

Schwarz, Alexandra

Working Paper

Measurement, monitoring, and forecasting of consumer credit default risk: An indicator approach based on individual payment histories

Schumpeter Discussion Papers, No. 2011-004

Provided in Cooperation with:

Schumpeter School of Business and Economics, University of Wuppertal

Suggested Citation: Schwarz, Alexandra (2011) : Measurement, monitoring, and forecasting of consumer credit default risk: An indicator approach based on individual payment histories, Schumpeter Discussion Papers, No. 2011-004, University of Wuppertal, Schumpeter School of Business and Economics, Wuppertal,
<http://nbn-resolving.de/urn:nbn:de:hbz:468-20110503-154809-9>

This Version is available at:

<http://hdl.handle.net/10419/68716>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



SCHUMPETER DISCUSSION PAPERS

Measurement, Monitoring, and Forecasting of Consumer Credit Default Risk –

An Indicator Approach Based on Individual Payment Histories

Alexandra Schwarz

The Schumpeter Discussion Papers are a publication of the Schumpeter School of Business and Economics, University of Wuppertal, Germany

For editorial correspondence please contact
SSBEditor@wiwi.uni-wuppertal.de

SDP 2011-004
ISSN 1867-5352

Impressum
Bergische Universität Wuppertal
Gaußstraße 20
42119 Wuppertal
www.uni-wuppertal.de
© by the author



**BERGISCHE
UNIVERSITÄT
WUPPERTAL**

Measurement, Monitoring and Forecasting of Consumer Credit Default Risk*

An Indicator Approach Based on Individual Payment Histories

Alexandra Schwarz

*German Institute for International Educational Research
Frankfurt am Main, Germany*

Abstract

The statistical techniques which cover the process of modeling and evaluating consumer credit risk have become widely accepted instruments in risk management. In contrast, we find only few and vague statements on how to define the default event, i. e. on the concrete circumstances that lead to the decision of identifying a certain credit as defaulted. Based on a large data set of individual payment histories this paper investigates a possible solution to this problem in the area of installment purchase. The proposed definition of default is based on the time due amounts are outstanding and the resulting profitability of the receivables portfolio. Furthermore, to assess the individual payment performance during the credit period, indicators for monitoring and forecasting default events are derived. The empirical results show that these indicators generate valuable information which can be used by the creditor to improve his credit and collection policy and hence, to improve cash flows and reduce bad debt loss.

Keywords: Credit Risk Analysis · Credit Default · Risk Management · Accounts Receivable Management · Performance Measurement

JEL Classification: C44 · G32 · M21

*The author gratefully acknowledges the support of Gerhard Arminger (University of Wuppertal) and helpful comments of Annette Köhler (University of Duisburg-Essen) and participants at the 4th European Risk Conference in Nottingham, September 2010.

1 Introduction

*In theory, a Good account is one that you are glad you took
and a Bad account is one that you are sorry you took.*

That may be true but isn't very helpful.

(Edward M. Lewis, 1992)

Offering customers installment purchase is a widely-used instrument to increase sales. For the firm implementing this instrument, payment by installments is a sale on credit with special conditions, especially including an extended credit period and an installment plan fixing the due dates for payments to be made by the customer. Sales on credit as one part of a selling concept, directly incorporate a conflict between marketing and financial objectives – between gaining customers and controlling the credit risk involved by extended, more customer-oriented payment conditions.

The operative control of these credits is assigned to accounts receivable management whose key tasks are to record and manage payments, to configure terms of payment and trading conditions, to induce collection procedures and to control loan securities, if available.¹ As every such credit involves a default risk, an effective receivables management aims for preventing bad debt loss and should therefore check the customers' creditworthiness (Hoss 2006, 35; Johnson/Kallberg 1986, 9 ff.). The analysis and prediction of this default risk is usually supported by a standardized process, often referred to as *credit scoring*. This process is based on statistical methods for estimating the individual probabilities of customers to default on credit which are one of the essential inputs for the financial evaluation of credit sales and of the impact these sales have on a firm's working capital and liquidity.

The techniques that cover the process of modeling individual credit risks are widely discussed topics in financial and statistical literature.² In contrast, we find only few statements on how the dependent variable *default yes/no* in a scoring model is defined. Even in statistical publications this definition is always said to be given, but not described. Nonetheless, defining credit default events is a critical task within the process of modeling credit risk as any such definition is needed to operationalize the key dependent variable (e. g. creditworthiness), to calculate default probabilities and to monitor them over time.

¹See Brigham (1992), Johnson/Kallberg (1986) and Mueller-Wiedenhorn (2006) for an introduction to accounts receivable management.

²For an introduction to these techniques see for example Caouette et al. (2008, 201 ff.), Hand/Henley (1997) and Thomas et al. (2002).

It can be assumed that this lack of information is due to confidentiality reasons because the definition of credit default gives direct insight into a bank's or a company's internal calculations, its marketing strategy and credit policy.

The present paper addresses this question as it deals with the definition, monitoring and forecasting of default events in the area of installment credits. The focus is on two questions:

- (1) How can a credit default event be defined? That is, what are the concrete circumstances, e. g. in terms of payment behavior, that lead to the decision of classifying a certain account as defaulted?
- (2) What are useful indicators for monitoring individual payment behavior and detecting default events during the payment process?

Hence, the paper is organized as follows: First, the credit scoring process is set in the context of risk analysis in section 2 where the information generated by a credit scoring system and its implications for accounts receivable and credit risk management are described. Section 3 deals with the need for defining credit default. We review and discuss existing definitions of default events and describe general characteristics any definition of credit default should fulfill. To arrive at a possible definition of credit default, the patterns of payment – a common measure for the control of accounts receivable – are adopted to the case of installment purchase (section 4). Events of default and non-default are classified based on a measure of profitability that can be derived from these payment patterns. In the empirical study, this approach is applied to a large, unique data set of payment histories originating from a company trading consumer goods. Section 5 deals with indicators of individual payment performance and the monitoring of payment behavior. The empirical study continues by evaluating the proposed indicators with respect to their potential to detect defaults on-line, i. e. during the payment process. The paper closes with a discussion in section 6.

