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## Intertemporal budget policies and macroeconomic adjustment in indebted open economies

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Reihe Ökonomie  
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# **Intertemporal Budget Policies and Macroeconomic Adjustment in Indebted Open Economies**

Marcelo Bianconi, Walter H. Fisher



INSTITUT FÜR HÖHERE STUDIEN  
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June 2011

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

## **Abstract**

We analyze the role of government intertemporal budget policies in a growing open economy including nominal assets in the presence of an upward sloping supply of debt. This introduces transitional dynamics that influence the effects of government policy instruments on the long term fiscal liability. In particular, shifts in capital income taxes can lead to dynamic scoring effects through the evolution of foreign debt. We show that a combination of tax-cumexpenditure, or government expenditure alone can balance the long term government budget constraint. However, for certain combinations of parameter values, the capital income tax alone cannot balance the intertemporal budget.

## **Keywords**

Government budget constraint, nominal assets, capital income tax

## **JEL Classification**

E5, E6, F4

### **Comments**

A previous version of this paper was presented at the workshop in honor of Stephen J. Turnovsky, May 20-22, 2010 at IHS Vienna. We thank the comments of the discussant, Ben J. Heijdra, and suggestions of several other participants. This paper was also presented at the Dynamic Macroeconomics Workshop, Free University of Bozen-Bolzano, May 26, 2011. We thank Stefan Shubert, Phil Brock and other participants for their very helpful comments and suggestion. The able research assistance of Conor Carney is gratefully acknowledged. Any errors are our own.

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# 1. Introduction

This paper analyzes the role of policy on the intertemporal government budget in a growing open economy including nominal assets. Specifically, we evaluate fiscal and monetary policies in terms of the public's intertemporal tax liability, measured by the present value of future lump-sum taxes scaled by the domestic capital stock. In a small open economy model, the constraint for the valuation of private and public financial assets is in terms of the exogenous foreign price level. Bianconi and Fisher (2005) show that this limits, under purchasing power parity, the scope of the government to influence the real value of financial assets using fiscal and monetary policy instruments.<sup>1</sup> We take a step further by considering budget policies in the presence of an upward sloping supply of debt. This introduces a premium on the interest service paid to domestic and foreign creditors and makes the interest rate in a small open economy vary with the level of foreign indebtedness. As in the open economy growth framework of Turnovsky (1997a), this provides an endogenous channel of interest movements, similar to the case of a closed economy. Hence, even though the private and public assets are denominated in terms of the foreign price level, the interest "premium" introduces transitional dynamics and convergence towards the balanced growth path where the growth rates of domestic capital and consumption are equated through an adjustment of the country's net foreign asset position.<sup>2</sup>

Our paper relates to several recent strands in the literature in this area. Our model is one in which domestic nominal assets are denominated in terms of the foreign good, and sometimes this is referred to as "dollarization", see, for example, the work of Calvo (2001), Yeyati and Sturzenegger (2001). In addition, our analysis of the government budget constraint and balanced growth relates to a literature on Laffer-style effects of fiscal policy, e.g. Laffer (1976), Slemrod (1994), Ireland (1994), Bruce and Turnovsky (1999), Bianconi (1999), Novales and Ruiz (2002), Bianconi and Fisher (2005) and Mankiw and Weinzierl (2006).

In this paper, we extend the results of Bianconi and Fisher (2005) to a framework where the net foreign asset position adjusts endogenously to take the economy to its balanced growth path. This adds a new channel for the possibility of dynamic scoring. In particular, the transitional dynamics of a change in the capital income tax can contribute significantly to the possibility of dynamic scoring taking place.

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<sup>1</sup> Their result hinges on the fact that the real stock of public debt is insulated from domestic price level shifts.

<sup>2</sup> Chen et al (2008) examines a related problem from the perspective of Taylor rules for interest rates.

A third recent strand of the literature is the interest burden in indebted economies and the possibility of erosion of government debts. This is the focus of the work of Engen and Hubbard (2004), Aizenman and Marion (2009), and Hall and Sargent (2010). In our paper, assets are denominated in foreign currency and domestic price level changes do not affect the value of the debt; however it does have an effect on future tax liabilities through the traditional inflation tax channel. Finally, our paper provides insight into the twin deficit phenomenon, a question addressed by Feldstein (1987), Chinn (2005) and Bartolini and Lahiri (2006), among others.

One goal of our model is to provide a direct link between the “twin” deficits within a framework of intertemporal solvency and endogenous growth. In this context, one of the key results is that the economy’s long-run tax liability depends not only the primary deficit net of inflation tax revenues, but also—in sharp contrast to Bianconi and Fisher (2005)—on the long-run accumulation of national debt in terms of the capital stock, as well as on the speed of adjustment to the long-run balanced growth path. The latter is a consequence of the fact that the economy borrows (and lends) subject to an upward-sloping interest rate relationship. Our novel result is that the possibility of dynamic scoring of a cut in capital income tax depends on the effect on national borrowing. A decrease in the capital tax increases the growth rate, which increases foreign debt; this will decrease the long run liability because higher growth increases capital tax revenues and reduces the long term tax liability. Hence, in our framework foreign deficits are negatively related to the long term liability of the government. This effect is enlarged in the case where a tax-cut-expenditure policy is used. In addition, our numerical simulations show that the introduction of an upward sloped supply of debt makes the capital income tax rate policy less likely to balance of the intertemporal government budget constraint in the long run. In particular, for a large part of the parameter space, the capital income tax rate alone cannot balance the intertemporal budget.

The paper is organized in the following. Section 2 we describe our model, based on the indebted open economy framework of Turnovsky (1997a), and derive the dynamic, growth equilibrium. In section 3 we calculate the public sector’s intertemporal budget constraint and derive the expression for present value of future lump-sum taxes in terms of the economy’s fiscal and monetary policy tools and long-run adjustment of national debt scaled by the domestic capital stock. We discuss the steady-state equilibrium in section 4 and describe the corresponding long-run fiscal and monetary policy multipliers, as well as the implications of a shift in the

interest “premium”. In section 5 we analyze the implications of several budget policies on the solvency of the public sector. In section 6 we conduct several numerical policy simulations and also consider shifts in the interest rate relationship. We close the paper in section 7 with brief concluding remarks and with a mathematical appendix that underlies some of the analytical results discussed in the main text.

## 2. The Model and Growth Equilibrium

In this section we outline the small open economy structure, which is based on the endogenous growth model of Bianconi and Fisher (2005). As indicated, a key difference, however, between this and the earlier framework is the assumption of an upward sloping supply of debt. Under this specification, the economy’s net foreign asset position adjusts so that all growth rates are equalized along the balanced growth path. Moreover, the macroeconomic equilibrium is characterized by saddlepath dynamics. As in Bianconi and Fisher (2005), this is a one-good open economy framework in which purchasing power parity (PPP) holds at all times. Letting  $p$  represent the rate of domestic inflation ( $p \equiv \dot{P}/P$ , where  $P$  is the domestic price level),  $p^*$  the exogenous foreign rate of inflation ( $p^* \equiv \dot{P}^*/P^*$ , where  $P^*$  is the given world price level), and  $e$  is the rate of depreciation of the domestic currency ( $e \equiv \dot{E}/E$ , where  $E$  is the nominal exchange rate), PPP corresponds to:

$$p = p^* + e \quad (1a)$$

We impose, as do Bianconi and Fisher (2005), nominal interest rate parity, but alter it by incorporating a “premium” term that is an increasing, convex function of the stock of the real national debt  $z \equiv Z/P^*$  scaled by the domestic capital stock  $k$ :<sup>3</sup>

$$i = i^* + e + v\left(\frac{z}{k}\right), \quad v' > 0, \quad v'' > 0, \quad (1b)$$

where  $Z$  is the nominal stock of national debt,  $i$  represents the domestic national interest rate, while  $i^*$  is the exogenous world nominal interest rate. Substituting (1a) into (1b), we obtain the corresponding real interest rate parity condition:

$$i - p = i^* - p^* + v\left(\frac{z}{k}\right) \equiv r\left(\frac{z}{k}\right), \quad r' > 0, \quad r'' > 0. \quad (1c)$$

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<sup>3</sup> The premium function can be rationalized on the basis of a factors model, here with one factor included; or as a ratings function (such as Moody’s, for example) so that the lower the rating the higher the interest paid at the margin.

The economy is modeled as a representative agent with instantaneous preferences for consumption  $c$  and real money balances  $m \equiv M/P$ , where  $M$  is the nominal stock money balances. We specify that instantaneous preferences take the simple separable logarithmic form:

$$U(c, m) = \log c + \gamma \log m, \quad \gamma > 0. \quad (1d)$$

The agent also accumulates real international financial assets (debt)  $b = B/P^*$ , where the nominal stock of bonds  $B$  is deflated by the foreign price level. The real return on international assets corresponds to (1c), respectively, while the (negative) real return on domestic money equals  $-(p^* + e)$ . As a producer, the agent has access to a technology that is linear homogenous in the domestic capital stock,  $Ak$ , which, under appropriate conditions detailed below, can sustain on-going growth.<sup>4</sup> To prevent instantaneous adjustment of the domestic capital stock, we assume that real investment  $I$  incurs installation costs modeled according to the standard quadratic specification:<sup>5</sup>

$$\Phi(i, k) = I \left( 1 + \frac{hI}{2k} \right), \quad h > 0. \quad (1e)$$

In addition, we follow authors such as Rebelo (1991), Bruce and Turnovsky (1999), Bianconi (1999) and Bianconi and Fisher (2005) by fixing the level of employment.

