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A Duty to Vote

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A Duty to Vote

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Abstract: A duty to vote may be many things. It may be no more than an obligation to cast one's ballot as self-interestedly or as altruistically as one pleases. It may be a requirement to vote for the political party most likely to yield the highest social welfare. It may be a requirement to choose between competing parties in the interest of one's social class or with recognition of the needs of the poor. It may include a requirement to inform oneself about the issues in an election. This paper begins with a critique of the argument denying any duty to vote because, as with participants in the market, there is no conflict between self-interest and public interest in the choice whether to vote or abstain. The core of the paper is a discussion of several interpretations of the duty to vote, and there is a brief review of pros and cons of compulsory voting.

JEL Classification D72

Keywords: Pivotal voting, duty to vote , compulsory voting

A duty, at least in the context of this paper, is an unenforced obligation to other people, a mutual obligation that makes no sense unless people are better off on average when most people are dutiful than when they are not. Five questions precede the analysis in this paper.

- 1) When, if ever, is it in a person's self-interest to vote rather than abstain?
- 2) If self-interest alone is insufficient to induce people to vote, is it still in the public interest for people to do so?
- 3) If it is in the public interest for a larger proportion of the electorate to vote than would do so from self-interest alone, would it also be in the public interest to make voting compulsory, or might a sense of duty be relied upon to supply a sufficient number of votes?
- 4) What exactly does a sense of duty compel the eligible voter to do: just to vote or to vote with the additional obligation to vote for the party that is best for some sub-group in society or for the nation as a whole.
- 5) Why might citizens be prepared to act dutifully when it is not in their immediate interest to do so.

The focus of this paper is upon the second, third and fourth questions. The first question is dealt with extensively elsewhere. A substantial literature, discussed in Usher (2011) and in many other places, is devoted to showing that self-interest alone is insufficient to induce an adequate number of people to vote rather than abstain. The standard formulation of the problem in Riker and Ordeshook's (1968) is that a person votes or abstains according to whether

$$\pi B + D > C \quad (1)$$

where B is one's personal benefit if one's preferred party wins the election
D is the value one places upon voting as a duty to the rest of the community.

C is one's cost of voting

and π is the probability of casting a pivotal vote, of swinging the outcome of an election from the political party one opposes to the political party one favours. The consensus is that the probability of casting a pivotal vote is simply too small to make voting worthwhile unless D is large, meaning that the voter places a considerable valuation upon behaving dutifully. The fifth question is outside the scope of this paper. Here, we go no further than to establish a necessary condition for a sense of duty, that government by majority rule voting works far better when a large portion of the electorate votes than when people vote or abstain as they please.

The fourth question is really a series of questions: Is it sufficient to show up at the ballot box and cast one's ballot randomly or in accordance with the last ad one saw on television? Is it sufficient to cast one's ballot in one's own interest exclusively without regard to anybody else and ignoring the possibility that the party which is better for oneself is worse for the great majority of people? Does duty trump self-interest altogether, or must a balance be struck between them, and, if so, what can the voter be said to maximize? Or is politics like the competitive market where society as a whole becomes as well off as it can be when each person acts in his own interest alone?

Duty is a difficult concept with no place in conventional economic analysis where a clear line is typically drawn between the private sector within which people are assumed to be unreservedly greedy and the public sector within which actions are taken in accordance with some notion of the common good. Citizens' obligations to pay taxes and obey the law are enforced by the threat of punishment. Nobody is assumed to do what is right for no other reason than that it is the right thing to do.

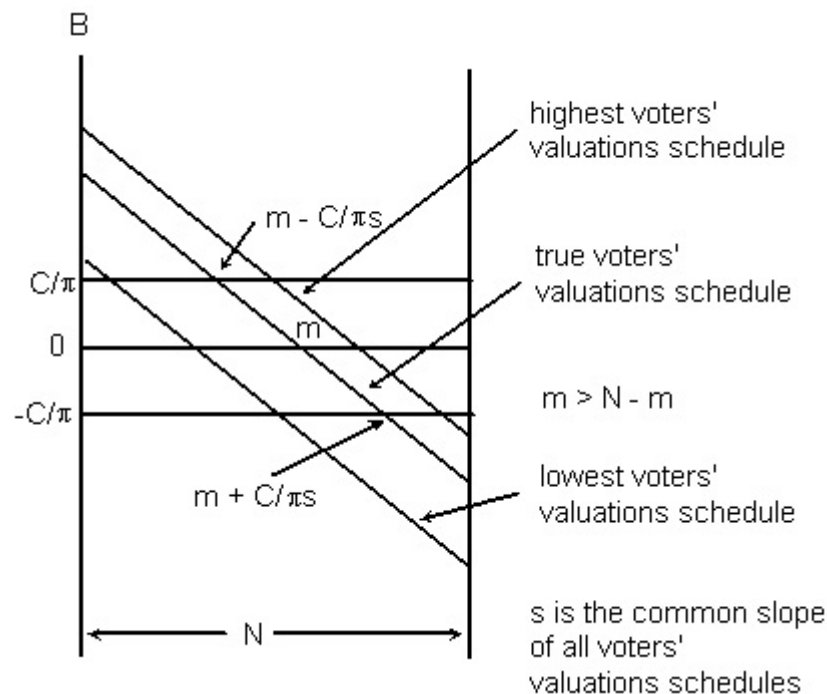
Voting conforms badly to this pattern. On the one hand, it is essential for the preservation of what we all see as a good society that large numbers of people choose to vote rather than to abstain. On the other hand, it is somewhere between difficult and impossible to compel one another to vote appropriately, and there is no voice from on high telling all of us what exactly a duty to vote requires of us, how to weigh public and private good and how one person's obligation is affected by another's refusal to recognize any such obligation.

The paper begins with the second question. As there can be no duty to vote unless some notion of the common good is served by voting, a convenient starting

place is with a special case where there is no duty to vote because a voter's self-interested decision whether to vote or abstain is in the public interest as well. Analysis of the assumptions in this case points to reasons why a duty to vote may be required; wide-spread abstentions may turn the election away from the party that the majority of citizens prefers and may make elections especially vulnerable to small, highly disciplined groups of people with interests opposed to those in the general population. There follows a discussion of the content of a duty to vote, views that one should vote in the common good, in the interest of all like-minded voters, in the interest of others with whom one is especially concerned, or that duty requires no more than voting regardless of whether one votes in one's own interest or in the interest of others. There is finally a brief discussion of the pros and cons of compulsory voting.

a) **An argument that there is no duty to vote:** Recognition of a duty to vote originates from the belief that people would be better off on average if more people chose to vote than when the decision whether to vote or abstain is based upon self-interest alone with no concern for the welfare of the community as a whole. An argument can be made that the belief is mistaken, that people would be better off on average if fewer people voted than would be inclined to do so voluntarily.

Figure 1: Why Abstentions are Desirable



To develop the argument, consider an election with two parties, left and right, where all voters can be ordered in accordance with their valuations, positive or negative, of a win for the left party. The valuation of the n^{th} voter is $B(n)$, so that if the 100th voter values a win for the left party at \$2,000, then $B(100) = 2,000$, and if the 3,000th voter values a win for the right party at \$500, then $B(3,000) = -500$. Three possible schedules of voters' valuations are shown as the three downward-sloping straight lines in Figure 1 with voters ordered by their valuations of a win for the left party on the horizontal axis and with the valuations themselves on the vertical axes. The common slope of these lines is signified by s .

