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United but (Un)Equal: Human Capital, Probability of Divorce and the Marriage Contract

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Abstract

This paper studies how the risk of divorce affects the human capital decisions of a young couple. We consider a setting where complete specialization (one of the spouses uses up all the education resources) is optimal with no divorce risk. Symmetry in education (both spouses receive an equal amount of education) then acts like an insurance device in case of divorce particularly when the institutions do not compensate for differences in earnings. But, at the same time symmetry in education is less efficient than the extreme specialization. This is the basic tradeoff underlying our analysis. We show that the symmetric allocation will become more attractive as the probability of divorce increases, if risk aversion is high and/or labor supply elasticity is low. However, it is only a “second-best” solution as the insurance protection is achieved at the expense of an efficiency loss. Efficiency can be restored through suitably designed marriage contracts because they can provide the appropriate insurance against divorce to a couple who opts for specialization. Finally, we study how the (economic) use of marriage is affected by the possibility of divorce.

JEL-Code: D130, J240, K360.

Keywords: post-marital education, marriage contract, divorce.

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1 Introduction

The family received little systematic treatment in economics before the 1950s with the exception of Malthus and his celebrated model of population growth. The work of Gary Becker (1965) initiated contemporary research on family economics, which quickly moved from the narrow study of fertility to an array of dimensions of family life, including marriage and divorce.\(^1\)

One of the most studied topics in family economics is the decision of marriage and of divorce. Typically, to the question why do we marry, family economists find a number of reasons such as sharing of public non rival goods, division of labor to exploit comparative advantages and increasing returns, extending credit and coordination of investment activities, risk pooling and coordinating child-care. To the question why do we divorce, they find answers, which mix exogenous shocks and institutional settings.

Our paper does not look at why we get married or why we divorce. It takes marriage as given and views divorce as a random event with a given probability. The main focus of our paper is how the risk of dissolution affects the human capital decisions of a young couple. We argue that the risk of divorce along with imperfect alimony rights may have a “distortive” impact on the allocation of capital at the start of the marriage. In our setting, if there were no risk of divorce or if the legal institutions would fully insure ex-spouses against the risk of divorce, we would have an efficient choice of human capital and of saving. With divorce and imperfect protection against losses that it may entail, both saving and education choices may be inefficient.

A number of existing studies are related to our paper. First, there is a literature showing that the probability of marriage or of divorce may depend on marriage legislation, matrimonial property regime, and divorce court sentencing practice; see Cigno (2007; 2011). A standard reference on divorce is Becker et al. (1977), who try to explain both theoretically and empirically the observed acceleration of separation and divorce. In particular they argue that couples are reluctant to invest in skills “specific” to their marriage if they anticipate dissolution. They also provide a lot of evidence on the various causes behind the steady growth in separations and divorces in the US and most Western countries.\(^2\) There is also a literature on the choice of education and marriage,

\(^1\)Among the textbooks which cover this evolving field, see, e.g., Browning et al. (2012), Apps & Rees (2009) and Cigno (1991).

\(^2\)See also Lommerud (1989) and Gonzalez and Ozcan (2008) who study the effect that an increase in
which assumes that education decisions are made before marriage. Education is then viewed as affecting the competitive strength of potential spouses and the spousal roles within the marriage.\(^3\)

The closest papers to ours are by King (1982), Borenstein and Courant (1989), Lommerud (1989), and Konrad and Lommerud (2000).\(^4\) King (1982) studies post marital education decisions and shows how these are affected by the risk of divorce. He focuses on investments in human capital and argues that the corresponding property rights are not clearly defined by the courts.\(^5\) Focusing on risk neutral individuals and abstracting from labor supply decisions, King shows that the couple may end up with inefficient low human capital investments because of the divorce risk. Borenstein and Courant (1989) analyze human capital investments of spouses who can finance these investments either by borrowing on the financial market or from their partner’s wealth. They argue that spouses are only willing to extend “credit” to their partners if they can expect to remain married later in life and thereby to profit from their investments. Consequently, investments in human capital are inefficiently low when there is a divorce risk and no marriage contract. Lommerud (1989) studies how the probability of divorce influences a couple’s (predivorce) allocation of time between market and home work. The institutional setting is such that spouses obtain no compensation in case of divorce, and there is no accumulation of assets. He shows that specialization becomes less likely for positive divorce risk. Konrad and Lommerud (2000) also study the education decision within a couple, but do not consider the effect of divorce. Their main finding is that non-cooperation leads to overinvestments in education.

Our paper studies how married couples choose the level of (tertiary) education of each spouse. Spouses behave cooperatively and they both maximize the sum of their lifetime expected utilities. They live for two periods. In the first they are married. At the beginning of this period education choices are made and then both spouses may work earning a wage that results from their education choice. At the end of the first period they face a given probability of divorce. In case of divorce, they have to count

\(^3\)This is well-summarized in Browning et al. (2012).

\(^4\)In a recent study Fernández and Wong (2011) study a related issue from an empirical prespective. They show that the reduction in the education gender gap can be explained to great extent by the increased probability of divorce risk.

\(^5\)At least not quite as clearly as for physical capital.
on their own earning and on a share of the couple’s accumulated wealth. That share depends on the family law in place. As is the rule in many countries, we assume that assets accumulated during marriage are equally shared between the two (ex-)spouse in case of divorce (see, e.g., §81 EheG for Germany). Additionally, we analyze how the availability of marriage contracts, which opens up the possibility to compensate for inequalities of resources resulting from differences in education, affects human capital investments.

Spouses’ education choices represent a challenging issue, even when the possibility of divorce is ignored. It has been studied by a number of authors and in particular by Cremer et al. (2011). The main question these authors focus on is whether there will be specialization (one of the spouse uses up all the education resources) or symmetry (both spouses receive an equal amount of education). They show (roughly speaking) that even when spouses are ex ante identical (same learning ability) specialization is efficient, unless the education technology involves a sufficient degree of decreasing returns.

