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Housing Liquidity, Mobility, and the Labour Market

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Housing Liquidity, Mobility and the Labour Market

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Abstract

The relationships among geographical mobility, unemployment and the value of owner-occupied housing are studied in an economy with heterogeneous locations and search frictions in the markets for both labour and houses. Differences in labour market conditions between cities affect the speed with which houses may be sold—that is, the *liquidity* of housing. At the same time housing market conditions affect employment decisions and thus the allocation of labour across cities. In equilibrium, unemployment rates for home-owners are higher than for otherwise identical renters. Unemployment and home-ownership rates are, however, negatively correlated across cities. In a parameterized example we find that, although renters are much more mobile than owners, the impact of home-ownership on aggregate unemployment is quantitatively small.

Journal of Economic Literature Classification: : J61, J64, R23

Keywords: Liquidity, mobility, home-ownership, unemployment.

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1 Introduction

In this paper, we study the relationships among geographical mobility, unemployment and the value of owner-occupied housing in an environment characterized by frictions in the markets for both labour and houses. The price of a house reflects its *liquidity* — i.e. the speed with which it can be transferred to another home owner— and this in turn affects both mobility and labour market outcomes. Our model is consistent with recent micro-evidence on the relationship between ownership and unemployment across cities and between individuals. It is also consistent with large observed differences in mobility between renters and owners. Nevertheless, we find that the impact of home ownership on aggregate unemployment is very unlikely to be economically significant.

In our economy a large number of *ex ante* identical households may choose to live in either of two cities which differ in regard to the productivity of jobs. Households require housing and may either rent in a competitive market or purchase in a market characterized by a search friction. All households, whether employed or unemployed, randomly receive offers of employment in both their city of current residence and the other city. In order to take a job in the other city, a household must move and acquire housing there. Migrating home owners put their houses up for sale and initially rent in the other city while searching for a house. In this environment, we establish the existence and uniqueness of a stationary equilibrium characterized by constant relocation and housing market activity.

The willingness of a home owner to accept a job in the other city depends not only on relative wages but also on rental rates and the market value of their current house. Since the latter depends on how quickly a buyer can be found, the liquidity of housing affects the distribution of households across cities and unemployment both at the city level and in the aggregate. At the same time, the frequency with which households choose to relocate affects the liquidity of the housing markets.

For a range of parameter values, our model can account for some puzzling and seemingly contradictory recent evidence on the relationship between home ownership and unemployment both across US cities and at the individual level.¹ In particular, Coulson and Fisher (2008) find that across U.S. metropolitan areas home ownership rates are correlated positively with average wages and *negatively* with unemployment.² In contrast, using micro-data

¹Unlike countries or regions, it seems reasonable to think of each metropolitan area as a distinct labour market.

²After controlling for other co-variates, Coulson and Fisher (2008, p.26) conclude that, “there is a negative correlation between unemployment and ownership and a positive correlation between wages and ownership

and controlling for demographic and locational characteristics, they find that the marginal impact of home ownership on the likelihood of unemployment for an individual is significant and *positive*, though modest in size. At the same time (and consistent with the aggregate cross-city results) the likelihood of unemployment is negatively correlated with the rate of home ownership in an individual's city of residence.

In our framework, home-ownership increases the likelihood of unemployment for an individual because, while separation rates and offer rates are the same for all, in equilibrium only home owners ever turn down offers. At the city level, however, there is a second effect which offsets the impact of home ownership on unemployment. Because the high wage city has the lower vacancy rate, it has the higher rate of home-ownership. It also, however, has the higher rental rate, making it unattractive to unemployed renters who may move to the low wage city even without a job offer. In contrast, employed renters in the low wage city move to the high wage (and high rent) city as long as the wage premium is sufficient and unemployed home-owners never re-locate to the high-wage city without an offer. Depending on parameters, these factors may combine to generate higher unemployment in the low-wage city, where the home ownership rate is also lower.

There is considerable evidence that owners tend to move less than renters, even after controlling for household and locational characteristics (see, for example, Rohe and Stewart, 1996, or Boheim and Taylor, 2002). Recently, a number of commentators have argued that because of its relationship with mobility, home-ownership may create frictions in the labour market that lead to inefficient outcomes.³ Indeed some have gone so far as to conjecture that differences in home-ownership rates across countries may be a leading factor in driving differences in unemployment.⁴

In general, the available evidence based on micro-data is not particularly favorable to this conjecture, finding at best only a small marginal effect of ownership on the likelihood of unemployment. It is, however, somewhat difficult to interpret these findings. Although unconditionally the unemployment rate amongst renters is significantly higher than that amongst owners, this largely reflects the differing characteristics of these households. For example, owners tend to be more educated, older and more likely to be married than renters.

across US metropolitan areas.”

³See, for example, Blanchflower (2007).

⁴This argument is typically based on the observation of a positive correlation between unemployment and home ownership across countries or regions. See Nickell (1998), Oswald (1999), Partridge and Rickman (1997), Pehkohnen (1999) and Cochrane and Poot (2007). Munch et al. (2006) and Rouwendal and Nijkamp (2006) critically review some of this work.

These characteristics also make them less likely to be unemployed, independent of any direct effect of ownership on mobility. Obviously, to test the conjecture one must control for all the relevant demographic and locational characteristics some of which may not be observed. Our framework allows us to isolate the effects of home ownership *per se* on both mobility and unemployment. We model all households as *ex ante* identical and so home ownership affects mobility, rather than the reverse.

Using a version of our model calibrated to match aggregate US labour market flows and mobility rates, we find that the fraction of home owners that turn down high wage offers in the other location is substantial. Consequently, in accordance with the empirical evidence, the mobility rate for owners is much lower than for renters. Despite these large effects on mobility, however, we find that the impact of ownership on aggregate unemployment is very small. Moreover, because the impact of home-ownership on the likelihood of unemployment is small compared to the effect of the rent differential on the mobility of unemployed renters, unemployment rates and home-ownership are *negatively* correlated across cities as observed by Coulson and Fisher (2008). These findings are robust to several alternative parameterizations and generalizations of our basic model.

Others have developed theories of the relationship between home ownership and the labour market. For example, Dohmen (2005) and Munch, Rosholm and Svarer (2006) present models of labour market search in which home-owners and renters are assumed to behave differently. Coulson and Fisher (2008) present a theory based on endogenous job creation that is consistent with their evidence on unemployment, but does less well with regard to wages. In particular, in their model home owners receive lower wages than renters as a result of their immobility. All of these theories, however, abstract from both housing choice and transactions in the housing market. Owners are either simply assumed to be immobile or to face higher moving costs than renters. Here, because the price of housing is endogenously determined, the relative degree of mobility depends on labour and housing market conditions.

Rupert and Wasmer (2008) also develop a theory of the relationship between unemployment and housing market frictions. They do not distinguish between ownership and renting, and they focus on the trade-off between commuting time and locational decisions within a single labour market. In contrast, our focus is on the role of housing markets in generating frictions *between* labour markets. In this sense, our paper is complementary to theirs. In a generalization of our basic model, we consider the role of within-city relocation. This extension is related to Wheaton (1990), who develops a model of housing markets but considers neither linkages to labour markets nor cross-city re-location. Albrecht, Axelrod, Smith, and

Vroman (2007) consider a search model in which the flow values of search to buyers and sellers change over time. Their work is related to ours but different in that whereas they focus on the relationship between prices and time on the market, we focus mainly on mobility and the relationship between the labour and housing markets.

A substantial literature also focuses on the relationship between the length of residence spells (which tend to be higher for home owners than for renters) and investments in social capital (*e.g.* see Rossi and Weber (1996) and DiPasquale and Glaeser (1999)). Empirically, Coulson, Hwang, and Imai (2002) find that the fraction of home owners in a neighborhood is associated with higher property values. While our model is consistent with this observation (as home ownership and house prices are both higher in the high-wage city) as well as the fact that home owners remain in a city longer than renters, we abstract from investment of all kinds.

This remainder of the paper is organized as follows: Section 2 describes the environment. Section 3 defines a symmetric stationary equilibrium, establishes existence and uniqueness, and characterizes the equilibrium under various assumptions. Section 4 considers the implications of the theory for the relationship between ownership, mobility and unemployment at the individual, city and aggregate level. We also consider a parameterized version of the model to assess its quantitative implications. In Section 5 we assess how robust our conclusions are to several generalizations of the basic model. Section 6 summarizes and describes future work. Proofs, longer derivations, and the details of some of the extensions are contained in appendices.

2 The Environment

Time is continuous. The economy is populated by a unit measure of infinitely lived, *ex ante* identical, risk-neutral *households* who discount the future at rate ρ . There are two locations, called *cities*, indexed by $i \in \{1, 2\}$. Households must reside in one and only one city at any point in time. They are free, however, to move between cities at any time at no direct cost. Each city contains two types of residential dwellings. At a point in time, let R_i denote the stock of *rental* housing in City i and H_i the stock of *owner-occupied* housing. Let π^R denote the flow utility received by a household which lives as a renter in either city. Similarly, π^H is the flow utility from living in an owned house. We assume that $\pi^H > \pi^R$.

Firms in City i produce output y_i using labour l_i according to the production function

$$y_i = (\phi_i l_i)^\eta - F \tag{1}$$

where $\eta \in (0, 1)$ and ϕ_i represents a city-specific productivity parameter. Taking wages as given, firms may hire as many workers as they like, provided that they have paid the per period fixed cost F to operate the production technology. Without loss of generality, we assume that productivity is higher in City 2 than in City 1. That is, $\phi_2 > \phi_1$.

