Pedersen, Pål Andreas

Working Paper

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Department of Economics Discussion Paper, No. 01,05

Provided in Cooperation with:
University of Kent, School of Economics

Suggested Citation: Pedersen, Pål Andreas (2001) : A game theoretical approach to road safety, Department of Economics Discussion Paper, No. 01,05, University of Kent, Department of Economics, Canterbury

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A GAME THEORETICAL APPROACH TO ROAD SAFETY

Pål Andreas Pedersen*

March 2001

Abstract
A theoretical model is adopted in order to explain incentives and actual safety behaviour for drivers, pedestrians and other road users which do not utilise motorised vehicles. A road user’s outcome is supposed to be dependent on her individual actions and cares decided upon by other individuals utilising the roads simultaneously, as well as on external traffic safety conditions. By varying the types of road users meeting in traffic and the order of moves taken, several different games are identified, analysed and compared. In addition to focussing on the possible strategic interactions between the road users and the outcomes most likely to be found in different situations, we discuss the existence and size of moral hazard effects caused by improvements in external safety conditions.

JEL Classification: R4, C7, D8.

Keywords: Road safety, strategic interaction, moral hazard.

Acknowledgements: I am indebted to Professor Roger Vickerman, University of Kent at Canterbury, and Professor Finn Jørgensen, Bodø Graduate School of Business, for helpful comments. The usual disclaimers apply.

Address for Correspondence: Pål Andreas Pedersen, Department of Economics, Keynes College, University of Kent at Canterbury, Canterbury, Kent, CT2 7NP, UK. Tel: (+44) 1227 827639; Fax: (+44) 1227 827850; Email: P.A.Pedersen@ukc.ac.uk.

* Bodø Graduate School of Business, Bodø, Norway and University of Kent at Canterbury, UK.
1. Introduction

Public authorities all over the world are concerned about how to take efficient steps towards lower accident rates on the roads as the volume of motorised traffic and the numbers of accidents have been growing significantly over the recent years. Although there has been a debate among researchers about how much accident costs really will increase as the traffic flows grow, see for instance the recent papers by Peirson et al (1998) and Dickerson et al (2000) and the references therein, it is still an important issue to improve our understanding of the traffic behaviour among road users in order to find ways to prevent traffic accidents. It seems reasonable to believe that increased investment in road safety and the continuing technological progress towards safer vehicles have contributed to lower accident rates on the roads. However, as pointed out by several researchers, such political and technological improvements have often not led to the fall in accident rates that one would expect. This is because the road users, facing a safer environment, would find it advantageous to change their behaviour in a way that has the opposite effect. Their less careful behaviour, as a response to external improvements in road safety, will lead to more accidents on the roads. Such kind of indirect effect on accident rates through changed individual behaviour therefore weaken the direct effect on accident rates following from external improvements in road and vehicle safety. In economic terms these responses from the road users on external safety improvements have been called “moral hazard” effects (Peltzman 1975). Alternatively, it has been said that external safety improvements are “incentive worsening” (Risa 1992, 1994) or are inducing “risk compensation” (Wilde 1992). The first main aim of this paper is, as economists, to contribute to the discussion on how different road safety improvements, politically or technologically induced, are likely to work by taking a closer look at how road users might be affected by such improvements, both directly and indirectly. Secondly, we are
interested in how different kind of road users will find it advantageous to react to each other’s behaviour. For instance, will less safe behaviour from one individual induce others to behave more or less carefully in the traffic?

In order to discuss our problems, we will consider a simple economic model consisting of rational individuals choosing their road behaviour in an environment where political and technological safety improvements as well as other road users’ actual behaviour are influencing each individuals’ outcome. In particular, it is supposed that actions taken by one road user, determining her level of care on the road might have a direct effect on the accident probabilities for others using the road simultaneously. This means that we, in contrast to most of the earlier research focusing on road users’ behaviour and the impact from different kinds of safety improvements which are looking at a representative driver (see for instance Pelzman 1975, O’Neill 1977, Blomquist 1986, Janssen and Tenkink 1988, Jørgensen and Polak 1993 and Jørgensen 1993), explicitly concentrate our analysis on the strategic interaction between several inter-related road users. By doing this, we will be able to see the conditions under which the possible strategic interactions will weaken and strengthen the moral hazard effects following from political and technological safety improvements.

It should be noted that there are researchers who earlier have discussed road safety in models consisting of several individuals simultaneously using the roads. Pioneering works explicitly discussing strategic interactions between several road users include Shavell (1980, 1982 and 1984), mainly concerned with situations involving one aggressor and one victim, and the game theoretic approach to road safety by Boyer and Dionne (1987) assuming identical drivers meeting on the roads. Boyer and Dionne discuss the mechanisms, which eliminate or weaken the inefficiencies caused by external effects among drivers, including different
insurance contacts and government taxation and subsidies. In their article it is also shown that
direct governmental regulation of road safety may reduce inefficiency following from
externalities among road users, but will generally not lead to socially optimal levels of road
safety. There are a lot of ways in which technological and political improvements in road
safety are taking place. In addition to direct investment in safer roads by improving their
technological standards, public authorities have the opportunity to improve traffic safety by
different kinds of interventions, such as orders and prohibitions given by law or by
informational and advertising campaigns. The introduction of compulsory seatbelts, speed
limits on roads, regulations governing the manufacturing and safety of automobiles and other
vehicles, and driving tests for granting and renewal of permits are all good examples on how
the public authorities by direct means can contribute to improve traffic safety. Furthermore,
investment in cycle paths, zebra crossings and traffic lights might be seen as public
interventions that also are improving road safety, particularly for non-motorised road users.
Although direct governmental regulations as those mentioned above may not be socially
preferred means in securing welfare-optimal road safety due to moral hazard effects, they are
commonly practised and deserve further attention from researchers.

Our model outlined in section 2 can be seen as an extended version of the model of Boyer and
Dionne (1987). Unlike Boyer and Dionne who presume identical drivers, it becomes possible
in our model to identify different kinds of road users (i.e. finding asymmetric equilibria). For
instance, it is believed that it is important to distinguish between two kinds of road users. The
first type of users can be thought of as persons travelling around by utilising different kinds of
motorised vehicles such as buses, trucks, lorries, cars, motorcycles and mopeds. For
simplicity we will in the rest of the paper use the notation *drivers* for members of this group.
The other group of road users is characterised as travelling without using any motorised
equipment. We might think of pedestrians, cyclists, or persons on skateboards, roller-blades or kick-scooters as typical actors belonging to this group, and in order to simplify, we use the term *pedestrians* as a common notion for members of this group.

The rest of the paper is organised as follows. In section 2 the model is outlined. Section 3 contains the analyses of the moral hazard effects given two kinds of one-shot games generally leading to two different equilibria. Firstly, the equilibrium (A) where the road users draw simultaneously is analysed, and secondly, the leader-follower equilibrium (B) is deduced and commented on where one of the road users chooses her level of care first. In section 4 some discussions are made by taking a closer look on some implications following from the analyses illustrated by examples involving different kinds of road users. Section 5 summarises the conclusions and gives some final comments.

