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Department of Economics Discussion Papers

**Testing for Granger (non-) Causality in a Time
Varying Coefficient VAR Model**

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TESTING FOR GRANGER (NON-)CAUSALITY IN A TIME VARYING COEFFICIENT VAR MODEL

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ABSTRACT:

In this paper we propose Granger (non-)causality tests based on a VAR model allowing for time-varying coefficients. The functional form of the time-varying coefficients is a Logistic Smooth Transition Autoregressive (LSTAR) model using time as the transition variable. The model allows for testing Granger non-causality when the VAR is subject to a smooth break in the coefficients of the Granger causal variables. The proposed test then is applied to the money-output relationship using quarterly US data for the period 1952:2-2002:4. We find that causality from money to output becomes stronger after 1978:4 and the model is shown to have a good out of sample forecasting performance for output relative to a linear VAR model.

KEYWORDS: Granger causality; Time-varying coefficients; LSTAR models.

JEL CLASSIFICATION: C51, C52.

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Testing for Granger (non-)causality in a time varying coefficient VAR model

1. Introduction.

The recent evolution of nonlinear time series models has shown that standard linear VAR models cannot capture adequately the dynamic behavior of many economic and financial time series (see, for instance, Granger and Teräsvirta, 1993, Frances and van Dijk, 2000, Stock and Watson, 1996 and van Dijk et al, 2003). In light of this, inference about information content based on Granger (non-)causality tests that ignore the nonlinear structure of the DGP can lead to misguided conclusions. Causality patterns may not be stable and change over time due to a variety of reasons such as changes induced by large shocks to the economy and changes in the economic environment. The non-constancy of causality patterns poses important problems for correct econometric inference. In this paper we focus our attention on a particular case of parametric non-constancy by analyzing causality patterns that change *over time*. This change is modeled as a single smooth regime shift in causality, although other forms such as temporary or multiple breaks can also be accommodated within this context.

Models that take into account potential endogenous breaks in the deterministic components of a VAR in the context of the Bai and Perron (1998) methodology are now common practice. However, these models are not designed to detect time-varying Granger-causality and the breaks are assumed to be sharp rather than smooth processes. Models of time varying causality that make use of Markov-Switching models have been developed in Psaradakis *et al* (2005), Lo and Piger (2005) and Warne (2000). In these models, time-varying causality is determined by a two states unobservable variable

governed by a discrete-time, discrete-state Markov stochastic process and hence causality is allowed to change sharply depending on time or the state of the economy. In a recent paper, Li (2006) analyzes single equation Granger causality tests allowing for threshold effects. Granger-causality changes depending on whether the endogenous variable is above or below a particular threshold.¹

Our paper takes a different route. We focus our attention on the potential time dependence of the results from causality tests and not on dependence on a particular exogenous variable. Time dependence in parameters is an important issue in its own and can be related to the time varying parameter model presented in Lütkepohl and Herwartz (1996). However, according to Lütkepohl and Herwartz (1996), their approach is a tool of preliminary data analysis and visualization rather a framework for statistical testing. In our paper we propose a simple test for time varying Granger-causality in which the coefficients of the model are subject to a one-off smooth structural change. In this context, Granger-causality patterns are allowed to change once due to a large regime shift, but this change is allowed to be a smooth function of time. There are several other attractive aspects in the test for economic applications. First, it uses a Logistic Smooth Transition (LSTR) function to describe the time varying behavior of the coefficients, which is a computationally simple representation. Second, it considers that the transition from one regime to the other is smooth as is common in many economic applications. Third, it treats changes in causality as random events and allows the data to select the change points. Fourth, the smooth transition process is allowed to differ between different lags of the Granger-causal variable. Fifth, the null hypothesis remains a linear VAR model. Sixth, it has a standard distribution under the null.

¹ See also Galvão (2006) for a Threshold-VAR model that allows for a structural change.