In each city there is labour market which functions much like that considered in Lucas and Prescott (1974). As a consequence of firms' demand for labour each city has a large number of potential employment opportunities, which we refer to as *jobs*. At any point in time, each household is either employed (*i.e.* holding a job) or unemployed. A household may hold at most one job and that job must be located in their city of current residence. Employed households in City i receive flow income equal to the *wage* w_i . Unemployed households receive flow consumption z .

All households, regardless of their employment status, randomly receive offers of employment both in their city of residence and in the other city. These may be seen as offers of admission to the labour market of a particular city, within which wages are determined in a Walrasian fashion. Let μ denote the Poisson rate at which households receive offers within their city of residence and μ^* denote the rate at which they receive an offer in the other city. We assume that these rates are symmetric across cities and that $\mu > \mu^*$. A household (employed or unemployed) which receives a job offer may either accept or reject it. Employed households in both cities lose their jobs at Poisson rate δ .⁵

There are also a large number of firms called *real estate managers* (REM's) which are owned by households and perform two functions: They rent out rental housing in city i in a competitive market at rate r_i , and they intermediate between buyers and sellers in city-specific markets for owner-occupied housing. We abstract from costs associated with the rental of houses so that $r_i \geq 0$. We assume that all households own equal shares in the economy's REM's and receive any profits as lump-sum transfer, κ .

Home-owners may sell their houses at any time to an REM in a competitive market. Let p_i denote the price received by a household that sells a house to a REM in City i . REM's receive no service flow from houses and hold them only for the purposes either of re-sale or for conversion into rental units. An REM can convert a formerly owner-occupied unit into a rental unit at a fixed per unit cost $C^R \geq 0$. Similarly, an REM can convert a rental unit to an owner-occupied one at cost $C^H \geq 0$.

The re-sale market for owner-occupied housing is characterized by a one-sided process

⁵It is possible to allow for differences in offer rates and separation rates across locations; we assume symmetry here for simplicity.

that matches potential buyers with REM's. Let S_i denote the stock of houses offered for sale by REM's in City i (and therefore vacant). Similarly, let D_i denote the stock of potential *home buyers* in City i . REM's match with potential buyers at rate γ_i , where⁶

$$\gamma_i = \frac{\lambda D_i}{S_i} \quad i = 1, 2. \quad (2)$$

For simplicity, we assume that a REM who matches with a potential buyer makes a take-it-or-leave-it offer, provided that the aggregate match surplus is positive.⁷ Let q_i^W and q_i^U denote the prices paid for houses in City i by employed and unemployed households respectively.⁸

Given the assumed structure of the markets for owner occupied housing, it takes time for houses to be transferred from one household to another. This friction results in houses being illiquid, as their market value depends on the speed with which a buyer can be found for a vacant house. Let the value of such a house in City i be denoted V_i^H . Then

$$\rho V_i^H = \gamma_i \mathbb{E}_{\{q_i^W, q_i^U\}} [\max\{q_i - V_i^H, 0\}] \quad i = 1, 2. \quad (3)$$

In each city there are four types of households, as each may be either employed or unemployed and may either rent or own a house. The measures of households in City i that are employed-owners, employed-renters, unemployed-owners and unemployed-renters are given by N_i^{WH} , N_i^{WR} , N_i^{UH} and N_i^{UR} respectively. The values associated with being in each of these states are given by W_i^H , W_i^R , U_i^H and U_i^R , respectively.

⁶In our basic model, we assume that unemployed and employed renters find houses at the same rate. In Section 6, we show that allowing unemployed renters to match at a lower rate makes little difference.

⁷It is straightforward to generalize the model to allow for a different division of the surplus. However, it makes no difference to our main conclusions. Calculations are available from the authors upon request.

⁸Since they earn zero profits when purchasing a previously owned house and make a take-it-or-leave it offer when re-selling, the role played by REM's in intermediating transactions is virtually equivalent to assuming that migrating households continue to own their vacant house until they match with and sell to another household. Assuming that this function is performed by REM's greatly simplifies the analysis, however, because it rules out the possibility of a migrating household returning to its previous location and moving back into its old house before selling. Allowing for this would expand the number of household states and complicate the analysis substantially. Since in the equilibria we consider these additional states would occur with very low probability, this complication would add nothing significant to our results.

3 Stationary, Symmetric Equilibrium

We consider equilibria which are stationary and symmetric in that all households of a given type behave in the same way. In this case, City i households' value functions satisfy

$$\begin{aligned} \rho W_i^R &= w_i + \pi^R - r_i + \kappa + \delta [U_i^R - W_i^R] + \mu^* \max \{W_j^R - W_i^R, 0\} \\ &\quad + \lambda \max \{W_i^H - q_i^W - W_i^R, 0\} \end{aligned} \quad (4)$$

$$\begin{aligned} \rho U_i^R &= z + \pi^R - r_i + \kappa + \mu [W_i^R - U_i^R] + \mu^* \max \{W_j^R - U_i^R, 0\} \\ &\quad + \lambda \max \{U_i^H - q_i^U - U_i^R, 0\} \end{aligned} \quad (5)$$

$$\rho W_i^H = w_i + \pi^H + \kappa + \delta [U_i^H - W_i^H] + \mu^* \max \{W_j^R + p_i - W_i^H, 0\} \quad (6)$$

$$\rho U_i^H = z + \pi^H + \kappa + \mu [W_i^H - U_i^H] + \mu^* \max \{W_j^R + p_i - U_i^H, 0\} \quad (7)$$

where the subscript j indexes the other city.

A *stationary symmetric equilibrium* for this economy is a collection of ten value functions, eight for the different types of households, W_i^R , W_i^H , U_i^R , and U_i^H , for $i = 1, 2$, and one for vacant housing in each city, V_i^H ; rental prices in each city, r_i ; house prices in each city, q_i^W , q_i^U , and p_i ; and measures of households in each of the eight states, N_i^{WR} , N_i^{UR} , N_i^{WH} , and N_i^{UH} , such that:

- i.** Given wages, firms choose employment levels l_i to maximize profits. Free entry into production implies that profits equal zero:

$$(\phi_i l_i)^\eta - w_i l_i - F = 0. \quad (8)$$

- ii.** Given prices and the value of houses in each city, the value functions satisfy (4)-(7).

- iii.** The rental prices, $r_i \geq 0$, clear the markets for rental housing in each city:

$$N_i^{WR} + N_i^{UR} = R_i \quad i = 1, 2. \quad (9)$$

- iv.** House purchase prices in both cities, p_i^W and p_i^U extract all of households' surplus.

- v.** The house sale price in each city is equal to the value of a vacant house:

$$p_i = V_i^H \quad i = 1, 2. \quad (10)$$

- vi.** The distribution of households over states is consistent with the population:

$$\sum_{i=1,2} [N_i^{WR} + N_i^{UR} + N_i^{WH} + N_i^{UH}] = 1. \quad (11)$$

vii. Rents are distributed equally as dividends to households:

$$\sum_{i=1,2} r_i R_i = \kappa$$

We begin by assuming at least one such equilibrium exists, and describe several characteristics that it must necessarily have. We then finish this section with a proposition establishing the existence and uniqueness of the equilibrium, subject to certain restrictions.

Profit maximization by firms in City i implies that each demands labour

$$l_i = \left[\frac{\eta}{w_i} \right]^{\frac{1}{1-\eta}} \phi_i^{\frac{\eta}{1-\eta}}. \quad (12)$$

Substituting (12) into (8) yields an expression for the equilibrium wage in city i :

$$w_i = \eta \left[\frac{1-\eta}{F} \right]^{\frac{1-\eta}{\eta}} \phi_i. \quad (13)$$

Thus, in each city the equilibrium wage is proportional to local productivity and is unaffected by conditions in the housing market. From now on we will therefore refer to City 1 and City 2 as the low and high wage cities, respectively.

We restrict attention to equilibria where employed renters in the low-wage city (City 1) who are offered a job in the high-wage city choose to relocate, but not *vice versa*. That is

$$W_2^R > W_1^R. \quad (14)$$

We also restrict attention to equilibria in which all renters, whether employed or not, buy houses when they get the chance. As real estate managers make take-it-or-leave-it offers to home buyers, this will happen as long as the surplus from such a transaction is positive:

$$W_i^H - W_i^R = q_i^W > p_i \quad U_i^H - U_i^R = q_i^U > p_i \quad i = 1, 2. \quad (15)$$

Below, we derive the conditions needed for (15) to hold.

Finally, we consider only equilibria that are interior in the sense that there are unemployed renters in each city. Because they are mobile, in any such equilibrium these households must be indifferent with regard to their city of residence. That is,

$$U_2^R = U_1^R = U^R. \quad (16)$$

Home owners, in contrast, face effective “moving costs” associated with the illiquidity of housing. Conditions (15) and (14) together imply that employed home owners are also unwilling to move from from the high wage city to the low wage one in equilibrium:

$$W_2^H - p_2 > W_2^R > W_1^R. \quad (17)$$

Making use of the facts that unemployed renters are indifferent between locations, that real estate managers extract all surplus from households who purchase houses, and that employed renters do not move from the high-wage city to the low-wage one, we may re-write the Bellman equations for renters:

$$\rho W_1^R = w_1 + \pi^R - r_1 + \kappa + \delta(U^R - W_1^R) + \mu^*(W_2^R - W_1^R) \quad (18)$$

$$\rho U^R = z + \pi^R - r_1 + \kappa + \mu(W_1^R - U^R) + \mu^*(W_2^R - U^R) \quad (19)$$

$$\rho W_2^R = w_2 + \pi^R - r_2 + \kappa + \delta(U^R - W_2^R) \quad (20)$$

$$\rho U^R = z + \pi^R - r_2 + \kappa + \mu(W_2^R - U^R) + \mu^*(W_1^R - U^R). \quad (21)$$

Subtracting (19) from (18) and (21) from (20), and solving in terms of U^R we have expressions for the values of unemployed renters in the two cities:

$$W_1^R = \underbrace{\left[\frac{w_1 - z}{\rho + \delta + \mu + \mu^*} \right]}_{\Gamma_1} + U^R. \quad (22)$$

$$W_2^R = \underbrace{\left[\frac{w_2 - z - \mu^* \Gamma_1}{\rho + \delta + \mu} \right]}_{\Gamma_2} + U^R. \quad (23)$$

Using (22) and (23) it is easily shown that

Lemma 1. *If $w_2 > w_1$, then $\Gamma_2 > \Gamma_1$ and in a stationary equilibrium, $W_2^R > W_1^R$.*

Note that equating (19) and (21), and using (22) and (23), it is also apparent that

$$r_2 - r_1 = \left[\frac{\mu - \mu^*}{\rho + \delta + \mu} \right] (w_2 - w_1) > 0 \quad (24)$$

Thus, the rental rate is higher in the high wage city by an amount proportional to the wage differential that depends on job offer arrival and destruction rates, and the interest rate. The *levels* of the rental rates themselves depend on the value of being an unemployed renter, U^R .

