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**Institut für Höhere Studien (IHS), Wien  
Institute for Advanced Studies, Vienna**

**Reihe Ökonomie / Economics Series**

**No. 1**

## **HOUSING MARKETS AND VACANT LAND**

**Christian Keuschnigg**

**Søren Bo Nielsen**



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## **Abstract**

The paper examines land use in an urban zone. We propose a partial equilibrium model of housing markets with both vacant and built up land. The existing literature assumes building decisions to be irreversible and treats any given stock of vacant land as an exhaustible resource. By way of contrast, we argue that vacant land is built upon in finite time rather than asymptotically, and reduces to a temporary phenomenon only. When housing stocks depreciate, previously built up land is released and may be rebuilt at any structural intensity dictated by the going market conditions. The continued replacement of existing structures allows to adjust the average structural intensity in the city even if all land is built upon. We explain analytically the long-run equilibrium with full land use and demonstrate numerically that vacant land may still be an important transitional phenomenon. In particular, we emphasize the implication for land prices.

## **Zusammenfassung**

Die Arbeit untersucht die Landnutzung in einer urbanen Zone. Wir stellen ein partielles Gleichgewichtsmodell der Wohnungsmärkte mit bebauten und unbebauten Flächen vor. Die existierende Literatur behandelt die Bauentscheidungen als irreversibel und behandelt freies Bauland als eine erschöpfbare Ressource. Im Gegensatz dazu argumentieren wir, daß freies Bauland in endlicher Zeit und nicht erst asymptotisch bebaut wird. Freies Bauland kann daher nur als temporäres Phänomen existieren. Im Zuge des physischen Verfalls wird bebautes Land frei und steht für neue Nutzungen mit einer beliebigen Baudichte wieder zur Verfügung. Die laufende Umwälzung des Häuseraltbestandes erlaubt eine fortwährende Anpassung der durchschnittlichen Baudichte, selbst wenn schon die gesamte Stadtfläche bebaut ist. Wir erklären analytisch das langfristige Gleichgewicht auf den Wohnungs- und Baulandmärkten bei voller Landnutzung und zeigen numerisch, daß freies Bauland immer noch ein quantitativ bedeutsames Übergangsphänomen sein kann. Insbesondere betonen wir die Implikationen für die Baulandpreise.

**Remarks by authors:**

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# 1 Introduction

Despite pressing needs for more apartments and steep housing rents, many cities actually experience that vacant land is not built upon. In several places, part of the area allocated for use in housing remains idle over long time spans. Obviously, tight housing markets alone will not suffice to induce investors to make full use of available land. But why would agents deliberately forego the productive use of land?

A more or less obvious reason lies in rapidly rising land prices. The associated capital gains may produce a return on holding vacant land that is as good as or even superior to the yield on alternative financial assets. Hence the owners of vacant land feel an incentive to postpone the sales in order to cash in the capital gains at some later date. Relying on this kind of argument, an innovative article by Sinn (1986) rationalizes the existence of vacant land as an equilibrium response on the part of markets to rapidly growing housing demand. Since he excludes any depreciation of housing stocks, the basic reason for vacant land ultimately derives from the fact that the decision on the structural density of housing (the quantity of housing structure on each piece of land) is irreversible. Once land is built upon, it never becomes available again for new construction activities with a possibly revised structural density.

Within Sinn's framework one might, at first sight, expect that it would be best to build up all land immediately. However, in case of rapid demand growth, housing markets are very tight in the future but relatively sluggish in the present. Given the comparatively weak state of current demand, a rather low structural intensity will be chosen. Unfortunately, the housing sector will then be locked into this structural intensity forever. Hence, it becomes optimal from a social viewpoint (as well as a private one) to build up only part of available land now with a low intensity and leave the rest for future construction activities with high intensities. By gradually depleting the stock of vacant land, cities may flexibly raise the structural density of new housing units in line with the growth in rental demand.

Is vacant land still viable when houses depreciate? The present paper endeavours to explain how the incentives for keeping vacant land are affected in the face of depreciation of houses that could be replaced with new structures. Even though it might appear to be a minor change of the model, depreciation in fact has major consequences for the operation of housing and land markets. When houses depreciate, land is released and may be newly built upon. Agents may replace old houses with new ones, choosing any desired structural intensity that is dictated by the going market conditions. The incentive to keep land vacant and to 'save' it for future building purposes is weakened since the average structural intensity of housing stocks may now be revised over time in the process of housing replacement. By analogy to natural resource economics, land assumes the characteristics of a renewable rather than an exhaustible resource. Indeed, we shall argue that a historically given stock of vacant land, if any, is depleted in finite time rather than



asymptotically as in Sinn's framework. Accordingly, vacant land is at most a temporary phenomenon when – quite realistically – housing depreciation is taken into account.

Under what circumstances will markets refrain from productively using all land? How important is the phenomenon? How rapidly is vacant land built up with structures? We show, analytically and by means of simulation exercises, that land may remain vacant over extended periods of time. Developing the model in an applied direction, we include adjustment costs in housing construction as well as land development costs. Thus, we capture the long gestation periods and long response times typically observed in housing markets. With the help of a realistically calibrated simulation model we then demonstrate how rapidly an historically given stock of vacant land gets built up with new structures in the face of sustained growth in housing demand. To demonstrate certain asymmetries in our model, we additionally investigate several reverse scenarios: housing markets start out with full productive use of all land but vacant land is created subsequently in response to serious negative shocks such as those envisaged in Mankiw and Weil (1989).

Apart from the obvious inspiration exerted by Sinn's (1986) paper, our approach to modeling housing markets is also related to various other contributions to the housing literature. As part of the housing stock depreciates, land is released and becomes available for new construction purposes. This turnover makes available a continuous flow of land which is in fixed supply once the city area is completely built up. In the long-run, our model thus resembles Feldstein's (1977) treatment of land as a fixed asset. Apart from the fixed amount of land consumption, the housing market then operates like in Poterba's (1984) model which simultaneously determines housing stocks and prices [see also Poterba (1991) and Mankiw and Weil (1989)]. The land price in our model capitalizes the flow of land rents in the construction activity. This mechanism is familiar from asset pricing with a fixed asset [see Calvo, Kotlikoff and Rodriguez (1979)]. Finally, Gavhari (1985) and Goulder (1989) address issues related to housing markets by means of simulation models. None of them considers the role of land, however.

The paper attacks the issues outlined above in five sections. Section 2 introduces the basic model of how tenants, landlords, and landowners interact in the markets for rental housing and land. Having described incentives facing market participants, we prove at the end of section 2 that land cannot remain vacant indefinitely when housing stocks depreciate. In section 3 we discuss how markets respond to exogenous demand and supply shocks. We develop analytical solutions of long-run equilibrium and transitional dynamics under conditions of full land use. Given the complexity involved, we are unable to characterize with the same rigour transitional adjustment in the presence of vacant land. We therefore calibrate in section 4 a simulation model with inertia in housing construction to get a more realistic picture of the transitional phase where vacant land is created and subsequently depleted. Section 5 concludes.

## 2 Housing Markets and Vacant Land

The easiest and most direct way to answer the questions posed in the introduction is to apply a partial equilibrium model of an urban zone. The model incorporates trading in the markets for rental housing and land and determines equilibrium prices that make the plans of tenants, landlords, and landowners consistent with each other. Our model is an extension of Sinn's (1986) paper to which we refer for a comparison of our results.

### 2.1 Landlords

Landlords earn an income by renting out the existing stock of houses  $H$  to tenants at a rental rate  $\Pi$ . To enhance future revenues, they build new houses  $N$  to expand existing stocks. Housing construction relies on a technology  $N(I, F)$  and requires residential investment goods  $I$  as well as a flow of land consumption  $F$ . Assuming linear homogeneity, one may write the production function in intensive form,

$$N = \phi(\epsilon)F, \quad \phi'(\epsilon) > 0, \quad \phi''(\epsilon) < 0, \quad \epsilon \equiv I/F. \quad (1)$$

The technology is assumed to require a constant share of land expenses  $\alpha = \epsilon\phi'(\epsilon)/\phi(\epsilon)$  in the total construction cost of a new house. The elasticity of the marginal product of capital is  $\beta = -\epsilon\phi''(\epsilon)/\phi'(\epsilon) = 1 - \alpha$ .

The existing stock of housing units depreciates at a constant rate  $\delta$ . Hence, part of housing construction merely serves to replace depreciated houses. Given a gross building activity  $N$ , the overall housing stock evolves according to

$$\dot{H} = N - \delta H. \quad (2)$$

Housing construction is typically characterized by long gestation periods. We capture stickiness in housing construction by including adjustment costs. The higher is the rate of new housing construction relative to existing stocks, the larger are adjustment costs. House builders can save on these costs by making stock adjustment more gradual, and indeed they will find it optimal to stretch their investment programs over time. Specifically, we formalize adjustment costs by assuming that builders need to purchase  $\kappa(n)I$  units of residential investment goods to make  $I$  units productive in the generation of new houses [see (1)] where the factor  $\kappa(n)$  increases progressively with the flow to stock ratio  $n \equiv N/H$ . With  $P^I$  denoting the price of residential investment goods, the total cost of rendering  $I$  units of investment goods productive amounts to

$$P^I \kappa(n)I, \quad \kappa(n) \geq 1, \quad \kappa''(n) \geq 0.$$

The analysis is simplified considerably by choosing a convenient normalization  $\kappa(\delta + g_h) = 1$  and  $\kappa'(\delta + g_h) = 0$ . Whenever the flow to stock ratio  $n$  is such that stocks expand at the balanced long-run growth rate  $g_h$  (i.e.  $n = \delta + g_h$ ), adjustment costs vanish.

