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Does the world real interest rate affect the real exchange rate? The South East Asian experience*

Karine Gente† and Miguel León-Ledesma‡

Abstract: We analyze the consequences of US real interest rate rises on the real exchange rate (RER) in a two-good overlapping generations model of a semi-small open economy. The equilibrium RER depreciates (appreciates) when the world interest rate increases in a debtor (creditor) country. We then study empirically the reaction of the RER in a set of South East Asian (SEA) countries to shocks in US real interest rates. The results support the conclusions of the theory model at least for Singapore, Thailand and South Korea during the period 1980-2001.

Key-words: Real exchange rate; overlapping generations; world interest rate shock.

Classification J.E.L.: D91; F31; F41.

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1. Introduction

Globalization has increased the inter-dependency between countries. Having a perfect access to the world capital market, small open economies have become highly dependent on the fluctuations of the world interest rate. For instance, the moderate rise in the US interest rate preceding the Asian Crisis is often considered as a factor which may have partly caused the crisis. Even if structural national factors such as a bad management of the banking system or bad quality loans are the main causes of the crisis, the US interest rate rise was considered as a push factor by the IMF\(^1\). Another example could be the rise in the German interest rate in 1990 preceding the two European exchange rate mechanism (ERM) crises. Hence, changes in the world interest rate in economies highly integrated in international financial markets may trigger currency and banking crises that could hinder the growth and transformation prospects of emerging markets.

This article deals with the influence of such world interest rate changes on small open economies. In a real setting, we study the equilibrium real exchange rate (RER) reaction to a rise in the world interest rate. We show that such a rise may lead to persistent misalignment [Hinkle and Montiel (1999)] if some rigidities prevent the current RER from being at its equilibrium level. This has notably been the case in South East Asian (SEA) countries whose currencies were pegged to the US Dollar at the time of the crisis or in the ERM in the early 1990s.

\[\text{Figure 1: Nominal and Real Exchange Rates}\]

\[\text{Thailand} \quad \text{South Korea} \quad \text{Malaysia} \quad \text{Singapore}\]

\(^1\text{See IMF World Economic Outlook (1997).}\)
As depicted by Figure 1, in SEA countries the RER is not fully flexible since nominal and real exchange rates seem to be highly correlated. A rise in the world interest rate that changes the equilibrium RER not followed immediately by the actual RER could then lead to an overvaluation in a debtor country. The aim of this paper is to identify some changes in RER caused by interest rate shocks which lead to misalignments. Such misalignments could entail crises.²

A few papers have been interested in studying the link between exchange rates and world interest rate. Agénor (1998) and Kollmann (2001), for instance, investigate the effects of a world interest rate shock on the nominal exchange rate in a monetary setting. However, they focus on the short-run behavior. Moreover, in an infinitely lived agents setting they do not distinguish the reaction in debtor and creditor countries. The main theoretical contribution of our paper is to use an overlapping generations model (OLG). Indeed, with selfish agents, the domestic rate of time preference need not be equal to the world interest rate. A low (high) time preference characterizes a creditor (debtor) country vis-à-vis the rest of the world. Hence, we confirm the intuition according to which the equilibrium RER reaction is different between a creditor and a debtor country.³

We consider a semi-small open economy as in Sen and Turnovsky (1989a), (1989b), (1990). This economy can borrow and lend at a given world real interest rate and purchase imported goods at given world prices, but it faces a downward sloping demand schedule for its exports because they are perceived to be imperfect substitutes for the tradable goods of other countries⁴. Thus, there is only one sector and the equilibrium RER clears leading to currency crises.

²See Kaminski et al (1998). These authors find misalignments of the RER as an important factor among crises.
³A related argument is developed in Kraay and Ventura (2000) in which the reaction of the current account to transitory shocks is different depending on the credit position of the country. Their setting, however, does not introduce overlapping generations.
⁴This concept of a semi-small open economy has become popular in CGE trade modelling literature. Appelbaum and Kohli (1979) provide empirical support for the semi small open economy hypothesis for Canada.
the domestic market. Investment is entirely of domestic origin. Consequently, the capital stock adjusts progressively to its long-run equilibrium level.

A rise in the world interest rate leads to a spread between domestic and world return on capital and hence following the interest rate parity the RER appreciates during adjustment in both creditor and debtor countries. This is a standard dynamic result which emerges even without the OLG structure. The main contribution of the OLG structure [Blanchard (1985)] concerns the steady-state RER reaction. Indeed, the long-run relation between the RER and the world interest rate is positive (negative) when the country is a debtor (creditor): the equilibrium RER depreciates (appreciates) following a rise in the world interest rate in a debtor (creditor) country.

From a reduced form derived from the theory we estimate an equilibrium RER with data for four SEA countries between 1980 and 2001. In most cases, the equilibrium RER is consistent with the theory and we find that the real US interest rate enters significantly a cointegrating vector of the RER, productivity and government expenditure. The impact of the US interest rate differs in creditor and debtor countries. We then use impulse response analysis to analyze the dynamic adjustment for the RER, and show that shocks to the US interest rate can trigger relatively long lived misalignments. This is supportive evidence for the common idea according to which the US interest rate rise of 1997 may have partly caused the SEA crisis especially in highly indebted countries such as Thailand.

The paper proceeds as follows. Section 2 develops the model and the theoretical effects of a permanent unanticipated rise in world interest rate. Section 3 presents the econometric tests. Section 4 provides some conclusions.

\(^5\)Nurkse (1945) defines the equilibrium RER as the long-run RER that is the RER consistent with the dual objectives of external and internal balance. In the model we develop, the labor market is always cleared, so the equilibrium RER is the long-run RER that is the level of the RER compatible with a constant stock of net foreign assets.
2. The Model

We present a two-good OLG model of a semi-small open economy. We assume that the country is big enough to influence its exports price but small in the world economy. Thus, the world interest rate $\bar{r}$ is given.

The economy consists of cohorts of Blanchard’s heterogeneous agents and a representative firm. At each instant, $p$ agents enter the economy with zero non-human wealth and $p$ agents leave, hence the size of the population is constant and normalized to one. The firm produces a unique commodity from a neoclassical production technology $F$ using inelastic labor supply $L$ and capital $K$. The aggregated consumption spending $\pi C$ is allocated between two substitutable commodities with Cobb-Douglas preferences: a domestically produced good $X$ and an imported good $Y$. $R$ is the relative price of the imported good in terms of the domestic good. All quantities are expressed in units of the domestic good. Under these conventions, a rise in $R$ means a real effective exchange rate depreciation.

Hereafter, we present briefly the model. We study the theoretical long-run effects of a rise in the world interest rate and describe the dynamics by means of a calibration exercise.