The representative agent's problem is formulated as follows:

$$\int_0^{\infty} (\log c + \gamma \log m) e^{-\delta t} dt, \quad \delta > 0, \quad (2a)$$

subject to:

$$\dot{m} + \dot{b} + I \left( 1 + \frac{hI}{2k} \right) = (1 - \tau)ak + r \left( \frac{z}{k} \right) b - c - (p^* + e)m - T, \quad (2b)$$

$$\dot{k} = I, \quad (2c)$$

where  $\tau$  = capital (output) tax rate ( $\tau \in [0, 1]$ ),  $T$  = lump-sum taxes,  $\delta$  = exogenous domestic rate of time preference. The agent's maximization problem is also subject to initial conditions on the stocks of domestic capital, nominal domestic money, and real international bonds:  $k(0) \equiv k_0 > 0$ ,  $M(0) \equiv M_0 > 0$ ,  $b(0) \equiv B_0/P_0^* > 0$ . In performing the optimization, the agent also

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<sup>4</sup> Some main references are Barro (1990), Jones and Manuelli (1990), Jones et al (1993), Rebelo (1991), Turnovsky (1996).

<sup>5</sup> The standard reference for the Tobin's  $q$  model of investment is Hayashi (1982).

takes the real interest rate  $r(z/k)$  as given. Standard techniques yield the following optimality conditions:

$$\frac{1}{c} = \lambda, \quad (3a)$$

$$1 + h \frac{I}{k} = q \Rightarrow \frac{I}{k} = \frac{\dot{k}}{k} = \frac{q-1}{h} \equiv \phi \Rightarrow k(t) = e^{\int_0^t \phi(s) ds}, \quad (3b)$$

$$\delta - \frac{\dot{\lambda}}{\lambda} = \frac{\gamma}{\lambda m} - (p^* + e) = r\left(\frac{z}{k}\right), \quad (3c)$$

$$\frac{(1-\tau)\alpha}{q} + \frac{(q-1)^2}{2hq} + \frac{\dot{q}}{q} = r\left(\frac{z}{k}\right), \quad (3d)$$

$$\lim_{t \rightarrow \infty} \lambda b e^{-\delta t} = \lim_{t \rightarrow \infty} \lambda m e^{-\delta t} = \lim_{t \rightarrow \infty} q \lambda k e^{-\delta t} = 0, \quad (3e)$$

where  $\lambda$  is the shadow value of international assets,  $q \equiv q'/\lambda$  is the shadow value of domestic capital in terms of international assets, and  $\phi$  denotes the economy's balanced growth rate that will be determined below. Equations (3a) – (3e) have the following straightforward interpretation: (3a) states that the marginal utility of consumption equals the shadow value of traded international assets; while (3b) indicates that the marginal cost of investment equals the shadow value (Tobin's  $q$ ) of domestic capital. Combining the optimality conditions for domestic money and international bonds, we derive in (3c) the real arbitrage condition for these assets, which, in turn, equals the rate of return of consumption, given by  $(\delta - \dot{\lambda}/\lambda)$ . Observe, again, that the real return to international assets includes the “premium”  $v(z/k)$  that depends on the national debt to capital ratio  $z/k$ . The arbitrage condition for domestic capital, given by (3d), must equal that of foreign bonds. Finally, the necessary transversality conditions for bonds  $b$ , domestic money  $m$ , and domestic physical capital  $k$  are stated in (3e).<sup>6</sup>

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<sup>6</sup> Substituting for (3b) and the solution for the shadow value  $\lambda$ , given by:

$$\lambda = \lambda(0) \exp \left\{ \delta t - \int_0^t r\left(\frac{z}{k}\right) ds \right\}$$

into the transversality condition (3e):

$$\begin{aligned} & \lim_{t \rightarrow \infty} q \lambda k e^{-\delta t} = \\ & = \lim_{t \rightarrow \infty} \lambda(0) \left\{ \exp \left[ \delta t - \int_0^t r\left(\frac{z}{k}\right) ds \right] \cdot \tilde{q} \cdot k_0 \exp \left[ \int_0^t \phi(s) ds \right] \cdot e^{-\delta t} \right. \\ & \quad \left. \tilde{q} \cdot k_0 \lim_{t \rightarrow \infty} \exp \left\{ \int_0^t \left[ \phi(s) - r\left(\frac{z}{k}\right) \right] ds \right\} \right\} = 0, \end{aligned}$$

which implies that the transversality condition is satisfied for  $q \rightarrow \tilde{q}$  and  $r \rightarrow \tilde{r}$  only if  $\tilde{\phi} < r(\tilde{z}/\tilde{k})$ , where  $\sim$  refers to the balanced growth equilibrium.

We next turn to the domestic public sector and describe the relationships defining the evolution of its financial liabilities. The public sector sells debt to foreign and domestic investors, assumed to be perfect substitutes for private assets traded internationally. Consequently, it bears a real rate of return equal to  $r(z/k) = i^* - p^* + v(z/k)$ . In contrast, money balances issued by the public sector are held only by domestic residents and erode in value at the rate equal to  $p^* + e$ . The flow of the government budget identity then corresponds to:

$$\dot{a} + \dot{m} = G + r\left(\frac{z}{k}\right)a - T - \tau\alpha k - (p^* + e)m, \quad (4a)$$

where  $a \equiv A/P^*$  is the real stock of government bonds evaluated in terms the exogenous foreign price level and  $G$  = real government expenditure. The evolution of government bonds is also subject to an initial condition corresponding to  $a(0) = a_0 = A/P^* > 0$ , where  $A$  denotes the nominal stock of government bonds in terms of foreign currency. To guarantee the intertemporal solvency of the public sector, we impose the following limiting condition on the path of government debt:  $\lim_{t \rightarrow \infty} \lambda a e^{-\delta t} = 0$ . Along with Bianconi (1999) and Bianconi and Fisher (2005), we assume that government expenditure and lump-sum taxes are set proportional to output. For government expenditure, this implies that  $G(t) = \bar{g}\alpha k(t)$ , where  $\bar{g}$  is the fraction of output devoted to public expenditures; while for lump-sum taxes, the fraction  $\bar{T}(t)$  corresponds to  $\bar{T}(t) = T(t)/\alpha k(t)$ . Finally, we specify that the public sector follows a simple constant *nominal* money growth rule, i.e., it sets  $\sigma = \dot{M}/M$ , which implies that the evolution of the *real* money supply equals:<sup>7</sup>

$$\dot{m} = (\sigma - p)m = (\sigma - p^* - e)m. \quad (4b)$$

Using the definition of real national debt,  $z \equiv a - b$ , combining (2b) and (4b) yields the expression for the current account balance:

$$\dot{z} = c + I\left(1 + \frac{hI}{2k}\right) - (1 - \bar{g})\alpha k + r\left(\frac{z}{k}\right)z, \quad (5a)$$

where we substitute  $G(t) = \bar{g}\alpha k(t)$  to obtain (5a). Substituting for  $I = h^{-1}(q - 1)k$  in (5a), we can express the current account balance in terms of  $q$ :

$$\dot{z} = (1 - \bar{g})\alpha k + c + \frac{(q^2 - 1)}{2h} + r\left(\frac{z}{k}\right)z. \quad (5b)$$

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<sup>7</sup> The constant rate of growth of money policy is not backed by any public debt, as in the contribution of Auernheimer (1974).

We next develop the open economy growth equilibrium, which we derive in terms of intensive variables. Specifically, we scale the variables of interest by the domestic capital stock  $k$  and employ the following notation, respectively, for consumption and national debt in terms of the domestic capital stock:  $\chi \equiv c/k$  and  $\psi \equiv z/k$ . The rates of growth of these ratios equal:

$$\frac{\dot{\chi}}{\chi} \equiv \frac{\dot{c}}{c} - \frac{\dot{k}}{k}, \quad \frac{\dot{\psi}}{\psi} \equiv \frac{\dot{z}}{z} - \frac{\dot{k}}{k}.$$

Calculating the time derivative of (3a) and combining with (3a) and (3c), we obtain:

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda} = r\left(\frac{z}{k}\right) - \delta. \quad (6)$$

Using the definitions  $\chi \equiv c/k$  and  $\psi \equiv z/k$ , we solve for the differential equation  $\dot{\chi}$  for consumption-capital ratio:

$$\dot{\chi} = \left[ r(\psi) - \delta - \frac{q-1}{h} \right] \chi. \quad (7a)$$

Using (3b) and (6) and the definitions  $\chi \equiv c/k$  and  $\psi \equiv z/k$ , the differential equation  $\dot{\psi}$  for the national debt-domestic capital ratio is also straightforward to obtain:

$$\dot{\psi} = \chi + \frac{(q^2 - 1)}{2h} - \frac{(q - 1)}{h} - (1 - \bar{g})\alpha + r(\psi)\psi. \quad (7b)$$

Clearly, the differential equation for Tobin's  $q$  is found directly in terms of national debt to capital ratio through the upward-sloping interest rate relationship:

$$\dot{q} = r(\psi)q - (1 - \tau)\alpha - \frac{(q - 1)^2}{2h}. \quad (7c)$$

For convenience, we collect (7a) – (7c) and state the system describing the dynamics of the small open economy

$$\begin{aligned} \dot{\chi} &= \left[ r(\psi) - \delta - \frac{q-1}{h} \right] \chi, \\ \dot{\psi} &= \chi + \frac{(q^2 - 1)}{2h} - \frac{(q - 1)}{h} - (1 - \bar{g})\alpha + r(\psi)\psi, \\ \dot{q} &= r(\psi)q - (1 - \tau)\alpha - \frac{(q - 1)^2}{2h}. \end{aligned} \quad (8)$$

Employing standard methods, the saddlepath solutions for the consumption-capital, national debt-capital, and Tobin's  $q$  correspond to:



$$\chi - \tilde{\chi} = \frac{\tilde{\chi} v'(\tilde{\psi})}{\xi_1} \left[ 1 + \frac{\frac{\tilde{q}}{h}}{\left[ r(\tilde{\psi}) - \frac{\tilde{q} - 1}{h} \right] - \xi_1} \right] \cdot (\psi - \tilde{\psi}), \quad (9a)$$

$$q - \tilde{q} = - \frac{v'(\tilde{\psi}) \tilde{q}}{\left[ r(\tilde{\psi}) - \frac{\tilde{q} - 1}{h} \right] - \xi_1} \cdot (\psi - \tilde{\psi}), \quad (9b)$$

where  $\psi = \tilde{\psi} - (\tilde{\psi} - \psi_0)e^{\xi_1 t}$  is the stable solution for the national debt-capital ratio, and  $\xi_1 < 0$  is the stable root of the system.<sup>8</sup> The saddlepaths are illustrated in Figure 1 and describe negatively-sloped relationships. The upper panel of Figure 1 depicts the locus XX describing the stable adjustment of  $\psi$  and  $\chi$ . It is negatively sloped, since a greater level of indebtedness lowers domestic wealth, which reduces consumption. Equally, the locus YY in  $(\psi, q)$  space is negatively-sloped, since a higher level of indebtedness relative to the domestic stock of capital raises the domestic real interest rate  $r(\tilde{\psi})$ , which, under arbitrage, requires a lower Tobin's  $q$ .