The stock of built up land  $L$  that is productive in the generation of housing services, is augmented or run down depending on the landlord's real estate decisions. They purchase a net amount of land  $F - \delta L$  and pay the prevailing market price  $P^B$ . Erosion of the existing housing stock at a constant rate  $\delta$  releases previously built upon land at the same rate. Thus, a flow  $\delta L$  may be sold on the market for vacant land, or may be rebuilt with new housing structures. On the other hand, construction activities require a gross acquisition rate of land  $F$ . Of course, it cannot become negative since land cannot be sold by landlords at a higher rate than what is currently released by housing depreciation.<sup>1</sup> Depending on real estate considerations, the stock of built up land accumulates according to

$$\dot{L} = F - \delta L, \quad F \geq 0. \quad (3)$$

Raw vacant land needs to be prepared before it becomes suitable for housing construction. Land development or retrofitting is assumed to entail a cost  $c$  per unit of newly developed land. Hence, the landlord's current flow of consumable income generated in the housing business adds up to

$$\chi = \Pi H - (P^I \kappa(n)\epsilon + c)F - P^B(F - \delta L). \quad (4)$$

Landlords require a rate of return on housing investments that matches the return on alternative assets,  $r = (\chi + \dot{V})/V$ , where  $V$  denotes the value of the housing stock, inclusive of built-upon land. Excluding any speculative bubbles,<sup>2</sup> landlords value housing stocks according to their fundamental value. They choose a level of new land and a capital intensity of new housing that generates maximum wealth,

$$V_t = \max \int_t^\infty \chi_s e^{-r(s-t)} ds, \quad (5)$$

subject to constraints

$$\begin{aligned} (P^H) \quad \dot{H} &= \phi(\epsilon)F - \delta H, \\ (P^L) \quad \dot{L} &= F - \delta L, \\ (\mu^F) \quad F &\geq 0. \end{aligned} \quad (6)$$

The shadow prices  $P^H$  and  $P^L$  refer to the laws of motion for houses and productive land, respectively, while the inequality multiplier  $\mu^F$  expresses the tightness of the non-negativity constraint on land consumption.

<sup>1</sup>Hence, we exclude the possibility that houses may be deliberately torn down to increase the amount of land that can be sold or newly rebuilt.

<sup>2</sup>The no bubbles condition requires  $\lim_{s \rightarrow \infty} V_s e^{-r(s-t)} = 0$ .

The landlord's optimal plans are easily derived upon operating on the current value Hamiltonian

$$\mathcal{H} = \chi + P^H[\phi(\epsilon)F - \delta H] + P^L[F - \delta L] + \mu^F F. \quad (7)$$

Among the necessary conditions we have, in addition to the laws of motion,

$$\begin{aligned} (a) \quad \epsilon &: \phi'(\epsilon)[P^H - \kappa'(n)P^I I/H] = P^I \kappa(n), \\ (b) \quad F &: m + P^L + \mu^F = P^B + c, \quad \mu^F \geq 0, F \geq 0, \mu^F F = 0, \\ (c) \quad H &: \dot{P}^H - (r + \delta)P^H = -[\Pi + P^I \kappa'(n)nI/H], \\ (d) \quad L &: \dot{P}^L - (r + \delta)P^L = -\delta P^B, \\ (e) \quad P_\infty^X &: \lim_{s \rightarrow \infty} P_s^X X_s e^{-r(s-t)} = 0, \quad X = \{H, L\}. \end{aligned} \quad (8)$$

We understand the marginal value product of land  $m$  as net of adjustment costs. Using (8a), we get  $m \equiv P^H \phi(\epsilon) - P^I \epsilon(\kappa(n) + \kappa'(n)n) = P^I \kappa(n) \epsilon \beta / \alpha$ . As is usual, the investment condition (8a) compares marginal costs of new structures (including adjustment costs) with their benefits. The marginal benefits stem from  $\phi'(\epsilon)$  units of new housing structures multiplied by their value  $P^H$ . The value of a new house consists of the present value of newly created rental income. The valuation condition (8c) governs the evolution of the shadow price of housing stocks. Building a new house creates a flow of future rental income. Furthermore, a higher stock today also yields future savings in adjustment costs. Both components of future income flows are capitalized in the shadow price of housing stock which is seen, upon forward solution of (8c), to be equal to

$$P_t^H = \int_t^\infty [\Pi + P^I \kappa'(n)nI/H] e^{-(r+\delta)(s-t)} ds. \quad (9)$$

The effective discount rate is increased by the depreciation rate, since the capacity of a house in existence at date  $t$  to generate future rental income diminishes due to depreciation.

A similar interpretation is appropriate for conditions (8b) and (8d). When using more land for construction purposes, agents pay a purchase price  $P^B$ , incur unit development costs  $c$ , and compare these costs with the benefits from using more land. On the benefit side, the marginal value product  $m$  captures the contribution of land to new housing units which generate a present value of additional rental income. Additionally, built up land may be resold when it is released in the future in the process of housing depreciation. Hence, landlords view built up land as part of their real estate property where  $P^L$  denotes its (appropriately discounted future) resale value. Upon direct integration of (8d),

$$\begin{aligned} (a) \quad P_t^L &= \int_t^\infty P_s^B [\delta e^{-\delta(s-t)}] e^{-r(s-t)} ds, \\ (b) \quad P_t^L &= \delta \int_t^\infty (m_s - c_s + \mu_s^F) e^{-r(s-t)} ds. \end{aligned} \quad (10)$$

The first interpretation of the shadow price of land emphasizes its 'speculative' value. Land that is newly built upon at date  $t$ , is released and becomes vacant at a constant

rate  $\delta$  due to housing depreciation. The amount of land released at date  $s$  is equal to the content of the square bracket, and it may be sold at the then prevailing market price of vacant land. According to (10a), the shadow value of land built upon at date  $t$  reflects the present value of the income stream from selling the future scrap values.

Formula (10b) offers an alternative interpretation of the shadow price which underlines the productive value of land. If built up land is not sold off but reinvested as soon as it is released, then its value derives from the marginal value products in housing construction. With continuous reinvestment of a unit of land, the fraction  $\delta$  will be able to earn the marginal value product at each instant.

While (10b) thus emphasizes the productive value of land, (10a) stresses its store of value function. To see more clearly these twin roles of built upon land, we may conceptually divide the landlord's operations into a housing and a real estate division with profits

$$\chi = \chi^H + \chi^L, \quad \chi^H = \Pi H - (m + P^I \kappa(n)\epsilon)F, \quad \chi^L = (m - c)F - P^B(F - \delta L_{-1}). \quad (11)$$

The real estate division obtains revenues from 'selling' land to the housing division at a 'competitive' price equal to the marginal value product of land in house building. It also incurs land development costs as well as net revenues or costs from augmenting or selling off the current stock of built up land. The housing division now calculates economic profits taking account of the real resource cost of using land in house building which is equal to  $m$ .

Both activities, of course, must generate acceptable rates of return. Imposing an arbitrage condition and solving forward, the value of the stock of built up land,  $V^L$ , is equal to the present value of the profits of the real estate division. Similarly, the value of the housing stock,  $V^H$ , amounts to the present value of the profits of the housing division. These profits are equal to the excess of rental income over the total resource costs of constructing new houses. Furthermore, the linearity assumptions imply  $V^L = P^L L$  and  $V^H = P^H H$ .

## 2.2 Landowners

Part of the total area  $\bar{L}$  is built up land, and part of it is vacant:  $\bar{L} = L + B$ . The landowner earns revenues by selling the existing stock of vacant land  $B$  and incurs costs by purchasing land that has newly become vacant and that is sold by landlords. Of course, she cannot sell more than what is available at any date. Hence, the landowner is constrained by  $B \geq 0$ . The optimal rate of selling land is chosen so as to maximize the present value of net revenues from sales,

$$V_t^B = \max \int_t^\infty P_s^B [F_s - \delta(\bar{L} - B_s)] e^{-r(s-t)} ds, \quad (12)$$

subject to

$$\begin{aligned} (\lambda^B) \quad \dot{B} &= \delta(\bar{L} - B) - F, \\ (\mu^B) \quad B &\geq 0. \end{aligned} \tag{13}$$

Using  $\mu^B$  to denote the multiplier pertaining to the non-negativity constraint and  $\lambda^B$  to multiply with the law of motion, the usual Hamiltonian approach reveals the necessary conditions

$$\begin{aligned} (a) \quad F &: P^B = \lambda^B, \\ (b) \quad B &: \dot{\lambda}^B - (r + \delta)\lambda^B = -[\delta P^B + \mu^B], \\ (c) \quad \lambda_\infty^B &: \lim_{s \rightarrow \infty} \lambda_s^B B_s e^{-r(s-t)} = 0. \end{aligned} \tag{14}$$

As long as the stock is not yet depleted, and the constraint is not yet binding the price of vacant land increases at the rate of interest [ $\mu^B = 0$  and, from (14a,b),  $g_{p^b} = r$  in this phase]. The incentives portrayed in the model, however, lead to depletion of vacant land in finite time as will be explained in subsection 2.4 below. In the long-run, therefore, the landowner will be engaged only in reselling immediately all the land that is released and becomes vacant. When the constraint becomes binding, the growth rate of the land price will fall short of the interest rate which is precisely the reason why landowners start to refuse holding vacant land as an asset. Furthermore, as long as vacant land exists, the value of the stock is equal to the present value of future dividends that are created by selling the stock over time. Once the stock is depleted, the value is zero, of course. Pure reselling of newly released land concerns flow operations only which yield zero profits.

