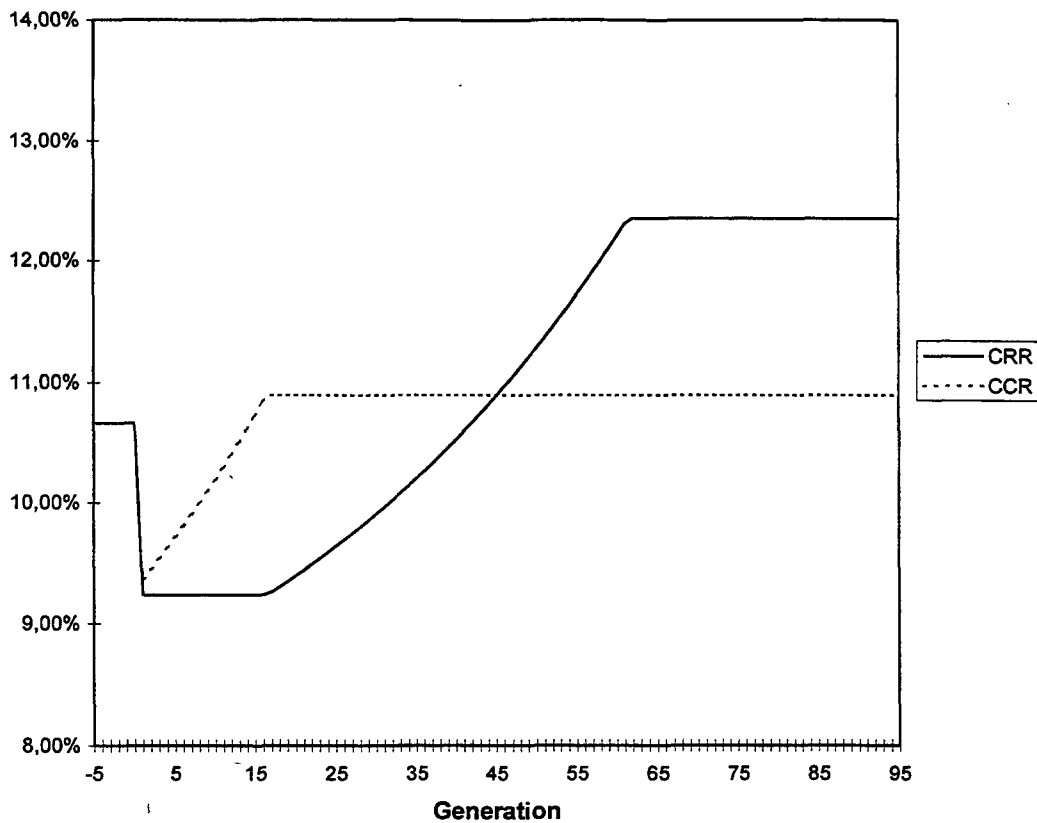
Proof: see Appendix.

The increase in the contribution rate is explained by a higher dependency ratio due to the increase in T . Under a constant replacement rate the expenditure of the pension system is therefore higher compared to a lower life expectancy. The contributions to the pension system and hence the contribution rate must be increased in order to guarantee a balanced budget of the pension system. The implicit tax rate therefore must rise as well because individuals invest more into a system with a lower yield than the interest rate.

Since the PAYG system is a zero-sum game, the steady state increase in the implicit tax rate implies that in the transition period some generations must be better off. These are the first generations who live longer. Under CRR, their implicit tax rate falls compared to those generations who live before as they face the low contribution rate under the old steady state but receive a pension for longer time. This is illustrated in Figure 1 which shows the results of a numerical simulation.⁶

⁶For the numerical simulation we use a discrete-time version of our model.

Fig. 1: Evolution of the implicit tax rate if life expectancy increases



We have chosen the parameters in such a way that they roughly describe the German case. Individuals born before $t = 1$ have a life expectancy of 60 periods while those born at $t = 1$ and after live for 62 periods. All generations work for 45 periods. As in the study for the Bundesministerium für Wirtschaft (1998) we assume that productivity growth per period equals $g = 2\%$ and the interest rate per period is $r = 4\%$. The rate of population growth per period is $m = 0\%$. The replacement level is set at $\bar{n} = 70\%$. This implies that the dependency ratio equals 33.33% in the old steady state. It rises after period 60 and reaches its new steady state value of 37.78% in period 62. The contribution rate follows the evolution of the dependency ratio and increases from 23.33% in period 60 to 26.44% in period 62.

Note first that all generations born from $t = 1$ until $t = 17$ face the same implicit tax rate which is lower than in the old steady state. They all pay the lower contribution rate of the old steady state and receive a pension for two more periods. Later-born generations, however, already have to pay the new higher contribution rate during part of their life. Hence their implicit tax rate is higher. For those generations born at $t = 42$ the implicit tax rate is already higher than in the old steady state. Generations born in $t = 62$ and after pay the new steady-state contribution rate during all of their life. Their implicit tax rate therefore reaches the new steady state level of 12.35%.

3.2 Constant contribution rate

A policy which keeps the contribution rate fixed in a pure PAYG system is not sufficient to keep the implicit tax rate constant:

Proposition 3.2: *Under a constant contribution rate \bar{b} , an increase in life expectancy leads to a decrease in the steady state replacement rate $n^*(\bar{b})$. If $r > m + g$, then the steady state implicit tax rate $\tau^*(\bar{b})$ increases.*

Proof: see Appendix

A constant contribution rate leaves the gross contributions to the PAYG system unchanged. Thus, as the dependency ratio increases due to a higher life expectancy, the replacement rate has to go down. There are two opposing effects. While the decrease in pensions leads to higher implicit taxes, the longer time-period in which pensions are received is favorable for the generations living longer. However, the first effect is dominant. The intuition is that an annuity is stretched in a system with a lower return than on the capital market. The lower annuity due to the cut in the replacement rate is simply a further investment in the PAYG system which yields the pension payments during the additional life expectancy. Thus, additional resources are invested in an asset with lower yield than the interest rate which leads to higher implicit taxes and thus a higher implicit tax rate.

Again, the steady state increase in the implicit tax rate implies that in the transition period some generations are better off. As under CRR, the first generations who live longer experience a fall in implicit taxes. They still receive high pensions during the first years of their retirement. Only as the first generations who live longer reach the former maximum life time, the dependency ratio begins to rise and pensions go down. Nevertheless, these generations still receive a pension in their additional time expectancy which lowers their implicit taxes. Consequently, later generations have to pay higher implicit taxes.

This is illustrated in Figure 1. The dotted line shows the implicit taxes under CCR for the same parameters as above. The contribution is set at $\bar{b} = 23.33\%$ which leads to a replacement rate of 70% in the initial steady state. Compared to CRR, the gains of the first generations are smaller because they receive a cut in pensions which leads to a lower long-run increase in the steady state implicit tax rate under CCR.⁷ In addition, the transition is faster than under CRR. Already generations born at $t = 15$ face a higher implicit tax rate than in the old steady state. The new steady state implicit tax rate of 10.89% is reached for the generation born in

⁷This can be shown formally. For the old steady state we set $\bar{b} = \bar{n}q^*$. Substituting in (A.7)

$t = 17$. This is the first generation for whom the dependency ratio has reached its new level during all of their retirement period and therefore only receives pensions under the new steady state replacement rate.

3.3 Constant implicit tax rate

Under both PAYG systems, the increase in life expectancy leads to a higher implicit tax rate. If it is to remain constant in the new steady state, then the PAYG system needs further adjustment:

Proposition 3.3: *If $r > m + g$ and the implicit tax rate is to be constant in the steady state before and after an increase in life expectancy, then both the replacement rate and the contribution rate have to fall.*

Proof: see Appendix.

This result can be explained by the annuity effect in the last section. An increase in life expectancy implies a longer time in which individuals invest in the PAYG system in form of lower annuities and therefore leads to an increase in implicit taxes and thus in the implicit tax rate even if the investment in form of wage contributions is kept constant. Thus, to keep the implicit tax rate constant, contributions must be decreased to compensate for the extra investment due to the annuity effect. This implies that the contribution rate has to fall.