2.1. Dynamics

Agents maximize their expected utility choosing both the time path of total consumption spending $\pi_t c_t$ and its allocation over the imported good $y_t$ and the domestic good $x_t$. We assume instantaneous Cobb-Douglas preferences. Let a capital letter $S$ stand for the aggregate variable $s$: aggregate variables are obtained by summing individual variables weighted by the number of individuals alive in each cohort at time $t$: $S(t) = \int_{-\infty}^{t} s(\sigma, t)pe^{-p(\sigma-t)}d\sigma$. Then, the aggregate consumption of the domestic good $X$ is an $\alpha$ share of the total consumption spending $\pi C$. Let $H$ denote the aggregated human wealth and $A$ denote the aggregated non-human wealth, we have $\pi C = \gamma [A + H]$ since the propensity to consume out of wealth $\gamma$ is age independent and equal to the effective discount rate $\beta + p$ that is the domestic rate of time preference $\beta$ plus the age independent

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6The instantaneous preferences are $c = x^\alpha y^{1-\alpha}$, $0 < \alpha < 1$. 5
risk of death $p$. Substituting the dynamics of human wealth $\dot{H} = (r + p) H - (1 - \tau) w$ where $p$ the risk of death is constant we obtain

$$\dot{X} = [r - \beta] X - p\gamma A$$  \hspace{1cm} (2.1)$$

Under the assumption of constant returns to scale, we have that $\theta F_K (K, L) = r + \delta$ and $\theta F_L (K, L) = w$ where $\theta$ denotes the total factor productivity, $r$ the domestic interest rate, $\delta$ the rate of capital depreciation and $w$ the wage. Since the labor supply is inelastic and population is constant and normalized to unity, the labor market is always cleared and $L = 1$. Under perfect capital mobility, the domestic interest rate is defined according to the Interest Rate Parity (IRP) relation $r = \bar{r} + \dot{R}/R$

$$\dot{R} = R [\theta F_K (K, 1) - (\delta + \bar{r})]$$  \hspace{1cm} (2.2)$$

with $\delta$ the rate of capital depreciation and $\bar{r}$ the world interest rate.

Production is either sold to residents who consume $X$ and invest $I$, or sold to the government $G_x$ or exported $Z (R)$

$$\dot{K} = \theta F (K, 1) - X - G_x - Z (R) - \delta K$$  \hspace{1cm} (2.3)$$

with $G_x = \nu G$ because government allocates a $\nu$-share of the global public spending $G = \tau \theta F_L (K, 1)$ to the consumption of the domestic good. Since there is no government debt the government has a balanced budget such that $G = \tau \theta F_L (K, 1)$.

Aggregate financial wealth $A$ consists of foreign assets expressed in units of the domestic good $RB$ and the capital stock $K$. Unless otherwise slated, quantities are expressed in units of the domestic good. We deduce the current account as $\dot{B} = \left[ \dot{A} - \dot{K} - \dot{RB} \right] / R$ and

$$\dot{B} = \bar{r} B + \frac{1}{R} \left[ Z (R) - \frac{1 - \alpha}{\alpha} X \right]$$  \hspace{1cm} (2.4)$$

7For simplicity we assume that government debt is zero. Nevertheless, this setting could be appropriate to study the relation between the RER and public debt because there is no debt-neutrality. Then, a public deficit would cause a current account deficit and finally a depreciation of the RER.

8Integrating the individual budget constraint $da (s, z) / dz = (r (z) + p) a (s, z) - \pi (z) c (s, z) + (1 - \tau) w (z)$ $\forall z \geq t$, $\lim_{z \to \infty} a (s, z) e^{-(r (z) + p) z} \geq 0$ the net financial wealth is $\dot{A} = rA + (1 - \tau) w - \pi C.$
Finally, this dynamic system has four variables $B, K, R, X$, two of these ($R$ and $X$) are forward ones. This system is not separable because from (2.3) the domestic good is both used for consumption, exports and investment implying that dynamics of capital and foreign assets are linked.

2.2. Steady State

A star * denotes the steady-state values. The long-run equilibrium exists when\(^9\) $\bar{r} \in ]0, r_p[$ with

$$r_p = \frac{\beta}{2} + \frac{\sqrt{\beta^2 + 4p\gamma (1 - \alpha)}}{2}$$

Then, this long-run equilibrium is unique and satisfies

$$\bar{r} + \delta = \theta F_K (K^*, 1) \tag{2.5}$$

$$X^* = \alpha \Gamma (1 - \tau) \theta F_L (K^*, 1) \tag{2.6}$$

$$R^* B^* = \frac{(\bar{r} - \beta) (1 - \tau) \Gamma \theta F_L (K^*, 1)}{p\gamma} - K^* \tag{2.7}$$

$$Z(R^*) = \bar{r} K^* + \theta F_L (K^*, 1) [1 - \alpha \Gamma + \tau (\alpha \Gamma - \nu)] \tag{2.8}$$

with the long-run propensity to consume out of wealth $\Gamma = \frac{p\gamma [p\gamma - \bar{r} (\bar{r} - \beta)]^{-1}}{\bar{r} K^*}$.

Equation (2.5) states that the marginal productivity of capital net of depreciation is equal to the world interest rate. Equation (2.6) expresses the long-run consumption of the domestic good as a function of the wage. From equation (2.7), the net foreign assets position depend on labor income and the accumulated capital stock. Equation (2.8) determines the equilibrium RER that is the level of the RER consistent with a constant

\(^9\)This condition guarantees that the dynamic system is saddlepath stable ($\bar{r} < p + \beta$ see Appendix 1) and that the exports are always positive. Indeed, we must impose that the share of domestic goods in consumption ($1 - \alpha$) is not too important to have $Z(R) > 0$. 

7
stock of net foreign assets. Assuming the economy consists of Blanchard’s agents, the world interest rate and the rate of time preference need not to be equal. In this setting, the spread between $\bar{r}$ and $\beta$ is a determinant of the country’s net financial position vis-à-vis the rest of the world. We show that a rise in the world interest rate will have different effects on RER whether the rate of time preference $\beta$ exceeds or not the world interest rate $\bar{r}$.

From equations (2.5) and (2.8), there exists a long-run relation between the RER, public spending (through the parameter $\tau$), productivity $\theta$ and world interest rate $\bar{r}$. From (2.5) we have $K^* = \phi(\bar{r})$ with $\phi = F_{\bar{r}}^{-1}$ and $\phi' < 0$. For small changes on $\bar{r}, \theta$ and $\tau$ the new long-run real exchange rate $R^*_1$ could be approximated by

$$R^*_1 - R^* = \left[ \frac{\phi(\bar{r}) + \bar{r}\phi'(\bar{r}) + \Theta \theta F_{L\bar{r}}(K^*, 1) \phi'(\bar{r})}{Z'(R^*)} (r_1 - \bar{r}) \right.
\left. + \frac{\theta F_{\bar{r}}(K^*, 1) (\alpha \Gamma - \nu)}{Z'(R^*)} (\tau_1 - \tau) + \frac{\Theta F_{\bar{r}}(\phi(\bar{r}), 1)}{Z'(R^*)} (\theta_1 - \theta) \right]$$

with $\nu_1$ the new level of variable $\nu$, $\nu = \bar{r}, \theta, \tau$. Table 1 presents the influence of these parameters on $R^*$

