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**Working Paper**

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Working Paper, No. 114

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*Suggested Citation:* Casajus, André (2012) : The Shapley value without efficiency and additivity, Working Paper, No. 114, Universität Leipzig, Wirtschaftswissenschaftliche Fakultät, Leipzig

This Version is available at:

<https://hdl.handle.net/10419/68233>

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UNIVERSITÄT LEIPZIG

**Wirtschaftswissenschaftliche Fakultät**  
**Faculty of Economics and Management Science**

Working Paper, No. 114

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**The Shapley value without efficiency  
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November 2012

ISSN 1437-9384

# The Shapley value without efficiency and additivity

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(November 2012, this version: November 30, 2012, 7:11)

## Abstract

We provide a new characterization of the Shapley value neither using the efficiency axiom nor the additivity axiom. In this characterization, efficiency is replaced by the gain-loss axiom (Einy and Haimanko, 2011, *Game Econ Behav* 73: 615–621), i.e., whenever the total worth generated does not change, a player can only gain at the expense of another one. Additivity and the equal treatment axiom are substituted by fairness (van den Brink, 2001, *Int J Game Theory* 30: 309–319) or differential marginality (Casajus, 2011, *Theor Decis* 71: 163–174), where the latter requires equal productivity differentials of two players to translate into equal payoff differentials. The third axiom of our characterization is the standard dummy player axiom.

Key Words: Shapley value; gain-loss axiom; differential marginality; efficiency, additivity

*JEL code:* C71, D60

*AMS subject classification:* 91A12

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We are grateful to Fank Huettner for comments on this note.

## 1. Introduction

The Shapley value (Shapley, 1953) probably is the most eminent one-point solution concept for TU games. Ever since its original characterization by Shapley himself, much effort has been put in the endeavor to provide alternative characterizations both for fixed player sets and for variable player sets as well as for certain subdomains, for example, the domains of superadditive games or of simple games.<sup>1</sup>

One aim of these attempts is to get rid of the additivity axiom. Young (1985) comes up with the very elegant marginality axiom, which requires a player's payoff to depend only on his *own* productivity measured by marginal contributions. Then, he characterizes the Shapley value by help of this axiom combined with efficiency and the equal treatment axiom. Recently, Casajus (2011) proposes a differential version of marginality, differential marginality, which demands two players' payoff differential to be affected only by the differential of *their own* productivities, again, measured by the difference of marginal contributions. It turns out that differential marginality is equivalent to the fairness property suggested by van den Brink (2001). Together with efficiency and the null player axiom, differential marginality/fairness characterize the Shapley value. While the former characterizations work within a fixed player set, the ingenious characterization due to Myerson (1977) refers to variable player sets. Besides efficiency, it relies on just a single other axiom, balanced contributions, which requires that the loss/gain caused on one player when another player leaves the game is the same as if the role of these player is reversed. Roth (1977) and Chun (1989), for example, suggest alternative foundations of the Shapley value without additivity.

Recently, Einy and Haimanko (2011) introduce the aesthetically (and otherwise) appealing gain-loss axiom as a substitute for efficiency in characterizations of Shapley value. Since their prior focus is on non-null simple monotonic games (henceforth, *voting games*), i.e., the worth generated by the grand coalition is 1, the main version of the gain-loss axiom just requires that one player can only gain when another player loses. When applied to general TU games, one has to restrict the axiom to situations where the worth generated by the grand coalition remains constant. What they show is that the Shapley value on the domain of voting games is characterized by the gain-loss axiom (in the narrow sense) and three standard axioms—the transfer axiom (Dubey, 1975, additivity adjusted to simple games), the equal treatment

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<sup>1</sup>In the literature, the term “simple game” is used both to games for which the worth is either 0 or 1 and to games that, in addition, are monotonic and non-null. We follow the first convention.

axiom or symmetry, and the dummy player axiom. In order to obtain a characterization on the full domain, they replace the transfer axiom by additivity and the gain-loss axiom with its broader-sense version. Since Shapley and Shubik (1954) apply the Shapley value to voting games as a measure voting power (also known as the Shapley-Shubik index), the former characterization is of particular interest. Other foundations of the Shapley value without efficiency have been proposed by Roth (1977), Blair and McLean (1990), and Laruelle and Valenciano (2001).

In this note, we “merge” the characterizations of Einy and Haimanko (2011) and van den Brink (2001)/Casajus (2011) and obtain a characterization of the Shapley value on the full domain of TU games via the dummy player axiom, the gain-loss axiom, and fairness/differential marginality (Theorem 2). This characterization can be restricted to certain subdomains, e.g., the domains of superadditive games or of convex games (Remark 2). Unfortunately, the domains of simple games and of voting games are not among these subdomains (Section 4). Yet, other than the van den Brink characterization, the Casajus characterization works within the domain of non-contradictory voting games, unless there are exactly two players (Proposition 3).

The plan of this note is as follows: Basic definitions and notation are given in the second section. The third section contains our characterization for general TU games. Our results on simple games can be found in the fourth section. Some remarks conclude this note.