2. The Model

We begin our modelling by assuming individual rational behaviour among risk neutral people using the roads. The preferences for a road user $i$ is assumed to be given by the expected net utility, $F^i$, defined as:

$$F^i = U^i(c^i) - p^i(c^i, ..., c^N, x)L^i(c^1, ..., c^N, x) = F^i(c^1, ..., c^N, x)$$  \hspace{1cm} (1)

$U^i$ is supposed to be the individual gross utility for road user $i$, $c^i$ is measuring the level of care chosen by user $i$, $x$ is a variable denoting external travelling conditions possibly influenced by public authorities, $p^i$ is the probability for accidents for driver $i$, $L^i$ is the loss

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1 In order to simplify our model reasoning we have chosen only to specify one such variable covering all kinds of possible safety improving conditions going on uncontrolled by the road users themselves.
experienced by user $i$ if an accident occurs and $N$ is the total number of road users supposed to meet in the traffic on a road, $i = 1,...,N$. Unlike in Jørgensen and Pedersen (2000), we are here only specifying one action variable for each road user, and we have adopted the common simplification that all kinds of traffic accidents can be treated uniformly, meaning that there exists one well-defined probability for a road user to be involved in an accident and one exact amount of individual loss which the road user will experience if an accident occurs. Furthermore, it is supposed that the functions in equation (1) satisfy the following conditions:

$$U'_i < 0, \quad U'_{ij} < 0, \quad p'_i < 0, \quad p'_{ij} < 0, \quad p'_1 \leq 0, \quad p'_i \geq 0, \quad p'_{ij} \geq (>)0, \quad p'_{ij} \geq 0, \quad p'_{ij} = p'_{ij} = 0$$

$$L'_i \leq 0, \quad L'_{ij} \leq 0, \quad L'_1 \geq 0, \quad L'_{ij} \geq 0, \quad L'_1 \geq 0, \quad L'_{ij} = L'_{ij} = 0,$$

$i, j = 1,...,N, \quad i \neq j$ where $Y'_i = \frac{\partial Y}{\partial z}$ and $Y''_i = \frac{\partial^2 Y}{\partial z \partial r}, \quad Y = U, p, L$ and $z, r = x, c, ..., c^N$.

The assumptions that are made above concerning the functions included in $F^i$ are quite common within models discussing road safety. Firstly, it is supposed that the gross utility for a road user is strictly decreasing and strictly concave in her effort to be careful, i.e. $U'_i < 0$ and $U'_{ij} < 0$. This implies that the road user experiences less utility by increasing her care and that the fall in utility becomes higher as the level of care is further stepped up. Secondly, own careful actions reduces the probability for the road user to be involved in accidents, i.e. $p'_i < 0$, but possibly at a lower rate as the initial level of care increases, i.e. $p'_{ij} \geq 0$. Other road users’ carefulness is also assumed to reduce the probability of being involved in an

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2 Most commonly the individuals’ choices influencing on safety are presumed to concern speed in the sense that the higher speed becomes, the less care is taken. However, it is seen in Assum et al (1999) and Jørgensen and Pedersen (2000), for instance, that concentration level, as a measure of care, can be included in both empirical and theoretical research on road safety.
accident, $p_{ij}^i < 0$, $i \neq j$. However, the impact an individual’s actual choice of care might have on her own probability for being involved in accidents might be weakened ($p_{ij}^i \geq 0$, $i \neq j$) or strengthened ($p_{ij}^i < 0$, $i \neq j$) as the level of care chosen by others is increased. In the first case more careful behaviour is less effective in reducing accident probability the more careful the other road users behave. In the latter case a more careful attitude from a road user will reduce accident probability more if the other road users are behaving more carefully.

Moreover, improvement in traffic conditions measured by increased $x$ might reduce the probability for accidents for a road user, $p_x^i \leq 0$, and the level of $x$ might be less important concerning the level of a probability of accident as the individual care is stepped up, $p_x^i \geq 0$.

External improvements in safety might influence all individuals’ accident probability or only a restricted number of road users. For instance, we might see that a particular external improvement in safety, such as safer cars, reduces the accident probability for a road user $i$ supposed to be an occupant of a safer car, i.e. $p_x^i < 0$. However, this external improvement need not influence another road user $j$’s accident probability if she is a pedestrian or a driver having the same “unsafe” vehicle as before, i.e. $p_x^j = 0$.

Also the individual losses in the case of an accident might be dependent on personal care, others’ care and external improvements in traffic conditions. Increased personal care, higher care from others and improvements in external safety might reduce the individual losses if an accident occurs, i.e. $L_x^i \leq 0$, $L_x^j \leq 0$, $L_x^i \leq 0$, $i \neq j$. Moreover, as the level of personal care increases, the marginal impact on losses by further improvements in personal safety,

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3 In Boyer and Dionne (1987) the cross derivatives of the probability function are presumed to be zero.
increased care by others and higher external safety might be reduced, i.e. \( L_{c,i}^i \geq 0, L_{c,i}^j \geq 0, L_{c,x}^i \geq 0, i \neq j \). Finally, in order to simplify the discussion, we have assumed that neither a road user’s initial level of care nor the level of external care will influence the strength a marginal increase in the road user’s care would have on other road users’ reduction in accident probabilities or losses, i.e. \( p_{c,i}^i = L_{c,i}^i = p_{c,i}^j = L_{c,i}^j = 0, i \neq j \).

In order to study road users’ behaviour and possible outcomes in traffic, we will further make the simplifying assumption that there are only two road users meeting on a road (simultaneously being inter-related) in the sense as described in (1). Moreover, it seems reasonable to assume that the actual outcome is an equilibrium in a one-shot game between these two road users choosing actions individually rationally. Before looking into possible outcomes of such games, let us take a look at the inter-relations between the two road users in our model. This can be seen by evaluating the change in a road user’s net utility caused by a small increase in the other individual’s chosen care. Differentiation of (1) with regard to \( c^j, j \neq i \), gives:

\[
F_{c,j}^i = -p_{c,j}^i L - p_{c,j}^j L_{c,j}^i > 0, \ i, j = 1,2, \ i \neq j .\]  

From (2) the externalities between the road users can be identified. According to our prior assumptions, an increase in a road user’s care will increase the net utility for the other. From (2) it is seen that there are two effects that must be considered. Firstly, an increase in a road user’s care reduces the probability for an accident for the other user, and, secondly, it might reduce the other road user’s loss if an accident happens. Implicitly, our result concerning externalities relies on the assumption that there exist accident losses which the road users,
involved in the accident, do not experience individually. When $L_i$ and $L_j$ are the accident costs for the two individuals, and $L^E$ is assumed to measure the external accident costs not covered by any of the road users in the case of an accident, we can define $L = L_i + L_j + L^E$ as the total accident costs. In our model reasoning it is assumed that $0 < L_j < L$, $j = 1, 2$, meaning that none of the actors faces the total losses if an accident occurs.

3. Possible Outcomes

As mentioned in the introduction we will analyse the outcomes when the road users are moving simultaneously and when one of them moves first. The first case is denoted Case A below, while the game characterised by non-simultaneous moves is denoted Case B.

3.1 Case A: Simultaneous Moves

For any given $x$, the first order conditions defining the general equilibrium on the roads with simultaneous moves where the two road users maximise their personal expected net utilities are given by:

$$F^i_{x} = U^i_{x} - p^i_{x} L - p^j L^j_{x} = 0, \ i = 1, 2. \quad (3)$$

Moreover, the second order conditions for equilibrium are given by:

$$F^{ii}_{x} = U^{ii}_{x} - p^{i} L - p^{j} L^{j}_{x} - 2 p^{i} L^{j}_{x} < 0, \ i = 1, 2. \quad (4)$$

Given our prior assumptions, the inequalities in (4) will be fulfilled such that the equations in (3) define a unique equilibrium. The interpretation of the conditions in (3) for a road user is

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4 This means that even though the road users might be insured or not, and without regard to who is to blame for an accident, there is supposed to be monetary and/or non-monetary losses to bear by the involved if an accident happens. Applying such an argument implies that insurances, if paid, can not be perfect in the sense that all monetary and non-monetary costs experienced by the road users are covered by the insurance companies if an accident occurs.
straightforward. The individual’s optimal care is found when the utility loss which the last unit of care causes for her, \( U'_c \), is equal to the expected gain that it gives her, \( p'_cL'_c + p'_cL'_{c'}. \) It is seen that the expected gain might come from two sources, either the reduction in probability for accidents that the last unit of care gives and/or the reduction in the losses it means for the particular road user.

