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## A note on the identification of dynamic economic models with generalized shock processes

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# Working Papers

Kiel Institute  
for the World Economy



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by Christopher Phillip Reicher

No. 1821 | January 2013

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DSGE models with generalized shock processes have been a major area of research in recent years. In this paper, I show that the structural parameters governing DSGE models are not identified when the driving process behind the model follows an unrestricted VAR. This finding implies that parameter estimates derived from recent attempts to estimate DSGE models with generalized driving processes should be treated with caution, and that there exists a tradeoff between identification and the risk of model misspecification.

Keywords: Identification, DSGE models, observational equivalence, maximum likelihood.

JEL classification: C13, C32, E00

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# A note on the identification of dynamic economic models with generalized shock processes

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## Abstract

DSGE models with generalized shock processes have been a major area of research in recent years. In this paper, I show that the structural parameters governing DSGE models are not identified when the driving process behind the model follows an unrestricted VAR. This finding implies that parameter estimates derived from recent attempts to estimate DSGE models with generalized driving processes should be treated with caution, and that there exists a tradeoff between identification and the risk of model misspecification.

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# 1 Introduction

In this paper, I show that attempts to estimate DSGE models whose shock (or wedge) process follows an unrestricted VAR will result in the nonidentification of deep model parameters. In particular, there always exists an unrestricted VAR process for model wedges which can mimic an unrestricted VAR for observables, and vice versa, for any valid set of deep parameters. This implies that the DSGE model has a flat concentrated likelihood and that the deep parameters of the model are hence unidentified. In practical terms, the estimates for model parameters derived from recent attempts to estimate such DSGE models should be treated with a great degree of caution, and there will always remain a tradeoff between identification and the risk of model misspecification.

The estimation of dynamic models which feature general shock processes through likelihood-based methods has become a major area of research in recent years. Cúrdia and Reis (2011), for instance, use Bayesian methods to estimate a large-scale dynamic model whose shock process follows a VAR(1). Their analysis is motivated by the intuitive idea that the orthogonality restrictions typically placed on shocks in DSGE models are arbitrary and restrictive, and that these restrictions carry with them a risk for misspecification. Ireland (2004) looks at a model with observation errors which follow a VAR process, which is isomorphic to the approach of Cúrdia and Reis. Ireland argues that a model with VAR errors can produce more realistic estimates of structural parameters than a model with simple, mutually uncorrelated AR(1) shocks. Most estimation exercises, such as that of Smets and Wouters (2007), have typically relied upon stronger restrictions upon the underlying shock processes. Cúrdia and Reis (2011) present results which suggest substantial differences between their estimates and those of Smets and Wouters, which they attribute to the more general nature of their estimated shock process.

While the motivation behind generalizing the shock process in DSGE models is appealing, I show in this paper that generalizing the shock process can result in a lack of identification if the generalization is taken too far. It turns out that a dynamic model with a driving process governed by an unrestricted VAR is unidentified using likelihood-based methods for the simple reason that a model with such a driving process can approximate an unrestricted VAR in the observables arbitrarily well for any valid set of structural parameter values. In order to achieve identification, it is therefore necessary to make meaningful restrictions on the driving process governing the model; otherwise, model parameters cannot put any meaningful restrictions on the law of motion for the observables. First I present some analytical results which show that this is a general problem. Then, I set up a concrete example. I show that in the classic three-equation New Keynesian model with errors governed by an unrestricted

VAR(1) driving process, that VAR process can trivially fit an unrestricted VAR(1) to the data regardless of the values of the structural parameters. It turns out that there remains some need to make restrictive identifying assumptions in order to achieve identification.