## 2. Basic definitions and notation

A **(TU) game** is a pair  $(N, v)$  consisting of a non-empty and finite set of players  $N$  and a **coalition function**  $v \in \mathbb{V}(N) := \{f : 2^N \rightarrow \mathbb{R} \mid f(\emptyset) = 0\}$ , where  $2^N$  denotes the power set of  $N$ . Since we deal with a fixed player set  $N$ , the latter is dropped as an argument whenever possible. In particular, we refer to  $v \in \mathbb{V}$  as a game. Subsets of  $N$  are called **coalitions**, and  $v(S)$  is called the worth of coalition  $S$ . For  $v, w \in \mathbb{V}$ ,  $\alpha \in \mathbb{R}$ , the coalition functions  $v + w \in \mathbb{V}$  and  $\alpha \cdot v \in \mathbb{V}$  are given by  $(v + w)(S) = v(S) + w(S)$  and  $(\alpha \cdot v)(S) = \alpha \cdot v(S)$  for all  $S \subseteq N$ . For  $T \subseteq N$ ,  $T \neq \emptyset$ , the game  $u_T$ ,  $u_T(K) = 1$  if  $T \subseteq K$  and  $u_T(K) = 0$  otherwise, is called a **unanimity game**; the game  $e_T$ ,  $e_T(T) = 1$  and  $e_T(K) = 0$  otherwise, is called a **standard game**. Any game  $v$  can be uniquely represented by unanimity games,

$$v = \sum_{T \subseteq N: T \neq \emptyset} \lambda_T(v) \cdot u_T, \quad \lambda_T(v) := \sum_{S \subseteq T: S \neq \emptyset} (-1)^{|T|-|S|} \cdot v(S). \quad (1)$$

A game  $v$  is called **simple** if  $v(S) \in \{0, 1\}$  for all  $S \subseteq N$ ; it is called **monotonic** if  $v(S) \leq v(T)$  for all  $S, T \subseteq N$  such that  $S \subseteq T$ ; it is called **superadditive** if  $v(S \cup T) \geq v(S) + v(T)$  for all  $S, T \subseteq N$  such that  $S \cap T = \emptyset$ ; it is called **convex** if  $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$  for all  $S, T \subseteq N$ . By  $\mathbf{0}$ , we denote the **null game**, i.e.,  $\mathbf{0}(S) = 0$  for all  $S \subseteq N$ . A non-null monotonic simple game is called a **voting game**. Let  $\mathbb{V}^{\text{si}}$ ,  $\mathbb{V}^{\text{vo}}$ , and  $\mathbb{V}^{\text{sa}}$  denote the sets of simple games, of voting games, and of superadditive games, respectively. For  $v, w \in \mathbb{V}^{\text{si}}$ , we define  $v \vee w \in \mathbb{V}^{\text{si}}$  and  $v \wedge w \in \mathbb{V}^{\text{si}}$  by  $(v \vee w)(S) = \min\{v(S), w(S)\}$  and  $(v \wedge w)(S) = \max\{v(S), w(S)\}$  for all  $S \subseteq N$ .

Player  $i \in N$  is called a **dummy player** in  $v$  iff  $v(S \cup \{i\}) - v(S) = v(\{i\})$  for all  $K \subseteq N \setminus \{i\}$ ; if in addition  $v(\{i\}) = 0$ , then  $i$  is called a **null player**; players  $i, j \in N$  are called **symmetric** in  $v$  if  $v(K \cup \{i\}) = v(K \cup \{j\})$  for all  $K \subseteq N \setminus \{i, j\}$ . Player  $i$  is called a **dictator** in  $v \in \mathbb{V}^{\text{si}}$  if  $v(S) = 1$  iff  $i \in S$  for all  $S \subseteq N$ .

A **value** is an operator  $\varphi$  that assigns a payoff vector  $\varphi(v) \in \mathbb{R}^N$  to any  $v \in \mathbb{V}$ . The **Shapley value** is given by<sup>2</sup>

$$\text{Sh}_i(v) = \sum_{T \subseteq N: i \in T} |T|^{-1} \cdot \lambda_T(v), \quad v \in \mathbb{V}, i \in N. \quad (2)$$

Below, we list the standard axioms that are referred to later on.<sup>3</sup>

**Efficiency, E.** For all  $v \in \mathbb{V}$ ,  $\sum_{i \in N} \varphi_i(v) = v(N)$ .

**Additivity, A.** For all  $v, w \in \mathbb{V}$ ,  $\varphi(v + w) = \varphi(v) + \varphi(w)$ .

**Transfer, T.** For all  $v, w \in \mathbb{V}^{\text{si}}$ ,  $\varphi(v \vee w) + \varphi(v \wedge w) = \varphi(v) + \varphi(w)$ .

**Null game, NG.**  $\varphi_i(\mathbf{0}) = 0$  for all  $i \in N$ .

**Null player, N.** For all  $v \in \mathbb{V}$  and all  $i \in N$  such that  $i$  is a null player in  $v$ ,  $\varphi_i(v) = 0$ .

**Dummy player, D.** For all  $v \in \mathbb{V}$  and all  $i \in N$  such that  $i$  is a dummy player in  $v$ ,  $\varphi_i(v) = v(\{i\})$ .

**Equal treatment, ET.** For all  $v \in \mathbb{V}$  and all  $i, j \in N$  such that  $i$  and  $j$  are symmetric in  $v$ ,  $\varphi_i(v) = \varphi_j(v)$ .

<sup>2</sup>Abusing notation, the restriction of the Shapley value to subdomains also is denoted by “Sh”.

<sup>3</sup>When restricted to a subdomain, an axiom is required to hold whenever all games involved belong to this subdomain.