To evaluate the intertemporal implications of fiscal, monetary policy and the interest premium, we derive the expression describing the dynamics of domestic real money balances scaled by the domestic capital stock. Combining (3c) with (4b) and substituting for (3a), we obtain

$$\frac{\dot{m}}{m} = \sigma + r\left(\frac{z}{k}\right) - \frac{\gamma c}{m}. \quad (10a)$$

Letting  $\mu \equiv m/k$  represent the real money-capital ratio, so that  $\dot{\mu}/\mu = \dot{m}/m - \dot{k}/k$ , and using (3b) and (10a), as well as the definitions  $\chi \equiv c/k$  and  $\psi \equiv z/k$ , we obtain the differential equation for  $\mu$ :

$$\dot{\mu} = \left[ \sigma + r(\psi) - \frac{q - 1}{h} \right] \mu - \gamma \cdot \chi. \quad (10b)$$

Linearizing (10b) about the steady-state equilibrium, substituting for the solutions (9a) – (9b), and integrating the resulting expression subject to the transversality condition yield the saddlepath solution for  $\mu$ :

$$\mu = \frac{\gamma \cdot \tilde{\chi}}{\sigma + \delta} + \frac{\Omega \cdot (\tilde{\psi} - \psi_0)e^{\xi_1 t}}{\sigma + \delta - \xi_1}, \quad (10c)$$

where

$$\tilde{\mu} = \frac{\gamma \cdot \tilde{\chi}}{\sigma + \delta}, \quad (10d)$$

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<sup>8</sup> The derivation of (9a) – (9b) is shown in the Appendix.

is the steady-state value of real money-domestic ratio and

$$\Omega \equiv v'(\tilde{\psi}) \left( \tilde{\mu} - \frac{\gamma \tilde{\chi}}{\xi_1} \right) \left[ 1 + \frac{\tilde{q}}{(\delta - \xi_1)} \right] > 0.$$

We observe that the latter implies that the long-run ratio of real money to consumption,  $\left( \frac{\overline{m/k}}{c/k} \right)$ , is  $\gamma/(\sigma + \delta)$ .

Finally, given our logarithmic preferences parameterization, using equations (10a-10d) and integrating, we obtain a measure of discounted welfare scaled by the capital stock given by:

$$W = \frac{1}{\delta} (\log \tilde{\chi} + \gamma \log \tilde{\mu}) + \left( \frac{1}{\delta - \xi_1} \right) (w_1 + \gamma w_2) (\tilde{\psi} - \psi_0), \quad (11)$$

where  $w_1 \equiv \frac{-v'(\tilde{\psi})}{\xi_1} \left[ 1 + \frac{\tilde{q}/h}{\delta - \xi_1} \right] > 0$ ;  $w_2 \equiv v'(\tilde{\psi}) \left( 1 - \frac{\gamma \tilde{\chi}}{\xi_1 \tilde{\mu}} \right) \left[ 1 + \frac{\tilde{q}/h}{\delta - \xi_1} \right] > 0$ .

### 3. Intertemporal Government Budget Constraint

We begin by substituting  $G(t) - T(t) = [\bar{g} - \bar{T}(t)]\alpha k(t)$  and  $\dot{m} = (\sigma - p^* - e)m$  into the government budget identity, yielding:

$$\dot{a} = [\bar{g} - \bar{T}(t)]\alpha k + r \left( \frac{z}{k} \right) a - \tau \alpha k - \sigma m. \quad (12a)$$

Defining real government debt in terms of the domestic capital stock  $\omega \equiv a/k$  (so that  $\dot{\omega}/\omega = \dot{a}/a - \dot{k}/k$ ) and using definitions  $\psi \equiv z/k$  and  $\mu \equiv m/k$ , we calculate the differential equation for  $\omega$ :

$$\dot{\omega} = [\bar{g} - \bar{T}(t)]\alpha + \left[ r(\psi) - \frac{q-1}{h} \right] \omega - \tau \alpha - \sigma \cdot \mu. \quad (12b)$$

Linearizing (12b), we obtain:

$$\dot{\omega} = \delta \omega + [\bar{g} - \bar{T}(t) - \tau]\alpha - \sigma \cdot \mu + v'(\tilde{\psi})\tilde{\omega} \cdot (\psi - \tilde{\psi}) - \frac{\tilde{\omega}}{h} \cdot (q - \tilde{q}). \quad (12c)$$

Substitution for the saddlepath solutions for  $\mu$ ,  $\psi$ , and  $q$ , we find:

$$\dot{\omega} = \delta \omega + [\bar{g} - \bar{T}(t) - \tau]\alpha - \frac{\sigma \gamma \cdot \tilde{\chi}}{\sigma + \delta} - \Phi \cdot (\tilde{\psi} - \psi_0) e^{\xi_1 t}, \quad (12d)$$

where

$$\begin{aligned} \Phi &\equiv \frac{\sigma \Omega}{\sigma + \delta - \xi_1} + v'(\tilde{\psi})\tilde{\omega} \left[ 1 + \frac{\tilde{q}}{\delta - \xi_1} \right] \\ &= -\frac{v'(\tilde{\psi})}{\delta} \left[ 1 + \frac{\tilde{q}}{\delta - \xi_1} \right] \left[ (\bar{g} - \tau)\alpha + \frac{\delta - \xi_1}{\xi_1} \cdot \sigma \tilde{\mu} \right] > 0 \end{aligned}$$

and represents a scale factor for the transitional dynamics effect. Imposing that the ratio of public debt in terms of the capital stock evolves from a given initial level,  $\omega(0) \equiv \omega_0$ , integration of (12d) results in the following general solution for  $\omega$ :

$$\begin{aligned} \omega = & \frac{(\bar{g} - \tau)\alpha}{\delta} + \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} + \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0)e^{\xi_1 t} - \alpha e^{\delta t} \int_0^t \bar{T}(s)e^{-\delta s} ds \\ & + \left[ \omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} - \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0) \right] e^{\delta t}. \end{aligned} \quad (12e)$$

Imposing the transversality condition  $\lim_{t \rightarrow \infty} \lambda \omega \exp - \left[ \delta t - \int_0^t \phi(s) ds \right] = 0$  on the general solution (12e), we solve for the intertemporal government budget constraint:

$$\alpha \int_0^\infty \bar{T}(s)e^{-\delta s} ds = \omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} - \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0). \quad (13a)$$

Equally, this long-run restriction can be expressed directly in terms of the path of lump-sum taxes,  $T(t)$ , scaled by the domestic capital stock:

$$\int_0^\infty [T(s)/k(s)]e^{-\delta s} ds = \omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} - \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0). \quad (13b)$$

Back substituting the intertemporal budget constraint into the general solution for  $\omega$ , we obtain the saddlepath relationship for the ratio of the stock of public debt in terms of the domestic capital stock:

$$\omega = -\frac{(\bar{g} - \tau)\alpha}{\delta} + \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} + \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0)e^{\xi_1 t} + \alpha \int_t^\infty \bar{T}(s)e^{-\delta s} ds. \quad (14a)$$

The latter can, of course, be written directly in terms of the path of lump-sum taxes scaled by the capital stock:

$$\omega = -\frac{(\bar{g} - \tau)\alpha}{\delta} + \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} + \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0)e^{\xi_1 t} + \int_t^\infty [T(s)/k(s)]e^{-\delta s} ds, \quad (14b)$$

where along the balanced growth path:

$$\tilde{\omega} = -\frac{(\bar{g} - \tau)\alpha}{\delta} + \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} = -\frac{(\bar{g} - \tau)\alpha}{\delta} + \frac{\sigma \cdot \tilde{\mu}}{\delta}, \quad (14c)$$

represents the long-run stock of government debt scaled by capital stock.

