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Markets, Income and Policy in a Unified Macroeconomic Framework

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Abstract

I construct a unified macroeconomic framework by incorporating frictional markets in a neoclassical environment. This framework is analytically tractable despite search frictions, income risks and endogenous money distributions. I use this framework to formalize a theory that the variety and the functioning of markets reflect the status of national income. In the model, households and firms have free access to goods markets with and without trading frictions, where the frictional markets are featured by competitive search. I characterize and prove the existence of a steady state. In equilibrium, the frictionless markets are generically used to smooth consumption, whereas the frictional markets are used only when households have sufficiently high expected real income. Uninsurable idiosyncratic income risks cause households to endogenously sort into various submarkets upon entry to a frictional market. Consequently, income inequality determines the dispersion of equilibrium trading protocols across frictional submarkets. In addition to the redistributive effects, both monetary and fiscal policies have their distinctive impacts on the intensive and extensive margins of frictional trading. The optimal policy program consists of money growth, proportional income taxes and proportional sales subsidies.

JEL Classification Codes: E0, E4, E5, E6, H2, H3.

Keywords: Market, Friction, Distribution, Competitive Search, Policy.

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1 Introduction

I construct a tractable macroeconomic framework that incorporates frictional trading in a neoclassical environment. I use this framework to develop a theory, which explains the key role of household income in determining the functioning of various markets. By the variety of markets, I refer to markets with or without trading frictions. By the functioning of markets, I refer to the specific trading protocols of markets. Economists have long been analyzing the frictionless markets, i.e., Walrasian markets. Encouragingly enough, recent research on the frictional markets, known as the search theory, has also gained significant advancement and wide recognition. Often in the real economies, both types of markets seem to exist and continue to flourish. For example, markets for groceries can be considered as frictionless, whereas markets for new cars or houses are typically regarded frictional. Such market variety does not even have to be tied to particular goods. For example, the market for houses for sale tends to display far more significant frictions than the rental market for housing. Given such observations, I address the following fundamental questions: Why, or under what circumstances, do frictional and frictionless markets coexist? What are their respective roles in an economy?

The variety and the functioning of markets seem to have an intricate connection with national income. To say the least, there tends to be a more sophisticated variety of markets in developed countries than in developing countries. In light of such observations, I build this model in a way that allows it to endogenously generate income distributions as well as individual choices of markets. In the model, there are idiosyncratic shocks to households' preference for labor supply. The uninsurable income risks give rise to diverse trading strategies of households, as well as decisions on consumption, savings and labor supply.

Households supply labor to competitive firms and purchase consumption goods from markets. There are two types of markets, i.e., frictionless and frictional. The frictionless markets are used to trade the general consumption goods. Such goods are general in the sense that all households can consume them and all firms can produce them. In essence, general goods have general availability but no variety. The frictionless markets are competitive and they clear in the Walrasian way. The frictional markets are used to trade the special consumption goods. These goods are special in the sense that only a fraction of the households can consume them and only a fraction of the firms can produce them. In other words, special goods are characterized by variety yet limited availability. A market for such goods has frictions because households and firms cannot coordinate and there lacks double coincidence of wants. Households and firms have free access to markets with and without frictions. In particular, households participate in both types of markets

if and only if it is optimal to do so.

A frictional market consists of a variety of submarkets. There is competitive search, also known as directed search, in each submarket. Search is competitive in that both households and firms take as exogenous the terms of trade of all submarkets, and choose which submarket to participate in. Individuals expect the trade-off between the terms of trade and the matching probabilities in a submarket. In particular, consider two submarkets that offer the same quantity of goods but require different amounts of money in exchange. The submarket involving a higher amount of money has a higher matching probability for a buyer but a lower matching probability for a seller. In equilibrium, free entry of firms ensures consistency of such expectations. Fiat money can be used for transaction purposes to overcome trading frictions and/or for precautionary purposes to insure oneself against income fluctuations.

This framework is tractable due to competitive search and free entry of firms. As a result, the endogenous money distribution does not affect the decision problems of any household or firm. This feature is called block recursivity, which is a concept first applied to economics by the seminal work of Shi (2009) on equilibrium wage-tenure contracts. With block recursivity, the state space of individual decision problems is drastically reduced, which makes the model exceptionally tractable. Tractability allows this model to generate a rich set of results, both analytically and quantitatively. I summarize the model results as follows.

First, households choose to participate in the frictionless markets under all circumstances. This is simply because there is no risk in obtaining consumption goods in such markets. Therefore, the frictionless markets are generically used for consumption-smoothing purposes. In contrast, households only use the frictional markets when their expected real income is sufficiently enough. This result is caused by risky trading in the frictional market. Intuitively, if income is generally low, then the household can only afford a low level of overall consumption. In any idiosyncratic state, taking a part of income to participate in the frictional market means that the household must endure a significant fluctuation in overall consumption because of the risks involved in getting special goods. This cannot be the optimal strategy if the expected income is sufficiently low. Therefore, households only pursue consumption variety if they have sufficiently high income. Overall, the variety of markets in an economy reflects the level of aggregate real household income.

Second, the functioning of frictional markets is critically determined by income inequality. The more dispersed the income distribution, the more varied the terms of trade and the matching probabilities across frictional markets. Therefore, the functioning of frictional

markets is a reflection of the severeness of income inequality in an economy.

Third, money serves two economic roles, as a medium of exchange and as a store of value. If the frictional market is used, money serves as a medium of exchange, which is the case when the expected income level is sufficiently high. If the idiosyncratic income levels are sufficiently diverse, households save money in a precautionary way, to fight against income fluctuations. Moreover, if household income is sufficiently volatile and the expected level is sufficiently high, households use money for both transaction and precautionary purposes. These results seem to be consistent with the historic evolution of money along economic progression.

Finally, policy can be used to influence the functioning of various markets. Inflation, income taxes and sales taxes on frictional trading all have their distinctive implications on the intensive and extensive margins of frictional trading. In particular, a higher money growth rate, a lower income tax rate and a lower tax rate on frictional sales, can all contribute to enhancing the extensive margin in the frictional markets. Therefore, households are more likely to participate in the frictional markets under all of these policies, which tends to improve welfare because of the expanded consumption variety. Moreover, inflation and income taxes both have a positive redistributive effect and a negative taxation effect on welfare. The latter is straightforward and the former is because these policies help mitigate income inequality across idiosyncratic states and thus stabilize income fluctuations. Overall, the optimal policy program consists of inflation, proportional income taxes and proportional sales subsidies. It is critical for the government and the monetary authority to coordinate policy decisions. In particular, it tends to be welfare-improving for the government to alleviate income taxes when the monetary authority is running deflation and to elevate income taxes under inflation.

This framework was inspired by Lagos and Wright (2005) and Menzio, Shi and Sun (2011). The influential work of Lagos and Wright (henceforth LW) was the first model structure that contained both frictional and frictionless markets. Alternating frictional and frictionless markets, together with quasi-linear preferences, give rise to a degenerate equilibrium money distribution, which makes their model analytically tractable. Because of tractability, the unique LW framework has prompted an exploding literature on micro-founded models of money with an emphasis on market frictions. This literature has recently been recognized as the *New Monetarist Economics* (Williamson and Wright, 2010a,b). In contrast to LW, the frictional and frictionless markets in my framework are not used to achieve tractability. Instead, they are the topical focus of my theory. Moreover, my framework is analytically tractable even with a non-degenerate equilibrium money distribution.

Menzio, Shi and Sun (2011) (henceforth MSS) was the first paper to explore the concept of block recursivity in a money search environment. In MSS, individuals only have access to frictional goods markets. Moreover, an individual cannot produce and purchase goods in the same time period. In equilibrium, all individuals go through a cycle, in which they work for one period and then stay as a buyer for one or more periods. The equilibrium money distribution is discrete by nature, which makes it challenging to analyze monetary policy in this model. In contrast, in my framework households can supply labor and purchase goods in every time period. More importantly, households have access to the frictionless markets to re-adjust their money balances, including the lump-sum government transfers. As a result, it is straightforward to have a stationary money distribution under monetary policy, whether this distribution is discrete or not. Furthermore, my framework can encompass idiosyncratic as well as aggregate uncertainty, in *i.i.d.* and persistent forms.

Overall, my framework contrasts with both LW and MSS in that it studies the functioning of various markets in a neoclassical environment. This helps bring the search theory one step closer to the mainstream macroeconomic literature. Furthermore, my framework can be used to study the effects of a comprehensive policy program, including monetary and fiscal policies such as inflation, income taxes and sales taxes.

The rest of this paper is organized as follows. Section 2 describes the physical model environment. Section 3 characterizes the monetary equilibrium and discusses the theoretical results. Section 4 examines the policy effects, both analytically and numerically. Finally, Section 5 concludes the paper.

2 A Unified Macroeconomic Framework

2.1 The environment

Time is discrete and continues forever. There is a large number of *ex ante* identical households. Each household consists of a couple, i.e., a worker and a buyer. All households can produce and consume general goods in the first sub-period. General goods are traded in perfectly competitive markets, called frictionless markets. Households can produce and consume special goods in the second sub-period. Special goods are traded in frictional markets in the sense that there is random matching between buyers and sellers in such a market. There are at least three types of special goods. The households are specialized in production and consumption of special goods, in a way that no double coincidence of wants can exist between any two households. The total measure of households is normalized to one.

The household members share income, consumption and labor cost. The preference of a household in a time period is

$$\mathcal{U}(y, q, l) = U(y) + u(q) - \theta l, \quad (1)$$

where y is consumption of general goods, q is the household's consumption of special goods and l is labor input in a time period. The parameter $\theta \in [\underline{\theta}, \bar{\theta}]$ measures the random disutility per unit of labor. It is *i.i.d.* across households and over time, where $0 < \underline{\theta} < \bar{\theta} < \infty$. It is drawn from the probability distribution $F(\theta)$. The value of θ is realized at the beginning of a time period, before any decisions are made. The functions u and U are twice continuously differentiable and have the usual properties: $u' > 0$, $U' > 0$; $u'' < 0$, $U'' < 0$; $u(0) = U(0) = u'(\infty) = U'(\infty) = 0$; and $u'(0)$ and $U'(0)$ being large but finite. Households discount future with $\beta \in (0, 1)$. There is no insurance on income risks. Nor is borrowing or lending feasible.