Our approach is inspired by their model. To get crisp results as to the effects of a potential divorce, we consider a setting where complete specialization is optimal when there is no possibility of divorce. This provides us with a simple benchmark. Symmetry in education then acts like an insurance device in case of divorce particularly when the institutions do not compensate for differences in earnings. But, at the same time symmetry in education is less efficient, namely it leads to less aggregate surplus (earnings net of disutility of labor) than the extreme specialization. This is the basic tradeoff underlying our analysis. We show that the symmetric allocation will become more attractive as the probability of divorce increases, if risk aversion is high and/or labor supply elasticity is low. However, it is only a “second-best” solution as the insurance protection is achieved at the expense of an efficiency loss. Furthermore, we show that suitably designed marriage contracts will restore efficiency because they can provide the appropriate insurance against divorce to a couple who opts for specialization in human capital investments. Returning to the case without optimal marriage contracts, the relationship between divorce probability and human capital decision is more complex than one would have expected, even in a simple setting like ours. For instance, it turns out that the relationship between a couple’s welfare and the education budget devoted to a given spouse is neither monotonic, nor concave or convex over the full
range. And while an increase in the divorce probability always makes the symmetric solution more attractive (compared to complete specialization), it may or may not be the optimal outcome even for divorce probabilities close to one. In some cases (with quadratic disutility of labor), the asymmetric solution remains optimal no matter what (even when risk aversion is very high). In other cases, an interior solution may obtain and both spouse are educated albeit to a different degree (in spite of the fact that their learning ability is the same).

We also study how the (economic) use of marriage is affected by the possibility of divorce. Whenever the educational budget is shared asymmetrically, it is possible that the spouse who will get less educated finds a marriage less profitable as compared to staying single and using up her own educational budget. As we will show the use of marriage again depends on risk aversion, labor supply elasticity and the possibility of a marriage contract. With a quadratic labor disutility, the surplus generated under specialization is so large that the worse-off spouse enjoys a higher utility with just half of the accumulated assets and no labor income than a spouse who stays single. Additionally, if individuals have the possibility to write a marriage contract which fully compensates the lower educated spouse in case of divorce, then, the use of a marriage is always positive since the couple is able to generate a higher surplus as compared to a single household.

Following Cremer et al. (2011) we make a number of simplifying assumptions to obtain specialization as the no divorce benchmark. In particular, both spouses have the same learning skill and the educational technology involves constant returns to scale. We are only concerned by human capital investments that are chosen at the start of the marital life. The total amount of human capital investments (the total education budget) is exogenously given and household decisions are made cooperatively, that is, both spouses share the same objective of maximizing the sum of their (expected) lifetime utilities.6

Our paper is organized as follows. Section 2 sets up the model. Section 3 defines the couple’s optimization problem and analyzes its first- and second-order conditions.  

6Besides Becker (1965), see his Treatise on the Family (1981; 1991), models of the household don’t distinguish between decision-making agents. The alternatives to this unitary model are models assuming that multi-person households include individual decision-makers. Such models have been coined ‘individual models’ (Apps and Rees, 2010) or ‘collective models’ (Bourguignon, Browning, and Chiappori, 1995; Browning, Chiappori and Weiss, 2012).
Section 3.5 provides a numerical example. Sections 4 and 5 analyze the optimal marriage contract and the use of marriage, while Section 6 concludes.

2 The model

Consider a male (subscript ‘m’) and a female (subscript ‘f’) who live for two periods \( t = 1, 2 \).\(^7\) In the first period the two are married while in the second period, they face a probability \( \pi \) of getting divorced. Divorce occurs for reasons exogenous to the analysis. For the time being, we assume that the reasons to enter into the marriage are exogenous too. In Section 5 we relax this assumption and analyze the individual’s economic use of the marriage. In both periods the two spouses cooperatively decide about labor supply \( \ell \) and consumption. While married the two share a common budget whereas when divorced each individual has to finance his/her own living. In the first period, the couple can save part of their income for the next period. Both the interest rate and time preferences are equal to zero. We consider an institutional framework such that in case of divorce savings \( s \) – or, more generally, wealth accumulated during marriage – are equally divided between the male and the female.\(^8\) Per period utility of each individual depends on consumption of a numeraire commodity \( c \) and on labor disutility \( v(\ell) \). Specifically, utility is given by \( u(c - v(\ell)) \) with \( u' > 0, v' > 0 \) and \( u'' < 0, v'' > 0 \). This quasi-linear specification is adopted for the ease of exposition.

Productivities or wages are endogenous. That is, at the beginning of the first period, the couple cooperatively decides about the human capital investments in each of them. We are only concerned by human capital investments that are made at the start of marital life. One can think of such investments as, \( e.g. \), the tertiary education choice, or resources invested for job specific skills like learning foreign languages. At the time of their marriage, each spouse already has some human capital or educational level \( \bar{w} \) which without loss of generality is normalized to zero. A couple’s feasible human capital choices are described by the following technology \( \Gamma \):\(^9\)

\[
\Gamma = \{(w_m, w_f) | w_m + w_f = 2\}.
\]

This amounts to assuming that wages are determined by educational expenditures

\(^7\)Throughout we assume that the couple consists of one women and one man. We make this assumption only for expositional convenience.

\(^8\)This presumes that a judge is able to observe the couple’s accumulated wealth.

\(^9\)See Cremer et al. (2011).
through a linear technology and that the total budget for human capital investments is fixed. The linear technology implies that productivities can be transferred between spouses on a one-by-one basis implying that both spouses have the same learning skills and that there are no decreasing returns to education. This technology incorporates two extreme scenarios: (i) complete equalization of wages between both spouses, \( w_m = w_f = 1 \) and (ii) maximum wage inequality, \( w_i = 2 \) and \( w_{-i} = 0 \) for \( i = m, f \). In the latter case all educational resources are concentrated on a single spouse.

The decisions of the couple are taken in a cooperative way. More precisely, the couple maximizes a common welfare function which is given by the sum of first- and second-period (expected) utility of both the male and female; see also Lommerud (1989). The two spouses’ utilities are weighted equally and the couple’s welfare function is thus given by

\[
W = u (e_{1c}^m - v(\ell_{1c}^m)) + u (e_{1c}^f - v(\ell_{1c}^f)) + \pi \left[ u (e_{2c}^m - v(\ell_{2c}^m)) + u (e_{2c}^f - v(\ell_{2c}^f)) \right] \\
+ (1 - \pi) \left[ u (e_{2c}^m - v(\ell_{2c}^m)) + u (e_{2c}^f - v(\ell_{2c}^f)) \right], \tag{2}
\]

where the first superscript denotes the period and the second superscript indicates the marital status of the individuals in the second period (‘c’ for couple and ‘s’ for single).

