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Search, liquidity and the dynamics of house prices and construction

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# Search, Liquidity and the Dynamics of House Prices and Construction

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# Search, Liquidity and the Dynamics of House Prices and Construction

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September 25, 2011

#### Abstract

We characterize the dynamics of relative house prices, construction rates and population growth across US cities. In response to fluctuations in relative incomes, we find that population growth rates adjust more rapidly than construction rates in the short run and that price appreciation exhibits considerable serial correlation in the short-run and mean reversion in the long-run. We develop a competitive search model of the housing market in which construction, the entry of buyers, house prices and rents are endogenously determined in equilibrium. Our theory generates dynamics that are qualitatively consistent with our empirical observations. In particular, in a version of the economy calibrated to match long-run features of the housing market in U.S. cities, variation in the time it takes to sell a house (i.e. the house's liquidity) induces house values and transaction prices to exhibit momentum, or serially correlated growth.

Journal of Economic Literature Classification: E30, R31, R10 Keywords: House prices, liquidity, search, construction, dynamic panel.

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# 1 Introduction

We explore the consequences of time-consuming search and matching for the dynamics of house prices and construction. We first characterize the impact of relative income shocks across U.S. cities on the short-run dynamics of average house prices, construction and population growth. We then develop an environment where the entry of new buyers and the construction of new houses are endogenously determined. Our model can generate short term momentum (i.e. serially correlated growth rates) for both construction and house prices, even if housing dividends are strictly mean-reverting following shocks. When calibrated to data on U.S. cities (with housing dividends represented by local incomes), a version of our model qualitatively accounts well for the short-run dynamics of house prices, construction, and population growth at the city level. Quantitatively, we find tension between accounting for both the momentum in house prices and construction and the volatility of fluctuations in these variables.

As a number of authors have noted, most time-series variation in house prices is local in nature, rather than national. This has motivated researchers to use local factors such as income, regulations and construction costs to account for price movements. In our empirical analysis, we estimate a structural panel VAR with city-level observations and use it to isolate the joint dynamics of relative prices, construction and population growth in response to persistent shocks to local income. We find that housing market dynamics in U.S. cities can be characterized by several key facts, many of which are consistent with those documented by other authors using different methods. First, houses prices are very volatile relative to per capita incomes and rents. More precisely, average city housing price appreciation is much more volatile than would be predicted by a simple asset pricing model using local rents or per capita incomes to proxy dividends. Secondly, there is strong positive serial autocorrelation in house price appreciation over the short term, but mean reversion in prices over longer periods.<sup>2</sup> A third finding is that population growth rates are more volatile than construction rates, especially in the short run. Finally, we find that construction rates are more persistent than population growth rates and that both are substantially more persistent than income fluctuations.

While movements in house prices like those described here have been documented pre-

<sup>&</sup>lt;sup>1</sup>This has been document for the U.S. by Abraham and Hendershott (1996), Del Negro and Otrok (2006) and Glaeser et al. (2010), and for Canada by Allen, Amano, Byrne and Gregory (2007).

<sup>&</sup>lt;sup>2</sup>See Abraham and Hendershott (1996), Capozza, Mack and Mayer (1997), Malpezzi (1999) and Meen (2002).

viously, Capozza, Hendershott and Mack (2004) point out that a well-developed behavioral theory which accounts for them has proved difficult to construct. Since the work of Case and Shiller (1989) and Cutler, Poterba and Summers (1991), it has been recognized that movements in house prices pose a challenge to strict versions of the efficient markets view. In particular, the fact that the strong positive autocorrelation of house price appreciation does not appear to be explained by fundamentals suggests that a simple asset-pricing approach alone is of limited value.<sup>3</sup> Glaeser et al. (2010) find that while a dynamic rational expectations model of housing with endogenous construction can generate long-run mean reversion, it "fails utterly at explaining high frequency positive serial correlation of price changes."

Several authors have argued that there are good reasons to suspect that search and matching play important roles in housing markets. For example, both the observed positive co-movement of prices and sales (Rios-Rull and Sanchez-Marcos, 2007) and the fact that prices and sales are negatively correlated with average time on the market (Krainer, 2008) are broadly consistent with search theories of housing markets. Moreover, as first noted by Peach (1983) and more recently documented by Caplin and Leahy (2008), there is significant negative correlation between vacancies and price appreciation. Diaz and Jerez (2010) suggest that movements in the division of surplus between buyers and sellers driven by changes in the tightness of housing markets (as is predicted by competitive search theory) may be a significant source of fluctuations in house prices.

In light of the above literature, we construct a framework that introduces search and matching into a housing market, where both the entry of new buyers and the construction of new houses evolve endogenously. The value of living in a particular city is determined by an exogenous housing dividend in the form of the income they can earn locally relative to that in other locations. New buyers enter a city whenever the expected value of doing so exceeds their next best alternative. These entrants need housing. They initially rent, but many then search for houses to buy, a process that takes time. New houses are constructed and offered either for sale or for rent by profit maximizing development firms. Resident homeowners may also put their houses up for sale or rent due to shocks that either render them dissatisfied with their current house or cause them to exit the city altogether. Exchange in the market for residential housing is characterized by competitive search as proposed by Moen (1997). In our environment, we establish the existence of a unique stationary growth

 $<sup>^{3}</sup>$ For example, Case and Shiller (1989) argue that serial correlation in rents cannot explain momentum in price changes.

path characterized by constant rates of population growth and construction.

We study the implications of shocks to local relative income by calibrating our model to data on U.S. cities. The model generates short-term price momentum in equilibrium even in the absence of persistent income growth (i.e. even if income follows a first-order auto-regressive process). In the model, an increase in the value of living in a city spurs an immediate increase in house search activity as households enter the city. It takes time, however, for these buyers to find a house because of the matching frictions, as well as for construction of new housing to respond. To meet the immediate housing demands of new entrants, some existing vacant houses are shifted to the rental market. As a result, the matching rate for individual house buyers initially declines, while both sales and the rate at which houses sell rise immediately. Therefore, although the value of house search begins to decline after just one period (due to mean reversion in income), the tightness of the housing market (i.e. the ratio of buyers to sellers) continues to rise for several periods. This ongoing increase in market liquidity causes the expected re-sale value of housing to grow. Because current transactions prices partly reflect the value of houses to their owners, these prices grow in anticipation. Over time, as income reverts to its long-run relative level, the stock of buyers declines as entry slows and residents become home-owners. Higher home values induce increased construction so that the decline in vacancies slows and is eventually reversed. The buyer-seller ratio, and hence housing liquidity, eventually fall. after initially growing for several periods, the house price falls in anticipation of a less tight market in the future, and eventually revert to its steady-state level.

Recently, van Nieuwerburgh and Weill (2010) and Glaeser et al. (2010) also develop dynamic housing models in which house prices reflect the interaction between local supply conditions and the willingness of households to pay to live and work in a particular location.<sup>4</sup> Van Nieuwerburgh and Weill (2010) study the role of cross-city wage dispersion in driving house price dispersion in the long run. Glaeser et al. (2010) study short term dynamics driven by an estimated process for city-level incomes. Neither considers the role of liquidity effects associated with search and matching in the housing market. The possibility of short-term price momentum and mean reversion in prices and construction arises in Glaeser et al. (2010) only because of the observed "hump-shaped" pattern of relative incomes.<sup>5</sup> Their model is reasonably successful in accounting for short term volatility in prices and construction but

<sup>&</sup>lt;sup>4</sup>In this respect these models build on the urban tradition of Alonso (1962), Rosen (1979) and Roback (1982).

<sup>&</sup>lt;sup>5</sup>Glaeser et al. (2010) also assume utility is decreasing in local population size which has a dampening effect on prices. However, in their calibration this effect is tiny so, in fact, the shock process drives everything.

fails to generate short-term momentum in price appreciation and cannot account for volatility over a longer time horizon.

Diaz and Jerez (2010) also develop and calibrate a competitive search model of the housing market. They consider a version of Wheaton (1990) with neither entry of buyers nor house construction. They suggest that competitive search may magnify the effects of exogenous changes on house prices due to movements in the shares of surplus accruing to buyers and sellers. In the context of our model, however, we find that allowing for this effect, by considering a matching function that implies variable shares (e.g. an urn-ball matching function), reduces the momentum of house prices but has no impact on their relative volatility.

Although a number of other researchers have studied the role of search and matching in housing markets (e.g. Wheaton, 1990; Krainer, 2001; Albrecht at al., 2007; and Head and Lloyd-Ellis, 2011), these papers generally treat the aggregate housing stock as fixed and/or consider steady-state implications. Caplin and Leahy (2008) consider the non-steady-state implications of their model, assuming a fixed housing stock. In contrast, we focus on the role of transitional dynamics of prices and construction of new homes in response to shocks. Furthermore, we allow for the turnover of existing homes, which turns out to be crucial for both the qualitative and quantitative nature of price and investment dynamics. Finally, models of housing investment and construction (e.g. Davis and Heathcote, 2007) generally abstract from search and matching in the market for houses in order to focus on supply-side factors. In contrast, in this paper we bring together aspects of both literatures.

Section 2 documents some of the key empirical features of the dynamics of housing markets at the city level. Section 3 develops the basic model structure. In Section 4, a competitive search equilibrium is characterized and a deterministic steady-state derived. Section 5 presents both a baseline calibration for our search economy and an alternative economy without search. Section 6 considers the dynamic implications of income shocks in the theory. Section 7 checks the robustness of our main findings by examining two modifications to the baseline search economy. Section 8 concludes. All proofs and extended derivations are contained in the appendix.

# 2 Empirical properties of relative house prices, income, construction and city populations

In this section, we characterize the dynamics of local (*i.e.* city-level) income, house prices, construction rates and population growth for a sample of 98 U.S. cities (metropolitan statistical areas [MSA's]). Our data is annual and runs from 1980 through 2008. We provide data details, including sources, in Appendix A. We focus on four variables: *per capita* income at the city level (total income less labor earnings from construction divided by population), the growth rate of the housing stock, the level of house prices, and population growth. Since we are interested in the dynamics of city-level measures relative to those in other cities, we first transform the data by removing common time effects. That is, we estimate a panel regression for the logarithm of each series with time dummies and study the residual components from these regressions.

### 2.1 A Structural Panel VAR

Relative movements in house prices, construction and city populations are likely affected by many factors. Here we seek to isolate the dynamics that result from changes to *income* at the city level, which we interpret as a dividend to residence in the city. We focus on such dynamics so as to be consistent with the theory we present in the next section. In our theoretical model, when income in a particular city rises relative to the average, the city's rate of population growth also rises as households from outside move into the city to take advantage of either the higher income itself or the factor(s) that caused it. Higher entry drives housing demand above trend, puts upward pressure on house prices, and over time spurs construction. As increased construction pushes costs higher and local income reverts toward its trend, entry slows relative to construction. Eventually, house prices decline and the city returns to its long-run trend.

Given our theory, we estimate a panel vector auto-regression (VAR) model of the following form:

$$\mathbf{B}\mathbf{X}_{ct} = \sum_{i=1}^{T} \mathbf{A}_{i}\mathbf{X}_{ct-i} + \mathbf{F}_{c} + \boldsymbol{\varepsilon}_{ct}, \tag{1}$$

where  $\mathbf{X}_{ct} = [Y_{ct}, P_{ct}, g_{ct}^H, g_{ct}^N]'$  denotes the vector of the log of income less construction earnings  $per\ capita$ , the log of house prices, the growth rate of the stock of houses (cumulative permits) and population growth in each city at each date.<sup>6</sup> Here  $\mathbf{B}$  and  $\mathbf{A}_i$  are matrices

<sup>&</sup>lt;sup>6</sup>The fact that we use log levels of incomes and prices and growth rates of population and construction

of parameters,  $\mathbf{F}_c$  is a vector of city fixed-effects and  $\boldsymbol{\varepsilon}_{ct} = [\varepsilon_{ct}^Y, \varepsilon_{ct}^P, \varepsilon_{ct}^H, \varepsilon_{ct}^N]'$  contains the structural shocks.

To estimate the structural parameters of this model, we must impose a set of identifying restrictions. Specifically, we assume that the shocks are orthogonal and adopt the ordering indicated by the definition of  $\varepsilon_c$  above. In particular, income does not contemporaneously depend on any of the other variables and house prices depend contemporaneously only on income. This ordering, which is consistent with the theory we develop below, emphasizes the importance of shocks to fundamentals that affect current and future income, as well as house prices, construction, and population growth contemporaneously. It also allows for shocks to prices that have no *contemporaneous* effect on income, but can affect construction, population growth, and *future* income.<sup>7</sup> Like our theory, this ordering rules out shocks to current and future income that have no contemporaneous effect on house prices.

We estimate equation (1) for T=2 using the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998).<sup>8</sup> This estimator is asymptotically consistent when the number of panels becomes large for a given time-dimension, thereby avoiding the incidental parameters problem associated with fixed-effects estimators (Nickell, 1981). There are several reasons for which we choose to focus on results using this estimator rather than others. Firstly, it is generally found to outperform other standard GMM estimators such as that of Arellano and Bond (1991) when the endogenous variables are persistent.<sup>9</sup> Secondly, its asymptotic properties are well understood and it has been extended to the context of panel VARs by Binder, Hsiao and Pesaran (2005). Finally, the standard fixed–effect estimator has been found to exhibit a significant finite-sample bias for samples with similar dimensions to ours (i.e. moderately large time and panel dimension; see Judson and Owen, 1999). There are, however, some potential pitfalls in using the system GMM estimator in finite samples. We discuss these in more detail in Appendix B and compare our

is consistent with our theoretical analysis. However, in Appendix B we report the results of using growth rates of all variables in the panel VAR. With this specification, the nature of the results remain unchanged.

<sup>&</sup>lt;sup>7</sup>Note that  $g_{ct}^H$  and  $g_{ct}^N$  are growth rates going forward (i.e.  $g_{ct}^H = \ln H_{t+1} - \ln H_t$ ). Thus it seems reasonable that these variables are able to respond to time t shocks to in income and prices. The ordering of  $g_{ct}^H$  and  $g_{ct}^N$  in the system makes very little difference to our results.

<sup>&</sup>lt;sup>8</sup>We have also estimated the system with more than two lags, although the results show little difference from the two-lag case.

<sup>&</sup>lt;sup>9</sup>Essentially, the system GMM estimator instruments the endogenous variables using lagged differences. In fact, we adopt an alternative, but asymptotically equivalent approach which has been found to perform better in finite samples. Specifically, we instrument levels with lagged deviations from the forward mean of the remaining sample (see Arellano and Bover, 1995). As such the model is "just identified" (i.e. the number of regressors equals the number of instruments), so that system GMM is numerically equivalent to equation-by-equation 2SLS.

estimates to those generated using other methods.

Parameter estimates are reported in Appendix B. Table 1 provides overall summary statistics from our baseline panel VAR. The first column shows the average standard deviation of each series relative to that of the growth in *per capita* income. The second column shows the correlation with *per capita* income growth. The remaining columns show the first four coefficients of autocorrelation.<sup>10</sup>

Table 1: Moments from Panel VAR — all shocks

	Relative	Corr. with	Autocorrelations			
	Std. Dev.	Inc. growth	year 1	year 2	year 3	year 4
Income growth	1.0000	1.0000	0.2425	0.0194	-0.0560	-0.0758
Price appreciation	2.6601	0.4086	0.4138	0.1164	-0.0252	-0.0826
Construction Rate	0.3547	0.1839	0.7643	0.5249	0.3372	0.2035
Population Growth	0.5271	0.2485	0.4335	0.3339	0.1905	0.1181

Several observations can be made based on Table 1. First, house prices are much more volatile than city-level incomes. Secondly, price changes are more persistent than those of income growth, with a first-order autocorrelation exceeding 0.4 as compared 0.24. Moreover, the level of house prices exhibits momentum. Thirdly, population growth rates are more volatile on average than construction rates. Finally, construction rates are more persistent than population growth rates and both, like price changes, exhibit substantially more persistence than income.