The nonidentification result fits into a rapidly growing literature on the identification of structural parameters in DSGE models.<sup>1</sup> The traditional way to assess identification has been to check that the information matrix is of full rank, following Rothenberg (1971). Canova and Sala (2009) provide a set of diagnostics intended to detect possible nonidentification when matching impulse responses. Consolo, Favero, and Paccagnini (2009) discuss the identification of DSGE models within the context of DSGE-VAR and DSGE-FAVAR approaches. Iskrev (2010) and Komunjer and Ng (2011) discuss further conditions under which economic models may or may not be locally identified in a classical setting. Cochrane (2011) discusses the ways in which the parameters which govern unstable eigenvalues in DSGE models may not show up in the data and hence are not identified. Koop, Pesaran, and Smith (2011) discuss the identification of Bayesian models and propose examining the rate of decay of posterior variances as subsamples get larger, in order to get a sense of whether the model seems to be converging toward some mode. These methods to assess identification are highly useful, but they generally require the actual estimation of a model, which can be difficult when the model is large or poorly-behaved. For instance, even the basic three-equation, New Keynesian model with a Taylor rule, aggregate supply equation, intertemporal asset pricing equation, and AR(1) errors has eleven parameters, and it has a poorly-behaved likelihood function. That model but with VAR(1) errors has twenty parameters in total, while a simple unrestricted VAR(1) in the observables has only fifteen parameters. Instead of presenting numerical results, I show analytically that a DSGE model is not identified when its errors follow an unrestricted VAR process. This is useful to know *a priori* to the extent that the full Bayesian estimation of such a DSGE model may still produce parameter estimates, and these parameter estimates may even conceivably pass certain tests of identification even though the model is identified by its priors and not by its likelihood.

The rest of this paper follows a simple structure. First I show that if the shock process behind a DSGE model follows a VAR process, then the linearized observables follow a VAR process, and vice versa. Then I show that these two propositions imply that the concentrated likelihood of the deep parameters governing the DSGE model is perfectly flat. I then give the example of what this looks like in case of the three-equation New Keynesian model. I conclude with a word of caution and some words about alternative approaches to dealing with potential model misspecification.

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<sup>1</sup>Cúrdia and Reis (2011) present an excellent review of the previous literature on the specification of shocks in DSGE models. I only give a brief summary here.

## 2 The issue of identification

In this section I show that meaningful restrictions on the law of motion for the driving processes (or wedges, in the language of Chari, Kehoe, and McGrattan (2007) and Šustek (2011)) behind a model are necessary for the identification of deep model parameters. First, I show that a VAR process for the model wedges implies a VAR process in the observables, and vice versa. Then I show that these two results imply nonidentification for the deep parameters of a DSGE model.

### 2.1 Mapping from wedges to the data

In this section I show that an unrestricted VAR process for the shocks to an economic model results in a VAR process for the observables. A dynamic economic model is governed by a parameter set  $\theta$ . Elements of  $\theta$  may include parameters which govern production technology, labor supply, frictions in price and wage formation, adjustment costs, and so on. A set of mean-zero exogenous structural wedges  $w_t$  of rank  $k$  follows a VAR process with the law of motion:

$$w_t = \sum_{i=1}^{\infty} F_i w_{t-i} + \zeta_t, \text{ where } E\zeta_t \zeta_t' = \Sigma_w. \quad (1)$$

The matrices  $\{F_i\}$  are unrestricted  $k$  by  $k$  matrices of VAR coefficients. Following the majority of the literature, I focus here on de-meaned, linearized approximations to DSGE models. While levels (e.g. of labor's share or of the real interest rate) can contain substantial information about model parameters, I focus on the information contained in the dynamic laws of motion for the observables.

A complete linearized DSGE model including wedges and observables can be represented using the notation of Sims (2002), treating the law of motion (1) of the wedges as given:

$$\Gamma_{0,0}x_{t+1} = \Gamma_{1,0}x_t + \Pi_0\eta_{0,t+1} + \Psi_0\zeta_{t+1}. \quad (2)$$

The matrices  $\Gamma_{0,0}$ ,  $\Gamma_{1,0}$ ,  $\Pi_0$ , and  $\Psi_0$ , as described by Sims (2002), are functions of the deep model parameters  $\theta$  and the VAR coefficients  $\{F_i\}$ . The complete state matrix  $x_t$  contains the observables  $z_t$ , the wedges  $w_t$ , and any other auxiliary variables included in the model. Endogenous expectational errors, which are functions of the shocks to the wedges, are given by  $\eta_{0,t+1}$ . I assume that  $\zeta_t$ ,  $w_t$ , and  $z_t$  have the same rank  $k$ , and that the model is stationary (to ensure invertibility) and locally determinate (which is not strictly necessary).