As a consequence of the positive externalities that exist, commented on above, the equilibrium defined by (3) means that both individuals choose to devote less personal effort to safety on the roads. The reason is simple. When each of the actors chooses personal care, they only consider their own gain from care, not the positive effect a careful attitude will have on the other road user. In Appendix 1, a formal proof is given, showing that the simultaneous equilibrium, defined by the equations in (3) gives rise to less care than the level which would be preferred from a welfare point of view. In order to study the inter-relations between the two road users further, one could ask how the marginal utilities with regard to own care level might be influenced by the other’s actual choice of care. By differentiation of equation (3) with regard to \( c^j \) it follows that:

\[
F_{c^i} = -p'_cL'_c - p'_cL'_{c'} - p'_cL'_c - p'_cL'_{c'} \leq (>)0, \quad i, j = 1, 2, \quad i \neq j
\]  

(5)

From (5) it is seen that, according to our prior assumptions, it is not possible to draw any unambiguous conclusion concerning what happens to the marginal utility with regard to personal care for a road user when another road user increases her care. However, in the case where \( p'_{c^i} > 0 \), it follows directly from our prior assumptions that the sum of the four terms on the right hand side of the equation in (5) is negative meaning that we can say for certain that the individual chosen levels of cares are substitutes in the utility functions, i.e. \( F_{c^i} < 0, \quad i \neq j \). This means that a road user who is increasing her level of care will decrease
the other’s marginal utility with regard to own care. Furthermore, in the case where care does not influence the accident loss, and the marginal impact of increasing personal care on the fall in accident probability is independent of the other’s chosen care, i.e. \( L'_i = L'_j = p^{ij} = 0 \), cares will be independent in the utility function for road user \( i \).

And finally, in the case where the fall in accident probability for higher personal care increases in size when the other road user improves her care, i.e. \( p^{ij} > 0 \), we may have a situation where the positive first term dominates the possible negative sum of the second, third and fourth terms on the right hand side of equation in (5), giving us that \( F^{ij} > 0, \ i \neq j \).

In this latter case, cares are complements in road user \( i \)'s utility function, implying that the marginal impact on utility by increasing own care is increasing when the other steps up her care.

It is now interesting to see whether improvement in external safety is causing the road users to behave less or more carefully. This is the same as asking whether there will be moral hazard effects and eventually finding what determine the sizes of these effects. In order to deal with this question, we will see what happens when the external traffic safety, measured by \( x \), is increased given that the actors move simultaneously. Differentiation of the equalities in (3) with respect to \( x \) and solving for the marginal impact on the personal cares chosen by the road users, give us:

\[
\frac{dc}{dx} = \frac{F^{ij} F^{ij} F^{ij} - F^{ij} F^{ij} F^{ij}}{F^{ij} F^{ij} F^{ij} - F^{ij} F^{ij} F^{ij}} = \frac{F^{ij} F^{ij} - F^{ij} F^{ij}}{D}, \ i, j = 1, 2, \ i \neq j. \tag{6}
\]

In order to discuss the sign and size of the expressions in (6), we make the reasonable assumption that the marginal expected utility with regard to personal care is more sensitive to changes in own care than to changes in the others care, i.e. that \( |F^{ij}| > |F^{ij}| \), which implies
that the denominator in (6), $D$, is positive. If more external safety affects a road user $i$ not to increase her own safety behaviour, this would mean that $\frac{dc_i}{dx} \leq 0$. When the denominator in (6) is supposed to be positive, this condition will hold if the nominator is non-positive, i.e. $F_{e_{ij}}^i F_{e_{ij}}^i \geq F_{e_{ij}}^i F_{e_{ij}}^j$, $i, j = 1, 2$, $i \neq j$. From the assumptions made, it follows from (5) that $F_{e_{ij}}^i < (>)0$ if the cares are substitutes (complements) in the utility function of road user $i$.

Furthermore, by differentiation of (3) with regard to $x$, it is seen that:

$$0 = x_{i\cdot} c_{i\cdot} c_{x} - x_{i\cdot} c_{i\cdot} c_{x} F_{p_{i\cdot} L_{i\cdot}} - x_{i\cdot} c_{i\cdot} c_{x} F_{p_{i\cdot} L_{i\cdot}} \leq 0.$$ (7)

This means that increased external safety, measured by $x$, normally will give a reduction in marginal utility of increasing personal care. However, in the case where road user $i$ is not directly affected by the external improvement in safety, i.e. $p_{i\cdot} = L_{i\cdot} = p_{i\cdot} = L_{i\cdot} = 0$, as for pedestrians when the cars become safer, the marginal growth in utility with regard to personal care is unaffected by the level of $x$, and (7) holds as an equality.

Furthermore, using (7) and the assumptions made, it is seen that in the case where the personal chosen levels of care are complements in individuals’ the utility functions, $F_{e_{ij}}^i > 0$, $i, j = 1, 2$, $i \neq j$, we will always find moral hazard effects for the involved road users. The moral hazard effects are, in the case of complements, strengthened by the strategic interaction between the road users. Firstly, an actor will reduce her level of care as a direct consequence of increased external care as long as she is directly affected by the improvement. Secondly, both road users will reduce cares as an indirect consequence of the less care chosen by the other road user.

5 This assumption is common in the literature and means that the equilibrium is stable; see for instance the footnote 4 in Bulow et al (1985).
In the case where cares are substitutes in the utility functions for both individuals, \( F_{c_{ij}} < 0, \ i, j = 1, 2, \ i \neq j \), moral hazard can be identified if the direct negative effects on the road users’ care from increased external safety, \( F_{c_{ij}}F_{c_{ix}}/D \), dominate the indirect positive effects on the road users’ cares which follows as a response to the other individuals’ choice of less care, \( F_{c_{ij}}F_{c_{xj}}/D \). If \( \frac{dc_i}{dx} \) is negative for both road users, we have the case where both road users behave “more dangerously” after the external improvements in traffic safety, i.e. we have identified moral hazard for both actors. However, in general, if cares are substitutes in the utility functions, it can not be excluded that one road user will find it advantageous to increase personal safety at the same time as the other choose to reduce her personal care. For the one who increases personal care we have then the situation that the positive indirect effect from the other’s reduced individual safety dominates the negative direct effect caused by the increase in external safety. The road user who increases care is then characterised by a marginal expected utility with regard to personal care which is relatively more sensitive to the other road user’s care than to external safety. However, as shown in Appendix 2, there can exist only one such road user, increasing her personal care when \( x \) is improved. The intuition for this is quite simple. By looking at the incentives belonging to the road user characterised by less careful behaviour, it is seen that both the direct and indirect effects in this special case will be negative. Firstly, she will find it advantageous to reduce her own care as a consequence of the increase in the external safety, and, secondly, as a consequence of the first road user’s increase in her personal care.