Recall that a divorce occurs with probability \( \pi \) in which case the (ex) spouses are single in the second period. The budget constraints of a couple who does not divorce (and thus pools resources in both periods) are given by

\[
e_{1c}^m + e_{1c}^f = w_m \ell_{1c}^m + w_f \ell_{1c}^f - s, \tag{3}
\]
\[
e_{2c}^m + e_{2c}^f = w_m \ell_{2c}^m + w_f \ell_{2c}^f + s. \tag{4}
\]

In case of divorce the two ex-spouses face separate budget constraints in the second period. They consume their own labor income plus half of the couple’s first period saving. Formally, we have

\[
e_{2s}^m = w_m \ell_{2s}^m + \frac{s}{2}, \tag{5}
\]
\[
e_{2s}^f = w_f \ell_{2s}^f + \frac{s}{2}. \tag{6}
\]
3 The couple’s optimization problem

3.1 Statement and first-order conditions

The couple maximizes (2) subject to the budget constraints, equations (3)–(6) and the education technology (1). For expository convenience, we decompose this problem into two stages. First, the couple chooses the education levels $w_m$ and $w_f$. Second, the couple chooses savings as well as labor supplies and consumption levels of each of the spouses in both periods and states of nature (divorce or not). This specific timing is of no relevance to our results.\footnote{Except for the assumption that human capital investments are made (once and for all) at the beginning of married life and thus before it is known if the couple will effectively divorce or not. Formally this implies that the levels of $w_m$ and $w_f$ are unique (there is no period/state of nature superscript).}

Let us first consider the second stage optimization problem, that is, the determination of savings, consumption and labor supply for a given educational decision. Solving equations (3) and (4) for $c^1_m$ and $c^2_m$ and substituting into the objective function (2) yields the following optimization problem $(t = 1, 2, j = c, s$ and $i = m, f)$.

$$
\max_{s, c^1_i, c^2_i} \mathcal{W} = u(w_m c^1_m + w_f c^1_f - s - c^1_f - v(c^1_f)) + u(c^1_f - v(c^1_f)) \\
+ \pi \left[ u(w_m c^2_m + \frac{s}{2} - v(c^2_m)) + u(w_f c^2_f + \frac{s}{2} - v(c^2_f)) \right] \\
+ (1 - \pi) \left[ u(w_m c^2_m + w_f c^2_f + s - c^2_f - v(c^2_f)) + u(c^2_f - v(c^2_f)) \right]. \tag{7}
$$

Differentiating, rearranging and defining $x_{ij}^{tj} \equiv c_{ij}^{tj} - v(t_{ij}^{tj})$ consumption net of labor disutility yields the following first-order conditions

$$
u'(x_{ij}^{1c}) = u'(x_{ij}^{1c}) \quad \forall t \tag{8}$$
$$v'(\ell_{ij}^{1f}) = w_i \quad \forall t, j, i \tag{9}$$
$$u'(x_{ij}^{1c}) = \frac{\pi}{2} \left[ u'(x_{ij}^{2c}) + u'(x_{ij}^{2f}) \right] + (1 - \pi) u'(x_{ij}^{2c}) \quad \forall i. \tag{10}
$$

Consumption levels of married spouses are set so as to equalize their marginal utilities. Labor supply is independent of time, marital status and gender and is an increasing function of wage. It is chosen to equalize marginal labor disutility with wages.\footnote{The simplification arises because preferences are quasi-linear. They make our argument crisper but are not essential for our results. In particular our results do not depend on the property that labor supply increases with wage; see Section A.} The optimal level of savings equalizes individual’s marginal utility in the first period and expected marginal utility in the second period. Note that savings only act as an insurance device for the case of divorce. They provide some measure of protection for an...
individual who ends up single and has low productivity (and thus low labor income). Except for the risk of divorce, the two periods are perfectly symmetrical and there is otherwise no need to accumulate wealth in this setting. Specifically when there is no risk of divorce ($\pi = 0$) consumption possibilities are the same in both periods and no wealth is (dis)accumulated as the interest rate and time preferences are zero.

Let us now turn to the first stage, the educational decision. Specifically, we want to study whether one of the extreme solutions (equalization or maximum differentiation of wages) emerges. With our quasi-linear specification we can reduce this problem to a single dimension, namely the choice of one of the spouses productivity level. To do so, we substitute the optimal consumption, labor supply and savings decisions as defined by equations (8) to (10) back into the welfare function (2). Additionally, we take the educational technology $w_i = 2 - w_{-i}$ as defined by (1) into account and generate the optimal value function $\Omega(w_i) = W(c^*, \ell^*, s^*, w_i)$, which relates individual-$i$'s wage rate $w_i$ with maximum welfare given optimally chosen consumption, $c^*$, labor supply, $\ell^*$, and savings, $s^*$.

Optimal wages $w_i$ solve

$$w_i^* \in \arg \max_{w_i} \Omega(w_i),$$

where $\Omega(w_i)$ is given by

$$\Omega(w_i) = 2u \left( \frac{(2 - w_i)\ell_{-i}^* + w_i\ell_i^* - v(\ell_{-i}^*) - v(\ell_i^*) - s^*}{2} \right) + \pi \left\{ u \left( \frac{(2 - w_i)\ell_{-i}^* - v(\ell_{-i}^*) + s^*}{2} \right) + u \left( w_i\ell_i^* - v(\ell_i^*) + s^* \right) \right\} + (1 - \pi)2u \left( \frac{(2 - w_i)\ell_{-i}^* + w_i\ell_i^* - v(\ell_{-i}^*) - v(\ell_i^*) + s^*}{2} \right).$$

(11)

In the remainder of this section, we study the properties of $\Omega$ in order to determine the optimal human capital investment decision and to study the impact of parameters like the divorce probability and risk aversion. Even though we have now reduced the problem to a single dimension this turns out to be a non-trivial exercise since the global behavior of $\Omega$ is more complex than one could have anticipated. We shall proceed in different steps by deriving first- and second-order conditions and by proving some

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12 In reality couples may save for various non-divorce related reasons, including retirement preparation. To account for this we could introduce a third period during which people retire. This would complicate the analysis without affecting the results.