### 2.3 Intertemporal Equilibrium

To close the model, we need to describe demand in the rental market for housing. For simplicity, we postulate a demand function  $H^D = Af(\Pi)$  with a constant price elasticity  $\eta$ . The shift parameter  $A$  fixes the position of the rental demand schedule and indicates the strength of demand. It captures the influence of demographics and general economic growth on housing demand as well as other exogenous demand shifts. It is assumed to grow at an exogenous constant rate  $g_a$ .<sup>3</sup> The inverted demand schedule relates the marginal willingness to pay rent to the existing amount of 'effective' housing  $H/A$ . The market clearing rent is thus given by

$$\Pi = \pi(H/A), \quad \eta \equiv -A\Pi/(H\pi') > 0. \tag{15}$$

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<sup>3</sup>In general, we require the following relationship for the model to yield a meaningful solution:  $g_c \leq g_a/[\alpha + \beta\eta] < r$ , where the middle term is a basic joint growth rate of a number of variables such as the capital intensity  $\epsilon$  and the marginal value product of land  $m$  in housing construction [see section 3]. The inequalities state that land development costs should not grow faster than this basic growth rate which, in turn, is set to be smaller than the interest rate.

An intertemporal equilibrium consists of price paths  $\{\Pi\}$  and  $\{P^B\}$  such that markets for land and rental housing clear at all dates and individual constraints and optimality conditions are simultaneously satisfied.

## 2.4 Vacant Land Is a Temporary Phenomenon

Vacant land vanishes well before housing markets approach a long-run equilibrium with balanced growth at constant rates. Land development requires resources that are available at some price  $c$ . If development costs grow moderately relative to the marginal value product of housing  $m$ , the latter will eventually exceed development costs. If the growth rate of  $c$  coincides with that of  $m$ , then we assume that the level of the marginal value product with full productive use of land exceeds the level of development costs. In the other case, land prices would drop to zero. Land would be employed only up to an amount that equates the marginal value product with development costs. With  $m = c$  and both components growing at the same rate, (10b) implies  $P^L = 0$  and (8b) in turn  $P^B = 0$ . The rest of the area would just remain idle. In particular, it would not be asymptotically depleted. This scenario reminds of the early American history with free taking of land. We can safely ignore it as an unrealistic special case that does not apply to the housing problems in modern cities.

Instead we assume that the marginal value product of housing exceeds the land development cost (at least from some point on). Suppose now, contrary to our statement, that vacant land is depleted asymptotically only rather than in finite time. If this were the case, the price of vacant land would have to grow at the rate of interest indefinitely to make it a worthwhile asset for landowners [ $\mu^B = 0$  and, from (14a,b),  $g_{pb} = r$ ]. Therefore, the speculative value of built up land would coincide with the price of vacant land [ $P_t^B = P_s^B e^{-r(s-t)}$  implying  $P_t^L = P_t^B$  by (10a)]. Apart from the land development cost, the net cost of using land in the production of new housing would be zero [see (8b)]! With the marginal value product of land exceeding the development costs, landlords would feel an unlimited desire to acquire new land because it would generate a positive net marginal value product in addition to its real estate value. Thus, vacant land must be depleted in finite time. Land prices therefore cannot increase at the rate of interest indefinitely. During some transitional phase, however, land may remain vacant.

Sinn (1986) ignored depreciation of housing stocks and, thus, introduced an irreversibility in the structural intensity of housing on each piece of built up land. Once built up, land would never become available again for building purposes. Consequently, he had to treat vacant land as an exhaustible resource that is depleted only asymptotically, provided the rate of growth of housing demand is sufficiently high. Instead, we argue that land becomes a renewable resource once one takes into account – quite realistically – physical depreciation of housing stocks. Depreciation releases previously built up land

for new construction purposes. Builders may choose any desired structural intensity dictated by the going market conditions. This breaks up the kind of irreversibility present in Sinn's model. In the long-run, vacant land is completely exhausted but productive land is continuously turned over. Apart from the exceptional circumstances noted above, vacant land is a temporary phenomenon only.

### 3 Housing Markets in the Long-Run

In the long-run, the city area is completely built up and land is in fixed supply. New land becomes available for construction only at the rate at which it is released on account of housing depreciation:  $L = \bar{L}$  and  $F = \bar{F} = \delta\bar{L}$  making  $\mu^F = 0$ . Conditions (8a,c) together with the demand schedule for rental housing, the law of motion for housing stocks and the transversality condition (8e) for housing value can be solved independently in  $(P^H, H)$  space just like in the familiar asset price model of Poterba (1984) except that the current model considers the productive role of land and allows for differing growth rates in model variables. The long-run solution of the model core also fixes the marginal value product of land and its growth rate which in turn implies levels and growth rates of land prices. The solution of the model core involves in itself a recursive procedure of finding first the equilibrium growth rates and, subsequently, the equilibrium levels of the price and stock of housing which are conditional on the growth rates. Finally, we provide a brief description of transitional dynamics in case of full land use.

#### 3.1 Solution in Growth Rates

Long-run equilibrium in housing markets reflects balanced growth at constant rates. The long-run growth rates of the core variables are tied together according to

$$\begin{aligned} (a) \quad g_h &= g_a - \eta g_\pi, & (c) \quad g_h &= \alpha g_\epsilon, \\ (b) \quad g_\pi &= g_{p^h}, & (d) \quad g_\epsilon &= g_{p^h} / \beta. \end{aligned} \tag{16}$$

The first relationship (a) emerges from the demand schedule (15), while (b) builds on the asset pricing relationship (8c) under constant growth. The price of residential investment goods  $P^I$  is constant. Dividing (8c) by  $P^H$ , one obtains on the r.h.s. a fraction  $\frac{P^I I}{P^H H}$  which will be verified below to remain constant in the long-run, too. The stock flow ratio is a constant  $n = \delta + g_h$ . Hence, (8c) implies a constant long-run price rental ratio giving (b). A balanced expansion of housing stocks [see (6)] implies a growth rate as indicated in (c). To be compatible with this, the housing price must grow at a rate that fulfills the investment condition (8a) at each date, hence the equality in (d). Substituting (b) into (a), (d) into (c) we derive demand and supply side relationships describing balanced growth rates in prices and stocks. Equating these two relationships gives the long-run



equilibrium growth rates in prices and quantities with immediate implications for the rental price of housing and the capital intensity in residential construction,

$$g_{ph} = g_a\beta/(\alpha + \beta\eta), \quad g_h = g_a\alpha/(\alpha + \beta\eta). \quad (17)$$

The price and stock of housing are the two core variables determining all other variables in the long-run. The value of the housing stock,  $V^H = P^H H$ , grows in line with a number of other variables at a basic common growth rate  $g_{ph} + g_h \equiv g = g_\epsilon = g_m = g_I$  which verifies the above mentioned statement of a constant ratio  $\frac{P^I I}{P^H H}$ .

### 3.2 Solution in Levels

Given the solution in growth rates, we can now proceed to solve for levels. The asset pricing condition (8c) yields the inverse of the price rental ratio  $\Pi/P^H$  that is compatible with a given housing price  $P^H$  growing at the balanced rate  $g_{ph}$  from then on. To show this, we define an elasticity  $\xi \equiv n\kappa'(n)/\kappa(n)$  and transform the equality stated in the definition of  $m$  following (8) into  $P^I \kappa(n)(1 + \alpha\xi) = P^H \phi(\epsilon)\alpha/\epsilon$ . Substitute this into the long-run version of (8c),  $\frac{\Pi}{P^H} = r + \delta - g_{ph} - \xi\kappa(n)\frac{P^I I}{P^H H}$ . Use the definitions of  $\epsilon$ ,  $N$  and  $n$  and obtain  $\frac{\Pi}{P^H} = r + \delta - g_{ph} - \frac{n\alpha\xi}{1+\alpha\xi}$ . Since the basic growth rate is  $g = g_{ph} + g_h$  and the flow stock ratio satisfies  $n = \delta + g_h$ , the long-run rental price ratio emerges as  $\frac{\Pi}{P^H} = r - g + n/(1 + \alpha\xi)$ . Inverting the demand schedule gives the demand for housing as a function of the rental rate and, thus, of the house price. In deriving the demand function, it is important to realize that growth rates remain constant on a balanced growth path. The demand function is declining in the first argument and increasing in the second:

$$H^D(P^H; A) = A\pi^{-1} \left[ (r - g + \frac{n}{1+\alpha\xi})P^H \right], \quad (18)$$

$$\frac{\partial H^D}{\partial P^H} < 0, \quad \frac{\partial H^D}{\partial A} > 0.$$

The supply function rests on the investment condition (8a) and the constant growth version of the law of motion given in (6). Doing the same kind of transformations as in the preceding paragraph, we may write the square bracket in (8a) as  $[\cdot] = P^H/(1 + \alpha\xi)$ . In a balanced growth equilibrium with a fixed  $n = \delta + g_h$ , the investment condition therefore implies an optimal capital intensity  $\epsilon(P^H, P^I)$  as a function of the shadow price of housing and the price of structures. Substituting into the stationary version of (6) and using  $F = \bar{F} (= \delta\bar{L})$  pins down long-run housing supply,

$$H^S(P^H; \bar{F}, P^I) = \frac{\phi[(\phi')^{-1}[P^I \kappa(n)(1 + \alpha\xi)/P^H]]\bar{F}}{\delta + g_h}, \quad (19)$$

$$\frac{\partial H^S}{\partial P^H} > 0, \quad \frac{\partial H^S}{\partial \bar{F}} > 0, \quad \frac{\partial H^S}{\partial P^I} < 0.$$

Equating the two relations fixes equilibrium quantities and prices in the housing market that obtain in the long-run with exhausted land in fixed flow supply [see figure 1]. Since the demand function is continuously shifted upwards due to exogenous demand growth  $A$ , both housing stocks and prices increase over time, and they increase at constant rates as is evident from the solution in growth rates.