The sale price of a house in City i may be shown to satisfy

$$p_i = \frac{\gamma_i}{\rho + \gamma_i} [\alpha_i(W_i^H - W_i^R) + (1 - \alpha_i)(U_i^H - U^R)] \quad (25)$$

where $\alpha_i \equiv N_i^{WR}/R_i$ represents the fraction of renters in City i that are employed. Since, in each city renters constitute the potential buyers in the housing market, we may write (2) as:

$$\gamma_i = \frac{\lambda R_i}{H_i - N_i^{WH} - N_i^{UH}} \quad i = 1, 2. \quad (26)$$

Combining (9), (11) and (26) we may derive a locus of values for γ_1 and γ_2 which are consistent with equilibrium conditions **iii.** (rental market clearing) and **vi.** (aggregation):

$$\lambda \left[\frac{R_1}{\gamma_1} + \frac{R_2}{\gamma_2} \right] = R_1 + R_2 + H_1 + H_2 - 1. \quad (27)$$

We depict this locus (labeled AM) in Figure 1.⁹ As we demonstrate below, one consequence of Lemma 1 is that $\gamma_2 > \gamma_1$. Thus, without loss of generality, the equilibria that we study here are all located on the the segment of the AM curve above the 45° line.

Recall that we consider equilibria in which employed renters will move from the low-wage city to the high-wage one if offered a job, but not *vice versa* and that in this case employed *home owners* will also not move from the high-wage city to the low-wage one. Assuming that an equilibrium exists, there are two different possible cases with regard to the movement of *home owners* between cities:

- I. Some fraction (possibly all) of unemployed home owners in the low-wage city move to the high-wage city if offered a job there, but employed home owners do not move.
- II. All unemployed home owners and some fraction of the employed home owners (again, possibly all) in the low-wage city move if offered a job in the high-wage city.

In Case I, we say that the *marginal home owner* (*i.e.* a home owner who is indifferent between moving from the low-wage to the high-wage city upon receiving a job offer there) is unemployed. Case II, in contrast, is that in which the marginal home owner is employed. We consider the two cases separately. We will see that which case obtains depends on the magnitude of the wage differential between cities and is reflected in the relative liquidity of the housing markets in the two cities.

⁹In the figure, $\underline{\gamma}_1$ and $\underline{\gamma}_2$ are asymptotic values below which the matching rates in each respective city cannot feasibly fall.

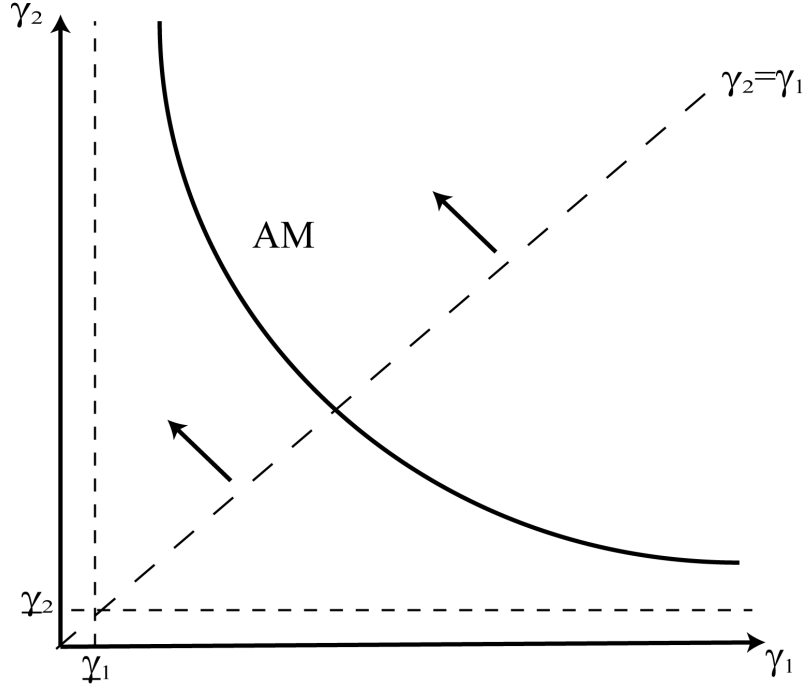


Figure 1: The AM Curve

3.1 Case I: The marginal home-owner in City 1 is unemployed.

Define θ_i^{UH} and θ_i^{WH} respectively as the probabilities with which unemployed and employed home owners in City i move if they receive a job offer in the other city within a unit of time. Alternatively, we may think of these as the *fractions* of these households that accept such offers conditional on receiving one. Case I equilibria are those in which

$$\theta_1^{UH} \in (0, 1], \quad \theta_1^{WH} = 0, \quad \text{and} \quad \theta_2^{UH} \in (0, 1]. \quad (28)$$

That is, equilibria in which unemployed home-owners in both cities accept job offers requiring relocation with some probability, but employed home owners in the low-wage city decline to relocate with probability one.

The steady-state flow of households between states in a Case I equilibrium is described

by (9), (26) and the following equations:

$$(\delta + \mu^* + \lambda)N_1^{WR} = \mu N_1^{UR} + \mu^* (N_2^{UR} + \theta_2^{UH} N_2^{UH}) \quad (29)$$

$$(\mu + \mu^* \theta_1^{UH}) N_1^{UH} = \delta N_1^{WH} + \lambda N_1^{UR} \quad (30)$$

$$\delta N_1^{WH} = \lambda N_1^{WR} + \mu N_1^{UH} \quad (31)$$

$$(\delta + \lambda) N_2^{WR} = \mu N_2^{UR} + \mu^* (N_1^{UR} + N_1^{WR} + \theta_1^{UH} N_1^{UH}) \quad (32)$$

$$(\mu + \mu^* \theta_2^{UH}) N_2^{UH} = \delta N_2^{WH} + \lambda N_2^{UR} \quad (33)$$

$$\delta N_2^{WH} = \lambda N_2^{WR} + \mu N_2^{UH}. \quad (34)$$

Equation (29) says that the measure of agents that cease being employed renters in City 1 (by losing their job, accepting an offer in city 2, or buying a house) equals the measure that *become* employed renters in that city (unemployed renters in either city who receive offers in City 1 or unemployed home-owners in City 2 that receive and accept City 1 job offers). Similarly, (30) equates the flows into and out of being an unemployed home owner in City 1, and (31) does the same for employed home owners in that city. Equations (32)-(34) represent the analogous conditions for city 2.¹⁰

Within Case I, there are three distinct possibilities. In what we refer to as the *interior* sub-case, a fraction of unemployed homeowners in each city accept job offers in the other city: $\theta_1^{UH} < 1$ and $\theta_2^{UH} < 1$. There are also two cases that we refer to as *corners*: In Corner *X* *all* unemployed homeowners in the low-wage city accept jobs in the high-wage city: $\theta_1^{UH} = 1$ and $\theta_2^{UH} < 1$. In Corner *Y* *all* unemployed home owners in the high-wage city accept jobs in the low-wage city: $\theta_1^{UH} < 1$ and $\theta_2^{UH} = 1$.

Consider Corner *X*, in which an unemployed home owner in the low-wage city accepts a high-wage offer with probability one. This implies that the measure of houses for sale in City 1 is at its highest within Case I (as it could only be higher if *employed* home owners began selling houses in order to relocate—*i.e.* in Case II). Thus, the matching rate for sellers in City 1 reaches a minimum value that we denote γ_1^X . Similarly, when all unemployed home owners in the *high-wage* city relocate to the low-wage city whenever possible (in Corner *Y*), the matching rate in City 2 reaches a minimum value that we denote γ_2^Y . It follows that in the interior sub-case, the matching rates for home buyers in each city must exceed these critical levels, *i.e.* $\gamma_1 > \gamma_1^X$ and $\gamma_2 > \gamma_2^Y$. Also, in each corner, when the matching rate for either city hits its critical level, the matching rate in the *other* city (which we denote γ_2^X or γ_1^Y) is determined by the AM curve, (27).

¹⁰ The asymmetry between (29) and (32) stems from (14).

We begin our analysis with the interior sub-case. In order an unemployed home owner to leave the low-wage city for a job in the high-wage one with interior probability ($\theta_1^{UH} \in (0, 1)$), it must be that they are indifferent between the two situations. That is

$$W_2^R = U_1^H - p_1. \quad (35)$$

Similarly, we require

$$W_1^R = U_2^H - p_2 \quad (36)$$

if unemployed home owners in the high-wage city accept jobs in the low-wage city with interior probability ($\theta_2^{UH} \in (0, 1)$).