**Symmetry, S.** For all  $v \in \mathbb{V}$ ,  $i \in N$ , and all bijections  $\pi : N \rightarrow N$ ,  $\varphi_{\pi(i)}(N, v \circ \pi^{-1}) = \varphi_i(N, v)$ , where  $v \circ \pi^{-1} \in \mathbb{V}$  is given by  $(v \circ \pi^{-1})(S) = v(\pi^{-1}(S))$ ,  $K \subseteq N$ .

### 3. General TU games

As their main result, Einy and Haimanko (2011, Theorem 3) show that the Shapley value on the domain voting games is characterized by the transfer axiom, the equal treatment axiom, the dummy player axiom, the gain-loss axiom, below. Note that we already state the general version of this axiom. For voting games, the requirement  $v(N) = w(N)$  is void and can be dropped. This may enhance appeal of the gain-loss axiom, but only if one blocks out the fact that this condition is met in voting games by definition.

**Gain-loss, GL.** For all  $v, w \in \mathbb{V}$  and  $i \in N$  such that  $v(N) = w(N)$  and  $\varphi_i(v) > \varphi_i(w)$ , there is some  $j \in N$  such that  $\varphi_j(v) < \varphi_j(w)$ .

This axiom demands that whenever the size of the pie does not change one player can only gain at the expense of another one. Although this axiom has some flavor of efficiency—efficiency obviously entails the gain-loss axiom—, it is much weaker. In particular, it does not demand gains and losses to match.

Einy and Haimanko (2011, Remark 2) also obtain a characterization of the Shapley value on the full domain of games. The proof relies on their results for voting games, in particular, on Einy (1987, Lemma 2.3). We first provide a short and elementary proof of this result.

**Theorem 1** (Einy and Haimanko 2011). *The Shapley value is the unique value that satisfies **A**, **D**, **ET**, and **GL**.*

**Proof.** It is well-known that Sh meets the axioms. Let  $\varphi$  obey **A**, **D**, **ET**, and **GL**. By **A**, it suffices to show  $\varphi_i(\lambda \cdot u_T) = \text{Sh}_i(\lambda \cdot u_T)$  for all  $i \in N$ ,  $T \subseteq N$ ,  $T \neq \emptyset$ ,  $\lambda \in \mathbb{R}$ . By **D**, the claim is immediate for  $i \in N \setminus T$ . Set  $v := \frac{\lambda}{|T|} \cdot \sum_{i \in T} u_{\{i\}}$ . Obviously,  $v(N) = \lambda = (\lambda \cdot u_T)(N)$ . By **D**,

$$\varphi_i(v) = \frac{\lambda}{|T|}, \quad i \in T \quad \text{and} \quad \varphi_i(v) = 0, \quad i \in N \setminus T. \quad (3)$$

Further, any  $i, j \in T$  are symmetric in  $\lambda \cdot u_T$ . By **ET**,

$$\varphi_i(\lambda \cdot u_T) = \varphi_j(\lambda \cdot u_T), \quad i, j \in T. \quad (4)$$

Suppose,  $\varphi_i(\lambda \cdot u_T) \geq \frac{\lambda}{|T|} = \varphi_i(v)$  for some  $i \in T$ . By (3) and (4), this entails  $\varphi_j(\lambda \cdot u_T) \geq \varphi_j(v)$  for all  $j \in T$ . Since  $\varphi_j(\lambda \cdot u_T) = 0 = \varphi_j(v)$  for all  $j \in N \setminus T$ , this contradicts **GL**. Hence,  $\varphi_i(\lambda \cdot u_T) = \frac{\lambda}{|T|} = \text{Sh}_i(\lambda \cdot u_T)$  for  $i \in T$ .  $\square$

van den Brink (2001) and Casajus (2011) characterize the Shapley value by efficiency, the null player axiom, and either the fairness axiom or differential marginality.

**Fairness, F.** For all  $v, w \in \mathbb{V}$  and  $i, j \in N$  such that  $i$  and  $j$  are symmetric in  $w$ ,  $\varphi_i(v + w) - \varphi_i(v) = \varphi_j(v + w) - \varphi_j(v)$ .

**Differential marginality, DM.** For all  $v, w \in \mathbb{V}$  and  $i, j \in N$  such that

$$v(S \cup \{i\}) - v(S \cup \{j\}) = w(S \cup \{i\}) - w(S \cup \{j\})$$

for all  $S \subseteq N \setminus \{i, j\}$ ,  $\varphi_i(v) - \varphi_j(v) = \varphi_i(w) - \varphi_j(w)$ .

Fairness requires two players' payoffs to change by the same amount whenever a game is added where these players are symmetric. This property is quite plausible because adding such a game does not affect the differential of these players productivities measured by marginal contributions. Differential marginality imposes this requirement directly—equal productivity differentials should entail equal payoff differentials, i.e., two players' payoff differential should only depend on *their own* productivity differential. Indeed, fairness and differential marginality are equivalent. More precisely, differential marginality implies fairness on arbitrary domains and is implied by fairness on any (linear) subspaces of the full domain of games (Casajus, 2011, Proposition 3).

While fairness quite often is more useful to work with, differential marginality is less technical and has more interpretational appeal. In particular, it is structurally similar to and shares some of the interpretational appeal of marginality employed by Young (1985) to characterize the Shapley value in combination with efficiency and the equal treatment axiom. Other than its differential cousin, marginality refers to a single player—a player's payoff should depend only on his *own* productivity.