The results here are robust to a larger system for the wedges or to local indeterminacy (for which some mapping from wedges into observables still exists but may be indeterminate); smaller systems will almost surely result in stochastic singularity.

The observables  $z_t$  are linked to the system through the observation equation  $z_t = Hx_t$ . The law of motion for the wedges (1) and the model equations form the rows of (2). The model has a solution of the form:

$$x_t = A_0 x_{t-1} + B_0 \zeta_t, \quad (3)$$

which usually has to be solved for numerically. Iterating the reduced-form law of motion (3) forward yields the following expression for the observables as a function of the history of the structural shocks:

$$z_t = Hx_t = H \sum_{i=0}^{\infty} A_0^i L^i B_0 \zeta_t. \quad (4)$$

The expectational error for  $z_t$ , denoted by  $\varepsilon_t$ , equals  $HB_0\zeta_t$ . I assume that the matrix  $H\Lambda B_0$  is of rank  $k$  for any full-rank matrix  $\Lambda$ , so that the model can explain the data. Substituting the relationship between innovations to the data and innovations to the wedges into (4) gives the infinite-order MA process which governs the evolution of the data:

$$z_t = H \sum_{i=0}^{\infty} A_0^i L^i B_0 (HB_0)^{-1} \varepsilon_t, \quad (5)$$

which, since the system implied by  $A_0$  is strictly stationary, can be written as  $z_t = H(I - A_0L)^{-1}B_0(HB_0)^{-1}\varepsilon_t$ . Because the matrix premultiplying  $\varepsilon_t$  is of full rank ensuring invertibility, the system can alternatively be written in a VAR form with some coefficients  $\Phi$  and a covariance matrix for its errors given by  $\Sigma_z$ :

$$z_t = \sum_{i=1}^{\infty} \Phi_i z_{t-i} + \varepsilon_t, \text{ where } E\varepsilon_t \varepsilon_t' = \Sigma_z = HB_0 \Sigma_w B_0' H'. \quad (6)$$

The matrices  $\{\Phi_i\}$  are the  $k$  by  $k$  matrices of VAR coefficients which govern the evolution of the data, given the driving process coefficients  $\{F_i\}$  and the model coefficients  $\theta$ . A VAR process for the wedges therefore implies a VAR process for the observables, which may, especially in the presence of endogenous state variables, possibly be of infinite order as shown by Ravenna (2007). This is not a new result—del Negro and Schorfheide (2009) also discuss this property of DSGE models, which forms the backbone of the DSGE-VAR literature—but it is important for what follows. It turns out that the converse of this statement is true—if



the data follow an unrestricted VAR process, then it is possible to recover the VAR process governing the wedges as well. By estimating an unrestricted VAR on the observables, one could always fit a wedge-generating process similar to (1).

## 2.2 Mapping from the data to the wedges

To show that (6) implies (1), I again represent the model using the notation of Sims (2002) in equation (2), but this time I treat the law of motion (6) as given instead of (1). The variables  $x_t$  again contain the observables  $z_t$ , the wedges  $w_t$ , and any other auxiliary variables included in the model. Endogenous expectational errors, which are functions of the shocks to the wedges or equivalently the shocks to the data, are given by  $\eta_{1,t+1}$ . I again assume that  $\varepsilon_t$ ,  $w_t$ , and  $z_t$  have the same rank  $k$  and that the model is stationary and locally determinate. Formally, the system contains the law of motion for the data (6) plus any model equations. The system in the notation of Sims (2002) now takes the form:

$$\Gamma_{0,1}x_{t+1} = \Gamma_{1,1}x_t + \Pi_1\eta_{1,t+1} + \Psi_1\varepsilon_{t+1}. \quad (7)$$