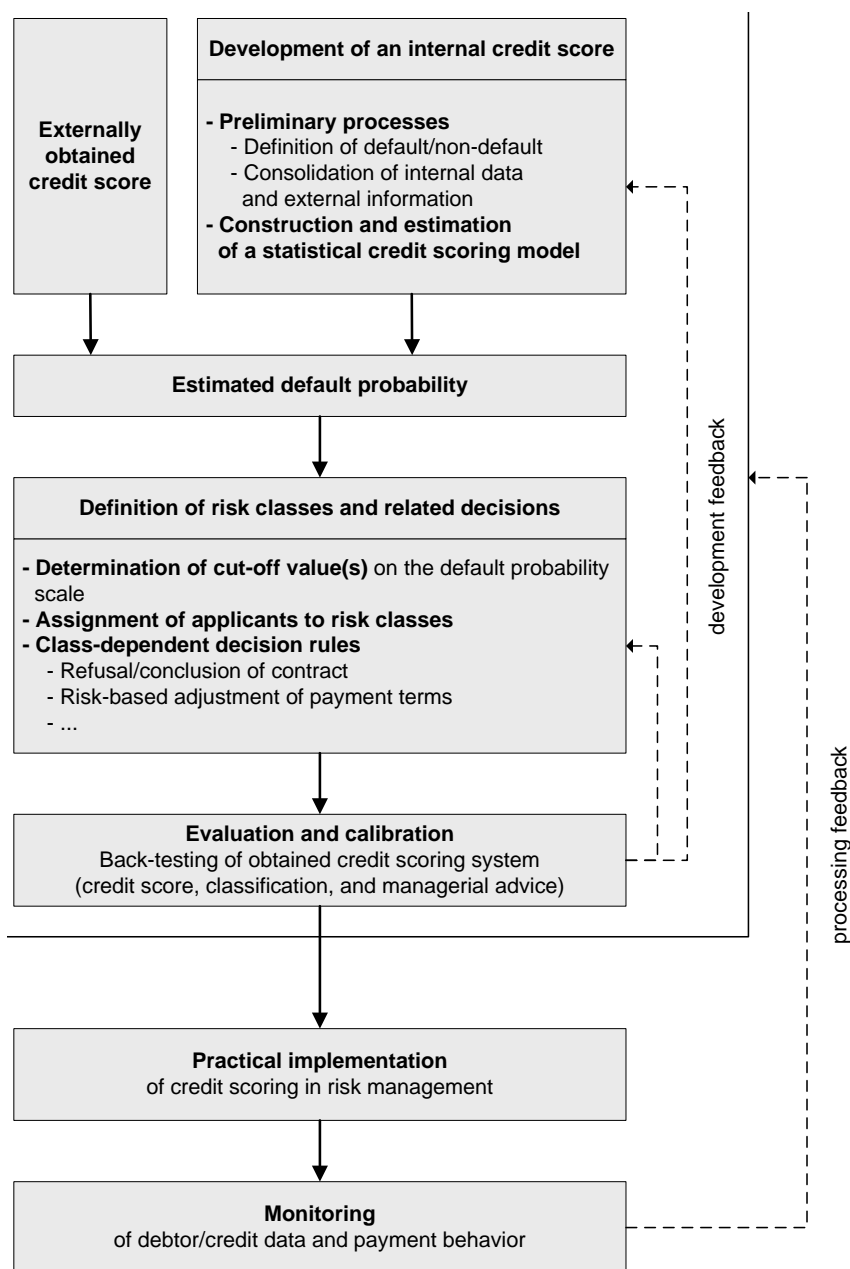
2 Credit scoring systems for evaluating sales on credit

In the ideal case, a credit scoring system for identifying, analyzing and monitoring customer credit risk is an integrative part of a company's risk management: on the one hand such a system depends on historical accounting data, on the other hand it generates useful information for controlling and managing credit risk. Consequently, evaluating sales on credit by means of a scoring system is a concurrent process as illustrated in figure 1.

To measure the default risk involved by sales on credit, customers are assigned to certain risk classes based on their individual propensities to default on payment. The required default probability can either be obtained externally or on basis of an internal scoring model. The main internal source of information on creditworthiness is a company's own accounting department which can provide data on a customer's previous payment behavior and individual characteristics like age, education, profession, residence etc. By means of statistical methods, this data can be used to construct and estimate a credit scoring model for the prediction of the default probability of new credits. Firms can also turn to commercial credit agencies which collect data on contractual and non-contractual processing of business connections. Companies which provide goods and services on credit can purchase information on criteria like outstanding accounts, requests to pay issued by court order, enforcement procedures and uncovered checks. These criteria normally serve as knock-out criteria as they deliver outright facts on a consumer's propensity to default on payment (Reichling et al. 2007, 56). Following the design of corporate ratings, some credit agencies provide consumer ratings. These are individual score point values which are assigned a certain default probability. Such an external credit score can also be used as an additional input feature in an internally developed credit scoring model.

Independent of the source of credit quality information, the next step consists in defining risk classes and in establishing decisions with respect to credit applications. In this regard, cut-off values have to be defined on the ordinal or metric default probability scale. Besides the number of risk classes this requires the systematic formulation of activities to be taken on customers who are assigned to a certain risk class. A simple example would be to establish two risk classes representing sufficient and insufficient payment, or creditworthy and not creditworthy customers, respectively. To avoid the involved risk, a firm's risk management may decide to refuse applications of customers who are not creditworthy. As firms always bear the risk involved by accepted sales on credit, more diversified risk strategies lead to a range of classes and interventions, accompanied by risk-based pricing, adjustment of payment terms and risk-adjusted interest rates.

Figure 1: Process of implementing a credit scoring system



Of course, the appropriate policy will be found by evaluating the profitability of alternative systems, that is, by assessing the benefit of increased sales against the direct and indirect costs of granting credit to customers with a varying likelihood to pay slowly or even end up as a bad debt loss (Brigham 1992, 799).

Whether the obtained credit scoring really fulfills the desired targets is evaluated by back-testing the whole system (scores, classifications and interventions) on the basis of a hold-

out sample of historical customer data. This calibration of the credit scoring provides useful hints for the improvement of the developed model and the resulting decisions (development feedback). Once the credit scoring has been implemented as part of the risk management it is necessary to document the debtor- and credit-related data as well as the individual processes of payment. This monitoring enables the technical and statistical maintenance of the scoring model and it allows for a concurrent evaluation of the risk involved by receivables, especially with respect to financing costs which reduce the firm's rate of return and liquidity (processing feedback). If, for example, slow or deficient payments exceed a certain level, this may force the firm to adjust its credit policy, e. g. it may increase the required financial strength of acceptable customers, or it may introduce a more insistent collection policy.

3 On the definition of credit default events

This section deals with the general concept of credit default and the definition of credit default events. It is discussed why it is inevitable to define the default event in a concrete context, e. g. consumer credits offered by a bank or installment purchases offered by a company, even if credit quality information is obtained externally. Afterwards, we review existing definitions of default events and summarize their general characteristics.

3.1 The need for defining the credit default event

In contrast to default risk in general and the statistical techniques that cover the process of modeling individual credit risks, the task of defining a credit as default or non-default is only rarely discussed so far. Nonetheless, from a methodological point of view, there are three main reasons why any such definition is strongly required in credit risk analysis.

First, if a firm³ decides to establish its own internal credit scoring model, an operationalization of the latent dependent variable 'creditworthiness' is required. As creditors seek for an estimation of default probability the dependent variable Z normally is binary coded, i. e. $Z \in (0, 1)$. Then $z_i = 1$ represents a bad account and a not creditworthy customer, and $z_i = 0$ represents a good account and a creditworthy customer, respectively. Second, even if a scoring system is based on an externally obtained credit score to predict individual default risk, it is necessary to evaluate whether the obtained score really measures what it is supposed to, i. e. if it really fits the individual credit risks of the customers at

³As throughout the whole paper, the focus is on non-banks. Yet, the described concepts of credit default and default events apply to banks as well, especially to the retail sector.

hand. The comparison of predicted and actual default risks requires an internal risk estimator like default rates and therefore a definition of default and non-default. Finally, the same reasoning applies to the validation of a scoring system once it has been implemented for practical use. By monitoring the customers' payment behavior and the appearance of default events, firms are able to appraise whether the internal or external credit risk model still fits the portfolio of customers at hand, and if the introduced business concept and credit policy are still affordable.

3.2 A review of default event definitions

Most generally speaking, a bad account is a matter of deficient payment. A consistent concept of the concrete circumstances which lead to the identification of a credit default does not exist: "Even deciding on the definition of what should be regarded as a good or bad risk may be far from straightforward." (Hand 1998, 71) In addition, Hand points out that the definition depends on the nature of the loan, i.e. the definitions of default will be different for a credit card account and the repayment of a mortgage loan. "The definition may be based on slow repayments (but is one month overdue to be regarded as 'bad' or should it be two, or...?), a combination of account balance below some level throughout the month and overdraft limit exceeded at some point, or some more sophisticated combination." (Hand 1998, 71) An indicator originating from the accounts receivable management process may be the institution of legal proceedings against the debtor (Fueser 2001, 45). Caouette et al. (2008, 208) suggest that the definition of a bad account is usually based on three payment delinquencies whereas good accounts are those who have not experienced these arrears. Lewis (1992) discusses the definition of credit default with respect to revolving credit like credit card or bank giro accounts. He suggests that in this context a good account "might be someone whose billing account shows:

- (1) On the books for a minimum of 10 months.
- (2) Activity in six of the most recent 10 months.
- (3) Purchases of more than \$50 in at least three of the past 24 months.
- (4) Not more than once 30 days delinquent in the past 24 months." (Lewis 1992, 36)

Here definitions (1) to (3) exclude those accounts from further investigations which belong to fairly new customers or to customers with low activity. Lewis (1992, 37) argues that a bad account is more difficult to describe but may be identified adequately by one of the following definitions:

- The debtor is delinquent for 90 days at any time with an outstanding undisputed balance of \$50 or more.
- The debtor is delinquent three times for 60 days in the past 12 months with an outstanding undisputed balance on each occasion of \$50 or more.
- The debtor has gone bankrupt while the account was open.