Proposition 3.3 only tells us how the contribution rate and the replacement rate have to change to keep the implicit tax rate constant in a steady state. Therefore, we now turn to the question how the implicit tax rate can be kept constant in the transition period as well. It turns out that this problem is not trivial. In particular, any policy which smooths the implicit tax rate must rely on generation-specific contribution or replacement rates. If the generations with a higher life expectancy face the same contribution and replacement rate as in the old steady state, then their implicit tax rate would be lower. Furthermore, the introduction of funded elements might be necessary if the contribution and replacement rates which smooth

yields

$$\left. \frac{\partial \tau^*}{\partial T} \right|_{b=\bar{b}} = \bar{n} \left(\frac{\partial q}{\partial T} \frac{u}{q^*} - \frac{\partial u}{\partial T} \right).$$

$r > m + g$ implies that $u < q^*$. Comparing with (A.5) we obtain

$$\left. \frac{\partial \tau^*}{\partial T} \right|_{b=\bar{b}} < \bar{n} \left(\frac{\partial q^*}{\partial T} - \frac{\partial u}{\partial T} \right) = \left. \frac{\partial \tau^*}{\partial T} \right|_{n=\bar{n}}$$

the implicit tax rate are not compatible with a balanced budget of the pension system.

We discuss three policies which keep the implicit tax rate constant for all generations after a change in life expectancy and illustrate our results with numerical simulations. The working period E is regarded as given.⁸ The parameters are the same as in Figure 1 unless different values are stated. The implicit tax rate is stabilized at its initial value of 10.66%. The first policy smoothes the implicit tax rate within a *pure* PAYG system, i.e. without funded elements. Under the second policy, there is a permanent capital stock while the third policy relies on a temporary deficit.

1. A pure PAYG solution with generation-specific replacement rates

Upon retirement, the contributions to the pension system of each generation can be determined. A simple way to keep the implicit tax rate constant is to calculate the replacement rate for each generation in such a way that the implicit tax rate reaches the exogenously given level of $\bar{\tau}$.⁹ Thus, the generation born at $t = 1$ (generation 1) will experience a cut in their replacement rate at the time of their retirement. This has feedback effects on the contribution rate which will fall. Therefore the replacement rate of the next generation must be cut as well causing a further fall in the contribution rate.

⁸An alternative way to compensate for an increased life expectancy is to increase E . Breyer *et al.* (1997) show in a similar model that changing E can lead to peculiar intergenerational redistribution effects. Analyzing a pension system with a constant replacement rate, they demonstrate that an increase in E combined with actuarial compensation for generations who work longer makes earlier-born generations who work longer better off at the cost of later-born generations. On the basis of this model, Breyer and Kifmann (1999) advocate the introduction of funded elements to smooth the implicit tax rate. The idea basically corresponds to the second policy we examine.

⁹Allowing for varying contribution rates and generation-specific replacement rates $n(t)$, the implicit tax rate is given by

$$\bar{\tau} = \frac{\int_t^{t+E} e^{-r(s-t)} b(s) w(s) ds - n(t) \int_{E+t}^{T+t} e^{-r(s-t)} w(s) ds}{\int_t^{E+t} e^{-r(s-t)} w(s) ds}$$

(see (11) - (13)). Solving for $n(t)$ yields

$$n(t) = \frac{\int_t^{t+E} e^{-r(s-t)} b(s) w(s) ds - \bar{\tau} \int_t^{E+t} e^{-r(s-t)} w(s) ds}{\int_{E+t}^{T+t} e^{-r(s-t)} w(s) ds}$$

This development will only come to a stop when generation 1 exceeds the old maximum life expectancy of $T = 60$. Only then the dependency ratio rises which leads to an increase in the expenditure of the pension system. This causes a sharp increase in the contribution rate.

Figures 2 and 3 show the evolution of the contribution rate for two numerical simulations.¹⁰ The straight line shows the new steady state rate for a constant implicit tax rate. In both cases, the contribution rate falls at $t = 46$, the period in which generation 1 retires and rises sharply in $t = 61$ when higher life expectancy has its first effect on the expenditure of the pension system. Then both simulations show further fluctuations of the contribution rate. These cycles are due to the first fluctuation of the contribution rate which leads to different contributions for each generation. For generations who face mostly low contribution rates, the replacement rate must therefore be low, while for generations confronted with high contribution rates, the replacement rates must be high. The balanced budget condition implies that the contribution rate will also fluctuate. When generations with high (low) replacement rates are retired, then the contribution rate is high (low). This gives rise to cycles whose length is 45 periods which corresponds to the time which each generation works. Whether the contribution rate converges to a steady state or not depends on the parameters. In Figure 2, the contribution rate fluctuations become smaller. In Figure 3, however, the cyclical behavior intensifies. This makes this option unattractive since eventually the contribution rate falls below 0% or rises above 100% which will be difficult to sustain.

2. *Constant contribution rate, generation-specific replacement rates and permanent additional funding*

The second idea to smooth the implicit tax rate fixes the contribution rate at the old steady state value. Similar to the first proposal, the replacement rate of the generations who live longer is calculated such that the implicit tax rate remains constant and therefore is lower than the old steady state replacement rate. Since the contribution rate is constant, however, the pension system makes a surplus when the first generations with a higher life expectancy retire. This surplus increases as more generations with a low replacement rate retire and replace generations with a high replacement rate. These surpluses are invested in the capital market. A fund is built up to finance the pensions for the additional life-time of the generations who live longer.

¹⁰In Figure 2, the parameters are as in Figure 1. Figure 3 shows the results for $r = 5\%$, $g = 0\%$ and $m = 0\%$.