Clearly,  $\tilde{\omega}$  depends directly on the primary fiscal deficit  $(\bar{g} - \tau)$  and on the revenues from money creation, represented by  $\sigma \cdot \tilde{\mu}$ , where we substitute for (10d) to obtain the second equality of (14c). In view of (14c), we can express the long-run restriction on the path of the

lump-sum taxes in terms of the long-run adjustment of government and national debt in terms of the domestic capital stock:

$$\int_0^\infty [T(s)/k(s)]e^{-\delta s}ds = [\omega(0) - \tilde{\omega}] - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0). \quad (13b')$$

For the remainder of the paper, we define (13b') the present discounted value of taxes required to maintain intertemporal solvency of the public sector budget as:

$$\begin{aligned} V(T/k) &\equiv \int_0^\infty [T(s)/k(s)]e^{-\delta s}ds \\ &= \omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} - \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0), \end{aligned} \quad (15)$$

where  $V(T/k)$  represents the measure of sustainable long-run fiscal balance discussed in Bruce and Turnovsky (1999), Bianconi (1999), and Bianconi and Fisher (2005). In addition to the initial stock of public debt (scaled by the capital stock)  $\omega(0)$ , observe that the measure  $V(T/k)$  of the long-run tax liability stated in (15) depends not only on the primary deficit  $(\bar{g} - \tau)$  net of inflation tax revenue (represented by the term  $\frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma + \delta)\delta} = \frac{\sigma \cdot \tilde{\mu}}{\delta}$ ), but also—in contrast to Bianconi and Fisher (2005)—on the long-run accumulation of national debt in terms of the capital stock  $(\tilde{\psi} - \psi_0)$ , as well as on the speed of stable adjustment  $\xi_1 < 0$ . The latter is a consequence of the fact that the economy borrows (and lends) subject to the upward-sloping interest rate relationship described in (1c).<sup>9</sup>

#### 4. Steady State, Long-Run Effects and Impact Effects

Letting  $\dot{\chi} = \dot{\psi} = \dot{q} = 0$ , the long-run equilibrium of the small open economy corresponds to:

$$r(\tilde{\psi}) - \delta = (i^* - p^*) + v(\tilde{\psi}) - \delta = \frac{\tilde{q} - 1}{h} = \tilde{\phi}, \quad (16a)$$

$$\tilde{\chi} + \frac{\tilde{q}^2 - 1}{2h} - \frac{(\tilde{q} - 1)\tilde{\psi}}{h} + r(\tilde{\psi})\tilde{\psi} = (1 - \bar{g})\alpha, \quad (16b)$$

$$\frac{(1 - \tau)\alpha}{\tilde{q}} + \frac{(\tilde{q} - 1)^2}{2h\tilde{q}} = r(\tilde{\psi}), \quad (16c)$$

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<sup>9</sup> In Bianconi and Fisher(2005), the formula for the long term tax liability is:  $V(T/k) \equiv \int_0^\infty [T(s)/k(s)]e^{-(g+\delta)s}ds = -\frac{(\bar{g}-\tau)\alpha}{d+\delta} + \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma+\delta)\delta}$  where  $g$  is the growth of consumption and  $d$  is the difference between the growth of consumption and the capital stock.

where  $x = \tilde{x}$  denotes a steady-state variable. The three equations determine the values of  $\{\tilde{q}, \tilde{\psi}, \tilde{\chi}\}$  as a function of the parameters of the model, independently of the initial conditions.

Equation (16a) is the long-run consumption-Euler relationship and indicates that the steady-state balanced growth  $\tilde{\phi}$  equals the real interest rate net of the rate of physical capital. Long-run market clearing is given in equation (16b), while (16c) is the steady-state version of the arbitrage equation (16c). As in Turnovsky (1997a), the endogenous adjustment of the national real interest rate  $r(\tilde{\psi})$  insures that in long-run equilibrium the ratios of consumption and national debt to domestic capital reach their steady-state values and, thus, that the economy ultimately attains a common growth rate  $\tilde{\phi}$ . Observe, moreover, that the steady-state equilibrium (16a) – (16c) is non-linear. It is straightforward to establish the conditions under which the economy's steady-state balanced growth rate is unique and positive.<sup>10</sup>

The model economic policy parameters are  $\{\bar{g}, \tau, \sigma; \omega(0)\}$  while the model parameters are  $\{h, \delta, \alpha, \gamma, r(i^*, p^*, v)\}$ .<sup>11</sup> In Table 1a we present some comparative statics of the fiscal and monetary policy variables and the parameter of the real interest function (including the risk premium relationship) on the balanced growth equilibrium. Table 1b presents the corresponding impact effects. All effects are evaluated at the initial balanced growth equilibrium. We first note that the order of the effects (from left to right in Table 1a) represents the order of impact on the overall economy. A shift in the capital tax rate affects all endogenous variables, followed by the real interest function, which, however, has no effect on growth  $\tilde{\phi}$ . Both of those exogenous factors do impact upon the long run national debt per unit of capital,  $\tilde{\psi}$ , and thus lead to transitional dynamics towards the stable adjustment path to the long run balanced growth path. This is followed by government spending which—while it crowds-out private consumption and real money balances—has no effect on growth. Finally, increasing the rate of money growth only affects money demand.

In this model, government spending and monetary policy do not impact upon the long-run national debt per unit of capital,  $\tilde{\psi}$ , and thus there are no transitional dynamics towards the

<sup>10</sup> See Turnovsky (1997a)—who uses a more general specification of preferences and productive government expenditure—for details. For a general discussion of these issues, see Turnovsky (1997b), chapter 5.

<sup>11</sup> The initial stock of public debt is included as a policy parameter for the potential case of a change in government policy parameters financed by a swap with initial public debt; see e.g. Novales and Ruiz (2002).

stable adjustment path to the long run balanced growth path in those cases. The capital tax rate has a negative effect on long run growth and on the marginal cost of capital. It also has a negative effect on the long run national debt and shifts resources to consumption and the accumulation of real money balances through a lower domestic price level. The effect of the capital tax rate on the long run stock of government debt is ambiguous, because while there is the direct, negative, effect on long term growth that raises the stock of government debt, there is also negative indirect effect through additional consumption and money balances (the inflation tax channel). The real interest function does not affect growth and the long-run marginal cost of capital, but it discourages foreign borrowing thus decreases the long-run stock of foreign debt. This shift increases long run consumption and money balances and raises the long run government debt through the additional real interest cost of debt. Government spending reduces long term money balances through higher price levels thus reducing long term government debt; but higher money growth has the opposite effect on long term government debt because there is no direct long term consumption effect in this case.

The last row of Table 1a presents the effects on discounted welfare. A change in the capital tax rate has two opposing effects on welfare. The positive effect of the tax rate on consumption and money balances raises welfare, while the negative effect on foreign debt, through the transitional dynamics, lowers welfare. Similarly for a change in the interest rate function. A change in government spending or in the rate of growth of money lowers welfare unambiguously through the consumption and money balances channel.

In Table 1b we note that the impact effect of a change in the capital tax rate is unambiguously negative on the marginal cost of capital, but ambiguous for initial consumption and money balances. The reason is that there is a positive effect from higher long term consumption but a negative effect from lower long term foreign debt. An increase in the real interest function also decreases the initial marginal cost of capital but has similar ambiguous effects in initial consumption and money balances. A change in government spending does not affect the initial marginal cost of capital, and it crowds out initial private consumption and reduces initial money balances thus increasing the initial domestic price level. Higher money growth reduces initial money balances as well. From Tables 1a and 1b, it is clear that changes in government spending and money growth do not give rise to transitional dynamics, the economy jumps from one balanced growth path to another directly.

## 5. Budget Policies and Analysis

In this section, we describe the implications of a change in the government policy parameters and the real interest function on the long-term tax liability.

There are several important economic questions that our framework can address. In Bianconi and Fisher (2005), we focused on effects of changes in fiscal and monetary policy on the balanced growth path, the potential for dynamic scoring effects on the intertemporal budget constraint and the use of fiscal and monetary policy to guarantee long run fiscal solvency. The novelty in this paper is that the interest premium on debt has effects on the growth equilibrium and transitional dynamics. Consequently, it adds an important linkage between fiscal policy and the current account, thus shedding light, among other things, on the twin deficits hypothesis discussed, for example, by Feldstein (1987), Chinn (2005), and Bartolini and Lahiri (2006). The key relationship that illustrates this issue is equation (15) which we rewrite in terms of the inflation tax:

$$V(T/k) \equiv \int_0^\infty [T(s)/k(s)] e^{-\delta s} ds = \omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} - \frac{\sigma}{\delta} \cdot \tilde{\mu} - \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0).$$

Rewriting (15) as:

$$\omega(0) + \frac{(\bar{g} - \tau)\alpha}{\delta} = V(T/k) + \frac{\sigma \cdot \tilde{\mu}}{\delta} + \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0), \quad (17)$$

we observe that, holding other factors constant, this model is Ricardian since public debt and lump-sum tax liabilities are perfectly correlated, and the primary fiscal deficit,  $\bar{g} - \tau$ , (net of inflation tax revenues) and foreign debt accumulation,  $(\tilde{\psi} - \psi_0)$ , tend to move in the same direction.

First, we examine the effects of a change in government policy parameters and the real interest function on the long term tax liability,  $V(T/k)$ , all evaluated at a given initial equilibrium. A change in the capital income tax  $\tau$  is obtained by evaluating (15) as follows:

$$\begin{aligned} \frac{\partial V(T/k)}{\partial \tau} &= -\frac{\alpha}{\delta} - \frac{\sigma \gamma}{(\sigma + \delta)\delta} \cdot \frac{\partial \tilde{\chi}}{\partial \tau} - \frac{\Phi}{\delta - \xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial \tau} \\ &= -\frac{\alpha}{\delta} - \frac{\sigma}{\delta} \cdot \frac{\partial \tilde{\mu}}{\partial \tau} - \frac{\Phi}{\delta - \xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial \tau} \geq 0. \end{aligned} \quad (18a)$$

There are three distinct effects on the long-run tax liability  $V(T/k)$ . The first is the negative effect on the primary deficit,  $-\frac{\alpha}{\delta}$ , which decreases the long-run tax liability. The next term,  $-\frac{\sigma\gamma}{(\sigma+\delta)\delta} \cdot \frac{\partial \tilde{\chi}}{\partial \tau} = -\frac{\sigma}{\delta} \cdot \frac{\partial \tilde{\mu}}{\partial \tau}$  captures the fact that an increase in the tax, because it discourages capital accumulation, increases the long-run consumption-capital ratio  $\tilde{\chi}$ , which, because it also results in an increase in the real money capital ratio  $\tilde{\mu}$ , increases inflation tax revenues and reduces  $V(T/k)$ . The last term  $-\frac{\Phi}{\delta-\xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial \tau}$  refers to the effect on national indebtedness and is part of the transitional dynamic adjustment induced by the tax change, but is of the opposite sign as the first two effects. An increase in the capital tax lowers the growth rate, which lowers  $\tilde{\psi}$  according to the steady state consumption Euler relationship (16a). This will increase the long run liability because lower growth lowers capital tax revenues. This is the source of the possibility of dynamic scoring in this paper. It is clear, then, the first two channels lead to a decrease in future tax liabilities; but the transitional dynamics channel increases the future tax liability.<sup>12</sup>