In addition to those cases commented on above, we have several other possibilities of personal responses which might be relevant. If one of the road users (\( j \)) is not directly
influenced by an external improvement in safety, i.e. $F_{c/x}^j = 0$, her response to the improvements depends only on whether cares are substitutes or complements in her utility function. In the case of complements, she will decrease her safety effort as a reaction to less care chosen by road user $i$, while in the case of substitutes she will increase her safety effort. Furthermore, if the cares are independent in the utility function for individual $j$, i.e. $F_{c/c}^j = 0$, she will respond by being less careful as long as she is directly affected by an improvement in external safety, i.e. $F_{c/x}^j < 0$. In the case where road user $i$, having cares as substitutes, meets an individual $j$, having cares as complements, we know from (6) that person $j$, as a consequence of both the direct and indirect effects, will reduce her care when $x$ increases. Individual $i$’s direct response on the improvements in external safety will also be to reduce her safety effort, but this effect is weakened by the indirect effect through individual $j$’s changed behaviour. Whether a moral hazard effect can be observed for road user $i$ in this case, then depends on whether the direct or indirect effect dominates.

**Result 1:**

*Given two road users both directly influenced by external safety improvements and both characterised by having cares as substitutes, the individual tendency of being less careful as the external safety is increased (moral hazard effects) can be observed by both individuals or by only one of them. In the situation where both actors are being less careful, however, the individuals’ tendency to behave more riskily as external safety is improved is weakened by the other road user’s similar response to behave less carefully. Moreover, if the indirect effect following from another’s less careful behaviour dominates the direct effect caused by increased external safety for an individual, she will behave less riskily as public safety is increased, giving the other road user an additional reason to behave more riskily. If the road users’ utility functions both have cares as complements, however, the strategic interaction*
going on between them always strengthens the moral hazard effects from increased external safety. Finally, when a road user characterised by cares as complements meets a road user with cares as substitutes, we know for certain that the first one will reduce her safety effort. However, the second one will react by reducing care as a consequence of improvements in external safety and will respond by increasing care as a consequence of the less care taken by the first one giving us an ambiguous conclusion regarding her final safety effort compared to the original level.

3.2 Case B: Non-simultaneous Moves

Suppose now that a road user chooses her personal effort in safety first, and the other, after having observed the first mover’s behaviour, is making a choice concerning safety. The condition defining the second road user’s (or the follower’s) optimal behaviour is given by the first equality in (3). This equality is implicitly defining the optimal action made by the follower, denoted as $c^f$, as a function of the action made by the leader, denoted by $c^l$, i.e.

$$c^f = c^{fr}(c^l, x)$$

where

$$\frac{\partial c^{fr}}{\partial c^l} = \frac{F_{c^l,c^l}^f}{F_{c^l,c^l}^f} \leq (>) 0 \quad \text{if} \quad F_{c^l,c^l}^f \leq (>) 0 \quad \text{and} \quad \frac{\partial c^{fr}}{\partial x} = -\frac{F_{c^l,c}^f}{F_{c^l,c}^f} \leq 0. \quad (8)$$

Equation (8) defines the follower’s choice of care for any given levels of $x$ and $c^l$. It is seen that the follower will be more (less) careful the less careful the leader is, ceteris paribus, if the cares are substitutes (complements) in the follower’s utility function. Furthermore, in the case where road user $f$’s utility function is characterised by cares being independent, she will not respond by taking any other choice of care when individual $l$ increases her effort in safety. In the terms of Bulow et al (1985) we may say that cares are strategic substitutes (complements) if they are substitutes (complements) in the utility functions. Furthermore, without regard to whether the cares are substitutes or complements in the utility function, for any given level of
the leader’s choice of care, the follower’s care will decrease for improvements in external
safety as long as she is directly influenced by the increase in external safety, i.e. \( F^f_{c',x} < 0 \).

The first moving road user (or the leader) is supposed to know the reaction from the follower
on any moves she takes, i.e. she is supposed to know the reaction function in (8). This means
that the leader maximises \( F^l[c^l, c^R(c^l, x), x] \) with regard to \( c^l \) for any predetermined value of
\( x \). The first order condition for the leader can then be written as:

\[
\frac{dF^l}{dc^l} = F^l_{c^l} + F^l_{c^l} \frac{\partial c^R}{\partial c^l} = F^l_{c^l} - F^l_{c^l} \frac{F^f_{c^l,c^l}}{F^f_{c^l,c^l}} = 0.
\]

(9)

The second order condition for the leader’s maximising problem is:

\[
\frac{d^2 F^l}{d(c^l)^2} = 2F^l_{c^l,c^l} \frac{\partial c^R}{\partial c^l} + F^l_{c^l,c^l} \left( \frac{\partial c^R}{\partial c^l} \right)^2 + F^l_{c^l,c^l} \frac{\partial^2 c^R}{\partial (c^l)^2} + F^l_{c^l,c^l}
\]

\[
= -2F^l_{c^l,c^l} \frac{F^f_{c^l,c^l}}{F^f_{c^l,c^l}} + F^l_{c^l,c^l} \left( \frac{F^f_{c^l,c^l}}{F^f_{c^l,c^l}} \right)^2 + F^l_{c^l,c^l} \frac{2F^f_{c^l,c^l} F^f_{c^l,c^l} F^f_{c^l,c^l} F^f_{c^l,c^l} - (F^f_{c^l,c^l})^2 F^f_{c^l,c^l} - (F^f_{c^l,c^l})^2 F^f_{c^l,c^l}}{(F^f_{c^l,c^l})^3} + F^l_{c^l,c^l} < 0
\]

(10)

where it follows from (2) and the assumptions made earlier that \( F^f_{c^l,c^l} = -2p^l L^l_{c^l} \leq 0 \). In
order to simplify the further analysis, it is supposed that this second order condition in (10)
holds such that the equation in (3), interpreted for the follower, and equation (9), interpreted
for the leader, define a unique solution. Let us now compare the equations in (9) and (3),
interpreted for actor \( l \) and \( f \) respectively, defining the leader-follower equilibrium, with
simultaneous case where the equations in (3) defines the optimal behaviour for both actors.\(^6\)

In the case where both individuals have cares as substitutes in their utility functions, it is then
seen that actor \( l \), being a leader, will choose a lower level of care and actor \( f \), being a follower,

\(^6\) Comparing the non-simultaneous equilibrium with the Pareto-optimal one would be possible
by following the same reasoning as in Appendix 1 where the simultaneous case is compared
to the Pareto-optimal solution. Doing this, it is easily seen that the individuals’ chosen levels
of cares are less than the welfare optimal ones also in the case of non-simultaneous draws.
will choose a higher level of care than if the two actors had to choose care simultaneously, ceteris paribus. The intuition behind this result is simple. The first moving road user is saving personal costs by taking a less careful action than in the case of simultaneous moves. At the same time this less careful action forces the second moving road user, watching her, to be more careful in order to reduce the expected accident loss. In the terms of Gal-Or (1985), there is a first mover’s advantage and a second mover disadvantage in this game where cares are substitutes because the leader will be better off and the follower will be worse off than if both players moved simultaneously. The formal reasoning behind this result is shown in Appendix 3.

If cares are complements for both road users, however, it is seen that the leader would prefer choosing a higher level of care than in the simultaneous game. The follower, having the opportunity to watch the first mover’s more careful action, answers by being more careful than she would have been in the simultaneous game. The reason behind this result is as follows. The leader, who has a marginal utility with regard to own care which is increasing in the follower’s actual choice of care, wants to stimulate the follower to be careful. To motivate the follower to be careful in this non-simultaneous game, the leader chooses a more careful behaviour than she would have done in a simultaneous game. It is shown in Appendix 3 that a road user will choose higher care as a leader than as a follower and that both individuals will prefer playing both of the non-simultaneous games compared to the simultaneous game. This is because both cares in the non-simultaneous games are higher than in the simultaneous game. It is also shown that it is better for a road user to be the follower than the leader in such a non-simultaneous game. The intuition behind this result is that being the leader means choosing a relatively high level of safety effort that reduces personal utility, giving the follower the opportunity to choose a relatively lower level of personal safety effort. This
means that in the case where cares are strategic complements, there is a second mover advantage; see Gal-Or (1985) and the formal proof in Appendix 3.