**Marginality, M.** For all  $v, w \in \mathbb{V}$  and  $i \in N$  such that

$$v(S \cup \{i\}) - v(S) = w(S \cup \{i\}) - w(S)$$

for all  $S \subseteq N \setminus \{i\}$ ,  $\varphi_i(v) = \varphi_i(w)$ .

In the following, we show that van den Brink/Casajus characterization can be modified as follows. While efficiency is weakened into the gain-loss axiom, the null



player axiom is strengthened into the dummy player axiom. This result does not simply drop from Theorem 1. On the one hand, the null game axiom together with fairness or differential marginality implies the equal treatment property (van den Brink, 2001, Proposition 2.4). But on the other hand, we cannot simply employ Casajus (2011, Proposition 6), which says that efficiency, the null game property, and differential marginality imply additivity, unless the player set contains exactly two players.

**Theorem 2.** *The Shapley value is the unique value that satisfies [F or DM], D, and GL.*

**Proof.** It is clear that Sh obeys **D** and **GL**. Further, van den Brink (2001) and Casajus (2011, Corollary 5) show that Sh satisfies **F** and **DM**. As mentioned above, both axioms are equivalent on  $\mathbb{V}$ . Let the value  $\varphi$  obey **DM**, **D**, and **GL**. If  $|N| = 1$ , then **D** already entails  $\varphi = \text{Sh}$ .

Let now  $|N| > 1$ . For  $v \in \mathbb{V}$ , set

$$\mathcal{T}_1(v) := \{T \subseteq N \mid |T| > 1 \wedge \lambda_T(v) \neq 0\}. \quad (5)$$

For  $v \in \mathbb{V}$  and  $T \in \mathcal{T}_1(v)$ , let  $v^T \in \mathbb{V}$  be given by

$$v^T := v - \lambda_T(v) \cdot \left( u_T - |T|^{-1} \cdot \sum_{i \in T} u_{\{i\}} \right). \quad (6)$$

This implies

$$\varphi_i(v) - \varphi_i(v^T) \stackrel{\mathbf{DM}}{=} \varphi_j(v) - \varphi_j(v^T) \quad \text{for all } i, j \in T \text{ and all } i, j \in N \setminus T. \quad (7)$$

We show  $\varphi = \text{Sh}$  by induction on  $|\mathcal{T}_1(v)|$ .

*Induction basis:* If  $|\mathcal{T}_1(v)| = 0$  for  $v \in \mathbb{V}$ , the claim follows from **D**. Let now  $|\mathcal{T}_1(v)| = 1$ , i.e.,  $\mathcal{T}_1(v) = \{T\}$  for some  $T \subseteq N$ ,  $T \neq \emptyset$ , i.e.,  $v = \lambda \cdot u_T$  for some  $\lambda \in \mathbb{R}$ ,  $\lambda \neq 0$ . If  $|T| = 1$ , **D** entails  $\varphi(v) = \text{Sh}(v)$ . Let now  $|T| > 1$ . We have

$$\varphi_i(v) \stackrel{\mathbf{D}}{=} \varphi_i(v^T) \stackrel{\mathbf{D},(2)}{=} \text{Sh}_i(v) \quad \text{for all } i \in N \setminus T. \quad (8)$$

Suppose,  $\varphi_i(v) \geq \text{Sh}_i(v) \stackrel{(2)}{=} \text{Sh}_i(v^T) \stackrel{\mathbf{D}}{=} \varphi_i(v^T)$  for some  $i \in T$ . By  $v^T(N) = v(N)$ , **GL**, and (8), there is some  $j \in T$  such that  $\varphi_j(v) \leq \varphi_j(v^T)$ , contradicting (7). Hence,  $\varphi_i(v) = \text{Sh}_i(v)$  for all  $i \in T$ . By (8), we thus have  $\varphi(v) = \text{Sh}(v)$ .

*Induction hypothesis (IH):*  $\varphi(v) = \text{Sh}(v)$  for all  $v \in \mathbb{V}$  such that  $|\mathcal{T}_1(v)| \leq k$ .

*Induction step:* Let  $v \in \mathbb{V}$  be such that  $|\mathcal{T}_1(v)| = k + 1 > 1$ . By (5) and (6), we have  $|\mathcal{T}_1(v^T)| = |\mathcal{T}_1(v)| - 1$  and therefore

$$\varphi(v^T) \stackrel{IH}{=} \text{Sh}(v^T) \stackrel{(2)}{=} \text{Sh}(v) \quad \text{for all } T \in \mathcal{T}_1(v). \quad (9)$$

Below, we find that

$$\varphi_i(v) - \text{Sh}_i(v) = \varphi_j(v) - \text{Sh}_j(v) \quad \text{for all } i, j \in N. \quad (10)$$

Suppose  $\varphi_i(v) \geq \text{Sh}_i(v) \stackrel{(9)}{=} \varphi_i(v^T)$  for some  $i \in N$ . By  $v^T(N) = v(N)$  and **GL**, there is some  $j \in N$  such that  $\varphi_j(v) \leq \varphi_j(v^T) \stackrel{(9)}{=} \text{Sh}_j(v)$ , contradicting (10). Hence,  $\varphi(v) = \text{Sh}(v)$ .