According to Lewis, it is important to leave some accounts indeterminate, namely those that do not fall in either group, because the lender may not be able to make a qualitative decision on the performance of the loan, for example for newly acquired accounts or accounts that are delinquent for 30 days.

An alternative approach to arrive at a definition of credit default may be to adopt the definitions settled for banks by the Basel Committee on Banking Supervision (BCBS 2004, sect. 452). Within this framework two alternative definitions of default are given:⁴

- *Unlikeliness to pay*: The bank considers that the obligor is unlikely to pay his/her credit obligations to the banking group in full, without recourse by the bank to actions such as realizing security (if held).
- *90 days past due*: The obligor is past due more than 90 days on any essential credit obligation.

Section 125 of the German Solvency Regulations (Deutsche Bundesbank 2008) concretizes the unlikeliness-to-pay clause by a list of indicators which may suggest the definition of default event, for example allowances for declined credit quality, sale of credit obligations with a substantial economic loss, or the debtor has gone bankrupt. In this regulation also the essential credit obligation mentioned in the 90-days-past-due clause is specified more precisely. An overdraft of any obligation is said to be essential if it amounts to more than 100 Euros and to more than 2.5% of the overall credit line. At least the 90-days-past-due clause provides a precise definition of default events, but has some fundamental drawbacks. These result from the fact that the Basel regulations, including the definition of default events, were set to harmonize the measurement of capital requirement. Hence great emphasis is placed on the evaluation of corporate credit, which makes up the bulk of banks' business. Therefore, the Basel 90-days-past-due clause need not necessarily lead to an adequate decision on default events in terms of profitable or not profitable accounts. Porath (2006), who discusses whether credit scoring models comply

⁴These definitions are still valid in the 'International framework for liquidity risk measurement, standards and monitoring' (BCBS 2010).

with the Basel II requirements for risk quantification, argues that a scoring model's primary aim is to support internal decisions and not to fulfill the supervisory requirements. Consequently, "the default event sets as soon as the loan becomes no longer profitable for the bank and this is usually not the case when the loan defaults according to the Basel definition. It depends, instead, on the bank's internal calculation." (Porath 2006, 31) Obviously, it can be assumed that the same applies to creditors in non-financial business and it would be interesting to examine whether a company's own definition of default events goes in line with the Basel one.

The existing definitions of default events are either formulated in a very general manner, or in case they are more precise they refer to a special type of loan like revolving credit. From a managerial point of view this result is quite obvious: Every company has to arrive at its own definition of default, depending on the nature of the loans and the company's internal calculation, its marketing strategy and credit policy. Consequently, the lack of information on any concrete definition of default is due to confidentiality reasons and the increased competition companies face in the industrial and commercial sector. This conclusion goes in line with Foster/Stine (2004) who build a predictive model for bankruptcy and claim that their research, especially with respect to the identification of relevant predictors, suffers from issues of confidentiality in the credit industry and from the resulting lack of exchange with credit analysts.

3.3 General characteristics of default event definitions

Based on the approaches reviewed above we can describe some general characteristics of a default event definition. At first, there are basic requirements that should be considered when developing a default definition.

- The definition of an account as good or bad is entirely based on the performance of the account once accepted (Lewis 1992, 31). This means that the evaluation of performance is only based on internal data concerning the individual payment process. External information on credit quality or the application itself (e.g. age, profession etc. of the credit applicant) is not included.
- The analysis of payment performance must lead to a definition that is consistent, precise and understandable, for the staff working with it as well as for internal and external reviewers (Fueser 2001, 45; Lewis 1992, 36).
- The definition should offer the opportunity of a computer-aided, automated detection of bad accounts (Lewis 1992, 37).

In addition, the more sophisticated methods of defining credit default, documented by the BCBS (2004) and Lewis (1992), agree on two major components used for defining default events.

- (1) *The temporal component*: The customer is in arrears for a certain *number of periods* (e. g. for 30 days). This means he failed to pay at a *due date* in the past.
- (2) *The monetary component*: The customer is in arrears with a certain *amount of money* (e. g. 100 Euros). This means he failed to pay at least one *due amount* in the past.

Independent of the type of loan, like revolving credit or installment purchase, these two components of deficient payment induce different types of additional costs concerning an accepted and open loan contract (Lauer 1998, 84 ff.; Salek 2005, 22):

- Every *delayed payment* induces additional costs in terms of interest charges for financing the capital fixed in outstanding accounts receivable.
- If outstanding amounts are *not paid in the long run*, this induces additional costs in terms of bad debt loss.
- Both delayed payments and payments never made cause *additional costs of managing* accounts receivable, not only overhead costs, but also costs of trying to collect accounts receivable individually.

These general characteristics and components of default event definitions form the basis of the approach to the classification of installment credits which is proposed in this paper.

4 A payment-pattern approach to the definition of credit default events on aggregate level

A credit in the special form of an installment purchase usually involves an installment plan which documents the due dates and due amounts of payment. These due and expected payments can be compared to the actual payments of a debtor by means of the individual account balances. The basic idea of the proposed classification is to balance expected and actual payments of debtors on an aggregate level (e. g. on company level) at every point in time at which payments are expected. By evaluating this pattern of payments and the resulting profitability of the involved accounts we can determine the maximum period of deficient payment which is acceptable for financial purposes.

4.1 Common approaches to the evaluation of accounts receivable

Most approaches to the control of accounts receivable follow a one-parameter technique: A single indicator is used to describe the current status of the portfolio of accounts receivable and to forecast its development in the near future. Well-known examples are the average days that sales are outstanding (DSO) and the reciprocal, i. e. the accounts receivable turnover (ART), which gives the number of times that receivables will turn over in one year (Johnson/Kallberg 1986, 28; Brigham 1992, 794 ff.; Lauer 1998, 57 ff.). Computed on a monthly basis, increasing values of DSO and ART may suggest problems in collecting receivables. To gain further insight into the composition of receivables one can calculate an aging schedule which is the proportion of accounts receivable that are in different age classes (Stone 1976, 70). To control the development of accounts receivable, any of these indicators is projected into the future, and to incorporate seasonal varying sales on credit, the aging schedule of a certain month may be compared to the respective aging schedule of the year before. More sophisticated methods, e. g. mover-stayer models or Markov chain approaches, estimate the probabilities of accounts to change their state, for example to make transitions among the state 'paid' and the state 'overdue'. Frydman et al. (1985) describe the application of both techniques to credit behavior. The result of these procedures is an estimated transition matrix which gives these probabilities for the total portfolio of analyzed accounts. The complexity of this procedure and the resulting transition matrix increases rapidly with the number of states defined, especially if not only states but also monetary components like outstanding amounts are considered.⁵

The main drawback of these approaches with regard to defining default events is that all of them give a more or less deep insight into the composition and the development of the *portfolio* of accounts or customers, respectively. This is due to the fact that the emphasis lays on once-only sales on credit which have to be paid until a certain payment deadline. In addition, the analysis of accounts receivable aims for an appropriate estimation of expected loss needed for a company's annual balance. Consequently, neither the analysis nor the control and forecasting of accounts receivable status refer to *individual* payment behavior. Nonetheless, we make use of the patterns of payment – an alternative way to measure the status and development of accounts receivable – to analyze payment behavior and derive a profit-oriented definition of credit default events.