<table>
<thead>
<tr>
<th>$R^*$</th>
<th>$\theta$</th>
<th>$\tau$</th>
<th>$\bar{r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>+ if $\bar{r} &lt; \frac{\beta}{2}$</td>
<td>- if $\bar{r} &gt; \frac{\beta}{2}$</td>
</tr>
</tbody>
</table>

A rise in productivity $\theta$ increases the supply of the domestic good and the agents income. However, since agents allocate a $\alpha$ share of their consumption spending to the imported good, an increase in productivity leads to a rise in exports meaning that the RER depreciates. Of course, the scale of this positive effect depends on the share of domestic goods in total consumption spending. Unlike in Balassa-Samuelson-type models we obtain that a positive productivity shock leads to a real depreciation because the model is unisectoral. A rise in $\tau$ with a balanced budget corresponds to an increase in public spending. This rise in public spending may lead to RER appreciation when $\nu > \alpha$. In this case
case, an increase in public spending makes the domestic good less available for exports because private agents consume less domestic goods than government. Hence, a rise in \( \tau \) transfers consumption spending from the private agents to the government. Since the government consumes more domestic good than agents, the demand for domestic goods increases with \( \tau \) and hence the RER appreciates. Nevertheless, the relation between \( \tau \) and \( R^* \) may be reversed when \( \alpha > \nu \Gamma^{-1} \). This could be the case in a creditor country even if the government spends more on domestic good than agents (\( \alpha < \nu \)). Indeed, when the world interest rate exceeds the domestic rate of time preference, the long-run propensity to consume \( \Gamma \) is higher than unity. Hence, an increase in public spending may lead to a depreciation because agents consume a higher share of the wage at steady state. A rise in \( \tau \) transfers consumption from agents to government and, when agents are rich, consumption is reduced and so do exports leading to a RER depreciation.

The long-run relationship between \( R^* \) and \( \bar{r} \) is less trivial. Indeed, the RER response to a world interest rate shock depends mainly on the time preference \( \beta \). We have

\[
Z(R^*) = \theta F(K^*, 1) - \delta K^* - X^* - \tau \theta F_L(K^*, 1)
\]

(2.9)

On the one hand, the government spending and the supply net of capital depreciation always decrease after a rise in the world interest rate. On the other hand, the domestic consumption reaction depends on the rate of time preference \( \beta \). As a result, the real exchange rate reaction depends on the relative magnitude of these two effects\(^{10}\). Since, the response of consumption depends on the level of \( \beta \), so does the real exchange rate. We characterize the long-run relation between \( R^* \) and \( \bar{r} \) assuming that \( \tau = 0 \) and \( \theta = 1 \).

\(^{10}\)We have that \( \partial X^*/\partial \bar{r} = \left[ \Gamma \alpha (\partial w^*/\partial \bar{r}) + w^* \Gamma (\nu \gamma)^{-1} \alpha (2 \bar{r} - \beta) \right] (1 - \tau) \). Hence, the consumption reaction depends on \( \beta \). Indeed, the first term of the derivative is a negative income effect while the second one is positive (resp. negative) when the time preference is low (resp. high). Hence, the domestic consumption increases more when the time preference is low (\( \beta < 2 \bar{r} \)). The figure 1 shows that when \( \beta > 2 \bar{r} \), the second effect could become so large that the domestic consumption rises.
\[ F(K, L) = K^\rho L^{1-\rho} \] is given by
\[
\frac{\partial Z(R^*)}{\partial \bar{r}} = \left( \frac{\Gamma}{p\gamma} \right)^2 \left( \frac{-\bar{r}\Omega_1(\bar{r}) + \Omega_2(\bar{r})}{(\bar{r} + \delta)(1 - \rho)} \right)
\] (2.10)

with \( \Omega_1(\bar{r}) = [p\gamma\Gamma^{-1}]^2 \) and \( \Omega_2(\bar{r}) = (\bar{r} + \delta)(1 - \rho) p\gamma \alpha [p\gamma\Gamma^{-1} + (\beta - 2\bar{r})(1 - \rho) \rho^{-1}(\bar{r} + \delta)] \).

The first term \(-\bar{r}\Omega_1(\bar{r})\) always negative represents the systematic fall in production that makes the domestic good less available for export. The second term \(\Omega_2(\bar{r})\) has an ambiguous sign and two distinct parts. First, \(p\gamma\Gamma^{-1} > 0\) indicates the negative effect of an increase in \(\bar{r}\) on the wage: the smaller \(\beta\) is the more this fall in the wage reduces the consumption the more the RER tends to depreciate. Second, the term \((\beta - 2\bar{r})(1 - \rho) \rho^{-1}(\bar{r} + \delta)\) depends on the spread between \(\bar{r}\) and \(\beta\). When the time preference is high \((\beta > 2\bar{r})\), the \(\Omega_2(\bar{r})\) term is positive: the long-run propensity to consume out of wealth decreases \(\partial\Gamma/\partial\bar{r} = -\Gamma^2(\beta - 2\bar{r}) < 0\). The fall in domestic consumption exceeds the fall in production and hence the real exchange rate depreciates. This result is very intuitive because the higher \(\beta\) is, the more the agents go into debt and the more they experiment an increase in the debt service.

On the other hand, when the time preference is low \((\beta < 2\bar{r})\), \(\Omega_2(\bar{r})\) becomes quickly\(^{11}\) negative and dominant. An increase in world interest rate has a positive effect on the long-run propensity to consume since the agents benefit from a higher return on net foreign assets. Then, domestic consumption increases or decreases less than the fall in production. Hence, the domestic good is less available for exports and the real exchange rate appreciates. These effects are represented on Figure 2 and are summed up in the

\(^{11}\text{We have that } \Omega_2(\bar{r}) < 0 \text{ if } \bar{r} \in [0, r_p] \text{ and } \beta < 2\delta (1 - \rho). \text{ Besides, } \Omega_2(0) > 0 \text{ and } \Omega_2(\beta/2) > 0 \text{ but } \Omega_2(r_p) < 0. \text{ Hence, there exists } \bar{r}_\Omega \in [\beta/2, r_p] \text{ such that } \Omega_2(\bar{r}_\Omega) = 0 \text{ (see Appendix 3.1).}\)
following proposition.

![Figure 2: Real exchange rate and real interest rate](image)

**Proposition** An increase in the world interest rate leads to an appreciation (resp. depreciation) of the REER when the time preference is low (resp. high): $\beta \leq 2\bar{r}$ (resp. $\beta > 2\bar{r}$).