There is a measure one of competitive firms. Firms hire workers from households, who own equal shares of all firms. The labor market is competitive and frictionless. All firms have free access to the frictionless and the frictional goods markets. Labor is hired in the first sub-period and is used in production for both general and special goods. Each firm can organize production and sales of the general goods and one particular type of special goods. Therefore, each firm only hires workers who are specialized in producing that particular type of special goods, in addition to producing general goods. A firm pays competitive wages and distributes profit to the households. In a frictional market, firms have free entry to a variety of submarkets, which differ in terms of trading protocols. A firm chooses the measure of shops to operate in each submarket. The cost of producing q units of special goods is $\psi(q)$ units of labor. The cost of operating a shop for one period is $k > 0$ units of labor. The function ψ is twice continuously differentiable with the usual properties: $\psi' > 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

In each period, markets with and without frictions open sequentially in two sub-periods. Trading in the frictionless goods market takes place in the first sub-period, followed by trading in the frictional market in the second sub-period.¹ All goods, general and special,

¹In the framework by Lagos and Wright (2005), it is critical to have the frictional and the frictionless markets operate sequentially, in order to make the model tractable. The frictionless market, together with quasi-linear preferences, generates a degenerate money distribution across individuals. In contrast, my framework does not require a degenerate money distribution to gain tractability. It is competitive search in the frictional market that significantly improves tractability. I use the sequential order of markets because fundamentally it is an efficient arrangement for households to go through alternating frictionless and frictional markets. In particular, households can use the frictionless market to re-adjust their money balances, which is important when households are facing income, as well as matching, risks. Therefore, it is

are perfectly divisible. They are also perishable and cannot be consumed across sub-periods. The worker of a household works for a firm, while the buyer goes shopping in the goods markets. In a frictional market, buyers and shops are anonymous to each other. There is no record-keeping technology for the actions of individual buyers or shops. Thus a medium of exchange is needed in every trade in the frictional market, to overcome the problem of no double coincidence of wants. This role can be served by a fiat object called *money*. Money is perfectly divisible and can be stored without cost.

Trading in a frictional market is characterized by competitive search across a variety of submarkets. Each submarket specifies particular terms of trade between the buyers and the shops, together with respective matching probabilities. Search is competitive in the sense that households and firms take as given the terms of trade and the matching probabilities of all submarkets, and choose which submarket to participate in. Matching in each submarket is random and pair-wise between a buyer and a shop because households and firms cannot coordinate. Once matched, the exchange of money for goods is conducted according to the terms of trade specified for that submarket. In equilibrium, firms' free entry is such that the terms of trade and the matching probabilities are consistent with the specified ones.

A submarket is characterized by its trading protocols, (x, q, b, s) , where (x, q) are the terms of trade and (b, s) are the respective matching probabilities for a buyer and a shop. In particular, x is the amount of money and q is the amount of goods traded in a match. Denote N_b/N_s as the *market tightness*, where N_b and N_s are respective measures of buyers and shops. The measure of matches in the submarket be given by a function $\mathcal{M}(N_b, N_s)$, which has constant returns to scale. Then one can express s as a function of b , denoted as $s = \mu(b)$, which I will refer to as the *matching function*. As households and firms choose which submarket to enter, the tightness in each submarket becomes a function of the terms of trade (x, q) . In turn, both s and b are functions of (x, q) , as is shown in (4). Therefore, a submarket can be sufficiently indexed by (x, q) . I impose the following assumption:

Assumption 1 For all $b \in [0, 1]$, the matching function $\mu(b)$ satisfies: (i) $\mu(b) \in [0, 1]$, with $\mu(0) = 1$ and $\mu(1) = 0$, (ii) $\mu'(b) < 0$, and (iii) $[1/\mu(b)]$ is strictly convex, i.e., $2(\mu')^2 - \mu\mu'' > 0$.

Part (i) is a regularity condition. Part (ii) requires that a submarket with a higher matching probability for a buyer must have a lower matching probability for a shop. This is

an optimal strategy for the households to visit the two types of markets sequentially in every period. This helps smooth consumption over time and across idiosyncratic states. In other words, forcing households to trade in the frictionless and the frictional markets at the same time is inefficient and significantly restricts the trading strategies of households.

because such a submarket has relatively more shops than buyers. With this assumption, for any given x , a firm's total cost of production is lower in submarkets with higher b . Part (iii) imposes that a firm's production cost be strictly concave in b for any given x . That is, firms must be compensated with increasingly larger reductions in the production cost so that a submarket can attract firms to set up more shops to increase buyers' matching probability. This requirement is necessary for the trade-off between the matching probability and the terms of trade across submarkets to yield a unique optimal choice of a submarket for a buyer.

The money stock per capita evolves according to $M' = \gamma M$, where $\gamma \geq \beta$ is the money growth rate. The variable M refers to the aggregate money supply in a period and M' is that of the next period. Money growth is achieved by a lump-sum transfer from the government to the households, and vice versa for money contraction. In this environment, firms are known identities even though individual buyers and shops are anonymous and untraceable. Therefore, the government can run a comprehensive fiscal policy program, which includes taxes on wage income, and sales taxes imposed on households as well as on firms. In particular, $\tau \in [0, 1]$ is the proportional income-tax rate, $\sigma_g, \sigma_s \leq 1$ are the proportional sales-tax rates (subsidy rates if $\sigma_g, \sigma_s < 0$) collected from firms and imposed respectively on general and special goods revenue, and $t_g, t_s \geq -1$ are the proportional sales-tax rates (subsidy rates if $t_g, t_s < 0$) collected from households and imposed respectively on general and special goods purchases.

It is necessary to elaborate on how the taxes are collected. Income taxes are remitted from firms to the government. That is, workers only receive after-tax wage income. Sales taxes imposed on firms are collected directly from firms. In other words, these taxes are included in goods prices. Sales taxes imposed on households are paid by buyers to shops at the point of sale. Firms bundle these tax payments from shops and then remit them to the government. Firms cannot evade taxes because the total revenue of a firm is deterministic even though its individual shops face idiosyncratic matching risks. All taxes are redistributed from the government to the households in a lump-sum manner, together with the transfers made for money growth purposes. Transfers are made in the first sub-period of each period.

I focus on steady state equilibria. I will suppress the time index throughout the paper. I use labor as the *numeraire* of the model. In particular, let m denote the real value of a household's money balance at a particular point in time, where the label "real" means that m is measured in terms of labor units. I assume that $m \in [0, \bar{m}]$, where \bar{m} is a sufficiently large but finite number. Let w denote the normalized wage rate, which is the nominal wage

rate divided by the per capita money stock M . Then the dollar amount associated with a balance m is $(wM)m$.

2.2 A firm's decision

In the frictionless market, a representative firm takes the general-goods price as given and chooses output Y to maximize profit. It takes Y units of labor to produce Y units of general goods. Let p be the price of general goods, measured in terms of labor units. The firm's revenue pY is subject to a proportional sales tax σ_g . In the frictional market, the firm takes the terms of trade for each submarket, (x, q) , as given and chooses the measure of shops, $dN(x, q)$, to set up in each submarket. Recall that a shop is matched by a buyer with probability $s = \mu(b)$. Once matched, production takes place and money and goods are exchanged according to the terms of trade specified for this particular submarket. For a particular shop in the submarket, the operational cost is k units of labor and the expected cost of production is $s\psi(q)$ units of labor.² A shop's expected revenue is sx , where the revenue x is measured in labor units and is subject to the sales tax rate σ_s . The firm's total profit in a period, π , is given by

$$\pi = \max_Y \underbrace{\{(1 - \sigma_g)pY - Y\}}_{\text{profit in frictionless}} + \max_{dN(x, q)} \underbrace{\left\{ \int \{(1 - \sigma_s)xs(x, q) - [k + \psi(q)s(x, q)]\} dN(x, q) \right\}}_{\text{profit in frictional}} \quad (2)$$

Free entry of firms implies that the firm earns zero profit in the frictionless market and $(1 - \sigma_g)p = 1$ in equilibrium.

²In this framework, workers are paid a competitive wage rate for their choices of expected labour input, rather than for the exact amount of labour input. For example, suppose the firm allocates n workers to a particular shop in a submarket (x, q) . Also assume that these workers have offered to supply the same amount of expected labour effort l . The total labour cost of maintaining the shop, k , occurs regardless of whether the shop is matched with a buyer. With probability $s(x, q)$, a buyer shows up and the workers exert a total of $\psi(q)$ units of labour to produce the goods. With probability $1 - s(x, q)$, the workers do not produce anything and avoid such labour cost. Overall, the wage income for each worker is deterministic and is given by $l = [k + s(x, q)\psi(q)]/n$. Households understand that there is uncertainty involved with the amount of labour required for a job. Nonetheless, they accept this arrangement because of risk neutrality in labour supply and risk aversion in consumption. This particular structure of the model is intended to capture the following observations. In reality, people seem to tolerate significant variations in the amount of work involved with a particular job. For example, Christmas season can be several times busier for a store salesperson than any other time of the year. A bank teller can be more occupied around noon than other time of the day. In many professions, people are paid a stable salary for working a certain number of hours a day and a certain number of weeks a year, given that the job is more demanding in certain times than others.

The expected profit of operating a shop is

$$s(x, q) [(1 - \sigma_s) x - \psi(q)] - k.$$

If this profit is strictly positive, the firm will choose $dN(x, q) = \infty$, but this case will never occur in equilibrium under free entry. If this profit is strictly negative, the firm will choose $dN(x, q) = 0$. If this profit is zero, the firm is indifferent across various non-negative and finite levels of $dN(x, q)$. Thus, the optimal choice of $dN(x, q)$ satisfies:

$$s(x, q) [(1 - \sigma_s) x - \psi(q)] \leq k \quad \text{and} \quad dN(x, q) \geq 0, \quad (3)$$

where the two inequalities hold with complementary slackness. As in Menzio, Shi and Sun (2011), I focus on equilibria where condition (2) also holds for submarkets not visited by any buyer.³ This implies that the firm also earns zero profit in the frictional markets in equilibrium.

For all submarkets such that $k < (1 - \sigma_s) x - \psi(q)$, the submarket has $dN(x, q) > 0$, and (3) holds with equality. For all submarkets such that $k \geq (1 - \sigma_s) x - \psi(q)$, the submarket has $dN(x, q) = 0$, in which case I set $s = 1$ and $b = 0$. Putting the two cases together, the matching probability for a particular shop is given by

$$s(x, q) = \mu(b(x, q)) = \begin{cases} \frac{k}{(1 - \sigma_s)x - \psi(q)}, & \text{if } k \leq (1 - \sigma_s)x - \psi(q) \\ 1, & \text{if } k > (1 - \sigma_s)x - \psi(q). \end{cases} \quad (4)$$

The free-entry condition pins down the matching probabilities in a submarket as functions of the terms of trade. Indeed, a submarket can be sufficiently indexed by the terms of trade, (x, q) .

2.3 A household's decision

2.3.1 Decision in the frictionless market

Let $W(m, \theta)$ be a representative household's value at the beginning of a period with money balance m and the realization of θ . Given θ , price p , policy and the terms of trade in all submarkets, a household maximizes its value by choosing consumption of general goods

³Given such beliefs off the equilibrium, markets are complete in the sense that a submarket is inactive only if the expected revenue of the only shop in the submarket is lower than its expected cost given that some buyers are present in the submarket. Such a restriction is common in the literature on competitive search, *e.g.*, Moen (1997) and Acemoglu and Shimer (1999). This restriction can be justified by a "trembling-hand" argument that an infinitely small measure of buyers appear in every submarket exogenously.