⁵See for example the analysis described by Kallberg/Saunders (1983).

4.2 The patterns of payment

The patterns of payment are closely related to the aging schedule mentioned above. Their main advantage over the previously described techniques is that they can be adopted adequately to the case of installment credits. At the same time, they offer the opportunity to assess the profitability of the accounts receivable portfolio.

In the context of sales on credit the receivable balance pattern “is the proportion of any month’s sales that remains outstanding at the end of each subsequent month” (Johnson/Kallberg 1986, 25). This proportion is expected to decay over subsequent months. Therefore, it is tracked by simply following the percentages over time. The collection pattern is the mirror image of the receivable balance pattern, giving the cumulative collections of the subsequent months in percent of credit sales. In the following, we define suitable patterns of payment for the case of installment credits. A description of the original procedures is given in Stone (1976).

Suppose we observe the complete payment history of n installment credits $i = 1, \dots, n$ with total financed amounts y_i . We also suppose that these credits are paid off by an equal number T of installments and that payments are due at regular intervals. Hence, payments are observed at points in time $t = 1, \dots, T, \dots, T + h, \dots, T + H$ with every t denoting an observation point of due installments. Then T denotes the total number of installments and at the same time the end of the agreed credit period, and H is the number of points in time $h = 1, \dots, H$ at which we observe payments after the end of the regular payment term. Hence, $T + H$ describes the end of our complete observation period.

Let $y_{i,t}$ denote the due amount of payment of credit i at time t , which is the installment to be paid at time t , and let $x_{i,t}$ denote the respective amount actually paid at time t . Then

$$X_k = \sum_{t=1}^k x_t \quad \text{with} \quad x_t = \sum_{i=1}^n x_{i,t} \quad (1)$$

are the cumulated payments actually made until k with $k \in \{1, \dots, T + H\}$. Equivalently, the cumulated expected payments until k are denoted by

$$Y_k = \sum_{t=1}^k y_t \quad \text{with} \quad y_t = \sum_{i=1}^n y_{i,t} \quad (2)$$

Consequently,

$$\Delta_k = \sum_{t=1}^k \delta_t \quad \text{with} \quad \delta_t = \sum_{i=1}^n (y_{i,t} - x_{i,t}) \quad (3)$$

are the cumulated outstanding payments at time k . Obviously, $Y_k = X_k + \Delta_k$ at each k . In this retrospective analysis of payments the collection pattern over $T + H$ points of observation can be calculated as the respective cumulated payments in % of the overall financed amount $Y = \sum_{t=1}^T y_t$, i. e. X_k/Y for all k . Respectively, the receivable balance pattern is given by Δ_k/Y , that are the respective cumulated outstanding amounts in % of the total expected payment.

4.3 Measurement of profitability

The proposed definition of credit default events is based on the profitability of accounts which can be measured using the patterns of payment described above. To illustrate this approach we assume that an account is (still) profitable at time k if the additional costs caused by deficient payment that occurred up to k are strictly smaller than the profit generated by payments made up to k . For measuring the profitability P_k of accounts at time k , we introduce a weight a (in %) for the cumulated payments and a weight c (in %) for the cumulated outstanding amounts Δ_k . Then $a \cdot X_k$ measures the profit made by the payments received until time k , and $c \cdot \Delta_k$ denotes the additional costs caused by the amounts not collected until time k . Then the profitability of the credit granting concept can be measured by the indicator

$$P_k = a \cdot X_k - \sum_{t=1}^k c \cdot \Delta_k \quad (4)$$

In addition, the weighted cumulated additional costs $\sum_{t=1}^k c \cdot \Delta_k$ incorporate the so-called revolving effect of credit which occurs if deficient payments are protracted over a certain period. Let t^* denote the minimum k of all observation points for which $P_k \leq 0$:

$$t^* = \min_{k=1, \dots, T+H} (k | P_k \leq 0) \quad (5)$$

This means, t^* is the point in time of the period of deficient payments at which the performance of credits is no longer acceptable, whereas $t^* - 1$ denotes the last point in time of the period of acceptable performance. This leads to the following classification rule concerning the definition of the default event Z : Credit i is assigned to the class of bad accounts if it contributes to the overall loss, that is, it shows an outstanding amount at t^* . Otherwise credit i is assigned to the class of good accounts. With

$$\delta_{i,t^*} = \sum_{t=1}^{t^*} (y_{i,t} - x_{i,t}) \quad (6)$$

denoting the sum of outstanding amounts for credit i at t^* the classification rule is

$$z_i = \begin{cases} 1 & \text{if } \delta_{i,t^*} > 0 \\ 0 & \text{else} \end{cases} \quad (7)$$

From a financial perspective, our assumption made above may not be realistic: Classifying credits as defaults based on t^* as defined in (5) means that the costs of financing the outstanding amounts exceed the profit made by the received payments. Hence, it may be useful to generalize the right hand side of (5) by $P_k \leq \tau$ where $\tau = 0$ in the above example. For practical purposes, an estimation of the parameters a and c based on internal calculations is required.

4.4 Empirical study: Individual payment histories of installment purchases

4.4.1 Description of the data set

The data set analyzed originates from a company which produces household appliances and offers its customers payment by installments. The company's management needs to improve the internal control of the cash flows from its accounts receivable portfolio because installment credits involve an increasing amount of bad debt loss. The data provided by this company is used to exemplify and evaluate the proposals for defining and monitoring default events. It has to be noted that this unique data set is not distorted by a credit scoring system, that is, until the date of the data retrieval no systematic screening of the customers' credit standing had been implemented and all applications for installment purchase were accepted.

The data set consists of 33,986 installment purchases of household appliances, each of them paid off by 15 regular installments. For the company granting these credits, 13.29% of the financed amounts remain uncollectible. The data set represents a complete cohort of credits: It consists of all credits for which the due date of the first rate of payment lies between March 01, 2004 and August 31, 2004. For each credit, the observed payment history runs from the due month of the first installment until March, 2007. The payment histories are given in the form of monthly account balances. This implies an exact installment plan with a due date of each installment, but the exact dates at which payments were made are not given. Consequently, the data related to each credit is analyzed on a monthly basis.

4.4.2 Definition of credit default events on aggregate level

For the data described above we observe the individual payment histories of $n = 33\,986$ credits with an agreed number of installments $T = 15$ and an additional observation period of $H = 17$ months where we expect accounts to be finally balanced. Therefore, the complete observation period is $(T + H) = 32$ months. Figure 2 shows the patterns of payment in % of the overall financed amount for each t , that are the expected pattern (Y_k/Y) , the collection pattern (X_k/Y) , and the receivable balance pattern (Δ_k/Y) . To exemplify the measurement of profitability we arbitrarily choose $a = 5\%$, $c = 1\%$ for the regular payment term $t = 1, \dots, 15$ and $a = 5\%$, $c = 2\%$ for the additional period $t = 16, \dots, 32$ after the end of the agreed payment term.⁶ In figure 2 the profitability is given in % of the expected profit $(P_k/(a \cdot Y_k))$. The left dashed, vertical line denotes the end of the regular payment term. The vertical line in the middle separates $t^* - 1$ and t^* , the two points in time which we would use for defining default events in terms of profitability, i. e. where the profitability becomes negative. The decision rule that was finally implemented by the company providing the data is illustrated by the right vertical line. Here the end of the period of acceptable performance is set to $t^* - 1 = 27$, which is twelve months after the end of the regular payment term. The resulting classification rule for the default event Z is

$$z_i = \begin{cases} 1 & \text{if } \delta_{i,28} > 0 \\ 0 & \text{else} \end{cases} \quad (8)$$

The numbers of detected defaults and non-defaults are given in table 1. On the basis of the classification rule (8) we detect 5,382 default events out of the 33,986 analyzed credits (15.84%). The remaining 28,604 accounts are defined as non-defaults (84.16%). As described in section 2, but not discussed in detail here, we can link these accounts – now classified as default and non-default – to further individual characteristics, e. g. age, profession and residence of the customer, to build up a scoring model. Such a model then leads to an estimation of default probabilities for new customers applying for an installment credit which depends on the specific combination of the individual characteristics.