13 $c^*$ and $\ell^*$ are vectors and are short for optimal consumption/labor supply of male and female in both periods and states.
additional properties (while discussing the underlying intuition). To show that various configurations can arise we will also make use of numerical examples. A proposition summarizing the findings will be presented at the end of the section.

Using the envelope theorem, the first-order condition (FOC) with respect to \( w_i \) is given by

\[
\frac{\partial \Omega(w_i)}{\partial w_i} = 2u'(x^{1c}) \left( \frac{-\ell^*_i + \ell^*_i}{2} \right) + \pi \left( u'(x^{2s})\ell^*_i - u'(x^{2c})\ell^*_{-i} \right) + (1 - \pi)2u'(x^{2c}) \left( \frac{-\ell^*_{-i} + \ell^*_i}{2} \right).
\]

Evaluating equation (12) at equal wages \( w_i = w_{-i} = 1 \) yields

\[
\frac{\partial \Omega(1)}{\partial w_i} = 0
\]

as \( \ell^*_m |_{w_m=1} = \ell^*_f |_{w_f=1} = \ell^* \) and \( s^* |_{w_m=w_f=1} = 0 \) implying \( x^t_{ij} = x^* \forall t, i, j \). In other words, the FOC is always satisfied for equal wages and one might be tempted to conclude that it is always optimal for the couple to equalize wage rates. However, we already know from Cremer et al. (2011) that when \( \pi = 0 \), wage equalization is never optimal. Consequently, we certainly cannot restrict our attention to the FOC at equal wages.

We also have to look at the second-order condition (SOC) and the global behavior of the objective function. A first interesting fact is revealed by evaluating the FOC (12) at unequal wages \( w_i = 2 \) and \( w_{-i} = 0 \) which yields

\[
\frac{\partial \Omega(2)}{\partial w_i} = 2u'(x^{1c}) \frac{\ell^*_i}{2} + \pi u'(x^{2s})\ell^*_i + (1 - \pi)2u'(x^{2c}) \frac{\ell^*_i}{2} > 0.
\]

Alternatively, considering the symmetric solution \( w_i = 0 \) and \( w_{-i} = 2 \) yields\(^{14}\)

\[
\frac{\partial \Omega(0)}{\partial w_i} = -2u'(x^{1c}) \frac{\ell^*_i}{2} - \pi u'(x^{2s})\ell^*_i - (1 - \pi)2u'(x^{2c}) \frac{\ell^*_i}{2} < 0.
\]

In other words, slightly moving away from the extreme solution with maximum wage differentiation always reduces welfare. To get further insight, we now turn to the second-order condition.

\(^{14}\)Where \( \ell^*_i \) for \( w_i = 2 \) and \( w_{-i} = 0 \) is equal to \( \ell^*_{-i} \) for \( w_i = 0 \) and \( w_{-i} = 2 \).
3.2 Second-order condition

The SOC is given by

\[
\frac{\partial^2 \Omega(w_i)}{\partial w_i^2} = 2\left(u''(x^1) + (1 - \pi)u''(x^2)\right) \left(-\frac{\ell^*}{2}\right)^2 \left(\frac{\partial \ell^*}{\partial w_i} - \frac{\partial \ell^*}{\partial w_i}\right) + \pi \left[u'(x^2) \frac{\partial \ell^*}{\partial w_i} - u'(x^2) \frac{\partial \ell^*}{\partial w_i}\right] + \pi \left[u''(x^2)\left(\ell^*\right)^2 + u''(x^2)\left(\ell^*\right)^2\right] + \left(u'(x^1) + (1 - \pi)u'(x^2)\right) \left(\frac{\partial \ell^*}{\partial w_i} - \frac{\partial \ell^*}{\partial w_i}\right).
\] (13)

Evaluating this expression at equal wage rates yields

\[
\frac{\partial^2 \Omega(1)}{\partial w_i^2} = 2\pi u''(x^1)\left(\ell^*\right)^2 + 4\pi u''(x^1) \frac{\partial \ell^*}{\partial w_i} = 2u'(x^1)\ell^* (2\varepsilon_{\ell,w} - \sigma \pi \ell^*) \geq 0,
\]

where \(\sigma = -u''/u'\) denotes absolute risk aversion and \(\varepsilon_{\ell,w} = (\partial \ell^*/\partial w)(w/\ell^*)\) is the labor supply elasticity. The couple’s welfare is thus a convex function of \(w_i\) at equal wages whenever

\[
\varepsilon_{\ell,w} > \frac{\sigma \pi \ell^*}{2}.
\] (14)

When this condition holds we can rule out wage equalization (as it is then a local minimum). This is the case when the divorce probability or the degree of risk aversion are sufficiently small (particularly when the couple is risk neutral), or when labor supply elasticity is sufficiently large. Absent the possibility of divorce (\(\pi = 0\)) we return to the setting of Cremer et al. (2011), who show that welfare (which with our quasi-linear specification reduces to surplus) is highest under maximum wage differentiation. Under risk neutrality, it is plain that this result remains valid when divorce is introduced (\(\pi > 0\)). While divorce affects individual utility of the (ex)-spouses it has no impact on total surplus which is all what matters in this case.