Remains to show (10). By (7) and (9), (\*) (10) holds for all  $i, j \in N$  such that there is some  $T \in \mathcal{T}_1(v)$  with  $i, j \in T$  or  $i, j \in N \setminus T$ . We consider a number of cases.

*Case 1:*  $\mathcal{T}_1(v) \neq \{T, N \setminus T\}$  for all  $T \subseteq N, T \neq \emptyset, N \setminus T \neq \emptyset$ . One of the following holds true: (i) There are  $S, T \in \mathcal{T}_1(v), S \neq T$  such that  $S \cap T \neq \emptyset$ . (ii) There are  $S, T \in \mathcal{T}_1(v), S \neq T$  such that  $S \cup T \neq N$ . Note that these subcases may not be mutually exclusive.

*Case 1(i):* Since  $S \neq T$ , w.l.o.g.,  $S \setminus T \neq \emptyset$ . Let  $i \in S \cap T, j \in S \setminus T, k \in T$ , and  $\ell \in N \setminus (S \cup T)$ . Note that such an  $\ell$  might not exist. By (\*), we have

$$\varphi_\ell(v) - \text{Sh}_\ell(v) \stackrel{j, \ell \notin T}{=} \varphi_j(v) - \text{Sh}_j(v) \stackrel{i, j \in S}{=} \varphi_i(v) - \text{Sh}_i(v) \stackrel{i, k \in T}{=} \varphi_k(v) - \text{Sh}_k(v),$$

i.e., (10) holds.

*Case 1(ii):* Since  $S \neq T$ , w.l.o.g.,  $S \setminus T \neq \emptyset$ . Let  $\ell \in S \cap T, j \in S \setminus T, k \in T \setminus S$ , and  $i \in N \setminus (S \cup T)$ . Note that such  $k$  or  $\ell$  might not exist. By (\*), we have

$$\varphi_\ell(v) - \text{Sh}_\ell(v) \stackrel{j, \ell \in S}{=} \varphi_j(v) - \text{Sh}_j(v) \stackrel{i, j \notin T}{=} \varphi_i(v) - \text{Sh}_i(v) \stackrel{i, k \notin S}{=} \varphi_k(v) - \text{Sh}_k(v),$$

i.e., (10) holds.

*Case 2:*  $\mathcal{T}_1(v) = \{T, N \setminus T\}$  for some  $T \subseteq N, T \neq \emptyset, N \setminus T \neq \emptyset$ . Fix  $i \in T$  and  $j \in N \setminus T$ . We have

$$v = \lambda_T \cdot u_T + \lambda_{N \setminus T} \cdot u_{N \setminus T} + \sum_{k \in N} \lambda_k \cdot u_{\{k\}}$$

for some  $\lambda_T, \lambda_{N \setminus T}, \lambda_k \in \mathbb{R}, k \in N$ . Let  $w \in \mathbb{V}$  be given by

$$w = \lambda_T \cdot u_T - \lambda_{N \setminus T} \cdot u_{((N \setminus T) \setminus \{j\}) \cup \{i\}} + \sum_{k \in N} \lambda_k \cdot u_{\{k\}}.$$

Note that (\*\*\*)  $w$  is as in *Case 1(i)* or  $|\mathcal{T}_1(w)| \leq 1$ . Thus, we have

$$\varphi_i(v) - \varphi_j(v) \stackrel{\text{DM}}{=} \varphi_i(w) - \varphi_j(w) \stackrel{(***)}{=} \text{Sh}_i(w) - \text{Sh}_j(w) \stackrel{\text{DM}}{=} \text{Sh}_i(v) - \text{Sh}_j(v).$$

In view of (\*), this entails (10). Done.  $\square$

*Remark 1.* Our characterization is non-redundant. The Banzhaf value (Banzhaf, 1965; Owen, 1975) meets **DM** and **D**, but not **GL**. The equal division value fails **D**, while satisfying **DM** and **GL**. The pre-nucleolous (Schmeidler, 1969) obeys **D** and **GL**, but not **DM**.

*Remark 2.* Casajus (2011, Theorem 1 and Proposition 4) shows that both his and the van den Brink characterization of the Shapley value can be restricted to any convex cone<sup>4</sup> within the full domain of games that contains all unanimity games. Combining the ideas of the proof of Theorem 2 and of Casajus (2011, Theorem 1), it is not too difficult to show that Theorem 2 also works within any convex cone that contains all unanimity games and, in addition, the “negative” unanimity games referring to singleton player sets, i.e., all  $-u_{\{i\}}$ ,  $i \in N$ . For example, the superadditive games are such cone. For notational parsimony, the details of the proof are left to the reader. As in the proof above, one employs  $\mathcal{T}_1(v)$  and  $v^T$  instead of  $\mathcal{T}(v)$  and  $v - \lambda_T(v) \cdot u_T$  from the original proof.

*Remark 3.* Within the Young characterization, efficiency also can be replaced by the gain-loss axiom. But then the dummy player axiom has to be added to the list of axioms, for example. In order to prove this claim, one combines the technique of the Young proof with the technique of the proof of Theorem 2.