Home owners' Bellman equations in this case are given by

$$\rho W_1^H = w_1 + \pi^H + \kappa + \delta (U_1^H - W_1^H) \quad (37)$$

$$\rho U_1^H = z + \pi^H + \kappa + \mu (W_1^H - U_1^H) + \mu^* (W_2^R + p_1 - U_1^H) \quad (38)$$

$$\rho W_2^H = w_2 + \pi^H + \kappa + \delta (U_2^H - W_2^H) \quad (39)$$

$$\rho U_2^H = z + \pi^H + \kappa + \mu (W_2^H - U_2^H) + \mu^* (W_1^R + p_2 - U_2^H) \quad (40)$$

Combining these with (22), (23) and (25) we can derive expressions for the relationship between the net surplus from being an unemployed homeowner in each city and the value of being an unemployed renter, U^R :

$$U_1^H - p_1 = \Upsilon_{U_1}^I(\gamma_1, \cdot) + \Xi_{U_1}^I(\gamma_1, \cdot)U^R \quad (41)$$

$$U_2^H - p_2 = \Upsilon_{U_2}^I(\gamma_2, \cdot) + \Xi_{U_2}^I(\gamma_2, \cdot)U^R. \quad (42)$$

Here $\Upsilon_{U_1}^I > 0$, $\Upsilon_{U_2}^I > 0$, $\Xi_{U_1}^I \in (0, 1)$ and $\Xi_{U_2}^I \in (0, 1)$ are functions of the underlying parameters of the model (the actual expressions are given the appendix). The Υ 's and Ξ 's are all decreasing in the values of the applicable γ_i , reflecting the fact that in each city the house sale price is increasing in the matching rate.

Figure 2 depicts the (linear) relationships represented in (22), (23), (41) and (42). In order for (1) unemployed home-owners to be indifferent with regard to relocating and (2) unemployed renters to be indifferent between cities, γ_1 and γ_2 must be such that (22) and (41) intersect at the same value of U^R as do (23) and (42). If, for instance, γ_1 were to increase, (41) would shift down. This would imply that $U_1^R < U_2^R$ inducing unemployed renters to move. Since indifference is required in equilibrium, in this case γ_2 would have to increase also.

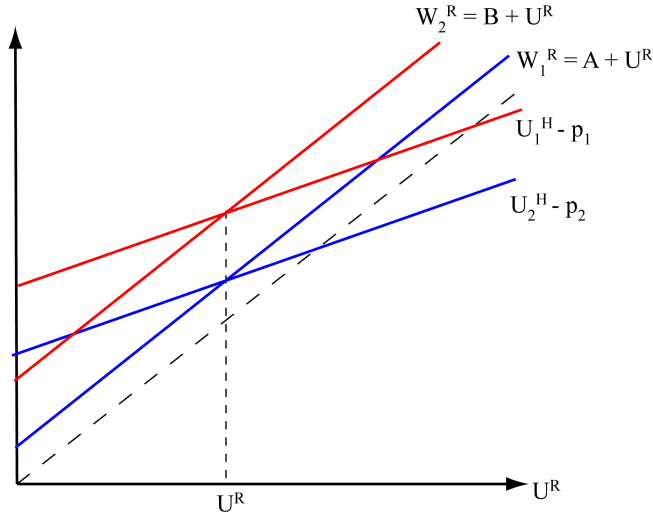


Figure 2: Determination of demand-side relationship between γ_1 and γ_2

The implied positive relationship between γ_1 and γ_2 in an interior Case I equilibrium is in fact linear (see appendix) and may be written:

$$\gamma_2 = \Omega^I + \Psi^I \gamma_1 \quad (43)$$

where Ω^I and Ψ^I are positive constants. An increase in γ_1 raises house prices in City 1, thereby lowering the cost of relocation to unemployed home-owners in that city. To maintain indifference, (35), this must be offset by an increase in the rental rate in City 2. To maintain the indifference of unemployed renters, (16), this must in turn be matched by an increase in the rental rate in City 1. As a result, migration from City 2 declines and the consequent decline in houses for sale pushes up γ_2 . Figure 2 depicts this relationship (labeled VVI) together with AM and illustrates an interior Case I stationary equilibrium.

We next consider the two corner sub-cases, and these are depicted in Figure 4. In Corner Y, unemployed home owners in the high-wage city accept jobs (and initially rent) in the low-wage city with probability one. In this case, $\gamma_2 = \gamma_2^Y$ and $\gamma_1 = \gamma_1^Y$. Diagrammatically, we can imagine moving toward this case when the w_1 rises toward w_2 (*i.e.* the differential *lessens*). In this case VVI shifts downward and to the right (see Figure 4). Intuitively, as the difference between wages in the two cities lessens, unemployed home owners in the high-wage city have less incentive to wait for a high-wage job and are thus more likely to accept an offer of employment in the low-wage city. This increases the number of homes for sale in the high-wage city and lowers γ_2 relative to γ_1 . The corner occurs at any relative wage such

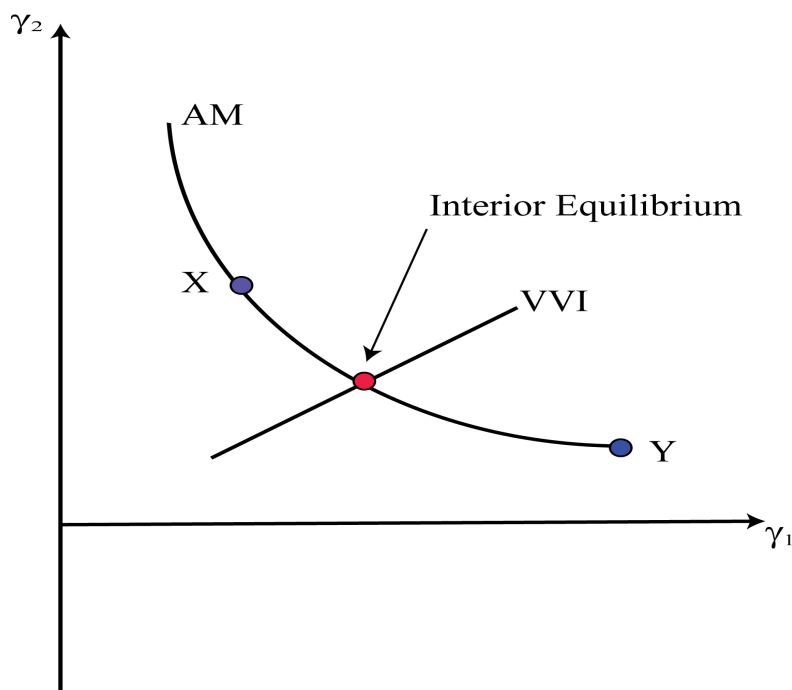


Figure 3: Case I Interior Equilibrium

that VVI lies to the right of Y.

Corner X occurs when all unemployed home owners in the *low-wage* city accept job offers in the high-wage city. In this case $\gamma_1 = \gamma_1^X$, and $\gamma_2 = \gamma_2^X$. We approach this corner as w_1 falls relative to w_2 (and the wage differential widens). In this case it is more attractive for home owners in the low-wage city to become renters in the high-wage city, despite not owning a home and paying higher rent. This results in VVI shifting upward and to the left, increasing the number of homes for sale in the low-wage city and lowering γ_1 relative to γ_2 . The corner occurs when VVI lies above and to the right of X . At this from this point, further increases in the wage differential eventually result in the marginal low-wage city home owner being employed—*i.e.* in Case II.

3.2 Case II: The marginal home-owner in City 1 is employed

Case II equilibria are those in which

$$\theta_1^{UH} = 1, \quad \theta_1^{WH} \in (0, 1], \quad \text{and} \quad \theta_2^{UH} \in (0, 1). \quad (44)$$

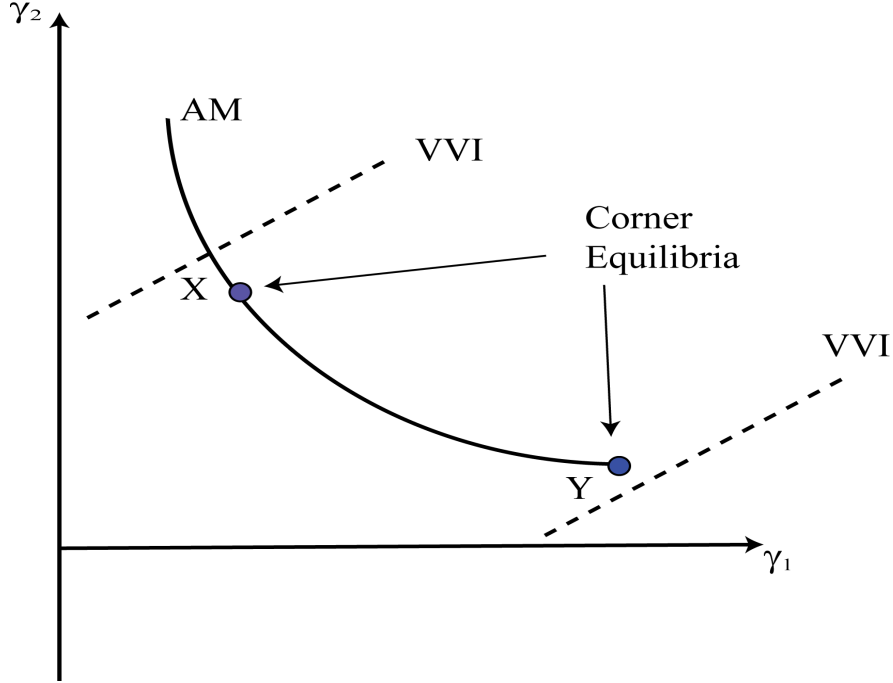


Figure 4: Case I Corner Equilibria

That is, those in which job offers in the high-wage city are accepted by home owners in the low-wage city with probability one if they are unemployed and with strictly positive probability even if they are employed. We now have two sub-cases:¹¹ (1) The *interior* sub-case, in which employed home owners in the low-wage city *reject* high-wage job offers with positive probability ($\theta_1^{WH} \in (0, 1)$) and (2) Corner *Z* in which they never do. Our analysis of these cases is essentially parallel to that presented in section 3.1.