(see Appendix 3.2 for the proof)

2.3. An unexpected rise in the world interest rate

To investigate the dynamic effects of a rise in the US interest rate we use a calibration based on the stable dynamic time paths obtained in Appendix 2. We simulate the model with a Cobb-Douglas production, $F(K, L) = K^\rho L^{1-\rho}$, $\theta = 1$ and a constant elasticity exports function $Z(R) = R^\eta$. Hereafter, we assume to simplify that $\nu = \alpha$. Table 2 collects the values used in our calibration.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\bar{r}$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$p$</th>
<th>$\rho$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02 / 0.05</td>
<td>0.027</td>
<td>0.5</td>
<td>0.1</td>
<td>0.015</td>
<td>0.33</td>
<td>3</td>
</tr>
</tbody>
</table>
The real world interest rate \( \bar{r} \) is around 2.7\% to match the mean value of the US real interest rate during the period 1981-2001. We use standard values of \( \alpha, \rho \) and \( \delta \). Life expectancy is around 67 years\(^{12}\) because it corresponds to the working life. To compare the creditor and debtor cases, \( \beta \) takes two different values: 2\% for the creditor country and 5\% for the debtor country\(^{13}\). Thus, when \( \beta \) is higher (resp. smaller) than the world interest rate, the country is a net debtor (resp. creditor). The world interest rate after the shock is \( \bar{r} = 0.028 \) implying a 5\% rise.

We are interested in describing the transition from one steady state to another after an unanticipated increase in the world interest rate of 5\%.

Figure 3 plots the RER, domestic consumption and net foreign assets transitions (continuous lines) to the new steady state (dotted lines) in both the debtor (right side) and creditor countries (left side). For the forward variables, the dash-dot lines represent the preceding steady state that is to say the long-run level of each variable before the shock. Numerical simulations show that all the variables except domestic consumption

\(^{12}\)The results are qualitatively similar with a shorter life expectancy provided that the stability condition \( \bar{r} < \beta + p \) is fulfilled.

\(^{13}\)These values of \( \beta \) are chosen to be different from \( \beta = 2 \bar{r} \) before and after the shock to avoid particular cases.
adjusts monotonically to their long-run levels (after initial jump in RER).

Figure 3: Transitional paths after a positive world interest rate shock of 5%

Since investment comes entirely from the domestic good, the capital stock adjusts gradually to its long-run level. Indeed, a permanent rise in world interest rate leads to a spread between home and foreign return on capital. Hence, the domestic interest rate experiences a monotonic rise since it converges toward its new long-run level and
the capital stock is reducing. This standard effect on capital entails a monotonic real appreciation according to the interest rate parity relation. Figure 3 does not depict the capital stock paths since the main differences between creditor and debtor countries is due to the fact that investment is entirely of domestic origin.

We know from the preceding sub-section that the equilibrium RER appreciates (depreciates) after the shock in a creditor (debtor) country. Let \( R_0^* \) be the initial steady state level. It follows that \( R(t) > R_0^* \) in a debtor country whereas \( R(t) < R_0^* \) in a creditor country. If we assume according to evidence (see Figure 1) that for exogenous reasons the current RER stays around \( R_0^* \) and does not adjust instantaneously to its equilibrium level \( R(t) \), the spread between \( R(t) \) and \( R_0^* \) represents misalignment. Then, the RER appears to be overvalued in a debtor country. In a creditor country, the scale of misalignment is smaller since it is first positive (until point \( E \) on Figure 3) and after negative.

Finally, the main consequences of the OLG structure is that we could consider either a debtor and a creditor country. In this setting, the RER values the net foreign assets and hence is a key-determinant of financial wealth \( A = RB + K \). Figure 3 shows that the initial RER jump is positive whatever \( \beta \) is. This means that the RER depreciates initially when world interest rate increases. Let \( R(0) \) be the initial level of RER, the net financial wealth is at time zero \( A(0) = R(0)B_0 + K_0 \) and hence \( \partial A(0)/\partial R(0) = B_0 \). In a creditor country, the real depreciation leads initially to a higher financial wealth. At the opposite, in the debtor country this depreciation increases the debt \( \text{vis-à-vis} \) the rest of the world and reduces financial wealth. As a result, domestic consumption initially jumps up in a creditor country while it jumps down in a debtor country. After this initial jump, the domestic consumption starts decreasing (resp. increasing) in the creditor country - the rate of time preference being smaller (resp. higher) - because agents prefer to save (resp. to consume). The constant real appreciation during adjustment has a negative (positive) effect on financial wealth in a creditor (debtor) country. Since the return on net foreign assets is higher, the creditor country becomes richer and after consumption starts increasing.
Based on this theoretical model, the next section analyzes empirically the equilibrium RER behavior for Thailand, South Korea, Malaysia and Singapore. Our aim is to test whether the reaction of the equilibrium RER is different depending on the level of indebtedness and to investigate to what extent world real interest rate shocks can lead to misalignments.

3. Empirical analysis

Our empirical application focuses on four SEA countries: Thailand (1981:1-2001:1), South Korea (1980:1-2001:1), Malaysia (1981:2-2000:4) and Singapore (1981-3 - 2001:3). The empirical analysis consists of estimating the long-run relationship derived from the reduced form summarized in Table 1. We consider this the equilibrium RER in the spirit of Edwards (1994), McDonald and Stein (1999), and McDonald and Ricci (2003) among others. We then present the impulse-response functions of the RER to a unit US real interest rate shock to analyze the dynamic adjustment of the RER and the possible misalignments arising from this shock.

3.1. Model and data

As the theory model is unisectoral we use an "external" real effective exchange rate (REER). The choice of effective exchange rates rather than bilateral is due to the fact that the model presents a one country case and, hence, it is more appropriate to empirically use one country against the rest of the World. We can use CPI-based or WPI-based REERs. The CPI-based REER, however, has the disadvantage of including tax distortions (see Hinkle and Montiel, 1999). Nevertheless, both gave similar outcomes, although the WPI-based gave more robust results for cointegration and we hence proceed to report these.

According to Table 1 the long-run equilibrium RER is determined by productivity, government consumption and the $\bar{r}$. Hence, the estimated model has the form $LREER = f(LY, LG, \bar{r})$. With $LREER$ being the log of the REER, $LY$ the log of output, $LG$ the log of government expenditure and $\bar{r}$ the real world interest rate.
We used quarterly data obtained mainly from the IFS database of the IMF. The REERs of South Korea, Malaysia and Singapore were obtained directly from the IFS. For Thailand, however, this data was not available and we constructed the REER as the trade weighted RER of Thailand with its main 5 trading partners. Regarding productivity, given that there was no long enough quarterly data on labor and capital stock to estimate TFP for Thailand, Malaysia and Singapore we had to use output indexes as a proxy (from the National University of Singapore updated with IMF data). In the case of Korea we used the manufacturing output index (from the National Bank of Korea). For Korea $LG$ is real government consumption, while for Thailand, Malaysia and Singapore we had to use government expenditure as disaggregated expenditure data was not available. We use the US real interest rate as the world real interest rate. This is based on the short run (3-month) Treasury-Bill rate and was calculated using 2 methods:

1) We subtracted from the quarterly nominal interest rate the quarter on quarter inflation rate during the period of maturity of the bill. Hence, this method assumes that agents have perfect foresight as we use the $ex-post$ real interest rate.