Figure 1 depicts the implied impulse response functions for a shock to relative local income together with the associated 95% confidence intervals.<sup>11</sup> In response to the shock, local income exhibits positively auto-correlated growth, peaking after one year, and is quite persistent. The resulting movement in the relative house price exhibits considerably more momentum, continuing to rise for three years before starting to revert to its mean. Mean reversion is, however, more rapid overall for house prices than for income. Population growth responds immediately to the shock then slows down, whereas the construction rate responds more sluggishly, peaking after two years. A key consequence of this is that the ratio of city population to the housing stock rises and remains persistently high following a shock to income.<sup>12</sup>

<sup>&</sup>lt;sup>10</sup>In the table we report statistics for price growth, while the model is estimated using the *level* of house

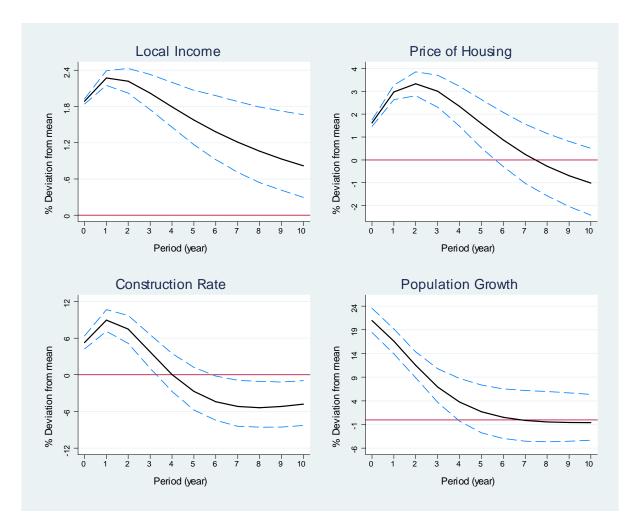


Figure 1: Impulse response functions from panel VAR: income shock

These observations are quantified in Table 2, which reports statistics associated with the effect of income shocks implied by the panel VAR. The unconditional standard deviation of house prices generated by income shocks alone is roughly half that observed in the data overall (see Table 1). The persistence of house price movements generated by local income shocks is, however, somewhat larger than what they exhibit overall. Similar statements hold for both construction and population growth rates. In response to a shock to local income, house price growth jumps by roughly 20% more in the first year, and is about 35% more

prices.

<sup>&</sup>lt;sup>11</sup>Confidence intervals are computed using a Monte Carlo simulation for panel VAR provided by Love and Zicchino (2006).

<sup>&</sup>lt;sup>12</sup>Once we move beyond five years, the confidence intervals become quite wide so we do not place too much emphasis on what happens beyond this point.

volatile than overall. In contrast, the construction rate exhibits only 11% as much volatility as income, and population growth just under 18% as much. All three measures, however, exhibit substantially more momentum than income in response to a shock to relative local income.

Table 2: Moments from Panel VAR — Income Shock only

	Relative	Corr. with	Autocorrelations			
	Std. Dev.	Inc. growth	year 1	year 2	year 3	year 4
Per capita Income growth	1.0000	1.0000	0.2480	0.0221	-0.0546	-0.0759
House Price Appreciation	1.3483	0.8049	0.7066	0.2988	0.0274	-0.1144
Construction Rate	0.1144	0.4600	0.8946	0.6589	0.4116	0.2117
Population Growth	0.1791	0.7115	0.7149	0.4533	0.2526	0.1219

In Appendix B, we report the results of re-estimating the panel VAR model for several regional sub-samples. These regional groupings, corresponding to those considered by Glaeser et al. (2010), are coastal cities within 50kms of a coastline, (non-coastal) sun-belt cities and the remaining interior cities. While the parameter estimates naturally vary across samples, the general picture provided by the statistics in Table 2, is unchanged (see Table B2). Moreover, the effects of a shock to local income, as depicted by the impulse responses in Figure 1, are similar qualitatively for all sub-samples considered.

Also in appendix B we report the result of several alternative specifications of the panel VAR. We consider alternative definitions of the key variables and alternative estimators. While parameter estimates and moments vary, the general picture that emerges is robust across specifications. Interestingly, when we restrict the VAR so that income is assumed to follows a univariate AR(2) process, the results are hardly changed. This suggests that feedback effects of construction, prices and population on per capita income are not very large and that, to a first approximation, it may be reasonable to think of per capita income as an exogenous process.

# 2.2 Implications of a linear asset pricing model

In Section 3 we proceed to develop a theory to determine the extent to which search and matching in the housing market can account for volatility and momentum in house prices as well as persistent movements in both construction rates and population growth, in response

to shocks to relative local income. Before doing so, however, it is useful to reflect on the implications of the simplest possible asset pricing model since this serves as a useful benchmark for evaluating the explanatory power of alternative models for price dynamics.<sup>13</sup>

Specifically, consider the price of an asset which represents a claim to relative local (city) income,  $y_t$ . If agents are risk-neutral (as they will be in our theory below), and if these claims are traded in a Walrasian market, the price of this asset,  $P_t^L$ , will equal the present discounted value of local income:

$$P_t^L = E_t \sum_{i=1}^{\infty} \beta^i y_{t+i}, \tag{2}$$

where  $\beta$  is the market discount factor. Imposing the transversality condition,  $\lim_{T\to\infty} \beta^T E_t P_{t+T}^L = 0$ , setting  $\beta = 0.96$ , and assuming that  $\ln(y_t)$  follows the univariate AR(2) process estimated in Appendix B, we generate implied moments for the price of this asset,  $P^L$ . Table 3 compares these moments with those documented for U.S. cities in Table 2. As may be seen the relative volatility of prices,  $\sigma_p/\sigma_y$ , implied by the linear asset pricing model (LAPM) is only about 30% of that in the data. Moreover, despite the fact that income growth does exhibit serial correlation, this does not translate into any momentum at all in prices. Indeed, for an income process to generate momentum in prices via this model, very high and persistent serial correlation in income growth would be needed. The reason is, of course, that price movements capitalize future income fluctuations.

Table 3: Moments from a linear asset pricing model – income shock

Moment	US Cities	LAPM
$\sigma_p/\sigma_y$	1.35	0.38
$\sigma_{py}$	0.80	0.95
$\rho_1^p$	0.71	-0.04
$ ho_2^p$	0.30	-0.04
$\rho_3^p$	0.03	-0.04
$\rho_4^p$	-0.11	-0.04

# 3 The model environment

Time is discrete and indexed by t. We consider economic activity in a single housing market (which we refer to as a city) and treat activity outside this market as exogenous. The

<sup>&</sup>lt;sup>13</sup>Of course, the models below account for other variables as well.

aggregate economy is populated by measure  $Q_t$  of ex ante identical households, which grow exogenously at net rate  $\mu$ . Each period, new households enter the city through a process described below. All households living in the city require housing, and they each may either own or rent a house to live in. On entry to the city, households are randomly and permanently differentiated into two types, those who derive utility from owning the house that they live in and those that do not. All households discount the future at rate  $\beta \in (0, 1)$ .

Each household is infinitely-lived and supplies one unit of labour inelastically. Labour earns  $y_t$  per unit supplied, where  $y_t$  is city-specific, exogenous and follows a stationary stochastic process in log-levels. We parameterize specific processes for  $y_t$  in Section 6.

At date t, preferences over consumption  $c_t$ , and housing  $z_t$  are given by:

$$U_t(c_t, z_t) = c_t + z_t, (3)$$

The variable  $z_t$  denotes a utility premium derived from owning the house in which the household lives. In particular,  $z_t = z^H$  if the household likes the house it owns, and  $z_t = 0$  either if the household does not like the house it owns or if the household is renting a house. We assume that  $z^H$  reflects the value of the quality of the house to its owner. This value is constant over time because any depreciation resulting from occupancy is assumed to be offset by maintenance. We let m denote the cost of maintenance incurred by the owner.

In period t, the city has a stock  $H_t$  of housing units, which are either occupied by a resident owner, rented to a resident, or vacant and offered for sale. The measure of resident home-owners is denoted by  $N_t$ , and that of resident renters by  $B_t + F_t$ . Here  $B_t$  is the measure of renters who would like to own a house (and so are currently searching for one to buy) and  $F_t$  is that of renters that are not interested in owning.<sup>14</sup> A measure  $S_t$  of houses are for sale, where  $S_t = H_t - N_t - B_t - F_t$ . Houses for sale include both newly built ones that are currently owned by developers, and houses put up for sale by resident owners who either do not want them anymore or are moving elsewhere. At the beginning of each period, a house that is not currently owner-occupied can either be rented or listed for sale. Let  $H_t^R$  denote the stock of houses available for rent. A rented house earns rent  $r_t$  less the constant maintenance cost m. The rental market is competitive.

In the city, there are a large number of *developers* who behave competitively and operate a technology for the construction of new housing units. Each new house requires one unit of land, which can be purchased in a competitive market at unit price  $q_t$ , and incurs a

<sup>&</sup>lt;sup>14</sup>These permanent renters do not play a crucial role in our analysis. We allow for them in order to capture the fact that not all renters are potential buyers.

construction cost,  $k_t$ . Houses constructed at time t become available either for sale or for rent at time t + 1 and do not depreciate over time.

The unit construction cost is assumed to be of the form

$$k_t = \frac{1}{\phi} \left( \frac{H_{t+1} - H_t}{N_t + B_t + F_t} \right)^{\frac{1}{\eta}}.$$
 (4)

where  $\phi$  and  $\eta$  are parameters. That is, unit costs increase with the construction of new houses *per capita*. This formulation of the construction cost function is consistent with those implicitly estimated in the housing literature (e.g. Green, Malpezzi and Mayo, 2005). As we show in Section 7 it can be derived from a simple labor market equilibrium. Here, however, we simply posit the cost function, (4).

The price of land depends on the stock of houses relative to its trend value  $H_t^*$ :

$$q_t = \bar{q} \left( \frac{H_t}{H_t^*} \right)^{\frac{1}{\xi}}. \tag{5}$$

Here  $\xi$  represents the elasticity of land supply with respect to its price. This elasticity could depend on many factors including topography, land regulations and local politics (see Saiz, 2010). Newly built houses are identical to pre-existing ones. Developers can either rent them out or designate them for sale, in which case they remain vacant for at least one period and have exactly the same value as existing vacant houses. Only houses that are occupied require maintenance to offset depreciation.

At the beginning of period t, a measure  $\mu Q_{t-1}$  of new households enter the economy and receive an alternative value  $\varepsilon$  to entering the city. Here  $\varepsilon$  is distributed across the new households according to a stationary distribution function  $G(\varepsilon)$  with support  $[0,\bar{\varepsilon}]$ . There exists a critical alternative value  $\varepsilon_t^c$ , at which a new household is just indifferent to entering the city:

$$\varepsilon_t^c = \bar{W}_t, \tag{6}$$

where  $\bar{W}_t$  is the value of being a new entrant to the city. All non-resident households with  $\varepsilon \leq \varepsilon_t^c$  enter the city and are immediately separated into two types. A fraction  $\psi$  of the new entrants derive utility from owning their own home per se and become potential buyers, while the rest do not and become perpetual renters. Let  $W_t$  denote the value of being a potential buyer and  $W_t^f$  the value of being a perpetual renter. It follows that

$$\bar{W}_t = \psi W_t + (1 - \psi) W_t^f. \tag{7}$$

Searching for a house to own takes at least one period, and during this time potential buyers also rent. At the end of each period, perpetual renters may, with probability  $\pi_f \in (0,1)$ , experience an exogenous shock that induces them to leave the city. On receiving this shock they move out immediately and receive a continuation value Z. Otherwise they remain as renters in the next period.

Home-owners are subject to two exogenous shocks. With probability  $\pi_h \in (0,1)$  owners receive a shock that causes them to want to leave the city. Like renters, upon receiving this shock they move out immediately and receive a continuation value Z. They now also have a vacant house which they either put up for sale or rent. With probability  $\theta \in (0,1)$  the remaining  $(1-\pi_h)N_t$  of owners at date t will find that they no longer derive the utility premium  $z^H$  from owning their current house. Such "mismatched" owners immediately move out of their current house and rent while searching for a new one.<sup>15</sup>

We assume that capital markets are perfect and that the gross interest rate is  $1/\beta$ . Because capital markets are complete and there is free entry into construction, households do not have any interest in owning houses either as a means of saving or for speculative purposes. As such, it makes no difference whether or not we allow for the trading of vacant houses in a Walrasian market. In order to receive utility  $z^H$  from owning, households must search for the right house through a time-consuming process that depends on the measures of buyers and houses/sellers in the market.

The market for houses for sale is characterized by competitive search, and may consist of a variety of sub-markets. Each sub-market is characterized by a pair  $(\omega_t, P_t)$ , where  $P_t$  is the price of a house in the sub-market and  $\omega_t$  is the tightness of the sub-market, i.e. the ratio of buyers to sellers. Search is competitive in the sense that buyers and sellers take the prices and tightness of all sub-markets as given, and then decide which single sub-market to enter and search for a trade. There is no cost to entering any sub-market.

Let  $N_t^b$  and  $N_t^s$  be the respective measures of buyers and houses/sellers present in a submarket. Thus  $\omega_t = N_t^b/N_t^s$ . The measure of matches the sub-market is determined by the matching function  $\mathcal{M}\left(N_t^b, N_t^s\right)$ , which is increasing in both arguments and exhibits constant returns to scale. It follows that a buyer will find a vacant house in the current period with probability

$$\lambda_t = \frac{\mathcal{M}\left(N_t^b, N_t^s\right)}{N_t^b} = \lambda\left(\omega_t\right). \tag{8}$$

<sup>&</sup>lt;sup>15</sup>As we shall see later, in equilibrium mis-matched owners are indifferent between moving out and remaining in their own house while searching. Assuming some or all mis-matched owners remain in their current home while searching yields almost identical results.

Similarly, a seller in this sub-market will find a buyer with probability

$$\gamma_{t} = \frac{\mathcal{M}\left(N_{t}^{b}, N_{t}^{s}\right)}{N_{t}^{s}} = \gamma\left(\omega_{t}\right) = \omega_{t} \,\lambda\left(\omega_{t}\right). \tag{9}$$

Let  $\epsilon(\omega_t)$  denote the elasticity of the measure of matches with respect to the measure of buyers, *i.e.* 

$$\epsilon\left(\omega_{t}\right) = \frac{N_{t}^{b}}{\mathcal{M}} \cdot \frac{\partial \mathcal{M}}{\partial N_{t}^{b}} = \frac{1}{1 - \frac{\gamma(\omega_{t})/\gamma'(\omega_{t})}{\lambda(\omega_{t})/\lambda'(\omega_{t})}}.$$
(10)

We impose the following assumption on the matching function:

**Assumption 1.** There exists an interval  $(\underline{\omega}, \bar{\omega})$ , with  $0 < \underline{\omega} < \bar{\omega} < \infty$ , such that for all  $\omega \in (\underline{\omega}, \bar{\omega})$ : (i)  $\lambda(\omega) \in [0, 1]$ ,  $\gamma(\omega) \in [0, 1]$ ,  $\lim_{\omega \to \bar{\omega}} \lambda(\omega) = \lim_{\omega \to \underline{\omega}} \gamma(\omega) = 0$  and  $\lim_{\omega \to \underline{\omega}} \lambda(\omega) = \lim_{\omega \to \bar{\omega}} \gamma(\omega) = 1$ ; (ii)  $\lambda'(\omega) < 0$ ,  $\gamma'(\omega) > 0$ ; (iii)  $\epsilon'(\omega) \leq 0$ .

Part (i) and Part (ii) are intuitive. Part (iii) is required to generate a positive relationship between the market tightness and the value of houses. Given part (iii), as the tightness increases, houses are sold at a higher rate. This drives up the value of a vacant house for a given selling price. We associate the rate at which houses sell with their *liquidity*. When this rate increases (decreases), houses become more (less) liquid. We parameterize a specific matching function as part of our calibration in Section 5.

# 4 Competitive search equilibrium

Linearity of preferences in consumption, together with the perfect capital markets, implies that households are indifferent with regard to the timing of their consumption.

**Lemma 1**. Each household makes its housing decisions so as to solve

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[ y_t + z_t - \Omega_t \right] \tag{11}$$

where  $\Omega_t$  represents the net value of all housing-related transactions that take place in period t.