In Corner *Z*, the measure of houses for sale in City 1 reaches its absolute maximum (because *all* home owners relocate if they get the chance), and this implies a lower bound on γ_1 which we denote γ_1^Z . At the other extreme (for Case II), in Corner *X*, no employed City 1 owners relocate, so that $\gamma_1 < \gamma_1^X$. Finally, since in all Case II equilibria unemployed home owners in City 2 reject low-wage offers with positive probability, house sales in City 2 are below their maximum so that $\gamma_2 > \gamma_2^Y$ and $\gamma_1 < \gamma_1^Y$. Thus, in a Case II equilibrium the matching rate for sellers in the low-wage City must satisfy

$$\gamma_1^Z < \gamma_1 < \min [\gamma_1^X, \gamma_1^Y]. \quad (45)$$

¹¹Note, however, that Corner case *X* may also be considered a sub-case of Case II.

As before we begin with the interior sub-case. As unemployed home owners in the high-wage city are indifferent with regard to relocation if they receive a low-wage offer, (36) continues to hold as in Case I. Now, however, it is employed home owners, rather than unemployed ones, in City 1 who are indifferent with regard to accepting a job in City 2. Thus, (35) is replaced by

$$W_1^H - p_1 = W_2^R. \quad (46)$$

The Bellman equations for home owners in City 2 and for unemployed owners in City 1 remain the same as in Case I. That for employed owners in City 1, however, is now given by

$$\rho W_1^H = w_1 + \pi^H + \kappa + \delta (U_1^H - W_1^H) + \mu^* (W_2^R + p_1 - W_1^H). \quad (47)$$

Following a similar procedure as in Case I, one can derive an expression the net value of being a employed home owner in City 1:

$$W_1^H - p_1 = \Upsilon_{W_1}^{II}(\gamma_1) + \Xi_{W_1}^{II}(\gamma_1)U^R \quad (48)$$

As before $\Upsilon_{W_1}^{II} > 0$ and $\Xi_{W_1}^{II} \in (0, 1)$ (see appendix) are decreasing in γ_1 . To characterize the relationship between γ_1 and γ_2 in this equilibrium, we can replace (41) with (48).

As in Case I, we may again derive a linear relationship between γ_1 and γ_2 given by

$$\gamma_2 = \Omega^{II} + \Psi^{II}\gamma_1 \quad (49)$$

where Ω^{II} and Ψ^{II} (see appendix) are constants which depend on labour market conditions. Figure 5 depicts this relationship (labeled VVII) together with AM and illustrates Case II stationary equilibria. Note that because $\Omega^{II} < \Omega^I$ and $\Psi^{II} < \Psi^I$, VVII always lies to the right of VVI.

We now turn to Corner Z , in which home owners in the low-wage city who receive a job offer in the high-wage city accept it with probability one regardless of their employment status. In this case $\gamma_1 = \gamma_1^Z$ and γ_2 is determined by the AM curve, (27). Intuitively, as w_2 rises relative to w_1 , employed home owners in the low wage city accept job offers in the high-wage city with higher probability, increasing the number of houses for sale in the low-wage city and driving γ_1 down relative to γ_2 . Diagrammatically, the corner occurs when VVII has shifted all the way to the left of AM (See Figure 5).

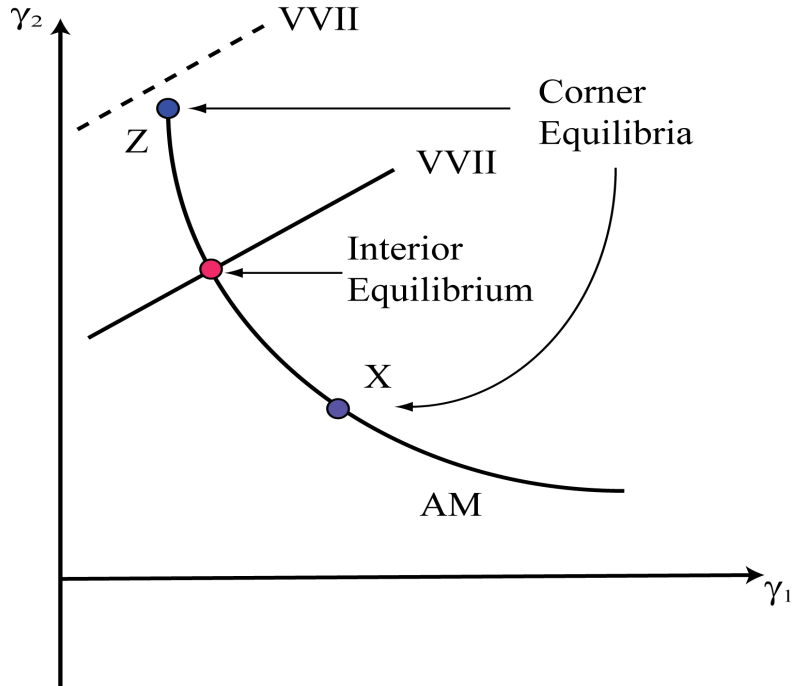


Figure 5: Case II Equilibria

3.3 Existence and Uniqueness

We now establish the existence of a unique stationary equilibrium within the class that we consider and have described above. As a preliminary, we first identify a restriction on parameters sufficient to ensure that in any stationary equilibrium the surplus is positive in any match between an REM and a potential home buyer:¹²

Proposition 1. *If $\mu > \alpha_1 \gamma_1^X$ then in a stationary equilibrium, both employed and unemployed renters buy houses when they get the chance. That is, (15) holds.*

We have our main existence result, which is proved in the appendix:

Proposition 2. *Subject to parameter restrictions (see appendix), there exists a unique stationary equilibrium.*

Note that stationarity of the equilibrium implies

¹²Note that this restriction is required only to ensure that in Corner X unemployed renters wish to buy houses. In our analysis of calibrated examples below, it is never binding.

Corollary 1. *In a stationary equilibrium, house sales take place in both cities. That is,*

$$\theta_1^{UH} > 0 \quad \text{and} \quad \theta_2^{UH} > 0. \quad (50)$$

4 Mobility, Home Ownership and Unemployment in the Basic Model

Implicit in our discussion of the various equilibrium cases above is the assertion that the rate at which REM's match with home buyers is highest in the high wage city. We now demonstrate this formally:

Proposition 3. *If the wage differential across cities, $w_2 - w_1$, is sufficiently high, then the matching rate is highest in the high-wage city: $\gamma_2 > \gamma_1$.*

Below, we will show in examples that the wage differential may indeed be very small.

In equilibrium, the home ownership rate in City i can be expressed as a function of the matching rate:

$$h_i(\gamma_i) = \frac{\frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}. \quad (51)$$

The rate of home ownership is increasing in the ratio of the stock of owned to rental housing. Moreover, since h_i is increasing in γ_i it follows that

Corollary 2. *If the ratio of the stocks of owned and rental houses is the same in both cities, then home ownership is greatest in the high-wage city: $h_2 > h_1$.*

Since home owners and renters receive offers and are separated from jobs at the same rates, they differ only with regard to the likelihood with which they accept offers. As only home owners turn down jobs in equilibrium, the following result is not surprising:

Proposition 4. *The unemployment rate among homeowners exceeds that among renters.*

This result is consistent with the empirical findings of Coulson and Fisher (2008) who, when controlling for demographic and locational differences between home owners and renters estimate a higher likelihood of unemployment for U.S. home owners. Because households in our model are *ex ante* identical, Proposition 4 is not in conflict with the observation that in the data unemployment is higher among *all* renters than among all home owners.

The following proposition characterizes the tendency of unemployed renters to move to the low-wage city:

Proposition 5. *There exists $\sigma \in (0, 1)$ such that if $R_1/R_2 > \sigma$, then the fraction of renters who are employed is greatest in the high wage city: $\alpha_2 > \alpha_1$.*

In any stationary equilibrium, the majority of households (all renters and some homeowners) resident in the low-wage city 1 that receive high-wage job offers move to accept them. In contrast, only unemployed renters and some fraction unemployed home owners resident in the high-wage city migrate to the low-wage city to accept a job offer. This asymmetry tends to drive up the rental rate in the high-wage city relative to that in the low-wage city. This in turn may induce unemployed households with no job offer to remain in or move to the low-wage (and low rent) city. Consequently, the proportion of renters who are unemployed tends to be higher in the low-wage city. Proposition 5 shows that this is true unless rental housing in the low-wage city is sufficiently scarce.

At the city level, then the unemployment rate reflects a trade-off between two effects. The unemployment rate in city i can conveniently be expressed as

$$\nu_i = \underbrace{\left[\frac{\delta}{\delta + \mu} \right] h_i}_{\text{home ownership effect}} + \underbrace{\left[1 - \left(\frac{\delta + \mu + \lambda}{\delta + \mu} \right) \alpha_i \right]}_{\text{rent differential effect}} (1 - h_i) \quad (52)$$

where h_i is given by (51). The first term reflects the positive impact of home-ownership to the city's unemployment rate due to the fact that some home owners turn down job offers rather than relocate. The second term reflects the fact that there is typically a higher concentration of unemployed renters (represented in (52) by α_i) in the city with the lower rental rate. The home ownership effect is typically larger in the high-wage city, as home ownership is higher there. In contrast, the rent differential effect is typically higher in the low-wage city, as unemployed households to some extent flow there in order to take advantage of relatively low rent. Overall the relationship between unemployment and home ownership at the city level depends on which of these effects dominate.