2) We subtracted from the quarterly annualized nominal interest rate the forecast of the inflation rate obtained from an ARMA (4, 4) model. Here we assume that agents are adaptive in forecasting future inflation rates.

We report the estimations using the first method as both gave essentially the same results. An important issue here is the choice of the US as the world interest rate. The theory model presented considers a one country case and hence, as commented earlier, we use REER rather than bilateral RER. Using a trade weighted world real interest rate, for instance, would be equivalent to using the foreign country real interest rate in a bilateral model. This, however, would induce endogeneity in the world real interest rate driven by trade shares. Using the US real interest rate alleviates this problem as the US is just one of the countries used to construct the REER of our four countries. All the variables are plotted in Appendix 4, where an increase in $LREER$ is a depreciation. The main prominent aspect is the strong impact of the 1997 crisis, especially for Thailand, but also
for South Korea and Malaysia. The 1997 crisis, as is well know, has an important impact on the RER and output of these countries.

3.2. Empirical modeling

In order to obtain an equilibrium REER and analyze the impact of $\bar{r}$ shocks we estimated a VAR model with the 4 variables and tested for cointegration (equilibrium) relations. The lag augmentation ($l$) of the VAR was based on a general to specific method ensuring no autocorrelation in the residuals. We chose 6 lags for Thailand and South Korea, 5 lags for Malaysia and 4 lags for Singapore. To ensure normality of the errors a dummy variable accounting for the 1997 crisis had to be used for Thailand, South Korea and Malaysia and this dummy enters any possible cointegration vector. The VAR includes a restricted intercept.

The adequacy of the REER equation and the whole VAR are reported in Table 3 containing the diagnostic tests on the errors of the model. It can be seen that, overall, the models behave well except for the case of Malaysia. In this case, the introduction of the dummy variable for the crisis was not able to normalize the errors. This lack of normality seems to arise from the REER equation. We proceeded with the rest of the steps for this country but the results from the cointegration analysis must be taken with caution. The only other problem that seems to arise is heteroscedasticity in the REER equation for Thailand. This problem, however, does not have an important distortion effect on cointegration tests. There are slight signs of autocorrelation for Singapore, but we decided not to add more lags to keep the model as parsimonious as possible. The rest of the diagnostic tests show that the errors can be treated as spheric and hence we can proceed to the analysis of cointegration.

14 Pre-tests of the order of integration showed that the variables involved are I(1). Results available.
3.3. Cointegration analysis

Given that the VAR model includes a shift dummy for three of the countries, inferences about the cointegration rank of the system cannot be efficiently obtained by directly applying the Johansen (1991) technique. Hence, we follow Saikkonen and Lutkepohl (2000) and adjusted the series for deterministic terms (including the dummy) and then performed a LR type test on the adjusted series. This method shows substantial advantages over Johansen and Nielsen (1993) in terms of local power and size properties as shown by Lutkepohl et al (2003). Once this is done, we carry out our estimations with the original series. Table 4 provides the cointegration analysis. It contains the eigenvalues, maximum eigenvalue tests, trace tests and the modulus of the 4 largest roots for \( r = 1 \) and \( r = 2 \).

For South Korea and Malaysia both tests indicate the existence of only one cointegration vector. For Thailand, though, the maximum eigenvalue test chooses no cointegration, whereas the trace test would admit 1 cointegration relation. The large eigenvalue of 0.322 indicates a relatively fast adjustment towards an equilibrium relation. Analyzing the modulus of the largest characteristic roots we can also see that the largest root for \( r = 1 \) is 0.90 and the second largest for \( r = 2 \) is 0.91. This seems to indicate that 0.97 is a unit root, but 0.90 is not. Hence we chose \( r = 1 \). For Singapore the Maximum Eigenvalue Test indicates just one cointegration relation, whereas the Trace Test points to two. Both Eigenvalues are high, but the 0.99 third largest root for \( r = 2 \) is very likely to be a unit root and we also chose \( r = 1 \) for Singapore.

In order to identify the cointegration vector, we normalized the two vectors by imposing the coefficient on the REER to be equal to one. We also imposed over-identifying restrictions on the adjustment coefficients based on economic theory. Particularly, given that the world interest rate is given exogenously and cannot be influenced by the home country, we imposed the restriction that \( \bar{r} \) does not adjust to the REER equilibrium relation and is hence weakly exogenous\(^{15}\). We do not use further restrictions to avoid imposing

\(^{15}\text{That is, we imposed that the element } \alpha_{4,1} \text{ of the speed of adjustment matrix } \alpha \text{ is equal to zero. The long run impact matrix of a VECM is } \Pi = \alpha \beta' \text{ with } \beta \text{ being a matrix of long run coefficients.} \)
too much structure on the data. This restriction is supported by the LR test for Thailand and Singapore, but was rejected for South Korea and Malaysia. This rejection may be due to the fact that for these countries the variability of the US nominal exchange rate and prices may be playing an important role in their REER, hence leading to endogeneity of $\bar{r}$. We hence report the results for Thailand and Singapore imposing the restriction and for South Korea and Malaysia without it. The cointegration vectors are reported in Table 5. The results show the following cointegration relations representing the equilibrium ER:

**Thailand:** $LREER = 5.583 + 0.406DUM97 + 0.103LYt - 0.176LGt + 0.010\bar{r}$

**South Korea:** $LREER = 3.394 + 0.425DUM97 - 0.832LYt - 0.465LGt - 0.045\bar{r}$

**Malaysia:** $LREER = 3.718 + 0.424DUM97 + 0.418LYt - 0.424LGt - 0.027\bar{r}$

**Singapore:** $LREER = 1.169 - 0.256LYt + 0.400LGt - 0.012\bar{r}$

For Thailand all variables show the expected sign and they are all significant, although government expenditure is only marginally so. An increase in home productivity leads to a REER depreciation, an increase in government expenditure to an appreciation and, finally, the effect of a world interest rate shock is a long-run depreciation. This is an expected result as Thailand is a large net debtor. The adjustment coefficient is very large, reflecting a rapid adjustment to deviations from equilibrium. For South Korea, the impact of home productivity is significant but produces the wrong sign. This may be due to the fact that Balassa-Samuelson effects may have a strong impact and these have not been considered in our unisectoral model.\(^{16}\) Another explanation could be that the share of domestic goods in total consumption spending is high (around 80% on average between 1980 and 2000). Hence, the income rise from a higher productivity is mostly used in the