We may also have the case where a road user, having cares as substitutes, meets a road user having cares as complements. If the road user, having cares as complements, draws first, she will choose a low level of care inducing the follower, having cares as substitutes, to choose a relatively high level of care. By comparing the different equations defining the simultaneous equilibrium and the non-simultaneous equilibrium when the road user having cares as complements moves first, it is seen that the leader’s care in Case B will be lower than in Case A, while the follower’s care in Case B will be higher than in Case A. Then, it is easily found that the first mover will be better off by being the leader than playing the simultaneous game, while the follower is worse off, see Appendix 3. However, if we change the orders of moves by assuming that the road user having cares as substitutes chooses safety effort first, it is seen that she will choose a relatively more safe behaviour in Case B than in Case A. This safer behaviour from the leader is inducing the follower to increase her safety effort in Case B compared to Case A. Hence, the non-simultaneous equilibrium in the case where a road user, having cares as substitutes draws first is characterised by more effort in safety from both road users than in the simultaneous case. This means a leader-follower equilibrium making them both better off than the simultaneous equilibrium. However, whether the road user having cares as complements prefers playing as a leader or a follower is not generally clear from our model. The formal reasoning behind these results is given in Appendix 3.

**Result 2:**

In non-simultaneous one-shot games between two road users, the follower’s chosen level of care will always be higher than if she was participating in a simultaneous game. The leader’s
chosen care will be lower (higher) in the leader-follower game than in the simultaneous game if the follower considers cares are strategic substitutes (complements). Additionally, in the case where both road users have cares as complements, the level an individual will choose as a follower will be lower than the level she will choose as a leader. Finally, it is found that if cares are strategic substitutes (complements) for both individuals there is a first-mover (second mover advantage) implying that the road user will prefer being a leader (follower) than the follower (the leader) in a non-simultaneous one-shot game. In the case where cares are substitutes for one road user and complements for the other one, the first one will have a first mover advantage (second mover disadvantage), while it is generally ambiguous whether the other one will prefer being a leader or a follower, but it is clear that both non-simultaneous games would be better for her than the simultaneous one.

The next interesting question is how the road users react to changes in $x$ if they play a non-simultaneous game. In order to discuss this, we have differentiated equation (3) and (9) with respect to $x$ and solved the equations with regard to changes in the leader’s and follower’s chosen care. Doing this, it follows that:

\[
\frac{dc^l}{dx} = -\frac{F^l_{c,c'}}{c,c'} \frac{\partial c_{c,c'}^{R} \partial c_{c,c'}^{R} + F^l_{c,c'} \frac{\partial c_{c,c'}^{R}}{\partial c_{c,c'}^{R}} + F^l_{c,c'} \frac{\partial^2 c_{c,c'}^{R}}{\partial c_{c,c'}^{R} \partial x} + F^l_{c,c'}}{2F^l_{c,c'} \frac{\partial c_{c,c'}^{R}}{\partial c_{c,c'}^{R}} + F^l_{c,c'} \frac{\partial^2 c_{c,c'}^{R}}{\partial c_{c,c'}^{R} \partial x} + F^l_{c,c'}}
\]

(11)

\[
\frac{dc^f}{dx} = \frac{\partial c_{c,c'}^{R} \partial c_{c,c'}^{R} + \partial c_{c,c'}^{R} \partial c_{c,c'}^{R}}{\partial x} \frac{dc^l}{\partial x} + \frac{\partial c_{c,c'}^{R} \partial c_{c,c'}^{R}}{\partial x} \frac{dc^l}{\partial x}
\]

(12)

where it follows from (2) and the assumptions made above that $F^l_{c,c'} = -2p_L^l \leq 0$. According to (10) the denominator in (11) is negative. However, the sign of the nominator is generally not unambiguous. It is seen that:

\[
\frac{dc^l}{dx} \leq (>)0 \text{ if } F^l_{c,c'} \frac{\partial c_{c,c'}^{R} \partial c_{c,c'}^{R} + F^l_{c,c'} \frac{\partial c_{c,c'}^{R}}{\partial c_{c,c'}^{R}} + F^l_{c,c'} \frac{\partial^2 c_{c,c'}^{R}}{\partial c_{c,c'}^{R} \partial x} + F^l_{c,c'}}{2F^l_{c,c'} \frac{\partial c_{c,c'}^{R}}{\partial c_{c,c'}^{R}} + F^l_{c,c'} \frac{\partial^2 c_{c,c'}^{R}}{\partial c_{c,c'}^{R} \partial x} + F^l_{c,c'}} \leq (>)0
\]

(13)
By taking a look on the left hand side of the right inequality in (13) it is seen that it consists of five terms pulling the sign and size of what happens to the leader’s care in opposite directions. In the case where cares are strategic substitutes (complements) for the follower, the first term is non-positive (non-negative) and the second term is non-negative (non-positive). Moreover, the sign of the third term depends on the sign of $\frac{\partial^2 c^R}{\partial c^l \partial x}$, where it follows that:

$$\frac{\partial^2 c^R}{\partial c^l \partial x} = \frac{F^l_{c^l c^l} F^l_{c^l x} - F^l_{c^l c^l} F^l_{c^l x}}{(F^l_{c^l c^l})^2} \geq (\langle, \rangle) 0 \quad \text{as} \quad F^l_{c^l c^l} F^l_{c^l x} \geq (\langle) F^l_{c^l c^l} F^l_{c^l x}.$$

Furthermore, if cares are substitutes (complements) for the leader, it follows that the fourth term is non-negative (non-positive) and the fifth term, no matter whether the cares are substitutes or complements, is non-positive. By taking a closer look at the sum of the fourth and fifth terms in (13), however, it is similar to the effects analysed in the simultaneous case in connection with equation (6). As in the simultaneous case the sign of the sum of the fourth and fifth terms will be negative (positive) if $F^l_{c^l c^l} F^l_{c^l x} - F^l_{c^l c^l} F^l_{c^l x}$ is positive (negative). In the case where the leader has cares as complements, this expression is clearly positive, showing that the strategic interactions between the road users strengthen the tendency to behave less carefully as external safety is increased. Furthermore, in the case where cares are substitutes for the leader, the expression above will still be positive as long as what we called the direct effect from increased external safety dominates what we called the indirect effect. In addition to these well-known effects similar to those we found in the simultaneous case, we have the sum of the first, second and third terms in (13) which is related to how the leader’s response will be when taking into account the reaction from the follower. Generally this effect, measured by the sum of the first, second and third terms in (13) might weaken or strengthen the “simultaneous” effect in terms four and five. In the special case where
Regarding the follower’s response to improvements in external safety, one has also here to consider both a direct and an indirect effect. Firstly, it is noticed from (12) that if we have the (unrealistic) situation where the leader will respond to a higher level of external safety by being more careful, i.e. that \( \frac{dc}{dx} > 0 \), the follower will surely be less careful as \( x \) grows. This is because both the direct effect and the indirect effect through changed behaviour from the leader will then be negative. However, if the leader becomes less careful when the external safety increases, the indirect effect of changed \( x \) would be positive for the follower, while the direct effect is still negative. As long as the direct effect dominates, we will then find that the follower’s response is to behave less carefully, implying that we have a moral hazard effect as a consequence of increased external safety.