The matrices  $\Gamma_{0,1}$ ,  $\Gamma_{1,1}$ ,  $\Pi_1$ , and  $\Psi_1$ , as described by Sims (2002), are functions of the deep model parameters  $\theta$  and the VAR coefficients  $\{\Phi_i\}$ . The wedges  $w_t$  are linked to the system through the observation equation  $w_t = Dx_t$ . The augmented model has a solution of the form:

$$x_t = A_1x_{t-1} + B_1\varepsilon_t, \quad (8)$$

which usually has to be derived numerically. Iterating (8) forward yields the wedges as a function of the history of the innovations to the data:

$$w_t = Dx_t = D \sum_{i=0}^{\infty} A_1^i L^i B_1 \varepsilon_t. \quad (9)$$

The expectational error for  $w_t$ , denoted by  $\zeta_t$ , equals  $DB_1\varepsilon_t$ . I assume that the matrix  $D\Lambda B_1$  is of rank  $k$  for any full-rank matrix  $\Lambda$ , which is equivalent to saying that the model is relevant to the data. Substituting this relationship into (9) gives the infinite-order MA process which governs the evolution of the wedges:

$$w_t = D \sum_{i=0}^{\infty} A_1^i L^i B_1 (DB_1)^{-1} \zeta_t, \quad (10)$$

which, since the system implied by  $A_1$  is strictly stationary, can be written as  $w_t = D(I - A_1L)^{-1}B_1(DB_1)^{-1}\zeta_t$ . Because the matrix premultiplying  $\zeta_t$  is of full rank ensuring invertibility, this system can be written in the VAR form given in (1), which again may be infinite in order.

It is therefore econometrically equivalent to treat the wedges as linear functions of the data as in this section or the data as a linear function of the wedges as in the previous section. Knowing the law of motion of one, conditional on  $\theta$ , gives the law of motion of the other.

### 2.3 Main result: Nonidentification under an unrestricted $F$

It is possible to prove based on the results from the previous two sections that estimating (1) by maximum likelihood results in a flat concentrated likelihood of  $\{z_t\}$  for any parameter set  $\theta$ , so that  $\theta$  is not identified through likelihood-based methods. Proof proceeds through construction. Equations (6) through (10) suggest that there is a simple way to estimate (1) by maximum likelihood (or by some other criterion such as GMM) when  $F$  and  $\Sigma_w$  are unrestricted. First, estimating (6) by OLS delivers the unrestricted maximum likelihood estimates of  $\Phi$  and  $\Sigma_z$ , given by  $\hat{\Phi}$  and  $\hat{\Sigma}_z$ , respectively. Then numerically solving for the values of  $F$  and  $\Sigma_w$  implied by  $\hat{\Phi}$  and  $\hat{\Sigma}_z$ , by going through steps (7) through (10), gives a set of estimates for the law of motion for the wedges  $\hat{F}$  and  $\hat{\Sigma}_w$ , for a given parameter set  $\theta$ . Since  $\hat{F}$  and  $\hat{\Sigma}_w$  imply a VAR system of the same form as (6), it is not possible to increase the likelihood of  $\{z_t\}$  any further, or else  $\hat{\Phi}$  and  $\hat{\Sigma}_z$  would not be maximum likelihood estimates of  $\Phi$  and  $\Sigma_z$  in the first place. Therefore  $\hat{F}$  and  $\hat{\Sigma}_w$  are maximum-likelihood estimates of  $F$  and  $\Sigma_w$  as well. Furthermore, since the density of  $z_t$  evaluated at  $\hat{\Phi}$  and  $\hat{\Sigma}_z$  does not depend at all on  $\theta$ ,  $\theta$  is unidentified in the sense that the concentrated likelihood of  $\theta$  is perfectly flat. The estimated parameters  $\hat{F}$  and  $\hat{\Sigma}_w$  are functions of  $\theta$ , but the parameters  $\hat{\Phi}$  and  $\hat{\Sigma}_z$  are not. As  $\theta$  varies,  $\hat{F}$  and  $\hat{\Sigma}_w$  have enough degrees of freedom to simply adjust in order to bring the estimated driving process (1) completely into line with the estimated law of motion (6).

This result should not be entirely surprising, since it mirrors a century of work on the identification of systems of equations. A simple static system of supply and demand is illustrative. To estimate a supply and demand system with two equations and data on quantity and prices, it is necessary to make additional identifying restrictions or to bring in outside information. One way to do this is through instrumental variables, where some set of shocks is assumed to be uncorrelated with another set of shocks—the classic example given by Wright (1928) involves taking shocks to the productivity of land as orthogonal to shocks