A change in the share of government spending  $\bar{g}$ , evaluating (15) at the initial equilibrium:

$$\frac{\partial V(T/k)}{\partial \bar{g}} = \frac{\alpha}{\delta} \left[ 1 + \frac{\sigma\delta}{\sigma+\delta} \right] \geq 0. \quad (18b)$$

In this case there are two direct effects and no transitional dynamics. The first is the positive effect on the primary deficit,  $\frac{\alpha}{\delta}$ , which increases the long-run tax liability. The next term,  $\frac{\sigma\delta}{\sigma+\delta}$  captures the fact that an increase in the government spending crowds out private consumption and reduces money balances thus increasing  $V(T/k)$ . Both channels lead to an increase in future tax liabilities.<sup>13</sup>

Similarly, a change in the rate of growth of money,  $\sigma$ , yields:

$$\frac{\partial V(T/k)}{\partial \sigma} = \frac{\gamma\tilde{\chi}}{(\sigma+\delta)^2} < 0. \quad (18c)$$

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<sup>12</sup> This result can be related to the earlier findings of Bianconi and Fisher (2005). See their Proposition 1, p. 12 that describes the conditions for dynamic scoring to take place in their framework in which the economy can borrow and lend at the fixed world interest rate.

<sup>13</sup> Government spending does not provide any external effects in private utility of production in this model, hence just crowds out private consumption and raises the price level with the decline in money balances.



There are no transitional dynamics in this case as well and only the inflation tax effect. An increase in the growth of money reduces money balances (increases the price level), but at the initial equilibrium it increases inflation tax revenues and reduces  $V(T/k)$ .

Next, we consider a balanced-budget change in the capital tax, i.e.,  $d\tau = d\bar{g}$ . We find:

$$\begin{aligned} \frac{\partial V(T/k)}{\partial \tau} \Big|_{d\tau=d\bar{g}} &= -\frac{\sigma}{\delta} \cdot \frac{\partial \tilde{\mu}}{\partial \tau} \Big|_{d\tau=d\bar{g}} - \frac{\Phi}{\delta - \xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial \tau} \Big|_{d\tau=d\bar{g}} \\ &= \frac{\alpha}{r'(\tilde{\psi})(\tilde{q} + \delta h)} \left\{ \frac{\Phi}{\delta - \xi_1} - \frac{\gamma\sigma[1 - hr'(\tilde{\psi})]}{(\sigma + \delta)} \right\} \geq 0. \end{aligned} \quad (19a)$$

Clearly, since the primary deficit is unaffected by the balanced budget tax cut, implications for long-run tax liabilities depend solely on the responses of  $\tilde{\mu}$  and  $\tilde{\psi}$ . It is straightforward to show that the change in the consumption-capital ratio is ambiguous, since an (de)increase in government expenditure crowds-in(out) consumption. Specifically, the shift in  $\tilde{\chi}$  is given by:

$$\frac{\partial \tilde{\mu}}{\partial \tau} \Big|_{d\tau=d\bar{g}} = -\frac{\gamma\alpha\delta[1 - hr'(\tilde{\psi})]}{r'(\tilde{\psi})(\tilde{q} + \delta h)(\sigma + \delta)} \geq 0. \quad (19b)$$

If the consumption-capital ratio rises(declines) on net, then so does the real money-capital ratio, which (de)increases inflation tax revenues and raises the possibility of dynamic scoring in response to a balanced-budget tax cut. In contrast, the response of national indebtedness is the same whether or not  $\bar{g}$  falls along with  $\tau$ :

$$\frac{\partial \tilde{\psi}}{\partial \tau} = \frac{\partial \tilde{\psi}}{\partial \tau} \Big|_{d\tau=d\bar{g}} = -\frac{\alpha}{v'(\tilde{q} + \delta h)} < 0. \quad (19c)$$

Thus, as in (18a), the rise(decline) in  $\tilde{\psi}$  contributes to the rise(decline) in long-run tax liabilities  $V(T/k)$ .

In summary, we can state the conditions for dynamic scoring under a capital income tax rate change.

**Proposition 1:** A cut in the capital income tax rate decreases long term government liability if

$$\frac{\alpha}{\delta} + \frac{\sigma}{\delta} \cdot \frac{\partial \tilde{\mu}}{\partial \tau} < -\frac{\Phi}{\delta - \xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial \tau};$$

i.e. the overall weighted effect on foreign borrowing is larger than the direct effect plus the inflation tax effect.

This result emerges directly from equation (18a).

**Proposition 2:** A necessary condition for a cut in the capital income tax rate balanced with a cut in the fraction of government spending to decrease the long term government liability is

$$\frac{\Phi}{\delta - \xi_1} > \frac{\gamma \sigma [1 - hr'(\tilde{\psi})]}{(\sigma + \delta)},$$

and a sufficient condition is that

$$1 - hr'(\tilde{\psi}) < 0 \quad \Leftrightarrow \quad r'(\tilde{\psi}) > \frac{1}{h}.$$

This finding is a direct consequence of (19a). Note that the sufficient condition implies that the slope of the premium function has to be greater than the inverse of the slope of the investment adjustment cost parameter. As long as the slope of the premium function is high enough to discourage a relatively high level of foreign borrowing, the likelihood of dynamic scoring is increased.

We next consider the implications of an external shock. A shift in the real interest function  $r$ , due, for example, to a rise in the world real interest rate, impacts on the long-term liability according to:

$$\frac{\partial V(T/k)}{\partial r} = \frac{1}{v'} \left[ -\frac{\sigma \gamma \cdot \tilde{\chi}}{(\sigma + \delta) \delta} + \frac{\Phi}{\delta - \xi_1} \right] \geq 0. \quad (20)$$

The first effect,  $\frac{-1}{v'} \cdot \frac{\sigma \gamma}{\sigma + \delta}$  refers to the effect on long-run consumption and money balances.

Because a higher real factor in interest does *not* affect the marginal cost of capital and growth, it must reduce the long run national indebtedness. This allows higher long run consumption and money balances to increase, which reduces the long-term tax liability through the inflation tax channel. However, the transitional dynamics effect of the lower long run national indebtedness,  $\frac{1}{v'} \cdot \frac{\Phi}{\delta - \xi_1}$  raises long term tax liability through the *higher* interest cost and lower growth during the transition. Those effects are in opposite direction and the net effect on long term liability is, thus, ambiguous. Both effects are scaled by  $\frac{1}{v'}$  which is the (inverse of) the slope of the interest premium, i.e. the change in the premium given a change in national indebtedness. The steeper the slope of the interest premium function, the smaller the overall net effect of the real interest rate on the long-term tax liability because, all else constant, the steeper slope implies a smaller level effect on the great ratios of the economy. Overall, the effects of the external shock operate along the transitional path that reflects the decline in national indebtedness. This is because the

fall in indebtedness leads to a reduction in the premium that causes the domestic interest rate return to its original long-term level.<sup>14</sup>

We now turn to the issue of the long run sustainability of budget policies. Bruce and Turnovsky (1999), Bianconi (1999), and Fisher and Bianconi (2005) define a “stricter” measure of sustainability of the intertemporal budget constraint as the choice of fiscal and monetary policies that guarantee  $V(T/k) = 0$ . In other words, fiscal and monetary policies are set so no *future* tax liabilities are needed to balance the intertemporal budget.<sup>15</sup> In the framework of this paper,  $V(T/k) = 0$  can be guaranteed by the choice of one (or more) of the government policy parameters  $\{\bar{g}, \tau, \sigma; \omega(0)\}$  under the restriction that  $V(T/k) = 0$ .

In the next section we use numerical simulations and sensitivity analysis to gain further insights into the effects of government policy on the long term tax liability under the interest risk premium function, including a welfare ranking of the alternative policies.

## 6. Numerical Simulations

We provide some numerical evaluations to illustrate some of the main results, with the benchmark set of parameter values used in the simulations for the balanced growth path given by:

$$h = 1, \quad \delta = 0.04, \quad \alpha = 0.1, \quad \tau = 0.31, \quad \bar{g} = 0.11, \quad \gamma = 0.20, \quad \sigma = 0.04, \quad (21a)$$

$$k_0 = 10 \text{ (so that } \alpha k_0 = 1), \quad b_0 = 0.50, \quad i^* = 0.10, \quad p^* = 0.04, \quad \alpha_0 = 0.55, \quad z_0 = 0.05,$$

where the implied value of Tobin’s  $q$  is  $1.0275 > 1$  so that the equilibrium is characterized by ongoing growth, e.g. 2.75%. We parameterize the interest premium by the function

$$v(\tilde{\psi}) = s_1 \exp(s_2 + s_3 \tilde{\psi}), \quad s_1, s_2, s_3 > 0, \quad (21b)$$

and use parameters  $s_1 = 0.00002$ ,  $s_2 = 5.75$ , and  $s_3 = 1.75$ . Figure 2 illustrates the convex relationship between the level of the national indebtedness in terms of the domestic capital stock  $\tilde{\psi}$ , and the interest premium. We also show the sensitivity of the interest rate function where  $s_3$  increases to 2.75. The initial equilibrium is one where the tax rate is large relative to the government share of output. This initial equilibrium implies that the initial tax liability,  $V(T/k)$

<sup>14</sup> For example, a rise in the world interest rate  $r^*$ , where  $r(\tilde{\psi}) = r^* + v(\tilde{\psi})$  yields

$$\frac{\partial V(T/k)}{\partial r^*} = -\frac{\sigma}{\delta} \cdot \frac{\partial \tilde{\mu}}{\partial r^*} - \frac{\Phi}{\delta - \xi_1} \cdot \frac{\partial \tilde{\psi}}{\partial r^*} = -\frac{1}{r'(\tilde{\psi})} \left[ \frac{\sigma \gamma}{\sigma + \delta} - \frac{\Phi}{\delta - \xi_1} \right] \geq 0.$$

<sup>15</sup> See also Agell and Persson (2001) and Fredriksson (2007). Ostry et al (2010) introduce an alternative concept of ‘fiscal space’ in reference to the difference between the stock of debt limit (requiring intertemporal balance) and the stock of current debt.

is 0.025, or 2.5% of output; the real interest rate from the interest rate function is 6.75%; the national debt of the nation is about 10% of output (the country is a net debtor to the ROW); and the half life to the balanced growth path is about 6.67 periods.

