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Taxation of Firms with Unknown Mobility

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Abstract

We analyze the optimal tax choices of a revenue-maximizing government that levies taxes from firms of which the true degree of mobility is ex ante unknown. Differential tax treatment of immobile and mobile firms is ruled out, but the government may learn from the firms’ location responses to past tax rate changes. Firms, however, may anticipate this and adjust their choices accordingly. We derive all symmetric Bayesian equilibria with a focus on the (so far neglected) one where the government sets a tax rate that triggers partial migration but full revelation of the true number of mobile firms. We show that, if tax competition is fierce (i.e., relocation cost and foreign tax rates are low), expected tax rates and expected firm migration are higher if the degree of mobility is unknown. There is a positive value of learning, i.e. commitment on future tax rates cannot increase the government’s expected revenue. However, if the government can commit to a rule-based learning mechanism, i.e. credibly tie its future tax policy to present policy outcomes, it may obtain a Pareto improvement.

JEL-Code: H250, H320, H870.

Keywords: corporate taxation, firm mobility, incomplete information.

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1 Introduction

The question how mobile resources are (optimally) taxed has been in the centre focus of tax-related research in the last twenty-five years. Mobile resources (e.g., firms, capital or workers) have an outside option which puts a binding restraint on tax policy. The question arises how governments actually learn about the mobility of the entities to be taxed. Given the large interest in this topic, it may seem surprising that there are few theoretical contributions that deal with this question. Most contributions implicitly or explicitly assume that policy-makers either know or correctly anticipate the true degree of mobility of the taxed entities. However, making mistakes in guessing the actual degree of mobility can be costly and may have persistent negative effects on tax revenues in subsequent periods.

In this paper, we analyze the optimal taxation of mobile firms of which the true degree of mobility is \textit{ex ante} unknown. We consider a two-period model of a small open economy where a revenue maximizing government levies taxes on firms in both periods. Before period 1, the number of mobile firms is unknown to the government. Mobile firms may respond to tax policy in both periods by migrating abroad. Thus, policy-makers may learn from the firms’ past responses to tax policy and induce estimates on the actual degree of mobility. Therefore, firm migration which is otherwise inefficient becomes valuable as it reveals information about the true state of the economy. However, firms will anticipate the government to learn from first period migration and adjust their location decision accordingly. For instance, an individual firm anticipating fellow firms to move abroad (due to high taxes) may decide to stay if it expects the government to respond to the loss in tax base by lowering tax rates.

We derive all symmetric Bayesian equilibria of which there are, in general, three types. The first type of equilibrium implies that the government sets the maximum tax rate and all mobile firms leave the country with certainty (which fully reveals the degree of mobility). After observing firm migration, the government chooses between attracting all mobile firms back by implementing a sufficiently low tax rate and taxing the remaining immobile firms at the maximum tax rate. Under the assumption of uniformly distributed mobility, this equilibrium most likely occurs if the tax rate abroad and mobility cost are low. The second type of equilibrium implies that the government sets tax rates in both periods that no mobile firm has an incentive to leave. Then, the fraction of mobile firms remains unknown. This equilibrium occurs if the tax rate abroad and mobility cost are high. The third – and maybe most interesting – type of equilibrium has the government setting a tax rate which triggers partial migration. The actual number of firms moving abroad reveals information that can be used to optimally adjust tax policy in the second period. This equilibrium prevents revenue losses associated with full firm migration at the cost of lower tax rates in
the first period. This equilibrium is most likely for medium foreign tax rates and low mobility cost.

In order to analyze the role of incomplete information, we start by considering a benchmark scenario in which the true degree of mobility is common knowledge. Then, we assume that information on mobility is incomplete, i.e. both government and firms do not know the true degree of mobility.

Our main findings are the following. First, if tax competition from outside is fierce, i.e. the cost of relocation and foreign tax rates are low, expected tax rates are higher under incomplete information. The reason is that, if the true degree of mobility is unknown, the government cannot condition its tax rate choice on the actual mobility of firms. If competition from outside is strong, it is optimal to allow all mobile firms to leave and tax the remaining ones at the highest tax rate. In contrast, under complete information, the government will only do so if the actual mobility is sufficiently low. Second, since information is incomplete for both firms and the governments, firms do not earn an information rent and, thus, do not benefit from incomplete information. They do not suffer, either, because the government’s policy is set to just compensate them for not leaving the country. In contrast, the government unambiguously loses in terms of tax revenue because incomplete information implies inefficient firm migration and, thus, real resource losses in terms of migration cost. Third, we can show that there is a positive value of learning. This implies that, even if the government were able to commit itself to future tax rates, it does not have an incentive to do so. However, if government can commit itself to a certain learning rule, i.e. if it can credibly tie its future tax policy to present policy outcomes, a Pareto improvement can be obtained. If the government is able to credibly announce a threshold of moving firms above which it lowers the tax in order to attract these firms back, it may reduce equilibrium migration and, thus, migration cost and tax revenue losses. All domestic agents benefit.

Our findings have a number of important implications for tax policy. First of all, the welfare cost of taxing mobile resources may be larger than usually assumed. If mobility is unknown, the government has to observe inefficient migration in order to estimate the degree of mobility. The cost related to this information revealing migration is usually ignored in studies that assume known mobility. Second, setting a tax rate that triggers partial migration may be an optimal choice for risk-averse policy-makers who shy away from the risk of losing all mobile firms but do not want to lower taxes too much. Third, political governance and institutions matter. In the analysis below, we assume that the government cannot commit on future tax rules. We show however, that doing so would imply a Pareto improvement. Real world tax policy may have (or develop) means to intertemporarily commit to certain tax rules.

To the best of our knowledge, there are only two papers that deal
with tax policy with unknown firm mobility. First, Osmundsen, Hagen & Schjelderup (1998) consider the optimal taxation of firms with different degrees of mobility. In contrast to our approach, they allow for firm-specific (Mirleesian) taxation. In their model, the government optimally applies a mechanism that induces firms to truthfully report their degree of mobility. Since immobile firms have incentives to mimic mobile firms, the investment decision by the latter is optimally distorted in order to make mimicking less attractive. In our framework, firm-specific taxation is ruled out (which we think is in line with most real world tax systems). Accordingly, in our model, the direct mechanism that induces truthful revelation is much simpler. We will sketch it in Section 3 while focussing in the rest of the paper on the government’s learning strategy when past responses to tax choices can be observed. The second paper on unknown firm mobility is by Becker & Fuest (2011) who explore the scope for indirect discrimination (in case that direct tax discrimination is ruled out) when firms also differ in profitability. Then, a decrease of capital depreciation allowances (tax base broadening) can be used to shift the tax burden from mobile firms to immobile ones. In our paper, indirect discrimination is not possible since there are no differences across firms other than the degree of mobility.

Our paper builds on the learning literature since it considers – as Aghion et al. (1991) put it – a “dynamic decision problem of an agent who is initially uncertain as to the true shape of his payoff function, but who obtains information about it over time by observing the outcome of his past decisions.” (p. 621). In our framework, there is an explicit decision to learn. As shown by Aghion et al. (1991) in a more general framework, learning can be perfect, because the government’s payoff function is analytical, smooth and quasi-concave (in the relevant parameter ranges). Our analysis abstracts from all political economy aspects like in Majumdar & Mukand (2004). We assume that the government maximizes tax revenue which may be in line with the voters’ interest if, for instance, firms are owned by foreigners. There are certain similarities between the tax setting government in our framework and the “ignorant monopolist” analyzed by Clower (1959). In this line of

\[2\text{Corporate taxation in the presence of firms with known mobility has been intensively studied in the literature, see Richter & Welisch (1996), Boadway, Cuff & Marceau (2002) and Fuest (2005).}
\]

\[3\text{Keen (2001) criticizes existing rules that outlaw special tax treatment of mobile tax bases, e.g., internationally mobile firms, arguing that these provisions may actually exacerbate tax competition (see also Janeba & Smart, 2003).}
\]

\[4\text{Baldwin & Okubo (2009), Davies & Eckel (2010) and Hauffler & Stähler (forthcoming) present models in which heterogeneous mobile firms sort themselves into high-tax and low-tax locations according to their profitability and cost structure. In contrast to these contributions, we consider homogeneous firms which only differ in mobility.}
\]

\[5\text{There is substantial empirical evidence for the cross-border mobility of firms and its tax sensitivity, see Zodrow (2010) and Feld & Heckemeyer (2011) for recent surveys of the literature.}
\]
literature, Mirman, Samuelson & Urbano (1993) consider a monopolist who experiments and, thus, sacrifices first period profits in order to improve her knowledge on the shape of the demand function and, thus, second period profits – a setting quite similar to the one analyzed below.