Having determined the long run levels of housing quantity and price, we may find the capital intensity  $\epsilon$  and the rental price of housing  $\Pi$  from the formulas

$$\epsilon = (\phi')^{-1}[P^I \kappa(n)(1 + \alpha\xi)/P^H]$$

and

$$\Pi = P^H(r - g + n/(1 + \alpha\xi))$$

In long-run equilibrium, built up land is released and rebuilt at a constant rate  $\delta$ . Land that becomes available for construction, is paid its marginal value product in housing,  $m = P^I \kappa(n)\epsilon\beta/\alpha$ . According to (10b), the price of productive land capitalizes these factor payments as far as they exceed development costs. Under constant growth, one obtains

$$P^L = \delta \left( \frac{m}{r - g} - \frac{c}{r - g_c} \right). \quad (20)$$

If development costs grow slowly ( $g_c < g$ ), they will asymptotically vanish, and the price of built up land will eventually grow at a rate close to  $g$ . If the growth rate of  $c$  is equal to  $g$ , then  $P^L$  will exactly increase at the basic growth rate.

From (8b) we derive the price of vacant land,

$$P^B = m \left( \frac{\delta + r - g}{r - g} \right) - c \left( \frac{\delta + r - g_c}{r - g_c} \right). \quad (21)$$

Again, the price of vacant land grows at a rate close or equal to  $g$ . The two prices for vacant and built up land are related according to

$$P^B \approx \left( 1 + \frac{r - g}{\delta} \right) P^L > P^L$$

With our maintained assumption of  $r > g$ , the price of vacant land will be close to proportional to the price of productive land with a proportionality factor in excess of one. They are exactly proportional if development costs grow at the basic rate,  $g_c = g$ .

### 3.3 Long-Run Dynamics with Fixed Land

In the long-run, all land is completely built up. As part of the existing housing stock erodes, a fixed flow of land becomes available at each instant and is immediately rebuilt

with new structures. We briefly analyze the local adjustment dynamics in a neighborhood of a balanced growth equilibrium of housing markets. The government, for example, may expand housing zones. When the new area allocated to housing is completely built up, housing depreciation releases a larger flow of built up land at each date, and the amount of land available for new construction activities is augmented correspondingly. Furthermore, we consider the effects of a price increase in residential investment goods and an increase in the level of demand for rental housing.

In the appendix, we log-linearize the model under the assumption of full land use. We obtain long-run comparative statics results as well as explicit solutions for adjustment dynamics. For simplicity, we assume that adjustment costs vanish in the long-run equilibrium with a balanced rate of housing construction:  $\kappa(g_h + \delta) = 1$  and  $\kappa'(g_h + \delta) = 0$ . As a notational convention, we employ small-letter variables to denote detrended versions of original variables while a ‘hat’ indicates percentage changes from the initial balanced growth path. Housing markets ultimately respond to shocks according to [see appendix for details]

$$\begin{bmatrix} \hat{h}_\infty \\ \hat{p}_\infty^h \end{bmatrix} = \frac{1}{1 + \alpha/(\beta\eta)} \begin{bmatrix} 1 & -\frac{\alpha}{\beta} & \frac{\alpha}{\beta\eta} \\ -\frac{1}{\eta} & \frac{\alpha}{\beta\eta} & \frac{1}{\eta} \end{bmatrix} \begin{bmatrix} \hat{F} \\ \hat{P}^I \\ \hat{A} \end{bmatrix}. \quad (22)$$

Not too surprisingly, the increased availability of land for housing construction (as a result of government zoning) expands housing stocks and depresses house prices. Price increases in residential investment goods have the opposite effect. Higher demand pressures increase both housing stocks and prices.

Prices of productive and vacant land are proportional in the long-run [see the preceding subsection] and depend on the marginal value product of land  $m$  which, in turn, depends on the capital intensity  $\epsilon$ . Hence, it suffices to characterize long-run effects on the rental price of housing and the capital intensity in construction,

$$\begin{bmatrix} \hat{\epsilon}_\infty \\ \hat{\pi}_\infty \end{bmatrix} = \frac{1}{1 + \alpha/(\beta\eta)} \begin{bmatrix} -\frac{1}{\beta\eta} & -\frac{1}{\beta} & \frac{1}{\beta\eta} \\ -\frac{1}{\eta} & \frac{\alpha}{\beta} & \frac{1}{\eta} \end{bmatrix} \begin{bmatrix} \hat{F} \\ \hat{P}^I \\ \hat{A} \end{bmatrix} \quad (23)$$

With an increased flow of land available for construction, the capital intensity of buildings, the rental price of housing, and the prices of productive and vacant land will go down in the long-run. If the price of investment goods goes up, this will raise the rental price of housing, but both the capital intensity of new housing stock and the price of productive or vacant land are bound to fall over time relative to the base case equilibrium. Furthermore, a once and for all upward jump in the level of housing demand will lead to long-run increases in the capital intensity of housing, the rental price, and the prices of vacant and productive land. Most of these effects accord well with one’s intuition.

Finally, with full productive use of land, adjustment is monotonic. Stocks gradually converge to their stationary equilibrium values. The deviation of housing prices from long-run values is tied to stocks according to

$$\begin{aligned} (a) \quad \hat{h}_t &= (1 - e^{\lambda_1 t}) \hat{h}_\infty, \\ (b) \quad \hat{p}_t^h - \hat{p}_\infty^h &= s(\hat{h}_t - \hat{h}_\infty), \end{aligned} \tag{24}$$

where  $\lambda_1 < 0$  is the stable eigenvalue and  $s < 0$  fixes the slope of the saddle-path [see the appendix].

## 4 Vacant Land

We argued that housing depreciation eliminates – in the long-run – all incentives to keep vacant land. Consequently, the city area is completely built upon in finite time. Vacant land is a transitional phenomenon at most. One may wonder how vacant land comes about, if at all. Is it an important phenomenon? How much time is required for any historically given stock to be completely soaked up in housing construction? We will first discuss in subsection 4.1 rather heuristically the factors that are most likely to create vacant land.<sup>4</sup> Then we turn to simulation since many aspects of our problem are essentially matters of magnitude and cannot be decided with analytical arguments alone.

### 4.1 A Heuristic Argument

The system given in (24) characterizes dynamic adjustment of housing markets in response to supply or demand disturbances provided that all city area is completely built up. Indeed, if the rate of return on holding vacant land is less than the interest rate, landowners would have no desire to hold back vacant land for speculative reasons. Such a situation would confirm the assumption underlying (24) that all available land is immediately supplied for productive use in housing construction. If, on the other hand, land prices increase so rapidly that the associated capital gains exceed the return on alternative assets, landowners would rather keep land vacant and sell it at some later date. Hence, the critical factor that may give rise to vacant land, is the growth rate in land prices.

Differentiating (A.12) in the appendix, we obtain the change in the rate of increase in the land price under the condition of full land use,<sup>5</sup>

$$\dot{\hat{P}}_0^B = -\frac{m}{m-c} \lambda_1 \left[ \frac{\rho s}{\beta} + \frac{1-\rho}{\alpha} \right] \frac{(r-g)(\delta+r-g-\lambda_1)}{(r-g-\lambda_1)(\delta+r-g)} \hat{h}_\infty. \tag{25}$$

<sup>4</sup>From now on we assume that the share of land development in total construction costs remains constant in the long-run. Consequently, we let these costs grow at the basic rate  $g_c = g$ .

<sup>5</sup>The parameter  $\rho$ ,  $0 < \rho \leq 1$ , measures the extent of adjustment costs (see the appendix). If  $\rho$  is unity, such costs are absent and the square bracket in (25) is negative. We assume that adjustment costs are not so large or, equivalently, that  $\rho$  is not so small as to make  $[\rho s/\beta + (1-\rho)/\alpha]$  positive.

Right after an unexpected shock, the *change in the growth rate* of the price of vacant land is just opposite and, subject to errors of approximation, proportional to the long run change in housing stocks. Consider, for example, a positive demand shock that raises long run housing stocks. Quite intuitively, such a market disturbance will raise housing rents, increase the marginal value product of land in housing construction, and shift up the entire time path of land prices. However, since residential construction is particularly strong in the initial adjustment phase when housing stocks are still too low, demand for land will be much stronger in the short run and land prices will increase on impact by much more than in the long run. Thus, land prices get a boost initially but relax as housing stocks accumulate and the construction activities slow down to more normal levels. While the *level* of land prices jumps up, the *rate of increase* actually *falls*, and so does the rate of return on holding vacant land. Hence, positive demand shocks are not conducive to land price speculation but favor, quite intuitively, the productive use of land.

Consider now the opposite case of a permanent negative shock that initiates a decumulation of housing stocks. This will surely depress the level of land prices over the entire time horizon. However, initially housing stocks are overly high as compared to long run levels, and construction activity will be much weaker. Hence, demand for land is much weaker in the short run rather than in the future when building activities return to more normal levels again. Hence, market forces tilt land prices towards the future and, thus, make for steep price increases for land in the initial adjustment phase, once the initial level effect has occurred. Consequently, negative shocks of sufficient magnitude may well generate rates of change in land prices to such an extent that land price speculation becomes a worthwhile financial activity. Full productive use of land becomes untenable, and vacant land emerges as an equilibrium response.<sup>6</sup>

Figure 2 illustrates what may be going on. It shows the evolution over time of the price of land when housing markets are in a state of balanced growth (line a). Line b is drawn under the assumption of full land use and shows the reaction of land prices in response to a serious negative shock. The path of land prices is shifted down and tilted towards the future. As the first part of curve b entails a rate of increase in the price of land that exceeds the interest rate, it is infeasible. Land is held back for speculative reasons. Builders who demand land for construction purposes and landowners who pursue financial investment opportunities bid up the price of land. The price is pushed to a level that allows for a capital gains rate exactly equal to the interest rate over the entire period where landowners keep vacant land (curve c). When housing stocks have decumulated enough, construction activities return to more normal levels and demand for land picks up again. Hence, a stock of vacant land is accumulated in an initial adjustment phase

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<sup>6</sup>The emergence of vacant land is in a sense related to overshooting phenomena in the housing market. Because the stock of housing is fixed in the short run, adjustment in the initial phase translates into disproportionately large jumps in rents as well as house and land prices.

as released land is held back for speculative reasons. Eventually, this stock is depleted again. Once all the city area is completely built up, the growth rate in land prices will taper off and fall below the rate of interest.