Perpetual renters never choose to search for a house and remain as renters until they exogenously move to another location. It follows that the value of being such a renter is

$$W_t^f = u_t^R + \pi_f \beta Z + (1 - \pi_f) \beta E_t W_{t+1}^f. \tag{12}$$

where

$$u_t^R = y_t - r_t. (13)$$

The stock of perpetual renters evolves according to

$$F_t = (1 - \pi_f)F_{t-1} + (1 - \psi)G(\varepsilon_t^c)\mu Q_{t-1}.$$
(14)

The value of being a home-owner  $J_t$  is given by

$$J_{t} = u_{t}^{H} + \pi_{n}\beta \left(Z + E_{t}\tilde{V}_{t+1}\right) + (1 - \pi_{n})\theta\beta \left(E_{t}W_{t+1} + E_{t}\tilde{V}_{t+1}\right) + (1 - \pi_{n})(1 - \theta)\beta E_{t}J_{t+1},$$

$$(15)$$

where

$$u_t^H = y_t + z^H - m. (16)$$

At the beginning of each period, a house that is not currently occupied by the owner can be either rented or listed for sale. Thus the value of a house which is not owner–occupied is given by

$$\tilde{V}_t = \max\left[r_t - m + \beta E_t \tilde{V}_{t+1}, \ V_t\right],\tag{17}$$

where  $V_t$  is the value of a house designated for sale.

House sellers are free to enter any sub-market without cost but can only enter one sub-market in each period. By entering a sub-market characterized by  $(\omega_t, P_t)$ , a seller sells a house at  $P_t$  with probability  $\gamma(\omega_t)$ . If the house is not sold in the current period, the seller holds on to it until the next period and receives the value of a house that is not currently owner-occupied. The seller chooses to enter a sub-market that maximizes his/her expected return. It follows that the value of a vacant house for sale satisfies

$$V_{t} = \max_{(\omega_{t}, P_{t})} \left\{ \gamma(\omega_{t}) P_{t} + \left[1 - \gamma(\omega_{t})\right] \beta E_{t} \tilde{V}_{t+1} \right\}.$$

$$(18)$$

Given (18), a seller is willing to enter only sub-markets that offer  $P_t \ge \beta E_t \tilde{V}_{t+1}$ .

Free entry of sellers implies that all active sub-markets in equilibrium must offer the sellers the same payoff  $V_t$ , although  $(\omega_t, P_t)$  may vary across sub-markets. It follows that (18) determines a relationship between the listed price and the market tightness that must be satisfied by all active sub-markets:

$$\gamma\left(\omega_t(P_t)\right) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}}.$$
(19)

Thus it is sufficient to index sub-markets by the posted price  $P_t$  alone, which will be the case in the rest of this paper. From (19) it is clear that the probability of a vacant house being sold in a given period is lower in sub-markets with higher prices. This in turn implies that an individual house's time-on-the-market is positively related to its price, as is extensively documented by empirical studies.<sup>16</sup>

Buyers also decide in each period which sub-market to enter. A buyer who is successfully matched in a sub-market pays the posted price  $P_t$ , and becomes a home-owner in the next period, receiving value  $J_{t+1}$ . One who remains unmatched continues to search in the next period. Buyers, who are by definition searching for a house, are subject to neither separation nor preference shocks. Recall that they are currently renting, and thus receiving the renter utility  $u_t^R$  for the current period. The value of being a buyer  $W_t$  is therefore given by

$$W_{t} = u_{t}^{R} + \max_{P_{t}} \left\{ \lambda \left( \omega_{t}(P_{t}) \right) \left( \beta E_{t} J_{t+1} - P_{t} \right) + \left[ 1 - \lambda \left( \omega_{t}(P_{t}) \right) \right] \beta E_{t} W_{t+1} \right\}.$$
 (20)

It is clear that a buyer is willing to enter a sub-market if and only if  $P_t \leq \beta (E_t J_{t+1} - E_t W_{t+1})$ .

In equilibrium, the set of active sub-markets is *complete* in the sense that there is no other sub-market that could improve the welfare of any buyer or seller. We focus on equilibria in which the total trade surplus in the housing market is strictly positive, *i.e.* 

$$E_t \left[ J_{t+1} - W_{t+1} - \tilde{V}_{t+1} \right] > 0. {(21)}$$

**Definition.** A competitive search equilibrium is a sequence

$$\left\{J_t, V_t, W_t, \bar{W}_t, W_t^f, P_t, \varepsilon_t^c, B_t, N_t, H_t, \omega_t, r_t, H_t^R\right\}_{t=0}^{\infty}$$

such that the following hold for any given evolution of  $\{y_t, Q_t\}_{t=0}^{\infty}$ :

- i. New households enter the market optimally so that (6) and (29) are satisfied;
- ii. The trade surplus in the housing market is strictly positive, i.e. (21) holds;
- iii. The value of being a home-owner satisfies (15); the value of a vacant house satisfies (18) and the value of a buyer satisfies (20);

<sup>&</sup>lt;sup>16</sup>The positive correlation between time-on-the-market and transaction price is found in the work of Forgey et al (1996), Kang and Gardner (1989), Leung, Leong and Chan (2002), Anglin et al (2003), and Merlo and Ortalo-Magné (2004), among others.

iv. The owner of a vacant house is indifferent between putting the unit up for rent and for sale:

$$\tilde{V}_t = r_t - m + \beta E_t \tilde{V}_{t+1} = V_t; \tag{22}$$

v. The market for rental housing clears:

$$H_t^R = B_t + F_t; (23)$$

vi. Given house prices and construction costs, there is free entry into construction:

$$\beta E_t \tilde{V}_{t+1} \le \kappa_t + q_t, \quad H_{t+1} \ge H_t, \tag{24}$$

where the two inequalities hold with complementary slackness;

- vii. Boundary conditions on value functions rule out bubbles:  $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$ ;
- viii. All active sub-markets must have  $\gamma(\omega_t)$ ,  $\lambda(\omega_t) \in (0,1)$  and satisfy the free-entry condition (19).

## 4.1 The equilibrium dynamic system

The buyer's optimization problem in (20) may be represented;

$$\max_{P_t} \left\{ \lambda \left( \omega_t(P_t) \right) \left( \beta E_t J_{t+1} - P_t - \beta E_t W_{t+1} \right) \right\}, \tag{25}$$

where

$$\omega_t(P_t) = \gamma^{-1} \left[ \frac{V_t - \beta E_t V_{t+1}}{P_t - \beta E_t V_{t+1}} \right]$$
(26)

as implied by the free-entry condition (19). We then have the following proposition:

**Proposition 1.** In a competitive search equilibrium, there is only one active sub-market. In this market, the share of the surplus from house transactions that accrues to the buyer is equal to the elasticity of the measure of matches with respect to the measure of buyers:<sup>17</sup>

$$s(\omega_t) = \epsilon(\omega_t). \tag{27}$$

Therefore, with competitive search, the respective shares of the trade surplus accruing to buyers and sellers in a transaction depend on the tightness of the market. Moreover, the house price satisfies

$$P_{t} = [1 - s(\omega_{t})] \beta (E_{t} J_{t+1} - E_{t} W_{t+1}) + s(\omega_{t}) \beta E_{t} V_{t+1},$$
(28)

<sup>&</sup>lt;sup>17</sup>This result is a special case of that derived in Moen (1997).

which is derived from equations (85) and (86) in Appendix C.

Given that there is a unique active (sub-)market of houses for sale, the stock of buyers at date t is given by:

$$B_{t} = \theta(1 - \pi_{h})N_{t-1} + \psi G(\varepsilon_{t}^{c})\mu Q_{t-1} + (1 - \lambda(\omega_{t-1}))B_{t-1}.$$
 (29)

The stock of owners evolves via

$$N_t = (1 - \pi_n)(1 - \theta)N_{t-1} + \lambda(\omega_{t-1})B_{t-1}.$$
(30)

Combining (18) and (22), it is apparent that, in equilibrium the stocks of rental and ownable housing must be such that the return to renting a house for a period equals the expected gain from holding it vacant for sale:

$$r_t - m = \gamma_t \left( P_t - \beta E_t V_{t+1} \right). \tag{31}$$

We focus on equilibria in which construction of houses is always positive, that is,  $H_{t+1} > H_t$ . It then follows from (4), (22) and (24) that the quantity of new housing constructed in period t is given by

$$H_{t+1} - H_t = \phi^{\eta} \left( N_t + B_t + F_t \right) \left( \beta E_t V_{t+1} - q_t \right)^{\eta}. \tag{32}$$

To obtain a stationary representation of the economy, we normalize the state variables by the total population  $Q_t$ . We use lower case letters to represent *per capita* (i.e. per household) values. Given (14), (29), (30) and (32), the dynamic equations for renters, buyers, owners and houses, *per capita*, respectively, can be written as

$$(1+\mu)f_t = (1-\psi)\mu G(\bar{W}_t) + (1-\pi_f)f_{t-1}$$
(33)

$$(1+\mu)b_t = \psi \mu G(\bar{W}_t) + [1-\lambda(\omega_{t-1})]b_{t-1} + \theta(1-\pi_n)n_{t-1}$$
(34)

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})b_{t-1}$$
(35)

$$(1+\mu)h_{t+1} = h_t + \phi^{\eta} (n_t + b_t + f_t) (\beta E_t V_{t+1} - q_t)^{\eta}.$$
 (36)

By definition, the tightness of the housing market is given by

$$\omega_t = \frac{b_t}{h_t - b_t - f_t - n_t}. (37)$$

Moreover, market-clearing in the rental market implies

$$h_t^R = b_t + f_t. (38)$$

<sup>&</sup>lt;sup>18</sup>It is straightforward to show that this will be the case in any competitive search equilibrium for an economy with sufficient population growth.

## 4.2 Deterministic steady state

We now consider a steady-state in which general non-construction income per capita is constant and normalized to unity:  $y_t = 1$ . In this setting,  $\bar{W}$  and  $\omega$  are constant. Then (33) implies that the normalized measure of renters is

$$f^* = \frac{(1 - \psi)\mu G(\bar{W}^*)}{\mu + \pi_f}.$$
 (39)

Similarly, from (34) the measure of buyers each period satisfies

$$b^* = \frac{\psi \mu G(\bar{W}^*)}{\mu + \lambda(\omega^*) - \frac{\theta(1-\pi_n)\lambda(\omega^*)}{\mu+\pi_n+\theta(1-\pi_n)}}.$$
(40)

Equation (35) implies that the steady-state fraction of the total population located in the city is

$$n^* = \frac{\lambda(\omega^*)}{\mu + \pi_n + \theta(1 - \pi_n)} b^*, \tag{41}$$

and (36) yields that the housing stock per capita satisfies

$$h^* = \frac{\phi^{\eta} (n^* + b^* + f^*)}{\mu} (\beta V^* - \bar{q})^{\eta}.$$
 (42)

**Lemma 2.** In the steady-state, there exists a negative "supply-side" relationship between the value of a house for sale and market tightness:

$$V^* = V^S(\omega^*) = \frac{1}{\beta \phi} \left[ \mu \left( 1 + \frac{\psi \left( \mu + \pi_f \right)}{A \gamma \left( \omega^* \right) + B \omega^*} \right) \right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}, \tag{43}$$

where  $A = \frac{\mu + \psi \pi_f + (1 - \psi) \pi_n}{\mu + \pi_n + \theta (1 - \pi_n)}$  and  $B = \mu + \psi \pi_f$ .

The relationship described in (43) can be interpreted as follows. As the value of vacant housing rises, new construction is stimulated and more houses become available for sale. This drives down the market tightness  $\omega$ , *i.e.* the ratio of buyers to houses for sale.

In the steady state, the values of owners, buyers and vacant houses, the house price and

rent, must satisfy the following set of equations:

$$J^* = \bar{u}^H + \pi_n \beta Z + \pi_n \beta V^* + (1 - \pi_n) \theta \beta (W^* + V^*) + (1 - \pi_n) (1 - \theta) \beta J^*$$
 (44)

$$W^* = \bar{u}^R + \lambda(\omega^*)(\beta J^* - P^*) + (1 - \lambda(\omega^*))\beta W^*$$
(45)

$$V^* = \gamma(\omega^*) P^* + (1 - \gamma(\omega^*)) \beta V^*$$
(46)

$$P^* = (1 - s(\omega^*))\beta (J^* - W^*) + s(\omega^*)\beta V^*$$
(47)

$$r^* = m + \gamma \left(\omega^*\right) \left(P^* - \beta V^*\right) \tag{48}$$

$$W_f^* = \bar{u}^R + \pi_f \beta Z + (1 - \pi_f) \beta W_f^* \tag{49}$$

$$\bar{W}^* = \psi W^* + (1 - \psi)W_f^*, \tag{50}$$

where  $s(\omega^*)$  in (47) is the share of surplus to a buyer expressed in terms of steady-state market tightness,  $\omega^*$ ,  $\bar{u}^H = \bar{y} + z^H - m$  and  $\bar{u}^R = \bar{y} - r^*$ . One can solve the first five equations of this system for  $J^*$ ,  $W^*$ ,  $V^*$ ,  $P^*$  and  $r^*$ . Then the last two equations can be used to determine  $\bar{W}^*$  and  $W_f^*$ . The above yields another relationship between the value of houses for sale and market tightness:

**Lemma 3.** In a steady-state, there exists a positive "demand-side" relationship between the value of a house for sale and market tightness:

$$V^* = V^D(\omega^*) = \frac{\gamma(\omega^*) (1 - s(\omega^*)) \beta z^H}{(1 - \beta) [1 - \beta (1 - \theta) (1 - \pi_n)] + (1 - \beta + \pi_n \beta) \beta \lambda(\omega^*) s(\omega^*)}.$$
 (51)

Intuitively, a higher ratio of buyers to sellers, *i.e.* a tighter market, has two effects. Firstly, it increases the rate at which houses sell,  $\gamma$ . For a given selling price, this drives up the value of a vacant house. Secondly, it *lowers* the rate at which buyers find houses, which increases the gain from becoming an owner. This raises the selling price of houses, and so the value of a house for sale. Lemmas 2 and 3 then yield the following proposition:

**Proposition 2.** There exists a unique steady-state equilibrium if the following condition holds:

$$0 < \frac{(1 - s(\bar{\omega}))\beta z^{H}}{(1 - \beta)\left[1 - \beta(1 - \theta)(1 - \pi_{n})\right] + (1 - \beta + \pi_{n}\beta)\beta\lambda\left(\bar{\omega}\right)s(\bar{\omega})} - \frac{1}{\beta\phi}\left[\mu\left(1 + \frac{\psi\left(\mu + \pi_{f}\right)}{A + B\bar{\omega}}\right)\right]^{\frac{1}{\eta}} - \frac{\bar{q}}{\beta}.$$

$$(52)$$

Figure 2 depicts the existence of a steady-state at the intersection of (43) and (51). The existence of the steady-state depends on the values of a variety of parameters which together

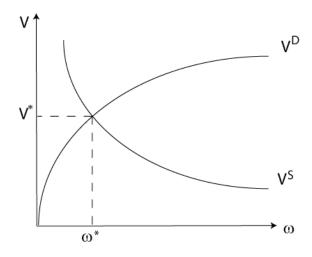


Figure 2: Steady State Equilibrium

ensure that the surplus from housing transactions is positive given that new houses are always being built in the city. For example, a land price  $\bar{q}$  that is too high, or an ownership premium  $z^H$  that is too low, may cause the surplus from housing transactions to become negative. In our calibration below, the parameters are such that this condition is never violated.

# 4.3 An alternative economy without search

It is useful to compare our baseline results to those from an economy in which new entrants to the city can either rent or purchase a house immediately and move in. Since households derive more utility from owning and construction costs are the same, only pure renters will choose to rent in equilibrium. In this case, the dynamic system is given by

$$(1+\mu)f_t = (1-\pi_f)f_{t-1} + (1-\psi)\mu G(\bar{W}_t)$$
(53)

$$(1+\mu)n_t = (1-\pi_n)n_{t-1} + \psi\mu G(\bar{W}_t)$$
(54)

$$(1+\mu)h_{t+1} = h_t + \phi^{\eta} (n_t + f_t) (\beta E_t P_{t+1} - q_t (h_t/h^*))^{\eta}$$
(55)

$$h_t = n_t + f_t (56)$$

$$J_t = u_t^H + \beta \pi_n (Z + E_t P_{t+1}) + \beta (1 - \pi_n) E_t J_{t+1}$$
 (57)

$$W_t^f = u_t^R + \pi_f \beta Z + (1 - \pi_f) \beta E_t W_{t+1}^f$$
 (58)

$$\bar{W}_t = \psi (J_t - P_t) + (1 - \psi) W_t^f$$
 (59)

$$r_t = m + P_t - \beta E_t P_{t+1} \tag{60}$$

This economy is comparable in several aspects to the model studied by Glaeser et al. (2010). An important difference, however, is that in their paper the alternative to living in the city yields a homogeneous payoff so that the elasticity of entry is effectively infinite. In response to a shock, this implies immediate entry of buyers until the price of housing adjusts to keep the value of entering constant. This tends to generate high variance in both prices and construction in response to income shocks. In our model there is a distribution of alternatives, so that as households enter, the critical outside value rises. This determines the flow of additional households into the city.