$y \geq 0$, expected labor input $l \geq 0$, and the money balance to carry into the frictional market, $z \in [0, \bar{m}]$. Given the household's choice of l , the firm allows the household to choose the timing of the wage payments. In particular, the household can choose to receive wage payment equivalent to $l_g \in [0, l]$ units of labor, in the first sub-period of the current period and to receive the rest $l_s = l - l_g \in [0, l]$, at the end of the second sub-period. Therefore, the household's choice of labor supply entails two choices, $l_g \geq 0$ and $l_s \geq 0$. The overall supply l follows trivially. Note, in particular, that the household's choice of wage payments does not reflect the actual timing of the worker's labor input. The firm has the liberty of allocating particular tasks to the worker. All wage income is subject to the tax rate τ . The dividend Π is paid to the household at the end of a period. In equilibrium $\Pi = 0$ because firms earn zero profit.

The choice of money balances z and l_s deserves more explanation. The balance z is used for trading in the frictional market, i.e., solely as a medium of exchange. The household receives the after-tax wage payment, $h = (1 - \tau) l_s$, at the end of the period. Therefore, the balance h is carried into the following time period solely as precautionary savings. If the household gets to trade in the frictional market, it spends z and its end-of-period money balance is h . If there is no trade, then the end-of-period money balance is $z + h$. Given the upper bound on households' money holdings \bar{m} , the choice of precautionary money balance satisfies $h \in [0, \bar{m} - z]$.

The value $W(m, \theta)$ satisfies the following Bellman equation:

$$W(m, \theta) = \max_{(y, l_g, h, z)} \left\{ U(y) - \theta \left(l_g + \frac{h}{1 - \tau} \right) + V(z, h) \right\} \quad (5)$$

$$s.t. \quad (1 + t_g)py + z \leq m + (1 - \tau)l_g + T,$$

where T is government transfer for various policy purposes. The household takes (τ, t_g, T) and p as given. The above decision problem is subject to a standard budget constraint. Note that I do not impose any cash-in-advance constraint. The function $V(z, h)$ is the household's value at the beginning of the second sub-period, i.e., before the frictional market opens. Because the analysis on the decisions of frictional trading is more involved, I will postpone fully characterizing V until the next section. In Lemma 4, I show that V is differentiable and concave in z . For now, I take such information of V as given. Throughout this paper, I assume $\bar{\theta}$ is sufficiently low that the choice of l_g is always interior. Given $U' > 0$, the budget constraint must hold with equality and it can be used to eliminate

l_g in the objective function:

$$W(m, \theta) = \frac{\theta(m + T)}{1 - \tau} + \max_{y \geq 0} \left\{ U(y) - \frac{\theta p y (1 + t_g)}{1 - \tau} \right\} + \max_{z, h} \left\{ V(z, h) - \frac{\theta(z + h)}{1 - \tau} \right\}. \quad (6)$$

The first-order condition on y is given by

$$U'(y) = \frac{p\theta(1 + t_g)}{1 - \tau}, \quad (7)$$

the first-order condition on z is

$$V_z(z, h) \begin{cases} \leq \frac{\theta}{1 - \tau}, & \text{and } z \geq 0 \\ \geq \frac{\theta}{1 - \tau}, & \text{and } z \leq \bar{m} - h, \end{cases} \quad (8)$$

and the first-order condition on h is

$$V_h(z, h) \begin{cases} \leq \frac{\theta}{1 - \tau}, & \text{and } h \geq 0 \\ \geq \frac{\theta}{1 - \tau}, & \text{and } h \leq \bar{m} - z \end{cases} \quad (9)$$

where the all sets of inequalities in the above hold with complementary slackness. Clearly, the household's current money balance m does not affect these optimal choices of y , z or h . Let the policy functions be $y(\theta)$, $z(\theta)$ and $h(\theta)$. Given that $U'(0)$ is sufficiently large, $y(\theta)$ is always interior. Condition (8) implies that the money distribution across households for trading in the frictional market is non-degenerate because z depends on the realization of θ . Given (6), the value function W is clearly continuous, differentiable and linear in m :

$$W(m, \theta) = W(0, \theta) + \frac{\theta}{1 - \tau} m, \quad (10)$$

where

$$W(0, \theta) = \frac{\theta T}{1 - \tau} + U(y(\theta)) - \frac{\theta(1 + t_g)p}{1 - \tau} y(\theta) + V(z(\theta), h(\theta)) - \frac{\theta[z(\theta) + h(\theta)]}{1 - \tau}. \quad (11)$$

The preceding exposition proves the following lemma:

Lemma 1 *The value function W is continuous and differentiable in (m, θ) . It is also affine in m .*

2.3.2 Decision in the frictional market

The household's decisions on frictional trading are non-trivial and deserve much attention. The household chooses whether to participate in the frictional market. If yes, then it chooses which submarket to enter and search for a trade. Given the choice of money balances z and h , the household is faced with the following problem of frictional trading:

$$\max_{(x,q)} \left\{ b(x,q) \left[u(q) + \beta E \left[W \left(\frac{h}{\gamma}, \theta \right) - W \left(\frac{z+h}{\gamma}, \theta \right) \right] \right] + \beta E \left[W \left(\frac{z+h}{\gamma}, \theta \right) \right] \right\}, \quad (12)$$

where $q \geq 0$, $x = z/(1+t_s)$ and $b(x,q)$ is determined by (4). It is convenient to use condition (4) to eliminate q in the above objective function. Given linearity of W , the problem in (12) simplifies to

$$B(z) + \beta E \left[W \left(\frac{z+h}{\gamma}, \theta \right) \right] \quad (13)$$

where

$$B(z) = \max_{b \in [0,1]} b \left\{ u \left(\psi^{-1} \left(\frac{1-\sigma_s}{1+t_s} z - \frac{k}{\mu(b)} \right) \right) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z \right\} \quad (14)$$

is the household's expected trade surplus. Note that I have used (4) to eliminate q in (14).

Consider the problem in (14). Let the policy function $b(z)$, which then implies the policy function $q(z)$ given condition (4). If $b = 0$, then the choice of q is irrelevant. In this case, the household chooses not to participate in the frictional submarket. If $b > 0$, it must be the case that $q > 0$ and that the surplus from trade is strictly positive, i.e.,

$$u \left(\psi^{-1} \left(\frac{1-\sigma_s}{1+t_s} z - \frac{k}{\mu(b)} \right) \right) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z > 0. \quad (15)$$

Given (14), the first-order condition on b is given by

$$u(q) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z + \left(\frac{u'(q)}{\psi'(q)} \right) \frac{kb\mu'(b)}{[\mu(b)]^2} \leq 0, \quad \text{and} \quad b \geq 0, \quad (16)$$

where the inequalities hold with complementary slackness, and

$$q = \psi^{-1} \left(\frac{1-\sigma_s}{1+t_s} z - \frac{k}{\mu(b)} \right) \quad (17)$$

is derived from (4). It has been taken into account in condition (16) that $b = 1$ cannot be an equilibrium outcome. This is because $b = 1$ implies $s = 0$, which implies the expected profit per shop is $-k$ in such a submarket. I will further characterize the value function B and its policy functions later in Lemma 3.

A household's lottery choice. It is necessary to mention that the value function $B(z)$ may not be concave in z because the objective function in (14) may not be jointly concave in its state and choice variables, (z, b) . This objective function involves the product between the choice variable b itself and a function of b . Even if both of these two terms are concave, the product may not be jointly concave. Above all, it is unclear whether either of the two terms is a concave function of z , given that b is a choice variable and is yet to be determined. To make the household's value function concave, I introduce lotteries with regards to households' money holdings, as is the case in Menzio, Shi and Sun (2011). In particular, lotteries are available every period immediately before trading in the frictional market takes place.

A lottery is characterized by (L_1, L_2, π_1, π_2) . If a household chooses to play this lottery, it will win the prize L_2 with probability π_2 . The household loses the lottery with probability π_1 , when it will receive a payment of L_1 . There is a complete set of lotteries available. Given z , a household's optimal choice of lottery solves:

$$\tilde{V}(z) = \max_{(L_1, L_2, \pi_1, \pi_2)} \{\pi_1 B(L_1) + \pi_2 B(L_2)\} \quad (18)$$

subject to

$$\begin{aligned} \pi_1 L_1 + \pi_2 L_2 &= z; & L_2 &\geq L_1 \geq 0; \\ \pi_1 + \pi_2 &= 1; & \pi_i &\in [0, 1] \text{ for } i = 1, 2. \end{aligned}$$

Denote the policy functions as $L_i(z)$ and $\pi_i(z)$, respectively, where $i = 1, 2$. If the household is better off not playing any lottery, it is trivial to see that $L_1(z) = L_2(z) = z$. Figure 1 illustrates how the lottery can help make the value function $\tilde{V}(z)$ concave, even though the function $B(z)$ has some strictly convex part. It is intuitive to see that a household will choose to play a lottery if it has very low money balance. As is shown in Figure 1, for any money balance $z \in (0, z_0)$, it is optimal for the household to participate in the lottery offering the prize z_0 . The lottery makes $\tilde{V}(z)$ linear whenever $B(z)$ is strictly convex. The properties of z_0 are presented in part (v) of Lemma 3. Recall the household's first-order condition (8) on the optimal decision of the money balance z . Given the lottery, the policy function $z(\theta)$ may not be unique because V has some linear segments. In what follows, I focus on the symmetric equilibrium where households with the same realization of θ will

choose the same value of z , whenever the optimal choice of z is not unique.