For there to be any chance that one's vote is pivotal (that is, for π to be other than 0 or 1), there must be uncertainty about the outcome of the election. Each person eligible to vote must have some knowledge about the preferences of the rest of the electorate, but not too much. Uncertainty is created here by the assumption that, by chance alone, the true voters' valuation schedule lies somewhere between the highest and the lowest schedule in the diagram and that all schedules between these limits are equally likely, so that the probability of a tied vote among all but one voter is the horizontal distance between these limits and, on the assumption that ties are broken by the flip of a coin, one's chance of casting a pivotal vote is half that. Voters are assumed to know the locations of the upper and lower voters' valuations schedules, but nobody knows which schedule has by chance been selected. Everybody knows that some people are left-leaning and other people are right-leaning, but nobody is quite sure which side will have the most votes on election day. Uncertainty about the location of the true voters' valuations schedule might be attributed to each person's ignorance about the preferences of the rest of the electorate or to nation-wide shifts of opinion unrecognized on election day.

Now suppose that chance has selected the in-between schedule crossing the horizontal axis at the point m . If voting were costless (that is, if $C = 0$), the left party would acquire m votes, and the right party would acquire $N - m$ votes. As the figure is drawn, $m > N - m$ so that left party wins the election.

With a cost of voting C and on the assumption that everybody is strictly selfish (so that $D = 0$), a person votes for the left party if and only if B is greater than C/π , votes for the right party (bearing in mind that B would be negative) if and only if $-B$ is greater than C/π , and abstains when B is in between C/π and

- C/π . The two horizontal lines in the figure are at a distance C/π above and below the horizontal axis. Whatever voters' valuation schedule has by chance been selected, people vote rather than abstain if and only if their values of B are above or below both of these lines. The number of votes for the left party is reduced from m to $m - C/\pi s$, and the number of votes for the right party is reduced from $N - m$ to $N - m - C/\pi s$. By comparison with a situation where nobody abstains, there is a total cost saving of $C^2/\pi s$, but the left party wins regardless. Here, cost-induced abstentions are unambiguously beneficial to society as a whole by reducing the cost of voting without affecting the outcome of the election.

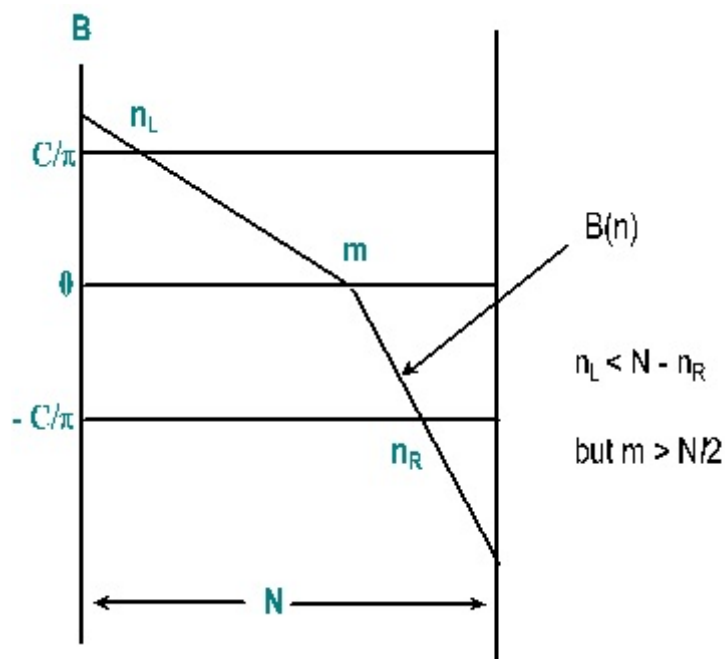
The general principle in this example is that abstentions drawn equally from both political parties can do no harm because they have no effect upon the outcome of an election. If the left party would win by 10,000 votes when there are no abstentions and if abstentions reduce numbers of votes for both parties equally, then the left party must win by 10,000 votes regardless. Anticipated electoral outcomes may be described as consisting of four distinct percentages: 1) the percent of sure votes for the left party, 2) the percent of sure votes for the right party, 3) the percent of votes that could go either way and 4) the percentage of abstentions. The argument that abstentions have no effect upon the outcome of an election relies upon the premise that whatever increases the percentage of abstentions, must necessarily reduce the percentages, (1) and (2), of votes both parties' votes equally and without affecting the percentage of votes, (3), that could go either way. It follows at once that, when this is so, public policy should be directed to *increasing* the number of abstentions. Voluntary voting becomes preferable to compulsory voting. A tax or fee on voting would make everybody better off as long as the revenue from the tax is appropriately redistributed, for the total cost of voting would be reduced without affecting the outcome of elections. The more abstentions the better, provided there is some mechanism, like that in figure 1, to apportion abstentions equally between political parties.

Essentially the same story can be told within a different model of randomization where everybody's absolute value of B is the same (in that everybody gains equally from a win by their preferred party), where the cost of voting varies randomly from 0 to some maximal value, where the distribution of voting cost is the same for supporters of both parties and where each person's party preference is randomly assigned. The optimal number of voters is where the marginal cost of voting just equals the marginal benefit to society from the

resulting increase in the probability of a win for the party preferred by the majority of population. As shown in Borgers (2004), voluntary voting may be preferable to compulsory voting in this case.

Notwithstanding these considerations, there are reasons why society as a whole might be better off when everybody, or almost everybody, votes than when large numbers of eligible voters abstain. The argument against the existence of a duty to vote depends critically on the assumption that the true voters' valuations schedule is linear. Otherwise, as illustrated in figure 2, the party preferred by a majority of the population and could easily fail to win the election.

Figure 2: The More Popular Party Wins the Election



To see how abstentions can swing the outcome of an election from one party to another, consider the voters' valuations schedule in figure 2 representing an electorate made up of a large number of people for whom B is positive but relatively small in absolute value, together with a small number of people for whom B is negative but relatively large in absolute value. The schedule cuts the horizontal axis at m , so that, if everybody votes, m people vote for the left party, $N - m$ people vote for the right party and the left party wins the election because, as the figure is drawn, $m > N - m$. Now introduce a cost of voting of C and suppose every voter has a certain chance, π , of casting a pivotal vote. [Imagine the

schedule on figure 2 to have been chosen at random from a set of schedules bounded by upper and lower schedules. The value of π becomes a half of the average distance between them.] The number of votes for the left party is n_L at which $\pi B = C$. The number of votes for the right party is $N - n_R$ at which $\pi(-B) = C$. As the figure is drawn, $N - n_R > n_L$ implying that the right party wins the election.

This is more than a theoretical curiosum. The shape of the schedule in Figure 2 is the natural consequence of a typical distribution of income with a small number of wealthy people and with a mean income well above the median. The shape of the voters' valuations schedule would be as shown in Figure 2 as long as the absolute value of B is more or less proportional to income. The party of the rich wins because each person's absolute value of B is less than C/π for a larger proportion of the poor than of the rich. The expected cost of swinging an election may be high enough to deter a great many poor people from voting but not so high as to deter a comparable proportion of rich people, enabling the party of the rich minority to win the election. Abstentions provoked by the cost of voting are beneficial on average as long as they have no impact on the outcome of elections, but there are good reasons why that may not be so.

It is arguable that this advantage to the rich is illusory because their cost of voting is higher too. If people's values of B are proportional to their incomes, if people's incomes per hour are proportional to their productivities of labour, and if the time required to vote is the same for everybody, then people's cost of voting should be proportional to income; B/C should be no higher on average for the rich than for the poor, and rich people should have no higher propensity to vote. However, this is not true of income derived from ownership of capital or where voting is sufficiently different from work that there is no one-to-one trade-off in hours devoted to each activity. The experience of working is very different from one person to the next. The experience of voting is not.