Fig. 2: Transition in a pure PAYG system, $r = 4\%$

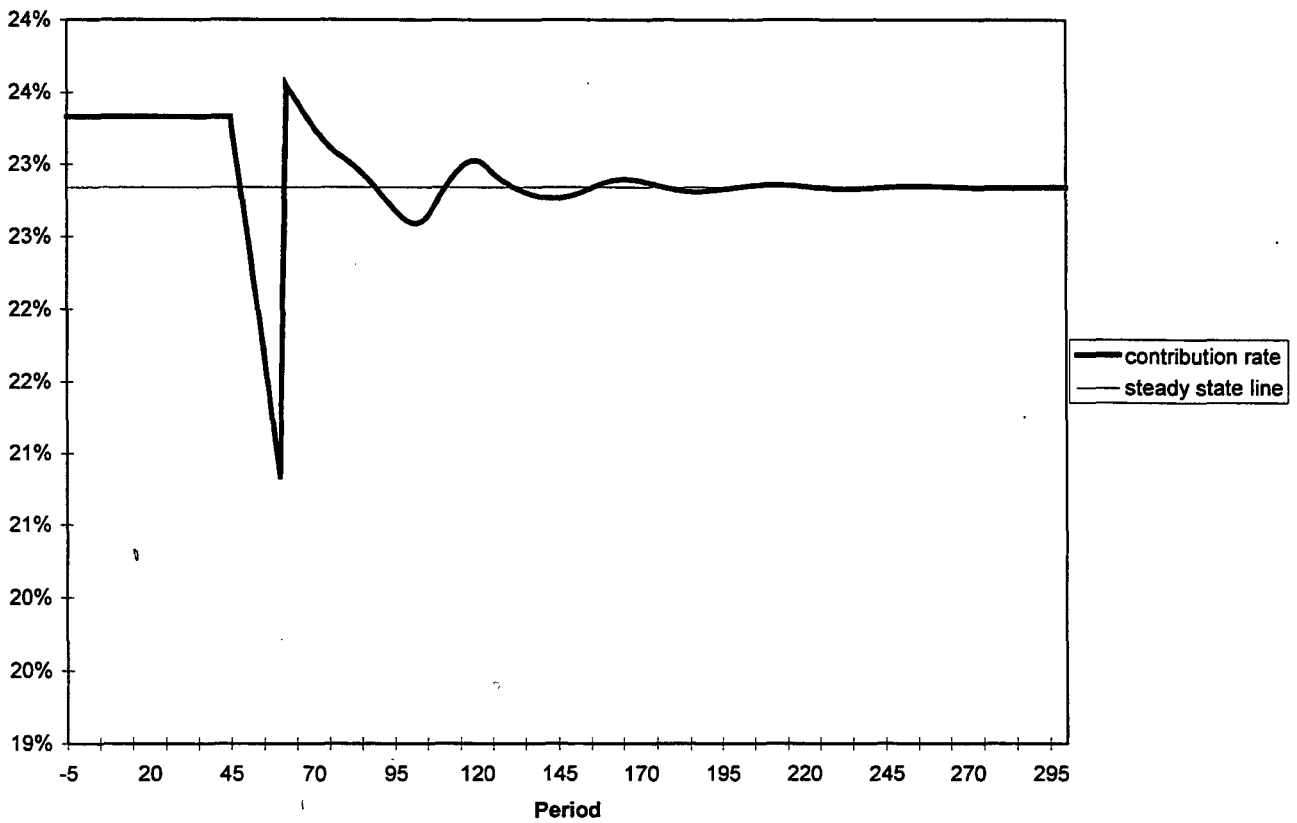


Fig. 3: Transition in a pure PAYG system, $r = 7\%$

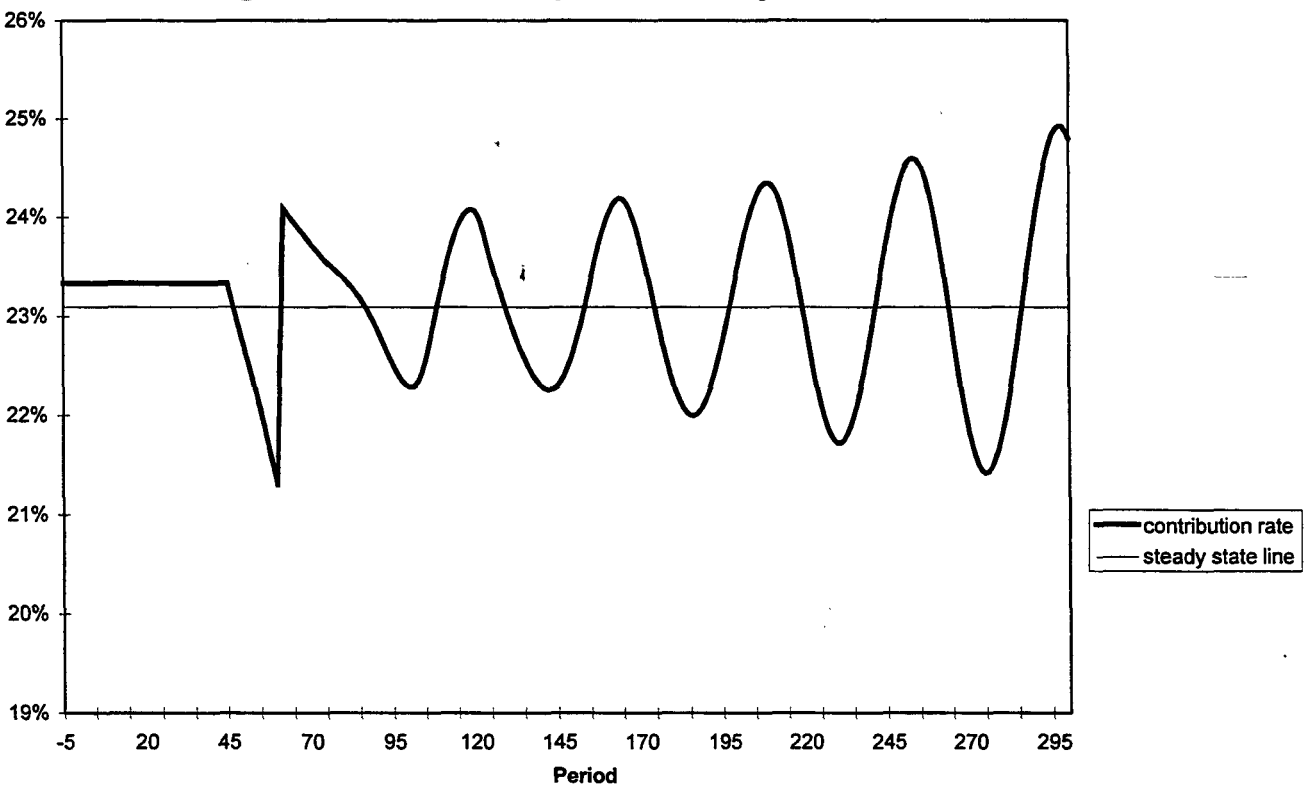
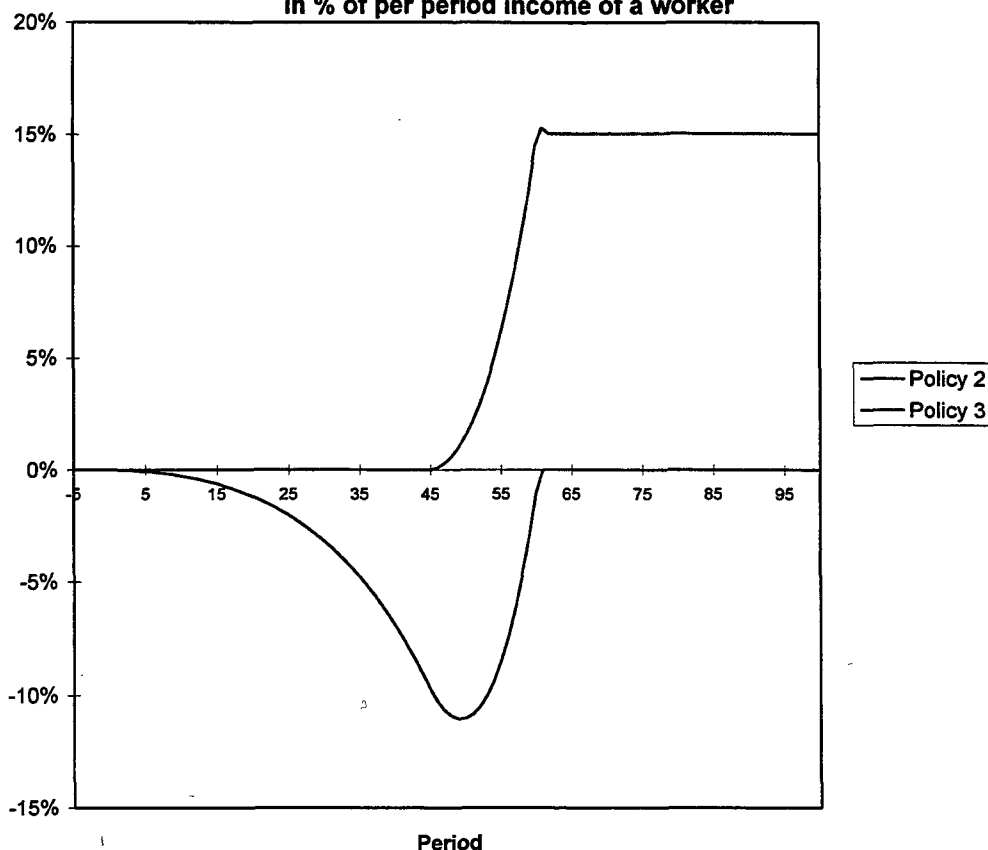


Fig. 4: Fund per capita under funded smoothing
in % of per period income of a worker



The fund grows at a high rate until the first generation reaches an age above the old life expectancy. Then the dependency ratio increases and the fund is used to pay for the pensions for the additional life-time. The surplus of the pension system falls and the fund grows at a lower rate. In the new steady state, the fund grows at rate g since pensions grow at this rate under the new constant replacement rate.

In Figure 4 the upper line shows the resulting capital stock as a percentage of per period wage income of a worker. As the first long-living generation enters retirement age in $t = 46$, the first savings are accumulated. Until $t = 60$, further savings⁴ increase the capital stock at a high rate. In $t = 61$, there is the first withdrawal as the pension of the first generation has to be financed out of savings. In $t = 62$, the build-up phase of the system is finished. From then on, the capital stock grows at the same rate as wages because pensions grow at this rate, too. Consequently, the capital stock as a percentage of per period wage income of a worker remains constant.