#### 4. Simple games

Unfortunately, Theorem 2 does not work within the domain of voting games or within the domain simple games for  $|N| > 1$ . To see this consider the value  $\varphi^{\text{si}} \neq \text{Sh}$  be given by

$$\varphi_i^{\text{si}}(v) = \begin{cases} 1, & v = u_{\{i\}}, \\ \frac{1}{2} \cdot \text{Sh}_i(v), & v \neq u_{\{i\}}, \end{cases} \quad i \in N, v \in \mathbb{V}^{\text{si}}. \quad (11)$$

In  $\mathbb{V}^{\text{si}}$ , the only non-null dummy players are dictators, i.e., the players  $i$  in the games  $u_{\{i\}}$ , which are dealt with explicitly. Afterwards,  $\varphi^{\text{si}}$  inherits **D** from **Sh**. To see **GL**, first consider  $u_{\{i\}}$  and  $u_{\{j\}}$ ,  $i \neq j$ . This gives  $\varphi_i^{\text{si}}(u_{\{i\}}) = 1 > 0 = \varphi_i^{\text{si}}(u_{\{j\}})$  and  $\varphi_j(u_{\{i\}}) = 0 < 1 = \varphi_j(u_{\{j\}})$ . Consider now  $u_{\{i\}}$  and  $v \in \mathbb{V}^{\text{si}}$ ,  $i \in N$ ,  $v \neq u_{\{i\}}$  for all  $j \in N$ . We have  $\varphi_i^{\text{si}}(u_{\{i\}}) = 1 > \frac{1}{2} > \frac{1}{2} \cdot \text{Sh}_i(v) = \varphi_i^{\text{si}}(v)$  and  $\varphi_j^{\text{si}}(u_{\{i\}}) =$

<sup>4</sup>A subset  $\mathcal{C}$  of  $\mathbb{V}$  is a convex cone if for all  $v, w \in \mathcal{C}$  and  $\lambda \in \mathbb{R}$ ,  $\lambda \geq 0$ , we have  $v + w \in \mathcal{C}$  and  $\lambda \cdot v \in \mathcal{C}$ .

$0 < \frac{1}{2} \cdot \text{Sh}_j(v) = \varphi_j(v)$  for some  $j \in N \setminus \{i\}$  because there is no dictator in  $v$ . If  $v, w \in \mathbb{V}^{\text{si}}$  are such that  $v, w \neq u_{\{j\}}$  for all  $j \in N$ , then  $\varphi^{\text{si}}$  inherits **GL** from Sh. If  $v, w \in \mathbb{V}^{\text{si}}$  are such that  $v, w \neq u_{\{j\}}$  for all  $j \in N$ , then  $\varphi^{\text{si}}$  inherits **DM** from Sh. For  $u_{\{i\}}$  and  $v \neq u_{\{i\}}$ ,  $i \in N$ , the hypothesis of **DM** can only be satisfied for  $k, \ell \in N \setminus \{i\}$ . In  $u_{\{i\}}$ , such players are null players. Hence in  $v$ , they have to be symmetric, which rules out  $v = u_{\{k\}}$  or  $v = u_{\{\ell\}}$ . Therefore, Sh passes **DM** to  $\varphi^{\text{si}}$ . Recall that **DM** implies **F**.

For  $|N| > 1$ , the value  $\varphi^{\text{si}}$  fails **E**. Now, one may wonder whether Theorem 2 would work in  $\mathbb{V}^{\text{si}}$  or some subdomain with **E** in place of **GL**. Since dictators are the only non-null dummy players in  $\mathbb{V}^{\text{si}}$ , one easily checks that **E** and **N** already imply **D** on  $\mathbb{V}^{\text{si}}$  or any subdomain. So, the question is whether the van den Brink characterization or the Casajus characterization work within  $\mathbb{V}^{\text{si}}$  or certain subdomains of  $\mathbb{V}^{\text{si}}$ . For  $\mathbb{V}^{\text{si}}$  itself, the answer is affirmative by van den Brink (2001, Theorem 3.1) together with Casajus (2011, Proposition 3).

We now turn to  $\mathbb{V}^{\text{vo}}$  and  $\mathbb{V}^{\text{vo}} \cap \mathbb{V}^{\text{sa}}$ , where the latter domain contains those voting games that are non-contradictory, i.e.,  $v(S) = 1$  implies  $v(T) = 0$  for all  $T \subseteq N \setminus S$ . First note that in these domains **F** has no bite because  $v + w \notin \mathbb{V}^{\text{vo}}$  for all  $v, w \in \mathbb{V}^{\text{vo}}$ . Since  $\mathbf{0} \notin \mathbb{V}^{\text{vo}}$ , **DM** combined with **N** does not entail **ET** within  $\mathbb{V}^{\text{vo}}$  for  $|N| = 2$ . Indeed, one easily checks that the value  $\varphi^\heartsuit \neq \text{Sh}$  on  $N = \{1, 2\}$  given by

$$\varphi_i^\heartsuit(v) = \begin{cases} i - 1, & v = u_N \vee v = u_N + e_{\{1\}} + e_{\{2\}}, \\ \text{Sh}_i(v), & v = u_{\{1\}} \vee v = u_{\{2\}}, \end{cases} \quad i \in N,$$

meets **E**, **N**, and **DM** in  $\mathbb{V}^{\text{vo}}$  or  $\mathbb{V}^{\text{vo}} \cap \mathbb{V}^{\text{sa}}$ .

