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Taste for variety and optimum product diversity in an open economy

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Abstract

We extend the Benassy (1996) 'taste for variety' model to an open economy setting. With the Benassy effect, the market equilibrium is inefficient, openness reduces the varieties provided in the unconstrained optimum and there are potential gains from international coordination.

Keywords: Taste for variety; monopolistic competition; Benassy effect; open economy
JEL classification: D43, F12

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1. Introduction

In a recent paper, Benassy (1996) presented an alternative to the representation of 'taste for variety' traditionally used in the monopolistic competition literature (see e.g., Dixit and Stiglitz, 1977 and Spence, 1976). The Benassy (1996) specification disentangles 'taste for variety' from market power and substitutability measures. As a result, the monopolistic competition market equilibrium may no longer be efficient. This is done in a closed economy context.

The purpose of this paper is to study the implications of the Benassy specification for the efficiency of the market equilibrium in an open economy and the potential gains from international coordination.

More specifically, we consider a two way model of trade where firms have market power owing to the consumers love for variety. We analyze the unconstrained open economy optimum and compare it with the market equilibrium and the global social planner outcome.[1] In addition, we study the impact of changes in openness on firm numbers. We will argue that the Benassy specification may provide an explanation for recent changes in concentration in industries such as the defence industry.

The paper is organized as follows, section 2 presents the model and finds the solution for the unconstrained small open economy equilibrium. Section 3 compares the different equilibria. Finally, section 4 concludes the paper.

2. The model

We consider a two country model. There are n_1 and n_2 firms in countries 1 and 2 respectively, each producing a single variety of a differentiated good. Consumers' preferences in country 1 are characterized by the following utility function:

$$U_1 = \left[w^{\frac{1}{\rho}} \left(n_1^{\nu+1-\frac{1}{\alpha}} \left(\sum_{i=1}^{n_1} (d_{1i})^{\alpha} \right)^{\frac{1}{\alpha}} \right)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}} \left(n_2^{\nu+1-\frac{1}{\alpha}} \left(\sum_{j=1}^{n_2} (m_{1j})^{\alpha} \right)^{\frac{1}{\alpha}} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}};$$

$$\alpha \in [0,1),\ \rho \in [1,\infty),\ \nu > 0,\ w \in [\tfrac{1}{2}, 1], \tag{2.1}$$

[1] Flam and Helpman (1985) made the first contribution to the analysis of an government policy under monopolistic competition in a small open economy. We use a partial equilibrium version of their model which will clearly illustrate the impact of the Benassy specification.

where, d_{1j} represents domestic consumption of domestic variety j and m_{1j} represents domestic consumption of foreign variety j, similarly for country 2. As in Benassy (1996), the utility function includes separate parameters to measure the elasticity of substitution across varieties, $\sigma = \frac{1}{1-\alpha} \in [1, \infty)$, which will later determine the mark up, and the taste of variety, determined by ν, which measures the impact on utility of an increase in the total number of varieties N,[2] $\frac{\partial U_1}{\partial N}\frac{N}{U_1} = v+1$. This specification allows for a more ($\nu > \frac{1-\alpha}{\alpha}$) or less ($\nu < \frac{1-\alpha}{\alpha}$) taste for variety than in the Dixit-Stiglitz case ($\nu = \frac{1-\alpha}{\alpha}$).

In (2.1) the parameter $w \in [\ \frac{1}{2}, 1]$ represents the degree of 'home bias' for domestically produced goods.[3] For $w = 1$ we have autarky whilst the lower bound $w = \frac{1}{2}$ gives us the case of the complete integration of the two economies. Thus we can associate a lower home bias w with an increase in 'openness'.

2.1. *Demand functions and equilibrium producer prices*

In this section, we obtain the demand functions using standard two stage maximization procedure. We define the following price index associated to the domestic varieties in country 1:

$$P_{d_1} = n_1^{-\left(\nu+\frac{1}{1-\sigma}\right)} \left(\sum_{i=1}^{n_1} \left(p_i^c\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{2.2}$$

where p_i^c is the price consumers pay for domestic variety i (superindex c is introduced so as to distinguish between consumer and producer prices). Straight-

[2] At the symmetric equilibrium across varieties, the utility function is equal to

$$U_1 = \left(w^{\frac{1}{\rho}} \left(n_1^{\nu+1} d_1\right)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}} \left(n_2^{\nu+1} m_1\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}.$$

We then consider an expansion in the total number of varieties, keeping the relation between domestic and foreign varieties $\frac{n_1}{n_2}$ fixed. Then, putting $n_1 = kN$

$$U_1 = \left(w^{\frac{1}{\rho}} \left((kN)^{\nu+1} d_1\right)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}} \left(((1-k)N)^{\nu+1} m_1\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}},$$

thus, $\frac{\partial U_1}{\partial N}\frac{N}{U_1} = v + 1$.