The aggregate unemployment rate, $\bar{\nu}$, in Case I is given by

$$\bar{\nu} = \frac{\delta(\delta + \mu) - \lambda(\mu + \mu^* + \lambda)}{(\delta + \mu)(\delta + \mu + \mu^* + \lambda)} + \left(\frac{\delta(\mu^* + \lambda) + \lambda(\mu + \mu^* + \lambda)}{(\delta + \mu)(\delta + \mu + \mu^* + \lambda)} \right) \bar{h} \quad (53)$$

where $\bar{h} = 1 - R_1 - R_2$ is the aggregate rate of home-ownership. Thus it may be seen that

Proposition 6. *Aggregate unemployment is monotonically increasing in the economy-wide aggregate home ownership rate.*

Home ownership thus contributes to aggregate unemployment. Whether or not this effect is quantitatively significant depends on parameters. We take up this issue next, using a parameterized version of the basic model.

4.1 A Baseline Example

We consider a parameterized version of the model in order to illustrate the characteristics of a “typical” equilibrium. As a baseline example, we choose parameters so that in the stationary equilibrium, our economy is consistent with several observed aspects of the U.S. economy. The parameter values and the relevant targets are given in Table 1. We base our calibration on monthly data and, where possible, draw estimates from the literature which reflect that frequency. In particular, target values for the discount rate, the hiring rate and the separation rate are taken from Shimer (2005), as are values for the income replacement rate and the unemployment rate. Target values for the home ownership rate and the vacancy rate for owner-occupied homes, which determine the total measure of rental and owner-occupied housing per capita, are taken from the most recent US Census. The difference between the flow utilities from owned and rented housing is chosen so that the average rent-mortgage differential in both cities is negative as suggested by the estimates of Campbell, Davis, Gallin, and Martina (2008).

Table 1 — Parameter Choices

Parameter	Value	Target	Value	Source
ρ	0.0038	Annual discount factor	0.953	Shimer (2005)
δ	0.026	Monthly separation rate	0.026	Shimer (2005)
z/w_1	0.4	Income replacement rate	0.4	Shimer (2005)
w_2/w_1	1.1	Dense/non-dense metro premium	0.1	Glaeser & Mare (2001)
$R_1 + R_2$	0.32	Home-ownership rate	0.68	US Census
$H_1 + H_2$	0.705	Homeowner vacancy rate	0.028	US Census
μ	0.43	$\left\{ \begin{array}{l} \text{Monthly hiring rate} \\ \text{Unemployment rate} \\ \text{Ann. mobility (counties)} \end{array} \right.$	0.44	Shimer(2005)
μ^*	0.0145		0.057	US post-war average
λ	0.0012		0.06	US Census
$\pi^H - \pi^R$	0.01			

We assume that the two cities contain equal housing stocks of each type and that the wage in City 2 is 10% higher than the wage in City 1. Our results in this subsection are not

highly sensitive to the particular values of these parameters. They were chosen, however, based on a useful classification of US cities discussed by Overman and Ioannides (1999), who applied earlier work by Knox (1994). There U.S. cities are grouped into four tiers: The top tier consists of 10 ‘nodal centers’¹³, the second tier consists of 14 regional centers¹⁴ and the third tier consists of 19 sub-regional centers.¹⁵ The remaining 291 cities are allocated to the fourth tier.

For calibration purposes we take City 2 to represent the top 3 tiers and City 1 to represent the bottom tier. The total population of the top three tiers is approximately equal to that of the fourth tier and this distribution is fairly stable over time. Glaeser and Mare (2001, Table 3) estimate a dense metropolitan wage premium for cities with more than 500,000 inhabitants of 0.24 log points and a non-dense metropolitan premium of 0.14 log points. We use the difference between these as our estimate of the wage premium between our two cities.

An annual mobility rate (the % of the population that change address in a given year) may be found in the US Census. Although more than 15% of the US population change addresses each year, this includes people who move short distances within a county. For our purposes, a more appropriate estimate of mobility is that between labour markets. We therefore use as a target the component of the mobility rate associated with people who move between counties, which is roughly 6%.¹⁶ We choose jointly values of μ , μ^* and λ to match target values for the monthly hiring rate, the unemployment rate and the cross-county annual mobility rate.

4.1.1 Labour Market Implications

For our baseline calibration the stationary equilibrium is Case I, interior. Table 2 describes the distribution of the total population over the eight possible household states. City 2 has a larger population (slightly) including more employed renters, employed owners and unemployed owners. City 1 has substantially more unemployed renters, reflecting their incentive to move to the city with the lower rental rate.

¹³These are Atlanta, Chicago, Denver, Houston, Los Angeles, New York City, Miami, San Francisco, Seattle and Washington D.C.

¹⁴Baltimore, Boston, Cincinnati, Cleveland, Columbus, Dallas, Indianapolis, Kansas City MO, Minneapolis, New Orleans, Philadelphia, Phoenix, Portland OR, and St. Louis.

¹⁵Birmingham, Charlotte, Des Moines, Detroit, Hartford, Jackson MS, Little Rock, Memphis, Milwaukee, Mobile, Nashville, Oklahoma City, Omaha, Pittsburgh, Richmond, Salt Lake City, Shreveport, Syracuse and Tampa.

¹⁶This is likely to be an upper bound on mobility.

Table 2: Allocation of workers by job and housing status

	Renter	Owner	Renter	Owner
Employed	0.146	0.318	0.156	0.323
Unemployed	0.014	0.019	0.004	0.020

The first column of Table 3 contains statistics on mobility and unemployment for this baseline example. Although the parameters have been chosen to match average mobility, the relative mobility of owners and renters is endogenous. According to the US Census, the unconditional mobility rate for renters averages around 10% and for owners it is 2%. Thus, our model overstates somewhat the mobility of renters and understates the mobility of owners. Our economy, however, is populated by *ex ante* identical households, and it is possible that the census figures could understate the difference between *conditional* mobility rates, for a variety of reasons.¹⁷ We have also ignored direct moving costs and these are likely to affect mobility.

The low-wage city has a higher unemployment rate, due entirely to a high rate of joblessness among renters. The overall unemployment rate among owners is only slightly lower than that for renters, in spite of the fact that unemployed owners turn down opportunities to relocate at a significant rate (about 30%). This is not surprising as given observed average mobility, the rate at which households receive offers from outside their city of residence is very low. Thus, although the mobility of renters is much larger than owners, the consequence for their relative unemployment rates is very small.¹⁸

Recalling (52), the home ownership effect on unemployment is both small and effectively equal across cities. In contrast, the rent differential effect is very large (*i.e.* α_1 is much larger than α_2). Overall, this results in significantly higher unemployment in the low-wage city. Thus, as observed by Coulson and Fisher (2008), our model implies a negative relationship between unemployment and homeownership across cities (as well as a positive one between wages and homeownership).

¹⁷For example, it could be that more educated workers are both more likely to own and to move than less educated ones.

¹⁸Interestingly, however, the implied increase in the likelihood of unemployed associated with home ownership of the same order of magnitude as that estimated by Coulson and Fisher (2008).

Table 3: Labour Market Statistics

	Baseline	Rental only	Asymmetric Cities	Europe	Intracity Relocation	Differential Matching
Mobility rate	0.061	0.158	0.027	0.022	0.059	0.060
– of renters	0.166	0.158	0.070	0.062	0.162	0.165
– of owners	0.007	–	0.007	0.004	0.007	0.006
Population ratio	1.022	1.031	4.213	1.016	1.018	1.022
Unemployment rate	0.057	0.056	0.057	0.100	0.057	0.057
– in low-wage city	0.066	0.086	0.062	0.113	0.066	0.066
– in high-wage city	0.047	0.027	0.055	0.087	0.047	0.047
– amongst renters	0.056	0.056	0.056	0.098	0.056	0.056
– amongst owners	0.057	–	0.057	0.101	0.057	0.057
Rejection rate	0.315	0	0.315	0.440	0.315	0.354
Per capita GDP relative to baseline	1.000	1.002	1.028	0.954	1.000	1.000

4.1.2 Housing Premia

Consider a setting in which there are no frictions in the housing market; that is, in which households are free to either rent or own at any time. In this case, the following no arbitrage condition relates the rental rate in city i to the flow cost of owning a house in that city:

$$\rho p_i - r_i = \pi^H - \pi^R + \dot{p}_i \quad (54)$$

We define the *housing premium* in city i , x_i , as the deviation from (54) due to frictions in the housing markets, measured relative to the purchase price of a house:

$$x_i = \frac{r_i + \pi^H - \pi^R - \rho p_i}{p_i}. \quad (55)$$

We express housing premia relative to the price level, so as to enable comparison to those calculated by Campbell, Davis, Gallin, and Martin (2008). These authors estimate quality-adjusted premia for US cities that vary between 1.84% and 6.45% and average 2.99%. Table 4 presents the values for rents, house prices and housing premia for our baseline example. While in our equilibrium housing premia are small relative to their estimates, that for City 1 is certainly of the right order of magnitude. Overall, our results suggest that a significant fraction of the observed housing premia may be accounted for by the illiquidity of housing.