The new method is used to investigate the causal relationships between money and output using quarterly data for the US over the period 1959:2-2002:4. The information content of monetary aggregates for output is a key question in Monetary Economics.² An important debate has focused on whether money has lost information content to predict real and nominal variables around the 1980's. On the one hand, authors such as Friedman and Kuttner (1992) argue that the correlation between monetary aggregates and output vanished after the 1980's. On the other hand, Eichenbaum and Singleton (1986) and Stock and Watson (1989) find that this correlation becomes stronger when data for the 1980s is included in the sample. This structural instability has been addressed in papers such as Psaradakis *et al* (2005) and Rothman *et al* (2001). Weise (1999) studies possible nonlinear effects of monetary policies using an LSTR model in which the transition between states depends on inflation, the business cycle or the stance of monetary policy itself. Lo and Piger (2005) have recently presented evidence on the different hypotheses that may explain a nonlinear relationship between money and output. Hence, there is now an important body of literature that acknowledges that the money-output relationship may be subject to important features of structural instability which makes it an appropriate field of application of our test.

The remainder of the paper is organized as follows. In Section 2 we present the smooth breaking VAR model. In Section 3 we discuss the Granger (non-)causality tests based on the smooth break VAR, while Section 4 discusses some identification problems. Section 5 presents the empirical application and Section 6 concludes.

² See the survey in Walsh (2003) and the evidence in Stock and Watson (2001).

2. The Model.

We assume the following bi-variate $VAR(Q)$ model.

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \gamma_1 \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 \\ \phi_1 & \phi_2 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} \alpha_{1q} & \alpha_{2q} \\ \phi_{1q} & \phi_{2q} \end{bmatrix} \begin{bmatrix} y_{t-q} \\ x_{t-q} \end{bmatrix} + \dots + \begin{bmatrix} \alpha_{1Q} & \alpha_{2Q} \\ \phi_{1Q} & \phi_{2Q} \end{bmatrix} \begin{bmatrix} y_{t-Q} \\ x_{t-Q} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (1)$$

where $q \in [1, Q]$ is the lag augmentation of the VAR, $t \in [0, T]$, $[y_t \ x_t]'$ is assumed to be stationary and ergodic and e_i , $i = 1, 2$, is a stationary vector of errors satisfying $E(e_{it} | \Omega_{t-1}) = 0$ where $\Omega_{t-1} = [y_{t-1}, \dots, y_{t-q}; x_{t-1}, \dots, x_{t-q}]'$.

We consider now that coefficients that determine causal relationships in the VAR (α_{2q} and ϕ_{1q}) are not stable but change over time following a Logistic Smooth Transition (LSTR) functional form as the one adopted in Leybourne *et al* (1998):³

$$\begin{aligned} \alpha_{2q} &= \alpha_{2q}^* + \alpha_{2q}^{**} F(\lambda_{1q}, c_{1q}; \tau) = \alpha_{2q}^* + \alpha_{2q}^{**} [1 + \exp(-\lambda_{1q}(\tau - c_{1q}T))]^{-1} \\ \phi_{1q} &= \phi_{1q}^* + \phi_{1q}^{**} F(\theta_{1q}, g_{1q}; \tau) = \phi_{1q}^* + \phi_{1q}^{**} [1 + \exp(-\theta_{1q}(\tau - g_{1q}T))]^{-1} \end{aligned} \quad (2)$$

where α_i^* , α_i^{**} , ϕ_i^* and ϕ_i^{**} are parameters, and $\tau = t - (T/2)$ is a time trend centered around the midpoint of the sample. $\lambda_{1q} \in [0, \infty]$ and $\theta_{1q} \in [0, \infty]$ measure the speed of transition between the two regimes, and c_{1q} and g_{1q} are threshold parameters. As the transition variable is time, c_{1q} and g_{1q} are interpreted as the timing of the transition midpoint. The midpoint break date would simply be equal to $c_{1q}T + T/2$ or $g_{1q}T + T/2$. These timing coefficients can be determined endogenously through a grid search procedure, as will be

³ See also Saikkonen and Lütkepohl (2002) for other potential forms of the transition function in the context of level shift in deterministic trends.

discussed later on, so the researcher does not need to know the break point *a priori*.