[3] For the case where $\rho = 1$, w is the share of domestically produced varieties in total consumption.

forward utility maximization subject to the standard consumer budget constraint results in the following demand function for each domestic variety i

$$d_{1i} = w\left(\frac{p_i^c}{P_{d_1}}\right)^{-\sigma} n_1^{(\sigma-1)v-1}\frac{P_{d_1}^{-\rho}}{P_1^{1-\rho}}Y_1, \tag{2.3}$$

where $P_1 = (wP_{d_1}^{1-\rho} + (1-w)P_{m_1}^{1-\rho})^{\frac{1}{1-\rho}}$, P_{m_1} is the price index associated to the foreign varieties (similar to P_{d_1}) and Y_1 is net domestic income, which is defined as $Y_1 = y + \pi_1 + T_1$, where y is an endowment income, identical for both countries, π_1 is the total profits of the domestic firms and T_1 is the domestic government lump sum transfer to the consumers.[4] Similarly, demand for imported variety j is

$$m_{1j} = (1-w)\left(\frac{p_j^c}{P_{m_1}}\right)^{-\sigma} {n_2}^{(\sigma-1)\nu-1}\frac{P_{m_1}^{-\rho}}{P_1^{1-\rho}}Y_1. \tag{2.4}$$

We assume that all firms in both countries have identical constant marginal costs c and fixed production costs F. At the symmetric firm equilibrium, the monopolistic competition assumption generates the mark-up equation $p = \frac{c}{\alpha}$,where p is the producer price or monopoly price any variety (this is also typical of the Dixit-Stiglitz-Spence monopolistic competition framework). This price will not be affected by domestic or foreign government policy. Therefore, the domestic government can only affect domestic consumer prices.

2.2. *Small open economy unconstrained optimum*

In this section, we compute the small open economy unconstrained non-cooperative equilibrium. In the unconstrained optimum, governments have three policy instruments: a subsidy to domestic consumption, an imports tariff and a fixed cost subsidy to the domestic firms. Governments will then be able to choose the number of domestic firms, their objective will be to maximize welfare subject to the government's budget constraint, for country 1 this can be expressed as follows (similarly for country 2):

$$T_1 = \sum_{i=1}^{n_1}(p_i^c - p)d_i + \sum_{j=1}^{n_2}\left(p_j^c - p\right)m_j - n_1 F. \tag{2.5}$$

[4]The assumption that governments can use lump-sum taxation to finance their industrial policy implies that their optimal policy will be determined by the welfare effect, not by the government revenue needs.

We represent the subsidies as a wedge between producer and consumer prices. Thus, $(p_i^c - p)$ is the subsidy to variety i and $\left(p_j^c - p\right)$ is the tariff on the imports of variety j. Finally, the government pays the fixed cost to domestic producers through lump-sum transfers $n_1 F$.

At the symmetric equilibrium, the domestic and foreign price indexes are $P_{d_1} = {n_1}^{-\nu} p_{d_1}^c$ and $P_{m_1} = {n_2}^{-\nu} p_{m_1}^c$ respectively. Here, $p_{d_1}^c$ and $p_{m_1}^c$ represent the consumer prices for domestic and imported varieties at the symmetric equilibrium (in what follows, we drop subindexes i and j to represent symmetric firm values). We can then write the welfare maximization problem for country 1 as that of maximizing its indirect utility function

$$V_1(n_1, n_2, p_{d_1}^c, p_{m_1}^c, Y_1) = \left[w \left({n_1}^{-\nu} p_{d_1}^c \right)^{1-\rho} + (1-w) \left({n_2}^{-\nu} p_{m_1}^c \right)^{1-\rho} \right]^{\frac{-1}{1-\rho}} Y_1, \quad (2.6)$$

where $Y_1 = y + n_1(p-c)(m_2 + d_1) + T_1$, subject to the budget constraint

$$T_1 = n_1(p_{d_1}^c - p)d_1 + n_2 \left(p_{m_1}^c - p\right) m_1 - n_1 F.$$

Substituting T_1 in Y_1, we can rewrite income as:

$$Y_1 = y + n_1(p-c)m_2 + n_1 \left(p_{d_1}^c - c\right) d_1 + n_2 \left(p_{m_1}^c - p\right) m_1 - n_1 F. \quad (2.7)$$