# 5 Calibration

# 5.1 Baseline parameterization

In this section we discuss the calibration scheme. For our baseline model, we suppose the matching function takes the Cobb-Douglas form:

$$M = \kappa B_t^{\delta} S_t^{1-\delta},\tag{61}$$

where  $\kappa > 0$  and  $\delta \in (0,1)$ . In this case,  $\epsilon(\omega) = \delta$  so that the share of the surplus accruing to each party in a housing transaction is constant and thus not sensitive to the tightness of the market.<sup>19</sup>

Table 4 gives the parameter values for our baseline calibration. Numbers above the line are set to match the indicated targets directly. Values below the line are set jointly so that the specified steady state values generated by the model match the given targets. For illustrative purposes, however, in the table we associate these parameters with a specific target for which it is particularly relevant. We define a period to equal one quarter. We set  $\beta$  to reflect an annual interest rate of 4% and  $\mu$  is chosen to match annual population growth during the 1990s.<sup>20</sup> We normalize  $\bar{y} = 1$ . Thus, present values and prices are all measured relative to the steady-state  $per\ capita$  income.

We set  $\pi_f$  to match the annual fraction of renters that move between counties, which is about 12% on average according to the Census Bureau. Similarly,  $\pi_n$  is set to match the annual fraction of home-owners that move between counties (12%) and  $\theta$  the fraction

<sup>&</sup>lt;sup>19</sup>In this case, the competitive search model is equivalent to a random search model with house prices determined by Nash Bargaining in which the Hosios condition is imposed, *i.e.* the buyer's bargaining weight is equal to  $\delta$ .

<sup>&</sup>lt;sup>20</sup>Population growth has slowed somewhat in recent years.

of owners that move but do not change counties (60%). Note that Dieleman, Clark and Deurloo (2000) estimate an overall housing turnover rate of 8% annually (see also Caplin and Leahy, 2008), which is consistent with our quarterly value of  $\pi_n + (1 - \pi_n)\theta \simeq 0.02$ . We set the continuation value received when moving to another city equal to the present value of being a renter in steady state:  $Z = \bar{u}^R/(1-\beta)$ .

An important parameter is the elasticity of new construction with respect to the price of housing,  $\eta$ . Most recent estimates of this using annual data, both at the national level and at the city levels, suggest that it is quite large. Early estimates range from about 1.6 to 5 at the national level (see Topel and Rosen (1988), Poterba (1991) and Blackley (1999)) and up to 25 for some cities. For our baseline calibration, we set the value equal to the median elasticity for the 45 cities studied by Green, Malpezzi and Mayo (2005), which they estimate to be  $\eta = 5$ . We consider, however, the sensitivity of our results to variations in this parameter.

A related parameter is the price elasticity of land supply,  $\xi$ . In our model, this effectively relates movements in the price of housing to the total stock of housing (as opposed to new construction). Saiz (2010) studies the relationship between house prices and the stock of housing based on a long difference estimation between 1970 and 2000 for 95 U.S. cities.<sup>21</sup> In particular, by instrumenting using new measures of regulatory restrictions and geographical constraints, he is able to infer city level price elasticities that vary due to natural and man-made land constraints. His supply elasticity estimates vary from 0.60 to 5.45 with a population–weighted average of 1.75 (2.5 unweighted). Therefore, we therefore set  $\xi = 1.75$  in our our baseline. Nevertheless, we also consider the sensitivity of our results to variations in this parameter.

The steady-state unit price of land  $\bar{q}$  is set so that the relative share of land in the price of housing is 30% (see Davis and Palumbo, 2008, and Saiz, 2010). The average price of a house is approximately 3.2 times annual income or 12.8 times quarterly income. This implies a ratio of the land price to income of  $0.3 \times 12.8 = 3.84$ .

<sup>&</sup>lt;sup>21</sup>In this sense, the estimated relationship picks up long term dynamics associated with  $\xi$ . In contrast, the estimates of Green, Malpezzi and Mayo (2005) relate to short run dynamics associated with  $\eta$ .

Table 4: Baseline Calibration Parameters: Steady State

Parameter	Value	Target
β	0.99	Annual real interest rate = $4\%$
$\mu$	0.003	Annual population growth rate = $1.2\%$
$\pi_f$	0.030	Annual mobility of renters = $12\%$
$\pi_n$	0.008	Annual mobility of owners = $3.2\%$
$\theta$	0.012	Fraction of moving owners that stay local = $60\%$
$\eta$	5	Median price elasticity of new construction = 5
ξ	1.75	Median price elasticity of land supply $= 1.75$
$\overline{q}$	3.84	Average land price-income ratio
$\psi$	0.43	Fraction of households that rent = $32\%$
m	0.0125	Average rent to average income ratio, $r^* = 0.137$
$z^H$	0.026	Zero net-of-maintenance depreciation
$\kappa$	0.76	Vacancy rate = $2\%$
δ	0.09	Months to sell = months to buy
$\phi$	0.038	$P^* = 12.8$

We choose the remaining parameters so that several key steady state statistics match their average counterparts in U.S. data. In particular, the value of  $\psi$  is calibrated so that the average fraction of households that rent in the steady-state is 32%:

$$\chi = \frac{b+f}{n+b+f} = 0.32. \tag{62}$$

The maintenance cost m is chosen so that the rent is 13.7% of median income. Note that the income of the average renter in the U.S. is less than half of that of the average owner, reflecting the fact that the characteristics of owners and renters differ systematically. On average, a renter in the U.S. allocates 24% of his after-tax income to rent (see Davis and Ortalo-Magne, 2011). Since in our model all agents are homogeneous, we target the ratio of rent to the median income of owners and renters, which is somewhat lower (see Head and Lloyd-Ellis, 2011 for details).

As described earlier, we assumed that the maintenance cost is just enough to offset depreciation. Let d denote the rate of depreciation. Then, under a simple optimal maintenance program, the implicit steady state flow utility derived from owning a house would be given by<sup>22</sup>

$$z^{H} = \left(1 + \frac{1 - \beta}{\beta d}\right) m. \tag{63}$$

<sup>&</sup>lt;sup>22</sup>Suppose  $z_t^H = z^H(Q_t)$  where  $Q_t$  denotes the quality of a house. The optimal maintenance program of a

Harding et al. (2007, p. 212) estimate the gross maintenance rate of depreciation for a house of median age in the U.S. to be about 3% annually.<sup>23</sup> It follows from (63) that  $z^h = 0.026$ . We also consider the sensitivity of our results to alternative values.

Given the other parameters of the model, those of the matching function,  $\kappa$  and  $\delta$ , jointly determine the steady-state values of the vacancy rate and market tightness. Average vacancy rates for the U.S. economy and by MSA are available from the Census Bureau's Housing Vacancy Survey (HVS). In our model, houses that are vacant in equilibrium are designated for sale. The HVS distinguishes the category "vacant units which are for sale only". In 2000, for example, this category constituted 1% of the overall housing stock. Since owned homes constituted approximately two-thirds of the housing stock, this corresponds to a home-owner vacancy rate of about 1.5%. Housing units that are in the category "vacant units for rent" actually consist, however, of vacant units offered for rent only and those offered both for rent and sale. In 2000, for example, houses both for rent and sale constituted a further 2.6% of the overall housing stock. In our model, vacant units are technically available for rent in the subsequent period, so it makes sense to include some of those vacant units offered for both rent and sale in our measure of vacancies. For this reason, we assume an additional 1% of the housing stock is vacant and for sale, so that v = 0.02. Again, we consider the sensitivity of our results to alternative values of v.

We assume that, in steady-state, the time taken to sell a house is equal to the time taken to buy, so that  $\omega^* = 1$ . This is approximately true in the long run according to surveys conducted by the National Association of Realtors (NAR). It follows that the overall vacancy rate is

$$v = \frac{h - b - f - n}{h} = \frac{b}{2b + f + n}. (64)$$

Using (41), this implies that

$$\gamma^* = \lambda^* = \left[\mu + \pi_n + \theta(1 - \pi_n)\right] \frac{n^*}{b^*} = \left[\mu + \pi_n + \theta(1 - \pi_n)\right] (1 - \chi) \frac{1 - v}{v}.$$
 (65)

home-owner can be espressed as

$$V(Q_t) = \max_{\{m_t, Q_t\}} z^H(Q_t) - m_t + \beta V(Q_{t+1})$$
  
s.t.  $Q_{t+1} = (1 - d)Q_t + m_t$ .

If  $z^{H}(Q_t)$  is approximately linear, then the steady-state solution to this program implies (63).

<sup>&</sup>lt;sup>23</sup>The resulting actual depreciation rate is rather less than 1% precisely because maintenance is undertaken.

<sup>&</sup>lt;sup>24</sup>This number is close to the average over the period 1980-2008. However, more recently homeowner vacancy rates have exceeded 2.5%.

<sup>&</sup>lt;sup>25</sup>Again, since rental units constitute about a third of the housing stock, this corresponds to a rental vacancy rate of about 8%.

Given the values of  $\mu$ ,  $\theta$  and  $\pi_n$  from Table 4 and our targets for  $\chi$  and v, the implied value is  $\gamma = 0.76$ , leading to an average time for a house to be on the market of just under 4 months. This may seem somewhat high given that according to the NAR, the time taken to sell a typical house is about 2 months.<sup>26</sup> Their estimate of "time on the market" is, however, somewhat misleading because houses are sometimes strategically de-listed and quickly relisted in order to reset the "days on market" field in the MLS listing. In their detailed analysis of the housing market in 34 Cook county (Illinois) suburbs over the period 1992-2002, Levitt and Syverson (2008) compute time-to-sale by "summing across all of a house's listing periods that are separated by fewer than 180 days." They estimate that the average time on the market for a house that eventually sells is 94 days (3.07 months). Moreover, in their sample of 127,000 houses, 22% of houses put up for sale never sell. In less active markets, it is likely that the time on the market is even longer.

Given the other parameters,  $\phi$  is chosen so that the price of a house is 3.2 times annual income or 12.8 times quarterly income. Note that the value of  $\delta$  required to hit these targets, given the other parameters, implies that the elasticity of matching with respect to the number of buyers is low relative to that with respect to the number of sellers. With competitive search, this also implies that over 90% of the surplus from housing transactions goes to the seller.<sup>27</sup>

The dynamics of the model depend crucially on the shape of  $G(\cdot)$  in the vicinity of  $\varepsilon^c$  because it determines the responsiveness of new entrants to changes in the value of search.<sup>28</sup> In the steady state, however, the only variable that depends on  $G(\cdot)$  is the measure of searching households per capita,  $b^*$ . This is not something that is likely to be directly observable and so the parameters determining the relevant characteristics of  $G(\cdot)$  are not possible to identify in this way. Our approach is to use our estimate of the relative standard deviation of population growth in response to income shocks from Table 2 to calibrate the elasticity of  $G(\cdot)$  evaluated at  $\varepsilon^c$ :

$$\alpha = \frac{\varepsilon^c G'(\varepsilon^c)}{G(\varepsilon^c)}.$$
 (66)

In our baseline calibration below, we set  $\alpha = 6.2$ .

For the economy with no search, the parameters are chosen as in Table 4 except that we

<sup>&</sup>lt;sup>26</sup>There are varying estimates of the time to buy and the time to sell. Diaz and Jerez (2008) use 2 months based on a report from the National Association of Realtors. Piazzesi and Schneider (2009) suggest using 6 months. Anglin and Arnott (1999) report estimates of up to 4 months.

<sup>&</sup>lt;sup>27</sup>With random search,  $\delta$  could be higher without affecting our result much.

<sup>&</sup>lt;sup>28</sup>Since we log-linearize the model around the steady state, it is not necessary to specify the entire function  $G(\cdot)$ .

re-set  $\psi$ , m,  $z^H$  and  $\phi$  so that the steady-state again matches the relevant targets.<sup>29</sup> Without search, in the deterministic steady state, the house price is given by

$$P^* = \frac{\mu^{\frac{1}{\eta}}}{\beta \phi} + \frac{\bar{q}}{\beta}.\tag{67}$$

We use (67) to derive the value of  $\phi$  such that  $P^* = 12.8$ .

# 6 Equilibrium Dynamics

In order to study the dynamics of the model, we linearize the dynamic systems for calibrated versions of both the baseline and no search economies in neighborhoods of their respective deterministic steady states. For our calibrations, the resulting systems of first-order linear difference equations for both economies satisfy the conditions for saddle-path stability. We solve numerically for the implied local dynamics driven by stochastic movements in  $y_t$  using a Generalized Schur decomposition due to Klein (2000).

## 6.1 A simple first-order auto-regressive income process

To begin with, we assume that the process followed by the log of non-construction income,  $\ln y_t$ , is a simple AR(1) process with persistence parameter  $\rho = 0.98$  and innovation standard deviation  $\sigma_{\varepsilon} = 0.01$ . We use this example to illustrate that the model's qualitative dynamics are not driven by the hump-shaped dynamics of income observed in the data. That is, we show that without time consuming search and matching, persistence in earnings of this form will not translate into positively auto-correlated growth in house prices. Rather, the impulse response of the house price simply inherits the shape of that for local general labour earnings, which may be seen as reflecting local housing dividends. In our calibrated search economy, this is not generally the case. Rather, the theory produces momentum in house prices even in cases when housing dividends exhibit none.

The implied impulse response functions (IRF's) following a shock to local income are depicted in Figures 3 and 4. The four panels of Figure 3 depict the IRFs for income, house prices, construction rates and population growth rates relative to trend for the economies with and without search. In both economies, the shock to income induces entry and population growth rises. Moreover, the responses of city population growth in the two cases are both qualitatively and quantitatively similar. The responses of both house prices and the

<sup>&</sup>lt;sup>29</sup>Obviously, the parameters of the matching function  $\kappa$  and  $\delta$  are not relevant in this case.

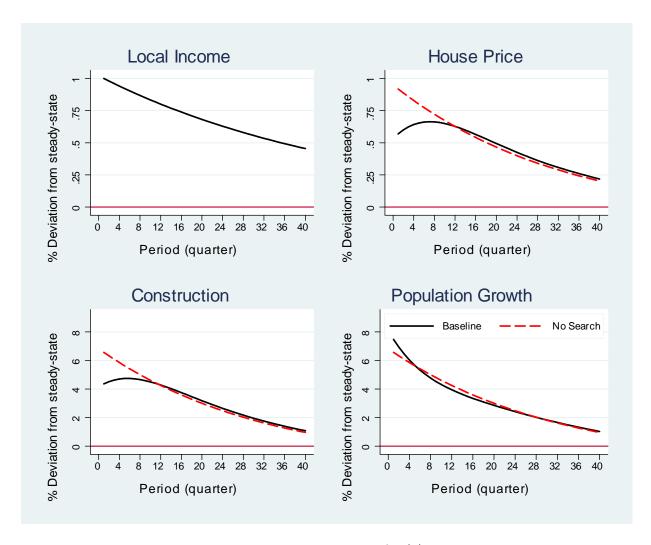


Figure 3: Impulse Responses, AR(1) Income

construction rate, however, differ qualitatively across the two economics. The search model generates momentum in both prices and construction which is qualitatively similar to the dynamics of the empirical model illustrated in Figure 1.<sup>30</sup> In contrast, the economy with no search generates no momentum in either prices or construction rates, in spite of the fact that it does generate substantial momentum in the housing stock.

The force which generates serial correlation in both house price appreciation and the construction rate in the economy with search is the *illiquidity* of housing. To see this, consider Figure 4, which depicts in its four panels market tightness,  $\omega$ , and the matching

<sup>&</sup>lt;sup>30</sup>These figures are not strictly comparable. The data is annual whereas the IRFs here relate to quarterly observations.

probability for house sellers,  $\gamma$  (labelled "absorption"), in the search economy and house sales in the no search economy, vacancies (for sale), for the search economy, and rent,  $r_t$ , for both economies.

Initially, an increase in the value of living in the city (due here to the income shock) generates an immediate increase in search activity as households enter and fraction  $\psi$  of the new entrants begin searching for a house. Ignoring, for now, any response of the measure of vacant houses for sale, the ratio of buyers to sellers (*i.e.* tightness) increases, reducing the rate at which buyers find homes through the matching process. The price of a house reflects in part its future resale value (as home-owners expect to eventually sell the house when they leave the city). Thus, an increase in tightness, by raising the per-period probability of sale, increases both the value of a vacant house, and the transaction price. Because newly-entering buyers are not all immediately matched with sellers, and because entry is persistent (owing to the persistence of the income shock), unmatched buyers "build up" in the market over time, generating future increases in both tightness and the rate at which houses sell in the search economy (see the two upper panels of Figure 4). As houses become more liquid (in this sense) over time, their value increases and transaction prices increase further as well, resulting in persistent house price appreciation in response to the income shock.