The remainder of the paper is organized as follows. The next section lays out the model and derives the equilibria. Our main results are then stated in a number of corollaries. Section 3 discusses the results from a various perspectives. Section 4 concludes.

2 The model

Assume a two-period model in a world with a large number of countries. We focus on policy questions in one of them, called the domestic country. The domestic country’s industrial sector consists of a unit mass of firms indexed by \( i \in [0,1] \). There are two types of firms, mobile ones and immobile ones.

The fraction of mobile firms, \( \mu \), is a random variable chosen by nature. The corresponding distribution function is denoted by \( F(\mu) \) and has a continuous corresponding density function \( F'(\mu) = f(\mu) \).

All firms have an exogenously given pre-tax profit which is independent of firm location and normalized to unity. Whereas immobile firms remain completely passive, mobile firms choose their location in both periods. Moving implies a relocation cost \( c \), independent of the direction of migration.

Firms are profit maximizers and, thus, choose the location in which after-tax profits are highest.

The domestic government is assumed to maximize its tax revenue\(^6\) by choosing tax rates \( \tau_t \in [0,1] \) where \( t \in \{1,2\} \) denotes the time index. Tax discrimination between immobile and mobile firms is ruled out, i.e. the government has to choose one tax rate for all firms. When firms move abroad, they choose the location with the lowest tax rate (since pre-tax profits are equal everywhere) which is denoted by \( \tau_r \in [0,1] \) and assumed to be constant over time. There are no strategic aspects which can be justified by the small country assumption. For simplicity, we assume that the domestic government does not want to (or is not able to) attract foreign firms. We discuss such an extension in Section 3.

The timing of decisions is as follows: At stage 0, nature draws \( \mu \in [0,1] \). At stage 1, the government sets the tax rate for the first period, \( \tau_1 \in [0,1] \). At stage 2, each mobile firm \( i \) sets \( \sigma_{1i} \in [0,1] \) which is the probability of moving abroad in period 1. After firms have migrated, after-tax profits as well as tax revenue of period 1 are realized. At stage 3, the government sets the tax rate for the second period, \( \tau_2 \in [0,1] \). Finally, at stage 4,\(^6\)This assumption may be reinterpreted as putting an infinitely large welfare weight on public goods provision, as maximizing national welfare when firms are owned by foreigners or, equivalently, as maximizing the utility of those who do not hold firm shares.
mobile firms that have not moved in the first period set $\sigma_{2i} \in [0, 1]$ which is the probability of moving abroad in period 2. Firms that have moved in period 1 set $\tilde{\sigma}_{2i} \in [0, 1]$ which is the probability of moving back. After the firms’ migration decision, after-tax profits as well as tax revenue of period 2 are realized.

We solve the game by backward induction. In principle, a truth-revealing mechanism could be implemented by asking each individual firm to report its mobility. We will discuss such a mechanism design approach in Section 3.

At stage 4, each firm that has not moved in the first period compares its potential after-tax profit at home and abroad. The former is given by $1 - \tau_2$ and the latter by $(1 - c) (1 - \tau^r)$. Similarly, each firm that has moved in the first period compares its profit of staying abroad, $1 - \tau^r$, with its after-tax profit when it moves back, $(1 - c) (1 - \tau_2)$.\(^7\) Lemma 1 summarizes the optimal decision at stage 4.

**Lemma 1.** Given $\tau_2$, each mobile firm that has not moved in the first period moves in the second period with equilibrium probability $\sigma^*_{2i}$ where

$$
\sigma^*_{2i} = \begin{cases} 
0, & \text{if } \tau_2 \leq \bar{\tau}_2 \\
1, & \text{if } \tau_2 > \bar{\tau}_2
\end{cases}
\quad (1)
$$

with

$$
\bar{\tau}_2 \equiv (1 - \tau^r)c + \tau^r, \quad (2)
$$

and each mobile firm that has moved in the first period moves back to the domestic country in the second period with equilibrium probability $\tilde{\sigma}^*_{2i}$ where

$$
\tilde{\sigma}^*_{2i} = \begin{cases} 
1, & \text{if } \tau_2 \leq \tilde{\tau}_2 \\
0, & \text{if } \tau_2 > \tilde{\tau}_2
\end{cases}
\quad (3)
$$

with

$$
\tilde{\tau}_2 \equiv \frac{\tau^r - c}{1 - c}. \quad (4)
$$

**Proof:** The proof follows directly by comparing the relevant net profits taking into account the cost of moving. \(\Box\)

This gives us some first results: First, the decision of each mobile firm in the second period does not depend on the fraction of mobile firms but is entirely determined by the government’s choice of $\tau_2$. Second, the order of the tax rates is $\bar{\tau}_2 \geq \tau^r \geq \tilde{\tau}_2$ which is a consequence of the relocation cost. Third, in order to avoid a degeneration of the problem, we assume that $\tilde{\tau}_2$ cannot become negative and $\bar{\tau}_2$ is always strictly lower than unity which requires Assumption 1:

\(^7\)Note that taxes are levied on profits net of relocation cost.
Assumption 1. \( c \leq \tau^r \) and \( \tau^r \in (0, 1) \).

We now turn to stage 3, the government’s second choice of the tax rate, \( \tau_2 \in [0, 1] \). At this stage, \( \tau_1 \) and \( \mu_1 \) have been determined. Given the firms’ moving decisions in the second period the government’s revenue function is:

\[
R_2(\mu_1, \tau_2) = \begin{cases} 
\tau_2(1 - \mathcal{E}(\mu|\mu_1)) & \text{if } 1 \geq \tau_2 > \bar{\tau}_2 \\
\tau_2(1 - \mu_1) & \text{if } \bar{\tau}_2 \geq \tau_2 > \hat{\tau}_2 \\
\tau_2(1 - c\mu_1) & \text{if } \hat{\tau}_2 \geq \tau_2 \geq 0 
\end{cases}
\]  

(5)

where \( \mathcal{E}(\mu|\mu_1) \) denotes the expected value of \( \mu \) given the observed number of moving firms, \( \mu_1 \). Thus, the choice of the second period tax rate is conditional on first period firm migration for two reasons. First, observing \( \mu_1 \) allows for a better estimate of \( \mu \). Second, even if \( \mu \) is known by the government, a sufficiently large number of moving firms may lead to the choice of \( \hat{\tau}_2 \), i.e. the tax rate that attracts migrated firms back to the domestic country.

The government chooses \( \tau_2 \) in order to maximize tax revenue. It actually chooses out of \( \{\hat{\tau}_2, \bar{\tau}_2, 1\} \) because increasing \( \tau_2 \) within the brackets indicated in (5) does not change firm behavior. We assume that the government sets the lower tax rate whenever two tax rates imply the same tax revenue.

Now, consider stage 2, the firm’s decision on \( \sigma_{1i} \). For simplicity, we assume for the rest of the paper that the moving decisions of the mobile firms in the first period are symmetric, i.e. the moving probabilities of the mobile firms in the first period \( \sigma_{1i} \) are equal, which is stated in Assumption 2:

Assumption 2. \( \sigma_{1i} \equiv \sigma_1 \) \( \forall i \).