Our small open economy assumption means that the government does not take into account the effect of $n_1$on m_2. The first order conditions are:

$$\frac{\partial V_1}{\partial p_{d_1}^c} + \frac{\partial V_1}{\partial Y_1} \left(n_1 d_1 + n_1 (p_{d_1}^c - c) \frac{\partial d_1}{\partial p_{d_1}^c} \right) = 0, \quad (2.8)$$

$$\frac{\partial V_1}{\partial p_{m_1}^c} + \frac{\partial V_1}{\partial Y_1} \left(n_2 m_1 + n_2 (p_{m_1}^c - p) \frac{\partial m_1}{\partial p_{m_1}^c} \right) = 0, \quad (2.9)$$

$$\frac{\partial V_1}{\partial n_1} + \frac{\partial V_1}{\partial Y_1} \frac{\partial Y_1}{\partial n_1} = 0. \quad (2.10)$$

Note that, applying Roy's identity[5] to (2.8) and (2.9), we get

[5] From Roy's identity ($x_i(p, Y)$ being a demand function) we know that

$$\frac{\partial V(p,Y)}{\partial p_i} + \frac{\partial V(p,Y)}{\partial Y} x_i(p, Y) = 0.$$

$$n_1(p^c_{d_1} - c)\frac{\partial d_1}{\partial p^c_{d_1}} = 0, \tag{2.11}$$

$$n_2(p^c_{m_1} - p)\frac{\partial m_1}{\partial p^c_{m_1}} = 0. \tag{2.12}$$

The above implies that the government will implement a consumption subsidy to make consumer price for domestic varieties equal to the marginal cost of production $p^c_{d_1} = c$, and the government will not implement a tariff, $p^c_{m_1} = p$. The reasons are that, under monopolistic competition, the government cannot use a tariff to reduce the price of foreign firms and since we have lump-sum taxation, we do not need to levy a tariff to generate revenue. This is also a feature of the Dixit-Stiglitz-Spence monopolistic competition framework.

We now turn to equation (2.10), there, $\frac{\partial V_1}{\partial n_1}$, represents the 'taste for variety' effect and $\frac{\partial V_1}{\partial Y_1}\frac{\partial Y_1}{\partial n_1}$ represents an income effect. To calculate the impact of a variation in the number of firms on domestic income $\frac{\partial Y_1}{\partial n_1}$, first note that substituting $p^c_d = c$ and $p^c_m = p$ in equation (2.7), we obtain

$$Y_1 = y + n_1(p - c)m_2 - n_1 F, \tag{2.13}$$

where

$$m_2 = (1 - w){n_1}^{-\nu-1+\rho\nu}\frac{\left(p^c_{m_2}\right)^{-\rho}}{w\left({n_2}^{-\nu}p^c_{d_2}\right)^{1-\rho}+(1-w)\left({n_1}^{-\nu}p^c_{m_2}\right)^{1-\rho}}Y_2. \tag{2.14}$$

Note that, given exports, an increase in number of varieties increases total profits, but, as can be seen from equation (2.14), changes in the number of the varieties would also affect the demand to each firm. For simplicity, we assume that individual countries are small in the sense that individual governments do not take into account the effect of a variation in n_1 on m_2.[6] In this case,

[6] The small country assumption can be justified as the limit of an economy with N countries. Variable export profits for country 1 would then be

$$\pi_1 = n_1(p - c)D_1(p)\frac{Y_1}{N}(N - 1),$$

where $D_1(p)$ is a general demand function and p is vector of prices. Then, the derivative of variable profits with respect to n_1

$$\frac{\partial \pi_1}{\partial n_1} = (p - c)D_1(p)\frac{Y_1}{N}(N - 1) + n_1(p - c)\frac{\partial D_1(p)}{\partial p_1}\frac{\partial p_1}{\partial n_1}\frac{Y_1}{N}(N - 1).$$

$$\frac{\partial Y_1}{\partial n_1} = (p-c)m_2 - F, \tag{2.15}$$

that is, an increase in the number of varieties increases the variable profits made by all the domestic firms through exports. Finally, introducing (2.15) in (2.10), substituting $p = \frac{c}{\alpha}$ and using all the first order conditions, we obtain the number of firms per country in the small open economy unconstrained equilibrium (see Appendix for details):

$$n^u = \frac{y\left[\alpha^{1-\rho} v w + (1-\alpha)(1-w)\right]}{F\left[\alpha^{1-\rho}(1+v)w + (1-w)\right]}. \tag{2.16}$$

3. Comparison of equilibria

Table 1 presents the number of varieties in the open economy unconstrained optimum, the market economy equilibrium n^m, the closed economy unconstrained equilibrium n^c (obtained by setting $w = 1$ in n^u) and the world first best n^s (with and without the Benassy effect).