When the speed of transition is close to ∞ the transition function $F(.) = [1 + \exp(-\lambda_{1q}(\tau - c_{1q}))]^{-1}$ becomes a heavy-side indicator function such that $F \rightarrow I(\tau < -c_{1q}T)$ ($F \rightarrow I(\tau < -g_{1q}T)$). When the speed of transition is close to zero then $F(.) \rightarrow 0.5$ and the baseline VAR model (1) reduces to a linear one. For large T s, the transition function $F(.)$ is well defined between 0 and 1. For a positive speed of transition, as $\tau \rightarrow -T/2$, $F(.) \rightarrow 0$, and as $\tau \rightarrow T/2$, $F(.) \rightarrow 1$. When $\tau = c_{1q}T$ then $F(.) = 0.5$. **Figure 1** plots the transition function for different transition speeds for a sample size of 200 and the breakpoint in the middle of the sample. For high transition speeds the function becomes a level shift.

Note that the baseline VAR model (1) under definition (2) allows for the fact that *variables at different lags* adjust differently towards the steady states when the elements of τ change. The assumption of common transition functions at the various lag lengths, which is equivalent to the expression that *variables at different lags* adjust simultaneously to the different steady states, can be tested by using the null hypothesis $H_0: \lambda_{i1} = \dots = \lambda_{iQ}, \theta_{i1} = \dots = \theta_{iQ}, c_{i1} = \dots = c_{iQ}$ and $g_{i1} = \dots = g_{iQ}$.

Although we only analyze here the case of a single smooth break of the LSTR form, it is also possible to think of a model with more than one break. In the case of two breaks we would have that $\alpha_{2q} = \alpha_{2q}^* + \alpha_{2q}^{**}F(\lambda_{1q}, c_{1q}; \tau) + \alpha_{2q}^{***}F(\lambda'_{1q}, c'_{1q}; \tau)$. In this case c_{1q} and c'_{1q} represent the two different timings of the transition midpoint for the two smooth breaks.^{4,5}

⁴ In the case of more than one break, an estimation strategy would consist of starting from a maximum number of breaks (N_{MAX}) and estimating the model for each number of breaks. Then the model selected would be the one that minimizes a criterion function such as AIC or SBC. This would endogenously select the number and dates of the breaks. An analysis of this strategy is beyond the scope of this paper.

⁵ It would also be possible to think of other functional forms for the time-varying Granger-causality structure of the VAR. An example is an exponential STR model (ESTR) in which we allow for a

3. Testing Granger (non-)causality.

An attractive feature of the VAR model (1) under definition (2) is that it allows us to test a situation where a structural break has occurred *in the causal relationships between the variables involved* possibly due to a permanent shift in the data to a new regime induced by, for instance, a policy or a structural change in the economy. The transition towards the new regime may not be immediate but a smooth function whose speed of transition can be estimated. This is a plausible situation when the covariates' predictive power before the occurrence of a structural break is different from that after the break, but this change in predictive power takes time to occur. Ignoring the fact that the casual relationships are not stable over time but might change as a result of some structural breaks could lead to erroneous inferences.

We test for Granger (non-)causality from x_t to y_t using two different hypotheses:

$$H_0^1 : \alpha_{21}^* = \alpha_{22}^* = \dots = \alpha_{2Q}^* = 0 \quad (5)$$

$$H_0^2 : \alpha_{21}^* + \alpha_{21}^{**} = \alpha_{22}^* + \alpha_{22}^{**} = \dots = \alpha_{2Q}^* + \alpha_{2Q}^{**} = 0 \quad (6)$$

Equally, testing for Granger (non-)causality from y_t to x_t we would have:

$$H_0^1 : \varphi_{11}^* = \varphi_{12}^* = \dots = \varphi_{1Q}^* = 0 \quad (7)$$

$$H_0^2 : \varphi_{11}^* + \varphi_{11}^{**} = \varphi_{12}^* + \varphi_{12}^{**} = \dots = \varphi_{1Q}^* + \varphi_{1Q}^{**} = 0 \quad (8)$$

temporary change in the parameters. This specification, however, presents some problems as the function becomes linear as the speed of transition tends to zero or infinity.

The alternative in all these tests is that the sums of the coefficients are different from zero. Hypotheses H_0^1 [(5) and (7)] and H_0^2 [(6) and (8)] are tests for Granger (non-)causality before and after the break respectively. The combination of these two tests allows us to address causality issues and analyze whether causal patterns have changed after the break.