3.3 The no divorce case: intuition

Before proceeding it is interesting to have a look at the intuition for the wage differentiation result when there is no divorce. When the couple remains married in period 2 for sure, the two period setting is of no relevance and the couple’s objective is equivalent to the maximization of total surplus. When the couple equally shares the education budget both spouses work, incurring labor disutility \(v(\ell(1))\), and their labor income amounts to \(1\ell(1) + 1\ell(1) = 2\ell(1)\); see Figure 1 (a). Total surplus is equal to twice the area of the upper triangle. On the other hand, when the entire education budget is
invested in a single spouse, total labor income (earned solely by the spouse with a wage of 2) amounts to $2\ell(2)$. In the process, one spouse incurs no labor disutility while the other one’s disutility is determined by a labor of $\ell(2)$; see Figure 1 (b). Total surplus is now equal to the area of the upper triangle which exceeds the level achieved in (a). One might at first be tempted to think that this result is based on the property that labor supply increases with wage so that $\ell(2) > \ell(1)$. However, this is not necessary. As a matter of fact, labor supply needs not to be adjusted in an optimal way. For instance, when we switch from equal wages to maximal wage differentiation, we could achieve a welfare improvement simply by maintaining labor supply of the productive spouse at $\ell(1)$ who would then earn $2\ell(1)$. In other words, total income is unchanged, but labor disutility is cut in half; see the dotted line in Figure 1 (b). This argument is presented more formally in Appendix A, where we show that the result continues to hold for general utility function (irrespective of the properties of labor supply).

This argument does not change when $\pi > 0$ but individuals are risk-neutral with $u'' = 0$ and thus $\sigma = 0$. However, when individuals are risk averse, the possibility of divorce may affect the couple’s human capital investment decisions. In particular, the equal wage solution now becomes more attractive. To see this, let us study how

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Figure 1: Optimal labor supply and surplus for (a) equal and (b) unequal wages.
Δ ≡ Ω(2) − Ω(1) changes as π increases. We have

\[ \begin{align*}
\Omega(1) &= 4u[\ell(1) - v(\ell(1))] \\
\Omega(2) &= 2u \left( \frac{2\ell(2) - v(\ell(2)) - s^*}{2} \right) \\
&\quad + \pi \left\{ u \left( \frac{s^*}{2} \right) + u \left( 2\ell(2) - v(\ell(2)) + \frac{s^*}{2} \right) \right\} \\
&\quad + (1 - \pi)2u \left( \frac{2\ell(2) - v(\ell(2)) + s^*}{2} \right).
\end{align*} \]

Differentiating with respect to π and making use of Jensen’s inequality yields

\[ \frac{\partial \Delta}{\partial \pi} = u \left( \frac{s^*}{2} \right) + u \left( 2\ell(2) - v(\ell(2)) + \frac{s^*}{2} \right) - 2u \left( \frac{2\ell(2) - v(\ell(2)) + s^*}{2} \right) \leq 0, \]

where the inequality is strict when \( u'' < 0 \).

While the couple’s utility under wage equalization does not depend on π, welfare under maximum wage differentiation decreases as the probability of divorce increases. Intuitively, when divorce is possible, under wage differentiation the spouses’ second period consumption levels are random variables which is not desirable when individuals are risk averse. More specifically, the total surplus of the couple in the second period is the same in both states of nature, but it is split unequally in the case of divorce. This decreases expected utility. Observe that this welfare loss will be larger the larger the degree of concavity of \( u \) (as measured for instance by the degree of risk aversion \( \sigma \)). Once again this result appears to rely on our specification of preferences because we use the property that the couple’s total surplus is the same in both states of nature. However, this is not effectively necessary for our result to obtain. A simple inspection of (16) suggest that \( \Delta \) will be decreasing in \( \omega \) as long as the couple’s utility in case of divorce is smaller than that when the couple persists. But this is necessarily true because the married couple can always choose the same consumption and labor profile than it would under divorce; see Appendix A for a formal proof.

To sum up, we have shown that wage equalization becomes more attractive as the divorce probability increases (provided that \( \sigma > 0 \)). In addition, the second order condition (14) shows that as π increases (and provided that σ is sufficiently large) wage equalization will eventually become a local maximum. Putting these two properties together one might be tempted to conjecture that when π and σ are sufficiently large, wage equalization would necessarily become the optimal policy. However, this conjecture is misleading as the following example with a quadratic disutility of labor shows.
3.4 Example: quadratic labor disutility

Assume for the time being that labor disutility is given by $v(t) = t^2/2$. Then, by equation (9) optimal labor supply is $t_{ij} = w_i \forall t, j, i$. From equation (8) we know that consumption levels while married are chosen so as to equalize marginal utilities of the male and female i.e., $x_{mc}^i = x_{fc}^i$. Substituting the education technology $w_{-i} = 2 - w_i$ into the objective function yields

$$W(w_i, s) = 2u \left( \frac{(w_i^2 + (2 - w_i)^2)/2 - s}{2} \right) + \pi \left[ u \left( \frac{w_i^2 + s}{2} \right) + u \left( \frac{(2 - w_i)^2 + s}{2} \right) \right]$$
$$+ (1 - \pi)2u \left( \frac{(w_i^2 + (2 - w_i)^2)/2 + s}{2} \right).$$

Welfare with equal wages, $W(w_i = w_{-i} = 1)$, and unequal wages, $W(w_i = 2, w_{-i} = 0)$, is given by the following two expressions

$$W(w_i = 1, s) = 2u \left( \frac{1 - s}{2} \right) + \pi \left[ u \left( \frac{1 + s}{2} \right) + u \left( \frac{1 + s}{2} \right) \right] + (1 - \pi)2u \left( \frac{1 + s}{2} \right)$$
$$W(w_i = 2, s) = 2u \left( \frac{2 - s}{2} \right) + \pi \left[ u \left( \frac{4 + s}{2} \right) + u \left( \frac{0 + s}{2} \right) \right] + (1 - \pi)2u \left( \frac{2 + s}{2} \right)$$

The optimal savings decision for equal wages is given by $s = 0$ implying $x_{ij}^t = 1/2 \forall t, i, j$. Consequently, each spouse has a utility level of $u(1/2)$ in both periods and states of nature. Now consider unequal wages and set savings equal to one i.e., $s = 1$; while this is not (in general) the optimal savings decision, it represents a feasible level. Then, utility for each spouse in the first period and for the single household with low productivity remains at $u(1/2)$. However, utility when married in the second period and utility for the high-productivity single household increases (exceeds $u(1/2)$). Consequently, extreme wage differentiation always dominates wage equalization, even when $\pi$ is equal or close to 1 and when the degree of risk aversion tends to infinity. In other words, it is never optimal for the couple to equalize wages with quadratic labor disutility. Intuitively this result arises because with quadratic labor disutility the gain in surplus achieved by wage differentiation (as compared to equalization) is so large that the couple can generate a sufficient amount of saving to ensure that each spouse is better off in any contingency.