Table 1. Small economy equilibrium number of firms.

	Benassy case: $\nu \neq \frac{1-\alpha}{\alpha}$	Dixit-Stiglitz case: $\nu = \frac{1-\alpha}{\alpha}$
Market economy equilibrium	$n^m = \frac{y(1-\alpha)}{F}$	$n^m = \frac{y(1-\alpha)}{F}$
Closed economy unconstrained optimum	$n^c = \frac{y\nu}{(1+\nu)F}$	$n^c = \frac{y(1-\alpha)}{F}$
World first best	$n^s = \frac{y\nu}{(1+\nu)F}$	$n^s = \frac{y(1-\alpha)}{F}$
Open economy unconstrained optimum	$n^u = \frac{y\left[\alpha^{1-\rho}vw+(1-\alpha)(1-w)\right]}{F[\alpha^{1-\rho}(1+v)w+(1-w)]}$	$n^u = \frac{y(1-\alpha)}{F}$

The world first best can be obtained using standard maximization procedure (see Appendix for a details). The Dixit-Stiglitz case for each equilibrium type is

Now, if $\lim_{N\to\infty} \frac{\partial D_1(p)}{\partial p_1} = 0$, then $\lim_{N\to\infty} \frac{\partial \pi_1}{\partial n_1} = (p-c)D_1(p)Y_1$. Therefore, the strategic effect of the variation in prices disappears. To obtain this result, it is important that we replicate the economy keeping the demand constant (see Chari and Kehoe, 1990).

obtained by setting $\nu = \frac{1-\alpha}{\alpha}$. It then becomes apparent that in the Dixit-Stiglitz case, firm numbers coincide at all the equilibrium types.[7]

Note that $\frac{dn^u}{dv} > 0$, therefore, the Benassy effect will increase the unconstrained optimum number of firms (relative to the Dixit-Stiglitz case) if $\nu > \frac{1-\alpha}{\alpha}$ and decrease it if $\nu < \frac{1-\alpha}{\alpha}$. This is consistent with the idea that these two cases represent more and less love for variety that in the Dixit-Stiglitz case.

The following results illustrate the impact of the Benassy effect on the efficiency of the market equilibrium, the impact of openness on firm numbers and the potential gains from international policy coordination.

Proposition 1. *If* $\nu > \frac{1-\alpha}{\alpha}$ $\left(\nu < \frac{1-\alpha}{\alpha}\right)$, *the unconstrained small open economy optimum number of firms is higher (lower) than the number of varieties generated by the market equilibrium* $n^u > n^m$ $(n^u < n^m)$.

Therefore, the Benassy effect ($\nu \neq \frac{1-\alpha}{\alpha}$) makes the small open economy market equilibrium inefficient. The intuition behind this result becomes apparent if we note that $\frac{1-\alpha}{\alpha}$ measures the magnitude of profits, whereas ν measures the social benefit of introducing new varieties.

Proposition 2. *If* $\nu > \frac{1-\alpha}{\alpha}$ $\left(\nu < \frac{1-\alpha}{\alpha}\right)$, *an increase in openness (a lower* w*) reduces (increases) the optimal number of firms* $n^c > n^u$ $(n^c < n^u)$ *in a small open economy.*

The above result implies that, with Benassy ($\nu \neq \frac{1-\alpha}{\alpha}$), changes in openness affect firm numbers. That is, the Benassy effect makes home bias relevant to the open economy unconstrained optimum. It is interesting to note that the increase in the size of the market could lead to a reduction in the number of firms.

This result is consistent with observed changes in concentration in industries where government subsidies are still prevalent. One such case is the military industry, where recent increases in openness have led to increases in concentration. This would suggest that a Benassy setting with $\nu > \frac{1-\alpha}{\alpha}$ would be a suitable representation of preferences in such industry (see Dunne et al., 2005).