Table 4: Housing Market Statistics

		Baseline	Asymmetric Cities	Europe	Intracity Relocation	Differential Matching
Low wage city	Rent (relative to wage)	0.03	0.01	0.07	0.04	0.02
	Price (rel. to ann. wage)	0.64	0.23	1.15	0.79	0.45
	Rent-mortgage differential	-0.0002	-0.0002	0.249	0.015	-0.0002
	Annual Housing Premium	1.6%	4.4%	2.7%	1.4%	2.2%
	Matching rate	0.015	0.007	0.008	0.03	0.014
	Ownership rate	67%	66.6%	67.8%	67.7%	67.7%
	Vacancy Rate (quarterly)	3.5%	7.9%	3.2%	3.2%	3.6%
High wage city	Rent (relative to wage)	0.12	0.10	0.16	0.13	0.11
	Price (rel. to ann. wage)	2.71	2.30	3.12	2.86	2.52
	Rent-mortgage differential	-0.048	-0.048	0.155	-0.033	-0.048
	Annual Housing premium	0.2%	0.3%	0.7%	0.3%	0.2%
	Matching Rate	0.090	0.080	0.027	0.122	0.088
	Ownership rate	69%	68.3%	68.3%	68.3%	68.3%
	Vacancy Rate (quarterly)	0.5%	0.5%	1.0%	0.6%	0.5%

The existence of positive housing premium in our equilibrium is indicative of a possible profit opportunity associated with the conversion of rental housing to owner-occupied units. Since in both cities the rent-mortgage differential, $r_i - \rho p_i$, is negative it is not profitable in equilibrium to convert ownable housing to rental, irrespective of the conversion cost.¹⁹ It would, however, be profitable to convert rental units to ownable housing unless the cost of conversion is sufficiently high, *i.e.* $C^H > p_i - r_i/\rho$. Using the rent-mortgage differential in each city, we can then determine the minimum conversion cost necessary to support the equilibrium of our baseline economy:

$$\frac{\rho p_i - r_i}{\rho} = \frac{0.048}{0.047} = 1.02.$$

That is, a (one-time) conversion cost approximately equal to the average monthly wage is sufficient.

Another possibility is for REMs to put rented houses on the market for sale, and then convert them to owner-occupied houses (and pay the conversion cost) only once they have matched with a buyer (this possibility has been excluded up to now).²⁰ The flow value of

¹⁹This is true whenever $\pi^H - \pi^R$ sufficiently large.

²⁰We exclude the possibility of the REM selling the house immediately to the current renter.

renting out a for sale house in City i is $\rho V_i^H(r) = r_i + \gamma_i(\bar{q}_i - C^H - V_i^H(r))$ where \bar{q}_i is the expected price at which the house will sell once a match is made. It follows that

$$V_i^H(r) = \frac{r_i + \gamma_i(\bar{q}_i - C^H)}{\rho + \gamma_i} \quad (56)$$

In a stationary equilibrium, the value of an *unrented* vacant house satisfies

$$V_i^H = \frac{\gamma_i \bar{q}}{\rho + \gamma_i} \quad (57)$$

It follows that the REM will not rent temporarily as long as $V^H > V^H(r)$ or if $C^H > r_i/\gamma_i$. In our baseline example, a conversion cost approximately one and a half times the the average monthly wage is sufficient to prevent REM's from putting rented houses up for sale.

4.2 Alternative Parameterizations

4.2.1 Impact of Aggregate Home Ownership

Qualitatively, as noted above, home ownership contributes to unemployment. Quantitatively, however, the effect is typically tiny. For example, in our baseline calibration a 10 percentage point increment in the aggregate rate of home ownership results in an increase in the unemployment rate of only 0.04 percentage points.²¹ This is the case despite the fact that the model *does* imply large differences in mobility between renters and owners. The high relative mobility of renters in our economy is driven by employed renters moving from the low-wage city to the high-wage one.²² In the equilibrium we consider, employed owners do not move.

It is useful to compare the baseline example with one in which all housing is rental. That is, suppose that the number of rental units in each city equals the total housing stock in the baseline example. In this case, there is excess supply of rental units and all the slack in this market arises in the low wage city. Consequently, the rental rate in city 1 falls to its lowest possible value which, in the absence of maintenance costs, is zero: $r_1 = 0$.

In equilibrium, the unemployed are indifferent between locations: $U_1^R = U_2^R = U^R$. Consequently, they will always accept offers of employment in either city. As before, the

²¹This effect is thus much smaller than that suggested by some commentators. For example, based on cross-country and cross-regional correlations, Oswald (1999) suggests that a 10 percentage point increase in home ownership is associated with a 1.3 percentage point increase in unemployment.

²²While the *fraction* of unemployed renters that move (to the low-wage city from the high-wage one) exceeds that of employed renters, there are many more employed renters.

employed in City 1 will accept offers from City 2 but not vice versa. The flows of workers in the stationary equilibrium therefore satisfy the following conditions:

$$\begin{aligned}
N_1^{WR} + N_1^{UR} &= 1 - R_2 < R_1 \\
N_2^{WR} + N_2^{UR} &= R_2 \\
(\delta + \mu^*)N_1^{WR} &= \mu N_1^{UR} + \mu^* N_2^{UR} \\
\delta N_2^{WR} &= \mu N_2^{UR} + \mu^* (N_1^{UR} + N_1^{WR})
\end{aligned}$$

Since $r_1 = 0$, the flow value of being an unemployed renter is simply

$$\rho U^R = \Gamma_2 + \pi^R + \kappa + \mu \Gamma_1 + \mu^* \Gamma_2$$

and the rent in city 2 is given by

$$r_2 = \left(\frac{\mu - \mu^*}{\rho + \delta + \mu} \right) (w_2 - w_1).$$

The second column in Table 3 presents labour market statistics for an example with rental housing only in which all other parameters remain at their baseline values. As can be seen, average mobility increases substantially (from 0.06 to 0.16). Unemployment has now shifted considerably to the low wage city, largely reflecting the flow of unemployed households to the location where rents are low. Nevertheless, the consequence for aggregate unemployment is very small, amounting to only a 0.1 percentage point decrease. Moreover, the consequence of this change for per capita GDP is also tiny.

4.2.2 Asymmetric Cities

In our baseline example we assumed that the housing stocks in the two cities are the same. The third column of Table 3 documents the implications of allowing the housing stock in the high wage city to be four times the size of that in the low wage city, while keeping the total stock of houses the same. The average mobility rate falls substantially as a result of this because most of the population are employed in the high wage city and are less likely to leave than those in the low wage city. Indeed, it is the reduction in the average mobility rate of renters that drives most of this effect.

While asymmetry of this type reduces the difference in unemployment rates across cities, it has no consequence for the difference between the rates of unemployment for renters and owners. It does, however, have significant consequences for the housing markets (Table 4). Although rents and prices fall in both cities, they fall by much more in the low wage city which also experiences a much higher vacancy rate.

4.2.3 A High Unemployment / Low-Mobility, “European” Example

Mobility in European countries tends to be much lower than for the US. At the same time, worker flow rates in European labour markets are markedly different from those observed for the US with lower separation rates and much lower hiring rates as well. At the same time, European unemployment rates tend to be higher than that of the US. To capture these features of a “typical European” economy, we adjust the separation, job offer, and housing market matching rates (see Table 5) holding the other parameters at their values in the baseline example.²³

As in the baseline, the resulting equilibrium is Case I interior. Labour market statistics are given in the fourth column of Table 3. In this case, 45% of home-owners turn down outside offers. As a result, the impact of home-ownership on the likelihood of unemployment is double that of the baseline (an increase of 3.1 % rather than 1.4%). The negative cross-city relationship between unemployment and ownership is also higher: Now a 1 percentage point higher ownership rate is associated with 2.7 percentage point drop in unemployment. Finally, the aggregate effect of home-ownership is more than twice as big as in the baseline example, with a 10% point higher ownership rate resulting in a 0.1 percentage point higher unemployment rate. This aggregate effect is still, however, quantitatively small.

Table 5: European Parameter Choices

Parameter Value	Target Value
$\delta = 0.011$	Monthly separation rate = 0.011
$\mu = 0.097$	$\left\{ \begin{array}{l} \text{Monthly hiring rate} = 0.10 \\ \text{Unemployment rate} = 0.10 \\ \text{Annual mobility rate} = 0.02 \end{array} \right.$
$\mu^* = 0.005$	
$\lambda = 0.0006$	

5 Generalizations of the Basic Model

In this section we consider a number of generalizations of the basic model so as to demonstrate the robustness of our main results.

²³This exercise is meant to be illustrative only, we do not view it as a calibration to any particular economy.

5.1 Intra-city Relocation

In the basic model, we abstract from housing transactions amongst households who do not migrate, but remain within a city. Since all owner-occupied houses within a city are identical, there is no reason for a home owner to sell one house in order to move to another. The existence of intra-city relocation may, however, be important for inter-city migration, as it affects the liquidity of housing. Moreover, most actual relocation is within rather than between cities (although only a small fraction of intra-city moves are job-related).²⁴ We now consider an extension of the model along the lines of Wheaton (1990) which allows for moves within a city. We show that while this results in the housing markets being more liquid overall, it does not substantially affect any of our main results.

Following Wheaton (1990), we assume home-owners experience *housing taste* shocks at rate ψ . On experiencing a shock, the service flow a home owner receives from their current house falls permanently to $\pi^H - \varepsilon$, while that potentially available to them from other houses remains π^H .²⁵ All such *mismatched* owners immediately become potential buyers and start searching for a new house using the same matching technology as renters. Once they find a new house, they immediately sell their old house to an REM at the market price.²⁶ The REM sells them the new house at a price which extracts all of the surplus from the trade:

$$q_i^{WH} = W_i^H - \tilde{W}_i^H + p_i \quad q_i^{UH} = U_i^H - \tilde{U}_i^H + p_i \quad i = 1, 2. \quad (58)$$

where \tilde{W}_i^H and \tilde{U}_i^H denote the values of being a mismatched owner who is employed and unemployed, respectively.