First, we present simulations of policy changes evaluated at the initial growth path; and run sensitivity analysis of those results and welfare rankings. Second, we obtain numerical evaluations of policies that guarantee intertemporal sustainability, or  $V(T/k) = 0$ . In this latter case, we use a shooting algorithm.<sup>16</sup> We evaluate the parameter that solves (15) and then re-evaluate the dynamic and balanced growth paths under the new parameter value. We, thus, obtain a new value of  $V(T/k)$  and a new value for the parameter that solves (15) for  $V(T/k) = 0$ . This process iterates until convergence is obtained. We run sensitivity analysis and welfare rankings of those policies as well.

Table 2 presents comparative static effects of government policy changes on the long term liability of the government and welfare evaluated at the initial equilibrium. The first two columns refer to the base parameter set while the last two columns refer to the case where the interest premium function is steeper, i.e.,  $s_3$  increases to 2.75.

First, an increase in the capital income tax under the base parameter set increases the long term tax liability and increases welfare by a small magnitude. The first result indicates that at the base parameter set, the change in capital income taxes has Laffer style effects. However, this result is not robust to a steeper premium function. At a higher premium slope, the dynamic scoring effect disappears and the long term liability falls by almost 10 percentage points. The second result shows that when evaluated at the initial equilibrium, higher capital taxes lead to higher consumption (and money balances) thus increasing welfare in both cases.

The second row is an increase in the government share of GDP. The effect on the tax liability is very large relative to the capital income tax rate and welfare declines because of the crowding out effect on private consumption. At the higher slope of the interest rate function, borrowing abroad is more costly, this mitigates the increase in liability and loss in welfare.

The case of an increase in the rate of growth of money decreases the future tax liability and welfare though the impact on lower real money balances (inflation tax effect). Those effects are robust across the interest premium, but again a steeper premium function mitigates the effect on the long term tax liability because it discourages borrowing abroad. The case of a higher real

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<sup>16</sup> See e.g. Judd (1998).

interest rate in Table 2 refers to either an increase in the foreign interest rate or a decrease in foreign inflation; in this case there is no change in the slope of the premium function, Figure 3 illustrates the interest premium function in this case. The long term liability increases dramatically and welfare also increases in the base set. The additional effect of changing the slope of the premium function mitigates the increase in both the long term liability and the welfare because of the associated lower long run national indebtedness.

The next row shows the effect of increase in the government share of GDP financed by higher capital taxes. This has the largest impact in the long term liability among all the policies considered and a moderate negative impact on welfare. The steeper interest premium function makes this effect on the long term liability much smaller because the dynamic scoring effect of the capital tax rate disappears. The effect on welfare is negative and very similar in both cases.

Overall, the short run results of Table 2 show that while under the capital income tax dynamic scoring is possible, the effect is sensitive to the slope of the premium function and the form of finance. The potential dynamic scoring gains, in terms of the long term liability, would occur at a flat interest premium function and high levels of initial tax rates combined with low levels of government spending. However, even under high tax/low spending levels, a steeper interest premium function can reverse the dynamic scoring result.

Table 3 presents results referring to policies that satisfy the long term constraint that tax liabilities are zero, or  $V(T/k) = 0$ . When the policy change leads to transitional dynamics, in order to obtain numerical evaluations of policies that guarantee intertemporal sustainability, or  $V(T/k) = 0$ , we need to evaluate the parameter that solves (15) for  $V(T/k) = 0$ . This process iterates until convergence is obtained. When policy change does not lead to transitional dynamics, the change occurs instantaneously.

The first row of Table 3 shows the case for the capital income tax rate as a single policy instrument. At the base parameter set, the capital income tax rate should be reduced by approximately 0.031 percentage points, with convergence achieved in 4 iterations and 10E-6 accuracy. This is obtained through the dynamic scoring effect under high tax/low spending levels initially, and a welfare loss of about 0.33% occurs. However, the convergence of the capital income tax instrument is not robust. At the steeper interest rate function, the capital income tax alone cannot achieve long term budget balance since there is no convergence. Figure 4 illustrates the problem of capital tax finance of the long-term liability for the steeper interest rate function.

The vertical axis is the long term liability, the lower axis is the tax rate and the upper axis is the stable root that determines the speed of adjustment to the long term growth path. It is clear that at alternative levels of the tax rate, the long term liability does not get to the lower zero bound. At low tax rates, the root declines and the long term liability also declines, but close to the 30% tax rate, this process reverses and the long term liability and the stable root increase.<sup>17</sup>

The next row of Table 3 shows the case in which government spending adjusts to balance the intertemporal budget. This policy does not give rise to transitional dynamics. Moreover, an instantaneous decrease of 0.02 percentage points from the initial level of government spending takes the economy to the new balanced growth path such that  $V(T/k) = 0$ . The steeper interest premium function requires a slightly smaller decrease in government spending. In both cases, the welfare gains are small but robust because of the additional private consumption and real money balances.

Finally, we consider the case where both the tax rate and government spending are used to balance the intertemporal budget. This policy converges in both interest premium functions. The decrease in government spending and taxes is very small under the base set and slightly larger under the alternative interest premium, while the welfare gains are robust for both cases. Unlike the single capital income tax instrument case, the combined tax-cum-spending mix does achieve long term balance with 4 iterations and plausible accuracy.

In sum, the nonlinearities in the model are enough to make the implementation of intertemporal budget balance difficult under single instrument policy when transitional dynamics emerge. In Bianconi and Fisher (2005), the infinite elasticity of supply of debt relationship does not generate transitional dynamics, implying that single instrument policies easily achieve the target of balancing the intertemporal budget balance according to the  $V(T/k)=0$  criterion. Transitional dynamics due to an upward sloping supply of debt leads to interactions between the real activity of the economy and the government budget balance through the speed of adjustment to the balanced growth path. This interaction is very sensitive to the set of initial conditions and the parameter space. We find that while dynamic scoring can occur under one set of parameter values, the capital income tax alone cannot balance the intertemporal budget for a significant part of the parameter space.

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<sup>17</sup> There is convergence if the parameter in the slope of the interest premium function  $s_3$  changes by an infinitesimal amount, and the result is similar to the base parameter set. However, at a more wide range of parameter values convergence is not guaranteed.

## 7. Concluding Remarks

This paper analyzes the role of nominal assets in intertemporal budget policies in a growing open economy facing an upward sloping supply of debt. We show that future public sector liabilities are a function not only of the fiscal and monetary policy tools, but also of the economy's long-run accumulation of debt scaled by the domestic capital stock. This change has major implications for the government budget intertemporal balance. A cut in capital income tax may lead to dynamic scoring through as the novel channel of foreign indebtedness. However, our simulations show that using capital income taxes alone to balance the intertemporal government budget constraint, may not be feasible for a large combination of the parameter values.

Intertemporal balance can, however, be achieved with a tax-cum-expenditure policy or government expenditure policy changes alone.

In future work we plan to extend this framework to include heterogeneity in the distribution of capital and assets in the model, with particular attention to the effects of inequality on the intertemporal government budget balance.

## Appendix

### Derivation of the Saddlepath Solutions (9a)-(9b)

Linearizing (8) about the steady-state equilibrium, we obtain the following Jacobian system:

$$\begin{pmatrix} \dot{\chi} \\ \dot{\psi} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & \tilde{\chi}v'(\tilde{\psi}) & \frac{-\tilde{\chi}}{h} \\ 1 & r(\tilde{\psi}) + v'(\tilde{\psi})\tilde{\psi} - \frac{\tilde{q}-1}{h} & \frac{\tilde{q}-\tilde{\psi}}{h} \\ 0 & v'(\tilde{\psi})\tilde{q} & r(\tilde{\psi}) - \frac{\tilde{q}-1}{h} \end{pmatrix} \begin{pmatrix} \chi - \tilde{\chi} \\ \psi - \tilde{\psi} \\ q - \tilde{q} \end{pmatrix}. \quad (\text{A.1})$$

The trace and the determinant of the Jacobian Matrix  $\mathbf{J}$  are:

$$\mathbf{Tr}(\mathbf{J}) = 2 \left[ r(\tilde{\psi}) - \frac{\tilde{q}-1}{h} \right] + v'(\tilde{\psi})\tilde{\psi} = \xi_1 + \xi_2 + \xi_3,$$

$$\mathbf{Det}(\mathbf{J}) = - \left\{ \left[ r(\tilde{\psi}) - \frac{\tilde{q}-1}{h} \right] \tilde{\chi}v'(\tilde{\psi}) + \frac{\tilde{\chi}v'(\tilde{\psi})\tilde{q}}{h} \right\} = \xi_1\xi_2\xi_3,$$

where  $\xi_i, i = 1, 2, 3$ , are the eigenvalues of  $\mathbf{J}$ . Since the condition  $r(\tilde{\psi}) - \frac{\tilde{q}-1}{h} > 0$  is required to satisfy the transversality condition (3e), we obtain  $\mathbf{Tr}(\mathbf{J}) > 0$  and  $\mathbf{Det}(\mathbf{J}) < 0$ . The fact that  $\mathbf{Tr}(\mathbf{J}) > 0$  rules-out the case that all the eigenvalues are negative. This implies that the steady state displays saddlepoint dynamics locally, with one negative and two positive eigenvalues, i.e.,  $\xi_1 < 0, \xi_2 > 0$ , and  $\xi_3 > 0$ .