Proposition 3. *If* $\nu > \frac{1-\alpha}{\alpha}$ $\left(\nu < \frac{1-\alpha}{\alpha}\right)$, *the unconstrained small open economy optimum number of firms is lower (higher) than the world first best* $n^u < n^s$ $\left(n^u > n^s\right)$.

[7] Note that the reason why n^u and n^m coincide is that (unlike the original Dixit-Stiglitz (1977)) we do not include a numeraire. The purpose of this simplification is to illustrate the impact of the Benassy on the equilibrium number of firms in the different equilibria in a neat way.

The above result shows that the unconstrained optimum is generally inefficient and there are then potential gains to be made from international coordination. The reason is that governments only take into account the effect of an increase in variety on the domestic consumer $w\nu$. This effect reduces the number of varieties below the global social optimum. At the same time, firms obtain monopolistic profits by exporting to the other country, the government tries to increase this profit, this is the reason by the number of varieties n^u, depends both on the love for variety parameter and on the mark-up and may be both above or bellow the first best.

4. Conclusions

In this paper, we have extended the Benassy (1996) framework to an open economy. Using a simple utility function, we provide a succinct illustration of the impact of the Benassy effect in an open economy. We prove that with Benassy, the small open economy market equilibrium becomes inefficient, openness reduces the varieties provided in the unconstrained optimum and there is potential gains from international coordination. If we relax the small economy assumption, the expression for the unconstrained optimum becomes more complex but, still, the Benassy effect, a higher ν, increases the number of firms. The market equilibrium, the closed economy optimum and the world first best remain (for any ν) as before. However, the unconstrained optimum for $\nu = \frac{1-\alpha}{\alpha}$ changes and hence, it is different from the other equilibria. Therefore, although the market equilibrium will still generate the world first best, there will be potential gains from policy coordination if countries try to maximize welfare individually, even if the Benassy effect is not present.

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6. Appendix

6.1. Small economy unconstrained optimum

First, we obtain the derivatives corresponding to the first order condition on the number of firms:

$$\frac{\partial V_1}{\partial n_1} + \frac{\partial V_1}{\partial Y_1}\frac{\partial Y_1}{\partial n_1} = 0.$$

Partially differentiating (2.6) we have

$$\frac{\partial V_1}{\partial n_1} = vwY_1\left(n_1^{v(\rho-1)-1}\right)\left(p_{d_1}^c\right)^{1-\rho}\left[w\left(n_1^{-v}p_{d_1}^c\right)^{1-\rho} + (1-w)\left(n_2{}^{-\nu}p_{m_1}^c\right)^{1-\rho}\right]^{\frac{-1}{1-\rho}-1},$$

$$\frac{\partial V_1}{\partial Y_1}\frac{\partial Y_1}{\partial n_1} = \left[w\left(n_1^{-v}p_{d_1}^c\right)^{1-\rho} + (1-w)\left(n_2{}^{-\nu}p_{m_1}^c\right)^{1-\rho}\right]^{\frac{-1}{1-\rho}}\frac{\partial Y_1}{\partial n_1}.$$

Given that $p_d^c = c$ and $p_m^c = p$, total income is

$$Y_1 = y + n_1(p-c)m_2 - n_1F.$$

When we differentiate Y_1, we assume that the country is small, therefore, the government does not take into account the effect of $n_1$on m_2. In this case,

$$\frac{\partial Y_1}{\partial n_1} = (p-c)m_2 - F.$$

Note that

$$m_2 = (1-w)\left(\frac{p_{m_2}^c}{P_{m_2}}\right)^{-\sigma} n_1{}^{(\sigma-1)\nu-1}\frac{P_{m_2}^{-\rho}}{P_2^{1-\rho}}Y_2.$$

Substituting for all the price indexes we get

$$m_2 = (1-w)n_1{}^{-\nu-1+\rho\nu}\frac{\left(p_{m_2}^c\right)^{-\rho}}{w\left(n_2{}^{-\nu}p_{d_2}^c\right)^{1-\rho} + (1-w)\left(n_1{}^{-\nu}p_{m_2}^c\right)^{1-\rho}}Y_2.$$

Next, we introduce $\dfrac{\partial Y_1}{\partial n_1}$ into the first order condition for n_1 to obtain

$$\frac{vwY_1\left(n_1^{v\rho-v-1}\right)(c)^{1-\rho}}{w\left(n_1^{-v}c\right)^{1-\rho} + (1-w)\left(n_2{}^{-\nu}p\right)^{1-\rho}} + (p-c)m_2 - F = 0.$$