To sum up, we have established that maximum wage differentiation is optimal when the probability of divorce is zero and that it may remain optimal even when divorce probability and risk aversion are high. On the other hand, we have shown that wage equalization becomes more attractive as $\pi$ increases. This leads quite naturally to the
following two questions. First, are there cases in which wage equalization effectively becomes the optimal policy? Second, can we ever have an “interior solution” that is a situation where neither of the extreme policies is optimal.

Since it turns out that the answer to both of these questions is affirmative it is easiest to show this by a series of numerical illustrations.

3.5 Numerical illustration

Our simulations are based on the following functional form for individual utility

\[
u(c, \ell) = \begin{cases} 
\frac{c^{1+1/\varepsilon} - 1}{1+1/\varepsilon} & \text{for } \sigma \neq 1 \\
\ln \left( \frac{c - \ell^{1+1/\varepsilon}}{1+1/\varepsilon} \right) & \text{for } \sigma = 1.
\end{cases}
\] (18)

Notation are chosen so that $\sigma$ and $\varepsilon$ effectively represents relative risk aversion and labor supply elasticity with this specific functional form.

Figure 2 depicts $\Omega(w)$ for $\sigma = 2$ and $\varepsilon = 0.5$ and for various levels of the divorce probability, ranging from 0 to 1. Not surprisingly all the curves are symmetric around $w = 1$ and welfare always decreases in the neighborhood of $w = 0$ or $w = 2$. When the divorce probability is zero, $\Omega$ is a “nice” u-shaped function; maximum wage differentiation is optimal and equal wages yield a local (and global) minimum. As $\pi$ increases, (extreme) wage differentiation becomes less attractive. First, wage equalization becomes a local (but not global) optimum (see $\pi = 0.4$) and eventually the global optimum ($\pi = 0.60$). For the parameter values considered in this example, there is never an intermediate solution with partial wage differentiation.

Figure 3 shows that such an intermediate wage differentiation is possible for $\sigma = 2$, $\varepsilon = 0.18$ and $\pi = 0.1$. However, the range of parameter values for which such a solution occurs is small. The bang-bang type of solution described in the previous figure is a more typical outcome. However, since our examples are have no pretense to be empirically realistic, the meaning of the word “typical” has to be qualified accordingly.

3.6 Section summary

The main results obtained in this section are summarized in the following proposition.

**Proposition 1** Consider a couple which determines its human capital investment according to the education technology (1) and whose welfare function is given by (2). We
Figure 2: $\Omega(w)$ for $\sigma = 2$ and $\varepsilon = 0.5$ and for various levels of the divorce probability.

Figure 3: $\Omega(w)$ for $\sigma = 2$ and $\varepsilon = 0.18$ and for various levels of the divorce probability.
have

(i) When there is no divorce ($\pi = 0$) and/or spouses are risk neutral ($\sigma = 0$), maximum wage differentiation ($w_i = 2$ and $w_{-i} = 0$) is always optimal. This solution continues to be a local maximum for all $0 < \pi \leq 1$ and $\sigma > 0$.

(ii) Welfare under maximum wage differentiation decreases with $\pi$ (provided that $\sigma > 0$), while welfare with equal wages does not depend on $\pi$.

(iii) A sufficiently large divorce probability may or may not make wage equalization optimal. Specifically, with a quadratic disutility of labor maximum differentiation remains optimal, independent how large the degree of risk aversion $\sigma$ and the divorce probability.

(iv) Intermediate levels of wage differentiation may be optimal for some parameter values, but a “bang-bang” solution where we switch from maximum differentiation to full equalization as $\pi$ increases is also possible.

4 The marriage contract

Our model shows that for a positive divorce probability, the optimal education decision faces a trade-off between maximization of net income and hedging against the risk of divorce and relying on one own’s income. While the former calls for maximum wage differentiation, the latter calls for equal wages. This result is contingent on the underlying institutional and legal framework which assumes that savings are equally divided in case of divorce. Wealth accumulation while being married thus also implies a hedge (although an imperfect one) against ending up with low human capital and, hence, low income in case of divorce. Now, assume the institutional framework also includes the possibility to write a marriage contract which can be enforced by law at no costs. That is in the first period the couple has the possibility to commit to a transfer scheme for the case of divorce in the second period. The optimal transfer scheme, or alimony, $\{T_m; T_f\}$ the couple agrees upon in the first period is determined by solving:

$$\max_{s, c_m^T, c_f^T, T_i} W = u (c_m^{1c} - v(\ell_m^{1c})) + u (c_f^{1c} - v(\ell_f^{1c})) + \pi \left[ u \left( c_m^{2s} - v(\ell_m^{2s}) \right) + u \left( c_f^{2s} - v(\ell_f^{2s}) \right) \right]$$

$$+ (1 - \pi) \left[ u \left( c_m^{2c} - v(\ell_m^{2c}) \right) + u \left( c_f^{2c} - v(\ell_f^{2c}) \right) \right]$$

s.t. (3), (4), $T_m + T_f = 0$

$c_m^{2s} = w_m \ell_m^{2s} + 0.5s + T_m$ and $c_f^{2s} = w_f \ell_f^{2s} + 0.5s + T_f$. 

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Again, labor supply and consumption are chosen according to (8) and (9). The optimal transfer in case of divorce equals marginal utilities of the male and female when single:

\[ T_f^* = -T_m^* = \frac{u_m \ell_m^* - v(\ell_m^*) - (2 - w_m)\ell_f^* + v(\ell_f^*)}{2}. \]

With the above marriage contract, savings are no longer needed to ensure a minimum consumption of the low-wage single household in case of divorce so that we now have \( s^* = 0 \). Thus, the optimization problem reduces to

\[ \max_{w_i} \mathcal{W}(w_i) = 4u \left( \frac{(2 - w_i)\ell_{-i}^* + w_i \ell_i^* - v(\ell_{-i}^*) - v(\ell_i^*)}{2} \right). \]

In other words we return to the case where the couple maximizes total surplus exactly like in the previous section when \( \pi = 0 \) and/or \( \sigma = 0 \). Consequently, if the couple agrees on a marriage contract in the first period the solution always implies maximum wage differentiation. Divorce, which was problematic for the low productivity ex-spouse in the absence of marriage contracts is now no longer a problem. The human capital decision can be based on efficiency only even if that results in concentrating all investments on a single individual. Thanks to the optimally designed marriage contract the less productive spouse is fully protected against the risk of divorce.