3. *Generation-specific contribution and replacement rates accompanied by a transitional deficit*

In our steady state model, we can determine the contribution and the replacement rate which keep the implicit tax rate constant in the new steady state. If all individuals with a higher life expectancy pay this contribution rate and receive pensions according to the new replacement rate, then their implicit tax rate is as in the old steady state. This also applies to the older generations with the lower life expectancy with respect to the old steady state values of b and n . Therefore, generation-specific contribution and replacement rates can smooth the implicit tax rate.

However, a deficit is created as soon as the younger generations enter the system because the pension system's income falls while expenditure remain unchanged. This is shown by the lower curve in Figure 4 which shows the aggregate deficit including interest payments as a percentage of per period wage income of a worker in the economy. In each period there is a deficit until the generations with longer life expectancy and hence lower pensions enter the retirement age. From $t = 47$ on, there are surpluses in each period because the first generation with a low replacement rate receives a pension and because the dependency ratio has not yet reached its new equilibrium value until generation 1 dies. In $t = 62$, the dependency ratio has reached its new steady state value and there are no more surpluses or deficits in the pension system because everybody pays the new contribution rate and receives pensions according to the new replacement rate. In addition, the previous surpluses are just enough to pay for the aggregate deficit plus interest, i.e. the aggregate deficit is zero in the new steady state. This result is a logical consequence of this policy of generation-specific contribution and replacement rates: No generations has been made better off due to demographic change. Therefore no deficit or surplus remains after the transition phase.

Thus, generation-specific contribution and replacement rates combined with a temporary deficit can smooth the implicit tax rate. But whereas the former two policies only require adaptations when the generations with higher life expectancy enter the retirement age, this policy has to treat these generations differently already upon entering the labor force.

To sum up, we have shown that the implicit tax rate can be perfectly smoothed upon an increase in life expectancy. Whatever system is used, generation-specific contribution or replacement rates are necessary to smooth the implicit tax rate. Thus, there is a strong argument for a demographic factor in either of these parameters. Partial funding is not necessary but desirable because the contribution

rate may fluctuate strongly in a pure PAYG system which can even lead to negative contribution rates or contribution rates above 100%.

Systems which introduce funded elements allow stable contribution and replacement rates. For example, a permanent capital stock can be built up to finance the pension of those individuals with a higher life expectancy. However, building up a fund is not a necessary condition for a constant implicit tax rate. The implicit tax rate can also be smoothed with a temporary deficit and a lower contribution rate for the generations with a higher life expectancy.

4 A decline in the rate of population growth

The second major demographic change we analyze is a decline in the rate of population growth m . Again, we examine the consequences of this demographic change under the different pension systems and consider how the implicit tax rate can be smoothed.

4.1 Constant replacement rate

Under CRR, the fall in m has the same effects as the increase in life expectancy:

Proposition 4.1: *Under a constant replacement rate \bar{n} , a fall in the rate of population growth leads to an increase in the steady state contribution rate $b^*(\bar{n})$ and the steady state implicit tax rate $\tau^*(\bar{n})$.*

This result follows immediately from equations (9) and (A.3) and Lemma 1:

$$\left. \frac{\partial b^*}{\partial m} \right|_{n=\bar{n}} = \bar{n} \frac{\partial q^*}{\partial m} < 0 \quad (16)$$

$$\left. \frac{\partial \tau}{\partial m} \right|_{n=\bar{n}} = \bar{n} \frac{\partial q^*}{\partial m} < 0. \quad (17)$$

The rise of the contribution rate is a consequence of the increase in the dependency ratio. Since benefits remain unchanged, this implies that the implicit tax rate must also increase. This result is due to the decline of $m + g$, the internal rate of return in the PAYG-pension system, as m falls. Note that Proposition 4.1 does not depend on $r > m + g$ since a fall in m generally lowers the rentability of the pension system.

Fig. 5: The implicit tax rate if the rate of population growth declines

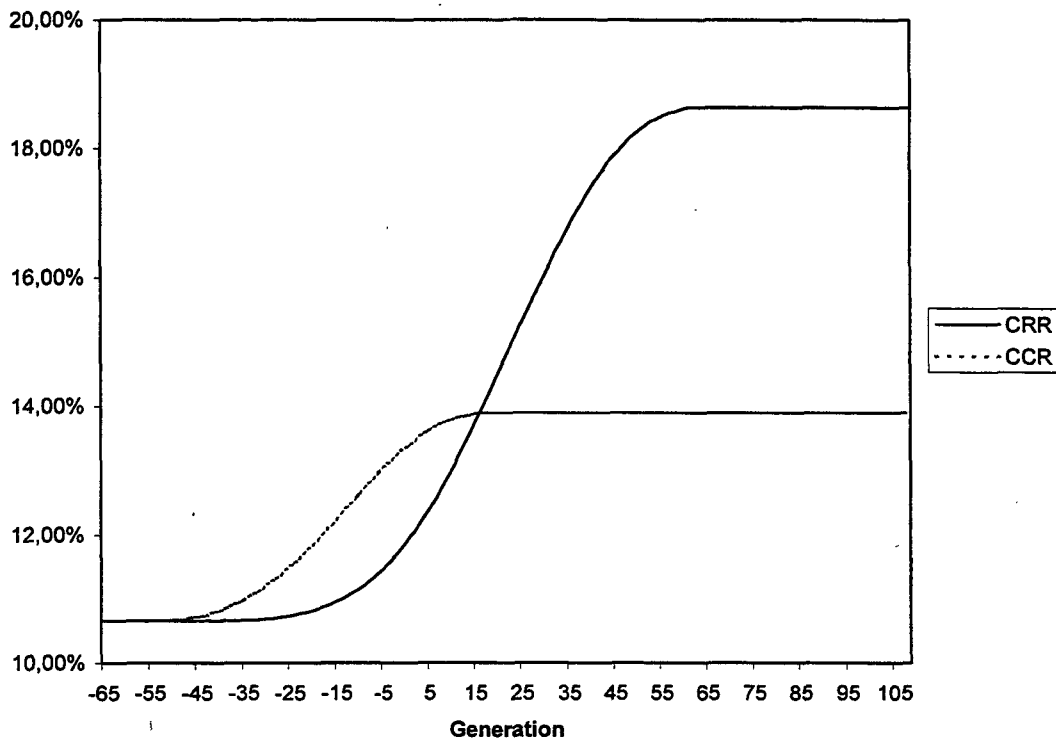


Figure 5 illustrates the evolution of the implicit tax rate when generation 1 is the first generation after the drop in m . The replacement rate is $n = 70\%$. Furthermore, $r = 4\%$, $g = 2\%$ per period and $T = 60$. The rate of population growth per period drops from 0% to -1% as generation 1 is born. This implies that the dependency ratio equals 33.33% in the old steady state. It rises after period 1 and reaches its new steady state value of 44.72% in period 60. As soon as the dependency ratio increases, the contribution rate rises, too. The generation born at $t = -43$ is therefore the youngest generation experiencing an increase in the contribution rate and the implicit tax rate. As more and more generations born after $t = 1$ join the system, the contribution rate and hence implicit taxes further increase until both reach their new steady state value in $t = 60$ when only generations born after the decrease in m are alive. The new steady state implicit tax rate is 18.64% . The time profile shows that all generations are worse off by the fall in m as the implicit burden of the PAYG system is shared by a smaller number of individuals.