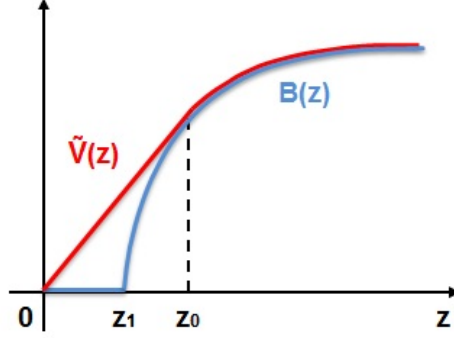


Figure 1. Lottery Choice

Recall that V is the value of a household at the beginning of the second sub-period, before trading decisions are made. Given (10), (13), (14) and (18), V is given by

$$V(z, h) = \tilde{V}(z) + \beta E[W(0, \theta)] + \frac{\beta E(\theta)}{\gamma(1-\tau)}(z + h). \quad (19)$$

Thus V is linear in h and

$$V_h(z, h) = \frac{\beta E(\theta)}{\gamma(1-\tau)}. \quad (20)$$

Then condition (9) implies that the optimal choice of h satisfies

$$h(\theta) \begin{cases} \geq 0, & \text{if } \theta \geq \frac{\beta}{\gamma} E(\theta) \\ \leq \bar{m} - z(\theta), & \text{if } \theta \leq \frac{\beta}{\gamma} E(\theta), \end{cases} \quad (21)$$

where the two sets of inequalities hold with complementary slackness. Given (m, θ) , the household's wage income in the first sub-period is

$$l_g(m, \theta) = \frac{1}{1-\tau} [(1+t_g)py(\theta) + z(\theta) - m - T], \quad (22)$$

which is derived from the household's budget constraint in (5). Obviously, l_g strictly decreases in m . Given concavity of V in z (as is shown in Lemma 4) and strict concavity of U , $l_g(m, \theta)$ is also strictly decreasing in θ because $z(\theta)$ decreases and $y(\theta)$ strictly decreases in θ . Intuitively, the more money one has, or the higher the disutility of labor, the lower

the incentives to work. Moreover, (21) and (22) imply

$$l(m, \theta) \begin{cases} = \frac{1}{1-\tau} [(1+t_g) py(\theta) + z(\theta) - m - T], & \text{if } \theta > \frac{\beta}{\gamma} E(\theta) \\ = \frac{1}{1-\tau} \left[(1+t_g) py\left(\frac{\beta}{\gamma} E(\theta)\right) + z\left(\frac{\beta}{\gamma} E(\theta)\right) + h\left(\frac{\beta}{\gamma} E(\theta)\right) - m - T \right], & \text{if } \theta = \frac{\beta}{\gamma} E(\theta) \\ = \frac{1}{1-\tau} [(1+t_g) py(\theta) + \bar{m} - m - T], & \text{if } \theta < \frac{\beta}{\gamma} E(\theta), \end{cases} \quad (23)$$

where $h(\beta E(\theta)/\gamma) + z(\beta E(\theta)/\gamma) \in (0, \bar{m})$. Hence follows a lemma:

Lemma 2 *Wage income, $l(m, \theta)$, strictly decreases in both m and θ .*

2.3.3 Properties of value and policy functions

I use this section to characterize the household's value functions and policy functions. Define $\Phi(q) \equiv u'(q)/\psi'(q)$. I have Lemma 3 and I provide a proof in Appendix A.

Lemma 3 *The value function $B(z)$ is continuous and increasing over $[0, \bar{m}]$. The value function $\tilde{V}(z)$ is continuous, differentiable, increasing and concave over $[0, \bar{m}]$. If and only if there exists $q > 0$ that satisfies*

$$u(q) - \frac{\beta E(\theta)(1+t_s)}{\gamma(1-\tau)(1-\sigma_s)} [\psi(q) + k] > 0, \quad (24)$$

there exists $z > 0$ such that $b(z) > 0$. In this case, the value function $B(z)$ is differentiable at z such that $b(z) > 0$. Given that (24) is satisfied by some $q > 0$, the following results hold:

(i) *For z such that $b = 0$, the value function $B(z) = 0$. In this case, the choice of q is irrelevant. For z such that $b > 0$, the function $B(z) > 0$ and $B'(z) > 0$;*

(ii) *For z such that $b > 0$, the policy functions $b(z)$ and $q(z)$ are unique and strictly increasing in z . In particular, $b(z)$ solves*

$$u(q(z)) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z + \left[\frac{u'(q(z))}{\psi'(q(z))} \right] \frac{kb(z)\mu'(b(z))}{[\mu(b(z))]^2} = 0, \quad (25)$$

where

$$q(z) = \psi^{-1} \left(\frac{1-\sigma_s}{1+t_s} z - \frac{k}{\mu(b(z))} \right); \quad (26)$$

(iii) There exists $z_1 > k/(1 - \sigma_s)$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$;

(iv) For any given z , $b(z)$ decreases in $E(\theta)$, while $q(z)$ increases in $E(\theta)$. If $b > 0$, then such monotonicity is strict;

(v) There exists $z_0 \in (z_1, \infty)$ such that a household with $z < z_0$ will play the lottery with the prize z_0 . Moreover, $B(z_0) = \tilde{V}(z_0) > 0$, $B'(z_0) = \tilde{V}'(z_0) > 0$ and $b(z_0) > 0$.

Lemma 3 summarizes the properties of the household's value and policy functions in the frictional market. Of particular interest are parts (ii) and (iv). According to part (ii), the optimal choices of (q, b) are strictly increasing in z when the household chooses $b > 0$. In this case, the more money the household spends, the higher a quantity it obtains in the frictional submarket and the higher matching probability at which it trades. As a result, households endogenously sort themselves into different submarkets based on their money balances.

According to part (iv) of Lemma 3, for any given z , a higher value of the expected disutility of labor, $E(\theta)$, implies a lower matching probability for a buyer and a higher amount of goods to be purchased by a buyer. The intuition is the following: The higher the expected value of θ , the higher the cost of labor. Thus firms respond to costly labor by setting up fewer shops in the submarkets. This leads to a lower matching probability for a buyer yet a higher matching probability for a shop. The latter causes firms to increase quantity per trade to increase revenue. Given Lemmas 1 and 3 and conditions (20) and (21), it is trivial to derive the following lemma:

Lemma 4 *The value V is continuous and differentiable in (z, h) . The function $V(\cdot, h)$ is also increasing and concave in $z \in [0, \bar{m}]$, with $V(z, h) \geq \beta E[W(0, \theta)] > 0$ for all z . If $\theta/E(\theta) \geq \beta/\gamma$, then $V_z(z, h) \geq \beta E(\theta) / [\gamma(1 - \tau)]$. If $\theta/E(\theta) < \beta/\gamma$, then $V_z(z, h) \geq 0$. Moreover, $V(z, \cdot)$ is affine in h .*

3 Stationary Equilibrium

In this section, I define and characterize the stationary equilibrium. Denote $G(m)$ as the money distribution immediately before lotteries take place.

3.1 Definition of a stationary equilibrium

A stationary equilibrium consists of a representative household's values (W, B, \tilde{V}, V) and choices $(y, l_g, h, z, (q, b), (L_1, L_2, \pi_1, \pi_2))$; a representative firm's choices $(Y, dN(x, q))$; price p ; wage rate w ; money distribution $G(m)$; policy program $(\gamma, \tau, \sigma_g, \sigma_s, t_g, t_s, T)$. These elements satisfy the following requirements:

- (i) Given the realizations of shocks, money holdings, general-good prices, policy and the terms of trade of all frictional submarkets, a household's choices solve (6), (14), (18) and (19), which induce the value functions $W(m, \theta)$, $B(z)$, $\tilde{V}(z)$ and $V(z, h)$;
- (ii) Given prices, policy and the terms of trade of all submarkets, firms maximize profit and solve (2);
- (iii) Free entry condition: The expected profit of a shop in each submarket is zero, and the function $s(x, q)$ satisfies (4);
- (iv) A stationary equilibrium is non-monetary if and only if $z(\theta) = h(\theta) = 0$ for all θ and monetary otherwise;
- (v) All labor markets, general-good markets and money markets clear;
- (vi) Stationarity: All quantities, prices and distributions are time invariant;
- (vii) Symmetry: Households in the same idiosyncratic state make the same optimal decisions;
- (viii) The government balances budget.

The above definition is self-explanatory. For parts (v), (vi) and (viii), I provide detailed formulas for the equilibrium money distribution, market-clearing conditions and the government transfer in Appendix D.

3.2 Characterization of a stationary equilibrium

Recall that the firm's free entry to the frictionless market implies that $p^* = 1/(1 - \sigma_g)$. Also recall that the household's optimal choice of y is given by (7). Given strict concavity of the function U and concavity of V , it is straightforward to obtain the following lemma from (7) and (8):

Lemma 5 *In equilibrium, $y(\theta)$ is uniquely given by*

$$U'(y(\theta)) = \frac{\theta(1 + t_g)}{(1 - \tau)(1 - \sigma_g)}. \quad (27)$$

Moreover, $y'(\theta) < 0$ and $z'(\theta) \leq 0$.

According to Lemma 5, a household's demand for general goods $y(\theta) > 0$ for all θ . This is important because it implies that all households participate in the frictionless goods market under all circumstances. Furthermore, the higher the disutility of labor, the lower the household's consumption of general goods, and the lower the money balance carried into the frictional market. Intuitively, the household supplies less labor and earns a lower income, given a higher utility cost of labor. This leads to a lower consumption level and lower money holdings. These are classic income effects.

Equivalence of household and firm sales taxation. It is worth mentioning that it does not matter whether sales taxes are imposed on firms or on households. These two channels are equivalent and can achieve the same results. For equilibrium output of general goods, it is clear from condition (27) that $y(\theta)$ is critically affected by the ratio of $(1 + t_g) / (1 - \sigma_g)$. Similarly, part (ii) of Lemma 3 shows that policy functions $b(z)$ and $q(z)$ critically depend on the ratio of $(1 + t_s) / (1 - \sigma_s)$. It follows that one proportional sales tax rate, be it on the households or the firms, is sufficient to generate desirable policy effects and it is not necessary to have both channels. Therefore, from this point on, I impose $t_g = t_s = 0$ and only consider sales taxes on the firms side, σ_g and σ_s .

In what follows, I have a theorem to characterize the steady state and a proof is available in Appendix B.

Theorem 1 *A stationary equilibrium exists. It is unique if and only if the lottery choices $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ are unique for all $z(\theta)$. Moreover, the following results hold: (i) The general-good consumption $y(\theta) > 0$ for all θ ; (ii) The money balance $h(\theta) > 0$ if $\theta/E(\theta) \leq \beta/\gamma$ and $h(\theta) = 0$ if $\theta/E(\theta) > \beta/\gamma$; (iii) If there does not exist $q > 0$ that satisfies condition (24), then $z(\theta) = 0$ for all θ . Otherwise, $z(\theta) \geq 0$ for all θ .*

Theorem 1 is the core of this theory and it sheds light on the reason why it is important to analyze an economy with both frictional and frictionless markets. In particular, the results in Theorem 1 uncover a theory on the role of household income in determining the variety and the functioning of markets. This theorem revolves around two critical conditions, one is essentially about a household's expected income level, and the other about idiosyncratic income levels:

Critical condition 1. There exists $q > 0$ such that condition (24) holds;

Critical condition 2. There exists θ such that $\theta/E(\theta) \leq \beta/\gamma$.