Other considerations lie outside of the models discussed in this paper. A so far unemphasized assumption of the model is that everybody knows his own value of B to the penny. If not, resources can be devoted to informing or bamboozling people about the virtues of one party or the vices of the other. When everybody, or almost everybody, votes, political activity can be redirected to persuading people of the virtues of one's political party, what it will do for particular groups of

people and why it is best for the nation as a whole. When large numbers of eligible voters are expected to abstain, political activity may be concentrated on getting out the vote. The rich may vote vicariously by contributing to the cost of electioneering and campaign advertising.

Also, by decreasing the range of uncertainty, abstentions tend to raise the return to coordination among like-minded voters. Ability to act in unison becomes increasingly influential as a determinant of the outcome of elections. Each two like-minded eligible voters can double the chance of casting a pivotal by a side-deal in which each promises the other to vote rather than to abstain. Each of a thousand like-minded voters can multiply the chance by a thousand. The logic of coordination is discussed below under the heading of “voting pacts”.

b) Voting Pacts: Among like-minded people, voting is the private provision of a public good. Nobody pays tax voluntarily because everybody's valuation of the extra services of the army or the police made possible by his tax alone is almost certainly less than the tax he pays. Everybody favours compulsory taxation because everybody's valuation of the entire services of the army and the police is almost certainly greater than the tax he pays. Similarly, within a group of like-minded eligible voters, all pro-left or all pro-right, everybody might agree to vote on condition that everybody else within the group is required to do so too.¹

Suppose each person has a cost of voting of \$10, a probability of casting a pivotal vote of $1/6,000$ and, consequently, an expected cost of swinging the election of \$60,000. A person abstains unless his benefit from a win by his preferred party exceeds \$60,000. The required benefit can be cut in half by a deal between any two like-minded voters who would not otherwise vote at all. Consider two eligible voters - both supporters of the left party or both supporters of the right

¹Voting pacts are first cousins, if not closer, to the voting behaviour of groups as modeled by Uhlander (1989) and Morton (1991). Their emphasis is upon leaders and followers, upon the interaction between candidates seeking office and citizens choosing whether and for whom to vote, but they must postulate some material or just psychic private advantage in voting to overcome the free rider problem. Uhlander discusses rewards to by successful candidates to groups small enough to make each person's vote advantageous. There is also a family resemblance between voting behaviour of groups and the probabilistic voting theorem demonstrating an equilibrium when candidates distribute goodies to groups so as to maximize their probability of being elected. See Mueller (2003, chapter 12).

- each of whom values a win for their preferred party at, say, \$50,000. Acting separately and self-interestedly, they would both abstain. Acting together, each voting on condition that the other does so too, they double the chance of being pivotal, from $1/6,000$ to $1/3,000$, cutting each of their expected costs of swinging the election from \$60,000 to \$30,000 at which it becomes individually advantageous for both of them to vote rather than to abstain. Acting together, 100 like-minded eligible voters can raise the probability of swinging the election from $1/6,000$ to $1/60$ at which voting becomes individually advantageous as long as one's benefit of a win for one's preferred party exceeds not \$60,000, but only \$600. If left-leaning voters are cooperative while right-leaning voters are libertarian, it may well happen that the left party wins the election despite the fact that more voters prefer the right party to the left.

Such "voting pacts" differ from ordinary public goods in two main respects: Their benefits accrue not to society as a whole, but to a part of society, people favouring one of two political parties, at the expense of the rest. More importantly, there is no legally-binding mechanism to ensure that all beneficiaries of voting pacts contribute their share of the cost. Agreements to share the cost of ordinary public goods are enforced by the state. Agreements to vote for one of two competing parties can only be enforced imperfectly and incompletely, through social pressure exerted by friends, neighbours, churches, labour unions and so on. Whether one votes or not may be observable, but not which party one votes for. Voting pacts rely on nothing more than the exhortation to "get out and vote" among people who recognize one another to be on the same side of the fence. That may be, but is not necessarily, sufficient depending on the degree of social cohesion.

Imperfect though they may be, voting pacts can have a profound effect upon the outcome of an election. Consider the extreme example summarized in table 1 of a two-class society with 100 rich people each placing a value of \$300 on a win for the right party, and with 900 poor people each placing \$100 on a win for the left party. If everybody votes, the left party would win nine-to-one, and society would be better off too because the dollar value of the gains from a win by the left party exceeds the dollar value of the gains from the right party.

Table 1: A Two-class Society
[The cost of voting, C , is \$10]

	number of eligible voters favouring	benefit per person of a win for the favoured party	total surplus
left	900 [$n = 1$ to 900]	$B(n) = \$100$	$S_L = \$90,000$
right	100	$B(n) = \$300$	$S_R = \$30,000$

In this society, 101 votes by left supporters is enough to ensure a win for the left party, and one extra vote, raising the total to 102, is enough to ensure that no voter, left or right, can be pivotal. With more than 101 votes for the left party, there is no chance of the right party winning the election, and supporters of the right party might as well abstain to save themselves the cost of voting in circumstances where their votes can have no effect.

Supporters of the left party are locked into a complex maze where each person is made better off by abstaining than by voting, but only on condition that enough others vote instead. Suppose once again that the cost of voting is \$10. Voters gain \$90 but abstainers gain \$100 as long as enough left-leaning people vote to ensure that the left party wins the election. How then are the voters selected? In a sense, the voters are suckers, bearing a burden that could very well be borne by somebody else instead. Each voter may well think to himself that, if he abstains, some other left-leaning person will be induced to vote in his place for fear of the emergence of a voting pact on the right. Perhaps supporters of one party or the other can devise a punishment for abstention. Exclusion from common activities or mere disapproval may be sufficient. Perhaps, if and when potential pact members can identify one another, they can devise a grim-trigger strategy of dissolving the entire pact if any member abstains. Pacts may be easier to enforce among all like-minded beneficiaries than among a proportion of them.

Despite the left's overwhelming majority, it remains possible for cooperation

among left-leaning people to break down altogether, and for the right party to win instead. Failure is especially likely if voting is expensive. Suppose the cost of voting were \$110 rather than \$10. It would then be in no left supporter's interest to vote without an agreement among a number of left supporters to share the cost. The right party could easily win despite the fact that there is a net gain of \$60,000 to society as a whole from a win for the left. There may be no equilibrium comparable to that illustrated in figure 1, and the outcome of the election may turn on who can cooperate with whom.

With costly participation, a voting pact is advantageous to its members if and only if

$$\Pi B > C + K \quad (2)$$

where Π is the probability that the *pact* is pivotal and K is the per person cost of participation. The value of Π would be 1 if the pact were sure to generate a win for the party it supporters. The value of Π would be less than 1 if there were just a chance of doing so. It is not unreasonable to suppose that the cost to each participant rises together with the size, s , of the pact

$$K = K(s) \quad (3)$$

where $K_s > 0$ because it is more expensive per member to hold together a large pact than to hold together a small one.

With reference to the example in table 1, it is at least possible that the right party wins the election because there is some pact of size, s , such that $\Pi = 1$ and

$$300 > 10 + K(s) \quad (4)$$

but there is no pact of size, s , such that $\Pi = 1$ and

$$100 > 10 + K(s) \quad (5)$$

Despite the left's overwhelming majority, it is possible for the right party to win

the election because an effective pact is feasible on the right side alone.²

For any given number of people eligible to vote, for any given proportions of left and right supporters in the population as a whole and as long as supporters of both parties are equally likely to abstain, it follows at once that an increase in abstentions increases the chance of a majority being overturned by a voting pact, for abstentions automatically reduce the size of the majority that must be overcome.