For  $|N| > 2$ , one easily checks that the value  $\varphi^{\text{vo}} \neq \text{Sh}$  on  $\mathbb{V}^{\text{vo}}$  given by

$$\varphi_i^{\text{vo}}(v) = \begin{cases} |N|^{-1}, & v = u_{\{j\}} + u_{N \setminus \{j\}} - u_N, \quad j \in N, \\ \text{Sh}_i(v), & \text{else,} \end{cases} \quad (12)$$

inherits **E** and **N** from Sh. If  $v, w \neq u_{\{j\}} + u_{N \setminus \{j\}} - u_N$  for all  $j \in N$ , then the implication of **DM** drops from Sh obeying **DM**. If  $v = u_{\{k\}} + u_{N \setminus \{k\}}$  and  $w = u_{\{\ell\}} + u_{N \setminus \{\ell\}}$ ,  $k, \ell \in N$ ,  $k \neq \ell$ , then the implication **DM** trivially is fulfilled. Let now  $w = u_{\{k\}} + u_{N \setminus \{k\}} - u_N$ ,  $k \in N$ . The hypothesis of **DM** is met by  $v$ ,  $w$ , and  $i, j \in N \setminus \{k\}$  only if  $i$  and  $j$  are symmetric in  $v$ . Since Sh obeys **ET**, the implication of **DM** follows. The hypothesis of **DM** is met by  $v$ ,  $w$ ,  $k$ , and  $i \in N \setminus \{k\}$  iff

$$v(S \cup \{k\}) - v(S \cup \{i\}) = w(S \cup \{k\}) - w(S \cup \{i\}) = \begin{cases} 0, & S = N \setminus \{i, k\}, \\ 1, & \text{else.} \end{cases}$$

Since  $v$  is monotonic, we have  $v(S \cup \{k\}) = 1$ ,  $v(S \cup \{j\}) = 0$ , and  $v(S) = 0$  for all  $S \subsetneq N \setminus \{i, k\}$  as well as  $v(N) = 1$  and  $v(N \setminus \{i\}) = v(N \setminus \{k\}) = 1$ , i.e.,  $v = w$ . Obviously, the implication of **DM** holds true.

Note that the games  $u_{\{j\}} + u_{N \setminus \{j\}} - u_N$  employed in (12) are not in  $\mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$ . Indeed, the Casajus characterization works within  $\mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  for  $|N| \neq 2$ .

**Proposition 3.** *For  $|N| \neq 2$ , the Shapley value is the unique value on  $\mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  that satisfies **E**, **N**, and **DM**.*

**Proof.** We know that Sh satisfies the axioms. Let the value  $\varphi$  on  $\mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  obey **E**, **N**, and **DM**. By **E**,  $\varphi = \text{Sh}$  for  $|N| = 1$ . Let now  $|N| > 2$ . Any  $v \in \mathbb{V}^{\text{vo}}$  is determined by the set  $\mathcal{W}(v)$  of its minimal winning coalitions. In particular,  $v = \bigvee_{T \in \mathcal{W}(v)} u_T$ . We show  $\varphi = \text{Sh}$  by induction on  $|\mathcal{W}(v)|$ .

*Induction basis:* If  $|\mathcal{W}(v)| = 1$  for  $v \in \mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$ , then  $v = u_T$  for some  $T \subseteq N$ ,  $T \neq \emptyset$ . By **E** and **N**,  $\varphi(v) = \text{Sh}(v)$ .

*Induction hypothesis (IH):*  $\varphi(v) = \text{Sh}(v)$  for all  $v \in \mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  such that  $|\mathcal{W}(v)| \leq k$ .

*Induction step:* Let  $v \in \mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  be such that  $|\mathcal{W}(v)| = k+1 > 1$ . For  $T \in \mathcal{W}(v)$ , let  $v^{(T)} \in \mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$  be given by  $v^{(T)} = \bigvee_{S \in \mathcal{W}(v) \setminus \{T\}} u_S$ . This implies

$$\varphi_i(v) - \varphi_j(v) \stackrel{\text{DM}}{=} \varphi_i(v^{(T)}) - \varphi_j(v^{(T)}) \stackrel{\text{IH}}{=} \text{Sh}_i(v^{(T)}) - \text{Sh}_j(v^{(T)}) \stackrel{\text{DM}}{=} \text{Sh}_i(v) - \text{Sh}_j(v) \quad (13)$$

for all  $i, j \in T$  and all  $i, j \in N \setminus T$ .

Since  $|\mathcal{W}(v)| > 1$  and  $v \in \mathbb{V}^{\text{sa}} \cap \mathbb{V}^{\text{vo}}$ , there are  $T_1, T_2 \in \mathcal{W}(v)$  such that  $T_1 \cap T_2 \neq \emptyset$ ,  $T_1 \setminus T_2 \neq \emptyset$ ,  $T_2 \setminus T_1 \neq \emptyset$ . Let  $i \in T_1 \setminus T_2$ ,  $j \in T_1 \cap T_2$ ,  $k \in T_2 \setminus T_1$ , and  $\ell \in N \setminus (T_1 \cap T_2)$ , where the latter may not exist. By (13), we have

$$\varphi_\ell(v) - \text{Sh}_\ell(v) \stackrel{\ell, k \notin T_1}{=} \varphi_k(v) - \text{Sh}_k(v) \stackrel{k, j \in T_2}{=} \varphi_j(v) - \text{Sh}_j(v) \stackrel{j, i \in T_1}{=} \varphi_i(v) - \text{Sh}_i(v).$$

Hence,  $\varphi_i(v) - \text{Sh}_i(v) = \varphi_j(v) - \text{Sh}_j(v)$  for all  $i, j \in N$ . Since both  $\varphi$  and Sh meet **E**,  $\varphi(v) = \text{Sh}(v)$ .  $\square$

*Remark 4.* The use of **DM** in Proposition 3 indicates that **DM** can be replaced by the following version of fairness.