An advantage of these tests is that, for a given set of estimated parameters of the transition function, they can be carried out using standard F statistics. In practice, as will be discussed below, these tests take the form of a *Sup-F* statistic. However, note that, under the null hypothesis H_0^2 , the parameters of the transition function are not identified. Testing these hypotheses would only make sense if a smooth break exists. These identification issues are discussed in the next section.

4. Identification and estimation issues.

An identification problem associated to VAR model (1) under definition (2) is that, since parameters λ_{1q} and θ_{1q} are not identified under the null, the null hypothesis H_0^2 cannot be tested. To overcome this problem we propose to test first for the existence of a nonlinear smooth breaking relationship by making use of a linearity test. We develop a Taylor series approximation of (1) and (2) around the origin which results in the following auxiliary regression:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \Gamma + \sum_{q=1}^Q \mathbf{B}_q \begin{bmatrix} y_{t-q} \\ x_{t-q} \end{bmatrix} + \dots + \sum_{q=1}^Q \mathbf{H}_q \begin{bmatrix} y_{t-q} \tau \\ x_{t-q} \tau \end{bmatrix} + \dots + \sum_{q=1}^Q \mathbf{P}_q \begin{bmatrix} y_{t-q} \tau^2 \\ x_{t-q} \tau^2 \end{bmatrix} + \sum_1^Q \mathbf{N}_q \begin{bmatrix} y_{t-q} \tau^3 \\ x_{t-q} \tau^3 \end{bmatrix} + e_t \quad (9)$$

where Γ and e_t are 2×1 vectors and \mathbf{B}_q , \mathbf{H}_q , \mathbf{P}_q and \mathbf{N}_q are 2×2 parameter matrices.

Equation (9) can be used to test the null of linearity $H_0 : \mathbf{H}_1 = \dots = \mathbf{H}_Q = \mathbf{P}_1 = \dots = \mathbf{P}_Q = \mathbf{N}_1 = \dots = \mathbf{N}_Q = 0$. This is a test for time dependence of the parameters that can be implemented by using a standard F statistic. If the null hypothesis of linearity is rejected against the alternative we can proceed to the estimation of the smooth break VAR model (1)-(2) and then test our two (non-)causality hypotheses.

As is standard in the literature on STR models, the estimation of (1)-(2) is not a trivial task. In particular the joint estimation of λ_{1q} and c_{1q} (or θ_{1q} and g_{1q}) can lead to several identification problems making the convergence of non-linear algorithms a difficult task. To overcome this problem we follow other relevant studies in the literature (see, for example, Frances and van Dijk, 2000 and Saikkonen and Choi, 2004).⁶ In particular, a grid search method can be used to select the initial value of λ_{1q} (or θ_{1q}) and c_{1q} (or g_{1q}). The selected starting values of λ_{1q} (or θ_{1q}) correspond to those yielding the smallest sum of squared residuals.

Given this estimation strategy, the F statistic for the two Granger (non-)causality hypotheses takes the following form:

$$\text{Sup-} F = \sup_{\lambda_{1q} \in \Lambda, c_{1q} \in C} \left\{ \frac{(RSSR - USSR) / m}{USSR / (T - k)} \right\} \quad (10)$$

where $RSSR$ is the sum of squared errors from the restricted model, $USSR$ is the sum of squared residuals from the unrestricted model, m is the number of restrictions, T the number of observations, k the number of parameters in the unrestricted regression, $\Lambda = [\underline{\lambda}_{1q}, \overline{\lambda}_{2q}]$, $0 < \underline{\lambda}_{1q} < \lambda_{1q} < \overline{\lambda}_{1q}$ and $C = [\underline{c}_{1q}, \overline{c}_{2q}]$, $0 < \underline{c}_{1q} < c_{1q} < \overline{c}_{1q}$.

⁶ See also Caner and Hansen (2001) in the context of TAR models.