The first critical condition is satisfied if and only if $E(\theta)$ is sufficiently low, that is, if households' expected income is sufficiently high. Part (iii) of Theorem 1 shows that the first critical condition is a necessary condition to have $z(\theta) > 0$ for some θ . That is, households choose to participate in frictional trading in some idiosyncratic states, only if their expected income is sufficiently high. If $E(\theta)$ is too high and the first critical condition fails to hold, then $z(\theta) = 0$ for all θ . In contrast, part (iii) of Theorem 1 shows that the frictionless markets are used in all circumstances, i.e., $y(\theta) > 0$ for all θ . The outcome that the use of the frictional market critically depends on households' expected income is caused by trading frictions. This outcome occurs even if general goods and special goods are identical in every way. To see this, suppose $u(c) = U(c)$ and $\psi(c) = c$ for all $c \geq 0$. Also let $k \rightarrow 0$. In this case, the general and special goods are identical in terms of household preferences and production technology. In this case, the first critical condition requires that there exists $q > 0$ such that

$$u(q) - \frac{E(\theta)}{\gamma(1-\tau)(1-\sigma_s)} q > 0,$$

Again, if $E(\theta)$ is too high, then the above condition is never satisfied and households never purchase any special goods. Now suppose special goods can also be traded in a frictionless market. Then it is straightforward to show that $q(\theta) = y(\theta) > 0$ for all θ . Thus it is the trading frictions that matter for the use of frictional markets. Even if general and special goods are identical in both household preferences and production technology, a household still chooses not to obtain any special goods if its expected income is too low. The intuition is simple. There are risks involved in getting special goods, while buying general goods is riskless. Therefore, frictionless markets are used at all times to smooth consumption. Nevertheless, if income is generally low, frictional markets are not used because it is too costly to sacrifice some consumption of general goods in hopes of getting a chance to purchase some special goods. Overall, the variety of markets in an economy, be it frictional and/or frictionless, is critically affected by expected household income.

Furthermore, the functioning of frictional markets critically depends on income inequality across idiosyncratic states. In particular, the equilibrium terms of trade and matching probabilities across all frictional submarkets, critically depend on households' choices of money balance, $z(\theta) > 0$, which is a result of the idiosyncratic realizations of θ . Therefore, the more dispersed the income distribution in an economy, the more diverse the trading protocols of frictional markets.

The second critical condition is about whether income inequality across idiosyncratic states is sufficiently severe. When it is the case, households choose to hold money as

precautionary savings. The use of money for precautionary purposes is active if and only if households have the need to fight against income fluctuations. The use of money for transaction purposes is active only if household expected income is sufficiently high such that households have incentives to engage in frictional trading. These conditions identified for the use of money seem to be consistent with the historic evolution of money along with economic progression. This framework not only highlights the two key roles of money, as a medium of exchange and as a store of value, but also identifies the quantity of money held by a household for each of the two distinct roles. In particular, $z(\theta) > 0$ is solely used for transaction purposes and $h(\theta) > 0$ is solely used for precautionary purposes.

Last but not the least, it might seem that the potential non-monetary equilibrium is not anything new. This outcome shows up even in the earliest search-theoretic models of money, Kiyotaki and Wright (1993) for example. Nevertheless, there is a major difference in what causes the non-monetary equilibrium between previous theories and mine. In previous literature, the non-monetary equilibrium arises because agents do not believe that others have the incentives to accept fiat money. It is essentially a self-fulfilling equilibrium outcome that fiat money may be rejected everywhere in the economy. In contrast, my model highlights the result that money has no value when income is generally low and income fluctuations are mild. Due to the commonly low income, households cannot afford to spend income in markets with trading risks. Because of mild income inequality across idiosyncratic states, households also have no incentives for precautionary savings.

Solving for the stationary equilibrium. To completely solve the equilibrium, one can first solve the optimization problems of the representative firm and the representative household, namely (2), (6), (14), (18) and (19). After obtaining the policy functions from the aforementioned decision problems, one can derive the equilibrium wage rate, money distribution, aggregate labor, aggregate output and government transfers, using the formulas presented in Appendix D.

4 Policy Effect

4.1 Analytical policy effects

I use this section to summarize the analytical results on the effects of monetary and fiscal policies. First, decisions in the frictional or frictionless markets are only affected by the respective sales tax rates. In particular, it is clear from Lemmas 3 and 5 that $y(\theta)$ is only affected by τ_g and σ_g and that $z(\theta)$ and $h(\theta)$ are only affected by τ_s and σ_s . Moreover,

as has been mentioned in the previous section, it does not matter whether sales taxes are imposed on firms or on households. The two channels through σ_g and σ_s are sufficient to achieve desirable policy results.

Secondly, condition (27) implies that $\partial y(\theta) / \partial \tau < 0$ and $\partial y(\theta) / \partial \sigma_g < 0$. These are classic income effects. Income is directly increased by a drop in the income tax rate and indirectly increased by a drop in the sales tax rate. The latter is because a reduction of the sales taxes increases firms' demand for labor and thus increases labor income. Therefore, households can afford higher consumption of general goods with a higher income. Note that monetary policy, a change in γ , has no effect on consumption of general goods. This is not surprising because in principle money is not need for transactions of general goods.

Finally, monetary and fiscal policies directly affect equilibrium trading protocols across frictional submarkets, in particular, intensive margin $q(z)$ and extensive margin $b(z)$ for a given money balance z . I have the following proposition to summarize such policy effects:

Proposition 1 *For all z such that $b(z) > 0$, the following results hold: (i) $\partial q(z; \sigma_s) / \partial \sigma_s < 0$ and $\partial b(z; \sigma_s) / \partial \sigma_s < 0$; (ii) $\partial q(z; \tau) / \partial \tau > 0$ and $\partial b(z; \tau) / \partial \tau < 0$; (iii) $\partial q(z; \gamma) / \partial \gamma < 0$ and $\partial b(z; \gamma) / \partial \gamma > 0$.*

Part (i) of Proposition 1 describes the effects of proportional sales taxes in the frictional market. All else equal, a lower sales tax rate σ_s increases a firm's revenue and thus profit. Free entry causes firms to increase the measure of shops, as well as quantity per trade in exchange for a given money balance z . The former leads to a higher matching probability for buyers, which is a positive extensive margin effect. The latter is a positive intensive margin effect. Overall, lower sales taxes in the frictional market improve trading protocols along both intensive and extensive margins. This effectively encourages households' consumption of special goods. Part (ii) of Proposition 1 summarize the effects of proportional income taxes. A higher income tax rate τ makes households frugal on money spending. For any given money balance, a household chooses to visit a submarket that offers a higher quantity of goods per trade, which is a positive effect on the intensive margin. In such a submarket, a firm's cost of production per trade is higher. Thus it reduces overall cost by setting up a smaller measure of shops in this submarket. This imposes a negative effect along the extensive margin.

All the results in part (i) and part (ii) are intensive and extensive effects of fiscal policies. These are novel results because current literature on search-theoretic models of money rarely analyzes fiscal policy in the frictional market. This is the case because

applying fiscal policy can pose a threat to the micro-foundations for a medium of exchange in these models. In a typical money search model, to justify a need for money, individual buyers and sellers are anonymous and there is no technology to keep track of their actions. These assumptions simply leave no ground for the government to collect taxes. If one assumes that the government can get ahold of the individuals and charge them taxes, then it must mean that there is a way to keep record of individual actions. It then implies that there is no need for money and the economy can operate on a credit system. My framework does not suffer from such a problem. The assumption here is that there is perfect record keeping for workers and firms, although no way to keep track of individual buyers or shops. As a result, this framework has a solid micro-foundation for money, without threatening the feasibility of a fiscal policy program.

Part (iii) of Proposition 1 lists monetary policy effects on intensive and extensive margins. In particular, the real value of money decreases with money growth. A household responds by sending its buyer to a submarket with a higher matching probability b , in order to increase the chance of spending money in the current period. In such a submarket, the matching probability for a shop is lower, which all else equal implies a lower profit for firms. Zero profit condition requires that firms must be compensated by producing a lower quantity per trade. These results of monetary policy are standard and have been well-documented in the money search literature.

It is clear from Theorem 1 that both monetary and fiscal policies can affect the variety of markets. All else equal, a higher money growth rate γ , a lower income tax rate τ and a lower sales tax rate σ_s tend to make “critical condition 1” more likely to hold. This is intuitive given results in Proposition 1, because all of these policies increase the extensive margin b . Therefore, they tend to generate $b(z) > 0$ for a given money balance z , which is crucial for the use of frictional markets. Furthermore, higher γ tends to make “critical condition 2” less likely to hold. Intuitively, households are less likely to save in precaution under higher inflation because of the alleviated income inequality across states. In contrast, the income tax rate τ has no effect on “critical condition 2”. This is because τ has the same proportional effect across all idiosyncratic income levels. “Critical condition 2” only depends on the idiosyncratic income level relative to the mean level, $\theta/E(\theta)$. Therefore, the effects of income taxation on both of these items cancel each other out. Finally, “critical condition 2” is not affected by the sales tax rate σ_s because this policy only affects trading protocols in the frictional market and has no implications on income fluctuations.

4.2 Numerical policy effects

I use numerical examples to further investigate welfare effects of the policy program. I employ the following functional forms:

$$u(c) = u_0 \frac{(c+a)^{1-\sigma} - a^{1-\sigma}}{1-\sigma}; \quad U(c) = u_0 \frac{(c+a)^{1-\sigma_u} - a^{1-\sigma_u}}{1-\sigma_u}$$

$$\psi(q) = \psi_0 q^\varphi; \quad \mu(b) = (1-b^\rho)^{1/\rho}; \quad F(\theta) \text{ is continuous uniform on } [\underline{\theta}, \bar{\theta}]. \quad (28)$$

The following table lists some of the key parameter values and policy choice sets:

$\beta = 0.95,$	$\rho = 1;$	$a = 0.001;$	$u_0 = 1;$
$\psi_0 = 1;$	$\underline{\theta} = 1;$	$\bar{\theta} = 2;$	$\gamma \in [\beta, 2];$
$\tau \in [0, 1];$	$\sigma_g \in [-1, 1];$	$\sigma_s \in [-1, 1];$	$t_g = t_s = 0.$

I report results for three cases, which differ in terms of preferences and technologies for the general and the special goods. Case I is where preferences for the two kinds of goods are identical and the respective production technologies are almost identical. Case II is where the two goods differ in terms of production technology but not preferences. Case III is where preferences differ but the production technologies are almost identical. In all three cases, I control parameters such that “critical condition 1” is satisfied and “critical condition 2” is also satisfied for some θ . Thus in all three cases, frictional markets are active and money is used both as a medium of exchange and as a store of value.

Case I. Identical preferences and almost identical technologies. The key parameters for this case are

Parameters for Case I

$\sigma = \sigma_u = 2;$	$k = 0.001;$	$\varphi = 1.$
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Given $\sigma = \sigma_u$, the utility functions for the general and the special goods are identical. Given $\psi_0 = \varphi = 1$, the production functions are also identical. Given $k = 0.001 \approx 0$, it

follows that the production technologies for the two types of goods are almost identical.