The stable solutions to the consumption-capital ratio,  $\chi$ , the national debt-capital,  $\psi$ , and Tobin's  $q$  are:

$$\chi = \tilde{\chi} + A_1 e^{\xi_1 t}, \quad (\text{A.1a})$$

$$\psi = \tilde{\psi} + B_1 e^{\xi_1 t}, \quad (\text{A.1b})$$

$$q = \tilde{q} + B_1 e^{\xi_1 t}, \quad (\text{A.1c})$$

where  $A_1, B_1$ , and  $C_1$ , are constants to be determined and where  $\xi_1 < 0$  is the stable eigenvalue. The constant  $B_1$  is determined by imposing the initial condition  $\psi(0) \equiv \psi_0 > 0$ , on the adjustment of the national debt to capital ratio. This yields  $B_1 = -(\tilde{\psi} - \psi_0)$ , substituting for (A.1a) – (A.1b) and  $B_1 = -(\tilde{\psi} - \psi_0)$  into the Jacobian matrix (9), we obtain:



$$\begin{pmatrix} -\xi_1 & \tilde{\chi}v'(\tilde{\psi}) & \frac{-\tilde{\chi}}{h} \\ 1 & \left[ r(\tilde{\psi}) + v'(\tilde{\psi})\tilde{\psi} - \frac{\tilde{q}-1}{h} \right] - \xi_1 & \frac{\tilde{q}-\tilde{\psi}}{h} \\ 0 & v'(\tilde{\psi})\tilde{q} & \left[ r(\tilde{\psi}) - \frac{\tilde{q}-1}{h} \right] - \xi_1 \end{pmatrix} \begin{pmatrix} A_1 \\ -(\psi_0 - \tilde{\psi}) \\ C_1 \end{pmatrix} \\
= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{A.2}$$

Evaluating (A.2), we derive:

$$-\xi_1 A_1 - \tilde{\chi}v'(\tilde{\psi}) \cdot (\tilde{\psi} - \psi_0) - \frac{-\tilde{\chi}}{h} \cdot C_1 = 0, \tag{A.3a}$$

$$A_1 - \left\{ \left[ r(\tilde{\psi}) + v'(\tilde{\psi})\tilde{\psi} - \frac{(\tilde{q}-1)}{h} \right] - \xi_1 \right\} \cdot (\tilde{\psi} - \psi_0) + \frac{(\tilde{q}-\tilde{\psi})}{h} \cdot C_1 = 0, \tag{A.3b}$$

$$v'(\tilde{\psi})\tilde{q}(\tilde{\psi} - \psi_0) + \left\{ \left[ r(\tilde{\psi}) - \frac{(\tilde{q}-1)}{h} \right] - \xi_1 \right\} \cdot C_1 = 0, \tag{A.3c}$$

equation (A.3c) then solves for  $C_1$ :

$$C_1 = \frac{v'(\tilde{\psi})\tilde{q} \cdot (\tilde{\psi} - \psi_0)}{\left[ r(\tilde{\psi}) - \frac{(\tilde{q}-1)}{h} \right] - \xi_1}, \tag{A.4}$$

and yields, by substitution in (A.1c) the saddlepath solution (10b) for Tobin's  $q$ , where  $\psi = \tilde{\psi} - (\tilde{\psi} - \psi_0)e^{\xi_1 t}$ . To determine  $A_1$ , we substitute for  $C_1$  from (A.4) into either (A.3a) or (A.3b). The convenient version is obtained by substituting into (A.3a) and yields the following expression for  $A_1$ :

$$A_1 = \frac{\tilde{\chi}v'(\tilde{\psi})}{\xi_1} \left[ 1 + \frac{\tilde{q}/h}{\left[ r(\tilde{\psi}) - \frac{(\tilde{q}-1)}{h} \right] - \xi_1} \right] \cdot (\tilde{\psi} - \psi_0), \tag{A.5}$$

which, given (A.1a), results in the saddlepath solution for (10a) for the consumption-capital ratio  $\chi$ .

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Table 1a: Long Run Comparative Statics

	$\tau$	$r$	$\bar{g}$	$\sigma$
$\tilde{q} \propto \tilde{\phi}$	$\frac{-h\alpha}{\tilde{q}+h\delta} < 0$	0	0	0
$\tilde{\psi}$	$\frac{-\alpha}{r'(\tilde{q}+h\delta)} < 0$	$\frac{-1}{r'} < 0$	0	0
$\tilde{r}(\psi)$	$\frac{-\alpha}{\tilde{q}+h\delta} < 0$	0	0	0
$\tilde{\chi}$	$\frac{\alpha}{r'}[\frac{r'\tilde{q}+\delta}{\tilde{q}+h\delta}] > 0$	$\frac{\delta}{r'} > 0$	$-\alpha < 0$	0
$\tilde{\mu}$	$(\frac{\gamma}{\sigma+\delta}) \frac{\alpha}{r'}[\frac{r'\tilde{q}+\delta}{\tilde{q}+h\delta}] > 0$	$\frac{\gamma}{\sigma+\delta} \frac{\delta}{r'} > 0$	$\frac{-\gamma}{\sigma+\delta} \alpha < 0$	$-\frac{\gamma \cdot \tilde{\chi}}{(\sigma+\delta)^2} < 0$
$\tilde{\omega}$	$\frac{\alpha}{\delta} + \frac{\sigma\gamma}{(\sigma+\delta)\delta} \frac{\partial \tilde{\chi}}{\partial \tau}$	$\frac{1}{r'}(\frac{\sigma\delta}{\sigma+\delta}) > 0$	$-\frac{\alpha}{\delta}[1 + \frac{\sigma\gamma}{\sigma+\delta}] < 0$	$\frac{\gamma \cdot \tilde{\chi}}{(\sigma+\delta)^2} > 0$
$W$	$\frac{\alpha}{v'}(\frac{1}{\tilde{q}+h\delta})[(\frac{1+\gamma}{\delta \cdot \tilde{\chi}})(v'\tilde{q}+\delta) - (\frac{w_1+\gamma \cdot w_2}{\delta-\xi_1})]$	$\frac{1}{v'}[(\frac{1+\gamma}{\tilde{\chi}}) - (\frac{w_1+\gamma \cdot w_2}{\delta-\xi_1})]$	$\frac{-\alpha}{\delta}(\frac{1+\gamma}{\tilde{\chi}}) < 0$	$\frac{-\gamma}{\delta(\sigma+\delta)} < 0$

Table 1b: Initial Impact Effects

	$\tau$	$r$	$\bar{g}$	$\sigma$
$q(0)$	$\frac{-\alpha[\tilde{q} + h(\delta - \xi_1)]}{(\tilde{q} + h\delta)(\delta - \xi_1)} < 0$	$\frac{-\tilde{q}}{\delta - \xi_1} < 0$	0	0
$\chi(0)$	$\frac{\alpha}{\tilde{q} + h\delta}[(\tilde{q} + \frac{h}{v'}) + \frac{\tilde{\chi}[h\delta - \xi_1(h + \tilde{q})]}{h\xi_1(\delta - \xi_1)}]$	$\frac{\delta}{v'} + \frac{\tilde{\chi}}{\xi_1}[1 + \frac{\tilde{q}}{h(\delta - \xi_1)}]$	$-\alpha < 0$	0
$\mu(0)$	$\frac{\gamma}{\sigma + \delta} \frac{\partial \tilde{\chi}}{\partial \tau} + \frac{\Omega}{(\sigma + \delta - \xi_1)} \frac{\partial \tilde{\psi}}{\partial \tau}$	$\frac{1}{r'}[\frac{\gamma\delta}{\sigma + \delta} - \frac{\Omega}{\sigma + \delta - \xi_1}]$	$\frac{-\gamma\alpha}{\sigma + \delta} < 0$	$-\frac{\gamma \cdot \tilde{\chi}}{(\sigma + \delta)^2} < 0$

Table 2: Policy changes from an initial equilibrium

Base	Sen		sensitivity: s3	
	parameter set		increases to 2.75	
	$\partial V(T/K)$ % from initial eq.	$\partial W$ % from initial eq.	$\partial V(T/K)$ % from initial eq.	$\partial W$ % from initial eq.
$\partial \tau > 0$ ; (*) $\tau = 0.36$	200.4 0.	020	-9.9	0.015
$\partial \bar{g} > 0$ ; $\bar{g} = 0.16$	510.4 -	3.280	304.3	-3.200
$\partial \sigma > 0$ ; $\sigma = 0.04125$	-8.9 -	0.097	-5.4	-0.097
$\partial r > 0$ ; $r = 0.073$ (*)	665.3 9.	980	276.9	6.180
$\partial \tau = \partial \bar{g} > 0$ ; $\tau = 0.36$ (*) $\bar{g} = 0.16$	717.6 -	2.740	299.4	-2.790
$\partial \tau = -\partial \omega(0) < 0$ ; $\tau = 0.26$ (*) $\omega(0) = 0.50$	93.4 -	0.030	169.4	-0.020
$\partial \bar{g} = \partial \omega(0) > 0$ ; $\bar{g} = 0.16$ $\omega(0) = 0.50$	710.6 -	3.280	423.6	-3.200

Note: (\*) indicates the policy change generates transitional dynamics.