Both the overall supply of housing and the allocation of houses between the rental market and vacancies for sale, respond to the income shock in a way affected by movements in liquidity. New entrants to the city require housing immediately. This causes the shifting of vacant houses into the rental market because the overall stock of housing units cannot respond instantaneously. Even if the equilibrium rent does not initially rise, or rises only slightly, owners of vacant houses are compensated for the temporary renting of their housing units by the return on houses associated with expected future increases in house sale prices. An increased relative supply of rental housing keeps the rental rate from rising too rapidly and reinforces the continued entry of buyers which drives the subsequent price appreciation. This self-fulfilling effect tends to support the underlying momentum in house prices. The increased value of a vacant house induces developers to build houses and leads to an increase in the housing stock. Moreover, momentum in the construction rate is generated through the same mechanism as momentum in the housing price; persistent growth of market tightness and thus reductions in the time required to sell a house to a resident owner.

Eventually, as *per capita* income reverts to its steady-state level, entry slows and the population growth rate returns to its trend. Increased construction lowers market tightness and causes both the value of a vacant house and the transaction price to return to their

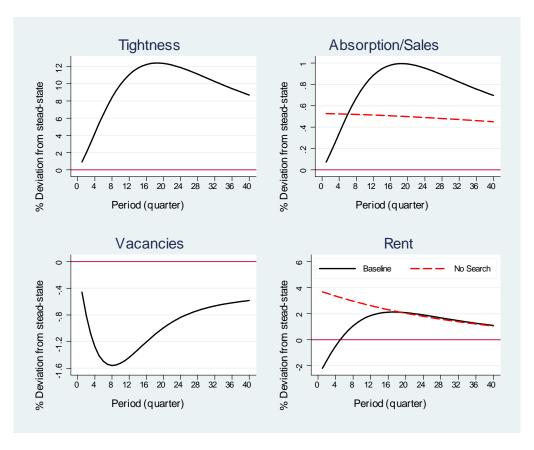


Figure 4: Impulse Responses: AR(1) Income

steady-state values.

Momentum in both the construction rate and the house price is thus generated by movements in the liquidity of housing associated with the population flows that are caused by the income shock. The economy with no search exhibits no such liquidity effect and thus has very different dynamics. In that economy, increased entry simply leads to a higher house price and increases in both the construction rate and house sales. All of these variables track the population growth rate and thus exhibit no momentum. In addition, rent (which here is paid only by those agents who do not derive utility from owning) behaves very differently in the two economies. Because in the no-search economy there are no vacant houses that can be shifted into the rental market, new entrants who do not want to buy houses bid up the rent immediately, which is necessary to induce developers to produce new rental housing. In the baseline model, the rent actually falls initially because the anticipated growth in prices temporarily induces houses that were previously vacant-for-sale to be supplied to the rental

market. Eventually, however, the growing demand counteracts this effect causing rents to rise.

## 6.2 An empirically relevant income process

In the previous section we used our calibrated model to illustrate the role played by search frictions in generating momentum in house prices. We now consider the quantitative implications of our theory by comparing the model's output under an empirically relevant process for local earnings with the stylized facts reported in Section 2.

Mechanically, this entails replacing the arbitrary AR(1) process for  $\ln y_t$  considered in the previous section with one based on our empirical findings in Section 2. This is complicated, however, by the mismatch between the frequency of available city-level income data and the period length assumed in our calibrated model. The former, which was used in our data analysis, is available annually, whereas the baseline calibration assumes that each period is a quarter. While the period length could be increased to one year in the model, this would be restrictive as it would require that houses for sale remain vacant for at least one year, which is clearly counter-factual.

Instead, we derive a quarterly process for income that shares key properties at annual frequencies with the process estimated in our panel VAR in Section 2. For a full description of procedure by which we construct this process, see the appendix. In Table 5 we report selected moments for both the estimated and constructed processes; specifically, the variance and autocorrelation of the growth in income at annual frequencies for the first four years.<sup>31</sup> Using the constructed process and the linearized model, we generate sample paths for the key variables of the model and use these to construct "annual" time series for the economy. Moments for these series, along with the corresponding moments for the U.S. economy, are presented in Tables 6 and 7.<sup>32</sup>

<sup>&</sup>lt;sup>31</sup>Clearly, these are not identical. The artificial process generates a somewhat flatter impulse response for the level of income than observed in the data.

 $<sup>^{32}</sup>$ Along a small fraction of sample paths, the buyer's matching probability,  $\lambda$ , exceeded unity for several periods. To address this we ran the simulation both with no constraints and with the constraint that  $\lambda \leq 1$ . This made no difference to our results.

Table 5: Calibration of Earnings Process

	$\sigma_y$	$\rho_1$	$ ho_2$	$\rho_3$	$ ho_4$
Data	0.02	0.25	0.02	-0.05	-0.06
Model	0.02	0.21	-0.03	-0.03	-0.03

First consider the moments in Table 6, which contains both the standard deviations of house price appreciation, housing stock growth, and population growth relative to that of local per capita income growth, and the correlations of those variables with local income growth. The first column of the table reports the numbers from our data analysis in Section 2. The second column reports the results for the linear asset pricing model (LAPM) discussed also in Section 2 and the third column reports the results for the model with housing construction but no search. The last columns contain results for our search model under the calibration presented in Section 5. Table 7 contains the first four autocorrelation coefficients for price appreciation, housing growth and population growth for each of these cases.

The calibrated search model generates price volatility that is about half what is observed in the data. This is less than that generated by the version with no search, but considerably more than the basic LAPM. On the other hand, the search model is able to account for a considerable amount of serial correlation in price appreciation, which the other models cannot. The model with search also does relatively well in terms of the rankings of volatility, correlation with income growth and serial correlation for the three variables. That is, it is consistent with the observation that price appreciation is the most volatile and most correlated with income growth, followed by population growth and then by construction rates, whereas for persistence they are ranked in the opposite order.<sup>33</sup> Quantitatively, however, the model with search substantially overstates the volatility of construction, understates the correlation of population growth with income growth and overstates the persistence of population growth rates.

<sup>&</sup>lt;sup>33</sup>For the model with no search, population growth and the construction rate are equal, since there are no vacancies.

Table 6: Volatilities and Co-movements: Calibrated Earnings Shocks

Moment	US Cities	LAPM	No search	Baseline
$\sigma_p/\sigma_y$	1.35	0.35	0.90	0.67
$\sigma_h/\sigma_y$	0.11	_	0.18	0.17
$\sigma_n/\sigma_y$	0.18	_	0.18	0.18
$\sigma_{py}$	0.80	0.94	0.91	0.96
$\sigma_{hy}$	0.46	_	0.38	0.35
$\sigma_{ny}$	0.71	_	0.38	0.37

Table 7: Autocorrelations: Calibrated Earnings Shocks

Moment	US Cities	LAPM	No search	Baseline
$\rho_1^p$	0.71	-0.01	-0.02	0.22
$\begin{array}{c c} \rho_1^p \\ \rho_2^p \\ \rho_3^p \\ \rho_4^p \end{array}$	0.30	-0.03	-0.04	0.08
$ ho_3^p$	0.03	-0.02	-0.04	0.0
$\rho_4^p$	-0.11	-0.02	-0.04	-0.05
$\rho_1^h$	0.89	_	0.89	0.94
$ ho_2^h$	0.66	_	0.79	0.85
$\begin{array}{c c} \rho_2^h \\ \rho_3^h \\ \rho_4^h \end{array}$	0.41	_	0.70	0.76
$ ho_4^h$	0.21	_	0.61	0.67
$\rho_1^n$	0.71	_	0.89	0.88
$\rho_2^n \\ \rho_3^n$	0.45	_	0.79	0.78
$\rho_3^n$	0.25	_	0.70	0.70
$\rho_4^n$	0.21	_	0.61	0.62

In summary, it is clear that the baseline calibration understates both the relative volatility of house prices and its serial correlation observed in the data. Search, however, seems to be necessary to generate any price momentum at all. Also while the search model does not generate as much price volatility as that without search, the price of houses traded through a time consuming matching process exhibit considerably more it exhibits considerably more volatility than would the price of a simple claim to local income traded in a frictionless market.

### 6.3 Alternative Parameterizations

We now depart from the baseline calibration of Section 5 and consider the sensitivity of our results to changes in the values of key parameters. Table 8 consider the implications of alternative choices of the specified parameters for the relative volatility and the first-order autocorrelation of price appreciation. In each case the other targets listed in Table 4 remain

fixed. This implies that some other parameters (i.e.  $\psi$ , m,  $z^h$ ,  $\delta$ ,  $\kappa$ ,  $\nu$  and  $\alpha$ ) must also be adjusted to match these targets. In this sense each case in Table 8 represents an alternative calibration of the model.

Table 8: Volatility and Persistence of Price Appreciation: Sensitivity Results

Moment	Baseline		New construction		Land	supply	Entry	
	Calibration		supply elasticity		elast	sicity	elasticity	
			$\eta = 2$	$\eta = 20$	$\xi = 0.5$	$\xi = 5$	$\alpha = 5$	$\alpha = 20$
$\sigma_p/\sigma_y$	0.67		1.71	0.16	0.74	0.65	0.51	1.52
$\rho_1^p$	0.22		0.07	0.48	0.19	0.24	0.33	0.05
	Vacancy		Homeowner exit		Housing		Matching elasticity	
Moment	Ra	ate	probability		Utility		(random matching)	
	v = .01	v = .03	$\pi_n = .004$	$\pi_n = .012$	$z^{H} = .01$	$z^{H} = .04$	$\delta = .01$	$\delta = .50$
$\sigma_p/\sigma_y$	0.83	0.53	0.81	0.58	0.46	0.72	0.74	0.41
$ ho_1^p$	0.11	0.33	0.08	0.35	0.58	0.15	0.15	0.40

Perhaps not surprisingly, increasing either the elasticity of new construction supply or the elasticity of land supply results in a decrease in price volatility and an increase in momentum. We have chosen the alternative values of these parameters to be at the extremes of the range of typical estimates. As may be seen within this range, these moments are much more sensitive to new construction supply elasticity than land supply elasticity.<sup>34</sup> Indeed for low values of  $\eta$  we obtain price volatilities that equal or exceed those observed in the data. This, however, comes at the expense of a substantial reduction in momentum, although it is still positive.

The trade-off between volatility and momentum may be seen for all the parameter changes considered in Table 8. Directly increasing the elasticity of entry, for example, implies a greater responsiveness of new entrants to current market conditions in the city and correspondingly less of a lag in entry. Consequently, prices become increasingly volatile and price momentum declines. Of course, when we adjust this parameter we not longer match the relative volatility of population growth.

Targeting a higher steady state vacancy rate of 3% implies a less effective matching process in the housing market,  $\kappa$ . As a result, it takes longer for households to find houses and market tightness grows to a higher level as buyers exit the pool of searchers more slowly. This tends to generate more momentum and less volatility in prices. When homeowners are

<sup>&</sup>lt;sup>34</sup>This is true even though the range of labour supply elasticities considered is proportionately larger than that of land supply elasticities.

more likely to exit the city (i.e. when  $\pi_n$  increases), they put more weight on future market conditions. If these conditions are expected to improve due to gradual entry, the persistence of current price growth increases in anticipation.

When we divorce housing utility,  $z^H$ , from the cost of maintenance, so that (63) no longer holds, we find that decreasing  $z^H$  raises price momentum and lowers volatility. The reason is that, in order to maintain the targets for the steady-state house price and vacancy rate, the decrease in  $z^H$  necessitates a reduction in the buyers' share of the surplus in house transactions. As a consequence, the house price becomes increasingly sensitive to future market conditions and the slow increase in the likelihood of sale translates into more gradual price growth.

Finally, we consider a case where we no longer impose that the buyers share of the surplus, s, must equal the elasticity of the matching function with respect to the number of buyers,  $\delta$ . Specifically we fix s=0.09, as in the baseline calibration, and allow  $\delta$  to vary. Doing this effectively drops the assumption of competitive search and changes the equilibrium to one involving (inefficient) random search. As may be seen, for all values of  $\delta$  the model continues to exhibit substantial price momentum. Thus competitive search  $per\ se$  is not necessary to generate serial correlation in price appreciation. As  $\delta$  becomes large, however, the relative volatility of prices starts to become quite low, falling below that for a simple claim to local income in the linear asset pricing model for  $\delta > 0.5$ .

## 7 Robustness

We now consider three alternative environments to assess further the robustness of our findings. Qualitatively, none of these affect our results significantly and thus we find our results to be very robust. Quantitatively, we find that our findings with respect to both volatility and the co-movements among the variables that we have considered are very robust. With regard to the degree of price momentum, however, our results are to some extent sensitive to fluctuations in the share of the surplus accruing to buyers and sellers in housing transactions.

## 7.1 Construction labour supply

In the baseline model we assumed a particular formulation for unit construction costs. Here we show that this formulation can be derived from more primitive assumptions on construc-

tion labour supply and housing production. Although, this changes the utility flow of houses somewhat, it does not affect the basic structure of the model.

Suppose that each household is endowed with two types of labour: general labour and construction labour. At each date t, a household supplies one unit of general labour inelastically and  $l_t$  units of construction labour endogenously, taking the construction wage  $w_t$  as given.<sup>35</sup> At date t, preferences over consumption  $c_t$ , construction labour  $l_t$  and housing  $z_t$  are given by:

$$U_t(c_t, l_t, z_t) = c_t - v(l_t) + z_t, (68)$$

where

$$v(l_t) = \frac{l_t^{1+\frac{1}{\eta}}}{\zeta^{\frac{1}{\eta}} \left(1 + \frac{1}{\eta}\right)} \tag{69}$$

and  $\eta$  and  $\zeta$  are constants. In equilibrium, the household construction labor supply function is

$$l(w_t) = \zeta w_t^{\eta} \tag{70}$$

Each new house requires one unit of land (as before) and  $1/\nu$  units of construction labour. The stock of houses thus evolves over time according to

$$H_{t+1} - H_t = \varpi L_t \tag{71}$$

and the labor market clears so that

$$(N_t + B_t + F_t)l_t = L_t (72)$$

If  $\phi = \zeta^{\frac{1}{\eta}} \varpi^{1+\frac{1}{\eta}}$ , it follows that the unit cost of construction is given by (4). It is then straightforward to show that all of the above results go through except that now

$$u_t^R = y_t + x(w_t) - r_t$$
 and  $u_t^H = y_t + x(w_t) + z^H - m$  (73)

where

$$x(w_t) = w_t l(w_t) - v(l(w_t)) = \frac{\zeta w_t^{1+\eta}}{1+\eta}.$$
 (74)

Thus the only difference to the overall model is that household utility has an extra term relating to construction wages. For reasonable parameter assumptions, however, the addition

<sup>&</sup>lt;sup>35</sup>It is straightforward to further generalize preferences so that general labour is also supplied endogenously. However, this would make no difference to our analysis, provided that the disutility of supplying each type of labour is separable.

of this term again has negligible effects on our results.<sup>36</sup> This is because construction income accounts for a very small proportion of overall household income.

#### 7.2 Mismatched owners remain in their houses

In our basic model, we assumed that mismatched owners put their houses up for sale immediately and become renters. Nevertheless, we have the following proposition:

**Proposition 3.** In equilibrium, mismatched owners are indifferent between the following two arrangements:

- (1) putting up their house for sale or rent immediately and renting while searching;
- (2) remaining in their current house while searching, then putting their vacant house up for sale once they are matched with a new one.