This result rests of course on the strong assumption that there is no uncertainty as to the enforcement of the marriage contract (nor is there any moral hazard in the most productive spouse’s labor force participation).

Comparing this solution with the one obtained in the previous section shows that the availability of (perfect) marriage contracts corrects two potential types of inefficiency brought about by divorce. First, it ensures that the surplus maximizing human capital allocation is implemented. Recall that without marriage contract this may or may not be true. Second, there is no longer any need for the couple to have positive saving. Remember that in our setting saving is useful only in that it may provide (partial) insurance to the less productive spouse. In other words, saving per se is inefficient in our setting and the availability of marriage contracts removes this source of inefficiency.

**Proposition 2** If the couple can commit to a marriage contract while married in the first period, the entire educational budget is invested in one spouse, yielding maximum wage differentiation \((w_i^* = 0 \text{ and } w_{-i}^* = 2)\) irrespective of the probability of divorce or the degree of risk aversion. Savings are equal to zero and the higher educated spouse
transfers half of his net income (consumption minus the monetary loss due to labor supply) to the other spouse in case of divorce.

5 The (economic) use of marriage

Our earlier analysis shows that the more unequal the educational budget is divided between the spouses, the higher the surplus the couple generates. We can now ask about the economic “use” (or benefit) of the marriage. Specifically, is it worth for both the male and the female from an economic point of view to marry in the first place. In Becker’s (1993; 1974) seminal theory of marriage, two individuals marry when marriage comes along with a positive surplus relative to the two remaining single. Assume each of the two individuals has half of the education budget without getting married, then wages are given by \( w_f = w_m = 1 \) and utility when staying single amounts to

\[
U_{\text{single}}^i = 2u(1\ell_i - v(\ell_i)) \quad \text{for} \quad i = m, f
\]

On the other hand, when getting married utility of each individual is given by

\[
U_{\text{married}}^i = u(c_i^{1c} - v(\ell_i^{1c})) + \pi u(c_i^{2s} - v(\ell_i^{2s})) + (1 - \pi) u(c_i^{2c} - v(\ell_i^{2c}))
\]

where consumption, labor supply, savings and wages are determined by equations (8) to (10) and (12). Obviously, if an equal wage distribution is optimal from the couple’s point of view then both partners are equally well off when marrying and when remaining single. However, whenever an unequal wage distribution is optimal, utility while married differs between the male and female unless \( \pi = 0 \). While the spouse who receives the higher share of the education budget is never worse off compared to his/her single status, the one who gets the smaller share of the cake may well be better off with staying single. From Subsection 3.4 we know that for quadratic labor disutility an unequal wage distribution Pareto-dominates equal wages. In other words, the economic use of a marriage is always positive for the female and the male if the labor supply elasticity is \( \varepsilon = 1 \). If the probability of divorce is zero, or the two can commit to a marriage contract, then, even the lower educated individual profits from the surplus generated by investing the whole education budget in one spouse as this surplus is equally divided between the two of them. We summarize our findings in the following proposition:

\[\text{Cigno (2009) shows that the decision to marry also depends on the choice of game after marriage. He finds that a couple will marry only if marriage serves as a commitment device for cooperation.}\]
Proposition 3 Both spouses necessarily profit from a marriage if (i) the divorce probability is zero, (ii) labor disutility is quadratic, or (iii) they can commit to marriage contract.

While this proposition provides a number of cases where marriage is beneficial for both spouses, there is no guarantee that this is always the case in our setting. As a matter of fact it is quite easy to provide counterexamples. To see this let us return to the numerical specification used in Subsection 3.5 which assumes that utility is defined by (18). Figure 4 illustrates the economic use for each individual for \( \sigma = 1 \) and \( \varepsilon = 0.5 \). For small divorce probabilities the expected utility of a marriage is higher for both individuals than when staying single. Even though in this case an unequal wage distribution is optimal, the additional surplus generated through specialization in the marriage exceeds the possible loss in case of divorce. For larger divorce probabilities the partner with the lower productivity is worse off with a marriage. The surplus created through specialization is too low to offset the possible utility losses in case of divorce. Finally, for even higher divorce rates (above 60 percent in the considered example) equal wages are optimal, so that utility when getting married and when staying single coincide.

To sum up, for low divorce probabilities marriage is beneficial for both partners; for intermediate levels, the low productivity spouse is worse off. Finally when the divorce probability is sufficiently large (and parameters are so that equal wages become optimal) marriage leaves utilities (and human capital investment decisions) unaffected.

6 Conclusion

This paper has examined how a couple’s (tertiary) education choices are affected by the possibility of divorce (seen as a random exogenous event). It has considered a simple setting where absent the possibility of divorce, it is optimal for the couple to specialize and invest the whole educational budget in one spouse. This maximum wage differentiation maximizes the couple’s overall surplus and hence welfare. If the probability of divorce becomes positive optimal human capital investments depend on risk aversion and the labor supply elasticity. A higher risk aversion thereby makes the symmetric solution (equal wage distribution) more likely, whereas a higher labor supply elasticity provokes more specialization (a more unequal wage distribution) between the husband
and wife. This result presumes that accumulated wealth is equally divided between the two individuals. If, additionally, the couple can commit to a marriage contract at the beginning of their marriage specialization is always optimal granted that the contract fully compensate the losers for his/her losses. While the marriage decision is always positive with a marriage contract, it may well turn negative for one of the spouses in case of medium divorce probabilities and no marriage contract. Throughout the paper we made some restricting assumption.

To obtain a clearcut benchmark we have assumed constant returns to education (following Cremer et al., 2011). Decreasing returns in the couple’s education technology could affect the no divorce benchmark. However, the degree of decreasing returns must be sufficiently strong to do away with the specialization result (see also Cremer et al., 2011). When this occurs, there is no longer any need to provide insurance against (the financial implications) of divorce. Additionally, we assumed a uniform learning ability. If we were to assume different learning abilities, then our results are only strengthened. The main difference would be that the spouse with the higher wage level would be the more able.