to the demand for butter and flaxseed. Wright uses this orthogonality assumption in order to estimate the elasticities of demand for these two commodities. The same situation holds in DSGE models with respect to orthogonality assumptions, in which case it is necessary to make meaningful restrictions on  $F$  and  $\Sigma_w$  in order to identify  $\theta$ . Identification requires finding a way to break the tight link between (6) and (1), and this is done through placing restrictions on (1). Typical restrictions placed in the macroeconomic literature are to assume that the off-diagonal elements of  $F$  and  $\Sigma_w$  are zero or that first moments contain useful information (e.g. using information from labor's average share of income to identify labor's share in a Cobb-Douglas production function). It is simply not possible to dispense with restrictions of this sort in the absence of other meaningful prior information. In a sense, the original critique made by Sims (1980) of the simultaneous equations literature cannot be fully reconciled with the DSGE approach. There will always be some degree to which DSGEs must place unbelievable restrictions on the data in order for the data to place believable restrictions on DSGE parameters.

### 3 A practical example of nonidentification

Here, I illustrate the problem of identification with a simple example based on the textbook 3-equation New Keynesian model mentioned in the introduction. The output gap  $y_t$  is related to the inflation gap  $\pi_t$  through an aggregate supply equation; the parameter  $\kappa$  reflects the effect of inflation on output, and  $\beta$  is the rate of time preference. Output is also related to future output, inflation, and current interest rates through an aggregate demand equation, where the parameter  $\sigma$  governs the willingness of consumers to substitute across time. Interest rates are governed by a Taylor Rule which relates interest rates to inflation and output through the Taylor rule coefficients  $\phi_\pi$  and  $\phi_y$  respectively. In the current example there is no interest rate smoothing, for the sake of simplicity.

The system, with wedges  $w_t$ , is expressed by the following three equations:

$$y_t = \kappa\pi_t - \kappa\beta E_t\pi_{t+1} + w_t^s; \tag{11}$$

$$y_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + E_t y_{t+1} + w_t^d; \tag{12}$$

and

$$i_t = \phi_\pi\pi_t + \phi_y y_t + w_t^i. \tag{13}$$

The wedges  $w_t^s$ ,  $w_t^d$ , and  $w_t^i$  represent reduced-form disturbances to aggregate supply, aggre-

gate demand, and monetary policy, respectively.

The system (11) through (13) written in the canonical form (7) takes the following form, assuming that the observables follow a VAR(1) with a coefficient matrix  $\Phi$ :<sup>2</sup>

$$\begin{aligned}
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \kappa\beta & 0 & 0 & 0 & 0 \\ -1 & -1/\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ w_{t+1}^s \\ w_{t+1}^d \\ w_{t+1}^i \end{bmatrix} &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & 0 & 0 \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & 0 & 0 & 0 \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & 0 & 0 & 0 \\ -1 & \kappa & 0 & 1 & 0 & 0 \\ -1 & 0 & -1/\sigma & 0 & 1 & 0 \\ \phi_y & \phi_\pi & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ i_t \\ w_t^s \\ w_t^d \\ w_t^i \end{bmatrix} \\
&+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^y \\ \varepsilon_{t+1}^\pi \\ \varepsilon_{t+1}^i \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1/\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_{t+1}^{ws} \\ \eta_{t+1}^{wd} \\ \eta_{t+1}^{wi} \end{bmatrix}. \tag{14}
\end{aligned}$$

It turns out in this case that the wedges  $w_t$  are a simple linear function of the observables  $z_t$ , and this fact greatly facilitates finding the law of motion for the wedges (which in a more general case may have to be solved for numerically). To see this, the bottom three lines of the system can be rewritten as obeying:

$$\begin{bmatrix} 0 & \kappa\beta & 0 \\ -1 & -1/\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t i_{t+1} \end{bmatrix} = \begin{bmatrix} -1 & \kappa & 0 \\ -1 & 0 & -1/\sigma \\ \phi_y & \phi_\pi & -1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} + \begin{bmatrix} w_t^s \\ w_t^d \\ w_t^i \end{bmatrix},$$

so after substituting in the law of motion for the observables and rearranging,

$$\begin{bmatrix} w_t^s \\ w_t^d \\ w_t^i \end{bmatrix} = \left( \begin{bmatrix} 0 & \kappa\beta & 0 \\ -1 & -1/\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \Phi - \begin{bmatrix} -1 & \kappa & 0 \\ -1 & 0 & -1/\sigma \\ \phi_y & \phi_\pi & -1 \end{bmatrix} \right) \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix},$$