\(^{16}\)For an overview of the impact of productivity on the RER see Lee and Tang (2003).
consumption of domestic goods. In this extreme case, a rise in $\theta$ could lead to a low depreciation and even a real appreciation. The rest of the variables show the expected sign and are significant. Notice that $\bar{r}$ has a strongly significant appreciation impact on South Korea’s REER. South Korea, for the majority of the period of estimation, has run current account surpluses. Korea’s assets were higher than its liabilities for most of 1988 onwards and, hence, we can treat it as either a net creditor or as a zero foreign wealth country. Finally, we can also see a relatively large speed of adjustment to equilibrium for South Korea. The results for Malaysia are consistent with the model except for the interest rate effect. In the case of Malaysia, a debtor country, we should be expecting a depreciation of the REER as a result of a world real interest rate shock. However, our results point to a significant appreciation, which is not supportive of the model’s predictions. Nevertheless, the lack of normality in the errors of the model for Malaysia renders the results unreliable.\footnote{It must be also noted that Malaysia’s management of the 1997 crisis was different as it relied heavily on capital controls, which are not accounted for in this model. This may be another factor influencing the results for this country.} For Singapore, all variables are significant, but $LY$ shows the unexpected sign according to the theory model. Again, Balassa-Samuelson effects may be playing a role in this direction or the share of domestic goods in consumption spending may be high. $LG$ shows the expected sign only if the government spends a higher share of its budget on imported goods than agents that is to say if $\alpha > \nu$. The US real interest rate shows the correct sign as Singapore is a net creditor country owing to its large current account surpluses and we would hence expect that a positive shock to $\bar{r}$ would lead to a REER appreciation.

The graphical representations of the actual and equilibrium REER are given in Figure 5 (see Appendix 5) where we can see that, for Thailand, South Korea and Malaysia the estimated equilibrium REER tracks the actual one relatively well although showing some periods of misalignment. The REER was overvalued especially before 1985, and then undervalued for the period of the large appreciation of the dollar in the 80s. Also, in the cases of Thailand and Malaysia, the 1997 crisis seemed to have more permanent
effects on the equilibrium REER than in South Korea. For Singapore the equilibrium REER appears to have a more volatile behavior, despite the fact that we should expect it to be more stable than the actual REER. This is mainly due to the large equilibrium depreciation around 1988 and appreciation around 1995.\textsuperscript{18}

Finally, we proceed to analyze the impact of a shock to $\bar{r}$ on the dynamic adjustment of the REER by performing orthogonalized impulse-response analysis. The impact of a 1 unit shock on the world interest rate equation on the REER is represented in Figure 6 (see Appendix 5) for the four countries. In the case of Thailand it shows that, initially, the REER suffers a small appreciation. After around 3 quarters, the REER starts appreciating. It takes around 10 quarters for the REER to reach the (depreciated) equilibrium level although it then overshoots for around 5 quarters. During the initial 10 quarters the price and/or wage rigidities prevalent in the data have led to an overvaluation with respect to the new equilibrium generating misalignment. We can thus see that the world interest rate shock has a similar impact to that predicted by the model in the long run but in the short run leads to misalignments. In the case of South Korea the response shows an initial large appreciation. It then depreciates slightly and then continues its appreciation from quarter 10 onwards towards equilibrium. The response for South Korea shows that the impact of a shock to $\bar{r}$ leads to a slight misalignment in the form of undervaluation. For Malaysia, as already mentioned, the behavior of the REER is an appreciation in the long-run contrary to our expectations. The adjustment process is relatively quick, leading to an undervaluation for around 5 quarters and then an overshooting effect that overvalues the RER for around 6 quarters. For Singapore the REER appreciates in the long run as predicted by the model. The adjustment in this case is similar to the one of South Korea, although in this case the RER is undervalued for around 5 quarters and then overvalued for around 10 quarters due to an overshooting effect. Hence, as stated before, as price rigidities of every kind prevent the actual REER to follow the equilibrium

\textsuperscript{18}Some authors, e.g. MacDonald and Ricci (2003), calculate the equilibrium RER by using smoothed values of the explanatory variables. This obviously generates a smoother estimate of the equilibrium RER.
one in the short run, our results would predict misalignment for Thailand in the form of an over-valued REER and under-valued in the case of South Korea and Singapore for the initial year approximately. If misalignment due to over-valuations are an important determinant of currency crises, an unexpected world real interest rate shock is more likely to induce currency crises in largely indebted countries such as Thailand.

Overall, the empirical evaluation of the model with data from SEA countries produces supportive results in the long run and is capable of capturing the misalignment effects of World interest rate shocks that may lead to crises. Only in the case of Malaysia we find unsupportive results. This is so even with a model which does not contain a nominal block. For instance, Rogers (1998) reports that between 19 and 60 per cent of the short run variance of the bilateral US dollar/UK sterling real exchange rate can be accounted for by monetary shocks. Our model abstracts from these shocks but even so is able to produce policy relevant conclusions about the dynamic response of REERs to world interest rate shocks.

<table>
<thead>
<tr>
<th>Table 3. Model Evaluation Diagnosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>LREER equation tests</td>
</tr>
<tr>
<td><strong>AR 1-5 test:</strong></td>
</tr>
<tr>
<td>F(5, 54 – 57 – 59 – 52)</td>
</tr>
<tr>
<td>Normality test:</td>
</tr>
<tr>
<td>(\chi^2(2))</td>
</tr>
<tr>
<td>Heteroscedasticity test:</td>
</tr>
<tr>
<td>F(41–49–32–41, 17–6–31–15)</td>
</tr>
<tr>
<td><strong>Vector tests</strong></td>
</tr>
<tr>
<td><strong>Vector AR 1-5 test:</strong></td>
</tr>
<tr>
<td>F(80,144 – 156 – 164 – 136)</td>
</tr>
<tr>
<td>Vector Normality test:</td>
</tr>
<tr>
<td>(\chi^2(8))</td>
</tr>
<tr>
<td>Vector Heteroscedast test:</td>
</tr>
<tr>
<td>F(410-490,480-69, 410-87)</td>
</tr>
</tbody>
</table>