Abstentions also magnify the importance of mobilization as compared with persuasion. If everybody votes, then the most a party can do to increase its chance of being elected is to persuade people that it is the better than its rival. Otherwise, if many people are prepared to abstain, there is likely to be a substantial cadre of people on the edge of voting left, voting right or not voting at all, people who can be induced to vote for a party not just by arguments about its virtues, but by meetings, personal contact, transportation to the polling station and so on.

At the extreme, voting pacts may be bands of fanatics - neo-Nazis being the obvious example but religion will do just as well - devoted exclusively to some cause and prepared to vote en masse for any party that will support it too. The more people who abstain, the stronger are such groups likely to be.

C) Utilitarianism: A person may choose to vote rather than to abstain, without any explicit or implicit contract to do so and without any quid-pro-quo from like-minded citizens, in the belief that it is the ethical thing to do. Ethical in this context is whatever is in the interest of the group with which the person identifies, the entire community of eligible voters or some sub-group based upon political persuasion, locality or social class. The choice to vote rather than to abstain may be based upon a principle of rule-utilitarianism (as opposed to act-utilitarianism) according to which behaviour is ethical in so far as it conforms to rules generating

²Consider Myerson's (1998) example based upon Palfrey and Rosenthal's (1985) model of person-to-person randomization where everybody's (absolute) values of B and C are the same and where equilibrium turnout is obtained by random voting. Everybody votes or abstains on the flip of a weighted coin, where weights party-specific. In Myerson's example, out of a population of 1 million right supporters and 2 million right supporters, the expected number of votes is no more than 32 for each political party. Were that so, a voting pact of as few as 33 voters would be virtually guaranteed to swing the election if supporters of the opposing party did not respond in kind.

the largest attainable aggregate utility. The rule need not require everybody to vote. A person's obligation to vote or abstain would be in accordance with a trade-off to the group, rather than the person himself, between the expected marginal benefit of an extra vote and the marginal cost of voting. Voting from a rule-utilitarian perspective has been analyzed by Harsanyi (1980), and developed by Feddersen (2004), Feddersen and Sandroni (2006) and by Coate and Conlin (2004), where the objective is respectively the welfare of society as a whole and that of all like-minded voters.

Recall that a purely self-interested person with no sense of duty whatsoever chooses to vote or abstain - in accordance with a variant of equation (1) above where $D = 0$ - depending on whether or not $\pi B > C$. One would like to derive a utilitarian analogue of this inequality with personal benefit, B , replaced by some measure of the voter's perceived benefit of a win by one party or the other for the nation as a whole.

Imagine an electoral context between a left party and a right party in a society of N people where a win for the left party generates a distribution of income is $(y_{iL}, y_{2L}, \dots, y_{NL})$ and a win for the right party generates a distribution of income $(y_{iR}, y_{2R}, \dots, y_{NR})$, and consider a person i whose (Neuman and Morgenstern) utility of income function is $u^i(y)$. As seen by person i , the welfare of the community can be measured as the income, which if person i had it for sure, would make person i as well off as he would be with equal chances of having the same income as each and every person in the community. For for person i , the welfare of the entire community - W_{iL} or W_{iR} depending on which party wins the election - is defined implicitly by the equations

$$W_{iL}^i = \sum_{n=1}^N u^i(y_{nL}) \quad (6a)$$

and
$$W_{iR}^i = \sum_{n=1}^N u^i(y_{nR}) \quad (6b)$$

To vote ethically as a true utilitarian, person i would have to vote for the left party if $W_{iL} > W_{iR}$ and for the right party if $W_{iR} > W_{iL}$.

One additional step is required in order to specify when one is duty-bound to vote and when not. Social welfare may be defined net or gross of the total cost of voting to everybody who votes rather than abstains. Let T be the total cost of voting, and redefine W_{iL} and W_{iR} as person i 's certainty equivalent incomes per head gross of the cost of voting and depending on which party wins the election. Let P be the probability that the left party actually wins. It follows that, prior to the election, the expected social welfare of the nation as a whole as seen by person i becomes

$$\begin{aligned}\text{expected social welfare} &= PNW_{iL} + (1 - P)NW_{iR} - T \\ &= NW_{iR} + PN(W_{iL} - W_{iR}) - T\end{aligned}\quad (7)$$

where W_{iL} and W_{iR} are what they are regardless of what person i chooses to do. By choosing whether to vote or abstain, person i can influence T and P but not N , W_{iL} or W_{iR} . If person i 's cost of voting is C , the effect of person i 's decision to vote rather than to abstain causes T to increase by C . If the chance of a person's vote being pivotal is π , the effect of person i 's decision to vote rather than to abstain causes P to increase by π . Then, if person i sees a win for the left party as conducive to the welfare of the community as a whole, that is if $W_{iL} > W_{iR}$, person i 's assessment of the change in social welfare resulting from voting rather than to abstaining becomes

$$\text{change in social welfare} = \pi N(W_{iL} - W_{iR}) - C \quad (8)$$

which is essentially equation (1) with private benefit of a win for the party one favours replaced by public benefit as seen by person i . An analogous rule can be constructed for a supporter of the right party.

Equation (8) provides utilitarian grounds for deciding whether to vote or abstain. The inclusion of total population, N , in the utilitarian criterion provides a strong case for voting. For a country with one million voters, consider a person whose cost of voting is \$10, whose estimate of his own benefit from a win for the left party is \$200 and whose estimate of the benefit *per person* to society as a whole from a win by the left party is equal to \$50. If exclusively self-interested, that person votes if and only if his chance of casting a pivotal vote exceeds 1/20, and he may well abstain because his chance of casting a pivotal vote is probably much

lower than that. If that person is ethical as defined here, his assessment of the gain from a win by the left party rises from \$200 to \$50,000,000, and the required probability of a pivotal vote falls accordingly from 1/20 to 1/5,000,000 which may well low enough to oblige a person to vote.³

On the other hand, the benefit per person, $(W_{iL} - W_{iR})$ in equation (8) could be very low, or vanish altogether, if the advantage of a win for the left party to left-leaning voters just canceled out with the advantage of a win for the right party to all right-leaning voters, so that, no matter how large the total population, person i sees no net gain to society as a whole from a win by either party. The counter-argument is that even a tiny value of $(W_{iL} - W_{iR})$ creates a large benefit when N is large. When people differ in their valuations of $(W_{iL} - W_{iR})$ and C , a utilitarian criterion may exempt from voting people with low values of the one or high values of the other.

To simplify the story and to connect with other writings on ethical voting, imagine all voters divided into two distinct groups, where political preferences are different between groups but identical within each group, and suppose that the common value of $W_{iL} - W_{iR}$ among everybody who believes social welfare is higher under the left party just equals the common value of $W_{iR} - W_{iL}$ among everybody who believes social welfare is higher under the right party. Then utilitarian votes as long as

$$\pi\Delta W > C \quad (9)$$

where ΔW is the common value of $(W_{iL} - W_{iR})$ among supporters of the left party and of $(W_{iR} - W_{iL})$ among supporters of the right party.