This corresponds to the F statistic of the regression using the values of λ_{1q} and c_{1q} yielding the smallest sum of squared residuals. Given that the F statistic is an increasing function of $RSSR$ the test is a Sup- F test (see Andrews, 1993). The value of λ_{1q} can be estimated using a grid search method over the space $\Lambda = [\underline{\lambda}_{1q}, \overline{\lambda}_{1q}] = [0.1, 100]$ using increments of 0.1. For the values of c_{1q} we recommend using $C = [\underline{c}_{1q}, \overline{c}_{1q}] = [0, 1]$ using increments of $1/T$.

5. Empirical illustration.

We apply the smooth break VAR model (1) under definition (2) to study the causal relationship between money (m_t) and output (y_t) in the USA over the period 1952:2–2002:4. Money is measured using quarterly data on M1 and output is proxied by the quarterly industrial production index. The data was taken from Lo and Piger (2005). To work with a stationary VAR, we took the first difference of the log of the variables involved so that we have a bi-variate VAR on $[\Delta y_t, \Delta m_t]'$. **Figure 2** plots the two variables against time.

We first consider Granger (non-)causality tests based on a conventional linear VAR. The results of the F -tests are shown in **Table 1**. The tests show strong evidence that bi-directional causality exists between money (Δm_t) and output (Δy_t) as both variables have significant predictive content for each other. We then check the temporal stability of the causal relationship between money and output to test for the existence of a structural change in the Granger causal parameters of the linear VAR. We applied the F -test based on equation (9) developed in Section 4. The computed F -statistic is equal to 2.68 with a p -value 0.02. This result indicates that the relationship between

money and output is not stable but changes over time⁷. In this case, inference from causality tests based on a linear VAR model may be inaccurate.

The next step is estimating the smooth break VAR model (1)-(2) using the grid search strategy explained above. As we explained in Section 4, the estimation of model (1) poses non-trivial problems due to the high nonlinearity in parameters that may lead to poorly identified estimates. A standard way to overcome this problem (see for example Caner and Hansen, 2001 and Saikkonen and Choi, 2004) is to obtain the values of $\Lambda = [\underline{\lambda}_{1q}, \overline{\lambda}_{2q}]$, and $C = [\underline{c}_{1q}, \overline{c}_{2q}]$, by using a grid search. We assume that λ_{iq} and c_{iq} , $i=1,2$, are between a lower and an upper bound such that $0 < \underline{\lambda}_{1q} < \lambda_{1q} < \overline{\lambda}_{1q}$ and $0 < \underline{c}_{1q} < c_{1q} < \overline{c}_{1q}$. Hence, λ_{iq} and c_{iq} are obtained as the values of the grid that yield the smallest sum of squared residuals. We searched for the value of λ_{iq} between 0.1 and 100 using increments of 0.01 while for the values of c_{iq} we searched between 0 and 1 using increments of 1/T.

The point estimates of the resulting model are presented in **Table 2**. We also provide F -tests for the existence of a common transition function at different lags of the VAR. The results suggest that we cannot reject the null of a common transition function for lags 1 and 2 for both the equation for output and the equation for money and we thus proceed by estimating the model with a single transition function for each equation. Focusing our attention on the parameters of interest we observe that in the equation for output the

⁷ It is possible, though, that other transition variables could drive the transition function. An obvious candidate would be inflation, as high(low) inflation regimes could influence the degree of information content of money on output. Following a referee's suggestion, we hence employed inflation as a potential transition variable. We replaced τ by the inflation series in the VAR system (9). The computed F -linearity tests show that the null hypothesis of linearity could not be rejected at 5% statistical level. In the equation of output the F -test was equal to 2.06 while for the equation of money the relevant figure was 1.76. The corresponding p -values were found to be equal to 0.07 and 0.12 respectively, which indicates that inflation does not drive time variation in the causal relationship.

midpoint of the structural break occurs in 1978:1 while in the equation of money it occurs in 1978:4. This suggests that the break happened around the same period of time for both equations and just before the second oil shock and the change in monetary policy stance with the appointment of Paul Volcker as Fed chairman.⁸ The speed of transition between regimes is high (7.58 for the output equation and 8.90 for the money equation). This suggests a sharp structural change as opposed to a smooth and persistent regime change.