Suppose instead that they remain in their houses until they find a new house, then put their vacant house up for sale. Since owners who become mismatched are indifferent between the two alternatives, the value functions remain unchanged. Let  $\tilde{n}_t$  denote these mismatched owners. Then the flows of households between states is now described by (33) and

$$(1+\mu)\tilde{n}_t = \theta(1-\pi_n)n_{t-1} + [1-\lambda(\omega_{t-1})]\,\tilde{n}_{t-1} \tag{75}$$

$$(1+\mu)b_t = \psi \mu G(\bar{W}_t) + [1 - \lambda(\omega_{t-1})] b_{t-1}$$
(76)

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})(b_{t-1}+\tilde{n}_{t-1})$$
(77)

Market tightness is given by

$$\omega_t = \frac{b_t + \tilde{n}_t}{h_t - b_t - \tilde{n}_t - f_t - n_t} \tag{78}$$

and the housing stock evolves according to

$$(1+\mu)h_{t+1} = h_t + \phi^{\eta} \left( n_t + \tilde{n}_t + b_t + f_t \right) \left( \beta E_t V_{t+1} - \bar{q} \right)^{\eta}. \tag{79}$$

When we change the model in this way and retain the same calibration targets as before we find that our results hardly change. The steady state probabilities of buying and selling increase somewhat but the effects on the model's dynamics are negligible.

<sup>&</sup>lt;sup>36</sup>We set  $\varpi = .00025$ , implying that it takes 4000 man hours to build a house. Given our target values for  $\phi$  and  $\eta$ , we set  $\zeta$  to match the steady-state house price.

### 7.3 Generalized urn-ball matching

We now consider an alternative matching function for which the equilibrium shares of the surplus received by the buyers and sellers are not constant. Specifically, consider

$$\mathcal{M}(B,S) = S\chi(1 - e^{-\tau \frac{B}{S}}). \tag{80}$$

If  $\tau=1$ , the matching probabilities are equivalent to the "urn-ball" matching process assumed by Diaz and Jerez (2010). Here we consider a somewhat more general form in order to calibrate the model to the same targets as for our baseline calibration above. This generalization could be motivated along the lines of Albrecht, Gauthier, and Vroman (2003), where  $\tau$  denotes the average number of applications to purchase made per period and  $\chi$  indexes the effort required to process each application. Given the other parameters of our baseline calibration, the matching function parameter values needed to achieve the same targets as above are  $\chi=0.792$  and  $\tau=3.295$ .

The surplus accruing to the buyer for this matching function is

$$s(\omega) = \epsilon(\omega) = \frac{\tau\omega}{e^{\tau\omega} - 1},\tag{81}$$

which is decreasing in market tightness  $\omega$ . That is, as the ratio of buyers to sellers increases, the share received by buyers falls.

Figure 5 illustrates the effect of a shock to general earnings on house prices and market tightness in both the baseline economy and in that with generalized urn-ball matching. Clearly, the form of the matching function has a significant effect on price momentum, and this can be traced to the effect of an increase in local earnings on the initial response of prices and the extent of entry. In the urn-ball matching case, the share of the surplus received by the buyer falls as tightness rises. Thus, the initial price increase in prices is greater, and this discourages entry as can be seen in the response of tightness. Since tightness responds less, prices peak earlier and return to their steady-state level faster than with Cobb-Douglas matching. Note that while the form of the matching function has a significant effect on momentum, it has little effect on either volatility or the co-movements among house prices, local earnings, tightness, and construction rates.

# 8 Concluding Remarks

This paper makes two main contributions. First, we provide a parsimonious characterization of the impact of relative income shocks across U.S. cities on the short run dynamics of average

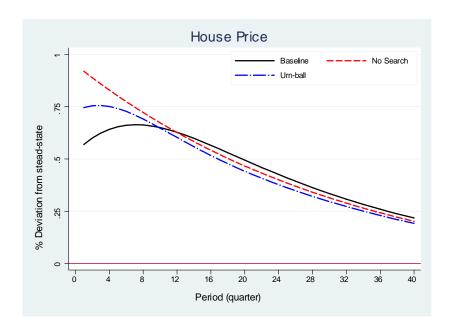


Figure 5: Response to an AR(1) Income Shock: Alternative Matching Functions

house prices, construction and population growth. Specifically we estimate a panel VAR with city level fixed effects and use it to isolate the impact of relative income shocks by making structural assumptions consistent with our theory. In particular, our estimates are consistent with previous findings (e.g. Glaeser et al. 2010) that house price appreciation exhibits substantial serial correlation in the short term and long-run mean reversion. Moreover, we find that the volatility of house price movements that occur in response to income shocks is high relative to the volatility of local incomes. City level population growth responds quite quickly in response to income shocks, whereas construction rates tend to be quite sluggish initially. Our interpretation of this is that although construction is quite elastic, net entry of the population into cities seems to be even more so.

Our second main contribution is to build a model that helps to understand these movements in house prices, construction and population. To do this we introduce competitive search into a dynamic model of housing markets with endogenous entry and construction. Three key features of the model are (1) that it takes time for potential buyers to match with a house they want, (2) that home buyers forsee that they will eventually sell, and (3) that unoccupied housing can be rented temporarily to new entrants who are searching for a home to own. In response to a persistent, relative increase in local income, entry into the market rises persistently. Because it takes time to match and new houses take time to build, the ratio of potential buyers to houses for sale rise slowly over time. Initially, therefore, the liquidity of houses is expected to rise over time. Since this causes the re-sale value of housing to grow and because current transactions prices reflect partly the value of houses to their owners, current transactions prices grow in anticipation. Higher home values induce increased construction so that the buyer–seller ratio, and hence housing liquidity, eventually fall. After initially growing for several periods, therefore, current house prices fall in anticipation. The fact that households can rent while searching implies that they can obtain the relative gains from living in the city without buying a house. Although an increase in local income therefore also increases the demand for rental housing, owners of unoccupied housing have an incentive to rent out their houses and delay selling them, if they expect prices to rise. Consequently entry continues to grow and is not immediately stemmed by a sharp rise in rents.

We calibrate the model so that its steady-state matches several long-run averages in U.S. data and assess the extent to which it can account quantitatively for the estimated moments in the data. We find that a calibrated version of our model captures qualitatively the observed movements in both the house price and construction quite well. In particular, the model generates serial correlation in price appreciation and relatively high volatility in the long run. Population growth responds quite rapidly in response to relative income shocks, whereas as the response of construction rates is more sluggish. Generally, however, our calibrated model understates both momentum and volatility quantitatively. Moreover, for a wide range of parameters, we find a tension between momentum and volatility in the model: the theory cannot simultaneously generate both the high persistence of price appreciation and long run volatility observed in cross-city data.

We believe that the model lends itself quite naturally to a number of extensions and applications. In the version developed here, households and houses are homogeneous. Allowing for heterogeneity along either or both dimensions would allow us to study the role of search frictions in driving the distribution of house prices both in steady state and in response to shocks. Another application would be to study the ability of the model to account for housing price dynamics on a city-by-city basis, using city level estimates of supply elasticities for new construction and land. Recently, Genesove and Han (2011) find evidence of increasing returns to matching in housing markets. This may also affect housing price dynamics in interesting ways.<sup>37</sup> We leave these issues for future research.

<sup>&</sup>lt;sup>37</sup>In a version of our model with increasing returns and random search, the price dynamics don't change much. However, with competitive search there may be more significant effects.

## 9 Appendix A: Data appendix

This appendix provides details on data sources, definitions and calculations. Our unit of observation is a core-based statistical area (metropolitan statistical area or MSA). We use the 2006 MSA definitions. Our sample consists of 98 MSAs.

**Populations:** City populations are taken from the REIS. Throughout we assume that city populations are proportional to the number of households.

Local incomes: We define local incomes as the total income for all sources less construction earnings. Our MSA level data are from the Regional Economic Information System (REIS) compiled by the Bureau of Economic Analysis (BEA, Table CA34). We subtract construction earnings because they are endogenous in our model. However, whether or not we include them make little difference to the empirical results. We could have defined local income as non-construction earnings. We chose not to because we would expect the incentives to move to a given location to depend on total income. However, when we estimate the panel VAR using this definition instead, the results were qualitatively similar.

House prices: Following van Niewenburgh and Weil (2010), we form a time series of home prices for each city by combining level information from the 2000 Census with time series information from the FHFA. From the 2000 Census, we use nominal home values for the median single-family home. From the FHFA we use the Home Price Index (HPI) from 1980 to 2008. The HPI is a repeat-sales index for single family properties purchased or refinanced with a mortgage below the conforming loan limit. As a repeat-sale index, it is a constant quality house price index. In contrast to Van Niewenburgh and Weil (2010), we combine prices for MSA divisions into those for MSAs by using population—weighted averages of the division level prices. We need to do this because the housing stock data (described below) can only be constructed using permits at the MSA level.

Housing Stocks: We form a time series for housing stocks for each city by combining information from the 2000 Census with times series information from the U.S. Department of Housing and Urban Development (HUD). From the 2000 Census, we use the estimated number of housing units. This data was only available at the county level, so we summed across the counties within the relevant MSAs. From HUD we used annual permits issued for each city from 1980 to 2008. According to the U.S. Census Bureau, approximately 97.5% of permits issued each year translate into housing starts, 96% of which are completed. We therefore constructed housing stocks  $H_t$  according to  $H_{t+1} = H_t + 0.936 \times Permits_t$ .

## 10 Appendix B: Empirical Results

#### 10.1 Full Panel VAR Results

Table B1 documents the parameter estimates for the baseline estimation of the Panel VAR discussed in Section 2. Estimating a panel VAR raises a number of econometric issues. A basic problem in dynamic panel data models with fixed effects is that the lagged dependent variables are, by construction, correlated with the individual effects. This renders the least squares estimator biased and inconsistent. Consistent estimation requires some transformation to eliminate fixed effects. A within transformation wipes out the individual effects by taking deviations from sample means, but the resulting within-group estimator is inconsistent when the number of panels becomes large for a given time-dimension (Nickell, 1981).

Table B1: System GMM (2SLS) estimates

			(					
	Y		P		$g^H$		$g^N$	
Y(-1)	1.22	(0.05)	0.45	(0.10)	0.01	(0.01)	0.07	(0.03)
P(-1)	-0.01	(0.01)	1.25	(0.05)	0.01	(0.00)	-0.02	(0.00)
$g^H(-1)$	0.57	(0.14)	1.19	(0.27)	0.74	(0.05)	0.40	(0.11)
$g^N(-1)$	-0.19	(0.18)	0.11	(0.26)	0.11	(0.05)	0.26	(0.19)
Y(-2)	-0.30	(0.05)	-0.60	(0.08)	-0.03	(0.01)	-0.07	(0.03)
P(-2)	0.01	(0.01)	-0.31	(0.06)	-0.01	(0.00)	0.02	(0.00)
$g^H(-2)$	-0.53	(0.11)	-0.88	(0.20)	-0.14	(0.03)	-0.22	(0.05)
$g^N(-2)$	0.13	(0.07)	0.59	(0.13)	0.05	(0.02)	0.17	(0.17)

Standard errors in parenthesis

Given this inconsistency, the literature focuses mainly on a first-difference transformation to eliminate the individual effect while handling the remaining correlation with the (transformed) error term using instrumental variables and GMM estimators (e.g. Arellano and Bond, 1991). However, the Arellano-Bond estimator is known to suffer from a weak instruments problem when the relevant time series are highly persistent, as they are in our case. As Blundell and Bond (1998) demonstrate this can result in large finite-sample biases. In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is consistent when the number of panels becomes large for a given time-dimension and is less likely to suffer from the weak instruments problem. Another reason for focusing on this estimator is that its properties

are fairly well understood and it has been studied in the context of panel VARs by Binder, Hsiao and Pesaren (2005).

### 10.2 Analysis of Regional Sub-samples

Table B2: Moments from system GMM estimation for regional sub-samples – income shock

	Region Relative Corr. with		Corr. with	Autocorrelation				
		Std. Dev.	Inc. Growth	year 1	year 2	year 3	year 4	
Income	Coastal	1.0000	1.0000	0.2433	-0.0147	-0.0813	-0.0843	
Growth	Interior	1.0000	1.0000	0.1405	-0.0305	-0.0660	-0.0683	
	Sunbelt	1.0000	1.0000	0.3263	0.0664	-0.0491	-0.0883	
Price	Coastal	2.1073	0.7783	0.6939	0.2314	-0.0361	-0.1413	
Appreciation	Interior	0.9218	0.8014	0.6152	0.2526	0.0420	-0.0679	
	Sunbelt	1.1937	0.7067	0.8168	0.4756	0.1820	-0.0144	
Construction	Coastal	0.0429	0.5512	0.7945	0.3724	-0.0192	-0.2572	
Rate	Interior	0.1453	0.4382	0.9141	0.7353	0.5629	0.4320	
	Sunbelt	0.1885	0.5229	0.8904	0.6431	0.3676	0.1287	
Population	Coastal	0.0987	0.4268	0.8082	0.6706	0.6278	0.6289	
Growth	Interior	0.1453	0.5258	0.7480	0.6036	0.4918	0.4334	
	Sunbelt	0.3194	0.8402	0.6704	0.3897	0.1535	-0.0031	

We now consider the results of estimating the panel VAR model on various sub-samples of both cities and time periods. Table B2 provides key moments for local earnings, house prices, construction rates, and ratios of housing stocks to city population based on shocks to local income in the panel VAR for each of the three sub-samples. Several, key observations are apparent. The standard deviation of house prices is roughly equal to that of local earnings in the full sample. Both construction rates and housing stock-population ratios are much less volatile than local earnings. House prices, construction rates and housing stock-population ratios are all strongly positively correlated with local earnings, although for inland cities these correlations are somewhat weaker. The higher and more persistent autocorrelation in both house price appreciation and population growth relative to earnings growth can also be observed in all the sub-samples.

Certain features of these moments and impulse response functions conform to a priori expectations regarding population and price movements. In particular, coastal cities typically have more inelastic land supply than sunbelt cities. Accordingly, in response to demand



Figure 6: Impulse responses from panel VAR for sub-regions: income shock

shocks, price volatility tends to be higher and population and construction volatility tend to be lower in the coastal cities.

#### 10.3 Alternative estimators

There are several potential problems with using the system GMM estimator for a sample with the dimensions considered here. While it is usually thought to be suitable for typical microeconometric panels, with only a few waves but a large number of individuals, here we have moderately large number of cities and a moderately long time series. Moreover, GMM estimators tend to have a larger standard error compared to the within-group estimator and may suffer from a finite sample bias due to weak instruments. Here we address these

issues by comparing our estimates with those of three alternative estimators: OLS with no fixed effects, a standard within-groups estimator (WGE) and an "orthogonal to backward mean" within-group estimator (OBM), inspired by Everaert (2011). Although the WGE is inconsistent as the number of panels becomes large, this should be less of a problem given the dimensions of our sample. While the OBM is also inconsistent, this inconsistency is expected to be negligibly small for moderately long time periods.

For the sake of brevity we do not report here all of the estimation results for each estimator. Instead Table B3 reports only the sum of the coefficients on the lagged dependent variables for each equation under each estimator, as suggested by Blundell and Bond (1998). As may be seem, the OLS estimates yields the most persistent processes for each variable. This reflects the upward bias due to the fact any fixed effect is attributed to persistent effects of the shocks. The WGE etimates yield the least persistent processes, which reflects the downward bias. Both the system GMM (2SLS) and OBM estimators imply persistence that lie between these two extremes.

Table B3: Implied persistence: sum of coefficients on lagged dependent variable

Equation	WGE	2SLS	OBM	OLS
Y	0.90	0.92	0.98	0.99
P	0.87	0.93	0.91	0.98
$g^H$	0.55	0.60	0.65	0.70
$g^N$	0.10	0.44	0.23	0.50

Table B4 documents the same set of moments as we have previously considered, for each of the estimators. While there are clearly some differences across estimators, the same broad pattern emerges as that depicted in Table 2. The biggest outliers come from those based on OLS estimation. This is because the omission of city level fixed effects forces any permament differences to show up as high persistence. The system GMM (2SLS) estimator implies a price appreciation response that is the most volatile and the least persistent.