There are several difficulties with the notion of utilitarian voting. The first is what might be called the problem of overkill. A utilitarian objective would seem to prescribe which party to vote for as well as whether to vote or abstain. No matter how much one has to gain from a win for the left party, no matter how large one's

³This is essentially the argument in Edlin, Gelman and Kaplan (2008). What Edlin ed. al. call "charity" as a motive for voting is only an inch away from what Feddersen and Sandroni (2006) call ethical voting on a utilitarian criterion. For a more detailed account of charity as a motive for voting, see Edlin, Gelman and Kaplan (2007)

value of B , it would be unethical to vote for the left party when social welfare is greater if the right party wins instead. In a community of utilitarians, one party would win unanimously unless people differed in their judgments about which party is best. A division based exclusively on opinions rather than interests is possible but odd, for it is hard to adhere to one's belief while recognizing that others as perceptive as oneself and no less disinterested believe something else. The best one can hope for in such cases is a variant of the Condorcet jury theorem where the best party is likely to win because every opinion has an equal chance of turning out to be right. Rule utilitarianism as distinct from act utilitarianism might exempt people from giving all they own to the poor, but would not exempt anybody who votes from voting in what they believe to be the interest of the community as a whole.

A distinction is required between what I am ethically obliged to do when everybody else acts ethically too, and what I am obliged to do in a society where some people act ethically but others do not. If a utilitarian objective exempts people from voting when the cost of voting exceeds some upper limit, that limit may actually be higher when not everybody feels compelled to vote than when a duty to vote is universally respected. The boy with his finger in the dyke would be obliged to hold it there for no more than half an hour a day if he were one of 48 boys with the same sense of obligation to protect the town, but he may be ethically obliged to stay at his post all day if there are no other boys nearby or if the 47 other boys refuse to do their share. Voting may be like that. The smaller the number of ethical voters, the more important each remaining vote becomes and the greater the cost of voting must be before one can be excused.

Degrees of duty may be recognized. For each person, there are two limits to the cost of voting, a lower limit (where $C = \pi B$) at which it becomes personally advantageous to vote and an upper limit (where $C = \pi \Delta W$) beyond which a utilitarian is exempted because the cost of voting would exceed the resulting gain in social welfare. Suppose the lower limit is \$10 and the upper limit is \$300. A completely ethical person votes as long as his cost of voting is less than \$300. A person may be said to be partially ethical if he votes as long as the cost is less than \$ x where x is somewhere between 10 and 300.

Most of people are probably somewhat ethical but less than completely so. The choice of which party to vote for and of whether to vote at all may be with reference to a utility function $U(Y, W)$ where Y is one's own income net, if one

votes, of the cost of voting and W is a measure of social welfare which would be dependent upon which party wins the election. This blend of selfish and utilitarian motives has something in common with Riker and Ordershook's rule in equation (1) above, but the specification of the criterion for voting and the meaning of duty are not quite the same. Consider a person who would vote left if he votes at all. His expected utility if he votes is

$$EU^V = p^V U(Y^L - C, W^L) + (1 - p^V) U(Y^R - C, W^R) \quad (10)$$

and his expected utility if he abstains is

$$EU^A = p^A U(Y^L, W^L) + (1 - p^A) U(Y^R, W^R) \quad (11)$$

where W^L and W^R represent social welfare depending on which party wins the election, where Y^L and Y^R represent this person's income depending on which party wins the election and where p^V and p^A represent the probability of the left party winning the election depending upon whether this person votes or abstains. The analogue to equation (1) is that this person votes if and only if $EU^V > EU^A$.

There is another possibility. In some circumstances, people's concern for others may extend no further than to others of the same political persuasion as oneself. A dubious assumption with regard to welfare as a whole, limited concern for others may be plausible in a referendum on such matters as Sunday shopping where the harm from being on the losing side of the vote is rigidly constrained. People who want stores open on Sunday may identify with one another, but have little or no concern, in this context, for the feelings of old fashioned folks who want to keep stores closed. Analytically, if support for Sunday shopping is identified as left and opposition to Sunday shopping is identified as right where k is the dividing line between them, equation (6) identifying person i 's sense of social welfare may be transformed into

$$Nu^i(W_i) = \sum_{n=1}^k u^i \quad (y_n) \quad (12a)$$

when person i is on the left and into

$$Nu^i(W_i) = \sum_{n=k+1}^N u^i \quad (y_n) \quad (12b)$$

when person i is on the right, on the understanding that people are ordered so that

the first k people would gain from a win by the left party and the remaining $N - k$ people would gain from a win by the right party.⁴

A distinction can be drawn here between rule-utilitarianism as a criterion for the choice of laws to be enforced by the state - what is sometimes called “government house” utilitarianism⁵ -and rules followed by ethically-inclined people in an environment without public enforcement and where some, but not all, other people are inclined to act the same way. A rich person may feel morally compelled to vote for a law mandating a transfer of income from all rich people to all poor people, but, in the absence of such a law, may feel no compulsion to transfer income voluntarily. A universally-applicable rule may exempt poor people from the cost of voting if and only if rich people feel morally compelled to vote for a appropriate degree of redistribution from rich to poor. An ethical rule may require one to act one way when everybody is expected to conform, but to act in a different way when they are not.

A duty to vote includes more than just voting. It includes an obligation to discover enough about competing political parties to make a reason judgment about which is best for oneself and for society as a whole. One would expect the cost of acquiring information about the difference between political parties to be greater, perhaps very much greater, than the cost of voting itself. The cost of actually voting is typically no more the hour or two one spends once every few years going to and from the ballot box. Information is acquired gradually in the hour or two one spends each day reading newspapers or listening to the news. Especially when there is an ethical motive for voting, one’s duty may be better performed by abstaining if one has not taken the trouble of acquiring enough information to make a reasoned judgment about which party is preferable. Voting based on little more than slogans and sound bites may be less dutiful than not voting at all.

From a utilitarian perspective, there are three voting crimes: to abstain when aggregate utility would be higher if one voted instead, to vote for the party which is best for oneself rather than for the party that is best society as a whole, and to allow

⁴A model along these lines, pertaining explicitly to referenda, has been developed in Coate and Conlin (2004). Such a model might describe preference in a society of workers and capitalists where members of each class care exclusively about other members of their class.

⁵On government house utilitarianism, see Goodin (1995) chapter 4.

oneself to vote foolishly because one has not taken the trouble to become well-informed about the virtues and vices of the competing parties.

Useful as they may be in focusing upon aspects of the voting process, the models of selfish voting, ethical voting for the nation as a whole or for one's fellow party supporters all suppress much of the detail of democratic politics. Left-leaning people may well have a sense of community with other left-leaning people, but they are also to some degree self-interested, with some concern for the welfare of others, left or right, in their town, province, church, occupation and the country as a whole, with a smaller degree of concern for everybody throughout the world and a still smaller degree of concern for generations to come. Choosing which of two parties to vote for is an immensely complex calculation, comparing the parties' mixtures of policies with effects on a wide range of people. Democracy may well require such diversity of concern. It is doubtful if democracy could be sustained in a society so polarized that concern for others extended no further than the membership of the political party one supports.

d) Altruistic Duty: Duty to the nation as a whole where every person has the same weight in one's objective function as everybody else and duty to one's social class where everybody within the class is weighted equally and people outside are given no weight at all are extremes of a more general concern for other people. The wealthy socialist is no contradiction in terms. One may be concerned about a poor person in Zimbabwe, somewhat more concerned about an equally poor person in one's own country, even more concerned if that person is a relative and most concerned if that person is oneself. There may be an income cutoff such that one takes account of the circumstances of other people if and only if their incomes fall below that critical level.