By deciding on \( \sigma_1 \), firms take the government’s second period behavior into account although each individual firm acts as a price-taker, i.e. does not assume that its individual behavior changes any other agent’s behavior. However, each firm anticipates that if the number of moving firms is large enough, the government may have an incentive to set \( \hat{\tau}_2 \) in the second period. Let \( p_{\hat{\tau}_2} \) denote the probability that the government sets \( \hat{\tau}_2 \) in the second period. The representative firm maximizes expected profits by choosing \( \sigma_1 \), i.e. it solves

\[
\max_{\sigma_1 \in [0,1]} (1 - \sigma_1) \left[ 1 - \tau_1 + \delta \left[ p_{\hat{\tau}_2} (1 - \hat{\tau}_2) + (1 - p_{\hat{\tau}_2}) (1 - \bar{\tau}_2) \right] \right] \\
+ \sigma_1 \left[ (1 - \tau^r) (1 - c) + \delta (1 - \tau^r) \right].
\]  

(6)

where \( \delta \) denotes the time preference parameter.

Note that, if the firm has stayed in the first period, the second period payoff is \( 1 - \hat{\tau}_2 \) independent of the firm’s second period location decision since \( 1 - \hat{\tau}_2 = (1 - \tau^r) (1 - c) \). Similarly, if the firm has moved in the
first period, the second period payoff does not depend on the second period location decision because 
\((1 - \tau') = (1 - \tilde{\tau}_2)(1 - c)\). Further note that each individual firm takes \(p_{\tilde{\tau}_2}\) as given while \(p_{\tilde{\tau}_2}\) depends on the aggregate choice of \(\sigma_1\).

Finally, at stage 1, the government decides which tax rate \(\tau_1\) to set in the first period. It takes firm behavior in the first and second period into account and anticipates that it can adjust its policy after observing the actual \(\mu\).

In the following, we consider two scenarios. As a benchmark scenario, we analyze the case of complete information where the mobility parameter \(\mu\) is common knowledge. Then, we assume that there is incomplete information on \(\mu\), i.e. neither the government nor the firms know \(\mu\) (of course, each mobile firm knows whether it is mobile or not but the aggregate number of mobile firms is unknown). Finally, we compare the equilibria derived under both scenarios.

### 2.1 Complete information

In this section, we analyze the subgame perfect equilibria for the case of complete information, i.e. the domestic government and all firms know \(\mu\).

Stage 4 is summarized in Lemma 1. We can therefore directly turn to stage 3. Here, complete information implies \(E(\mu|\mu_1) = \mu\). With \(\mu_1 = \sigma_1\mu\), it follows from (5) that the government chooses an optimal second period tax rate \(\tau_2^*\) according to Lemma 2.

**Lemma 2.** (i) If no mobile firm has moved in the first period, i.e. \(\sigma_1 = 0\), government sets \(\tau_2^* = \tilde{\tau}_2\) iff \(\mu \geq 1 - \tilde{\tau}_2\) and \(\tau_2^* = 1\) otherwise.

(ii) If at least some firms have moved in the first period, i.e. \(\sigma_1 > 0\), government sets

\[
\tau_2^* = \begin{cases} 
\tilde{\tau}_2 & \text{if } \mu_1 \geq \tilde{\mu}_1 \\
\tilde{\tau}_2 & \text{if } \mu_1 < \tilde{\mu}_1 \text{ and } \mu_1 \geq \frac{\sigma_1(1-\tilde{\tau}_2)}{1-\sigma_1\tilde{\tau}_2} \\
1 & \text{otherwise}
\end{cases}
\]  

(7)

with

\[
\tilde{\mu}_1(\sigma_1) \equiv \max \left\{ \frac{\sigma_1(1-\tilde{\tau}_2)}{1-\tau_2c\sigma_1}; \frac{\tilde{\tau}_2-\tilde{\tau}_2}{\tilde{\tau}_2-\sigma_1\tilde{\tau}_2} \right\}.
\]  

(8)

---

8There are, of course, scenarios with asymmetrically incomplete information. For instance, we might consider the case in which the government knows \(\mu\), but the firms do not. It is straightforward to show that the results do not differ from the benchmark scenario. Alternatively, we might assume that the firms do know \(\mu\), but the government does not. We do not consider this case here because we believe it to lack empirical plausibility. The government can always pay a firm to reveal the information.
Proof: The proof follows directly by comparing tax revenues as given in equation (5) applying \( \mu \sigma_1 = \mu_1 \).

For later use, it is helpful to deal with some properties of the above defined threshold \( \bar{\mu}_1(\sigma_1) \). First note that \( \bar{\mu}_1(\sigma_1) \) cannot fall below

\[
\bar{\mu}_1^{\text{min}} = \frac{\bar{\tau}_2 - \tilde{\tau}_2}{\tilde{\tau}_2 - \bar{e}\tilde{\tau}_2}
\]

as the first argument on the right hand side of (8), \( \sigma_1(1 - \bar{\tau}_2) \), monotonically increases in \( \sigma_1 \). Secondly, the largest level of \( \sigma_1 \) that ensures \( \bar{\mu}_1^{\text{min}} \) is given by \( \sigma_1^{\text{min}} = \frac{\bar{\tau}_2 - \tilde{\tau}_2}{\tau_2 - \bar{\tau}_2 + \bar{\tau}_2(1 - \bar{\tau}_2)(1 - c)} \) (the level of \( \sigma_1 \) equating the two arguments in (8)). If \( \sigma_1^{\text{min}} < \bar{\mu}_1^{\text{min}} \), firms in aggregate cannot reach the threshold by a further increase in \( \sigma_1 \).

Now, turn to stage 2 where firms decide on the optimal moving behavior in the first period, \( \sigma_1^* \), solving equation (6). To start with, assume that \( \mu \) is large enough such that firms can ensure \( p_{\bar{\tau}_2} = 1 \) given an adequate moving probability. If all firms stay, i.e. \( \sigma_1^* = 0 \), they anticipate that the government will never have an incentive to set \( \bar{\tau}_2 \), i.e. \( p_{\bar{\tau}_2} = 0 \). Therefore expected payoffs are given by \( 1 - \tau_1 + \delta (1 - \tilde{\tau}_2) \). Each individual firm now compares the profit from staying with the profit from leaving and, depending on the level of \( \tau_1 \) chooses the location which maximizes its payoff. If all firms leave, i.e. \( \sigma_1^* = 1 \), expected payoffs are \( (1 - \tau^*) (1 - c) + \delta (1 - \tau^*) \) which is compared to domestic profit with \( p_{\bar{\tau}_2} = 1 \). As a third alternative, firms can choose \( \sigma_1^* = \bar{\mu}_1^{\text{min}} / \mu \) and, thus, trigger a second period tax rate of \( \tilde{\tau}_2 \). Then, expected payoffs are

\[
(1 - \sigma_1^*) [1 - \tau_1 + \delta (1 - \tilde{\tau}_2)] + \sigma_1^* [(1 - \tau^*) (1 - c) + \delta (1 - \tau^*)] \tag{9}
\]

which requires, though, that the two terms in square brackets, i.e. the payoff from staying and leaving be equal. Thus, firms choose\(^9\)

\[
\sigma_1^* = \begin{cases} 
0 & \text{if } \tau_1 \leq \bar{\tau}_1 \equiv \tilde{\tau}_2 - \delta [\tilde{\tau}_2 - \tau^*] \\
1 & \text{if } \tau_1 > \bar{\tau}_1 \equiv \tilde{\tau}_2 + \delta [\tau^* - \tilde{\tau}_2] \\
\frac{\bar{\mu}_1^{\text{min}}}{\mu} & \text{if } \tau_1 = \tilde{\tau}_1
\end{cases} \tag{10}
\]

For our purpose, it is useful to restrict the analysis to cases in which \( \bar{\tau}_1 \leq 1 \), which requires that the cost of relocation is sufficiently low.\(^{10}\) Precisely,

---

\(^9\)For \( \tau_1 \in (\bar{\tau}_1, \tau_1) \) there cannot be a symmetric equilibrium. The reason is that, if \( \tau_2 = \tilde{\tau}_2 \), all firms strictly prefer to stay (which makes \( \tau_2 = \tilde{\tau}_2 \) impossible). If \( \tau_2 > \tilde{\tau}_2 \), all firms prefer to leave which triggers \( \tau_2 = \tilde{\tau}_2 \). Note that choosing a mixed strategy does not yield a symmetric equilibrium.