4. Discussion with Some Examples

In this section we will take a closer look at three different examples illustrating various outcomes and reactions at external improvements in external safety. Examples A and B discuss symmetric equilibria involving two road users responding to each others’ behaviour in a similar way, while in example C, two road users responding differently to each others’

\[ F_{c',c'}^f = F_{c',c'}^e = F_{c',c'}^f = F_{c',c'}^e = 0 \]

it is seen that the sum of these first three terms is zero, bringing us to the same conclusions in Case B as in Case A concerning the follower’s response to improvements in external safety.

\(^7\)According to the assumptions made, it should be noticed that sufficient conditions for the first two derivatives to be zero are that the accident loss for the leader is unaffected by any changes in external safety and the level of care chosen by the follower, i.e. \( L_{c,c}^f = L_c^f = 0 \).
chosen cares meet in traffic. Examples A and B illustrate road users finding advantages by choosing *forwarding* and *awaiting* attitudes, respectively. Hence, in example A there might be a race between the road users to be the first mover, while in example B the individuals might compete to be the second mover. Example C, however, comes up with a different story that may end up with a non-simultaneous equilibrium where both road users obtain their most preferred position. The explanation is that the road user, having cares as substitutes, finds it most preferable to be *forwarding* her decision, while the other one, having care as complements, at the same time finds it most preferable to be *awaiting* her decision.

### 4.1 Example A: Both Road Users having Cares as Substitutes

In this example it is supposed that for both road users meeting in traffic, increased level of care from the other actor will reduce the marginal utility of being more careful, i.e. $F_{c|c}^i < 0$, $i, j = 1, 2$, $i \neq j$. This means that the levels of cares are substitutes in the utility functions leading to reaction functions which are decreasing in the other’s level of care, or cares as strategic substitutes. Considering speeds as decision variables for drivers this means that a marginal speed reduction for a driver is less effective in order to decrease expected accident costs as the level of the other one’s speed is decreased. In other words, a low speed chosen by a driver means that the fall in expected accident costs for the other driver by reducing her speed marginally will be low compared to a situation where the first one had decided to drive fast. As seen in section 3 above, comparing the simultaneous and non-simultaneous games, such an inter-relation between the driver’s net utility functions results in a situation where both individuals would wish to be the first mover in order to obtain the first mover advantage. In practice this might be done by showing the other one, as soon as possible, a relatively fast speed on the road. By doing this, the driver signals a low level of care trying to force the other road user to be more careful by slowing her speed. In many
games of this kind there will be an *ex ante* common understanding who is the leader and who is the follower. For instance in a meeting between a driver, being relatively highly protected sitting in or on her vehicle, and a pedestrian, relatively more vulnerable if an accident occurs, it is not unlikely to believe that both persons comprehend the driver as the leader and the pedestrian as the follower. If so, the driver obtains the first mover advantage by taking a relatively unsafe action, forcing the pedestrian to be careful. Another possibility is that the driver considers herself as the follower of the game. In particular, we believe that in meetings in traffic between a child (of course being a pedestrian) and a driver, the driver, as a consequence of facing an individual who may not act rationally, awaits the action made by the child before choosing what to do.

However, when the road users do not have any clear common understanding of the roles, the possible gains they may have by being the first mover might cause a race to be the leader. This means that if both road users are forwarding a relatively unsafe behaviour, there will be a competition not to be the second mover, thus having to depart from the originally unsafe attitude. If no one retreats from this competition by choosing a safer behaviour and signalling she accepts being the follower, both road users’ aggressive behaviour would mean high risks for accidents and, of course, no equilibrium is obtained. On the other hand, if both road users retreat simultaneously, pretending to accept the role as follower, this would mean that both players choose the personal optimal level of care for a given value of the other road user’s care. Then, being aware of each other’s new and less aggressive behaviour, we end up with an outcome identical to the simultaneous case analysed above.
4.2 Example B: Both Road Users having Cares as Complements

In this example we explicitly consider the case where increased care from one road user increases the marginal utility for the other one in supplying higher care, and this holds for both road users, i.e. \( F_{ij}^i > 0, \ i, j = 1, 2, \ i \neq j \). In this case cares are complements in the utility functions leading to reaction functions which are increasing in the other’s chosen care, or the case of strategic complements. If the road users are drivers and the actions are speed levels, this means that a driver’s marginal gain by slowing down her speed a bit (measured in less expected accident costs) is higher the less speed the other driver has chosen. For instance, the description might be suitable for a situation where both persons initially are driving fast. In such a situation a partial reduction in speed from one of the drivers might be less effective, in order to reduce expected accident costs, the higher speed the other one has chosen. As shown in section 3 above, such a situation means that the leader chooses a relatively high level of care (or slow speed) inducing the other to be relatively careful (or drive slowly) compared to the case where the road users choose cares simultaneously. Furthermore, it is shown that in this case both actors prefer being the follower. In many situations the road user’s positions as first and second mover, respectively, might be well defined. For instance, as in example A in a meeting between an unprotected pedestrian and a driver one may see that the pedestrian takes the less preferred position of being the leader in the sense that she acts relatively safely, followed by a relatively less safe behaviour from the driver. However, it is found that both road users are being more careful than they would have been if they had to move simultaneously. Furthermore, in a meeting between two drivers with these kinds of utility functions, one may see that the individual driving the vehicle that is easiest to manoeuvre takes the leader’s position by choosing a relatively careful behaviour first.
However, in many cases there might not be an *ex ante* common understanding of the road users’ roles. In such cases there might be a competition to be the second mover, i.e. that each driver wishes to know what the other one is doing before taking an action. If both road users initially drive fast and they compete for being the second mover, we may end up with drivers, adopting a waiting behaviour, continuing driving fast waiting to see what the other one does before taking any action. In the case where one retreats, by taking the leader’s role of choosing a safer behaviour, the follower will also choose to reduce speed, but not as much as the leader. If both road users continue waiting, and at the end they both retreat (being aware that the other also will do), we will see an outcome of the game similar to the case of simultaneous draws. However, this outcome is less preferred than a non-simultaneous outcome, both in the case where the road user is playing the follower and in the case where she is forced to play the leader’s role.

### 4.3 Example C: One Road User having Cares as Substitutes - The Other has Cares as Complements

In this case let us think of an asymmetric situation where one of the road user’s marginal utility with respect to her own care is decreasing in the other’s chosen level of care, while the opposite holds for the other one. This means that $F_{c_i c_j} < 0$, $F_{c_i c_j} > 0$. From (8) it then follows that the reaction function for road user $i$ is downwards sloping telling us that individual $i$ will reduce her personal effort in traffic safety when road user $j$ increases her personal care. However, it is upwards sloping for individual $j$, implying that road user $j$ would find it advantageous to be more and more careful as road user $i$ improves her care. If individual $i$ is the leader, in order to force road user $j$ to be more careful, she will be more careful than in the simultaneous case, and the result, of course, is that both are more careful. This means that the non-simultaneous case, where road user $i$ is leader and individual $j$ is follower, is preferred by
both individuals to the outcome of a simultaneous game. However, if road user \( j \) is the leader, she will choose a relatively low level of care forcing the follower to behave more carefully. This outcome is preferred by individual \( j \) to the outcome in the simultaneous case, but the follower \( i \) is worse off. As mentioned in the discussions related to example A and B there might be a common *ex ante* understanding of who is playing the leader’s, and who is playing the follower’s, role. However, in situations where it is unclear what positions the two road users have, we might see that the individuals either compete for being the leader or the individuals both prefer that road user \( i \) becomes the leader and road user \( j \) becomes the follower. Whether there will be a race to be the leader of this game is dependent on whether road user \( j \) is better off by being the leader in her meeting with \( i \) than being the follower. As in the examples above, if the road users compete to be the leader, we may end up with the simultaneous case. If individual \( j \) is better off by being the follower than the leader, we will see individual \( i \), taking an early decision to be relatively careful, and an awaiting individual \( j \) matching this relatively careful behaviour by behaving relatively safely herself. In particular, we know that road user \( j \) will prefer being the follower instead of the leader as long as individual \( i \) as a leader will choose a level of care that is at least as high as she would have chosen as a follower, see Appendix 3 for details.