4.2 Constant contribution rate

As above, a policy which keeps the contribution rate constant in a pure PAYG system is not able to stabilize the implicit tax rate:

Proposition 4.2: *Under a constant contribution rate \bar{b} , a fall in the rate of population growth lowers the steady state replacement rate $n^*(\bar{b})$ and increases the steady state implicit tax rate $\tau^*(\bar{b})$.*

This result follows from equations (10) and (A.4) and Lemma 1:

$$\left. \frac{\partial n^*}{\partial m} \right|_{b=\bar{b}} = -\frac{\bar{b}}{q^{*2}} \frac{\partial q^*}{\partial m} > 0 \quad (18)$$

$$\left. \frac{\partial \tau}{\partial m} \right|_{b=\bar{b}} = \bar{b} \frac{u}{q^{*2}} \frac{\partial q^*}{\partial m} < 0. \quad (19)$$

The higher dependency ratio leads to a lower replacement rate if the contribution rate is fixed. The benefits of the PAYG system are lower while contributions remain unchanged. Thus, the implicit tax rate increases. This is demonstrated in Figure 5 which shows the evolution of the implicit tax rate for a constant contribution rate of $\bar{b} = 23.33\%$. Again, we observe a permanent increase in the implicit tax rate. Compared with CRR, the increase starts earlier but is less pronounced. On the one hand, already generation -58 faces a higher implicit tax rate. This is the first generation who experiences a cut in the replacement rate as generation 1 enters the labor force which leads to an increase in the dependency ratio. Under CRR, generation -43 was the first generation affected. On the other hand, the new steady state implicit tax rate reached in period 16 is only 13.89% and therefore much smaller than under CRR.¹¹

¹¹The smaller steady state increase in the implicit tax rate under CCR is a general result for $r > m + g$. Setting $\bar{b} = \bar{n}q^*$ and using (17) and (19) we obtain

$$\begin{aligned} \left. \frac{\partial \tau^*}{\partial m} \right|_{b=\bar{b}} &= \bar{n} \frac{u}{q^*} \frac{\partial q^*}{\partial m} \\ &= \frac{u}{q^*} \left. \frac{\partial \tau^*}{\partial m} \right|_{n=\bar{n}} \end{aligned}$$

Due to $r > m + g$, we have $u < q^*$ and therefore

$$\left. \frac{\partial \tau^*}{\partial m} \right|_{b=\bar{b}} < \left. \frac{\partial \tau^*}{\partial m} \right|_{n=\bar{n}}$$

4.3 Constant implicit tax rate

As above both the contribution and the replacement rate need to be adjusted to keep the implicit tax rate constant in the new steady state:

Proposition 4.3: *If the implicit tax rate is to be constant in the steady state before and after a fall in the rate of population growth, then both the replacement rate and the contribution rate have to fall.*

This result follows from equations (A.8) and (A.9) and Lemma 1:

$$\left. \frac{\partial n^*}{\partial m} \right|_{\tau=\bar{\tau}} = -\frac{\bar{\tau}}{(q^* - u)^2} \frac{\partial q^*}{\partial m} > 0$$
$$\left. \frac{\partial b^*}{\partial m} \right|_{\tau=\bar{\tau}} = -\frac{\frac{\partial q^*}{\partial m} u}{(q^* - u)^2} \bar{\tau} > 0.$$

This result is due to the fact that a decline in m lowers the rate of return of the PAYG system. Therefore, the implicit tax rate can only be kept constant in the new steady state if the amount invested in the PAYG system is reduced. This implies that the contribution rate has to fall.

The decline in the rate of return of the PAYG system also has another straightforward implication. It is not possible to keep the implicit tax rate constant for all generations as in the case of rising life expectancy in which $m + g$ remains constant. For example, if the contribution or replacement rate are kept constant, then the implicit tax rate constantly increases until it reaches its new steady state value. Since later-born generations face the highest increases in the implicit tax rate under these policies, we look at alternatives which use funded elements to smooth the implicit tax rate by transferring a part of the inevitable burden to earlier-born generations.

4.4 Smoothing the implicit tax rate

We examine two basic and non-exclusive approaches to smooth the implicit tax rate. Both use a temporary fund to lower the new steady state contribution rate of the “smaller” generations. This allows to reduce their implicit tax rate compared to a policy which keeps the contribution rate constant. The generation-specific contribution rate applies for the smaller generations as soon as they enter the labor force. The resulting deficit is paid by a surcharge on the “big” generations. Under the first approach, an additional contribution of the big generations is collected and saved. Under the second approach, a part of the pensions of earlier generations is retained and saved. Savings are essential under both approaches because otherwise a fall in the contribution rate would also be to the benefit of earlier generations.

Of course, both approaches can be combined, i.e. for big generations the contribution rate can be increased and the replacement rate can be cut. In our numerical simulation we distinguish between the approaches to point out the different effects of both policies.

1. *An additional contribution for earlier generations and generation-specific contribution rates*

In our numerical simulation, the old steady state value of the implicit tax rate is $\tau_0^* = 10.66\%$. If the contribution rate is kept constant at $\bar{b}_0 = 23.33\%$, the new steady state value is $\tau_b^* = 13.89\%$. To decrease the new steady state implicit tax rate, all big generations born in $t = -43$ or later pay a surcharge of 3.27% on their income which is used to build up a temporary fund. In addition, they continue to pay the old steady state contribution rate and receive pensions according to the old replacement rate of $n_0 = 70\%$. Small generations face a contribution rate of $\bar{b}_1 = 23.26\%$ and have a replacement rate of $n_1 = 52\%$ which leads to a balanced budget of the pension system in the new steady state with an implicit tax rate $\tau_1^* = 13.84\%$. This is below the one under CCR. Since the contribution rate is lowered for small generations, a temporary deficit of the PAYG pension system is created until the new steady state. The present value of the surcharge on big generations covers exactly the present value of these deficits. If the surcharge would exceed 3.27% of income, then the contribution rate and the implicit tax rate could furthermore be reduced.¹²

¹²A general increase of the surcharge might increase the implicit tax rate of the latest-born big generations above the new steady state implicit tax rate. To avoid this, the surcharge would have to be the higher, the closer a big generation is to retirement.

Fig. 6: Smoothing the implicit tax rate if the rate of population growth declines