\(^{10}\)For purposes of illustration, assume for a moment that \( \bar{\tau}_1 > 1 \). Then, if the government sets the first period tax rate at its largest level, \( \tau_1 = 1 \), each individual firm has the incentive to stay, given that all firms leave (and trigger \( \tilde{\tau}_2 \)). However, if all firms stay, \( p_{\bar{\tau}_2} = 0 \). Since firms cannot coordinate (by assumption), there is no symmetric equilibrium.
the level of relocation cost must not exceed a certain threshold, denoted by $c$, which is given by $\bar{c} = (\delta + 2)/(2) - \sqrt{(\delta^2 + 4\delta)/4}$. With $\delta \in [0, 1]$, $\bar{c}$ has a range of $\bar{c} \in [0.38, 1]$. The intuition is that, if the relocation cost is too large, the government’s strategy space is effectively reduced since, for instance, forcing firms to move back and forth is just too expensive as a policy choice.

If $\mu$ is small, i.e. $\mu < \tilde{\mu}_{\text{min}}^1 / \sigma_{\text{min}}^1$, probability $p_{\tau_2}$ is zero. Then, choosing a mixed strategy like in (9) is not a rational choice. Accordingly, firms choose out of $\sigma^*_1 \in \{0, 1\}$ depending on whether $\tau_1$ is below or above $\Sigma_1$. For $\tilde{\mu}_{\text{min}}^1 / \sigma_{\text{min}}^1 \leq \mu < \tilde{\mu}_1 (1)$, firms may actually force the government to choose $\tau_2$. However, if they all choose $\sigma_1 = 1$, government will not choose $\tilde{\tau}_2$.\textsuperscript{11}

At stage 2, consequently, firms choose $\sigma^*_1$ out of $\{0, \tilde{\mu}_{\text{min}}^1 / \mu, 1\}$ if there are enough mobile firms to ensure $\tau_2 = \tilde{\tau}_2$ and out of $\{0, 1\}$ otherwise.

Accordingly, at stage 1, government chooses the optimal tax rate $\tau^*_1$ out of $\{\Sigma_1, 1\}$ if $\mu < \tilde{\mu}_{\text{min}}^1 / \sigma_{\text{min}}^1$ and out of $\{\Sigma_1, \tilde{\tau}_1, 1\}$ otherwise. With $R(\tau_1, \tau_2)$ denoting the tax revenue as a function of tax rates the government’s optimization problem in the first period is given by

$$\max_{\tau_1} R(\tau_1, \tau_2) \quad \text{s.t. Lemma 2}.$$  

We can now state the following Proposition 1.

**Proposition 1.** Given the firms’ second period strategy as described in Lemma 1, for every set up of tax rate $\tau^r$ in the rest of the world, relocation costs $c$ and time preference $\delta$ there is a unique subgame perfect equilibrium:

- All firms stay\textsuperscript{12}: $\sigma^*_1 = 0$ and $(\tau^*_1, \tau^*_2) = (\Sigma_1, \tilde{\tau}_2)$ if $\mu \geq (1 - \tau^r) \left(1 - \frac{c}{1+\delta}\right)$.

- All firms move: $\sigma^*_1 = 1$ and $(\tau^*_1, \tau^*_2) = (1, 1)$ otherwise.

Firms’ optimal behavior in period 2 is described in Lemma 1.

**Proof:** See the Appendix. \hfill $\square$

The intuition behind the above proposition is as follows. Under complete information, the government has no need to learn from firm migration. Thus, forcing firms to move back and forth cannot be an equilibrium strategy. For this reason there are only two possible equilibria: A first one, in which all firms stay in both periods, and a second one, in which all mobile

\textsuperscript{11}Here, if $\tau_1 \in (\Sigma_1, \tilde{\tau}_1)$, firms choose $\sigma_1 = 1$ although choosing $\sigma_1 = \tilde{\mu}_{\text{min}}^1 / \mu$ would make them better off as a group. However, due to a lack of coordination capacity the symmetric choice of $\tilde{\mu}_{\text{min}}^1 / \mu$ is not feasible.

\textsuperscript{12}Note that, since $(1 - \tau^r) \left(1 - \frac{c}{1+\delta}\right) > 1 - \tilde{\tau}_2$, the government’s strategy to set $\tilde{\tau}_2$ in the second period is subgame perfect.
firms leave and then stay abroad. Which of these two equilibria is optimal depends on the actual number of mobile firms. The Appendix gives a more detailed and formal proof of the proposition.

Figure 1 illustrates the above proposition for the case of $\mu = 0.5$ and $\delta = 0.9$ which requires $c \leq \bar{c} = 0.4$ (i.e. we rule out the parameter space on the right hand side of the dashed line). The shaded area (low foreign tax rates and low relocation cost) represents equilibria where all firms leave, whereas the white area above the angle bisector (high foreign tax rates and high relocation cost) shows equilibria where all firms stay. The higher the preference for the future, $\delta$, the lower the incidence of the all firms stay equilibrium.

2.2 Incomplete information

In this section we modify the above presented model by assuming that the actual number of mobile firms, $\mu$, is unknown to all agents. Thus, both the government and the firms can only infer an estimation of $\mu$ from the actual number of moving firms in the first period, $\mu_1$. Due to this change in the information structure of the game, the concept of subgame perfect equilibria used above becomes insufficient. Instead, we apply the concept of Bayesian Equilibrium. Strictly speaking we have to define the government’s and the
firms’ expectations on the actual number of mobile firms for the first and second period decisions. However, since the government’s and the firms’ expectations in the first period are always given by the unconditioned expected value, $\mathbf{E}(\mu)$, and the firms’ second period decisions do not depend on the expected number of mobile firms, we only need to describe the government’s expectations in period 2.

At stage 4, the second period tax rate is known and Lemma 1 applies. We may therefore directly turn to stage 3. Since firms do not know $\mu$ either, the government may take the perspective of an individual firm and induce the level of $\sigma_1$. As a consequence, observing $\mu_1 = 0$ may mean that $\sigma_1 = 0$ or $\mu = 0$. Therefore, in (5), $\mathbf{E}(\mu|\mu_1) = \mu_1/\sigma_1$ if $\sigma_1 > 0$ and $\mathbf{E}(\mu|\mu_1) = \mathbf{E}(\mu)$ if $\sigma_1 = 0$.

Lemma 3 summarizes the government’s tax setting behavior in the second period.

**Lemma 3.** (i) If no mobile firm has moved in the first period, i.e. $\sigma_1 = 0$, the government sets a second period tax rate $\bar{\tau}_2$ iff $\mathbf{E}(\mu) \geq 1 - \bar{\tau}_2$ and $\tau_2^* = 1$ otherwise.

(ii) If at least some mobile firms have moved in the first period, part (ii) of Lemma 2 applies.

**Proof:** See proof of Lemma 2. 

Now turn to stage 2 where firms decide on their relocation probability $\sigma_1$. As in the case of complete information, the payoff is $1 - \tau_1 + \delta (1 - \bar{\tau}_2)$ if all firms stay, and $(1 - \tau^r) (1 - c) + \delta (1 - \tau^r)$ if all firms leave, see (6). Again, firms may opt for a mixed strategy in order to trigger $\tilde{\tau}_2$ in the second period, however this time without knowing the actual number of mobile firms, $\mu$. Then, expected payoffs are

$$
(1 - \tilde{\sigma}_1^* \left[1 - \tau_1 + \delta \left[p^{p}_{\tilde{\tau}_2} (1 - \tilde{\tau}_2) + (1 - p^{p}_{\tilde{\tau}_2}) (1 - \bar{\tau}_2) \right]\right] + \tilde{\sigma}_1^* \left[(1 - \tau^r) (1 - c) + \delta (1 - \tau^r)\right])
$$

(11)

where $\tilde{\sigma}_1^*$ denotes the choice of $\sigma_1$ under the mixed strategy and $p^{p}_{\tilde{\tau}_2}$ the probability that the government chooses $\tilde{\tau}_2$ (superscript $p$ denotes the scenario with incomplete information). Again, $p^{p}_{\tilde{\tau}_2}$ depends on the aggregate choice of $\sigma_1$. For (11) to describe a mixed strategy payoff, the payoffs of staying and leaving (the two terms in square brackets) have to be equal. $\tilde{\sigma}_1^*$ equates the two payoffs by adjusting $p^{p}_{\tilde{\tau}_2}$, with $p^{p}_{\tilde{\tau}_2} = 1 - F (\tilde{\mu}_1/\sigma_1)$. $p^{p}_{\tilde{\tau}_2}$ reaches a minimum at $\sigma_1 = \tilde{\sigma}_1^{\min}$ implying $p^{p}_{\tilde{\tau}_2} (\tilde{\mu}_1^{\min}) = 0$ and a maximum, denoted by $\tilde{p}^{p}_{\tilde{\tau}_2}$, at $\sigma_1 = \sigma_1^{\min}$ yielding $\tilde{p}^{p}_{\tilde{\tau}_2} = 1 - F (\tilde{\mu}_1^{\min}/\sigma_1^{\min})$ (recall that $\tilde{\mu}_1$ rises in $\sigma_1$).