It seems quite counterintuitive that the growth in land prices slows down once all vacant land is built upon. However, even if the city area is completely built up, depreciation of old housing stocks releases a continuous flow of land that becomes available for new construction. With continuous building, housing stocks eventually grow at a basic rate that is lower than the interest rate. In the long run, the marginal value product of land as well as development costs grow at this rate. Eventually, land prices must reflect their fundamental value rather than merely their speculative value that arises when agents view land as a purely financial asset. With continuous building, land prices correspond just to the value that derives from the marginal value product of land in new housing construction. Speculative land price ‘bubbles’ cannot be supported forever but may be a temporary market reaction in our model.<sup>7</sup>

## 4.2 Simulations with the model

Our characterization of housing dynamics in (24) as well as the change in the growth rate of the price of vacant land in (25) both result from a linear approximation around an initial balanced growth path. At the same time, we are particularly interested in the effects of large rather than small shocks to housing and land markets. Hence, the arguments given in the preceding section cannot substitute for a rigorous proof of when vacant land occurs. Indeed, they were meant only to provide some intuition into the matter. We now run several simulation experiments proving that vacant land is a genuine and potentially important phenomenon over extended time periods.<sup>8</sup>

Table 1 reports how we calibrate the simulation model. The empirical foundation is admittedly rather crude. Since the simulation exercises are intended mainly for theoretical purposes, such a quick approach is sufficient for the time being. The econometrics literature reports a rather wide range of values for some of the important parameters. We assign a value of 1.0 to the price elasticity  $\eta$  of rental demand for housing [Rosen’s (1985) estimate is unity, Poterba (1984, 1991) reports values ranging from 0.5 to 2 and chooses a value of unity, too, while Mankiw and Weil (1989) decide for a lower value of 0.5]. A 3.5 per cent real interest rate and a 1 percent depreciation rate for housing stocks are commonly used values in the empirical housing literature. However, we are not aware of any empirical work on the production function which combines structures and land in

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<sup>7</sup>One may, of course, doubt that real world land prices in a completely built up city correctly reflect fundamental values. However, bubbles seem not to go on forever but eventually burst which may be evidence of market forces such as those portrayed by our model.

<sup>8</sup>The simulation model is in discrete time but otherwise exactly analogous to the continuous time version presented here. The discrete time version is available in a separate appendix.

building new housing units. On the contrary, the role of land is largely ignored in the empirical housing literature.

We parameterize the production function for housing such that the long-run cost share of structures equals 75 per cent with the rest being absorbed by land consumption. The cost shares for structures and land are supported by evidence for Germany.<sup>9</sup> We calibrate a value  $\alpha \approx .83$  which just makes the long-run model solution support the prespecified cost share. We arbitrarily set the total stock of land at 10 units and find that value of the demand shift parameter  $A$  which normalizes rental income to 1. House prices exceed housing rents by a factor of 24, roughly. Again, this is within the range of values reported for some German cities.<sup>10</sup> The basic demand growth rate is set at 2.5 per cent which determines – together with the output elasticity  $\alpha$  and the demand elasticity  $\eta$  – the growth rates in housing stocks and house prices of around 2.1 and 0.4 per cent, respectively. Finally, the extent of adjustment costs is determined by a requirement that following a shock, half of the adjustment in the housing stock will be accomplished within twelve years. The calibration implements a long-run steady state equilibrium where all of the city area is completely built up. Starting from the balanced growth base case, we now simulate a number of unexpected as well as expected, future shocks to key parameters.<sup>11</sup>

Figures 3 a-h report how the model responds to an unexpected demand shock in  $A$ . The figures show the evolution of key quantity and price variables deflated by their own long-run growth rates. Hence, all of them will approach a constant value in the long-run, similar to the dashed lines which show detrended initial steady state values. The exception is the price of vacant land which we deflate by the interest rate. Hence, a flat portion indicates that the land price grows with the rate of interest reflecting the basic no-arbitrage condition that must be fulfilled for the landowner to be willing to hold land. All vacant land disappears in the long-run equilibrium since land price increases and, thus, the return on holding land eventually will fall short of the interest rate. Hence, the deflated land price is downward sloping in a steady state equilibrium.

We consider a large negative shock and let the demand shift parameter fall by 15 per cent. Housing construction stops for a few periods which makes the housing stock decline at the rate of depreciation. Previously built up land is released and is kept vacant rather than being rebuilt again. In period 3, vacant land amounts to almost 2 percent of the city area. The shortfall in demand triggers a dramatic crash in land prices which from then on grow at the rate of interest to make it a worthwhile asset for landowners. This is indicated by the flat portion in figure 3e. Once the building activity picks up again, land prices are

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<sup>9</sup>Our cost shares roughly correspond to values reported by the Institut fuer Wohnen und Umwelt, Darmstadt, Germany.

<sup>10</sup>These ratios vary widely across cities. The Institut fuer Wohnen und Umwelt reports ratios of 10 to 12 for Essen and 26 to 28 for Stuttgart.

<sup>11</sup>In the simulations, we assume for simplicity that  $\kappa(n) = 1$  for  $n < \tilde{n} = g_h + \delta$ , i.e. that adjustment costs appear only whenever the rate of housing construction  $n$  exceeds its steady state value.

**Table 1: Basic Model Parameters**

$N = k\epsilon^\alpha F$		housing production
$\kappa(n)^*$		adjustment cost for $n > \tilde{n}$
$\Pi = (A/H)^{1/\eta}$		rental demand
$r$	0.035	real interest rate
$g$	0.025	basic growth rate
$\eta$	1.000	price elasticity demand
$s^I$	0.750	cost share structures
$s^F$	0.200	cost share land
$s^C$	0.050	cost share development
$\delta$	0.010	housing depreciation rate
$H/L$	0.800	average structural intensity
$\Pi H$	1.000	normalization rental income
$t_{0.5}$	12.000	half life of adjustment
$\alpha^{**}$	0.834	output elasticity structures
$\kappa_0^{**}$	7.276	adjustment cost parameter
$A^{**}$	1.000	demand shift parameter
$g_a^{**}$	0.025	growth rate demand
$g_h$	0.021	growth rate housing stock
$g_{ph}$	0.004	growth rate house price
$N/F$	2.466	marginal structural intensity
$P^H/\Pi$	24.523	price rental ratio

\* $\kappa(n) = 1$ ,  $n \leq \tilde{n}$  and

$$\kappa(n) = \frac{\tilde{n}}{(1+\xi)} \left[ \left( \frac{n}{\tilde{n}} \right)^{(1+\xi)} + \xi \right] - n, \quad \xi > 0 \text{ for } n > \tilde{n}$$

\*\* indicates a calibrated value.



rather low relative to the cost of structures which induces builders to make intensive use of land in new housing construction. Consequently, land consumption overshoots the long-run turnover flow that becomes available from the depreciation of existing housing stocks. After construction activity starts again, the stock of vacant land is rapidly depleted and it takes about 12 years to completely rebuild the city area. From that moment on the supply of new land is a fixed flow. Taking the housing stock at that moment as an initial condition, the subsequent adjustment phase is completely characterized in qualitative terms in section 3. In particular, the rate of increase in the land price starts to fall short of the interest rate and will eventually coincide with the basic growth rate of the housing sector. Landowners would not consider vacant land any more as a worthwhile financial asset.

We may note in passing that vacant land creates asymmetric adjustment patterns of the housing sector. While a 15 per cent decline in demand creates vacant land, a corresponding increase in demand will certainly not do so. In fact, vacant land emerges only upon a truly large negative demand shock. Given our parameterization, a drop in the demand shift parameter  $A$  of, say, 5 per cent will not create any vacant land at all but a 9 per cent drop will do so.

Figures 4 a-h illustrate possible consequences of an expected future shock in the price of structures. The scenario is that 10 years ahead, in year 11, the price of investment goods increases by 10 per cent. Section 3.3 reports that, in the long-run, the cost increases are reflected in higher rents and house prices while housing stocks decline. Furthermore, the price of productive land stands to fall in the long-run. While the long-run results are quite intuitive, we observe rather interesting adjustment patterns in the transitional phase that result from intertemporal substitution and from creation and subsequent depletion of vacant land. Preceding the increase in the investment goods price, housing construction is significantly cheaper than later on, inducing landlords to build now rather than in the future. Hence, housing stocks first pick up somewhat before they fall later on. The increased construction activities in the first phase pushes up the price of vacant land. When the increase in the price of structures materializes, construction is depressed and land consumption drops. Vacant land emerges for a period of five years. The price of vacant land crashes but has to rise subsequently with the rate of interest to generate sufficient returns to landowners. Once construction of new houses returns to more normal levels, land consumption overshoots reflecting land intensive construction in the face of low land prices. The stock of vacant land is rapidly depleted again until the housing sector enters a final monotonic adjustment phase with full land use.