⁶We increase c after the end of the regular payment term to incorporate the fact that the accounts receivable management gets more and more involved over time; as the original financing period has expired, the costs of financing the accounts receivable may increase as well.

Figure 2: Empirical study: patterns of payment and profitability

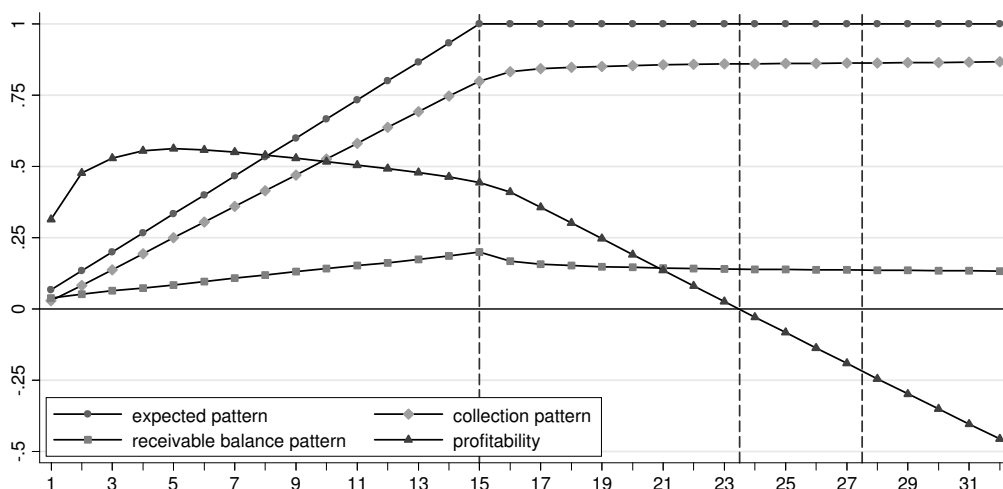


Table 1: Empirical study: identified defaults and non-defaults

Defaults ($z_i = 1$)	5,382	15.84%
Non-defaults ($z_i = 0$)	28,604	84.16%
Total	33,986	100.00%

5 Monitoring of credit default events based on individual payment histories

So far, a cohort of payment histories has been used to define the acceptable period for which a customer can stay in arrears prior to his account being classified as a default event. This section deals with the evaluation of the individual payment behavior of customers. First, the approach is introduced by describing some fundamental drawbacks of the general payment-pattern approach on aggregate level. Afterwards, two performance indicators for the real-time evaluation of payment behavior are defined. Their application is again exemplified by means of the empirical data set.

5.1 Drawbacks of the general payment-pattern approach on aggregate level

The payment-pattern approach to identifying default events shows fundamental drawbacks with respect to the practical evaluation of credit risks. We still analyze a *portfolio* of customers. The analysis of the payment patterns and the profitability of the total cohort of accounts over time does not tell us anything about the *individual* performance

of credits during the period $t = 1, \dots, t^*$. During this determined period of acceptable payment performance, customers paying off regularly compensate deficient payment of other customers. This compensation effect is intended and optimized in portfolio analysis, but conceals significant effects when detecting credit default events. Therefore, it may be suitable to evaluate the profitability of accounts individually. But the identification of default events based on the account's profitability strongly depends on the assumed cost ratio. As in the context of credit scoring models, reliable figures of profit and costs are required, but they can only be estimated as they are usually not given by credit analysts. In addition, it has to be considered that especially the additional costs induced by deficient payment vary over time, i. e. they increase with the period of deficiency.

As mentioned in section 3.1, the definition of credit default events is also needed for monitoring payment performance once a scoring system has been implemented in practice. The definition of default described in the previous section is based on a retrospective analysis of complete histories which are observed for a very long period, even after the end of the agreed payment term. This is not suitable for monitoring and forecasting default events because such a data base is not available during the actual payment process when payments are observed in real time. As a possible solution to these problems, two indicators of individual payment performance are proposed which can be used for on-line monitoring, and which are independent of an underlying cost function.

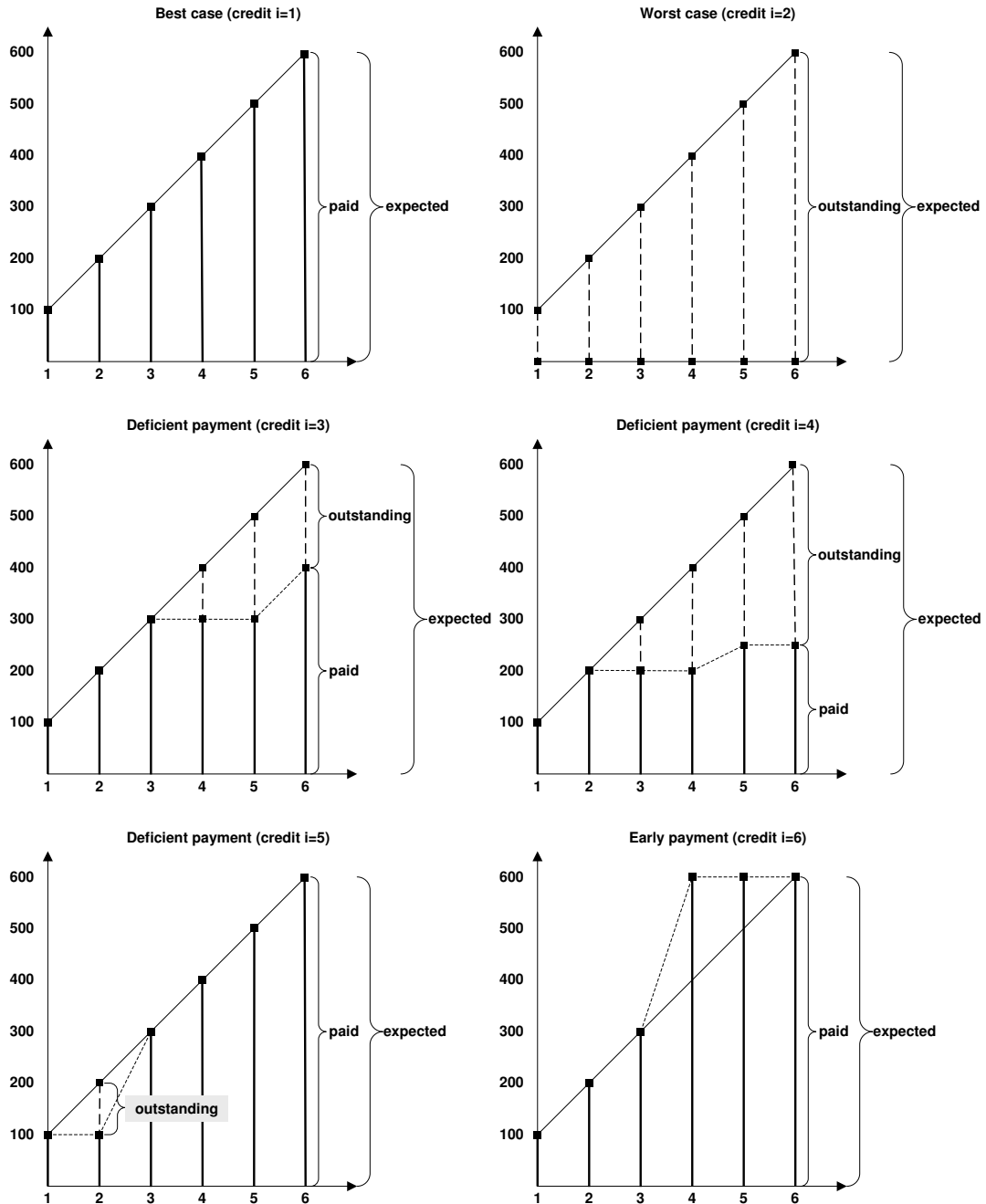
5.2 Indicators of individual payment performance

The basic idea of the indicators of individual payment performance is to evaluate the development of paid amounts with respect to the expected amounts and the time line of the individual payment process. This concept is illustrated by figure 3 which gives hypothetical examples of individual payment histories where a total financed amount of 600 Euros has to be paid off by six regular installments of 100 Euros each. In each plot, the solid line indicates the cumulated paid amount, the dashed line indicates the cumulated outstanding amount at each t . The diagonal line represents the expected payment process.