Note that, if all firms leave, $\sigma_1 = 1$, the probability is given by $p^{p}_{\tilde{\tau}_2} (1) < \tilde{p}^{p}_{\tilde{\tau}_2}$. 

12
The equilibrium moving probability in the first period, \( \sigma_1^* \), depends on the first period tax rate choice. Thus, firms choose

\[
\sigma_1^* = \begin{cases} 
0 & \text{if } \tau_1 \leq \underline{\tau}_1 \\
1 & \text{if } \tau_1 > \bar{\tau}_1 - \delta [\bar{\tau}_2 - \bar{\tau}] (1 - \bar{p}_{\bar{\tau}_2}) \equiv \bar{\tau}_1 \equiv \bar{\tau}_1 \quad (12)
\end{cases}
\]

with \( \underline{\tau}_1 \) and \( \bar{\tau}_1 \) defined in (10). Again, we assume that \( c \) is below a certain threshold, \( \hat{c} \) (equivalently defined as \( c \)) ensuring that \( \bar{\tau}_1 \leq 1 \). If the government chooses a first period tax rate out of the interval \((\underline{\tau}_1, \bar{\tau}_1]\), firms adjust their relocation probability such that they are indifferent between staying and moving.\(^{13}\)

Now, consider stage 1 where the government decides on the optimal first period tax rate \( \tau_1^* \). If \( \tau_1^* = 1 \), the second period tax rate equals, according to Lemma 3, \( \tau_2^* = \tau_2 \) if \( \mu \geq \frac{1 - \bar{\tau}}{\bar{\tau} - \bar{\tau}_2} \) and \( \tau_2^* = 1 \) otherwise. (Recall that firm migration at stage 2 fully reveals the true value of \( \mu \).) Expected tax revenue is then given by

\[
R'(1, \tau_2^*) = 1 - \mathbf{E}(\mu) + \delta(1 - p_{\bar{\tau}_2})(1 - \mathbf{E}_1'(\mu|1)) \\
+ \delta p_{\bar{\tau}_2} \bar{\tau}_2 (1 - c\mathbf{E}_2'(\mu|1)).
\]

where \( \mathbf{E}_1'(\mu|1) = \mathbf{E}(\mu|\sigma_1^* = 1, \tau_2 = 1) \) and \( \mathbf{E}_2'(\mu|1) = \mathbf{E}(\mu|\sigma_1^* = 1, \tau_2 = \bar{\tau}_2) \), respectively, denote the expected number of mobile firms when firms choose \( \sigma_1^* = 1 \) and optimal second period tax rates are \( \tau_2^* = 1 \) and \( \bar{\tau}_2 \), respectively.

If no firm leaves, tax revenue is given by

\[
R'(\underline{\tau}_1, \tau_2) = \underline{\tau}_1 + \delta \bar{\tau}_2 = \bar{\tau}_2 + \delta \tau'.
\]

Note that such a policy is only feasible if \( \mathbf{E}(\mu) > 1 - \bar{\tau}_2 \). Otherwise, government cannot commit to \( \bar{\tau}_2 \) in the second period.

In the mixed strategy equilibrium, the government chooses an optimal tax rate \( \tau_1^* \) out of \((\underline{\tau}_1, \bar{\tau}_1]\). Using \( \bar{\sigma}_1(\tau_1^*) \leq \sigma_1^{\text{min}} \), tax revenue is then given by

\[
R'(\tau_1^*, \tau_2) = \tau_1^*(1 - \mathbf{E}(\mu|\bar{\sigma}_1^*) + \delta(1 - p_{\bar{\tau}_2} - p_{\bar{\tau}_2}) (1 - \mathbf{E}_1'(\mu|1)) \\
+ \delta p_{\bar{\tau}_2} \bar{\tau}_2 (1 - \bar{\sigma}_1^* \mathbf{E}_1'(\mu|\bar{\sigma}_1^*)) + \delta p_{\bar{\tau}_2} \bar{\tau}_2 (1 - c\bar{\sigma}_1^* \mathbf{E}_2'(\mu|\bar{\sigma}_1^*))
\]

where \( \mathbf{E}_1'(\mu|\bar{\sigma}_1^*) \equiv \mathbf{E}(\mu|\sigma_1^* = \bar{\sigma}_1, \tau_2 = 1) \), \( \mathbf{E}_2'(\mu|\bar{\sigma}_1^*) \equiv \mathbf{E}(\mu|\sigma_1^* = \bar{\sigma}_1, \tau_2 = \bar{\tau}_2) \), and \( \mathbf{E}_2'(\mu|\bar{\sigma}_1^*) \equiv \mathbf{E}(\mu|\sigma_1^* = \bar{\sigma}_1, \tau_2 = \bar{\tau}_2) \). \( p_{\bar{\tau}_2} \) denotes the probability that the government chooses \( \bar{\tau}_2 \) in period 2. Of course, both probabilities

\(^{13}\)Precisely, a small increase of \( \tau_1 \) with \( \tau_1 \in (\underline{\tau}_1, \bar{\tau}_1] \) leads to a change in \( \bar{\sigma}_1 \) of \( \frac{d\bar{\sigma}_1}{d\tau_1} = \left[ \frac{\bar{\tau}_2 - \bar{\tau}_2}{\delta (\bar{\tau}_2 - \bar{\tau})} \right]^{-1} \) with \( \frac{d\bar{\sigma}_1}{d\tau_1} = -f \frac{d\bar{\sigma}_1}{d\tau_1} = f \left( \frac{1}{\bar{\tau}_1} \right) \frac{\tau_2 - \bar{\tau}_2}{\bar{\tau}_2 - \bar{\tau}_2} \).
depend on the equilibrium moving decision $\bar{\tau}_1$. In the case of $\bar{\sigma}_1(\bar{\tau}_1)$, $p_{\bar{\tau}}$ equals zero.

At stage 1, government solves

$$\max_{\tau_1} R^t(\tau_1, \tau_2) \quad \text{s.t. Lemma 3}$$

where the relevant tax revenues are given in (13)-(15).

Proposition 2 summarizes the three different types of Bayesian Equilibria that may occur.

**Proposition 2.** Assume that $\mu$ is unknown to both firms and the government. For each environment $(\delta, c, \tau^r, F(\mu))$ there exists a unique symmetric Bayesian Equilibrium which refers to one of the three following types:

1. All firms stay, i.e. $\sigma^*_1 = 0$ with tax rate choices $\tau^*_1 = \tau_1$ and $\tau^*_2 = \bar{\tau}_2$. Government’s expectations are $E(\mu|\mu_1) = E(\mu)$.

2. All firms move, i.e. $\sigma^*_1 = 1$ with tax rate choices $\tau^*_1 = 1$ and $\tau^*_2 \in \{\bar{\tau}_2, 1\}$, depending on the first period migration $\mu_1$. Government’s expectations are $E(\mu|\mu_1) = \mu_1$.

3. Some firms move, i.e. $\sigma^*_1(\tau^*_1) \in [\bar{\mu}^{\min}_1, \bar{\sigma}^{\min}_1]$ with tax rate choices $\tau^*_1 \in (\tau_1, \bar{\tau}_1]$ and $\tau^*_2 \in \{\bar{\tau}_2, 1\}$, depending on the first period migration $\mu_1$. Government’s expectations are $E(\mu|\mu_1) = \bar{\mu}_1$.