Notes: whenever the degrees of freedom of each test are different, they are presented in parenthesis where the first, second and third numbers correspond to Thailand, Korea and Singapore respectively. ** and * mean significant at the 1% and 5% level respectively.
<table>
<thead>
<tr>
<th>Eigenv Rank</th>
<th>Max. Eig Rank</th>
<th>95% critical value</th>
<th>Trace test</th>
<th>95% critical value</th>
<th>Modulus: 4 largest roots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>r = 2</td>
<td></td>
<td>r = 1</td>
</tr>
<tr>
<td>Thailand</td>
<td></td>
<td></td>
<td>r = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.322 0</td>
<td>27.87</td>
<td>28.270</td>
<td>54.26*</td>
<td>53.480</td>
<td>1.0</td>
</tr>
<tr>
<td>0.201 1</td>
<td>15.83</td>
<td>22.040</td>
<td>28.37</td>
<td>34.870</td>
<td>1.0</td>
</tr>
<tr>
<td>0.161 2</td>
<td>7.79</td>
<td>15.870</td>
<td>12.54</td>
<td>20.180</td>
<td>0.97</td>
</tr>
<tr>
<td>0.096 3</td>
<td>4.76</td>
<td>9.160</td>
<td>4.76</td>
<td>9.160</td>
<td>0.91</td>
</tr>
<tr>
<td>South Korea</td>
<td></td>
<td></td>
<td>r = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.356 0</td>
<td>38.256*</td>
<td>28.270</td>
<td>71.145*</td>
<td>53.480</td>
<td>1.0</td>
</tr>
<tr>
<td>0.181 1</td>
<td>17.327</td>
<td>22.040</td>
<td>32.977</td>
<td>34.870</td>
<td>1.0</td>
</tr>
<tr>
<td>0.161 2</td>
<td>15.245</td>
<td>15.870</td>
<td>15.239</td>
<td>20.180</td>
<td>0.99</td>
</tr>
<tr>
<td>0.059 3</td>
<td>5.298</td>
<td>9.160</td>
<td>5.298</td>
<td>9.160</td>
<td>0.93</td>
</tr>
<tr>
<td>Malaysia</td>
<td></td>
<td></td>
<td>r = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.466 0</td>
<td>49.530*</td>
<td>28.270</td>
<td>77.166*</td>
<td>53.480</td>
<td>1.0</td>
</tr>
<tr>
<td>0.159 1</td>
<td>13.673</td>
<td>22.040</td>
<td>27.636</td>
<td>34.870</td>
<td>1.0</td>
</tr>
<tr>
<td>0.098 2</td>
<td>8.112</td>
<td>15.870</td>
<td>13.963</td>
<td>20.180</td>
<td>0.99</td>
</tr>
<tr>
<td>0.071 3</td>
<td>5.851</td>
<td>9.160</td>
<td>5.851</td>
<td>9.160</td>
<td>0.83</td>
</tr>
<tr>
<td>Singapore</td>
<td></td>
<td></td>
<td>r = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.304 0</td>
<td>29.339*</td>
<td>28.270</td>
<td>69.004*</td>
<td>53.480</td>
<td>1.0</td>
</tr>
<tr>
<td>0.238 1</td>
<td>20.065</td>
<td>22.040</td>
<td>39.665*</td>
<td>34.870</td>
<td>1.0</td>
</tr>
<tr>
<td>0.106 2</td>
<td>9.039</td>
<td>15.870</td>
<td>17.600</td>
<td>20.180</td>
<td>0.99</td>
</tr>
<tr>
<td>0.100 3</td>
<td>8.560</td>
<td>9.160</td>
<td>8.560</td>
<td>9.160</td>
<td>0.86</td>
</tr>
</tbody>
</table>
### Table 5. Restricted cointegration vector

<table>
<thead>
<tr>
<th>Country</th>
<th>( \beta' ) (s.e.)</th>
<th>( \alpha )</th>
<th>LR test of restriction ( \chi^2(1) = 0.104 [0.747] )</th>
</tr>
</thead>
</table>
| Thailand | \[\begin{array}{cccccc}
1.000 & -0.103 & 0.176 & -0.010 & -0.406 & -5.583 \\
0.000 & (0.041) & (0.107) & (0.004) & (0.043) & (0.156) \\
\end{array}\] | \[\begin{array}{c}
-0.469 \\
(0.061)
\end{array}\] | - |
| South Korea | \[\begin{array}{cccccc}
1.000 & 0.832 & 0.465 & 0.045 & -0.425 & -3.394 \\
0.000 & (0.157) & (0.101) & (0.013) & (0.070) & (0.395) \\
\end{array}\] | \[\begin{array}{c}
-0.286 \\
(-0.066)
\end{array}\] | - |
| Malaysia | \[\begin{array}{cccccc}
1.000 & -0.418 & 0.424 & 0.027 & -0.424 & -3.718 \\
0.000 & (0.078) & (0.079) & (0.007) & (0.031) & (0.355) \\
\end{array}\] | \[\begin{array}{c}
-0.281 \\
(0.049)
\end{array}\] | - |
| Singapore | \[\begin{array}{cccccc}
1.000 & 0.256 & -0.400 & 0.012 & - & -1.699 \\
0.000 & (0.085) & (0.083) & (0.005) & (0.321) & (0.321) \\
\end{array}\] | \[\begin{array}{c}
-0.090 \\
(0.017)
\end{array}\] | - |

### 4. Conclusion

This article investigates the effects of a US interest rate rise on small open economies. Our main theoretical result is that, with selfish agents, a rise in the world interest rate leads to a long-run real appreciation (depreciation) when the country is a net creditor (debtor). The long-run relation between the world interest rate and the RER depends thus on the net financial position of the domestic country vis-à-vis the rest of the world.

This long-run relation between the US real interest rate and real effective exchange rate (REER) is confirmed by data on Singapore, Thailand and Korea between 1980 and 2001. Our estimations also show that world interest rate shocks may entail misalignment. This can be the case if there is some price or wage sluggishness which prevents an immediate REER adjustment process. This misalignment comes in the form of an overvalued RER for highly indebted countries hence leading to an increased probability of currency (and
possibly banking) crisis.
1. Dynamic system

The dynamic system is

\[
\begin{align*}
\dot{B} &= \bar{r}B + \frac{1}{R} \left[ Z(R) - \frac{1-a}{\alpha}X \right] \\
\dot{K} &= \theta F(K,1) - X - Z(R) - \delta K - \tau \nu F_L(K,1) \\
\dot{R} &= R [\theta F_K(K,1) - (\bar{r} + \delta)] \\
\dot{X} &= [\theta F_K(K,1) - (\beta + \delta)] X - p\gamma \alpha [RB + K]
\end{align*}
\]

Around the steady state, the linearized system is:

\[
\begin{pmatrix}
\dot{B} \\
\dot{K} \\
\dot{R} \\
\dot{X}
\end{pmatrix} =
\begin{pmatrix}
\bar{r} & 0 & \frac{1}{R} [Z'(R^*) + \bar{r}B^*] & -\frac{1-a}{\alpha} \frac{1}{R^*}
\end{pmatrix}
\begin{pmatrix}
B - B^* \\
K - K^* \\
R - R^* \\
X - X^*
\end{pmatrix}
\]

and \( \text{Jac} \) denotes the Jacobian matrix. Its determinant is:

\[
\det \text{Jac} = \theta F_K(K^*,1) R^* Z'(R^*) (\bar{r} + p)(\bar{r} - \beta - p)
\]

There are two backward variables \( B \) and \( K \). Consequently, saddle path stability requires that two eigenvalues have negative real parts. Therefore \( \det \text{Jac} \) must be positive which implies that \( \bar{r} < \beta + p \).

2. Paths

The linearized system can be written as:

\[
\dot{M}(t) = \text{Jac} \cdot M(t)
\]

with \( M(t) = [B(t) - B^*, K(t) - K^*, R(t) - R^*, X(t) - X^*]^T \).
General solution of the linearized system is: \( M(t) = P e^{Dt} P^{-1} M(0) \) with \( P \) the passage matrix and \( P_{ij} \) the passage matrix elements, \( P^{-1} \) the inverse passage matrix and \( P_{ij}^{-1} \) the inverse passage matrix elements. Hence, the diagonal matrix is \( D = P^{-1} A P \):

\[
D = \begin{pmatrix}
\lambda_1 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & \lambda_4
\end{pmatrix}
\]

with \( \lambda_1 < 0 \) and \( \lambda_2 < 0 \).