The first two examples of payment histories represent the best case and the worst case of payment behavior. For credit $i = 1$ all installments are paid at the due date, for credit $i = 2$ the customer fails to pay from the beginning of the payment term. The payment histories related to credits $i = 3, 4, 5$ show different types of late and deficient payment. Whereas the customers paying off credits $i = 3$ and $i = 4$ are still in arrears at the end of the regular payment term, the customer paying off credit $i = 5$ only fails to pay one

installment at $t = 2$ and balances his account immediately at $t = 3$. The figure related to credit $i = 6$ gives an example of early, complete payment where the total financed amount is paid off at an earlier stage of the payment process, here at $t = 4$ instead of $t = 6$.

Figure 3: Examples of individual payment histories



5.2.1 Individual liquidity

The first indicator $L_{i,k}$, which we call the *individual liquidity*, relates the cumulated amounts paid for credit i until time k to the respective cumulated expected amounts:

$$L_{i,k} = \frac{X_{i,k}}{Y_{i,k}} \quad \text{with} \quad X_{i,k} = \sum_{t=1}^k x_{i,t} \quad \text{and} \quad Y_{i,k} = \sum_{t=1}^k y_{i,t} \quad (9)$$

The individual liquidity at time k is therefore the proportion of due paid amounts until k . The respective figures for the exemplary payment histories introduced above are given in table 2. For the best case ($i = 1$), the individual liquidity is $L_{i,k} = 1 \forall k \in \{1, \dots, T\}$, for the worst case this indicator is $L_{i,k} = 0 \forall k \in \{1, \dots, T\}$, respectively. Early payment is indicated by $L_{i,k} > 1$, and this holds as long as the already paid amounts exceed the expected ones. For credit $i = 6$ for example, this indication holds for two periods, with $L_{6,4} = 1.50$ and $L_{6,5} = 1.20$. For $t = 6$ we again get $L_{6,6} = 1.00$. For the customer who misses to pay at $t = 2$ and pays off the outstanding amount immediately at $t = 3$ (credit $i = 5$), we observe $L_{5,2} = 0.50$ whereas the indicator equals 1.00 for all other t during the payment term. The two customers who are still in arrears at the end of the payment term (credits $i = 3, 4$) show a different individual liquidity. The indicator $L_{i,k}$ for credit $i = 3$ increases again at the end of the payment period because the customer pays the due installment at $t = 6$. This leads to $L_{3,6} = 0.67$ which means that the customer paid two-thirds of the expected amounts until the end of the regular payment term. For credit $i = 4$ we observe decreasing individual liquidity from $t = 3$ until the end of the payment term.

5.2.2 Individual payment career

The main disadvantage of the individual liquidity is that we still need the complete history of this indicator to get insight into the individual payment performance of a customer at a certain point of the payment process. If the payment behavior is monitored on-line, we calculate $L_{i,k}$ at a certain t . If we compare, for example, credits $i = 1$ and $i = 5$ at $t = 3$, both show an individual liquidity of $L_{1,3} = L_{5,3} = 1.00$, that is, both customers perform well. But in fact, the customer paying off credit $i = 5$ was in arrears once. This information is lost when we use $L_{i,k}$ for monitoring the payment process at a specific time without looking at the entire payment history.

To overcome this lack of information on the payment history, we define a second indicator $PC_{i,k}$ which we call the *individual payment career*. This indicator relates the total stock

Table 2: Performance indicators for the examples of individual payment histories

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
$L_{1,k}$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$PC_{1,k}$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$L_{2,k}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$PC_{2,k}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$L_{3,k}$	1.0000	1.0000	1.0000	0.7500	0.6000	0.6667
$PC_{3,k}$	1.0000	1.0000	1.0000	0.9000	0.8000	0.7619
$L_{4,k}$	1.0000	1.0000	0.6667	0.5000	0.5000	0.4167
$PC_{4,k}$	1.0000	1.0000	0.8333	0.7000	0.6333	0.5714
$L_{5,k}$	1.0000	0.5000	1.0000	1.0000	1.0000	1.0000
$PC_{5,k}$	1.0000	0.6667	0.8333	0.9000	0.9333	0.9524
$L_{6,k}$	1.0000	1.0000	1.0000	1.5000	1.2000	1.0000
$PC_{6,k}$	1.0000	1.0000	1.0000	1.2000	1.2000	1.1429

of paid amounts to the total stock of expected payments:

$$PC_{i,k} = \frac{\sum_{t=1}^k X_{i,k}}{\sum_{t=1}^k Y_{i,k}} \quad (10)$$

This approach is similar to considering the revolving effect of credit in equation (4). From the results for the exemplary payment histories given in table 2 we see that $PC_{i,k} = 1 \forall t$ only if every installment is paid at the due date. If a customer gets in arrears at least once during the payment period, this is kept in mind by his individual payment career. For credit $i = 5$, where the customer pays late at $t = 2$, we observe that this memory decreases over time, but is not lost completely. Here $PC_{5,k}$ would only equal or exceed 1.00 at some $t > 2$ in the case of an early payment. The same concept applies to the early payer (credit $i = 6$) who shows an individual liquidity of $L_{6,6} = 1.00$ at the end of the payment term, but an individual payment career of $PC_{6,6} = 1.1429$, still indicating that payment in advance occurred.

But in contrast to the individual liquidity, the individual payment career can not directly be interpreted in financial terms, e. g. as proportion of due payments. For credit $i = 4$ for example, $L_{4,k}$ and $PC_{4,k}$ decrease, but the payment career still gives figures > 0.5 at the end of the payment term, although the customer did not pay off half of the financed amount. The respective value of $PC_{4,6} = 0.5714$ (for credit $i = 4$ at $t = 6$, cf. table 2) means that the total stock of payments made amounts to 57.14% of the total stock of expected payments. Hence, it would be suitable to evaluate both indicators in parallel.

5.3 Empirical study: Monitoring and forecasting of credit default events

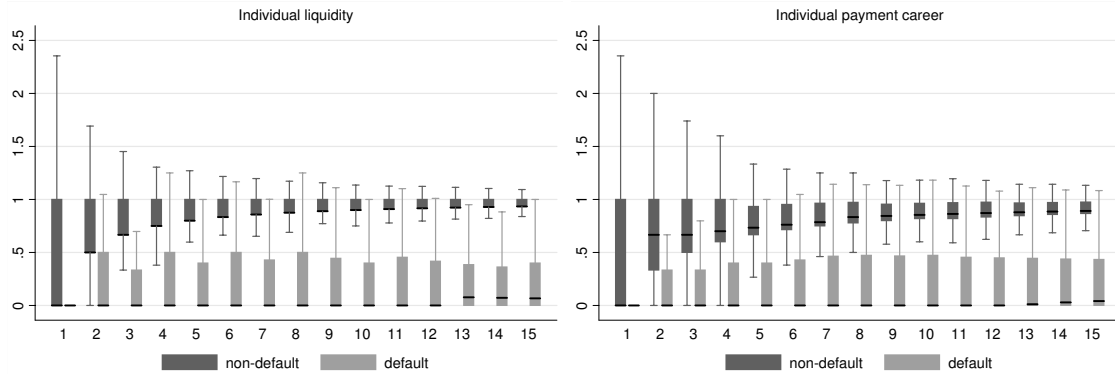
The defined indicators of individual payment performance are illustrated by applying them to the empirical data set. As the emphasis in this section lies on monitoring purposes, calculations are restricted to the regular payment term. For all $n = 33,986$ credits, we calculate the individual liquidity and the individual payment career at each $t = 1, \dots, 15$. If we perceive these numbers as realizations of random variables L and PC , a distribution of the two indicators at each t is generated. As every credit i has already been defined as a default or non-default event (see table 1), we can condition the distribution of each indicator on the default or non-default event. The separation of the two conditional, group-specific distributions shows how well the indicators perform in monitoring the default event.