The optimal actions in period 2 are described in Lemma 1 and Lemma 3.

Proof: Existence and uniqueness of the symmetric Bayesian Equilibria directly result from the fact that, for both the government and the firms, there is a unique best response at each of the four stages. Furthermore, there are no other optimal government strategies than those indicated in the proposition by the following argument: As in the case of full information, $(\tau_1, \bar{\tau}_2)$ and $(1, \bar{\tau}_2)$ cannot be optimal for reasons explained in the proof of Proposition 1 (see appendix). Strategy $(\tau_1, 1)$ can never be optimal by the following argument: $R^t(\tau_1, 1) = \tau_1 + \delta(1 - E(\mu)) > R^t(\tau_1, \bar{\tau}_2)$ iff $E(\mu) < (1 - \bar{\tau}_2)$ and $R^t(\tau_1, 1) > R^t(1, 1)$ iff $E(\mu) > 1 - \tau_1 + \delta p_{\bar{\tau}_2} (1 - c\bar{\tau}_2) \left[ E_{\bar{\tau}_2}(\mu|1) - \bar{\mu}_1(1) \right]$. As $\tau_1 - \delta p_{\bar{\tau}_2} (1 - c\bar{\tau}_2) \left[ E_{\bar{\tau}_2}(\mu|1) - \bar{\mu}_1(1) \right] < \bar{\tau}_2$ the strategy $(\tau_1, 1)$ is revenue dominated by either $(\tau_1, \bar{\tau}_2)$ or $(1, 1)$. □

Figure 2 illustrates the results of Proposition 2 when $\mu$ is uniformly distributed with support $[0, 1]$ and $\delta = 0.9$. The critical migration cost that ensures $\bar{\tau}_1 \leq 1$, $\bar{c}_t$, depends on the foreign tax rate. The parameter space above the dashed line implies $\bar{\tau}_1 > 1$ and is therefore ruled out. Like in the case of complete information, the all firms move equilibrium (shaded area) occurs for low foreign tax rates and low migration cost and the all firms stay
equilibrium (white area above the angle bisector) for high foreign tax rates and high migration cost. As a novelty, the government finds it optimal to trigger the mixed strategy equilibrium (grey area) for medium foreign tax and low migration cost. The higher the preference for the future, the larger the range of parameters where the mixed strategy equilibrium occurs.

In the above described setting, the government cannot commit to tax rates or policy rules. The absence of commitment enables the government to learn from firm reactions to past policy choices. This raises the question whether the government would like to trade the opportunity to learn against the ability to commit to tax rates in both periods.

**Proposition 3.** (Value of learning) *If information on firm mobility is incomplete, the opportunity to learn (i.e. to adjust policy in the second period) unambiguously increases welfare.*

**Proof:** For a proof, consider the case that the government has to fix both tax rates in the first period, i.e. we assume that it can (and it has to) commit to a second period tax rate.\(^{14}\) Then, learning is not possible. As is shown in the proof of Proposition 1, it is never optimal to set \(\tilde{\tau}_2\) in

\[ \tau_1 = \tau^r + \frac{1}{1 + \delta} (1 - \tau^r). \]

However, in expected terms, the value of tax revenue is the same as setting \(\sum_1\) in the first period and \(\tilde{\tau}_2\) in the second:

\[ \tau_1 + \delta \tilde{\tau}_2 = \tau^r (1 + \delta) + c (1 - \tau^r). \]

---

\(^{14}\)A stronger assumption would be to force the government to commit to a single tax rate in both periods. Then, the all firms stay equilibrium would imply to set a first period tax rate that makes firms indifferent between moving and staying: \(\tau_1 = \tau^r + \frac{1}{1 + \delta} (1 - \tau^r).\) However, in expected terms, the value of tax revenue is the same as setting \(\sum_1\) in the first period and \(\tilde{\tau}_2\) in the second: \(\tau_1 + \delta \tilde{\tau}_2 = \tau^r (1 + \delta) + c (1 - \tau^r).\)
period 2 with certainty. Thus the government effectively chooses between the all firms stay and the all firms leave equilibrium. In the all firms stay equilibrium, the government will choose a pair of tax rates that makes firms indifferent between moving and staying from the viewpoint of period 1 and at least indifferent from the viewpoint of period 2, i.e. \( \tau_2 \leq \bar{\tau}_2 \). Since the discount rate is assumed to be equal for the government and the firms, the expected value of tax revenue cannot exceed \( \tau_1 + \delta \bar{\tau}_2 \). In the all firms leave equilibrium, the largest possible tax revenue – given that firms cannot be attracted back – can be achieved by setting \((1, 1)\). Since \((\bar{\tau}_1, \bar{\tau}_2)\) and \((1, 1)\) are part of the government’s strategy space when learning is allowed for, it follows that tax revenue is at least as large as under the scenario with learning. However, in the all firms leave equilibrium, the government may increase its revenue by adjusting its tax rate to \(\bar{\tau}_2\) if first period migration is large enough. Given that firms have equal profits in all equilibria, see Corollary 3 below, we can state the following: Whenever the government chooses the some firms move equilibrium or adjusts the second period tax rate to \(\bar{\tau}_2\) in the all firms leave equilibrium, tax revenue and, thus, welfare is larger than in the case where the two tax rates have to be determined in period 1. Otherwise, tax revenue and welfare are equal.

The reason why commitment on tax rates does not improve welfare is that there is no hold-up problem; firms can always react on the government’s tax rate choices. Commitment to tax rates only reduces the number of available instruments and, thus, the level of attainable welfare.

We can show, however, that the commitment to a learning rule can improve welfare.

**Proposition 4.** (Rule based learning) *In the mixed strategy equilibrium, a credible commitment to a rule that links the number of observable moving firms to a second period tax rate can be a Pareto improvement.*

**Proof:** Assume the government commits to a lower threshold at which it opts for \(\bar{\tau}_2\), i.e. it chooses \(\bar{\tau}_2\) for some \(\mu_1 < \bar{\mu}_1\). Everything else held constant, this increases the probability of \(\bar{\tau}_2\) in the second period and, thus, the value of staying. Firms will therefore lower their moving probability \(\hat{\sigma}_{i1}\) until \(p_{i2} = \bar{p}_{i2}\) reaches its initial level. Thus, a decrease in the threshold only reduces the moving probability. Firms are as well off as in the scenario without rule based learning, but the government’s ability to adhere to a learning rule increases its first period tax base (as well as its second period tax base) and, thus, tax revenue.

Note that announcing a lower threshold does not prevent the government from learning the true level of \(\mu\) by observing the number of moving firms, \(\mu_1\).
2.3 Comparison

In this subsection, we compare different aspects of the equilibria derived above and establish a couple of corollaries.

To start with, we analyze the role of information for tax policy. As discussed in the introduction, most studies on optimal taxation of mobile firms (implicitly) assume that it suffices that the government has correct expectations of tax base mobility. Our model, however, shows that this assumption has to be qualified. Under complete information, government chooses the strategy \((\tau_1, \bar{\tau}_2)\) whenever \(\mu \geq (1 - \tau^r) \left(1 - c/(1 + \delta)\right)\). Under incomplete information, the government prefers \((\tau_1, \bar{\tau}_2)\) over \((1, \tau^*_2)\) if \(R^i(\tau_1, \bar{\tau}_2) \geq R^i(1, \tau^*_2)\), i.e. if

\[
E(\mu) > (1 - \tau^r) \left(1 - \frac{c}{1 + \delta}\right) + \frac{\delta}{1 + \delta} p_{\bar{\tau}_2} (1 - c\bar{\tau}_2) \left(E_{\bar{\tau}_2}(\mu|1) - \hat{\mu}_1 (1)\right) \tag{16}
\]

with \(E_{\bar{\tau}_2}(\mu|1) - \hat{\mu}_1 (1) \geq 0\). With \(p_{\bar{\tau}_2} \geq 0\), it follows that the right hand side of (16) cannot fall short of \((1 - \tau^r) \left(1 - c/(1 + \delta)\right)\). Thus, even if the government correctly estimates the degree of mobility, \(E(\mu) = \mu\), the policy under incomplete information may substantially differ from the one under complete information. Precisely, if, under complete information, the government chooses \((\tau_1, \bar{\tau}_2)\), the optimal policy under incomplete information may be \((1, \tau^*_2)\).