An altruistic variant of ethical duty is closer to the typical assumption in economics when a concern for others is introduced. Social welfare, W_i , as seen by person i may be a function of the incomes of other people weighted in accordance with person i 's degree of concern can be defined implicitly in a variant of equation (12) above.

$$\sum_{n=1}^N f_n^i u^i(W_i) = \sum_{n=1}^N f_n^i u^i(y_n) \quad (13)$$

where f_n^i is a diminishing function of n when everybody is ordered according to person i 's degree of concern for them. The weights themselves may be function of income. In political redistribution of income, person i may attach no weight anybody whose income is larger than his own, so that $f_n^i = 0$ whenever $y_n \geq y_i$, and, below that limit, weights may be assigned inversely proportional to income, so that f_n^i becomes an decreasing function of y_n .

A priori, any pattern of concern for others is possible. In practice, virtually everybody is somewhat selfish in that u is increased more by a rise in one's own income than by an equal rise in the income of anybody else, but a degree of altruism is common too. Typically, people's concerns for others are such that one's marginal utility of the income of a poor person exceeds the marginal utility of the income of a rich person, accounting for the willingness of many prosperous people to support progressive income taxation or social welfare programs. Thus, though a political party appealing primarily to rich voters would be expected to support policies relatively favourable to the rich and a political party appealing to poor voters would be expected to support policies relatively favourable to the poor, both parties may supply more redistribution of income than can be accounted for by purely selfish voting.

It is arguable that the balance between selfishness and altruism swings toward the latter when the chance of casting a pivotal vote is very small. If one gets a "warm glow" from doing the right thing - which in this context means voting for the party one believes to be best for society as a whole or for whatever segment of society one is especially concerned about - if the warm glow arises from the act of ethical voting regardless of whether or not one's vote turns out to be pivotal and if the chance one's vote turns out to be pivotal is small enough to wipe out the expected private benefit from a win by the party that is best for oneself, then the choices between voting and abstaining and of which party to vote for are reduced to a weighing of the benefit of "expressive voting" against the cost of casting one's ballot. "Expressive" in this context refers to supporting what you believe to be right, where the gain from doing what you believe to be right is what it is regardless of the chance that your action is effectual. The argument depends upon the premise that the chance of one's action being effectual matters for the evaluation of private

benefit but not for one's satisfaction in doing the right thing.⁶

e) Patriotic Duty: A duty to vote may be just that and no more, a duty to cast one's ballot with no additional obligation to vote for any particular party, on the understanding that most people will vote for the party that is best for themselves. For want of a better term, such voting may be called *patriotic*. Though full-fledged ethical voting - a moral obligation to vote for the party that seems best for society as a whole - may be the better rule if it is widely obeyed, the modest obligation to vote rather than to abstain may be preferable in a political environment where the more demanding rule would be disobeyed by a significant proportion of the population.

Unlike other forms of duty, patriotic duty has virtue of observability. One cannot know whether another person has voted selfishly, ethically, contractually or altruistically, but one can know whether that person has voted at all. Also, in an environment where some people vote and others abstain, some people are well-informed and others are not, it is not always obvious to oneself or to others what exactly a duty to vote requires and it may be best left to the individual voter how to balance the different considerations affecting the choice between one party and another. The best attainable outcome in this imperfect world may be a large turn-out of people who vote as they do for a variety of reasons know only to themselves. A patriotic duty to vote may be reinforced by a willingness to participate in the ceremony of democracy, to take one's place in the parade on which most people's sense a good society depends.

Patriotic duty may in practice be close to utilitarianism because, as long as everybody votes, the outcome is quite likely to be best for the community as a whole. If the absolute value of B is the same for everybody, a community of patriotic voters must maximize total welfare because the party with the most supporters automatically wins. There are exceptions. If a win for the left party is worth a penny to three quarters of the population and a win for the right party is worth a dollar to the remaining quarter of the population, then patriotic duty and utilitarian duty must point in opposite directions. With a universally-respected patriotic duty to vote, voting imposes a small fixed cost to the nation as a whole. Otherwise, politics becomes as it would be if there were no cost to voting at all.

⁶Feddersen et.al.(2009) provide evidence that people vote expressively. The paper also reviews the literature on expressive voting.

A duty to vote with no additional obligation about which party to vote for has the virtue that it can be obeyed and can be seen to be obeyed. It requires no great sacrifice on the part of the citizen. It protects democracy from the worst ravages of voting pacts among extremists. It focuses politics upon persuasion rather than getting out the vote. It allows the voter to be as selfish or as altruistic as he pleases in the choice of which party to vote for. It may be the best we can hope for in this imperfect world. Especially when the income distribution is skewed in the usual way, society can hobble along without an altruistic duty to vote, but a patriotic duty to vote may be indispensable because government by majority rule voting is only workable when a significant portion of the population chooses to vote rather than abstain.

f) Compulsory Voting: If and in so far as voting is a public good not just to like-minded voters, but to society as a whole, might it not be desirable to make voting compulsory? Perhaps people should be obliged to vote, just as they are obliged through taxation to pay a share of the cost of ordinary public goods. The case for compulsory voting depends very much on the strength of the duty to vote. If everybody votes from a sense of duty, because voting is fun or as a way of identifying oneself with the community of which one is a part, then compulsory voting is at best superfluous. At the other extreme where nobody votes unless it is individually advantageous to do so, compulsory voting could be as necessary as compulsory taxation to finance the police force. In between, the case for compulsory voting runs parallel to the enumeration of the reasons why there may be a duty to vote.

Among defects of voluntary voting that might be mitigated by compulsory voting are the bias when widespread abstention would favour one party over another, the emergence of voting pacts among groups or classes best able to vote in unison and the tilting platforms of political parties toward groups of people who are more inclined to vote when not compelled to do so.⁷ Together, these effects of widespread abstention tend to favour the rich over the poor, for the rich are less likely to be deterred by any given cost of voting, more likely to vote in unison and, a matter

⁷Compulsory voting is advocated by Lijphart (1997). Lijphart claims that turnout is declining in most countries, that the propensity to vote rather than to abstain increases with education and wealth, and that there is a corresponding bias in public policy against the uneducated and the poor.

not discussed in this paper, much more able to bear the cost of campaign advertising.

On the other side of the ledger are several disadvantages of compulsion. Foremost among these is that what is called a duty to vote is not just an obligation to show up at the ballot box. It is a duty to inform oneself about what is at stake in the election, to investigate the platforms of the competing parties, and to learn which party is best for oneself and for the nation as a whole. Voting can be made compulsory, but nobody can be compelled to vote thoughtfully. We cannot compel one another to watch the news on TV, to read newspapers and to become informed about the issues at stake in an election. Casual or uninformed voting could be massively counter-productive, excessively responsive to prejudicial attack ads and to mendacious campaign propaganda.

The case for compulsory voting is less compelling when support for one party or the other originates from opinions rather than interests. By contrast with the situation considered so far where each party is generally understood to be best for some subgroup of the population, it may instead be generally understood that one of two competing parties will turn out to be best for everybody, but people may disagree about which party that will be. Supporters of the right party believe that everybody would be better off if the right party is elected. Supporters of the left party believe that everybody would be better off if the left party is elected. A person with no opinion either way or who is very unsure of his opinion might well abstain, leaving the choice between parties to others who seem to be better informed than he.

An argument against compulsory voting might be made on the ground that voting is expensive. As discussed in connection with figure 1 above, the cost-saving in abstentions is pure gain to society as a whole if abstentions are evenly divided between political parties, but there is no guarantee that abstentions will be equally divided and the cost saved by not voting - the twenty or so dollars per abstention - is likely to be trivial by comparison with the cost to society of choosing the wrong political party and of the gradual weakening of democracy when people do not bother to vote.