5. **Concluding Remarks**

By assuming a rational model of human behaviour based on inter-related utility functions between several road users, simple one-shot games are applied in order to analyse possible strategic interactions between individuals choosing their levels of cares, and the influence on their behaviour from external improvements in safety. When the road users’ utilities are dependent on each other’s safety efforts, the chosen cares would be less than what is preferred
from a social welfare point of view as long as the road users are not faced with the total costs of being involved in traffic accidents. How external improvements in safety affect the individuals’ chosen cares, depends on whether the individual cares are substitutes or complements in the utility functions and possibly on the order of moves taking place. In simultaneous games, external improvements in safety will clearly weaken the personal incentives to be careful when cares are complements. This is because both the direct effect from increased external safety and the indirect effect, through less careful behaviour from the other, will cause the individuals to behave more hazardously in traffic. However, in the case where cares are substitutes, the indirect effect, through changed behaviour from another road user, will normally weaken the moral hazard effect caused by improvements in external safety. We have also seen that in the case of non-simultaneous moves by road users, the same mechanisms as in simultaneous games must be considered. However, the sign and the size of moral hazard effects might also be influenced by changes in the reaction function, possibly both weakening and strengthening the incentives to behave more riskily in traffic as a consequence of external improvements in safety. The lesson from a policy point of view must be that moral hazard is likely to appear when improvements in external safety take place, even when one controls for changed behaviour from all road users. The size of the moral hazard effects, however, may vary with regard to what kind of people meet and the orders of moves taken.

When there is no clear order of moves *ex ante*, we have seen that the road users may compete for preferable positions. In the case where both road users consider cares as substitutes, there will be an incentive for both individuals to forward their moves, which may end up with a race to be the first mover. On the other hand, if cares are considered as complements by both road users, there will be an incentive to wait and see the other’s actual move before taking an
action, possibly ending up in a competition to be the second mover. And if cares are substitutes for one of the road users while they are complements for the other, we may see an outcome, overall preferred by both, where the individual having cares as substitutes, moves first and the individual having cares as complements moves secondly.

As far as we can see, there are at least two problems arising when applying our game theoretical approach to road safety. First, one may argue that the most important choices road users make in traffic are regulated by traffic laws, meaning that the utility maximising model is not relevant in order to prescribe the individuals’ behaviour. For instance there are speed limits and rules prescribing who has the right to go first when two road users are simultaneously competing for road space. Even though traffic is in many senses strongly regulated, there is, however, still a lot of choices that must be taken by road users’ when they actually “face each other” on the roads. If so, focusing on what explains the individuals’ choices is relevant. But then another problem applying our game theoretical model arises. When two road users meet in traffic, they do not normally know what kind of person they face in the different situations, and, therefore, it becomes difficult to calculate an optimal behaviour. Rather than playing “new games” whenever another road user is met, it seems likely that individuals search for a norm that represents a suitable behaviour no matter what type is the road user being met. If so, one would expect that patterns of sustainable attitudes in traffic develop, making the direct empirical application of our model less relevant in explaining road users’ actual choices of care. However, if our rational model holds, where the utilities are inter-related, the behavioural patterns or the social norms, rather than single meetings in traffic, might be understood in the light of our model. If this is true, it becomes an interesting question in order to understand traffic behaviour, whether cares are mostly substitutes or complements in the utility functions. In order to establish the connection
between our model results, based on single analyses of equilibria and the behavioural norms that exist, further theoretical research is needed. And finally, one has to do empirical research in order to see whether such an understanding of traffic behaviour is relevant. For instance, one way of doing an empirical study might be to register levels of cares actually chosen by the road users when meeting (for instance speed, concentration levels, etc.), and use the collected data to find out what kind of implicit utility functions the involved persons must have had.
REFERENCES


APPENDIX 1

Let us now compare the Pareto-optimal solution, here denoted by \((\hat{c}^i, \hat{c}^j)\) and the simultaneous Nash-equilibrium denoted by \((\check{c}^i, \check{c}^j)\). If \((\hat{c}^i, \hat{c}^j)\) is a unique Pareto optimal solution, then:

\[
F^i(\hat{c}^i, \hat{c}^j, x) \geq F^i(\check{c}^i, \check{c}^j, x), \quad i, j = 1, 2, \quad i \neq j
\]

(A1)

where (A1) holds as a strict inequality for at least one of the road users.

Let us suppose that:

\[
\hat{c}^i \leq \check{c}^i, \quad i = 1, 2.
\]

(A2)

If (A2) holds, it then follows that:

\[
F^i(\hat{c}^i, \hat{c}^j, x) \geq F^i(\check{c}^i, \check{c}^j, x) \geq F^i(\check{c}^i, \check{c}^j, x), \quad i, j = 1, 2, \quad i \neq j.
\]

(A3)

The first inequality in (A3) follows from the definition of the simultaneous Nash equilibrium, where \(\check{c}^i\) defines road user \(i\)’s optimal care for road user \(j\)’s chosen level \(\check{c}^j\). The second inequality in (A3) is satisfied if (A2) holds given the assumption that \(F^j_{ij} > 0\). However, (A3) contradicts the inequality in (A1), which must hold as a strict inequality for at least one road user due to our prior assumptions. Hence, our reasoning shows that (A2), suggesting that the cares are lower in Pareto-optimum than in the simultaneous Nash equilibrium, does lead to a contradiction. Therefore, it is shown that cares must be higher in the welfare maximum solution than in the simultaneous Nash equilibrium, i.e. we have found that:

\[
\check{c}^i > \hat{c}^i, \quad i = 1, 2.
\]

(A4)
APPENDIX 2

If both actors would increase their safety when \( x \) is increased, by using (6) it is seen that this would mean that:

\[
\frac{F_{i\ell}^j}{F_{i\ell}^j F_{\ell \ell}^i} < 1 \quad \text{and} \quad \frac{F_{j\ell}^i}{F_{j\ell}^i F_{\ell \ell}^j} < 1
\]  

(A5)

Multiplying the left hand sides of the inequalities in (A5) gives the product \( \frac{F_{i\ell}^j F_{j\ell}^i}{F_{i\ell}^j F_{j\ell}^i} \).