For a systematic evaluation of the role of information for tax policy, we compare the situations under complete and incomplete information from the viewpoint of before stage 0 where nature draws \(\mu\). We can now state Corollary 1.

**Corollary 1.** (First period tax rates) If the foreign tax rate, \(\tau^r\), and the cost of relocation, \(c\), are low (i.e., if tax competition is fierce), the expected first period tax rate (across all \(\mu\)) under incomplete information is higher than under complete information.

**Proof:** Under complete information, there is, due to Assumption 1, always some \(\mu\) at which the government chooses \(\tau^*_1 = \bar{\tau}_1\) and always some \(\mu\) at which it chooses \(\tau^*_1 = 1\). Thus, the expected first period tax rate is an element of \((\bar{\tau}_1, 1)\). Under incomplete information, the government does not know \(\mu\) and, therefore, cannot condition its first period tax rate choice on \(\mu\). As a consequence, the expected first period tax rate is an element of \({\bar{\tau}_1, 1}\) where \(\bar{\tau}_1 \in (\bar{\tau}_1, \bar{\tau}^*_1]\). Thus, the first period tax rate under incomplete information is lower (and migration is lower) if \(\tau^*_1 = \bar{\tau}_1\), and it is higher if \(\tau^*_1 = 1\). It remains to show that, under incomplete information,

\[^{15}\]If choosing \((\bar{\tau}_1, \tau^*_2)\) dominates some of the two other strategies, the argument still holds since \(\bar{\tau}_1 > \tau_1\).
the government chooses 1 if $\tau^r$ and $c$ are low. Tax revenues in the all firms stay equilibrium and in the mixed strategy equilibrium tends to zero when $\tau^r$ (and, thus, also $c$) approaches zero. Expected tax revenue in the all firms leave equilibrium is positive, though. Thus, if tax competition is fierce, the government will prefer the all firms leave equilibrium.

It follows Corollary 2.

**Corollary 2. (First period migration)** If $\tau^r$ and $c$ are low (i.e., if tax competition is fierce), expected first period migration is higher under incomplete information.

*Proof:* The proof of Corollary 2 follows directly from the proof of Corollary 1.

It is worthwhile to compare firm profits under both scenarios. This is done in Corollary 3.

**Corollary 3. (Firm profits)** In equilibrium, firm profits (in expected terms) are equal under both complete and incomplete information.

*Proof:* In the all firms stay and the all firms move equilibrium, firms have after-tax profits equivalent to those when they move abroad. This is trivially true in the all firms leave equilibrium. In the all firms stay equilibrium, the government sets first period tax rates such that firms are indifferent between staying and moving. In the mixed strategy equilibrium, after-tax profits from staying and leaving have to be equal for firms to be indifferent. Since the after-tax profit abroad is independent of the scenario, the profit if the firm stays is, too.

Now, consider the question in which scenario tax revenue is largest. The answer is stated in Corollary 4.

**Corollary 4. (Expected tax revenue)** Expected tax revenue is lower if information on mobility is incomplete.

*Proof:* The above corollary is a consequence of the fact that the government does not make mistakes if information is complete, i.e. it never makes firms move and attracts them back. If, under incomplete information, the government chooses $(\tau_1, \bar{\tau}_2)$, tax revenues are equal to or lower than revenues under complete information. Proposition 1 demonstrates that triggering a mixed strategy is always revenue dominated by either $(\tau_1, \bar{\tau}_2)$ or $(1, 1)$. Finally, if the government chooses $(1, \tau^*_2)$, the revenue is equal in both scenarios if $\tau^*_2 = 1$ and lower if $\tau^*_2 = \bar{\tau}_2$. Thus, in expected terms, expected revenue is lower if information is incomplete.
3 Discussion

In this section we critically discuss aspects of the above analysis and explore the implications and the boundaries of our results.

**Time horizon** Our model ends after two periods. Real world tax policy, however, presumably has a longer time horizon. With more than two periods, the value of learning becomes larger and the equilibrium where all firms stay (and the government foregoes the opportunity of learning) becomes less attractive. In the other two equilibria, all information is revealed after period 2 and a stationary state is reached – as long as the level of mobility is constant over time.

If the mobility parameter changes over time, the game described in Section 2 is repeated. If current mobility contains information on future mobility, this may render the analysis more complex. However, as a rough tendency, this assumption would again increase the value of learning and, thus, decrease the attractiveness of the all firms stay equilibrium.

**Foreign firms** In the above model, the domestic government only deals with domestic firms. In principle, it may try to attract foreign firms as well. We have abstracted from this possibility although integrating foreign firms is straightforward. Assuming that foreign firms and domestic firms are equal with regard to pre-tax profits and mobility cost, the domestic government will consider setting a first period tax rate \( \tau_1 = \tilde{\tau}_2 \) in order to attract an expected number of foreign firms. Then, it does not learn about the mobility of domestic firms and has to base the decision on \( \tau_2 \) on unconditional estimates of domestic mobility, \( \mu \).

Thus, learning takes place either with domestic firms or with foreign firms. Since learning about mobility is in the centre focus of this paper, adding foreign firms simply extends the model without adding new aspects to the problem of learning. Of course, if the first period tax rate exceeds \( \tau_1 \) and – at least partial – migration of domestic firms is triggered, setting \( \tilde{\tau}_2 \) in the second period becomes more attractive in the presence of foreign firms.

**Coordination among firms** The assumption of an infinitely large number of firms implies that each firm acts as a price-taker and firms cannot coordinate. Allowing for the coordination of firms among themselves fundamentally changes the game, and an intensive analysis is beyond the scope of this paper. Although we do not think that such a scenario is of great relevance for real world tax policy, it is nevertheless worthwhile to outline potential implications.

Consider first the all firms stay equilibrium where, without coordination, firms are indifferent between staying and moving but weakly prefer to stay.
With coordination, firms may agree upon an arrangement where some firms move and, thus, trigger a second period tax rate $\tilde{\tau}_2$. Then, the moving firms are as well off as before and the staying firms are better off.

A similar effect occurs in the mixed strategy equilibrium under incomplete information whenever $\tau_1 < \tilde{\tau}_1$. Again, firms may coordinate upon some of them moving and, thus, increasing the probability of having $\tilde{\tau}_2$ in the second period. Again, the moving firms are as well off as before and the staying firms are better off.

In the mixed strategy equilibrium with $\tau_1 = \tilde{\tau}_1$ and in the all firms leave equilibrium, coordination among firms cannot be used for the firms’ advantage.

**Maximum revenue losses** Incomplete information on $\mu$ implies that the government has to make decisions based on expected values given the distribution of $\mu$, summarized in $F(\mu)$. From a policy perspective, it may be important how large the potential losses are if actual levels of $\mu$ deviate from expected values. The maximum loss in the all firms stay equilibrium occurs if $\mu = 0$ and is given by $1 - \tilde{\tau}_1 + \delta (1 - \tilde{\tau}_2)$. In the all firms move equilibrium the maximum loss occurs at $\mu = 1$ and equals $\tilde{\tau}_1 + \delta (\tilde{\tau}_2 - \tilde{\tau}_2 (1 - c))$. In the mixed strategy equilibrium, maximum losses are smaller. Precisely, if $\mu = 0$ they equal $1 - \tilde{\tau}_1$, and if $\mu = 1$ maximum losses are $\tilde{\tau}_1 - \tilde{\tau}_1 (1 - \tilde{\sigma}_1) + \delta (\tilde{\tau}_2 - \tilde{\tau}_2 (1 - \tilde{\sigma}_1 c))$. The finding that the mixed strategy equilibrium has lower maximum losses may be discussed in the context of loss-averse policy-makers. In the all firm stay equilibrium, losses may be huge but voters may never find out. In the all firm move equilibrium, losses are instantaneously revealed. The mixed strategy equilibrium implies lower observable and unobservable losses.