Table 3 reports the results of the (non-)causality tests before and after the break as in equations (5) and (6) [(7) and (8)]. The results indicate that causality from output to money cannot be rejected in both periods at confidence levels that are similar to those of the linear VAR. However, the causality from money to output appears to have become stronger after the break in 1978:1. Although causality cannot be rejected at the 10% level before the break, after the break the causal relationship appears to have become stronger and (non-)causality can be rejected at the 1% level. This result is in accordance with previous studies such as Eichenbaum and Singleton (1986) and Stock and Watson (1989) who find that money appears to have more predictive power for output if we include data for the 1980s. Psaradakis et al (2005) also find that M1 does have predictive power during most of the Volcker disinflation period including a brief period in the early 1990's. The results, however, contradict the evidence in Friedman and Kuttner (1992) who argue that including data from the 1980's sharply reduces the information content of money on real variables.⁹

⁸ See Kim et al (2005) for an analysis of changing patterns of monetary policy reaction functions of the US Fed.

⁹ Recently, Sims and Zha (2006) have argued that a good part of the regime changes in monetary policy in the US have been reflected in volatility changes rather than mean changes. This issue, however, goes beyond the scope of this paper.

As a last step in checking the adequacy of the smooth break VAR model against a linear VAR, we carry out a model selection exercise based on the out-of-sample forecast performance of the linear and nonlinear VAR models for a forecast horizon of 4 quarters.¹⁰ In **Table 4** we report the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE) and the p-value of the Diebold and Mariano (1995) test for comparing the predictive accuracy of the linear and nonlinear models. Our non-linear VAR model outperforms the linear VAR for both measures of accuracy in the two equations although the improvement is only significant for output forecasts. Overall, these results are consistent with our previous statement that the smooth break VAR model fits better the dynamics of the relationship between output and money than a linear VAR.

6. Concluding remarks.

Testing for Granger (non-)causality between economic time series has become part of the basic tools of applied macroeconomics. These tests are usually based on VAR models that assume that the Granger causal pattern is stable and does not change over time. In this paper we have developed a simple Granger (non-)causality test in the context of a bi-variate VAR model when causal relationships are subject to a smooth break. This can potentially be a common situation when causal patterns change due to a large shock or change in macroeconomic policy design.

We model the time-varying parameters as a Logistic Smooth Transition (LSTR) function of time where the coefficients of the variables are subject to a one-off smooth break with an endogenously determined breakpoint. We propose using an F test for Granger (non-)causality that takes the form of a

¹⁰ The results using 6- and 8-step ahead forecasts are similar although marginally better for the nonlinear model.

Sup- F test. The test allows for the break function to be different at different lag lengths of the variables involved and is distributed as a standard F statistic under the null. We propose a testing strategy that involves first a test for time-varying coefficients and then a causality test. The test is flexible and allows for other possible forms of breaks such as temporary or multiple breaks and is computationally simple.

We applied the test to the money-output relationship in the USA for the period 1959:2-2002:4 and found that a sharp break occurs in the second half of 1978. Output appears to Granger-cause money both before and after the break. Money also appears to be Granger causal for output in both periods but causality becomes much stronger after 1978:4. The out-of-sample forecast performance of the smooth break VAR model is superior to that of a linear VAR model, especially for predicting output.

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Table 1 : F – causality tests based on a linear VAR model.

$\Delta m \rightarrow \Delta y$	$\Delta y \rightarrow \Delta m$	LogLikelihood
2.96 [0.02]	6.78 [0.01]	1155.66

Note: Figures in brackets are p – values associated with the tests. The optimal lag order for the VAR model was set equal to 2. The AIC's test was minimized for $q = 3$ and Schwartz's by $q = 1$. As a compromise solution we set $q = 2$.

Table 2 : Point estimates of the non-linear VAR model (1).