Relative merits of voluntary and compulsory voting depend upon considerations not fully explored in this paper or, for that matter, in much of the literature on the propensity to vote. a) What exactly is the probability of an electoral

mistake where, for example, a majority of eligible voters prefers the right party but a majority of votes cast is for the left? In the example in figure 1 there is no such probability, but the probability of a mistake emerges when voters' valuations schedules are twisted as illustrated in figure 2 and is especially likely when one party tends to favour the rich and the other tends to favour the poor. b) What exactly is a mistake? Is the only objective to elect the party supported by a majority of voters, i.e. to elect the left party if and only if $m > N - m$. If so, how might the cost of a failure to do so be compared with the cost of voting? Or is the ultimate criterion to maximize social welfare by electing the party yielding the larger value some average over the entire population of W_{iL} and W_{iR} as defined in equation (6). Usually the party favoured by the larger number of people yields the larger social welfare as well, but that is not invariably so.

The ultimate objective in majority rule voting is not so much to make the right choice between two or more not too dreadful political parties as to provoke political parties to choose socially advantageous platforms and to govern well, to preserve government by majority rule voting and to keep the dictator away.⁸ The case for compulsory voting may well depend on the proportion of citizens who are inclined to vote voluntarily. If the proportion of abstentions is small, the balance of costs and benefits is probably against compulsory voting. If the proportion of abstentions is large, compulsion may be required. It is hard to say where exactly the line should be drawn.

Conclusion

Voting can be looked upon as a complex multi-person chicken game where it does not much matter if a few people abstain and there can be a slight gain to the abstainers from doing so, but where widespread abstention is harmful to everybody. To emphasize this aspect of voting, imagine an electorate of just two people, A and B, as illustrated in Figure 3 where both persons gain 10 as long as somebody votes, neither person gains at all if nobody votes, but there is a slight additional gain of 0.1 to whichever person abstains as long as the other votes. The sole voter is essentially a sucker.

⁸ Riker makes this distinction in the title of his book, "Liberalism against Populism" (1988).

Figure 3: Voting as a Chicken Game

	Person A votes	Person A abstains
Person B votes	10 / 10	10.1 / 10
Person B abstains	10 / 10.1	0 / 0

With just two people, there is obviously room for a deal in which they vote in alternative elections, but this solution to the chicken game is not open in an election with many voters and two political parties. Alternatively, if one person stubbornly refuses to vote at all, it is in the interest of the other to do so instead. Disaster looms if both people are stubborn.

With a large electorate, the alternatives are compulsion and reliance upon a duty to vote, and there is no well-recognized line between numbers of voters that are and are not sufficient to avert the voting analogy to the outcome in the south east box of Figure 3. The social cost of excessive abstention is a combination of 1) a risk that the party which is best for the community as a whole, and which would be elected if almost nobody abstained, fails to win the election, 2) a diversion of political activity from informing the electorate to getting out the vote, 3) the rise of extremism and the accentuation of the influence of small groups of fanatics who are especially likely to vote at their leader's command and 4) the gradual deterioration of democratic government as these trends accentuate. These evils are averted when a large enough proportion of the electorate chooses to vote rather than to abstain because they are compelled to do so or from a sense of duty. The principal drawbacks of compulsion is that people cannot be compelled to vote thoughtfully. Voluntary voting is best if and only a sufficient proportion - where it is difficult to say in practice where the line should be drawn - of the population is prepared to vote from a sense of duty alone.

Two great requirements differentiate political markets differ from commercial

markets, the requirement for compromise among politicians in the formation of platforms of political parties and in passing bills in the legislature, and the requirement among citizens for a duty to vote when voting may not be personally advantageous.

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Appendix: Why There is Less Redistribution of Income when the Propensity to Vote is Higher among the Rich than among the Poor.

It is commonly believed that abstentions influence political outcomes, in particular, that a relatively high rate of abstention among the poor tends to produce the amount of redistribution that the government supplies. Presented in this appendix is an example of precisely that. The ingredients of the example are a propensity to vote that increases steadily with income, a skewed distribution of pre-tax income, redistribution by a negative income tax, tax evasion and a tax rate determined by majority rule voting where the first preference of the median voter prevails. The chosen tax rate is lower, and the corresponding redistribution is diminished. Society's chosen tax rate is lower, and redistribution is diminished, when the propensity to vote increases with income than when everybody's propensity to vote is the same.

These are the assumptions:

a) People's pre-tax incomes, Y , vary steadily from 0 to 1. There is no harm in thinking of units as millions of dollars. Specifically, the density of population, $p(Y)$, is inversely proportional to income.

$$p(Y) = 1 - Y \quad (A1)$$

so that the average income is $\frac{1}{2}$.

b) The higher a person's income, the more likely is that person to vote rather than to abstain.

Specifically, people's propensity to vote is directly proportional to income.

$$v(Y) = Y \quad (A2)$$

A person with no income does not vote at all, a person with the highest income, 1, always votes and people's propensity to vote varies steadily in between. This is a very strong assumption chosen to simplify calculation and to produce a strong result.

c) Redistribution of income is arranged through a negative income tax. Everybody is

taxed at a fixed rate, t , and the entire revenue from the tax is redistributed in equal amounts to everybody. All other uses of public funds are ignored. The preferred tax rate, $t(Y)$, of a person with income Y is the rate at which that person's disposable income, I , is maximized, where disposable income is pre-tax income, less tax paid, less expenditure to reduce the amount of income declared, plus the equalized transfer of all public revenue. It will be shown that $t(Y)$ falls as income rises. The exact form of the function, $t(Y)$, will be derived.

d) People evade tax to the extent that it is in their interest to do so. Only a portion of one's income is hidden from the tax collector because the cost of hiding income increases steadily with the amount of income one hides. Specifically the cost of hiding the first 1% of one's income is 1% of the amount hidden, the cost of hiding the second 1% is two percent of the amount hidden, and so on until the cost of concealing the final 1% is 100% of the amount hidden. Tax evasion is one of several ways of reducing one's tax bill. Others are to work less and to engage in more do-it-yourself activities, but these are ignored here. The important consideration is that each person hides income up to the point where cost of hiding an extra dollar of income the cost of the tax that would otherwise be paid. This assumption ensures that nobody, not even the very poorest person, wants a tax rate as high as 100%.

e) The tax rate is chosen by majority rule voting where the first preference of the median voter prevails.

f) In voting about the tax rate and in choosing how much of one's income to declare, everybody acts in his own interest exclusively.

On these assumptions, a distinction can be drawn between two median incomes, the median, M , in the entire population and the median, R , among people who choose to vote rather than to abstain. The median M is the income of the person in the middle when everybody, regardless of whether or not he votes, is lined up in accordance with pre-tax income Y . The median R is the income of the person in the middle when only those people who vote are lined up in accordance with pre-tax income.

It will be shown that, on these assumptions,

- the income of the median *person*, M , is less than the income of the median *voter*,

R. Specifically, the pre-tax income of the median person is .293, while the pre-tax income of the median voter is .5.

- the preferred tax rate of the median person is 29.3%, raising the disposable income, I , of the median person by about 10% from .239 to .324. [That the preferred tax rate of the median person is the same as his pre-tax income is an accidental consequence of the assumptions, a quirk with no economic significance.]