**Migration cost** In our model, mobility cost $c$ is common knowledge whereas, depending on the scenario, the number of mobile firms is not. This modeling decision is, of course, to a certain degree arbitrary. In principle, assuming $\mu$ as common knowledge and $c$ as unknown is possible, too. More complex (and, perhaps, more realistic) settings in which firms differ continuously in mobility cost $c$ and certain parameters of the underlying distribution function are unknown are alternative modeling options.

For our model, we have chosen (one of) the simplest setting with only two types of firms, a mobile one and an immobile one. It is, thus, a fairly basic question which has the shortcoming of neglecting some real world complexities but the advantage that it may easily be transferred to related question in distinct settings (i.e. other than taxation of firms).

**A mechanism design approach** The model above describes optimal policy when the government can learn about unknown mobility by observing
past reactions to tax rate changes. Of course, there are alternative ways to extract information on unknown parameters such like \( \mu \). For instance, the government may set out to implement a mechanism that makes firms truthfully report their mobility (like in Osmundsen, Hagen & Schjelderup, 1998). How would such a mechanism look like?

The government may simply ask each individual firm whether it is mobile or not. If the reported number of mobile firms is above the threshold indicated in Proposition 1, government chooses \((\tau_1, \tau_2)\). Otherwise, it sets \(\tau_1 = 1\), and all mobile firms leave the country.

At first sight, it seems plausible to assume that, being asked by the government, each firm has the incentive to indicate that it is a mobile one. If the government assumes that all firms are mobile, the only rational policy response is to set \((\tau_1, \tau_2)\). No firm can get larger profits. However, strictly speaking, each firm is infinitely small and, thus, assumes that it has no impact on policy, i.e. the probability that a firm is pivotal is zero. Under these circumstances, a small incentive may be sufficient to make firms reveal their true type. Thus, the government can give an incentive to truthfully report their true type if it taxes the remaining immobile firms that have truthfully reported at a second period tax rate lower than 1. Note, however, that this implies that government can commit itself to a second period tax rate (see Proposition 4). Note also that firms being taxed at a rate of 1 cannot be further punished.\(^{16}\)

4 Conclusion

In this paper, we shed light on the role of information for tax policy. We analyze the tax setting behavior of a revenue maximizing government that faces a sector of firms of which the degree of mobility is \textit{ex ante} unknown. Since there is more than one period, the government – as well as the firms – can adjust its policy choice after observing policy outcomes in the past. In the stylized model we consider here, there are three different equilibria: a first one in which all firms stay and the true degree of mobility is never revealed; a second one in which all mobile firms leave and are eventually attracted back by low future tax rates; and a third one in which some firms move abroad and, thus, reveal the true degree of mobility in the whole sector. Depending on the economic environment (i.e. mobility cost and after-tax profits abroad), each of these equilibria can occur.

Whereas the first two equilibria are well known from much simpler model settings, the third one (in which some firms move revealing information on mobility) is novel. It emphasizes a role of firm migration that has mostly been neglected so far. By triggering firm migration, the government is able

\(^{16}\)If the government chooses \((\tau_1, \tau_2)\), potential misreporting cannot be detected. However, in expected terms, there is a reward of telling the truth.
to learn and to improve policy in the future. Even if firm migration is completely inefficient from a static point of view, it may become valuable if future policy can be based on better (i.e. more precise) information.

How do governments learn about the mobility of their tax bases in the real world? They ask lobby associations or empirical economists. Both of them can only credibly give an estimate of the actual mobility if there has been migration in the past. Thus, for lobby groups demanding lower tax rates, inefficient migration is a kind of costly signal. The analysis of tax policy in the presence of mobile tax bases has – to the best of our knowledge – ignored this aspect so far.

Our results suggest that information on mobility is of crucial importance. First, if tax competition from outside jurisdictions is strong (which is the case if relocation cost and foreign tax rates are low), expected tax rates are higher than under incomplete information. Incomplete information may thus slow down the race to the bottom. Moreover, the expected number of migrating firms is higher if mobility is unknown. The related cost in the form of lower tax revenue and lower profits due to relocation cost adds to the efficiency loss due to the underprovision of public goods. Second, expected tax revenue is lower under incomplete information. Finally, if the government can commit itself, the learning process can be made more efficient, i.e. less migration is necessary to extract the relevant information.

Of course, the stylized model above has its limitations. We do believe, though, that it is suitable to demonstrate the crucial role of information and learning for tax policy. Economic research itself may affect the government’s information and, thus, the efficiency properties of tax policy.

References


Appendix

Detailed proof of Proposition 1:

The proof proceeds in two steps. As a first step, we show that the government prefers \((\tau_1, \tau_2)\) over \((1, 1)\) if \(\mu \geq (1 - \tau^r) \left(1 - \frac{r}{1+r}\right)\). This follows

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directly from comparing the corresponding revenue functions \( R(\bar{\tau}_1, \bar{\tau}_2) = \bar{\tau}_1 + \delta \bar{\tau}_2 \) and \( R(1, 1) = (1 + \delta)(1 - \mu) \). As a second step, we make sure that all other government strategies – given the firms’ strategy choices described above – are inferior. Strategy \((\tau_1, \bar{\tau}_2)\) can never be optimal, since increasing the second period tax rate from \(\bar{\tau}_2\) to \(\bar{\tau}_2\) increases tax revenue without any loss in tax base. Similarly, \(R(1, \bar{\tau}_2)\) is never optimal since increasing the second period tax rate from \(\bar{\tau}_2\) to \(1\) increases revenue without losing part of the tax base. Strategy \((\tau_1, 1)\) can never be optimal by the following argument: \(R(\tau_1, 1) = \tau_1 + \delta (1 - \mu) > R(\tau_1, \bar{\tau}_2)\) iff \(\mu < (1 - \bar{\tau}_2)\) and \(R(\tau_1, 1) > R(1, 1)\) iff \(\mu > 1 - \tau_1\). As \(\tau_1 < \bar{\tau}_2\) the strategy \((\tau_1, 1)\) is revenue dominated by either strategy \((\tau_1, \bar{\tau}_2)\) or strategy \((1, 1)\). Strategy \((1, \bar{\tau}_2)\) is always revenue dominated which can be demonstrated as follows. \(R(1, 1) \geq R(1, \bar{\tau}_2)\) iff \(\mu \leq \frac{1 - \bar{\tau}_2}{1 - c\bar{\tau}_2}\) and \(R(\tau_1, \bar{\tau}_2) \geq R(1, \bar{\tau}_2)\) iff \(\mu \geq \frac{1 - \bar{\tau}_2 - \delta (\bar{\tau}_2 - \bar{\tau}_2)}{1 + c\bar{\tau}_2}\). Since \(\frac{1 - \bar{\tau}_2}{1 - c\bar{\tau}_2} > \frac{1 - \bar{\tau}_2 - \delta (\bar{\tau}_2 - \bar{\tau}_2)}{1 + c\bar{\tau}_2}\), it follows that choosing a strategy with \(\bar{\tau}_2\) is always dominated by either \((1, 1)\) or \((\tau_1, \bar{\tau}_2)\). Finally, it remains to show that \(\bar{\tau}_1\) is not part of an optimal strategy. Note that, if \(\mu < \mu_1^{\min} / \sigma_1^{\min}\), firms set \(\sigma_1 = 1\) in response to a first period tax rate of \(\bar{\tau}_1\). Then, revenue could be increased by choosing \(1\) instead of \(\bar{\tau}_1\). In contrast, if \(\mu \geq \mu_1^{\min} / \sigma_1^{\min}\), firms choose \(\sigma_1 = \frac{\mu_1^{\min}}{\mu}\), yielding tax revenue of \(R(\bar{\tau}_1, \bar{\tau}_2) = \bar{\tau}_1 (1 - \mu_1^{\min}) + \delta \bar{\tau}_2 (1 - c\mu_1^{\min})\). It is straightforward to show that \(R(\bar{\tau}_1, \bar{\tau}_2) > R(\bar{\tau}_1, \bar{\tau}_2)\) as long as \(\mu_1^{\min} > 0\).

The last step of the above proof implies that it is never optimal to force out some firms if it is certain that they will be attracted back in the second period. This is plausible since migration is costly and government and firms have the same discount factor. Thus, triggering partial migration is not an option under complete information.