5.3.1 Group-specific development of payment behavior

In a first step, we generate the conditional distributions of the individual liquidity and the individual payment career for both groups (defaults and non-defaults) separately. To visualize these conditional distributions of the indicators, figure 4 shows box plots of the individual liquidity (left) and the individual payment career (right). At each t for each of the two groups (default and non-default), the box ranges from the 25%-quantile to the 75%-quantile of the specific distribution. The black bar within the box denotes the median of the distribution. The range of the whiskers is calculated by the lower (upper) quartile minus (plus) 1.5 times the inner quartile range, but is restricted to the minimum and maximum observed value of the indicator. The plots exclude outside values, i. e. extremely high values of the indicators, which may emerge from high early payments.

Both indicators develop similarly over time – they give nearly the same picture of payment behavior. For each indicator, the conditional distributions become more separated for increasing t . Credits defined as non-defaults show high values of individual liquidity and individual payment career, with decreasing variation for large t . The respective customers have a higher tendency to pay regularly or even in advance (indicated by the upper whiskers). At the end of the regular payment term ($T = 15$), the distribution of both indicators of non-defaulted credits concentrates on the area around 1.00, which indicates high payment performance. In contrast to the individual liquidity, the individual payment career shows a slightly higher variation even for large t which results from deficient payment at an earlier stage of the payment process.

Figure 4: Empirical study: box plots of individual liquidity and individual payment career



For the group of default events, the median of both indicators equals 0.00 for all $t \leq 12$. This means that at least 50% of the customers paying off these credits show an individual liquidity and an individual payment career of 0.00, i. e. they fail to pay every installment. For $t = 1$ we find that no installment is paid for credits which are later defined as default events. But within the group of defaults also early payments occur, again indicated by the whiskers reaching or even exceeding indicator values of 1.00.

Obviously, the proposed indicators can be used to describe payment behavior and they show the ability to separate good and bad accounts. Although this ability increases with t , the conditional distributions also show an area of intersection even for large t . Therefore, the discriminatory power of the indicators is subsequently analyzed in more detail.

5.3.2 Forecasting of credit default events

An important application of the proposed indicators of individual payment performance is the forecast of default events at a very early stage of the payment process. For an early detection of default events we analyze the payment performance at $t = 6$. Again, both indicators are evaluated separately. Using the conditional distributions over $n = 33,986$ credits described above, we consider every unique value of $L_{i,6}$ ($PC_{i,6}$) as a possible cut-off value κ_L (κ_{PC}) for detecting default events. The default event is detected by the following classification rule: Credit i is assigned to the class of default events, if the individual indicator shows a value $\leq \kappa_L$ ($\leq \kappa_{PC}$). Otherwise, the credit is assigned to the class of non-default events. That is

$$z_i = \begin{cases} 1 & \text{if } L_{i,6} \leq \kappa_L \\ 0 & \text{else} \end{cases} \quad \text{or} \quad z_i = \begin{cases} 1 & \text{if } PC_{i,6} \leq \kappa_{PC} \\ 0 & \text{else} \end{cases} \quad (11)$$

Figure 5: Empirical study: ROC-curves of individual liquidity and individual payment career at $t = 6$

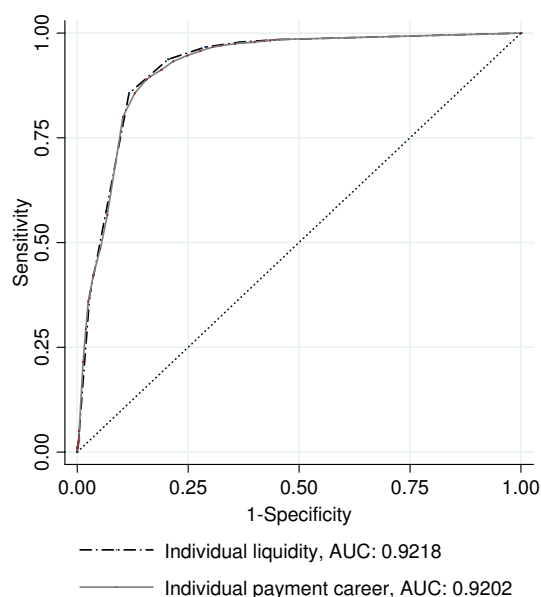


Figure 5 shows the two ROC-curves which result from classifying default events by means of all possible cut-off values of the two indicators. Every point generating one of the curves denotes the classification induced by a certain cut-off value. In this classification task, the sensitivity is the proportion of correctly detected default events, and the 1–specificity is the proportion of good accounts identified as default events. The cut-off values are chosen from the range of the respective indicator in descending order. According to (11), we find higher values of sensitivity and 1–specificity for lower values of the indicator used for detecting default events. The diagonal reference line denotes the trivial solution of the classification task, i. e. the purely random decision on classifying a credit as default or non-default. The area under the curve (AUC) is an overall measure of classification performance. The AUC can be interpreted as the probability that a randomly selected default event shows a lower value of $L_{i,6}$ ($PC_{i,6}$) than a randomly selected non-default event.

Obviously, both indicators perform well in identifying default events already at $t = 6$, with $L_{i,6}$ slightly dominating $PC_{i,6}$. The indicators show similar values of the AUC, 92.18% using the individual liquidity and 92.02% using the individual payment career for classifying credit defaults. This general result is not surprising since a weighted sum of the cumulated expected ($Y_{i,k}$) and the cumulated actual payments ($X_{i,k}$) is used in (4) to appraise the profitability of accounts and to define default events. Nonetheless, it becomes

Table 3: Empirical study: detection of credit default events at $t = 6$

cut-off value	detected defaults		detected non-defaults		total detected
	absolute	% of all defaults	absolute	% of all non-defaults	
$L_{i,6} = 0$	2,949	54.79%	447	1.56%	3,396
$L_{i,6} \leq 1/6$	3,410	63.36%	615	2.15%	4,025
$L_{i,6} \leq 1/3$	3,821	71.00%	950	3.32%	4,771
$L_{i,6} \leq 1/2$	4,276	79.45%	1,787	6.25%	6,063
$PC_{i,6} = 0$	2,949	54.79%	447	1.56%	3,396
$PC_{i,6} \leq 0.1$	3,042	56.52%	500	1.75%	3,542
$PC_{i,6} \leq 0.2$	3,233	60.07%	624	2.18%	3,857
$PC_{i,6} \leq 0.5$	4,219	78.39%	1,939	6.78%	6,158
$L_{i,12} \leq 1/3$	3,923	72.89%	396	1.38%	4,319
$L_{i,15} \leq 1/3$	4,002	74.36%	242	0.85%	4,244

clear that the definition of default events based on the profitability on aggregate level at $t^* = 28$ can be substituted by a definition using the proposed indicators of individual payment performance. This definition and detection of default events can take place at a very early stage of the payment process (here $t = 6$) and is independent of a concrete cost function.