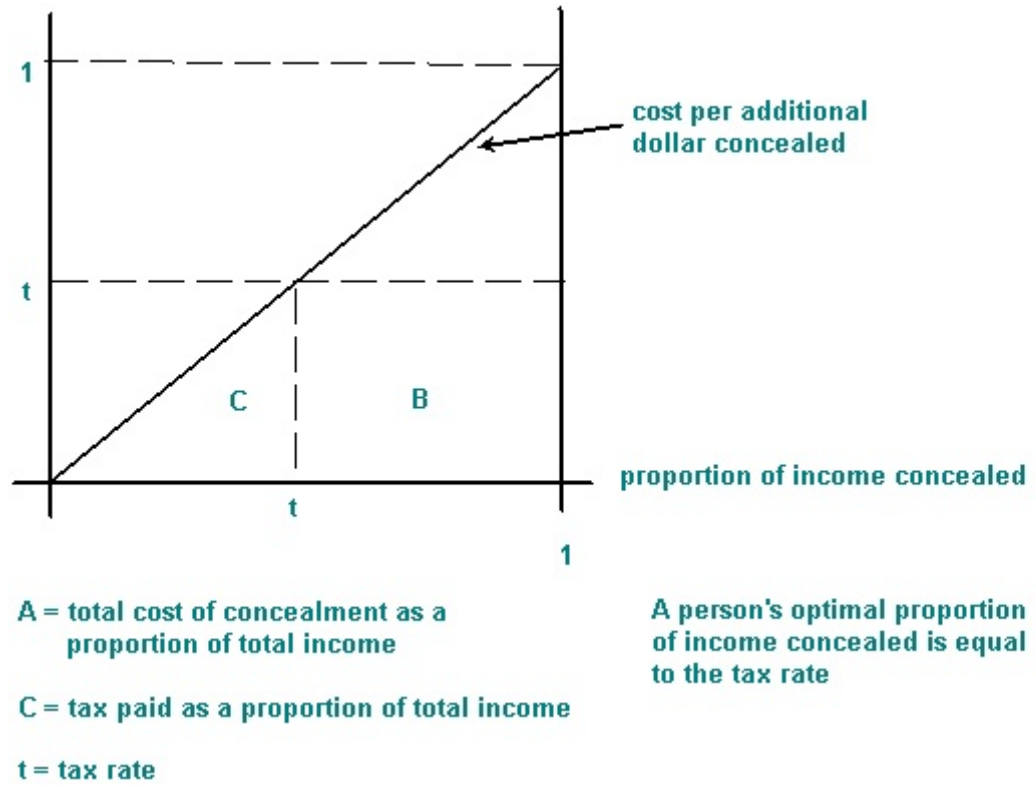
- The preferred tax rate of the median voter is 0%.

In this example, abstentions decreasing with income block redistribution altogether. The effect of abstentions would be in the same direction but less dramatic if the correlation between income and the propensity to vote were less pronounced.

The median voter's preferred tax rate, $t(M)$, is computed in three stages. First the gap between disposable income, I , and declared income, Y , is computed as a function of the tax rate. Then a person's preferred tax rate is computed as a function of income, i.e. the function $t(Y)$ is derived. Finally, the income, M , of the median voter is computed together with the median voter's preferred tax rate.

In accordance with assumption (d), the taxpayer's cost of tax evasion is shown in figure A. The premise behind assumption (d) is that there are inexpensive ways to conceal small amounts of income, but, the more income is concealed, the more expensive does extra concealment become. Both axes are proportions between 0 and 1, the horizontal axis of income concealed, the vertical axis of the cost of concealment per dollar concealed. The height of the diagonal line is the marginal cost of concealment as a function of the proportion concealed. In choosing the proportion of income to conceal, the taxpayer equates the marginal cost of concealment to the tax that would otherwise be paid. A person conceals income up to the point where it would be more costly to conceal an extra dollar of income than to pay the tax on that amount. It then follows that a tax rate of t induces people to conceal a fraction t of their incomes, so that declared income becomes a fraction $(1 - t)$ of true income and tax paid as a proportion of true income becomes $t(1 - t)$, shown as the area B on the figure. The full cost of concealment per dollar of income is indicated by area C which is equal $t^2 / 2$. Denote average income by Y^A . It follows from assumption (a) that Y^A equals $1/2$. Total tax revenue per person is $t(1 - t)Y^A$.

Figure A: Taxpayers' Cost and Benefit of Concealment of income from the Tax Collector



It then follows that the disposable income, I , of a person with pre-tax income, Y , becomes

$I = \text{pre-tax income} - \text{tax paid} - \text{cost of concealment} - \text{transfer (equal to average tax revenue)}$

$$= Y - t(1 - t)Y - (t^2/2)Y - t(1 - t)Y^A \quad (A3)$$

If empowered to choose the tax rate all by himself, a person with income Y would choose a rate, called $t(Y)$, to maximize disposable income, I . The rate would be set so that $\delta I / \delta t = 0$. Specifically,

$$\delta I / \delta t = -Y + 2tY - tY + (1 - 2t)Y^A = 0 \quad (A4)$$

implying that

$$t(Y) = (Y^A - Y) / (2 Y^A - Y)$$

$$= (1 - 2Y) / (2 - 2Y) \quad \text{because } Y^A = 1/2 \quad (A5)$$

This is the exact specification of the function $t(Y)$ in assumption (c) above. It follows at once that the preferred tax rate of a person for whom $Y = 0$ is 50%, and that the preferred tax rate declines steadily with income until it falls to 0 at Y^A . Anybody with more than the average income would prefer the tax rate to be negative, but that is not relevant here.

Now consider the median incomes, M and R , for all the entire population and for voters only. The median income, M , for the entire population is the solution of the equation

$$\int_0^M p(Y) dY = \int_M^1 p(Y) dY \quad (A6)$$

or equivalently

$$\int_0^M (1 - Y) dY = \int_M^1 (1 - Y) dY \quad (A7)$$

Since the integral of $(1 - Y)$ is $(Y - Y^2 / 2)$, equation (A7) implies that

$$[M - M^2/2] = 1/2 - [M - M^2/2] \quad (A8)$$

or

$$2 M^2 - 4M + 1 = 0$$

so that

$$M = 1 - 2^{-1/2} = .293$$

and the median person's preferred tax rate in accordance with equation (A5) is

$$t(M) = (Y^A - M) / (2 Y^A - M) = (1/2 - .293) / (1 - .293) = .293$$

If everybody voted, causing redistribution to reflect the preference of the

median voter, the tax rate would be .293 and the per capita transfer from the government .1036, raising the income of the poorest person from nothing to .1036 equivalent to 20.72% of average income.

From equation (A3), it follows that redistribution raises the disposable income of the median person from .293 to .323, but lowers the disposable income of the richest person from 1 to .856.

By contrast, the income, R , of the median income among all voters is identified by the equation

$$\int_0^R p(Y)v(Y)dY = \int_R^1 p(Y)v(Y)dY \quad (A9)$$

where $p(Y)v(Y)$ is the density of *voters* with income Y . With $p(Y)$ and $v(Y)$ in assumptions (a) and (b), equation (A9) becomes

$$\int_0^R (1-Y)Y dY = \int_R^1 (1-Y)Y dY \quad (A10)$$

Since the integral of $(1-Y)Y$ is $(Y^2/2 - Y^3/3)$, equation (A10) reduces to

$$[R^2/2 - R^3/3] = (1/2 - 1/3) - [R^2/2 - R^3/3]$$

or
$$4R^3 - 6R^2 + 1 = 0$$

implying that $R = 1/2$ which just happens to equal the pre-tax, pre-transfer income in the population as a whole. The median income among all voters just happens to equal the average income among all people, whether or not they vote. But, as is immediately evident from equation (A5), the person with the average income prefers a tax rate of 0% with no redistribution at all.

On the assumptions in this example, redistribution of income is blocked altogether by the greater propensity to vote rather than to abstain among the rich than among the poor. Milder assumptions would yield a less dramatic result, but the general direction of this result would remain. The higher the correlation between income and the propensity to vote, the less redistribution there would be.