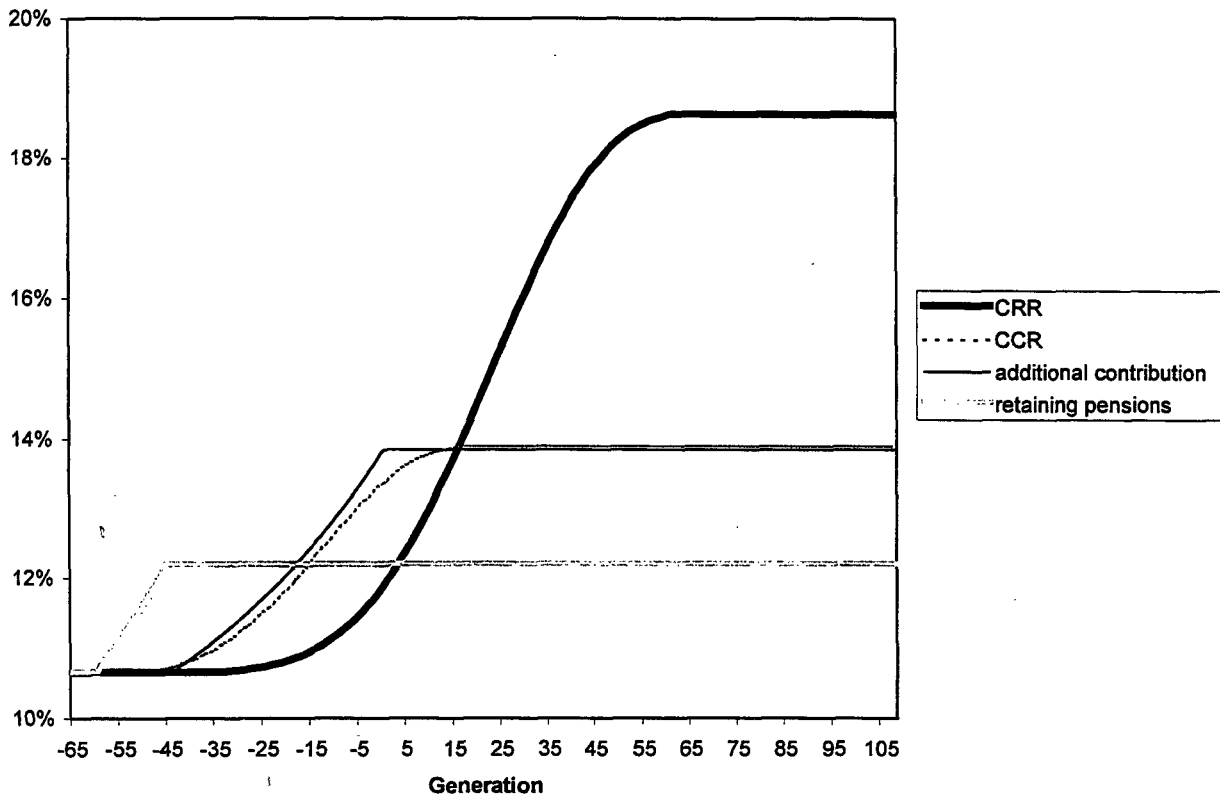
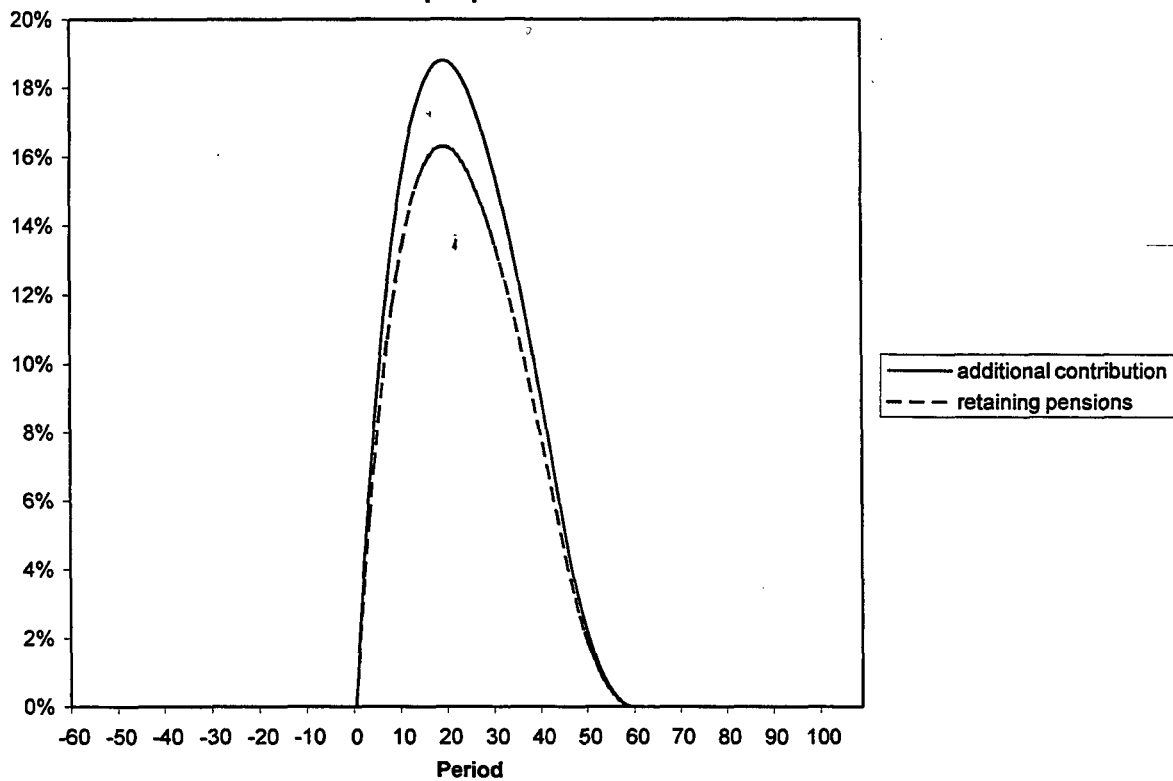


Fig. 7: Fund per capita under funded smoothing in % of per period income of a worker



The evolution of the implicit tax rate under this policy is shown in Figure 6. Compared with CRR which also keeps the replacement rate constant for big generations, the surcharges shift the burden to earlier generations. Compared with CCR, however, the implicit tax rate in the new steady state is only slightly smaller compared to CCR because this approach does not cut the replacement rate for big generations. Figure 7 demonstrates the size of the resulting capital stock as a percentage of per period wage income of a worker. In the beginning there is a build-up phase as only savings are collected. As more big generations retire, the increasing dependency ratio and the lower contributions of small generations are not sufficient anymore to cover pension expenditure and the capital stock shrinks. At $t = 60$, when only small generations are alive, the capital stock is completely used because the new steady state contribution rate has been chosen appropriately.

2. *Retaining a fraction of pensions of earlier generations and generation-specific contribution rates*

The second alternative also introduces generation-specific contribution and replacement rates but finances the resulting deficit by the retention of pensions. The old steady state replacement rate is 70%. Lowering the replacement rate for big generations to 61.52% and saving the difference builds up a capital stock. If the new contribution rate for the small generations is set at $\bar{b}_2 = 20.50\%$ with a corresponding replacement rate of $n_2 = 45.83\%$ which leads to a balanced budget of the pension system in the new steady state. The temporary deficits of the PAYG system are fully financed by the savings and the capital stock vanishes in $t = 60$ (see Figure 7). This is the first period in which only small generations are alive. Figure 6 illustrates the evolution of the implicit tax rate. Compared with the other policy, already generations born at $t = -58$ participate in sharing the burden of the fall in m which allows to reduce the new implicit tax rate considerably. In the new steady state the implicit tax rate is only $\tau_2^* = 12.20\%$.

Thus, collecting additional contributions and / or retaining pensions and saving the proceeds to lower the contribution and replacement rate for small generations allows to shift forward the inevitable burden caused by the decline in m . Retaining a fraction of pensions is most effective in lowering the implicit tax rate of later-born generations. However, it may not be possible to cut replacement rates below a certain level as retirees have no means to compensate for lower pensions. In this respect, collecting an additional contribution has an advantage because individuals can change their private savings for old age. In both cases, funded elements and generation-specific contribution and rates are necessary. On the one hand, savings

are needed to transfer resources to the smaller generations. On the other hand, only a demographic factor in the contribution and replacement rates can decrease the implicit tax rate in the new steady state.

5 Conclusion

For a pay-as-you-go pension system in a small open economy in which the interest rate is higher than the sum of the rates of population and productivity growth, we have analyzed how the implicit tax rate of a PAYG pension system responds to an increase in life expectancy and a fall in the rate of population growth. Both demographic changes lead to a higher steady state implicit tax rate if either the replacement rate or the contribution rate is kept constant. In addition, we have examined how the implicit tax rate can be smoothed under both demographic changes. Our main results are

- Generation-specific contribution or replacement rates are necessary to smooth the implicit tax rate.
- If life expectancy increases, then the implicit tax rate can in principle be kept constant. However, if the rate of population growth declines, then the implicit tax rate must rise for some generations.
- If life expectancy increases, then the implicit tax rate can be smoothed without funded elements. However, the contribution rate fluctuates and may not converge to the new steady state level. If a transitory deficit or a permanent capital stock is used, then the implicit tax rate can be smoothed with stable contribution and replacement rates.
- If the rate of population growth declines, then funded elements are essential to smooth the implicit tax rate.

Thus, the paper makes a strong case for the use of funded elements and a demographic factor in the contribution rate or the replacement rate if the smoothing of the implicit tax rate is regarded as desirable. Of course, in practice the fine tuning of these instruments will be difficult. However, it is important to note that generally both, generation-specific contribution or replacement rates and partial funding, are needed to smooth the implicit tax rate. This also implies that the current reform proposals are *a priori* unable to smooth the implicit tax rate because they rely on contribution and replacement rates which apply to all generations.

Finally, we want to point to some limitations of our analysis. First, our results have been derived for a small open economy for which the interest rate and the

rate of productivity growth were assumed to be constant. Further research might examine the consequences of relaxing this assumption. For example, our model could be used to perform a sensitivity analysis with respect to changes in the interest rate and the rate of productivity growth. In addition, these variables could be endogenized in a general equilibrium model. In such a framework, it could also be examined how differences in implicit tax rates influence investment into private pension plans. Finally, we have focused on a one-time change in the demographic variables. Future work could examine the implications of permanent changes in the demographic variables. This is particularly interesting for life expectancy which is still rising according to the United Nations population projections.