Variables	Estimates	p – values
Equation for Output		
Constant	0.005	0.01
Δy_{t-1}	0.008	0.93
Δy_{t-2}	0.31	0.01
Δm_{t-1}	0.03	0.73
Δm_{t-2}	-0.04	0.60
$\Delta m_{t-1} F(\lambda_{1q}; c_{1q}; T)$	0.24	0.02
$\Delta m_{t-2} F(\lambda_{1q}; c_{1q}; T)$	-0.19	0.01
λ_{1q}	7.58	0.85
c_{1q}	0.44 1978:1	0.01
F – Test for equal transition function for lags 1 and 2	1.65	0.85
Equation for Money		
Constant	0.006	0.69
Δm_{t-1}	0.52	0.01
Δm_{t-2}	-0.39	0.01
Δy_{t-1}	0.245	0.60
Δy_{t-2}	-0.105	0.07
$\Delta y_{t-1} F(\theta_{1q}; g_{1q}; T)$	-0.222	0.60
$\Delta y_{t-2} F(\theta_{1q}; g_{1q}; T)$	-0.159	0.01
θ_{1q}	8.90	0.85
g_{1q}	0.46 1978:4	0.01
F – Test for equal transition function for lags 1 and 2	1.21	0.91

Note: The optimal lag order for the VAR model was set equal to 2.

Table 3: F – causality tests based on the non-linear VAR model.

Before the break: Regime 1		After the break: Regime 2	
$\Delta m \rightarrow \Delta y$	$\Delta y \rightarrow \Delta m$	$\Delta m \rightarrow \Delta y$	$\Delta y \rightarrow \Delta m$
2.57	6.95	5.91	7.25
[0.10]	[0.01]	[0.01]	[0.01]

Note: Figures in brackets are p – values associated with the tests. Boldface values denote evidence against the null hypothesis of no causality.

Table 4: Out of sample forecasting performance. Time horizon=4.

	Linear VAR		Non-linear VAR		Diebold-Mariano p-value	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
Output	0.005	0.006	0.004	0.005	0.032	0.044
Money	0.009	0.011	0.008	0.010	0.142	0.268

Note: MAE and RMSE denote the mean absolute and the root mean square root error respectively. Diebold-Mariano is the p-value of the forecast comparison test of the linear model against the nonlinear. The null hypothesis is that both linear and nonlinear models have the same forecast accuracy and the alternative is that the nonlinear model outperforms the linear model. A value close to zero indicates better forecast performance of the nonlinear model.

Figure 1: Transition functions

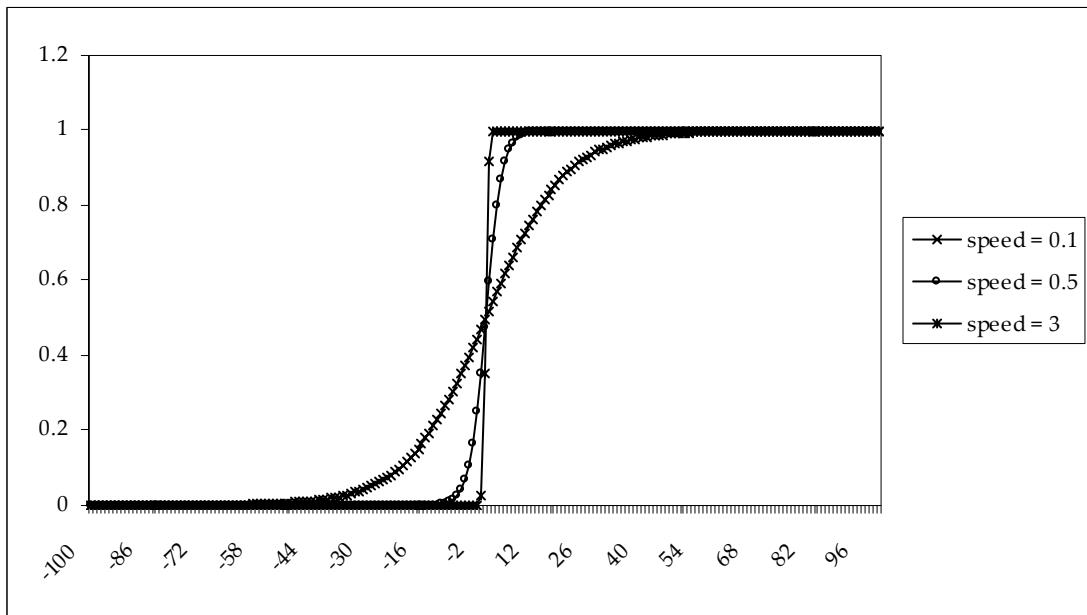


Figure 2: First difference of money and income