Table B4: Moments from estimation using alternative estimators – income shocks

	Estimator	Relative	Corr. with	Autocorrelation			
		Std. Dev.	Inc. Growth	year 1	year 2	year 3	year 4
Income	WGE	1.0000	1.0000	0.1980	0.0051	-0.0568	-0.0858
Growth	2SLS	1.0000	1.0000	0.2480	0.0221	-0.0546	-0.0759
	OBM	1.0000	1.0000	0.2340	0.0735	0.0200	-0.0082
	OLS	1.0000	1.0000	0.2203	0.0705	0.0230	0.0021
Price	WGE	1.1657	0.4851	0.8359	0.4780	0.1114	-0.1698
Appreciation	2SLS	1.3483	0.8049	0.7066	0.2988	0.0274	-0.1144
	OBM	1.2411	0.5224	0.8359	0.4803	0.1286	-0.1325
	OLS	1.1257	0.5011	0.8550	0.5545	0.2843	0.1027
Construction	WGE	0.1314	0.2425	0.8601	0.5475	0.2312	-0.0087
Rate	2SLS	0.1144	0.4600	0.8946	0.6589	0.4116	0.2117
	OBM	0.1706	0.4104	0.8991	0.6729	0.4332	0.2341
	OLS	0.2800	0.2512	0.9721	0.9090	0.8402	0.7782
Population	WGE	0.1829	0.8224	0.5952	0.2835	0.0595	-0.0887
Growth	2SLS	0.1791	0.7115	0.7149	0.4533	0.2526	0.1219
	OBM	0.2176	0.8609	0.6604	0.3642	0.1713	0.0651
	OLS	0.3200	0.5775	0.8771	0.7730	0.6949	0.6415

### 10.4 Alternative Specifications

Table B5 documents the relevant moments due to income shocks from the panel VAR for four alternative specifications.<sup>38</sup> The first specification, labelled "AR(2) Income", restricts the equation for income so that income depends only on its own lagged values. The specification labelled "All growth" uses growth rates of per capita incomes and prices in the VAR rather than levels. The specification labelled "Wage (levels)" uses (non-construction) wage per job instead of income per capita and the specification labelled "Wage (growth)" uses the same variable for income but both it and prices are specified in growth rates in the VAR.

As may be seen by comparing to Table 2, restricting the income process to be univariate has negligible effects. This suggest thats lagged feedback effects of prices and population on per capita income are of second order importance. Specifying the VAR so that incomes and prices are in growth rates rather than in log levels has somewhat larger effects on our results, but does not change the broad conclusions. Note that, by construction, the level of relative income under this specification is permanently high following a shock. However, this

<sup>&</sup>lt;sup>38</sup>We have considered others including alternative definitions of the construction rate and other definitions of income. Similar patterns emerge in all cases.

has little impact on the moments that we consider here. In contrast, replacing the income per capita variable with wages per job has substantial effects. The wage per job variable is considerably more volatile and less persistent than income. Moreover, shocks to this variable appears to account for much less of the variation in prices, construction and population.

Table B5: Moments from system GMM estimation for alternative specifications – income shocks

	Specification	Relative	Corr. with	Autocorrelation			
	_	Std. Dev.	Inc. Growth	year 1	year 2	year 3	year 4
Income	AR(2) Income	1.0000	1.0000	0.2473	0.0173	-0.0490	-0.0642
Growth	All growth	1.0000	1.0000	0.2429	0.0753	0.0498	0.0161
	Wage (levels)	1.0000	1.0000	0.0769	-0.0342	-0.0474	-0.0502
	Wage (growth)	1.0000	1.0000	0.0929	0.0152	0.0287	0.0274
Price	AR(2) Income	1.3600	0.7961	0.7107	0.3095	0.0429	-0.0944
Appreciation	All growth	1.1989	0.4833	0.8742	0.5986	0.3277	0.1318
	Wage (levels)	0.6505	0.5668	0.7947	0.4609	0.1209	-0.1397
	Wage (growth)	0.8413	0.4275	0.8815	0.6362	0.3842	0.2028
Construction	AR(2) Income	0.1250	0.4512	0.9049	0.6918	0.4675	0.2848
Rate	All growth	0.1559	0.4094	0.9134	0.7114	0.4866	0.3043
	Wage (levels)	0.0753	0.3679	0.8499	0.5347	0.2393	0.0208
	Wage (growth)	0.0899	0.3182	0.9135	0.7144	0.4988	0.3225
Population	AR(2) Income	0.1800	0.7179	0.7176	0.4605	0.2632	0.1377
Growth	All growth	0.1935	0.8626	0.6860	0.4147	0.2467	0.1279
	Wage (levels)	0.0753	0.7518	0.6942	0.3676	0.1069	-0.0699
	Wage (growth)	0.0847	0.7808	0.6712	0.4832	0.2480	0.1134

## 11 Appendix C: Math appendix

#### 11.1 Proofs and Derivations

Proof of Lemma 1. The household's optimization problem can be expressed as

$$\max_{c_t, l_t} E_t \sum_{t=0}^{\infty} \beta^t U_t(c_t, z_t) \quad \text{s.t.} \quad E_t \sum_{t=0}^{\infty} \beta^t c_t \le E_t \sum_{t=0}^{\infty} \beta^t \left[ y_t - \Omega_t \right]$$
 (82)

It follows their dynamic optimization problem is equivalent to

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[ y_t + z_t - \Omega_t \right] \tag{83}$$

**QED** 

**Proof of Proposition 1.** The first-order condition to the optimization problem in (25) yields

$$\lambda'(\omega_t)\omega_t'(P_t)\left(\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}\right) - \lambda\left(\omega_t(P_t)\right) = 0,\tag{84}$$

where  $\omega_t(P_t)$  and  $\omega_t'(P_t)$  are determined by (26). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma(\omega_t(P_t))/\gamma'(\omega_t(P_t))},\tag{85}$$

which can be used together with (26) to solve for  $P_t$ , given the values  $E_t J_{t+1}$ ,  $E_t W_{t+1}$  and  $E_t \tilde{V}_{t+1}$ . Then one can solve for  $\omega_t$  from (26). Note that (26) implies that  $\omega'_t(P_t) < 0$  given  $\gamma'(\omega) > 0$  from Assumption 1.

Recall from (21) that the trade surplus in the housing market is strictly positive. Given the boundary condition  $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$ , it is clear that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Thus  $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$ , where we have incorporated that  $V = \tilde{V}$  in the equilibrium. Recall condition (ix) of the equilibrium definition that  $\gamma(\omega_t)$ ,  $\lambda(\omega_t) \in (0, 1)$  for all active sub-markets. Also recall from part (ii) of Assumption 1 that  $\lambda'(\omega) < 0$ ,  $\gamma'(\omega) > 0$ . These conditions imply that  $\epsilon(\omega) \in (0, 1)$  by (10). Define  $LHS(P_t)$  as the left-hand side of (85) and  $RHS(P_t)$  the right-hand side. Given (10), it is clear that

$$RHS(P_t) = \frac{\epsilon(\omega_t(P_t))}{1 - \epsilon(\omega_t(P_t))}.$$
(86)

Because  $\epsilon(\omega) \in (0,1)$ , we have  $RHS(P_t) \in (0,\infty)$  for all  $P_t$ . Moreover, recall  $\omega'_t(P_t) < 0$  from (26) and  $\epsilon'(\omega) \leq 0$  from Assumption 1. Thus  $RHS'(P_t) \geq 0$ .

For any given  $V_t$ ,  $J_t$ ,  $W_t$ , one can verify that  $LHS'(P_t) < 0$  because  $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$ . Recall from (18) and (20) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \le P_t \le \beta E_t J_{t+1} - \beta E_t W_{t+1}.$$
 (87)

It follows that

$$LHS(P_t = \beta E_t V_{t+1}) = \infty > RHS(P_t = \beta E_t V_{t+1})$$
(88)

$$LHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}) = 0 < RHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}), \quad (89)$$

where the two inequalities are because  $RHS(P_t) \in (0, \infty)$  for all  $P_t$ . The above results imply a unique  $P_t^* \in (\beta E_t V_{t+1}, \ \beta E_t J_{t+1} - \beta E_t W_{t+1})$  that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*)) / \lambda'(\omega_t^*(P_t^*))}{\gamma(\omega_t^*(P_t^*)) / \gamma'(\omega_t^*(P_t^*))},\tag{90}$$

and a unique  $\omega_t^*(P_t^*)$  that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1} \left( \frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}} \right). \tag{91}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (90) may be written as

$$\frac{s(\omega)}{1 - s(\omega)} = \frac{\epsilon(\omega)}{1 - \epsilon(\omega)},\tag{92}$$

where  $s(\omega)$  denotes the buyer's share of the surplus in a sub-market with tightness  $\omega$ . The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that  $s(\omega) = \epsilon(\omega)$ . **QED** 

**Proof of Lemma 2.** First use (37) and (38) to derive

$$h^* = \frac{b^*}{\omega_t} + b^* + n^* + f^*. \tag{93}$$

Then use the above and (42) to eliminate  $h^*$ :

$$\frac{b^*}{\omega_t} + b^* + n^* + f^* = \frac{\phi^{\eta} (n^* + b^* + f)}{\mu} (\beta V^* - \bar{q})^{\eta}. \tag{94}$$

It follows that (43) can be obtained by substituting (39), (40) and (41) into the above. It follows that  $V^S(\omega^*)$  is strictly decreasing in  $\omega^*$  because  $\gamma'(\omega) > 0$  from Assumption 1. **QED** 

**Proof of Lemma 3.** Re-arranging (47) yields

$$P^* = (1 - s(\omega^*))\beta (J^* - W^* - V^*) + \beta V^*. \tag{95}$$

Substituting the above into (44), (45), (46) and (48) yield

$$V^* = \gamma(\omega^*) (1 - s(\omega^*)) \beta (J^* - W^* - V^*) + \beta V^*$$
(96)

$$W^* = Z + \frac{\lambda(\omega^*)s(\omega^*)}{1-\beta}\beta (J^* - W^* - V^*)$$
(97)

$$r^* = m + \gamma(\omega^*) (1 - s(\omega^*)) \beta(J^* - W^* - V^*)$$
(98)

and

$$J^{*} - W^{*} - V^{*} = \bar{u}^{H} + \pi_{n}\beta Z + \pi_{n}\beta V^{*} + (1 - \pi_{n})\theta\beta (W^{*} + V^{*}) + (1 - \pi_{n})(1 - \theta)\beta J^{*} - \bar{u}^{R} - \lambda(\omega^{*})s(\omega^{*})\beta (J^{*} - W^{*} - V^{*}) - \beta W - \gamma(\omega^{*})(1 - s(\omega^{*}))\beta (J^{*} - W^{*} - V^{*}) - \beta V^{*}.$$
(99)

Given (13), (16) and (98), we have

$$\bar{u}^{H} - \bar{u}^{R} = z^{H} + \gamma (\omega^{*}) (1 - s(\omega^{*})) \beta (J^{*} - W^{*} - V^{*}).$$
(100)

The above two equations yield

$$J^* - W^* - V^* = z^H - \pi_n \beta (W^* - Z) + [(1 - \pi_n) (1 - \theta) - \lambda(\omega^*) s(\omega^*)] \beta (J^* - W^* - V^*).$$

Using (97) to eliminate  $(W^* - Z)$  in the right-hand side of the above equation yields

$$J^* - W^* - V^* = z^H + \left[ (1 - \pi_n) (1 - \theta) - \left( \frac{1 - \beta + \pi_n \beta}{1 - \beta} \right) \lambda(\omega^*) s(\omega^*) \right] \beta \left( J^* - W^* - V^* \right),$$

which implies

$$J^* - W^* - V^* = \frac{z^H}{1 - \beta(1 - \pi_n)(1 - \theta) + \left(\frac{1 - \beta + \pi_n \beta}{1 - \beta}\right) \beta \lambda(\omega^*) s(\omega^*)}.$$
 (101)

Substituting the above into (96) yields (51). Recall from (27) that  $s(\omega) = \epsilon(\omega)$ . Also recall from Assumption 1 that  $\lambda'(\omega) < 0$ ,  $\gamma'(\omega) > 0$  and  $\epsilon'(\omega) \le 0$ . It follows that the right-hand side of (51) is increasing in  $\omega^*$ . **QED** 

**Proof of Proposition 2.** Because  $V^S(\omega^*)$  is decreasing in  $\omega^*$  and  $V^D(\omega^*)$  is increasing in  $\omega^*$ , a steady-state equilibrium must be unique if it exists. Existence basically requires

that the curves intersect at a value of  $\omega \in (\underline{\omega}, \bar{\omega})$ . That is, an (interior) equilibrium exists if  $V^D(\underline{\omega}) < V^S(\underline{\omega})$  and  $V^D(\bar{\omega}) > V^S(\bar{\omega})$ . Recall the definition of  $\epsilon(\omega)$  from (10). Also recall from Assumption 1 that  $\lambda(\bar{\omega}) = \gamma(\underline{\omega}) = 0$ ,  $\lambda(\underline{\omega}) = \gamma(\bar{\omega}) = 1$ ,  $\lambda'(\omega) < 0$  and  $\gamma'(\omega) > 0$ . It follows that  $V^S(\underline{\omega}) = \infty$ ,  $V^D(\underline{\omega}) = 0$  and

$$V^{S}(\bar{\omega}) = \frac{1}{\beta \phi} \left[ \mu \left( 1 + \frac{\psi \left( \mu + \pi_{f} \right)}{A + B\bar{\omega}} \right) \right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}$$
(102)

$$V^{D}(\bar{\omega}) = \frac{(1 - s(\bar{\omega}))\beta z^{H}}{(1 - \beta)\left[1 - \beta(1 - \theta)(1 - \pi_{n})\right] + (1 - \beta + \pi_{n}\beta)\beta\lambda(\bar{\omega})s(\bar{\omega})}.$$
 (103)

It follows that the condition for the existence and uniqueness of a steady state is given by (52). **QED** 

Solving the dynamic system: The dynamic system is given by

$$\ln y_t = (1 - \rho) \ln \bar{y} + \sum_{i=1}^{T} \rho_i \ln y_{t-i} + s_t$$
 (104)

$$(1+\mu)f_t = (1-\psi)\mu G(\bar{W}_t) + (1-\pi_f)f_t \tag{105}$$

$$(1+\mu)b_t = \psi \mu G(\bar{W}_t) + [1-\lambda(\omega_{t-1})]b_{t-1} + \theta(1-\pi_n)n_{t-1}$$
(106)

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda(\omega_{t-1})b_{t-1}$$
(107)

$$(1+\mu)h_{t+1} = h_t + \phi^{\eta} (n_t + b_t + f_t) (\beta E_t V_{t+1} - \bar{q})^{\eta}.$$
(108)

$$\omega_t = \frac{b_t}{h_t - b_t - f - n_t} \tag{109}$$

$$J_{t} = u_{t}^{H} + \beta \pi_{n} Z + \beta \left[ \frac{(\pi_{n} + (1 - \pi_{n})\theta) E_{t} \tilde{V}_{t+1}}{+(1 - \pi_{n})\theta E_{t} W_{t+1} + (1 - \pi_{n})(1 - \theta) E_{t} J_{t+1}} \right] (110)$$

$$W_{t} = u_{t}^{R} + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t}\right) + (1 - \lambda(\omega_{t})) \beta E_{t} W_{t+1}$$
(111)

$$V_t = \gamma(\omega_t)P_t + (1 - \gamma(\omega_t))\beta E_t V_{t+1}$$
(112)

$$P_{t} = (1 - s(\omega_{t}))\beta E_{t} (J_{t+1} - W_{t+1}) + s(\omega_{t})\beta E_{t} V_{t+1}$$
(113)

$$r_t = m + \gamma(\omega_t) \left( P_t - \beta E_t V_{t+1} \right). \tag{114}$$

$$W_t^f = u_t^R + \pi_f \beta Z + (1 - \pi_f) \beta E_t W_{t+1}^f$$
(115)

$$\bar{W}_t = \psi W_t + (1 - \psi) W_t^f.$$
 (116)

**Proof of Proposition 3.** The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_{t} = y_{t} + x_{t} - m + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t} + \beta E_{t} V_{t+1}\right) + (1 - \lambda(\omega_{t})) \beta E_{t} \tilde{J}_{t+1}.$$
(117)

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$W_{t} + V_{t} = u_{t}^{R} + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t}\right)$$

$$+ (1 - \lambda(\omega_{t})) \beta E_{t} W_{t+1} + \gamma(\omega_{t}) P_{t} + (1 - \gamma(\omega_{t})) \beta E_{t} V_{t+1}$$

$$= u_{t}^{R} + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t}\right)$$

$$+ (1 - \lambda(\omega_{t})) \beta E_{t} W_{t+1} + \gamma(\omega_{t}) (1 - s(\omega_{t})) \beta E_{t} \left(J_{t+1} - W_{t+1}\right)$$

$$+ \gamma(\omega_{t}) s(\omega_{t}) \beta E_{t} V_{t+1} + (1 - \gamma(\omega_{t})) \beta E_{t} V_{t+1}$$

$$= y_{t} + x_{t} - r_{t} + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t} + \beta E_{t} V_{t+1}\right)$$

$$+ \gamma(\omega_{t}) (1 - s(\omega_{t})) \beta E_{t} \left(J_{t+1} - W_{t+1} - V_{t+1}\right)$$

$$+ (1 - \lambda(\omega_{t})) \beta \left(E_{t} W_{t+1} + E_{t} V_{t+1}\right).$$

$$(118)$$

Given (31), the above implies that

$$W_{t} + V_{t} = y_{t} + x_{t} - m + \lambda(\omega_{t}) \left(\beta E_{t} J_{t+1} - P_{t} + \beta E_{t} V_{t+1}\right) + (1 - \lambda(\omega_{t})) \beta E_{t} \left[W_{t+1} + V_{t+1}\right].$$
(119)