Figure 2. Policy Anatomy - Case I

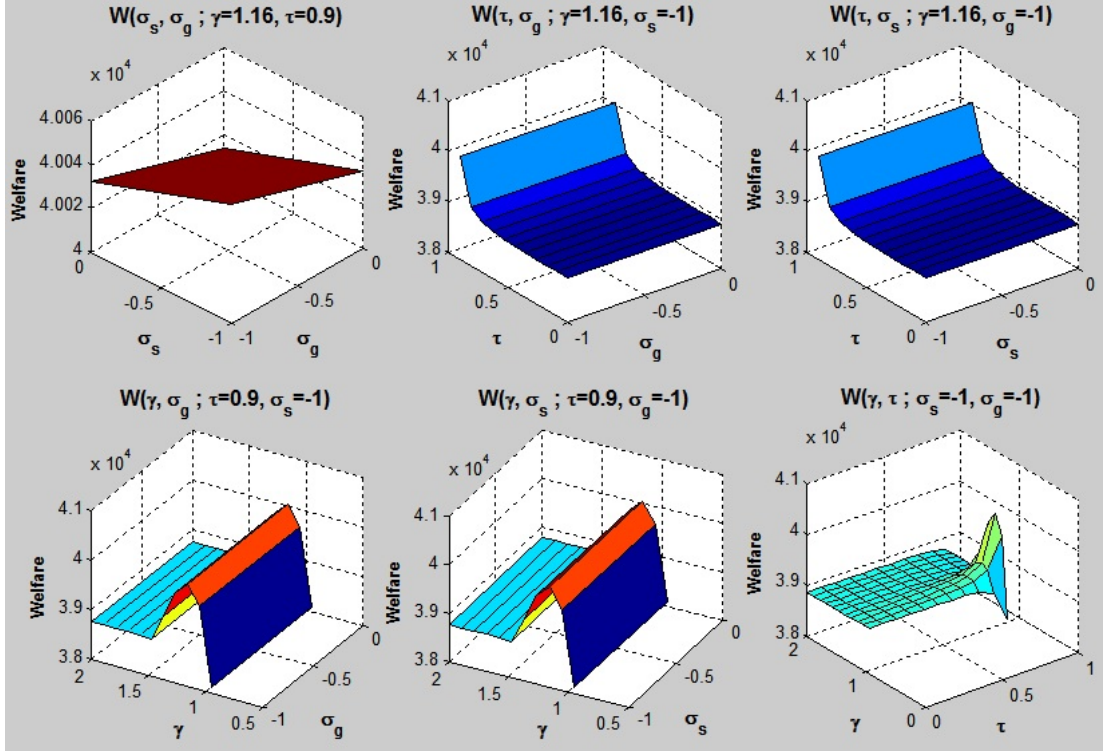


Figure 2 illustrates the anatomy of respective policy effects. In each panel, I control two of the four policy variables $(\sigma_s, \sigma_g, \tau, \gamma)$ and show the welfare effects of the other two. An obvious result from the top left panel is that the welfare effects of sales-tax rates (σ_s, σ_g) are almost identical, both of which tend to have a negative effect on welfare. Intuitively, a lower sales-tax rate tends to increase firms' production and entry incentives, which ultimately benefits the households by promoting consumption. In this case, the effects of sales taxes are quantitatively negligibly small. This result is clearly illustrated by all the panels except for the bottom right one, where both sales-tax rates are fixed. Such quantitative equivalence and insignificance seem to be driven by the fact that here general and special goods are made almost identical in every way.

The three panels in the second row of Figure 2 illustrate that all else equal, there tends to be a hump-shape relationship between inflation and welfare. Welfare tends to increase at mild inflation rates yet decrease at severe inflation rates. In addition to the effects on intensive and extensive margins reported by part (iii) of Proposition 1, inflation also has a positive redistributive effect and a negative taxation effect. The former is because inflation mitigates income inequality and the latter is because of the inflation tax. Then

the hump-shape relationship seems to suggest that the positive extensive-margin effect and the positive redistributive effect tend to dominate when inflation is relatively low but become dominated when inflation gets severe.

The bottom right panel best illustrates the welfare effect of income taxes. Under deflation, welfare tends to decrease with income-tax rates. This effect seems to be particularly significant under Friedman rule. Nevertheless, under inflation, welfare tends to increase with income taxes, which is especially prominent at mild inflation rates. To understand such non-trivial effects of income taxation, note that income taxes have a positive redistributive effect through alleviating income inequality, together with a negative taxation effect. In addition, income taxes also affect frictional trading protocols as shown in part (ii) of Proposition 1. The bottom right panel suggests that the positive redistributive effect of income taxes can be highly augmented when the positive redistributive effect of inflation dominates its negative inflation-tax effect. The middle and right panels on the top of Figure 2 can also confirm such results given $\gamma = 1.16$. Nevertheless, when there is deflation, income inequality is exacerbated. In this case, households in a low-income state are made much worse off by income taxes due to the negative extensive-margin effect of income taxes. Hence, Friedman rule is far from optimal when the proportional income tax is in effect. These quantitative results clearly indicate that it is important for the government and the monetary authority to coordinate policy decisions. It tends to be welfare-improving for the government to impose higher income-tax rates given inflation and lower income-tax rates given deflation.

Case II. Identical preferences and different technologies. The key parameters for this case are

Parameters for Case II

$\sigma = \sigma_u = 2;$	$k = 0.1;$	$\varphi = 2.$
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Obviously, the utility functions for the general and the special goods are identical, but the production technologies differ drastically. Figure 3 illustrate policy anatomy for this case. The welfare effects of inflation and income taxes are similar to those in Case I. However, the effects of sales taxes on special goods seem to be much more quantitatively significant than those of the general-good sales taxes. This is clearly driven by the difference in the production technology in that the production of special goods is more costly than that of general goods. This result follows from part (i) of Proposition 1 because a lower sales tax rate improves both intensive and extensive margins of frictional markets. Welfare is

enhanced because this policy promotes households' consumption of special goods.

Figure 3. Policy Anatomy - Case II

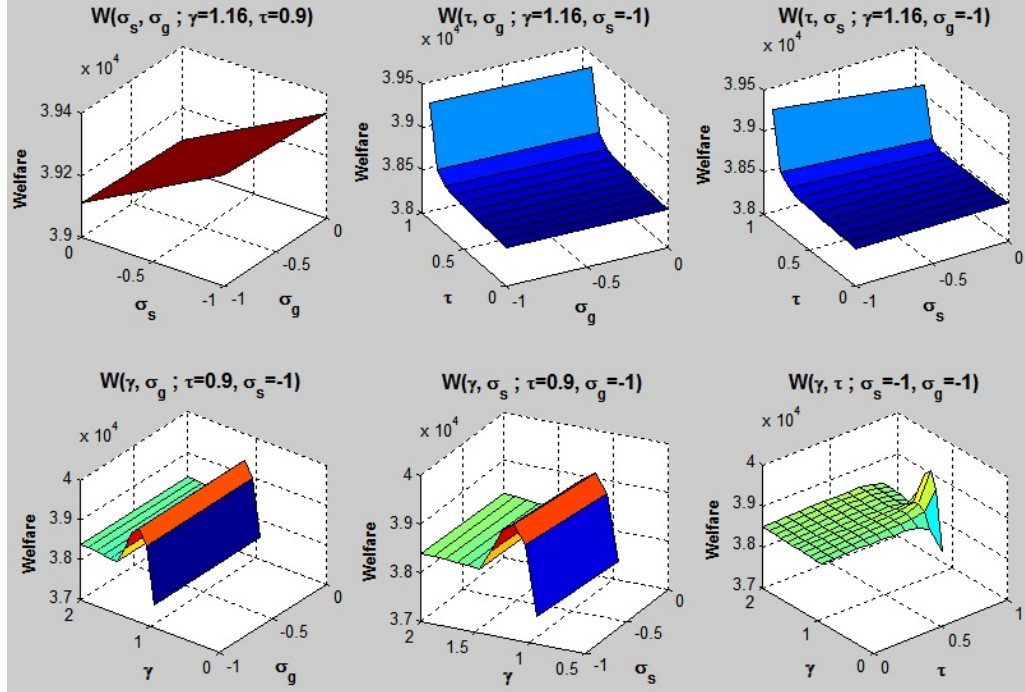
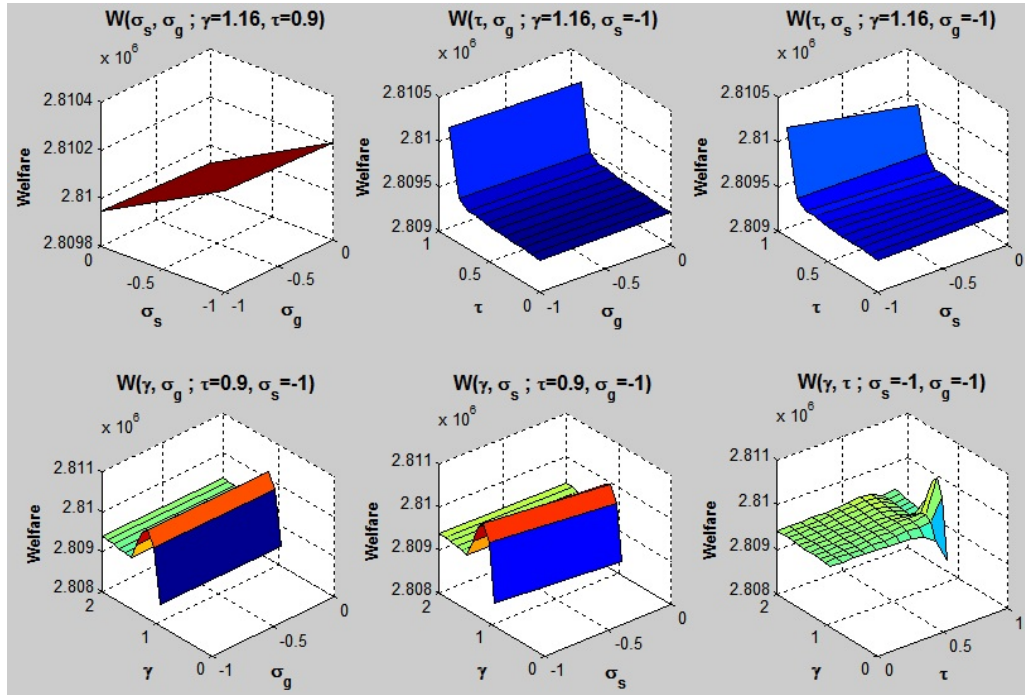


Figure 4. Policy Anatomy - Case III



Case III. Different preferences and almost identical technologies. The key parameters for this case are

Parameters for Case III

$\sigma = 2.8;$	$\sigma_u = 2;$	$k = 0.001;$	$\varphi = 1.$
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This is the case where the utility functions differ for the general and the special goods while the production technologies are almost identical. It is clear from Figure 4 that the welfare effects of inflation and income taxes are robust and similar to the previous two cases. As in Case II, the effects of a change in σ_s are more quantitatively significant than those of a change in σ_g . This seems to be driven by the difference in the utility functions.