As a last scenario, we consider a government zoning decision that expands the city area  $\bar{L}$  by 10 per cent. While such a decision tautologically creates vacant land, we ask: how fast will it be built up with new houses? The simulations tell us that it actually takes about 26 years for the newly allocated land to be completely built up with structures. Quite obviously, the prices of both vacant and productive land fall relative to the previous

steady state. Again, the price of vacant land grows at the rate of interest as long as some of the city area remains vacant. In the face of low land prices, land consumption jumps upwards reflecting the more land intensive building techniques. Starting from low levels, land prices increase quite rapidly at the rate of interest inducing builders to build with with increasingly lower land intensity. Eventually, land consumption is reduced to a level which corresponds to the turnover of the now larger city area. Even though land prices increase rapidly in the early adjustment phase, the price levels are rather low. In fact, the rapid increase in land prices indicates that land is abundant and cheap now whereas it is tight and expensive in the future. As a kind of intertemporal substitution, the building activity is shifted towards the present giving rise to overly high stocks at the date when the city area is completely built up. From then on, the housing sector enters a final monotonic adjustment phase with housing stocks declining to long-run levels that are, however, still higher than in the initial equilibrium.

## 5 Concluding Remarks

Sinn (1986) first suggested a partial equilibrium model of housing and land markets that explicitly allows for both vacant and built up land. This paper extends the model by introducing physical decay of housing stocks. At first sight this seems to be a minor change but it has actually quite dramatic consequences for the operation of housing and land markets. According to Sinn's approach, the structural intensity of housing is determined forever once land is built upon. In face of strong demand growth, this kind of irreversibility dictates that the stock of vacant land be depleted only asymptotically. Land needs to be kept available for future building purposes with high intensities. In this sense, vacant land is like an exhaustible resource. By way of contrast, our approach breaks up this irreversibility. As part of the housing stock depreciates, built up land is released and becomes available for new construction at any desirable intensity. With a continuous turnover and rebuilding of the city area, the necessity to keep land vacant and to conserve it for future building is weakened. We show that any stock of vacant land is depleted in finite time rather than asymptotically.

Vacant land is a transitory phenomenon only. Our simulations show that it may be quite important quantitatively over extended periods of time but will eventually be completely soaked up. We also find quite large variability in land prices. As long as some vacant land remains, land prices are depressed but grow rapidly at the rate of interest in order to provide a sufficient return for financial investors. Once the city area is completely built upon, the growth in land prices for new buildings slows down and correctly reflects long-run fundamentals. Eventually, the growth rate in land prices is tied to the growth rate in the marginal value product of land in new housing construction.

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## Appendix:

In the appendix we derive in some detail transitional dynamics in the neighborhood of an initial balanced growth equilibrium when the model is shocked by some exogenous demand and supply disturbances. Linearization gives valid conclusions only if the growth rates remain unaffected. By way of contrast, a higher growth rate for rental demand also affects the growth rates of the core variables. If they change, the shadow price and stock of housing would trend at different rates. Even the transformed variables that are detrended with *initial* growth rates and are stationary in the initial equilibrium, would eventually grow out of the interval for which linearization remains valid. Hence, we confine our analysis to those experiments that leave growth rates unaffected and induce pure level effects only.

In the following analysis, we indicate initial values by a tilde,  $\tilde{H}$  for example. Since the initial equilibrium is assumed to be one of balanced growth, variables trend at a constant rate. When the model is shocked, the new values differ from the initial trend by some deviation factor which we denote by lower case letters,  $H = h\tilde{H}$ . Hence,

$$\tilde{H}_t = \tilde{H}_0 e^{g_h t}, \quad \dot{\tilde{H}}_t = g_h \tilde{H}_t, \quad \dot{H}_t = (\dot{h}_t + g_h h_t) \tilde{H}_t, \quad (\text{A.1})$$

and similarly  $P^H = p^h \tilde{P}^h$  etc.

One is back at the initial equilibrium if the deviation factor satisfies  $h_t = 1$  and  $\dot{h}_t = 0$ . When log-linearizing the model, we get deviations in terms of percent changes from initial values. Define  $\hat{H}_t \equiv dH_t/H_t$  and  $\dot{\hat{H}}_t \equiv \partial \hat{H}_t / \partial t = d(\dot{H}_t/H_t)$ . Together with (A.1), these definitions imply  $\hat{H} = \hat{h}$  and  $\dot{\hat{H}} = \dot{\hat{h}}$ .

Given these definitions, we may proceed to log-linearize the law of motion for housing stocks in (2) of the text. Note that  $\tilde{N}/\tilde{H} = \tilde{n} = (\delta + g_h)$  in the initial balanced growth equilibrium and obtain

$$\dot{\hat{h}} = \tilde{n}(\hat{N} - \hat{h}). \quad (\text{A.2})$$

As housing construction relies on the technology  $N = \phi(\epsilon)F$ , log-linearization yields  $\hat{N} = \alpha \hat{\epsilon} + \hat{F}$ . Since we wish to express the dynamic system in  $(h, p^h)$  space, the change in capital intensity needs to be expressed in terms of the endogenous dynamic variables. For simplicity, we assume that adjustment costs become unimportant when the transition is effectively completed and housing construction proceeds with a balanced long-run rate:  $\kappa(\tilde{n}) = 1$  and  $\kappa'(\tilde{n}) = 0$ . Hence, the long-run version of the investment condition is  $\phi'(\epsilon)P^H = P^I \kappa(n)$  implying  $\frac{P^I I}{P^H H} = n\alpha/\kappa(n)$ . This simplifies the log-linearized version of (8a):

$$\hat{P}^I = -\beta \hat{\epsilon} + \hat{p}^h - \frac{\tilde{n}^2 \alpha \kappa''(\tilde{n})}{\kappa(\tilde{n})} \hat{n}. \quad (\text{A.3})$$

Note  $\hat{n} = \alpha \hat{\epsilon} + \hat{F} - \hat{h}$  in (A.3) and isolate  $\hat{\epsilon} \beta / \rho = \hat{p}^h - \hat{P}^I - \frac{(\tilde{n}\alpha)^2 \kappa''}{\alpha \kappa'} (\hat{F} - \hat{h})$ . Use this and get  $\hat{n} = \rho [\frac{\alpha}{\beta} (\hat{p}^h - \hat{P}^I) + \hat{F} - \hat{h}]$ . Since  $\hat{N} - \hat{h} = \hat{n}$ , we obtain from (A.2) the first dynamic equation

$$\begin{aligned} \dot{\hat{h}} &= \tilde{n} \rho [(\hat{p}^h - \hat{P}^I) \alpha / \beta + \hat{F} - \hat{h}], \\ \rho &\equiv \beta \kappa(\tilde{n}) / [\beta \kappa(\tilde{n}) + (\alpha \tilde{n})^2 \kappa''(\tilde{n})] \leq 1. \end{aligned} \quad (\text{A.4})$$

The next step finds the log-linearized version of (8c). Given the normalization  $\kappa'(\tilde{n}) = 0$ , the initial balanced growth path supports  $\tilde{\Pi} = (r + \delta - g_{p^h}) \tilde{P}^H$ . Using this and remembering  $\frac{P^I I}{P^H H} = n\alpha/\kappa(n)$ , log-linearization of the asset valuation equation (8c) results in

$$\dot{\hat{p}}^h = (r + \delta - g_{p^h})(\hat{p}^h - \hat{\pi}) - [\tilde{n}^2 \kappa''(\tilde{n}) / \kappa(\tilde{n})] \tilde{n} \alpha \hat{n}. \quad (\text{A.5})$$

Finally, the rental demand schedule (15) implies deviations of the rental price from the initial equilibrium values according to  $\hat{\pi} = (\hat{A} - \hat{h})/\eta$ . Substituting into the previous equation gives

$$\dot{\hat{p}}^h = (r + \delta - g_{ph})[\hat{p}^h + (\hat{h} - \hat{A})/\eta] - \tilde{n}(1 - \rho)[(\hat{p}^h - \hat{P}I) + \frac{\beta}{\alpha}(\hat{F} - \hat{h})]. \quad (\text{A.6})$$

The two dynamic equations approximate the dynamic adjustment of housing prices and stocks in response to small shocks. Note that the system refers to a situation where land is completely built up in each period and no truly vacant land exists. Writing  $a \equiv (r + \delta - g_{ph})$  for short, the dynamical system is

$$\begin{bmatrix} \dot{\hat{h}} \\ \dot{\hat{p}}^h \end{bmatrix} = \begin{bmatrix} -\tilde{n}\rho & \tilde{n}\frac{\rho\alpha}{\beta} \\ \frac{a}{\eta} + \tilde{n}\frac{\beta(1-\rho)}{\alpha} & a - \tilde{n}(1-\rho) \end{bmatrix} \begin{bmatrix} \hat{h} \\ \hat{p}^h \end{bmatrix} + \begin{bmatrix} \tilde{n}\rho & -\tilde{n}\frac{\rho\alpha}{\beta} & 0 \\ -\tilde{n}\frac{\beta(1-\rho)}{\alpha} & \tilde{n}(1-\rho) & -\frac{a}{\eta} \end{bmatrix} \begin{bmatrix} \hat{F} \\ \hat{P}I \\ \hat{A} \end{bmatrix} \quad (\text{A.7})$$

Since the housing stock is predetermined and the shadow price is forward-looking, we require one stable and one unstable eigenvalue for the dynamic system to be saddle-point stable. Since the determinant  $D$  of the coefficient matrix is negative, the system is well determined with one negative and one positive root.