**Transfairness, TF.** For all  $v, w \in \mathbb{V}^{\text{si}} \cap \mathbb{V}^{\text{sa}}$  and  $i, j \in N$  such that  $i$  and  $j$  are symmetric in  $w$  and  $v \vee w \in \mathbb{V}^{\text{si}} \cap \mathbb{V}^{\text{sa}}$ ,

$$\varphi_i(v \vee w) - \varphi_i(v) = \varphi_j(v \vee w) - \varphi_j(v).$$

## 5. Concluding remarks

We conclude this note by establishing a relation between the equal treatment axiom and symmetry on the domain of voting games—the equal treatment axiom combined with the transfer axiom and the null player axiom already yields symmetry. Note that Malawski (2008, Theorem 2) shows a similar relation for general TU games—the equal treatment axiom together with additivity entails symmetry. Given the former relation, Einy and Haimanko (2011, Theorem 3) is immediate from their second theorem.

**Lemma 4.** *If a value  $\varphi$  on  $\mathbb{V}^{vo}$  satisfies **T**, **N**, and **ET**, then  $\varphi$  also satisfies **S**.*

**Proof.** Let the value  $\varphi$  on  $\mathbb{V}^{vo}$  meet **T**, **N**, and **ET**. We first show (\*)  $\varphi_i(u_S) = \varphi_j(u_T)$  for all  $i, j \in N$  and  $S, T \subseteq N$  such that  $|S| = |T|$  and  $[i \in S \text{ and } j \in T]$  or  $[i \in N \setminus S \text{ and } j \in N \setminus T]$ . Since  $\varphi$  meets **N**, the claim drops from the following chain of reasoning. Let  $i, j \in N$ ,  $i \neq j$ , and  $T \subseteq N \setminus \{i, j\}$ , we have

$$\begin{aligned} \varphi_i(u_{T \cup \{i\}}) &\stackrel{\mathbf{N}}{=} \varphi_i(u_{T \cup \{i\}}) + \varphi_i(u_{T \cup \{j\}}) \\ &\stackrel{\mathbf{T}}{=} \varphi_i(u_{T \cup \{i\}} \vee u_{T \cup \{j\}}) + \varphi_i(u_{T \cup \{i\}} \wedge u_{T \cup \{j\}}) \\ &\stackrel{\mathbf{ET}}{=} \varphi_j(u_{T \cup \{i\}} \vee u_{T \cup \{j\}}) + \varphi_j(u_{T \cup \{i\}} \wedge u_{T \cup \{j\}}) \\ &\stackrel{\mathbf{T}}{=} \varphi_j(u_{T \cup \{i\}}) + \varphi_j(u_{T \cup \{j\}}) \stackrel{\mathbf{N}}{=} \varphi_j(u_{T \cup \{j\}}). \end{aligned}$$

Recall  $v = \bigvee_{T \in \mathcal{W}(v)} u_T$  for all  $v \in \mathbb{V}^{vo}$ . Thus, Einy (1987, Lemma 2.3) entails

$$\varphi(v) = \sum_{\mathcal{I} \subseteq \mathcal{W}(v): \mathcal{I} \neq \emptyset} (-1^{|\mathcal{I}|+1}) \cdot \varphi\left(u_{\bigcup_{T \in \mathcal{I}} T}\right). \quad (14)$$

For any bijection  $\pi : N \rightarrow N$ , we have

$$v \circ \pi^{-1} = \left( \bigvee_{T \in \mathcal{W}(v)} u_T \right) \circ \pi^{-1} = \bigvee_{T \in \mathcal{W}(v)} u_{\pi(T)}. \quad (15)$$

For  $i \in N$ , this yields

$$\begin{aligned} \varphi_{\pi(i)}(v \circ \pi^{-1}) &\stackrel{(15)}{=} \varphi_{\pi(i)}\left(\bigvee_{T \in \mathcal{W}(v)} u_{\pi(T)}\right) \\ &\stackrel{(14)}{=} \sum_{\mathcal{I} \subseteq \mathcal{W}(v): \mathcal{I} \neq \emptyset} (-1^{|\mathcal{I}|+1}) \cdot \varphi_{\pi(i)}\left(u_{\bigcup_{T \in \mathcal{I}} \pi(T)}\right) \\ &= \sum_{\mathcal{I} \subseteq \mathcal{W}(v): \mathcal{I} \neq \emptyset} (-1^{|\mathcal{I}|+1}) \cdot \varphi_i\left(u_{\bigcup_{T \in \mathcal{I}} T}\right) \stackrel{(14)}{=} \varphi_i(v), \end{aligned}$$

where the third equality is due to (\*) and the fact that for all  $\mathcal{I} \subseteq \mathcal{W}(v)$ , we have  $i \in \bigcup_{T \in \mathcal{I}} T$  iff  $\pi(i) \in \bigcup_{T \in \mathcal{I}} \pi(T)$ . Thus,  $\varphi$  meets **S**.  $\square$

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