Optimal policy program. In all three cases, the optimal policy program is given by $\gamma^* = 1.16$, $\tau^* = 0.9$, $\sigma_s^* = -1$ and $\sigma_g^* = -1$, with γ^* varying in the third decimal place. The welfare-maximizing policy program consists of inflation, proportional income taxes and proportional subsidies on sales. Given such policy combination, the positive redistributive effects of inflation and income taxes, together with the positive effects of sales taxes on trading protocols, are crucial in boosting welfare. All the above qualitative results on the optimal policy program are robust to various functional forms and parameter values, including various symmetric, left-skewed and right-skewed distributions of θ other than the uniform distribution. The optimal inflation rate seems to be sensitive to the values of the discount factor.

5 Conclusion

I have constructed a tractable macroeconomic framework that allows for frictional goods markets in a neoclassical environment. With this framework, I propose a theory that the variety and the functioning of markets reflect the status of national income. In particular, the variety of markets, in terms of trading frictions, reflects the level of national income. The functioning of frictional markets, in terms of diversity of trading protocols, reflects the severeness of income inequality in an economy. This model generates a rich set of analytical and numerical policy effects. The optimal policy program consists of money growth, proportional income taxes and proportional subsidies. It is important to coordinate monetary and fiscal policies. In particular, it tends to be welfare-improving for the government to alleviate income taxes when the monetary authority is running deflation and to elevate income taxes given inflation.

This framework is tractable and versatile. Because of competitive search, the decision problems of individual households and firms are independent of the equilibrium money distributions. This drastically reduces the state space of individual decision problems and makes the model tractable. Moreover, this model can encompass idiosyncratic and aggregate uncertainty of various forms, without losing tractability. Another important feature of this framework is that it allows one to study frictional goods markets in a standard macroeconomic setting. Frictional trading is a natural way of generating equilibrium price dispersion, which can be crucial for examining the macro performance of an economy. The versatile structure of a neoclassical environment makes this framework adaptable for various macroeconomic analysis.

Appendix

A Proof of Lemma 3

Given (14), it is straightforward to see that the value function $B(z)$ is continuous. Moreover, $B(z) \geq 0$ for all $z \geq 0$, where the equality holds if and only if $b = 0$. It is obvious that $B'(z) \geq 0$ and that $B'(z) > 0$ if and only if $b > 0$. If $b = 0$, the choice of q is irrelevant. Since B is continuous on a closed interval $[0, \bar{m}]$, the lotteries in (19) make V concave (see Appendix F in Menzio and Shi, 2010, for a proof). To see that V is differentiable in z , first note that $V'(0)$ exists because V is linear at $z = 0$ due to the lottery. Now consider any $z > 0$. Recall that a concave function has both left-hand and right-hand derivatives (see Royden, 1988, pp113-114). Let $V'(z^-)$ and $V'(z^+)$ be the left-hand and right-hand derivatives, respectively. Suppose $V'(z^-) > V'(z^+)$ for some z . Then z must be the joint of two lottery segments. However, $V'(z^-) > V'(z^+)$ implies that V is strictly concave at z , which contradicts the use of a lottery at z . Finally, it is obvious that the value function $B(z)$ is differentiable if $b > 0$. This finishes the proof of the leading part and part (i) of this lemma.

For part (ii), consider z such that $b > 0$. Define the left-hand side of (16) as $LHS(b)$:

$$LHS(b) \equiv u(q) - \frac{\beta E(\theta)}{\gamma(1-\tau)}z + \left(\frac{u'(q)}{\psi'(q)} \right) \frac{kb\mu'(b)}{[\mu(b)]^2}, \quad (29)$$

where q is given by (17). It is straightforward to derive that

$$\begin{aligned} LHS(b=0) &= u\left(\psi^{-1}\left(\frac{1-\sigma_s}{1+t_s}z - k\right)\right) - \frac{\beta E(\theta)}{\gamma(1-\tau)}z, \\ &= u(q) - \frac{\beta E(\theta)(1+t_s)}{\gamma(1-\tau)(1-\sigma_s)}[\psi(q) + k], \end{aligned}$$

where (17) yields $q = \psi^{-1}((1-\sigma_s)z/(1+t_s) - k)$ given $b = 0$. Thus the above implies that $LHS(b=0) > 0$ if and only if there exists $q > 0$ such that condition (24) holds. Moreover, one can further derive $LHS(b=1) = -\infty$, and

$$\begin{aligned} LHS'(b) &= u'(q)q'(b) + \frac{u''(q)\psi'(q) - u'(q)\psi''(q)}{[\psi'(q)]^2}q'(b) \left(\frac{kb\mu'(b)}{[\mu(b)]^2} \right) \\ &\quad + k \left(\frac{u'(q)}{\psi'(q)} \right) \frac{\mu(b)[\mu'(b) + b\mu''(b)] - 2b[\mu'(b)]^2}{[\mu(b)]^3} \\ &< 0. \end{aligned}$$

Given all the above results, condition (24) implies that there exists $z > 0$ such that $b > 0$. Furthermore, the above results imply that the policy function $b(z)$ is unique, which further implies that $q(z)$ is also unique given (17). Given $x = z/(1 + t_s)$, it is obvious that the objective function in (14) must be strictly increasing with the money balance z . It follows that

$$(1 - \sigma_s) \left[\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)(1 + t_s)}{\gamma(1 - \tau)(1 - \sigma_s)} \right] > 0. \quad (30)$$

Therefore, for z such that $b > 0$,

$$\begin{aligned} & \frac{\partial LHS(b; z)}{\partial z} \\ &= \frac{1 - \sigma_s}{1 + t_s} \left[\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)(1 + t_s)}{\gamma(1 - \tau)(1 - \sigma_s)} + \frac{kb\mu'(b)[u''(q)\psi'(q) - u'(q)\psi''(q)]}{[\mu(b)]^2[\psi'(q)]^3} \right] \\ &> 0. \end{aligned}$$

This implies that an increase of z shifts the entire function $LHS(b)$ upwards. Because $LHS'(b) < 0$, it follows that $b'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, (16) holds with equality. Total differentiating (16) by z yields

$$\begin{aligned} 0 &= u'(q)q'(z) - \frac{\beta E(\theta)}{\gamma(1 - \tau)} + \frac{kb\mu'(b)[u''(q)\psi'(q) - u'(q)\psi''(q)]}{[\mu(b)]^2[\psi'(q)]^3}q'(z) \\ &\quad + k\frac{u'(q)}{\psi'(q)} \left[\frac{\mu(b)[\mu'(b) + b\mu''(b)] - 2b[\mu'(b)]^2}{[\mu(b)]^3} \right] b'(z). \end{aligned}$$

Given $b'(z) > 0$ and Assumption 1, rearranging the above yields $q'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, one can derive that

$$\begin{aligned} B'(z) &= b'(z) \left[u(q(z)) - \frac{E(\theta)z}{\gamma(1 - \tau)} \right] + b(z) \left[\left(\frac{1 - \sigma_s}{1 + t_s} \right) \frac{u'(q(z))}{\psi'(q(z))} - \frac{E(\theta)}{\gamma(1 - \tau)} \right] \\ &> 0. \end{aligned}$$

This is because $b'(z) > 0$ and the trade surplus, $u(q(z)) - zE(\theta)/[\gamma(1 - \tau)]$, is strictly positive given $b > 0$, and also condition (30). This completes the proof of part (ii).

For part (iii), note from all previous parts that $b(z)$ is continuous and increasing in m . In particular, $b(z)$ is strictly increasing in z if $b > 0$. It is obvious from (17) that $b(z) = 0$ for all $z \in [0, k/(1 - \sigma_s)]$. Continuity of $b(z)$ implies that there exists $z_1 > k/(1 - \sigma_s)$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$. The rest of this part follows trivially.

For part (iv), if z is such that $b = 0$, then all the results in this part is trivial. Consider

$b > 0$. In this case, $b(z)$ is determined by (25). Obviously, $b(z)$ is strictly decreasing in $E(\theta)$, given the results about $LHS(b)$ in part (ii). Then (26) implies that $q(z)$ is strictly increasing in $E(\theta)$.

For part (v), it is straightforward to see that $B'(z) > 0$ for all $z \geq z_1$ and $B'(z) \rightarrow 0$ as $b \rightarrow \bar{b}$, where $\bar{b} \leq 1$ is the upper bound. Therefore, the value function $B(z)$ eventually becomes concave. That is, there must exist $\hat{z} < \infty$ such that B is concave for all $z \geq \hat{z}$. I assume that the upper bound on money holdings, \bar{m} , is sufficiently high such that $B(\bar{m}) = V(\bar{m})$ and $B'(\bar{m}) = V'(\bar{m}) > 0$. Therefore, there exists $z \in (0, \bar{m}]$ such that $B(z) = V(z) > 0$ and $B'(z) = V'(z) > 0$. Moreover, given that B is concave for all $z \geq \hat{z}$ and by construction of the lottery, it must be the case that $B'(z) \leq V'(z)$ for all z such that $B(z) = V(z)$, where the strict inequality holds if and only if $z = 0$. To see this, consider the two respective cases with $z = 0$ and $z > 0$. If $z = 0$, it is trivial that $B(0) = V(0)$ and $B'(0) < V'(0)$ because of the lottery. Consider any $z > 0$ such that $B(z) = V(z)$. By construction of the lottery, the function V serves as an upper envelope of the function B . It follows that $B'(z^-) \geq V'(z^-)$ and $B'(z^+) \leq V'(z^+)$. Moreover, $V'(z^-) \geq V'(z^+)$ because V is concave in z . Because B is differentiable, $B'(z^-) = B'(z^+)$, which then implies that $B'(z) = V'(z)$. Therefore, there must exist $z_0 \in (z_1, \bar{m})$ such that a household with $z \in (0, z_0)$ will play the lottery with the prize z_0 . Moreover, $B(z_0) = V(z_0) > 0$, $B'(z_0) = V'(z_0) > 0$ and $b(z_0) > 0$. **QED**