Table 3: Policy changes to balance long term liability:  $V(T/K)=0$

Base Parameter Set				Sensitivity: s3 increases to 2.75			
#	of iterat.	Accur.	$\partial W$ % from initial eq.	#	of iterat.	Accur.	$\partial W$ % from initial eq.
$\partial \tau < 0$ ; $\tau = 0.279^{(*)}$	4 1	0E-6	-0.337	$\partial \tau^{(*)}$	No Conv.	-- --	
$\partial \bar{g} < 0$ ; $\bar{g} = 0.090$	-- --		0.656	$\partial \bar{g} < 0$ ; $\bar{g} = 0.093$	-- --		0.517
$\partial \tau = \partial \bar{g} < 0$ ; $\tau = 0.303^{(*)}$ $\bar{g} = 0.103$	4 1	0E-7	0.264	$\partial \tau = \partial \bar{g} < 0$ ; $\tau = 0.290^{(*)}$ $\bar{g} = 0.090$	4 1	0E-7	0.281

Note: (\*) indicates the policy change generates transitional dynamics.



Figure 1: Equilibrium Dynamics

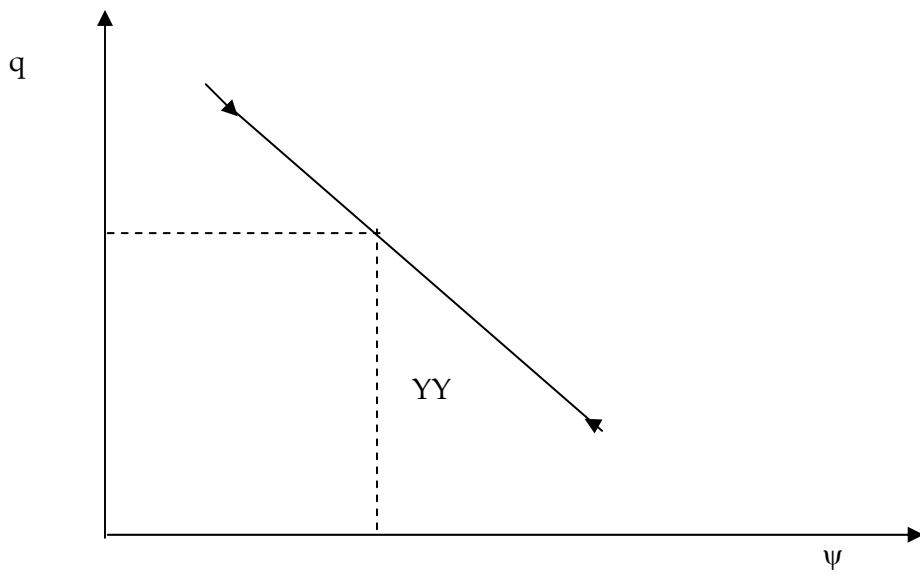
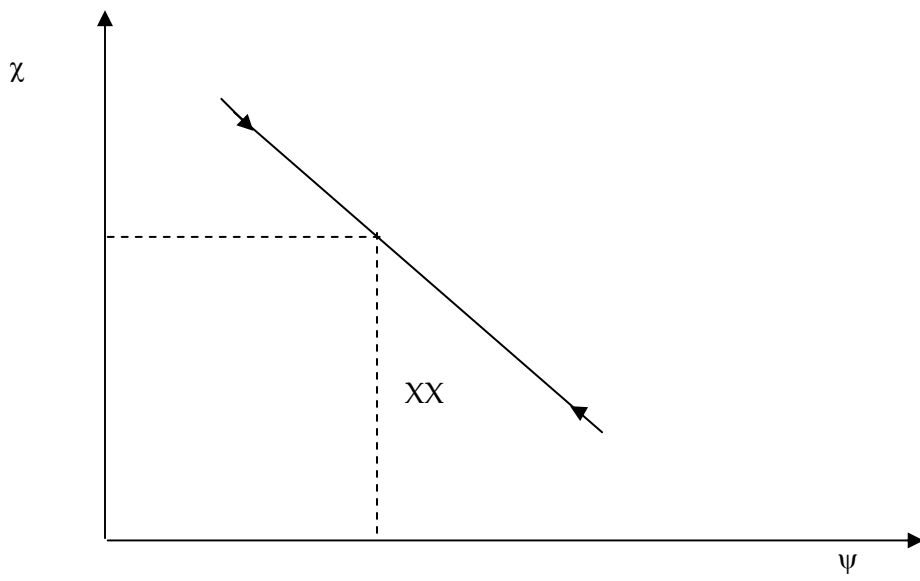


Figure 2: Interest Premium Function

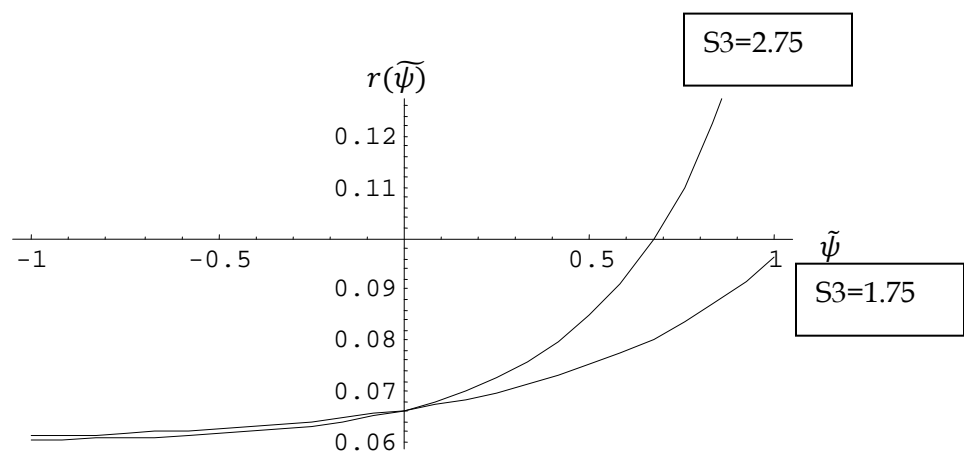


Figure 3: Interest Premium Function;  $\partial r$

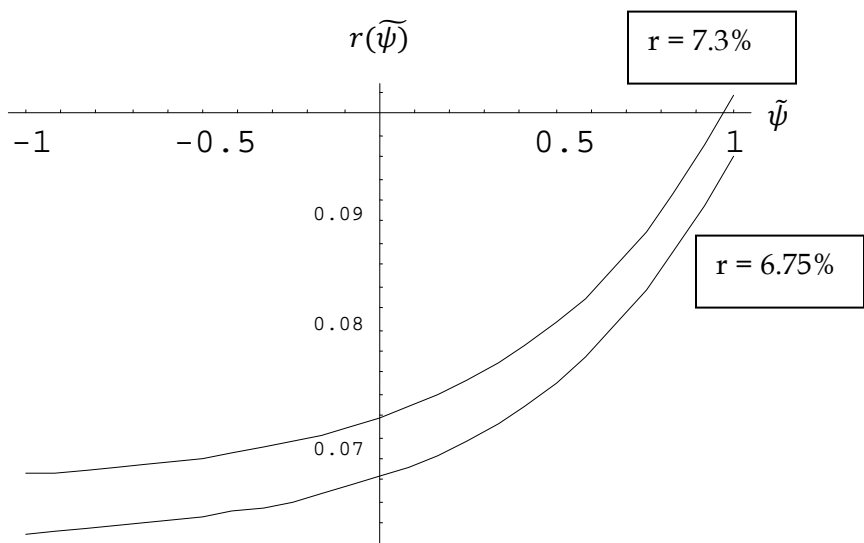
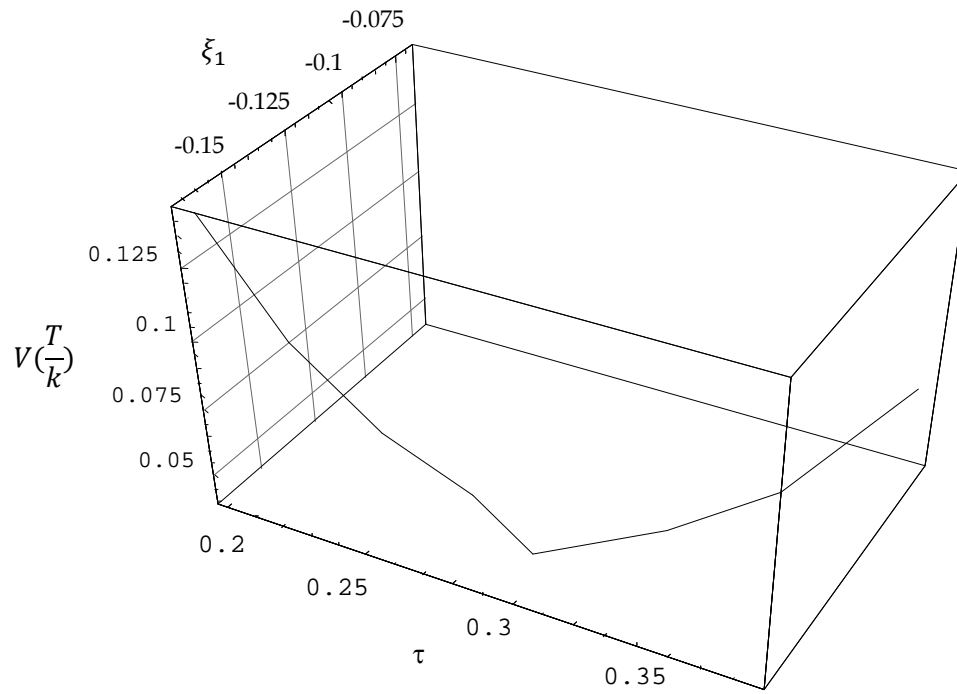


Figure 4: Capital Income Tax under Long Term Balance:  $V(T/k)=0$



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