Furthermore, it is seen that if both inequalities in (A5) are going to hold simultaneously, the product \( \frac{F_{i\ell}^j F_{j\ell}^i}{F_{i\ell}^j F_{j\ell}^i} \) has to be below 1. However, according to the existence of a stationary Nash equilibrium in the case of simultaneous draws, \( D \) in equation (6) is positive, which implies that the product \( \frac{F_{i\ell}^j F_{j\ell}^i}{F_{i\ell}^j F_{j\ell}^i} \) is above 1. This means that the two conditions in (A5) cannot both be satisfied as long as we restrict ourselves to discuss situations where there is a stationary equilibrium. Hence, we can conclude that both actors cannot increase their level of cares as the external care measured by \( x \) is stepped up.
APPENDIX 3

In the case of strategic substitutes, i.e. $F_{c_{i,j}}^i < 0$, $i, j = 1, 2$, $i \neq j$, we have seen directly by comparing the equations in (3), defining the equilibrium in the simultaneous case, by equation (3), interpreted for the follower, and equation (9), interpreted for the leader, defining the equilibrium in the non-simultaneous case, that:

$$c^{i_{f}} < \hat{c}^i < c^{i_{l}}, \quad i = 1, 2$$  \hspace{1cm} (A6)

where $c^{i_{f}}$ and $c^{i_{l}}$ symbolise road user $i$’s chosen levels of care as follower and leader respectively. Using (A6) gives us:

$$F^i(c^{i_{l}}, c^{i_{f}}, x) > F^i(\hat{c}^i, c^{i_{f}}, x) > F^i(\hat{c}^i, \hat{c}^i, x), \quad i, j = 1, 2, \quad i \neq j$$  \hspace{1cm} (A7)

where the first inequality follows from the fact that $c^{i_{l}}$ is the level of care maximising utility for the leader given the follower’s choice of $c^{i_{f}}$ and the second inequality follows because $c^{i_{f}} > \hat{c}^i$ and $F^i_{c_{i,j}} > 0$. (A7) implies that an actor, in the case where cares are strategic substitutes, prefers being a leader than playing the simultaneous game. Furthermore:

$$F^j(\hat{c}^j, \hat{c}^i, x) > F^j(c^{i_{f}}, c^{i_{f}}, x) > F^j(c^{i_{l}}, c^{i_{f}}, x)$$  \hspace{1cm} (A8)

where the first inequality follows from the fact that $\hat{c}^j$ is the level of care maximising utility for $j$ given the other’s choice of care $\hat{c}^i$. The second inequality follows because $c^{i_{l}} < \hat{c}^i$ and $F^j_{c_{i,j}} > 0$. (A8) then implies that a road user, in the case of cares as strategic substitutes, rather would play the simultaneous game than being the follower in a non-simultaneous game. Altogether (A7) and (A8) mean that we in the case of cares as strategic substitutes can write:

$$F^i(c^{i_{l}}, c^{i_{f}}, x) > F^i(\hat{c}^i, \hat{c}^j, x) > F^i(c^{i_{f}}, c^{i_{f}}, x), \quad i, j = 1, 2, \quad i \neq j$$  \hspace{1cm} (A9)

In the case of strategic complements, i.e. $F_{c_{i,j}}^i > 0$, $i, j = 1, 2$, $i \neq j$, it is found by comparing the simultaneous and the non-simultaneous solutions that:
\[ \hat{c}^i < e^{i=\text{f}} < e^{i=\text{l}}, \quad i = 1, 2. \quad (A10) \]

The first inequality in (A10) is easily seen by comparing equation (3) and (9) and concluding that in the case of strategic complements, \( c^{i=\text{l}} > \hat{c}^i, \ i = 1, 2. \) Then it follows directly from (8) that \( c^{i=\text{f}} > \hat{c}^i, \ i = 1, 2. \) However, in order to see that \( c^{i=\text{f}} < e^{i=\text{l}}, \ i = 1, 2, \) or the second inequality in (A10), some more reasoning is needed. Firstly, it is seen from (3) and (9) that in the case of cares as complements we have that:

\[ (\hat{c}^i, \hat{c}^j) \leq (e^{i=\text{f}}, e^{j=\text{f}}), \quad i, j = 1, 2, \quad i \neq j. \quad (A11) \]

Suppose now that the following inequality holds:

\[ c^{i=\text{f}} \geq c^{i=\text{l}}, \quad i = 1, 2. \quad (A12) \]

If (A12) holds, it means that the following inequalities are satisfied:

\[ F^i_{\hat{c}}(e^{i=\text{l}}, e^{j=\text{f}}, x) < F^j_{\hat{c}}(e^{i=\text{f}}, e^{j=\text{l}}, x) = 0, \quad i, j = 1, 2, \quad i \neq j. \quad (A13) \]

The first inequality in (A13) follows from the assumption that \( F^i_{\hat{c}} < 0 \) and (A12), and the second inequality in (A13) must be satisfied if (A12) holds, given that cares are complements, i.e. \( F^j_{\hat{c}} > 0. \) However, it is seen that (A13), based on (A12), leads to the opposite of (A11) which we know is a condition that must hold. Therefore, (A12) must be wrong, implying that the second inequality in (A10) holds, i.e. \( c^{i=\text{f}} < e^{i=\text{l}}, \ i = 1, 2. \) Furthermore, using similar arguments as in (A7) and (A8) above, it follows that we can deduce the following inequalities:

\[ F^i(e^{i=\text{l}}, e^{j=\text{f}}, x) > F^i(e^{i=\text{f}}, e^{j=\text{l}}, x) > F^i(\hat{c}^i, \hat{c}^j, x), \]

\[ F^i(e^{i=\text{f}}, e^{j=\text{l}}, x) > F^i(e^{i=\text{l}}, e^{j=\text{f}}, x) > F^i(\hat{c}^i, \hat{c}^j, x), \quad (A14) \]

\[ F^i(e^{i=\text{f}}, e^{j=\text{l}}, x) > F^i(e^{i=\text{l}}, e^{j=\text{f}}, x) > F^i(c^{i=\text{l}}, c^{j=\text{f}}, x). \]

From (A14) it is seen that a road user, in the case of cares as strategic complements, is preferring being the follower in a non-simultaneous game than the leader, and that both
positions in the non-simultaneous game are preferred to the outcome of a simultaneous game, i.e.

\[ F^i(c^{i=f}, c^{i=l}, x) > F^i(c^{i=l}, c^{i=f}, x) > F^i(\hat{c}^i, \hat{c}^j, x), \quad i, j = 1, 2, \quad i \neq j. \]  

(A15)

Suppose now that we have the case where one of the road user’s utility function is characterised by cares as substitutes and the other one’s utility function having cares as complements, i.e. \( F_{ij}^i < 0, \quad F_{ij}^j > 0 \). Then it follows from comparing the equations in (3), defining the equilibrium in the simultaneous case, with equation (3) and (9), interpreted for the two different situations of non-simultaneous moves that:

\[ \hat{c}_{ij}^f > \hat{c}_{ij}^j > c_{ij}^f \quad \text{and} \quad \hat{c}_{ij}^i > c_{ij}^i. \]  

(A16)

(A17)

Using the inequalities in (A16) and (A17), practising similar reasoning as above then gives us that (A8) above holds for individual \( i \), while the following inequalities hold for road user \( j \):

\[ F^j(c^{i=l}, c^{i=f}, x) > F^j(c_{ij}^j, \hat{c}^f, x) \quad \text{and} \quad F^j(c^{i=f}, c^{i=l}, x) > F^j(\hat{c}^j, \hat{c}^i, x). \]  

(A18)

Generally, it is impossible to know whether individual \( j \) prefers being a leader or a follower. However, it is seen from our assumptions that in all cases where individual \( i \)’s level of care as a leader at least is as high as the level she would have chosen as a follower, i.e. \( c^{i=l} \geq c^{i=f} \), road user \( j \) prefers being the follower rather then the leader, i.e.

\[ F^j(c^{i=l}, c^{i=f}, x) > F(c^{i=f}, c^{i=l}, x). \]