B Proof of Theorem 1

Recall the normalized wage rate w^* as given in (37). Note that all the policy functions in the right-hand side of (37) are independent of w^* . It is obvious that $w^* > 0$ exists. It is unique if and only if the lottery choices $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ are unique for all $z(\theta)$. Therefore, a stationary equilibrium exists and is characterized by w^* . Part (i) follows condition (21). For part (ii), recall from Lemma 3 that there exists $z > 0$ such that the policy function $b(z) > 0$ if and only if condition (24) holds for some $q > 0$. Therefore, if (24) does not hold, then $b(z) = 0$ for all z . In this case, $B(z) = B'(z) = \tilde{V}(z) = \tilde{V}'(z) = 0$ for all z . In this case, the household does not trade in the frictional market. Therefore, there is no need to hold a positive money balance for transaction purposes, i.e., $z(\theta) = 0$ for all θ . Consider the case where condition (24) holds for some $q > 0$. In this case, there exists $z > 0$ such that the policy function $b(z) > 0$ because condition (24) holds for some $q > 0$. Note that condition (8) implies that $z(\theta) > 0$ if $V_z(0, h) \geq \theta/(1 - \tau)$. If $V_z(0, h) < \theta/(1 - \tau)$, then $z(\theta) = 0$ is optimal. If $z(\theta) > 0$, $b(L_2(z(\theta))) > 0$ follows from construction of the lottery. **QED**

C Proof of Proposition 1

Recall equations (25) and (26) from part (ii) of Lemma 3. Substituting (26) into the left-hand side of (25) yields (29). Given $b > 0$, all results in this proposition follow trivially from the property of the function $LHS(b)$ derived in the proof of Lemma 3. In particular, $LHS'(b) < 0$. **QED**

D Distributions, Transfers and Market clearing

In this Appendix, I further characterize the equilibrium money distribution, market-clearing conditions and the formula for the government transfer. According to part (vi) of the equilibrium definition in Section 3.1, the money distributions must be stationary over time. Immediately after trading in the frictionless market, a household's money balance is $z(\theta)$, which is given by (8). The function $z(\theta)$ is decreasing in θ because of concavity of V . Lotteries are played before trading in the frictional market takes place. A household makes the choice of lottery based on its money balance $z(\theta)$. Recall that $G(m)$ is the money distribution immediately before lotteries take place. It is the measure of households holding a balance less than or equal to the amount m , which is given by

$$G(m) = \int_{z^{-1}(m)}^{\bar{\theta}} dF(\theta). \quad (31)$$

For part (viii), the total dollar amount of transfers that a household receives in a period consists of the transfer for monetary policy purposes and the transfers for fiscal policy purposes. For money growth, the household receives a dollar amount of $(\gamma - 1)M$, which is equivalent to $(\gamma - 1)M / (wM') = (\gamma - 1) / (w\gamma)$ units of labor. For income taxation, the amount of the government transfer in terms of labor units is τLS . Finally, consider the sales taxes. Because of the equivalence of the sales taxes on firms and on households, as explained in Section 3.2, I impose $t_g = t_s = 0$. The amount of real transfers from sales taxes on firms is sum of sales taxes from frictionless and frictional markets. In the frictional market, the measure of shops corresponding to the households holding $L_i(z(\theta))$ is given by $N_s = \pi_i(z(\theta)) dF(\theta) b(L_i(z(\theta))) / [\mu(b(L_i(z(\theta))))]$. For each of the shops, the probability of getting a trade is $\mu(b(L_i(z(\theta))))$. Given a trade, the amount of sales

taxes is $\sigma_s L_1(z(\theta))$. Altogether, the total real transfer is given by

$$\begin{aligned}
T^* = & \frac{\gamma - 1}{w^* \gamma} + \tau LS + \frac{\sigma_g}{1 - \sigma_g} \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) \\
& + \sigma_s \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) L_1(z(\theta)) b(L_1(z(\theta))) dF(\theta) \\
& + \sigma_s \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) L_2(z(\theta)) b(L_2(z(\theta))) dF(\theta). \tag{32}
\end{aligned}$$

For part (v), the market-clearing condition for the general-good market is

$$Y = \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta). \tag{33}$$

The market-clearing condition for the labor market is aggregate demand for labor, LD , is equal to aggregate supply of labor, LS . Consider LD first. A household's realization of θ determines the money balance $z(\theta)$. Given this money balance, the resulted money balance after lotteries is $L_i(z(\theta))$, $i = 1, 2$, which takes place with probability $\pi_i(z(\theta))$. Thus the measure of such households is $N_b = \pi_i(z(\theta)) dF(\theta)$. The measure of shops corresponding to the households holding $L_i(z(\theta))$ is given by $N_s = \pi_i(z(\theta)) dF(\theta) b(L_i(z(\theta))) / [\mu(b(L_i(z(\theta))))]$, which is derived from $b/\mu(b) = N_s/N_b$ given the constant-return-to-scale matching technology. Then for each shop, the expected labor demand is $k + \psi(q) \mu(b)$, which is used to compute the aggregate demand for labor in the frictional markets. Note that such calculation is also valid for cases when some households do not use lotteries and when $L_i(z(\theta))$ is the same for some different realizations of θ . Thus, LD is given by

$$\begin{aligned}
LD = & Y + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\pi_1(z(\theta)) b(L_1(z(\theta)))}{\mu(b(L_1(z(\theta))))} [k + \psi(q(L_1(z(\theta)))) \mu(b(L_1(z(\theta))))] dF(\theta) \\
& + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\pi_2(z(\theta)) b(L_2(z(\theta)))}{\mu(b(L_2(z(\theta))))} [k + \psi(q(L_2(z(\theta)))) \mu(b(L_2(z(\theta))))] dF(\theta). \tag{34}
\end{aligned}$$

The firm's zero-profit condition (3) implies that for $i = 1, 2$,

$$k + \psi(q(L_i(z(\theta)))) \mu(b(L_i(z(\theta)))) = (1 - \sigma_s) L_i(z(\theta)) \mu(b(L_i(z(\theta)))).$$

Then (34) can be transformed to

$$\begin{aligned}
LD &= \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) + (1 - \sigma_s) \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) b(L_1(z(\theta))) L_1(z(\theta)) dF(\theta) \\
&\quad + (1 - \sigma_s) \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) b(L_2(z(\theta))) L_2(z(\theta)) dF(\theta).
\end{aligned} \tag{35}$$

The aggregate labor supply is given by

$$LS = \int_{\underline{\theta}}^{\bar{\theta}} \int l_g(m, \theta) dG_a(m) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} l_s(\theta) dF(\theta),$$

where $G_a(m)$ is the money distribution at the beginning of a period. Recall $l_s(\theta) = h(\theta) / (1 - \tau)$ and $l_g(m, \theta)$ from (22) given $t_g = 0$. Thus,

$$LS = \int_{\underline{\theta}}^{\bar{\theta}} \int \frac{1}{1 - \tau} [py(\theta) + z(\theta) - m - T^*] dF(\theta) dG(m) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{h(\theta)}{1 - \tau} dF(\theta).$$

Use (32) to substitute for T^* in the above. Also recall the constraint for the household's lottery choice, $\pi_1(z(\theta)) L_1(z(\theta)) + \pi_2(z(\theta)) L_2(z(\theta)) = z(\theta)$. It follows that

$$\begin{aligned}
LS &= \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) + (1 - \sigma_s) \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) b(L_1(z(\theta))) L_1(z(\theta)) dF(\theta) \\
&\quad + (1 - \sigma_s) \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) b(L_2(z(\theta))) L_2(z(\theta)) dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] L_1(z(\theta)) dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] L_2(z(\theta)) dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} h(\theta) dF(\theta) - \int mdG_a(m) - \frac{\gamma - 1}{w^* \gamma}.
\end{aligned} \tag{36}$$

The labor-market clearing requires $LD = LS$. Thus (35) and (36) imply

$$\begin{aligned}
\int mdG_a(m) + \frac{\gamma - 1}{w^* \gamma} &= \int_{\underline{\theta}}^{\bar{\theta}} h(\theta) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] L_1(z(\theta)) dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] L_2(z(\theta)) dF(\theta).
\end{aligned}$$

Because m is a household's money balance at the beginning of a period, it consists of the money balance carried over for precautionary purposes and if any, the transactional

balance unspent due to matching frictions. Thus,

$$\begin{aligned} \int mdG_a(m) &= \int_{\underline{\theta}}^{\bar{\theta}} \frac{h(\theta)}{\gamma} dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \\ &\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta). \end{aligned}$$

The above two equations together yield

$$\begin{aligned} (w^*)^{-1} &= \int_{\underline{\theta}}^{\bar{\theta}} h(\theta) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) [1 - b(L_1(z(\theta)))] L_1(z(\theta)) dF(\theta) \\ &\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) [1 - b(L_2(z(\theta)))] L_2(z(\theta)) dF(\theta), \end{aligned} \tag{37}$$

which determines the normalized wage rate in the steady state. Given that the labor market and the general-good market clear, the money market clears by Walras' law. Note that $(w^*)^{-1}$ is essentially the normalized price of money in terms of labor. Thus (37) clearly indicates that money has no value, i.e., $(w^*)^{-1} = 0$, if $z(\theta) = h(\theta) = 0$ for all θ .

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A List of the Notation

β : discount factor;
 $U(y)$: a household's utility of consuming y units of general goods;
 $u(q)$: a household's utility of consuming q units of special goods;
 l_g : a household's wage income in the first sub-period;
 l_s : a household's wage income in the second sub-period;
 z : a household's money balance for frictional transactions;
 h : a household's money balance for precautionary purposes;
 $\psi(q)$: labor input needed for producing q units of special goods;
 θ : a household's random disutility per unit of labor;
 $F(\theta)$: CDF of the random shock θ ;
 k : a firm's cost of operating a shop in a frictional submarket, measured in labor units;
 N_s, N_b : numbers of shops and buyers, respectively, in a submarket;
 $\mathcal{M}(N_b, N_s)$: aggregate number of matches in a submarket with N_b buyers and N_s shops;
 $s = \mu(b)$: primitive matching function;
 M : aggregate stock of money per capita in a period;
 p : price of general goods in terms of labor;
 m : a household's real money balance, measured in terms of labor;
 x : money spending in a frictional trade, measured in labor;
 $W(m, \theta)$: a household's value at the beginning of the first sub-period;
 $V(z, h)$: a household's value at the beginning of the second sub-period;
 $\tilde{V}(z)$: a household's value of the lottery choice;
 $B(z)$: a household's value immediately after the lottery takes place but before trading in the frictional submarket;
 L_i : the realization in a lottery;
 π_i : the probability with which L_i is realized in the lottery;
 z_0 : the prize in a lottery participated by the households with low balances of z ;
 $G(m)$: measure of households whose holdings are less than or equal to m immediately before lotteries are played;
 γ : money growth rate;
 τ : proportional income tax rate;
 σ_g : proportional sales tax rate on general goods collected from firms;
 σ_s : proportional sales tax rate on special goods collected from firms;
 t_g : proportional sales tax rate on general goods collected from households;
 t_s : proportional sales tax rate on special goods collected from households.