$$D = -\rho\tilde{n}a[1 + \alpha/(\beta\eta)] < 0. \quad (\text{A.8})$$

After completing adjustment, housing stocks and prices eventually emerge as

$$\begin{bmatrix} \hat{h}_\infty \\ \hat{p}_\infty^h \end{bmatrix} = \frac{1}{1 + \alpha/(\beta\eta)} \begin{bmatrix} 1 & -\frac{\alpha}{\beta} & \frac{\alpha}{\beta\eta} \\ -\frac{1}{\eta} & \frac{\alpha}{\beta\eta} & \frac{1}{\eta} \end{bmatrix} \begin{bmatrix} \hat{F} \\ \hat{P}I \\ \hat{A} \end{bmatrix}. \quad (\text{A.9})$$

Knowing long-run results, transitional dynamics in the linearized model is characterized by  $X_t - X_\infty = [1, s]'ke^{\lambda_1 t}$ . From the coefficients matrix in (A.7), we compute the eigenvector that corresponds to the stable root  $\lambda_1 < 0$ :  $s = (\lambda_1 + \tilde{n}\rho)\beta/(\tilde{n}\rho\alpha) < 0$ . The initial condition on housing stocks pins down the scalar  $k = -\hat{h}_\infty$ . Adjustment dynamics are thus governed by

$$\begin{aligned} (a) \quad \hat{h}_t &= (1 - e^{\lambda_1 t})\hat{h}_\infty, \\ (b) \quad \hat{p}_t^h - \hat{p}_\infty^h &= s(\hat{h}_t - \hat{h}_\infty). \end{aligned} \quad (\text{A.10})$$

The remaining variables such as  $\epsilon$  and  $\pi$  are tied to the core variables:

$$\begin{aligned} \hat{\epsilon}_t &= \frac{\rho}{\beta}[\hat{p}_\infty^h - se^{\lambda_1 t}\hat{h}_\infty - \hat{P}I] - \frac{1-\rho}{\alpha}[\hat{F} - (1 - e^{\lambda_1 t})\hat{h}_\infty] \\ \hat{\pi}_t &= \frac{1}{\eta}[\hat{A} - (1 - e^{\lambda_1 t})\hat{h}_\infty]. \end{aligned}$$

Finally, with the help of formulas (10b) and (8b) in section 2 we compute expressions for the rate of change in the two price variables for land; the general formulas for the case of a positive cost of land development, which grows at the same rate as the marginal value product of land are somewhat complicated:

$$\hat{p}_t^l = \frac{m}{m-c}(\hat{P}^I + \frac{\rho}{\beta}(\hat{p}_\infty^h - \hat{P}^I) - \frac{1-\rho}{\alpha}(\hat{F} - \hat{h}_\infty) - \frac{r-g}{r-g-\lambda_1}\hat{h}_\infty e^{\lambda_1 t}[\frac{\rho s}{\beta} + \frac{1-\rho}{\alpha}]) - \frac{c}{m-c}\hat{c}, \quad (\text{A.11})$$

$$\hat{p}_t^b = \frac{m}{m-c}(\hat{P}^I + \frac{\rho}{\beta}(\hat{p}_\infty^h - \hat{P}^I) - \frac{1-\rho}{\alpha}(\hat{F} - \hat{h}_\infty) - \frac{(r-g)(\delta+r-\lambda_1-g)}{(r-\lambda_1-g)(\delta+r-g)}\hat{h}_\infty e^{\lambda_1 t}[\frac{\rho s}{\beta} + \frac{1-\rho}{\alpha}]) - \frac{c}{m-c}\hat{c} \quad (\text{A.12})$$

Upon differentiation of the last formula one arrives at formula (25) in the text.

Fig.1 Housing markets in the long run.

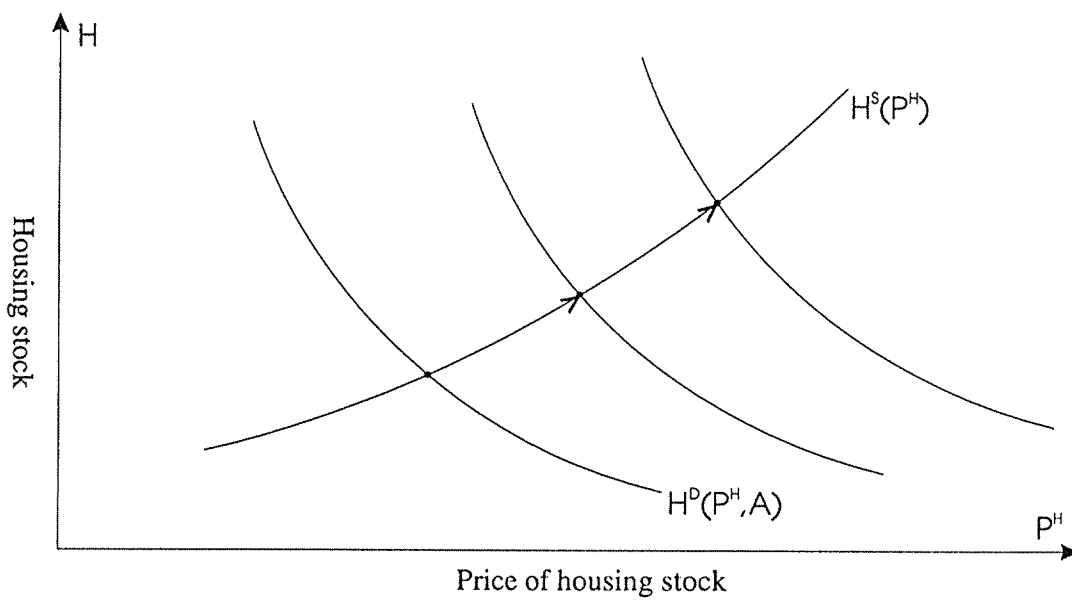
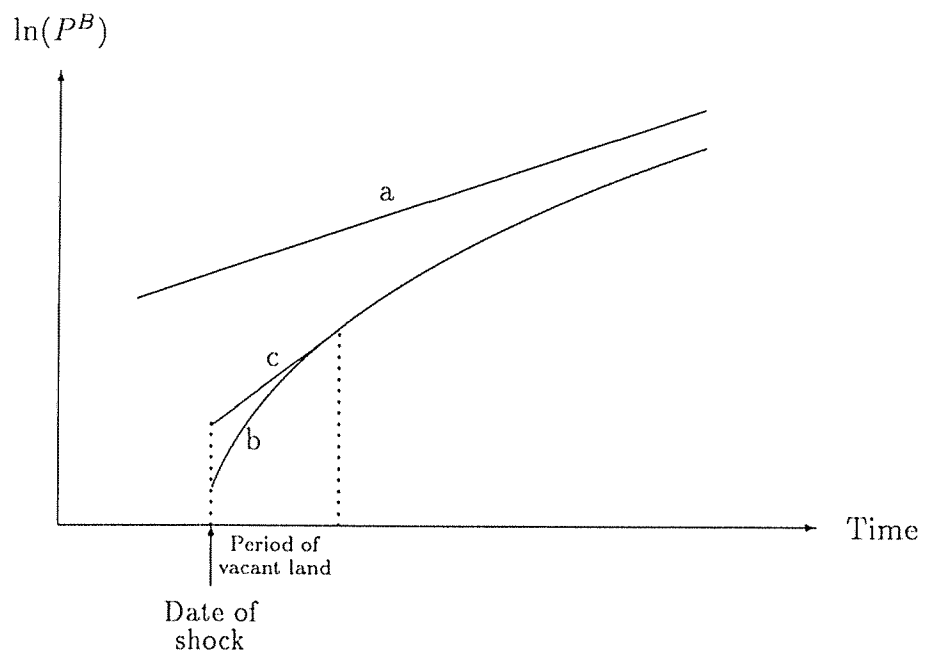
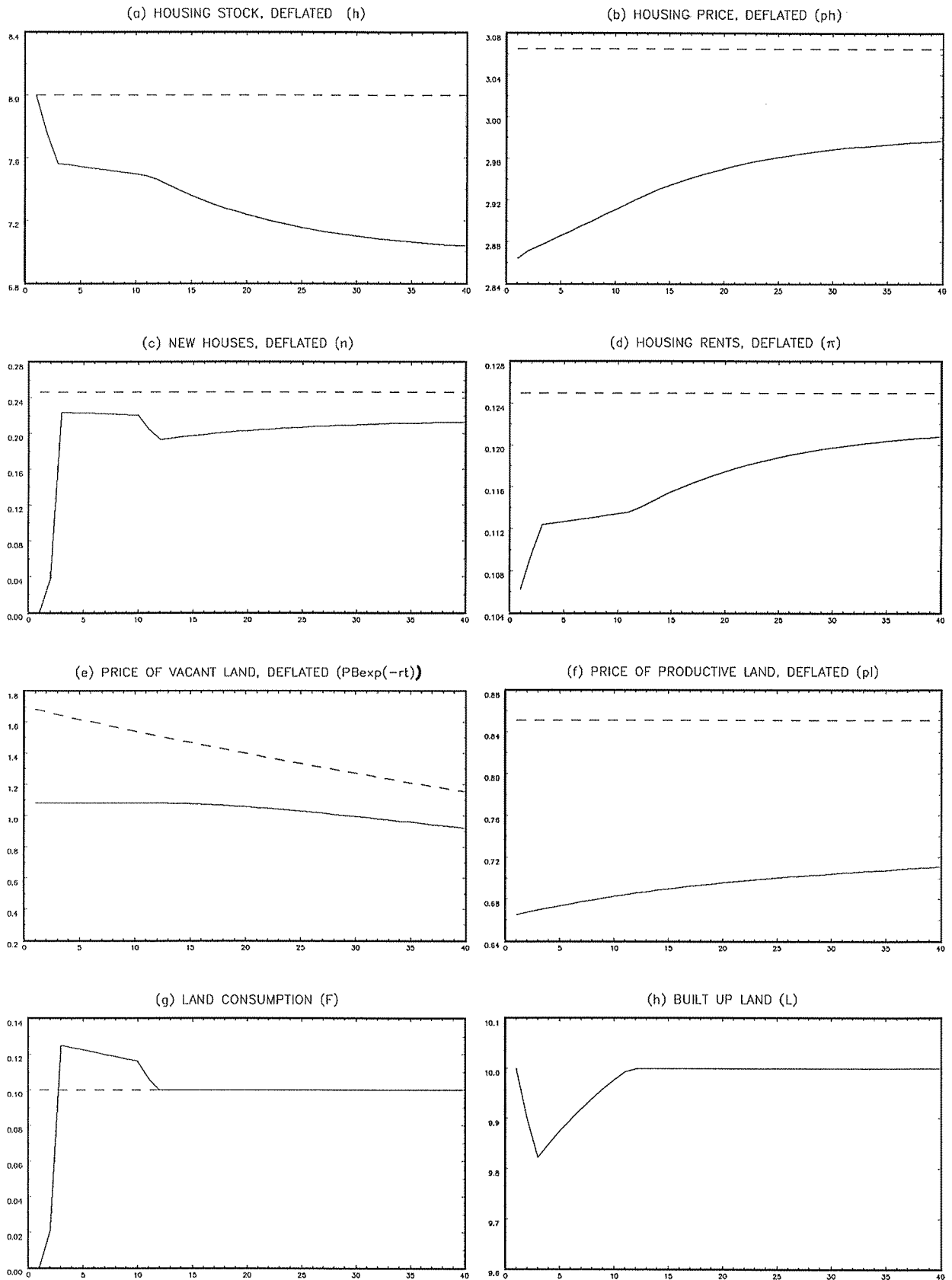