Since  $\lim_{T\to\infty} \beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty} \beta^T E_t [W_{T+1} + V_{T+1}] = 0$ , solving forwards implies that

$$\tilde{J}_t = W_t + V_t. \tag{120}$$

**QED** 

## 11.2 Stationary Equilibrium with no search

In a stationary equilibrium there are no shocks so that  $u_t^H = \bar{u}^H$ . Housing market clearing implies

$$h^* = n^* + b^*. (121)$$

It follows directly that the stationary equilibrium price is

$$P^* = \frac{\mu^{\frac{1}{\eta}}}{\beta \phi} + \frac{\bar{q}}{\beta}.\tag{122}$$

The value of being a home-owner is then

$$J^* = \frac{\bar{u}^H + \beta \pi Z + \beta \pi P^*}{1 - \beta (1 - \pi)}.$$
 (123)

Given stationary values for J and P, the new entrants per period is

$$b^* = \frac{\mu}{1+\mu}G(J^* - P^*) \tag{124}$$

and the steady-state fraction of the total population located in the city is

$$n^* = \frac{1}{\mu + \pi} b^*. \tag{125}$$

Finally, the housing stock per capita is

$$h^* = \frac{\phi^{\eta} (n^* + b^*)}{\mu} (\beta P^* - \bar{q})^{\eta}.$$
 (126)

### 11.3 Calibration of quarterly income shock process

If we now think of a period as a quarter, we can write an annual AR(2) process as

$$x_t = b_1 x_{t-4} + b_2 x_{t-8} + \varepsilon_t. (127)$$

Let  $y_t = x_{t-4}$ . Then we can write this as a stacked system given by

$$X_t = \mathbf{B}X_{t-4} + \mathbf{e}_t \tag{128}$$

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}.$$
 (129)

Now consider a VAR(1) given by

$$X_t = \mathbf{A}X_{t-1} + \mathbf{v}_t, \tag{130}$$

where  $\mathbf{v}_t = \begin{bmatrix} v_t & 0 \end{bmatrix}'$ . Iterating on this yields

$$X_{t} = \mathbf{A}^{4} X_{t-4} + \mathbf{A}^{3} \mathbf{v}_{t-3} + \mathbf{A}^{2} \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_{t}.$$
(131)

It follows that  $\mathbf{A} = \mathbf{B}^{\frac{1}{4}}$  and  $\mathbf{e}_t = \mathbf{A}^3 \mathbf{v}_{t-3} + \mathbf{A}^2 \mathbf{v}_{t-2} + \mathbf{A} \mathbf{v}_{t-1} + \mathbf{v}_t$ . We can decompose the VAR(1) as

$$x_t = a_{11}x_{t-1} + a_{12}y_{t-1} + v_t (132)$$

$$y_t = a_{21}x_{t-1} + a_{22}y_{t-1}. (133)$$

Since  $y_t = x_{t-4}$ , it follows that

$$x_t = a_{11}x_{t-1} + a_{12}x_{t-5} + v_t (134)$$

$$x_{t-4} = a_{21}x_{t-1} + a_{22}x_{t-5}. (135)$$

Substituting out  $x_{t-5}$  yields

$$x_t = a_{11}x_{t-1} + \frac{a_{12}}{a_{22}}(x_{t-4} - a_{21}x_{t-1} - v_{2t}) + v_t$$
 (136)

$$x_t = \left(a_{11} - \frac{a_{12}a_{21}}{a_{22}}\right)x_{t-1} + \frac{a_{12}}{a_{22}}x_{t-4} + v_t. \tag{137}$$

Thus the AR(2) process at the annual frequency translates into a particular AR(4) process at the quarterly frequency. There is of course a loss of information.

## 11.4 Full results of sensitivity analysis

Table C1: Volatilities and Comovements: Sensitivity Results

		New construction		Land supply		Vacancy Rate		Entry (demand)	
		supply elasticity		elasticity				elasticity	
Moment	Baseline	$\eta = 2$	$\eta = 20$	$\xi = .5$	$\xi = 5$	v = .01	v = .03	$\alpha = 5$	$\alpha = 20$
$\sigma_p/\sigma_y$	0.67	1.71	0.16	0.74	0.65	0.83	0.53	0.51	1.52
$\sigma_h/\sigma_y$	0.17	0.17	0.17	0.17	0.17	0.175	0.166	0.14	0.34
$\sigma_n/\sigma_y$	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.15	0.35
$\sigma_{py}$	0.97	0.94	0.73	0.97	0.97	0.96	0.96	0.97	0.94
$\sigma_{hy}$	0.35	0.29	0.42	0.38	0.34	0.39	0.30	0.35	0.34
$\sigma_{ny}$	0.37	0.32	0.39	0.39	0.36	0.37	0.37	0.38	0.34
$\alpha$	6.5	12	5	6.8	6.4	7.0	6.1		

Table C2: Autocorrelations: Sensitivity Results

		Labour supply		Land supply		Vacancy Rate		Entry (demand)	
		elasticity		elasticity				elasticity	
Moment	Baseline	$\eta = 2$	$\eta = 20$	$\xi = .5$	$\xi = 5$	v = .01	v = .03	$\alpha = 5$	$\alpha = 20$
$ ho_1^p$	0.22	0.07	0.48	0.19	0.24	0.11	0.33	0.33	0.05
$\rho_2^p$	0.08	0.02	0.04	0.07	0.07	-0.02	0.18	0.12	0.00
$\rho_3^p$	0.0	-0.01	-0.10	-0.01	-0.01	-0.06	0.06	-0.01	-0.03
$ ho_4^p$	-0.05	-0.03	-0.11	-0.04	-0.06	-0.06	-0.01	-0.06	-0.04
$ ho_1^h$	0.94	0.94	0.94	0.93	0.94	0.91	0.95	0.94	0.92
$ ho_2^h$	0.85	0.87	0.83	0.84	0.85	0.81	0.88	0.86	0.84
$ ho_3^h$	0.76	0.80	0.71	0.74	0.76	0.71	0.80	0.76	0.75
$ ho_4^h$	0.67	0.74	0.61	0.65	0.67	0.63	0.72	0.67	0.68
$\rho_1^n$	0.88	0.88	0.89	0.87	0.88	0.88	0.88	0.88	0.88
$\rho_2^n$	0.78	0.79	0.78	0.77	0.78	0.78	0.78	0.78	0.79
$ ho_3^n$	0.70	0.72	0.69	0.68	0.70	0.69	0.69	0.69	0.72
$ ho_4^n$	0.62	0.67	0.60	0.60	0.62	0.61	0.62	0.61	0.65

## References

- [1] Abraham, J. and Hendershott, P. H. (1996), "Bubbles in metropolitan housing markets," Journal of Housing Research, vol. 7, pp. 191–207.
- [2] Albrecht, J., A. Anderson, E. Smith and S. Vroman (2007), "Opportunistic matching in the housing market," *International Economic Review*, vol. 48, pp. 641-664.
- [3] Albrecht, J., P. Gautier and S. Vroman (2003), "Matching with Multiple Applications," *Economics Letters* vol. 78, pp. 67-70.
- [4] Alonso, W. (1962), Location and Land Use. Cambridge, MA: Harvard University Press.
- [5] Allen, J., B. Amano, D.P. Byrne and A. W. Gregory (2007), "Canadian City house prices and Urban Market Segmentation," Bank of Canada Working Paper, No. 2006-49.
- [6] Anglin, P. and R. Arnott (1999), "Are brokers' commission rates on home sales too high? A conceptual analysis," *Real Estate Economics*, vol. 27.
- [7] Anglin, P., R. Rutherford and T. M. Springer (2003), "The Trade-off Between the Selling Price of Residential Properties and Time-on-the-Market: The Impact of Price Setting," *Journal of Real Estate Finance and Economics*, vol. 6, pp. 95-111.
- [8] Arellano, M. and S. Bond (1991), "Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations," *Review of Economic Studies*, vol. 58, pp. 277-297.
- [9] Arellano, M. and O. Bover (1995), "Another look at the instrumental variable estimation of dynamic models using panel data," *Journal of Econometrics*, vol. 68, pp. 29-51.
- [10] Binder, M., C. Hsiao and M.H. Pesaran (2005), "Estimation and inference in short panel vector autoregressions with unit roots and cointegration," *Econometric Theory*, vol. 21, pp. 795-837.
- [11] Blackley, D. M. (1999), "The Long-Run Elasticity of New Housing Supply in the United States: Empirical Evidence for 1950 to 1994," *Journal of Real Estate Finance and Economics*, vol. 18, pp. 25-42.
- [12] Blundell, R. and S. Bond (1998), "Initial Conditions and Moment Restrictions in Dynamic Panel Data Models," *Journal of Econometrics*, vol. 87, pp. 115-143.

- [13] Caplin, A. and J. Leahy (2008), "Trading frictions and house price dynamics," NBER Working Paper 14605.
- [14] Case, K. and R. Shiller (1989), "The efficiency of the market for single–family homes," *American Economic Review*, vol. 79, pp. 125–137.
- [15] Capozza, D. R., P. C. Hendershott and C. Mack (2004), "An anatomy of price dynamics in illiquid markets: Analysis and evidence from local housing markets," *Real Estate Economics*, vol. 32, pp. 1–32.
- [16] Capozza, D. R., C. Mack, and C. J. Mayer (1997), "The Dynamic Structure of Housing Markets," mimeo.
- [17] Cutler, D., J. Poterba and L. Summers (1991), "Speculative Dynamics," *Review of Economic Studies*, vol. 58, pp. 529–46.
- [18] Davis, M. A. and J. Heathcote (2007), "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, vol. 54, pp. 2595–2620.
- [19] Davis, M. A. and F. Ortalo-Magne (2011), "Household Expenditures, Wages, Rents," Review of Economic Dynamics, vol. 14, pp. 248-261.
- [20] Davis, M. A. and M.G. Palumbo (2008), "The price of residential land in large U.S. cities," *Journal of Urban Economics* 63 (2008), pp. 352–384.
- [21] Del Negro, M. and C. Otrok (2007). "99 Luftballons: Monetary policy and the house price boom across U.S. states," *Journal of Monetary Economics*.vol. 54(7), pp. 1962-1985.
- [22] Diaz, Antonia and Jerez, Belen (2010), "House Prices, Sales and Time on the Market: A Search Theoretic Framework," mimeo, Universidad Carlos III.
- [23] Dieleman, F., W. Clark, and M. Deurloo (2000), "The geography of residential turnover in twenty-seven large U.S. metropolitan housing markets, 1985-1995," *Urban Studies*, vol. 37, pp. 223-245.
- [24] Everaert, Gerdie (2011), "Orthogonal to Backward Mean Transformation for Dynamic Panel Data Models," mimeo, Ghent University.

- [25] Forgey, F. A., R. C. Rutherford and T. M. Springer (1996), "Search and liquidity in single-family housing," *Real Estate Economics*, vol. 24, pp. 273-292.
- [26] Genesove, D., and L. Han. (2011), "Measuring the Thinness of Real Estate Markets", working paper, Rotman School of Management, University of Toronto.
- [27] Glaeser, Edward L., Joseph Gyourko, Eduardo Morales and Charles G. Nathanson (2010), "Housing Dynamics," mimeo, Harvard University.
- [28] Green, Richard K., Stephen Malpezzi, Stephen K. Mayo (2005), "Metropolitan-specific estimates of the price elasticity of supply of housing, and their sources," *American Economic Review*, Vol. 95, Papers and Proceedings, pp. 334-339
- [29] Gyourko, J. and A. Saiz (2006). "Construction Costs and the Supply of Housing Structure." *Journal of Regional Science*, vol. 46, No. 4, pp. 661-680.
- [30] Harding, John P., Stuart S. Rosenthal and C.F. Sirmansa (2007), "Depreciation of housing capital, maintenance, and house price inflation: Estimates from a repeat sales model," *Journal of Urban Economics*, vol. 61 (2), pp. 193-217
- [31] Head, A. and H. Lloyd-Ellis (2011), "Housing Liquidity, mobility and the labour market," forthcoming, *Review of Economic Studies*. (QED Working Paper # 1197, Queen's University).
- [32] Hosios, A. J. (1990), "On the efficiency of matching and related models of search and unemployment," *Review of Economic Studies*, vol. 57, pp. 279–98.
- [33] Judson R.A. and A. L. Owen (1999), "Estimating dynamic panel data models: a guide for macroeconomists," *Economics Letters*, vol. 65(1), pp. 9-15.
- [34] Kang, H. B. and M. J. Gardner (1989), "Selling Price and Marketing Time in the Residential Real Estate Market," *Journal of Real Estate Research*, vol. 4, pp. 21-35.
- [35] Klein, P. (2000), "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control*, vol. 24, pp. 1405-1423.
- [36] Krainer, J. (2001), "A theory of liquidity in residential real estate markets," *Journal of Urban Economics*, vol. 49, pp. 32–53.

- [37] Krainer, J. (2008), "Falling house prices and rising time on the market," FRBSF Economic Letter, Federal Reserve Bank of San Francisco, issue Mar 21.
- [38] Leung, C., Y. Leong and I. Chan (2002), "TOM: Why Isn't Price Enough?" *International Real Estate Review*, vol. 5, pp. 91-115.
- [39] Levitt, S. D. and C. Syverson (2008), "Antitrust Implications of Home Seller Outcomes when using Flat-Fee Real Estate Agents," *Brookings-Wharton Papers on Urban Affairs*.
- [40] Love, I. and Lea Zicchino (2006), "Financial development and dynamic investment behavior: Evidence from a panel VAR," The Quarterly Review of Economics and Finance, vol. 46, pp. 190-210.
- [41] Malpezzi, S., (1999), "A Simple Error Correction Model of House Prices," *Journal of Housing Economics*, vol. 8, pp. 27-62.
- [42] Meen, G., (2002), "The Time-Series Properties of House Prices: A Transatlantic Divide?" *Journal of Housing Economics*, vol. 11, pp. 1-23.
- [43] Merlo, A. and F. Ortalo-Magne (2004), "Bargaining over residential real estate: evidence from England," *Journal of Urban Economics*, vol. 56, pp. 192-216.
- [44] Moen, E. (1997), "Competitive Search Equilibrium," *Journal of Political Economy*, Vol 105, pp. 385-411.
- [45] Nickell, S. J. (1981), "Biases in Dynamic Models with Fixed Effects," *Econometrica*, vol. 49, pp. 1417-26.
- [46] Peach, R. (1983), "An econometric model of investment in single family homes," University of Maryland.
- [47] Piazzesi, M. and M. Schneider (2009), "Momentum Traders in the Housing Market: Survey Evidence and a Search Model," American Economic Review, vol. 99, pp. 406-411.
- [48] Poterba, J.M. (1991), "House Price Dynamics: The Role of Tax Policy," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 22, pages 143-204.
- [49] Rios-Rull, J. and V. Sanchez-Marcos (2007), "House price movements," mimeo.

- [50] Roback, J. (1982), "Wages, rents and the quality of life," *Journal of Political Economy*, vol. 90 (4), pp. 1257-78.
- [51] Rosen, S. (1979), "Wage-based indexes of urban quality of life," in P. Mieszkowski and M. Straszheim (eds.) Current Issues in Urban Economics, Baltimore: Johns Hopkins University Press.
- [52] Saiz, A. (2010), "The Geographic Determinants of Housing Supply," *The Quarterly Journal of Economics*, vol. 125 (3), pp. 1253-1296.
- [53] Topel, R. H. and S. Rosen (1988), "Housing Investment in the United States," *Journal of Political Economy*, vol. 96, pp. 718-40.
- [54] Van Nieuwerburgh, S. and P. Weill (2010), "Why Has House Price Dispersion Gone Up?," Review of Economic Studies, vol. 77, pp. 1567-1606.
- [55] Wheaton, W. C. (1990), "Vacancy, search and prices in a housing market matching model," *Journal of Political Economy*, vol. 98 (6), pp. 1270–92.