6 References

- Aaron, H.J. (1966). "The Social Insurance Paradox," *Canadian Journal of Economics and Political Science* 33, 371 - 374.
- Abel, A., N.G. Mankiw, L. Summers and R. Zeckhauser (1989). "Assessing Dynamic Efficiency: Theory and Evidence," *Review of Economic Studies* 56, 1 - 19.
- Blanchet, D. and D. Kessler (1991). "Optimal Pension Funding with Demographic Instability and Endogenous Returns on Investment," *Journal of Population Economics* 4, 137 - 54.
- Bonin, H. and K. Feist (1999). "Pension Reform in Germany and Redistribution between Living Generations," *Diskussionsbeiträge des Instituts für Finanzwissenschaft der Albert-Ludwigs-Universität Freiburg im Breisgau* Nr. 78/99.
- Breyer, F. (1989). "On the Intergenerational Pareto Efficiency of Pay-as-You-Go financed Pension Systems," *Journal of Institutional and Theoretical Economics* 145, 643 - 58.
- Breyer, F., M. Kifmann and K. Stolte (1997). "Rentenzugangsalter und Beitragsatz zur Rentenversicherung," *FinanzArchiv* 54, 187 - 202.
- Breyer, F. and M. Kifmann (1999). "Erhöhung der Regelaltersgrenze oder Kürzung des Rentenniveaus," *Wirtschaftsdienst* 79, 288 - 92.
- Breyer, F. and K. Stolte (2000). "Demographic Change, Endogenous Labor Supply and the Political Feasibility of Pension Reform," *Journal of Population Economics*, forthcoming.
- Brunner, J.K. (1994). "Redistribution and the Efficiency of a Pay-as-you-go Pension System," *Journal of Institutional and Theoretical Economics* 150, 511-23.

- Brunner, J.K. (1996). "Transition from a Pay-as-you-go to a Fully Funded Pension System: The Case of Differing Individuals and Intragenerational Fairness," *Journal of Public Economics* 60, 131-46.
- Bundesministerium für Wirtschaft (1988). *Grundlegende Reform der gesetzlichen Rentenversicherung : Gutachten des Wissenschaftlichen Beirats beim Bundesministerium für Wirtschaft*. Bonn: Studienreihe des Bundesministeriums für Wirtschaft, Nr. 99.
- Fehr, H. (2000). "Pension Reform during the Demographic Transition," *Scandinavian Journal of Economics*, forthcoming.
- Feldstein, M.S. and Samwick, A.A. (1998). "Potential Effects of Two Percent Personal Retirement Accounts," *Tax Notes* (May 4, 1998), 615 - 620.
- Fenge, R. (1995). "Pareto-Efficiency of the Pay-as-You-Go Pension System with Intragenerational Fairness," *FinanzArchiv* 52, 357 - 63.
- Gordon, R.H. and H.R. Varian (1988). "Intergenerational Risk Sharing," *Journal of Public Economics* 37, 185 - 202.
- Hagemann, R.P. and G. Nicoletti (1989). "Population Ageing: Economic Effects and some Policy Implications for Financing Public Pensions," *OECD Economic Studies* 12, 51 - 96.
- Keyfitz, N. (1985). "The Demographics of Unfunded Pensions," *European Journal of Population* 1, 5 - 30.
- Keyfitz, N. (1988). "Some Demographic Properties of Transfer Schemes: How to Achieve Equity Between the Generations," in Lee, R.D. *et al.*, (eds.) *Economics of Changing Age Distributions in Developed Countries*. Oxford: Clarendon.
- Schnabel, R. (1998). "Rates of Return of the German Pay-as-you-go Pension System," *FinanzArchiv* 55, 374 - 99.
- Sinn, H.-W. (2000). "Why a Funded Pension System is Needed and Why it is not Needed," *International Tax and Public Finance* 7, 389 - 410.

Smith, A. (1982). "Intergenerational Transfers as Social Insurance," *Journal of Public Economics* 19, 97 - 106.

Thum, M. und J. von Weizsäcker (1999). "Implizite Einkommensteuer als Messlatte für die aktuellen Rentenreformvorschläge," *Perspektiven der Wirtschaftspolitik* 1, forthcoming.

United Nations (1998). *Demographic Indicators 1950 - 2050*. New York: United Nations.

7 Appendix

7.1 Proof of Lemma 1

We show this result for a function of the type

$$f(A, T, E) = \begin{cases} \frac{A^{T-E} - 1}{A^T - A^{T-E}} & \text{if } A \neq 1 \\ \frac{T-E}{E} & \text{if } A = 1 \end{cases} \quad (\text{A.1})$$

of which the dependency ratio is a special case with $A = e^m$.

An increase in T

For $A = 1$, the result obviously holds. For $A \neq 1$ we obtain

$$\frac{\partial f}{\partial T} = \frac{\ln A}{A^T - A^{T-E}} > 0 \quad (\text{A.2})$$

because for $A > 1$ we have $\ln A > 0$ und $A^T - A^{T-E} > 0$, while for $A < 1$ the signs of $\ln A$ and $A^T - A^{T-E}$ are both negative.

An increase in m

It can easily be shown that the function $f(A, T, E)$ is continuous in A . Thus, if $\partial f / \partial A < 0$ for $A \neq 1$, f is monotonically decreasing in A and therefore in m . This result is obtained by simplifying $\partial f / \partial A$ for $A \neq 1$:

$$\begin{aligned} \frac{\partial f}{\partial A} &= \frac{-EA^{2T-E-1} + TA^{T-1} - (T-E)A^{T-E-1}}{(A^T - A^{T-E})^2} \\ &= \frac{TA^{T-E-1} \left(A^E - \left(\frac{E}{T}A^T + \frac{T-E}{T}A^0 \right) \right)}{(A^T - A^{T-E})^2} \end{aligned}$$

The sign of $\partial f / \partial A$ is therefore equivalent to the sign of $A^E - \left(\frac{E}{T}A^T + \frac{T-E}{T}A^0 \right)$. Because the function $h(x) = A^x$ is strictly convex for $A \neq 1$ we have

$$\frac{E}{T}A^T + \frac{T-E}{T}A^0 > A \left(\frac{E}{T}T + \frac{T-E}{T}0 \right) = A^E$$

Therefore $\partial f / \partial A < 0$ for $A \neq 1$. \square

7.2 Proof of Proposition 2

Under CRR, we have $b^*(\bar{n}) = \bar{n}q^*(m, T, E)$ from equation (9). Substituting into (14) yields

$$\tau^*(\bar{n}) = \bar{n}(q^*(m, T, E) - u(r, g, T, E)). \quad (\text{A.3})$$

Under CCR, we have $n^*(\bar{b}) = \bar{b}/q^*(m, T, E)$ from (10). Inserting into (14) we obtain

$$\tau^*(\bar{b}) = \bar{b} \left(1 - \frac{u(r, g, T, E)}{q^*(m, T, E)} \right) \quad (\text{A.4})$$

Hence, under both systems

$$\begin{aligned} \tau^* > 0 &\Leftrightarrow T^*(t) > 0 \\ &\Leftrightarrow q^*(m, T, E) > u(r, g, T, E) \\ &\Leftrightarrow \frac{e^{m(T-E)} - 1}{e^{mT} - e^{m(T-E)}} > \frac{e^{(r-g)(T-E)} - 1}{e^{(r-g)T} - e^{(r-g)(T-E)}} \end{aligned}$$

In Lemma 1, we have shown that a function of the type

$$f(A, T, E) = \frac{A^{T-E} - 1}{A^T - A^{T-E}}$$

is decreasing in A . Hence, $q^*(m, T, E) > u(r, g, T, E)$ if and only if $e^m < e^{(r-g)}$ which is equivalent to $r > m + g$. \square

7.3 Proof of Proposition 3.1

By equation (9) the effect on b^* is given by

$$\left. \frac{\partial b^*}{\partial T} \right|_{n=\bar{n}} = \bar{n} \frac{\partial q^*}{\partial T} > 0$$

which is positive due to Lemma 1.