Figure 2: The creation of vacant land





**Figure 3: The effects of a 15 pct. drop in A**



**Figure 4:** The effects of a 10 pct. increase in  $P^I$  in year 11

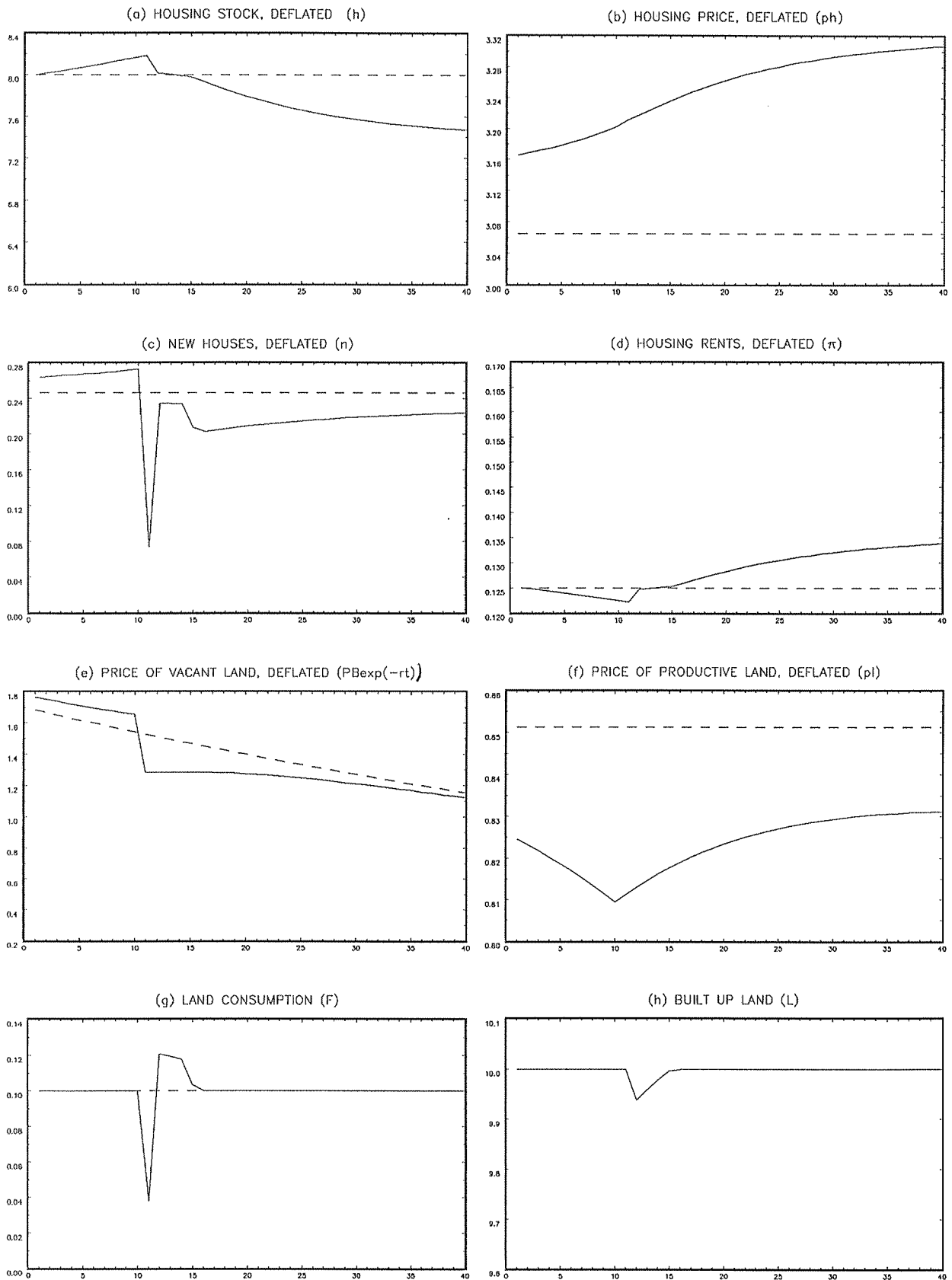
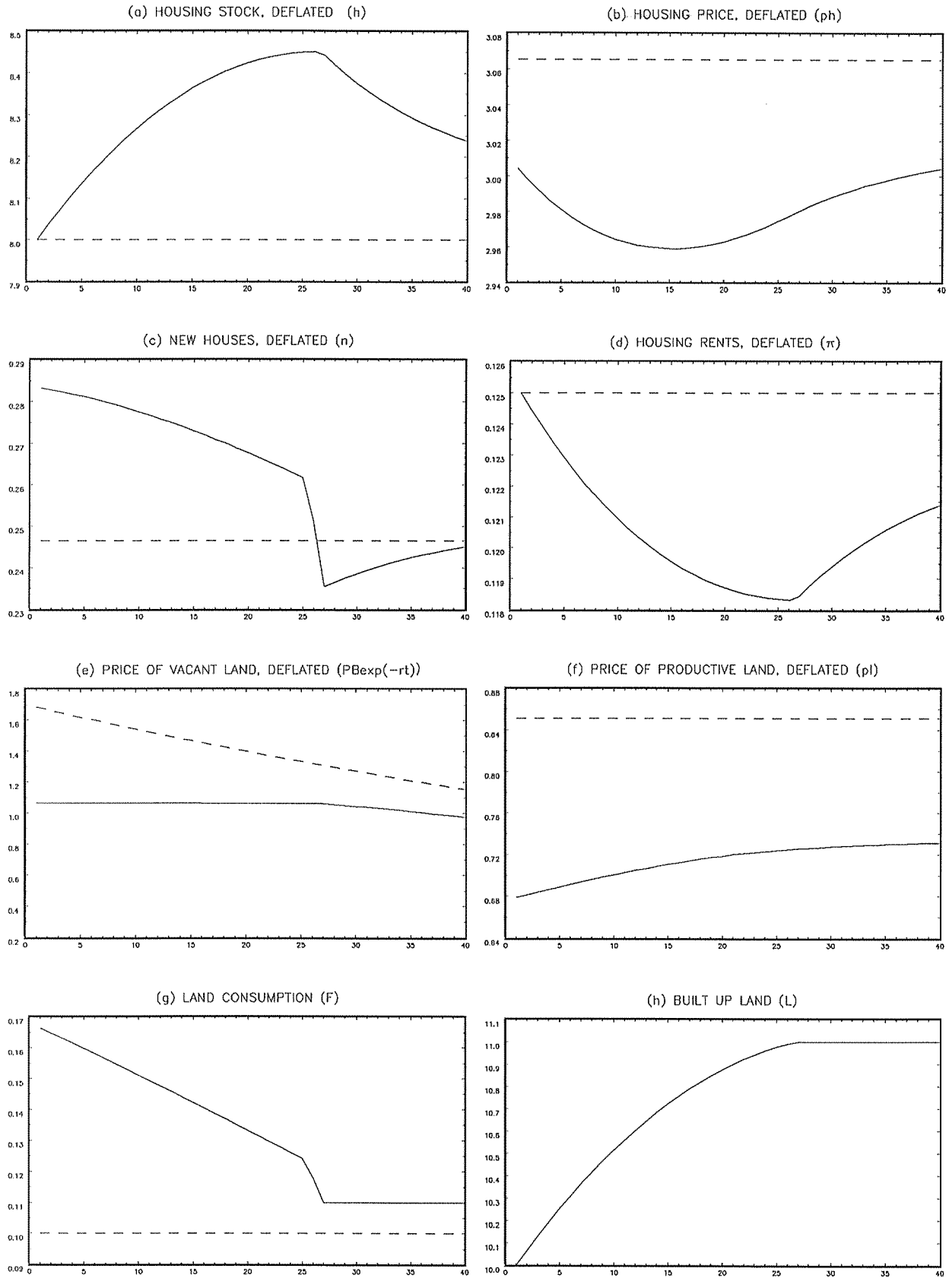


Figure 5: The effects of a 10 pct. increase in I





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