In the symmetric equilibrium, we get

$$\frac{vwc^{1-\rho}}{n\left[w\left(c\right)^{1-\rho}+\left(1-w\right)\left(p\right)^{1-\rho}\right]}Y+(p-c)m-F=0.$$

where the imports demand and budget constraint equations become

$$m=\frac{\left(1-w\right)\left(p\right)^{-\rho}}{n\left[w\left(c\right)^{1-\rho}+\left(1-w\right)\left(p\right)^{1-\rho}\right]}Y.$$

and

$$Y=y+n(p-c)m-nF.$$

Substituting m into Y, we get

$$Y=\frac{\left[y-nF\right]\left[w\left(c\right)^{1-\rho}+\left(1-w\right)\left(p\right)^{1-\rho}\right]}{\left[w\left(c\right)^{1-\rho}+c\left(1-w\right)\left(p\right)^{-\rho}\right]}$$

and substituting the above into m, we get

$$m=\frac{\left[y-nF\right]\left(1-w\right)p^{-\rho}}{n\left[w\left(c\right)^{1-\rho}+c\left(1-w\right)\left(p\right)^{-\rho}\right]}.$$

We now substitute the above two equations for Y and m in the first order condition for n_1 to get

$$\left[vwc^{1-\rho}+(p-c)(1-w)p^{-\rho}\right]\frac{\left[y-nF\right]}{n\left[w\left(c\right)^{1-\rho}+c\left(1-w\right)\left(p\right)^{-\rho}\right]}-F=0,$$

which gives

$$n=\frac{\left[vwc^{1-\rho}+\left(p-c\right)\left(1-w\right)\left(p\right)^{-\rho}\right]y}{\left[\left(1+v\right)w\left(c\right)^{1-\rho}+\left(1-w\right)\left(p\right)^{1-\rho}\right]F}.$$

The final expression for n^u, (2.16), is obtained by substituting $p=\frac{c}{\alpha}$ above.

6.2. World social planner optimum

We drop subindexes to indicate a world social planner who, since countries are identical, will choose the same variables for both countries from the outset to maximize the utility of the representative country U

$$U=n^{\nu+1}\left[w^{\frac{1}{\rho}}\left(d\right)^{\frac{\rho-1}{\rho}}+\left(1-w\right)^{\frac{1}{\rho}}\left(m\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}};$$

subject to:

$$y = nc(m+d) + nF.$$

The Lagrangian is:

$$L = n^{\nu+1}\left[w^{\frac{1}{\rho}}(d)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} + \lambda\left[y - nc(m+d) - nF\right].$$

and the focs are:

$$\frac{\partial L}{\partial d} = n^{\nu+1}\frac{\rho}{\rho-1}\left[w^{\frac{1}{\rho}}(d)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}-1}\left[w^{\frac{1}{\rho}}\frac{\rho-1}{\rho}(d)^{\frac{\rho-1}{\rho}-1}\right] - \lambda nc = 0,$$

$$\frac{\partial L}{\partial m} = n^{\nu+1}\frac{\rho}{\rho-1}\left[w^{\frac{1}{\rho}}(d)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}-1}\left[(1-w)^{\frac{1}{\rho}}\frac{\rho-1}{\rho}(m)^{\frac{\rho-1}{\rho}-1}\right] - \lambda nc = 0,$$

$$\frac{\partial L}{\partial n} = (\nu+1)\, n^{\nu}\left[w^{\frac{1}{\rho}}(d)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} - \lambda\left[c(m+d)+F\right] = 0,$$

$$\frac{\partial L}{\partial \lambda} = 0 : y = nc(m+d) + nF.$$

Dividing the first and second focs above and simplifying we get

$$m = \frac{(1-w)}{w}d,$$

which we introduce in the budget constraint to find

$$d = w\frac{(y-nF)}{nc},$$

$$m = (1-w)\frac{(y-nF)}{nc}.$$

Now, we divide the second and third equations to get

$$\frac{\left[(1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}-1}\right]}{(\nu+1)\left[w^{\frac{1}{\rho}}(d)^{\frac{\rho-1}{\rho}} + (1-w)^{\frac{1}{\rho}}(m)^{\frac{\rho-1}{\rho}}\right]} = \frac{c}{[c(m+d)+F]}$$

and substitute d and m to find the optimal number of firms per country

$$n^s = \frac{y\nu}{(\nu+1)F}.$$

as in Table 1.