Compared to the cost-based definition of default on aggregate level, which serves as a benchmark in our analysis, the early detection also involves classification errors. Based on the group-specific distributions underlying the ROC-curves we can determine suitable cut-off values for the on-line detection of credit default events. Table 3 gives some examples of possible cut-off values and the resulting correct and incorrect classification of default events. The total number of default events identified in this population of credits is 5,382 (cf. table 1). Setting $L_{i,6} = 0$, which corresponds to the worst case of payment behavior, we detect 2,949 out of these 5,382 default events (54.79%) already at $t = 6$. At the same time we identify 447 credits to be defaulted although they turn out to be non-default events by means of the classification rule (8). Raising the cut-off value, e. g. to $L_{i,6} = 1/3$, leads to an improved detection of default events (71.00%), but at the same time increases the error rate, i. e. classifying non-defaults as default events (3.32%).

The same applies to the detection of default events based on the individual payment career. With $PC_{i,6} \leq 0.5$ for example, we detect 78.39% of the defined default events already at $t = 6$ with an error rate concerning the non-defaults of 6.78%. Furthermore, the discriminatory power of the two indicators increases with t . Using a cut-off value of $L_{i,k} \leq 1/3$, we detect 72.89% of the credit defaults at $t = 12$. For $t = 15$ this proportion

increases to 74.36%. The proportion of non-defaults detected by these cut-off values for the individual liquidity decreases again from 1.38% at $t = 12$ to 0.85% at $t = 15$.

Linking the information given in table 3 to the definition of default given by equation (8) we find, for example, that 2,949 out of all 3,396 credits showing on indicator of $L_{i,6} = 0$ will not be balanced until one year after the end of the agreed payment term. That is, 86.84% of the installment credits, for which nothing has been paid off within the first six months, will later be classified as defaults. Therefore, the proposed indicators may be used as an additional input factor for calculating expected cash flows, the costs of financing installment purchase, and risk-adjusted product prices. Besides a differentiated collection policy, the company granting these credits may also draw conclusions on customer value. At an early stage they are able to estimate how the relation to a certain customer will develop and – in case the customer orders again – to decide whether he should be rejected, granted another credit, or switched to cash sale. On the other hand, but not shown in detail here, well-performing customers can be identified as well, which may profit from incentives or special payment conditions in the future.

6 Discussion

From the review and discussion of existing definitions of credit default events it can be concluded that a consistent concept of credit default does not exist. This is especially the case in the retail segment and for credits granted by non-banks. To contribute to the discussion on this topic, a definition of credit default events is proposed, which is based on the payment patterns and the profitability of a customer portfolio in the context of installment purchases, a special type of sales on credit. Here the retrospective analysis of customer accounts leads to the identification of the acceptable period customers can stay in arrears before they are identified as defaulted. This information can be used for improving the customer selection process and the management of payment conditions: Firms may decide whether or not to grant credit, or if an installment payment is only granted under more restrictive terms (shorter credit period, higher interest rate etc.). As the definition of credit default on aggregate level does not tell us anything about the individual payment behavior of a customer during the credit period, we define indicators for monitoring and forecasting default events already during the payment process.

The procedures are exemplified by a large, unique data set of individual payment histories, which originates from a company trading consumer goods. In this population of

credits, we identify the default events and examine the payment processes of the respective customers. The empirical results show that the proposed indicators are useful tools for measuring the individual payment performance and the profitability of a certain account in each installment payment period. They allow for a forecast of the expected development of each account at a very early stage of the payment process, e. g. in the sixth month of a total credit period of fifteen months. This information gives direct indication of how to improve the credit and collection policy and hence, how to improve cash flows and reduce bad debt loss. In addition, the update of a credit risk model or of parameters like the default probability may be operated much earlier and need not solely depend on the information of closed credits, which becomes available at the end of the credit period at the earliest.

It must be considered that the analysis presented in this paper is based on empirical, historical data of credit histories, and this implies that the company which provided the data took action to collect outstanding amounts. We have to assume that the results are biased as these actions have (or at least are expected to have) influence on the individual payment behavior of customers. Hence, the proposed measures and indicators should be improved by relating the payment histories to the respective histories of collection activities. By combining the payment histories used here and the concurrent history of collection activities, for example by an appropriate model of event history analysis, it would be possible to test the effect a certain activity (e. g. letter of reminder) has on the individual payment behavior. This would again have implications for an improvement of accounts receivable management.

References

Basel Committee on Banking Supervision (2004): International convergence of capital measurement and capital standards. Bank for International Settlements, Basel.

Basel Committee on Banking Supervision (2010): International framework for liquidity risk measurement, standards and monitoring. Bank for International Settlements, Basel.

Brigham, Eugene F. (1992): Fundamentals of financial management. 6th ed., Dryden, Forth Worth.

Caouette, John B., Altman, Edward I., Narayanan, Paul, Nimmo, Robert (2008): Managing credit risk. 2nd ed., Wiley, Hoboken.

Deutsche Bundesbank (2008): Solvabilitaets- und Liquiditaetsverordnung. Frankfurt am Main.

Foster, Dean P., Stine, Robert A. (2004): Variable selection in data mining: building a predictive model for bankruptcy. *Journal of the American Statistical Association*, Vol. 99, pp. 303-313.

Frydman, Halina, Kallberg Jarl G., Kao, Duen-Li (1985): Testing the adequacy of Markov chain and mover-stayer models as representations of credit behavior. *Operations Research*, Vol. 33, No. 6, pp. 1203-1214.

Fueser, Karsten (2001): *Intelligentes Scoring und Rating*. Gabler, Wiesbaden.

Hand, David J. (1998): Consumer credit and statistics. In: Hand, David J., Jacka, Saul D. (eds.): *Statistics in finance*. Arnold, London, pp. 69-81.

Hand, David J., Henley, William E. (1997): Statistical classification methods in consumer credit scoring: a review. *Journal of the Royal Statistical Society, Series A*, Vol. 160, pp. 523-541.

Hoos, Andrea (2006): Debitorenmanagement und Zahlungsabwicklung. In: Mueller-Wiedenhorn, Andreas: *Praxishandbuch Forderungsmanagement*. Gabler, Wiesbaden.

Johnson, Robert W., Kallberg, Jarl G. (1986): Management of accounts receivable and payable. In: Altman, Edward I. (ed.): *Handbook of corporate finance*. Wiley, New York.

Kallberg, Jarl G. and Saunders, Anthony (1983): Markov chain approaches to the analysis of payment behavior of retail credit customers. *Financial Management*, Vol. 12, No. 2, pp. 5-14

Lauer, Hermann (1998): *Konditionen-Management. Wirtschaft und Finanzen*, Duesseldorf.

Lewis, Edward M. (1992): *An introduction to credit scoring*. 2nd ed., Athena Press, San Rafael.

Mueller-Wiedenhorn, Andreas (2006): *Praxishandbuch Forderungsmanagement*. Gabler, Wiesbaden.

Porath, Daniel (2006): *Scoring models for retail exposures*. In: Engelmann, Bernd, Rauhmeier, Robert (eds.): *The Basel II risk parameters*. Springer, Berlin, pp. 25-37.

Reichling, Peter, Bietke, Daniela, Henne, Antje (2007): *Praxishandbuch Risikomanagement und Rating*. 2nd rev. ed., Gabler, Wiesbaden.

Salek, John G. (2005): *Accounts receivable management best practices*. Wiley, Hoboken.

Stone, Bernell K. (1976): *The payments-pattern approach to the forecasting and control of accounts receivable*. *Financial Management*, Vol. 5, No. 3, pp. 65-82.

Thomas, Lyn C., Edelman, David B., Crook, Jonathan N. (2002): *Credit scoring and its applications*. SIAM, Philadelphia.