Among possible extensions to this paper, myopia and taxation could be considered.
It is often argued that love is blind, in other words, when just married the couple may (partly) ignore their probability of getting divorced later in life. Witness the reluctance that young couples have to sign a marriage contract that would cope with the possibility of divorce and its unpleasant outcomes. In France, 16% of all couples draw up a contract when they get married and 3% do so in the years that follow, making a total of 19% of married couples (see Barthez and LaFerrere, 1996). Myopia has consequences for human capital investments. Assume the couple completely ignores a possible divorce in the second period; then, the optimization problem reduces to maximization of the couple’s common budget and this is maximized whenever the couple puts all eggs in one basket. In other words, myopia concerning the divorce probability leads to a more unequal wage distribution between the male and the female.

Another natural extension concerns public policy, which is assumed away in this paper with the exception of the marriage contract that needs a public authority to be enforced. In this paper we just considered two types of contracts. A richer variety of contracts and alimony rules could be analyzed. Further in the case couples decide to choose an equal investment in human capital and excessive saving, through an appropriate tax/transfer policy one could achieve a more efficient level of saving and educational choice.

Appendix

A General Utility

Throughout the paper we assume quasi-linear preference (with no income effect) and a utility function given by \( u(c_i - v(l_i)) \). This specification implies that labor supply is always increasing in wage and one might be tempted to think that this is crucial for our results. However, this is not the case. We use this specification for simplicity and to be able to reduce the problem to a single dimension. But this is just a matter of exposition. Proposition 1 continues to be valid for general utility functions. This is obvious for items iii) and iv), but needs to be established for i) and ii).

The result that maximum wage differentiation is optimal for \( \pi = 0 \) with general utility functions follows directly form Cremer et al. (2011). The theoretical argument effectively formalizes the intuition explained in Subsection 3.3 above. To make this paper self-contained we briefly sketch this main argument. Assume the general utility
function $u(c_i, \ell_i)$, then, for equal wages and $\pi = 0$ we will have $c_i = w_i\ell_i = \ell_i$ and $s = 0$ (spouses consume their own incomes, which are equal and saving is zero). Welfare is then given by

$$W^E = 4u(\ell^E, \ell^E),$$

where

$$\ell^E = \arg \max_{\ell} u(\ell, \ell).$$

(A1)

With unequal wages, $w_i = 2$ and $w_{-i} = 0$, the spouse with $w_i = 0$ does not work and earns no income but he/she gets a compensation from the spouse with higher human capital (as optimally marginal utilities are equalized). Denoting this transfer $T$, the couple’s welfare is given by

$$W^{MD} = \max_{\ell_i, \ell_{-i}, T} 2u(w_i\ell_i - T, \ell_i) + 2u(w_{-i}\ell_{-i} + T, \ell_{-i}),$$

$$= 2u(2\ell^E_i - T, \ell^E_{-i}) + 2u(0 + T, 0),$$

where we have $\ell_{-i} = 0$. Observe that as long as $\pi = 0$ we have $s = 0$ and the two periods are perfectly symmetrical. Now assume the individual with higher human capital gives simply half of her income to her spouse ($T = \ell^E_{-i}$) so that consumption levels are equalized (which is generally not the optimal level). Additionally, set $\ell^E_{-i} = \ell^E_{-i}$, so that the individual with $w_i = 2$ works the same number of hours as with equal wages (which is generally also not optimal). With these two assumptions, we have

$$W^{MD} = 2u(\ell^E, \ell^E) + 2u(\ell^E, 0) > W^E.$$

In words, under wage differentiation the couple can achieve the same consumption levels as under wage equalization by having only a single individual work (the same amount as under wage equalization). Intuitively this is simply a generalization of the argument discussed in Subsection 3.3 and represented in Figure 1. This establishes the item i) of Proposition 1 continues to be valid with more general utility function.

Introducing a positive $\pi$ the couple’s welfare with general utility functions is re-defined as

$$W = u(c^1_m, \ell^1_m) + u(c^1_f, \ell^1_f) + \pi [u(c^2_m, \ell^2_m) + u(c^2_f, \ell^2_f)]$$

$$+ (1 - \pi) [u(c^2_m, \ell^2_m) + u(c^2_f, \ell^2_f)].$$

(A3)
Budget constraints are unchanged and continue to be given by (3)–(6). Under equal wages, maximizing (A3) subject to (3)–(6) yields $c_{ij} = \ell_{ij} = \ell^E$ defined by (A2) so that welfare is given by $W^E = 4u(\ell^E, \ell^E)$ which does not depend on $\pi$. Under unequal wages ($w_i = 2$ and $w_{-i} = 0$), differentiating welfare with respect to $\pi$, while using the envelope theorem yields

$$\frac{\partial W}{\partial \pi} = [u(c^2_m, \ell^2_m) + u(c^2_f, \ell^2_f)] - [u(c^2_m, \ell^2_m) + u(c^2_f, \ell^2_f)] \leq 0.$$ 

To establish the inequality, observe that the second term in brackets is the utility of the married couple while the first term is the utility of the divorced spouses. Since savings and productivities are the same in both cases the utility of the married couple is always at least as large as that of the divorced spouses. This is because the $(c, \ell)$ bundles chosen by the divorced spouses are feasible for the married couple (while the opposite is not true). Consequently, item ii) of the proposition stating that wage differentiation becomes less attractive as $\pi$ increases remains valid with general utility. To sum up, none of these results requires an increasing labor supply function.

References


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16 Differentiating the Lagrangian expression, $\mathcal{L}$, associated with this problem (after substituting $w_{-i}$ by $1 - w_i$, one easily shows that at $w_i = 2$ and $w_{-i} = 0$ we have $\partial \mathcal{L}/\partial w_i > 0$ so that a local deviation from maximum differentiation decreases welfare irrespective of $\pi$ and $\sigma$ even with general utilities. However, unlike in the quasi-linear setting the problem cannot be reduced to a single dimension so that this property is no longer very meaningful.