From equation (A.3) we can derive the effect on τ^* :

$$\left. \frac{\partial \tau^*}{\partial T} \right|_{n=\bar{n}} = \bar{n} \left(\frac{\partial q^*}{\partial T} - \frac{\partial u}{\partial T} \right) \quad (\text{A.5})$$

To show that $\left. \frac{\partial \tau^*}{\partial T} \right|_{n=\bar{n}} > 0$, it is sufficient that

$$\frac{\partial f}{\partial T} = \begin{cases} = \frac{\ln A}{A^T - A^{T-E}} & \text{if } A \neq 1 \\ \frac{1}{E} & \text{if } A = 1 \end{cases} \quad (\text{A.6})$$

is monotonically decreasing in A . Then the fact that $q(m, T, E) > u(r, g, T, E)$ if $r > m + g$ implies that $\frac{\partial q}{\partial T} - \frac{\partial u}{\partial T} > 0$.

First, we show that $\frac{\partial f}{\partial T}$ is continuous in A . Since this function is differentiable but for $A = 1$, the only point in question is $A = 1$. Using the rule of l'Hôpital, we obtain

$$\begin{aligned} \lim_{A \rightarrow 1} \frac{\partial f}{\partial T} &= \lim_{A \rightarrow 1} \frac{1/A}{TA^{T-1} - (T-E)A^{T-E-1}} \\ &= \lim_{A \rightarrow 1} \frac{1}{TA^T - (T-E)A^{T-E}} \\ &= \frac{1}{T - T + E} \\ &= \frac{1}{E}. \end{aligned}$$

Secondly, we prove that $\frac{\partial f}{\partial T}$ is monotonically decreasing in A . For $A \neq 1$, we show that

$$\frac{\partial^2 f}{\partial T \partial A} < 0$$

which is sufficient for $\frac{\partial f}{\partial T}$ to be monotonically decreasing due to continuity.

Differentiating (A.2) with respect to A yields

$$\frac{\partial^2 f}{\partial T \partial A} = \frac{A^{T-1}}{(A^T - A^{T-E})^2} [1 - A^{-E} - \ln A [(1 - A^{-E})T + EA^{-E}]]$$

The first term is always positive. $\frac{\partial^2 f}{\partial T \partial A} < 0$ is therefore equivalent to

$$1 - A^{-E} - \ln A [(1 - A^{-E})T + EA^{-E}] < 0$$

or

$$\underbrace{\ln A [(1 - A^{-E})T + EA^{-E}]}_{k(A)} > \underbrace{1 - A^{-E}}_{g(A)}.$$

For $A \neq 1$ this inequality is fulfilled. Let us first examine the case $T = E$. Then $k(A) = \ln(A^E)$ and

$$\begin{aligned}\frac{\partial k}{\partial A} &= EA^{-1} > 0 \\ \frac{\partial g}{\partial A} &= EA^{-E-1} > 0.\end{aligned}$$

This implies

$$\frac{\partial k}{\partial A} = \begin{cases} > \frac{\partial g}{\partial A} & \text{for } A > 1 \\ = \frac{\partial g}{\partial A} & \text{for } A = 1 \\ < \frac{\partial g}{\partial A} & \text{for } A < 1 \end{cases}$$

Hence $k(A) > g(A)$ for $A \neq 1$. This also holds for $T > E$ as

$$\frac{\partial k}{\partial A} = A^{-1} [(1 - A^{-E})T + EA^{-E}] + \ln A [TE - E^2] A^{-E-1}$$

and

$$\frac{\partial^2 k}{\partial A \partial T} = A^{-1} (1 - A^{-E}) + \ln A (EA^{-E-1})$$

which implies

$$\frac{\partial^2 k}{\partial A \partial T} = \begin{cases} > 0 & \text{for } A > 1 \\ < 0 & \text{for } A < 1 \end{cases}$$

Thus, also for $T > E$, we have $k(A) > g(A)$ for $A \neq 1$. \square

7.4 Proof of Proposition 3.2

From equation (10) we obtain

$$\left. \frac{\partial n^*}{\partial T} \right|_{b=\bar{b}} = -\frac{\bar{b}}{q^{*2}} \frac{\partial q^*}{\partial T} < 0$$

by Lemma 1.

Equation (A.4) gives the implicit tax rate as a function of \bar{b} . Hence

$$\left. \frac{\partial \tau^*}{\partial T} \right|_{b=\bar{b}} = \bar{b} \frac{\frac{\partial q^*}{\partial T} u - \frac{\partial u}{\partial T} q^*}{q^{*2}} \quad (\text{A.7})$$

To show that this term is positive if $r > m + g$, we use

$$\begin{aligned} \frac{\partial \tau^*}{\partial T} \Big|_{b=\bar{b}} > 0 &\Leftrightarrow \frac{\partial q^*}{\partial T} u > \frac{\partial u}{\partial T} q^* \\ &\Leftrightarrow \frac{\frac{\partial q^*}{\partial T}}{q^*} > \frac{\frac{\partial u}{\partial T}}{u}. \end{aligned}$$

and define the function

$$h(A, T) \equiv \frac{\frac{\partial f(A, T)}{\partial T}}{f(A, T)}$$

Therefore

$$\frac{\partial \tau^*}{\partial T} > 0 \Leftrightarrow h(m, T) > h((r - g), T)$$

and it is sufficient to show that $h(A, T)$ is monotonically decreasing in A . Substituting from (A.1) and (A.2) yields

$$h(A, T) = \begin{cases} = \frac{\ln A}{A^{T-E} - 1} & \text{if } A \neq 1 \\ \frac{1}{T-E} & \text{if } A = 1 \end{cases}$$

As in the proof of Proposition 2.1 it can be shown that this function is continuous. For $A \neq 1$, we have

$$\frac{\partial h}{\partial A} = \frac{1}{A(A^{T-E} - 1)^2} [A^{T-E} - 1 - \ln A (T-E) A^{T-E}]$$

The first term is always positive. The second term is negative and thus $h(A, T)$ monotonically decreasing in A if

$$k(A, T) \equiv \ln A (T-E) A^{T-E} > A^{T-E} - 1 \equiv l(A, T).$$

If $T = E$, then for all A : $k(A, E) = l(A, E) = 0$. For $T > E$ we find the solution with the help of the following derivatives:

$$\begin{aligned} \frac{\partial k}{\partial T} &= A^{T-E} \ln A + (T-E) A^{T-E} (\ln A)^2 \\ \frac{\partial l}{\partial T} &= A^{T-E} \ln A. \end{aligned}$$

Thus $\frac{\partial k}{\partial T} > \frac{\partial l}{\partial T}$ and therefore for $T > E$ and $A \neq 1$

$$k(A, T) > l(A, T)$$

Thus the function h is monotonically decreasing in A . \square

7.5 Proof of Proposition 3.3

From equation (A.3) we obtain

$$n^*(\bar{\tau}) = \frac{\bar{\tau}}{(q^*(m, T, E) - u(r, g, T, E))}. \quad (\text{A.8})$$

Therefore

$$\left. \frac{\partial n^*}{\partial T} \right|_{\tau=\bar{\tau}} = -\frac{\bar{\tau}}{(q^* - u)^2} \left(\frac{\partial q^*}{\partial T} - \frac{\partial u}{\partial T} \right) < 0$$

because the function $\frac{\partial f}{\partial T}$ is monotonically decreasing in A as we showed in the proof of Proposition 3.1.

Similarly, we can solve equation (A.4) for b and obtain

$$b^*(\bar{\tau}) = \frac{q^*(m, T, E)}{(q^*(m, T, E) - u(r, g, T, E))} \bar{\tau}. \quad (\text{A.9})$$

Hence for $r > m + g$

$$\begin{aligned} \left. \frac{\partial b^*}{\partial T} \right|_{\tau=\bar{\tau}} &= \frac{\frac{\partial q^*}{\partial T} (q^* - u) + q^* \left(\frac{\partial q^*}{\partial T} - \frac{\partial u}{\partial T} \right)}{(q^* - u)^2} \bar{\tau} \\ &= \frac{\frac{\partial u}{\partial T} q^* - \frac{\partial q^*}{\partial T} u}{(q^* - u)^2} \bar{\tau} < 0 \end{aligned}$$

because the function $\frac{\partial f(A, T)}{f(A, T)}$ is monotonically decreasing in A as shown in the proof of Proposition 3.2. \square