We note \( P^{-1} M(0) = [\varphi_1(0), \varphi_2(0), \varphi_3(0), \varphi_4(0)] \). The terms in front of \( e^{\lambda_3 t} \) and \( e^{\lambda_4 t} \) in \( P e^{Dt} P^{-1} M(0) \) must be zero so that the system converges asymptotically toward its steady state:

\[
M(t) = Pe^{Dt} \begin{pmatrix}
\varphi_1(0) \\
\varphi_2(0) \\
0 \\
0
\end{pmatrix}
\]

With \( \varphi_3(0) = \varphi_4(0) = 0 \), we obtain the following system with two equations and two unknown variables \( R(0) \) et \( X(0) \):

\[
P_{31}^{-1} [B_0 - B^*] + P_{32}^{-1} [K(0) - K^*] + P_{33}^{-1} [R(0) - R^*] + P_{34}^{-1} [X(0) - X^*] = 0
\]

\[
P_{41}^{-1} [B_0 - B^*] + P_{42}^{-1} [K(0) - K^*] + P_{43}^{-1} [R(0) - R^*] + P_{44}^{-1} [X(0) - X^*] = 0
\]

As a result, all variables can be written as linear combinations of \( (B_0 - B^*) e^{\lambda_1 t} \) and \( (K_0 - K^*) e^{\lambda_2 t} \). We have:

\[
B(t) = B^* + \Gamma_1 (B_0 - B^*) e^{\lambda_1 t} + \eta_1 (K_0 - K^*) e^{\lambda_2 t}
\]

\[
K(t) = K^* + \Gamma_2 (B_0 - B^*) e^{\lambda_1 t} + \eta_2 (K_0 - K^*) e^{\lambda_2 t}
\]

\[
R(t) = R^* + \Gamma_3 (B_0 - B^*) e^{\lambda_1 t} + \eta_3 (K_0 - K^*) e^{\lambda_2 t}
\]

\[
X(t) = X^* + \Gamma_4 (B_0 - B^*) e^{\lambda_1 t} + \eta_4 (K_0 - K^*) e^{\lambda_2 t}
\]
3. Long-run real exchange rate reaction

3.1. Sign of $\Omega_2 (\bar{r})$

$$
\Omega_2 (\bar{r}) = (\bar{r} + \delta) (1 - \rho) p\gamma \alpha [p\gamma \Gamma^{-1} + (\beta - 2 \bar{r}) (1 - \rho) \rho^{-1} (\bar{r} + \delta)]
$$

$$
\Omega'_2 (\bar{r}) < 0 \text{ if } \bar{r} \in ]0, r_p[ \text{ and }
$$

$$
\Omega_2 (0) > 0, \quad \Omega_2 (\beta/2) > 0, \quad \Omega_2 (r_p) < 0
$$

Hence, there exists $r_{\Omega_2} \in ]\beta/2, r_p[ \text{ such that } \Omega_2 (r_{\Omega_2}) = 0,$

$$
r_{\Omega_2} = \frac{1}{2 (\rho - 2)} \left( -\beta + 2\delta (1 - \rho) - \sqrt{\beta^2 + (1 - 2\rho) 4\delta (\beta + \delta) + 4\rho^2 (\beta \delta + p\gamma) + 8\rho p\gamma} \right)
$$

3.2. The bell on Figure 1

Assuming a Cobb-Douglas production, we have in the long-run the following relation between $R^*$ and $\bar{r}$

$$
Z (R^*) = \rho (\rho a)^{1/(\rho - 1)} P (\bar{r})
$$

with $P (\bar{r}) = (\bar{r} + \delta) \frac{1}{\rho - 1} [\bar{r} + (1 - \rho) \delta - (1 - \rho) \rho^{-1} (\bar{r} + \delta) p\gamma (p\gamma - \bar{r} (\bar{r} - \beta))].$ The derivative is

$$
\frac{\partial Z (R^*)}{\partial \bar{r}} = \rho (\rho a)^{1/(\rho - 1)} \left[ -\frac{P (\bar{r})}{(1 - \rho) (\bar{r} + \delta)} + (\bar{r} + \delta)^{1/(\rho - 1)} \left( 1 + \frac{(1 - \rho) \alpha p\gamma \left(-1 + \frac{(\bar{r} + \delta)(\beta - 2\bar{r})}{p\gamma - \bar{r} (\bar{r} - \beta)} \right)}{\rho (p\gamma - \bar{r} (\bar{r} - \beta))} \right) \right] \equiv \Delta (\bar{r})
$$

(3.1)

We have $\Delta (0) > 0$ and

$$
\Delta (r_p) = \rho (\rho a)^{1/(\rho - 1)} \left[ 1 + r_p (1 + \delta) (r_p + \delta)^{-1} + (\beta - 2 r_p) (1 - \rho) \rho^{-1} (p\gamma - \bar{r} (\bar{r} - \beta))^{-1} (r_p + \delta) \right]
$$

which sign remains indeterminate.

Expanding equation (3.1), terms of order $\bar{r}^2$ or $\bar{r}\beta$ can be ignored. Then, vanishing all these small terms, we obtain that

$$
\Delta (r) \simeq (1 - \rho) \rho (\beta - 2\bar{r})
$$

(3.2)
and hence $\Delta (\beta/2) \to 0$. Let $\Phi$ be the long-run relation between the real exchange rate and the world interest rate $\bar{r} : R^* = \Phi (\bar{r})$ with $\Phi (\bar{r}) = Z^{-1} \left[ \rho (\rho a)^{1-\rho} P (\bar{r}) \right]$. From $\Delta (0) > 0$, the $\Phi$ curve starts increasing in $\bar{r} = 0$. From $\Delta (\beta/2) \approx 0$, we have a maximum for the function $\Phi$ around $\bar{r} = \beta/2$, and hence the $\Phi$ curve starts decreasing after $\bar{r} = \beta/2$. According to the approximation of $\Delta (\bar{r})$ in (3.2), the $\Phi$ curve's slope is negative when $\bar{r} < \beta/2$ as depicted on Figure 2.

4. Data Description
5. Econometric Results

Figure 5.1 LREER and estimated equilibrium exchange rate
Thailand

Figure 5.2 LREER and estimated equilibrium exchange rate
South Korea
Figure 5.3 LREER and estimated equilibrium exchange rate
Malaysia

Figure 5.4 LREER and estimated equilibrium exchange rate
Singapore
Figure 6.1 Impulse-response of the REER to a unit US real interest rate shock
Thailand

Figure 6.2 Impulse-response of the REER to a unit US real interest rate shock
South Korea

Figure 6.3 Impulse-response of the REER to a unit US real interest rate shock
Malaysia
Figure 6.4 Impulse-response of the REER to a unit US real interest rate shock
Singapore
References