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<sup>2</sup>In typical implementations of the Sims (2002) algorithm, the bottom row of coefficients governing the Taylor rule is placed on the  $t + 1$  side and not on the  $t$  side.

or equivalently,

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \left( \begin{bmatrix} 0 & \kappa\beta & 0 \\ -1 & -1/\sigma & 0 \\ 0 & 0 & 0 \end{bmatrix} \Phi - \begin{bmatrix} -1 & \kappa & 0 \\ -1 & 0 & -1/\sigma \\ \phi_y & \phi_\pi & -1 \end{bmatrix} \right)^{-1} \begin{bmatrix} w_t^s \\ w_t^d \\ w_t^i \end{bmatrix},$$

which can be represented by writing  $w_t = Jz_t$  or  $z_t = J^{-1}w_t$ , respectively.

Substituting the latter representation of the mapping between the data and wedges into the law of motion for the data (6) gives the law of motion for the wedges:

$$J^{-1}w_t = \Phi J^{-1}w_{t-1} + \varepsilon_t,$$

so that the wedges follow a VAR(1) of their own:

$$w_t = J\Phi J^{-1}w_{t-1} + J\varepsilon_t. \tag{15}$$

The important thing to note is that any VAR process for  $z_t$  implied by  $\Phi$  and  $\Sigma_z$  maps one-to-one into a valid VAR process for  $w_t$  implied by  $J\Phi J^{-1}$  and  $J\Sigma_z J'$ , unless one puts some meaningful restriction on the latter objects. This is true no matter what the contents of  $J$  turn out to be, so long as  $J$  is not singular. The parameters  $\{\kappa, \beta, \sigma, \phi_\pi, \phi_y\}$  do not place any meaningful restrictions on the wedge process needed to perfectly match an unrestricted VAR(1) on the data. In this particular case, a DSGE model with twenty parameters, only five of which are structural, can exactly replicate an unrestricted VAR model which has fifteen parameters. The fifteen parameters of the VAR driving process for the wedges can match the fifteen parameters of an estimated VAR which governs the observables.

## 4 Conclusion

In this paper, I have shown that DSGE models with a VAR shock process suffer from serious problems with identification. If the observables follow a VAR process as they do under such models, then there is always a set of parameters governing the shock process which can replicate an unrestricted VAR arbitrarily well, for any values for the model parameters  $\theta$ . In the case of the textbook three-equation New Keynesian model, the mapping between wedges and observables is perfectly linear. Using likelihood-based methods or the method of moments does not make it possible to identify  $\theta$  in this circumstance, since there is always some shock process governing the wedges which can generate the patterns seen in the data. The problems identified by Cúrdia and Reis (2011) and others with existing identification

schemes therefore does not appear to have a satisfactory solution. It does not seem to be possible to dispense with *a priori* statements regarding the nature of the wedge process without losing the ability to identify model parameters. With identifying assumptions comes a risk of model specification.

In order to deal with potentially misspecified models, it remains necessary to exercise considerable caution and judgment. An and Schorfheide (2007), for instance, discuss the role that posterior predictive checks and posterior odds comparisons can play in diagnosing potential misspecification and in building better models, while del Negro and Schorfheide (2009) discuss how to deal with potential misspecification in different ways when performing policy analysis. They argue that it is particularly hazardous to treat the behavior of a generalized shock process as exogenous to policy. Interestingly, they allude to the identification issues associated with allowing exogenous dynamics to drive the dynamics of the observables. They also discuss the approach taken by del Negro and Schorfheide (2004) in using DSGE models as priors for structural VAR systems (the DSGE-VAR approach), and how this approach can be used to formally discuss model misspecification. The idea behind DSGE-VARs is that a restricted DSGE model can produce a higher posterior data density than an unrestricted VAR. Analyzing the hyperparameter governing the strength of the DSGE prior for the VAR provides a formal way to discuss model misspecification. More work remains to be done in finding parsimonious ways to balance misspecification with identification in the context of estimating model parameters in a computationally efficient way, but such an approach seems more likely to yield fruitful results than allowing for additional parameters to govern the driving process for model wedges.

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