Let \tilde{N}_i^{WH} and \tilde{N}_i^{UH} denote the stocks of mismatched employed and unemployed owners, respectively, in city i . Since the stock of potential buyers now includes mismatched owners as well as renters, it follows that the matching rate in city i 's housing market is given by

$$\tilde{\gamma}_i = \frac{\lambda \left(R_i + \tilde{N}_i^{WH} + \tilde{N}_i^{UH} \right)}{H_i - N_i^{WH} - N_i^{UH} - \tilde{N}_i^{WH} - \tilde{N}_i^{UH}} \quad i = 1, 2. \quad (59)$$

The sale price of a house in City i now satisfies

$$p_i = \frac{\tilde{\gamma}_i}{\rho} \left\{ \eta_i \left[\alpha_i (q_i^{WR} - p_i) + (1 - \alpha_i) (q_i^{UR} - p_i) \right] + (1 - \eta_i) \left[\zeta_i (q_i^{WH} - p_i) + q_i^{UH} - p_i \right] \right\} \quad (60)$$

²⁴Rupert and Wasmer (2008) document some of these facts.

²⁵This shock could represent a change in tastes or an additional child being born, etc.

²⁶As before, this is equivalent to having the seller hold the house vacant until it sells as in Wheaton (1990), but limits the number of states we must consider.

where $\eta_i = R_i / (R_i + \tilde{N}_i^{WH} + \tilde{N}_i^{UH})$ denotes the fraction of buyers that are renters and $\zeta_i = \tilde{N}_i^{WH} / (\tilde{N}_i^{WH} + \tilde{N}_i^{UH})$ the fraction of mismatched owners that are employed.

We focus on an equilibrium similar to Case I above. In particular, parameters are such that the marginal owner in each city is unemployed and satisfied with their current house.²⁷ To illustrate the implications of this generalization, we again consider a numerical example. Relative to the baseline example, we introduce two new parameters: ε and ψ and retain the baseline values for all others. We set $\psi = 0.0015$ so that in the stationary equilibrium the fraction of moving owners who remain within the same city (rather than changing cities) is roughly 60%. This corresponds to the fraction of owners who move but remain within a county in the US census. It implies that 2% of owners become dissatisfied with their current house each year. We set $\varepsilon = 0.005$, half of the difference between π^H and π^R .²⁸

As can be seen in Tables 3 and 4 very little changes in this extension relative to the baseline economy. In particular, while the matching rates increase substantially and house prices in both cities increase reflecting this, the labour market statistics are largely unchanged.

5.2 Differential Matching Rates

In the basic model, unemployed renters match with house-sellers at the same rate, λ , as employed renters. Since unemployed owners are indifferent between remaining in one location and moving to accept an offer in the other, unemployed renters are always willing to buy a house if the price is sufficiently low. There are, however, a variety of reasons why the unemployed may not be able to buy houses as easily as the employed (e.g. the unwillingness of banks to provide them with mortgages). Here we generalize our model to capture this by assuming different matching rates for employed and unemployed renters: λ^w and λ^u respectively, where $\lambda^w > \lambda^u$.

This extension changes the model very little and has minimal effect on the equilibrium. The probability that the home buyer in a match is an employed renter is now given by

$$\hat{\alpha}_i = \frac{\lambda^w N_i^{WR}}{\lambda^w N_i^{WR} + \lambda^u N_i^{UR}} \quad (61)$$

²⁷For some parameter configurations, the marginal home owner could be one dissatisfied with their current match. Thus, this extension of the environment introduce several additional equilibrium cases. A full analysis of all the possible cases is omitted for brevity.

²⁸Our results are largely insensitive to the exact value of ε , provided it is less than $\pi^H - \pi^R$.

and the seller's matching rate is now

$$\hat{\gamma}_i = \frac{\lambda^w N_i^{WR} + \lambda^u N_i^{UR}}{H_i - N_i^{WH} - N_i^{UH}}. \quad (62)$$

The extension is straightforward and the qualitative results from the basic model are essentially unaffected.²⁹

As may be seen in the last columns of Tables 3 and 4, the quantitative effects of this extension are also small, even if $\lambda^u = 0$. There is a small increase in the steady state measure of unemployed renters. But because these make up only a small fraction of the population, this has no significant quantitative effect.

5.3 Rental Vacancies

In the basic model, we abstract entirely from frictions in the rental market. As a result, all rental units are occupied and there are no vacancies unless the rental market is slack (and $r_i = 0$). In reality, vacancy rates for rental units are often higher than for owner-occupied units, so one may wonder whether this would affect the nature of our results. The key issue, however, is whether vacancies in the rental market are associated with costs to households of moving and therefore affect mobility. Vacancies in the rental market are more likely to be symptomatic of the fact that, once a rental unit is vacated, it may not immediately be available to the rental market. For example, maintenance and decorating may be needed before it is ready to be rented again.

Here we show that it is straightforward to accommodate rental vacancies in the model without changing any of our results. We assume that, once it is vacated, a rental unit can only be returned to the market at an exogenous rate τ . The main consequence of this friction is that the effective rental stock is less than the actual stock of rental units, with the difference consisting of rental vacancies. Specifically, one can show that in the steady-state

$$N_1^{WR} + N_1^{UR} = \frac{\tau R_1}{\tau + \lambda + \mu^*} = \hat{R}_1 \quad (63)$$

and

$$N_2^{WR} + N_2^{UR} = R_2 - \left(\frac{\mu^* + \lambda}{\tau + \lambda + \mu^*} \right) R_1 = \hat{R}_2. \quad (64)$$

where \hat{R}_i represents the rental housing that is available for rent in City i . We can therefore simply replace the actual rental stocks, R_1 and R_2 , with \hat{R}_1 and \hat{R}_2 respectively, throughout the analysis. Note finally that if $R_1 = R_2$, (64) and (63) are symmetric.

²⁹Calculations may be obtained from the authors upon request.

6 Concluding Remarks

We have developed a two-city model that allows for interactions between search frictions in both housing and labour markets. Housing liquidity—the time it takes to sell a house to an appropriate buyer—determines the value that the seller can get for the house in the event that he/she wishes to move. This determines the different cities’ populations and rates of home-ownership. These, in turn, determine vacancy rates and, hence, the liquidity of housing in each city.

We show that in equilibrium, homeowners are substantially less mobile than renters even though there are no direct barriers or costs to moving. Homeowners turn down job offers in certain circumstances, even if they are currently unemployed or are offered a higher wage than their current one, because the price they can get for their house is insufficient to make migration worthwhile. In particular, the likelihood of unemployment for homeowners exceeds that for otherwise identical renters. In contrast, unemployment is negatively related to ownership rates across cities because unemployed renters tend to move disproportionately to the low rent (low wage) city, where home ownership is also lower.

A baseline version of the model, calibrated to match US labour market flows and average mobility, generates relative mobility rates and unemployment rates for homeowners and renters that accord reasonably well with the evidence. Moreover, we find that unemployment is negatively related to ownership rates across cities. Despite large differences in overall mobility rates between owners and renters, however, we find that the impact of ownership on aggregate unemployment is very small. In a low-mobility calibration, intended to capture the lower mobility and higher unemployment typical of some European economies, we find that all of these effects are magnified to some extent, but that the relationship between home ownership and aggregate unemployment remains weak.

We view the framework developed here as a useful starting point to study the interactions between labour markets, housing markets and the broader economy. It can be built upon in a number of ways that we plan to consider in future research. These include introducing various forms of heterogeneity amongst households and allowing for productivity growth, population growth and housing construction. Moreover, extending the model to incorporate multiple (*i.e.* more than two) cities would allow for a more exhaustive quantitative evaluation. Finally, since housing frictions are likely to play a larger role in transitions than they do in the steady-state, an analysis of the effects of shocks is likely to be especially interesting.

7 Appendix

7.1 Derivation of Case I Stationary Equilibrium

The solution to the 10 equation system described by (9), (26), and (29) – (34) can be expressed recursively as

$$N_2^{WR} = \frac{\mu R_2 + (\mu^* + \lambda)R_1}{\delta + \mu + \lambda} \quad (65)$$

$$N_1^{WR} = \frac{\mu R_1 + (\mu^* + \lambda)R_2 - \mu^* N_2^{WR}}{\delta + \mu + \mu^* + \lambda} \quad (66)$$

$$N_i^{UR} = R_i - N_i^{WR} \quad (67)$$

$$N_i^{UH}(\gamma_i) = \left(\frac{\delta}{\delta + \mu} \right) \left(H_i - \frac{\lambda}{\delta} N_i^{WR} - \frac{\lambda}{\gamma_i} R_i \right) \quad (68)$$

$$N_i^{WH}(\gamma_i) = \left(\frac{\mu}{\delta + \mu} \right) \left(H_i + \frac{\lambda}{\mu} N_i^{WR} - \frac{\lambda}{\gamma_i} R_i \right) \quad (69)$$

$$\theta_i^{UH}(\gamma_i) = \frac{\lambda}{\mu^*} \frac{R_i}{N_i^{UH}(\gamma_i)}. \quad (70)$$

The house prices in City 1 must satisfy

$$\begin{aligned} \rho p_1 &= \gamma_1 [\alpha_1 (W_1^H - p_1 - W_1^R) + (1 - \alpha_1) (U_1^H - p_1 - U^R)] \\ &= \gamma_1 [\alpha_1 (W_1^H - p_1) + (1 - \alpha_1) (U_1^H - p_1)] - \gamma_1 [\alpha_1 \Gamma_1 + U^R] \end{aligned} \quad (71)$$

Subtracting (71) from (37) and rearranging yields

$$(\rho + \delta + \alpha_1 \gamma_1) (W_1^H - p_1) = w_1 + \pi^H + \kappa + \gamma_1 [\alpha_1 \Gamma_1 + U^R] + (\delta - (1 - \alpha_1) \gamma_1) (U_1^H - p_1). \quad (72)$$

Similarly subtracting (71) from (38) and rearranging yields

$$(\rho + \mu + \mu^* + (1 - \alpha_1) \gamma_1) (U_1^H - p_1) =$$

$$\Gamma_2 + \pi^H + \kappa + \gamma_1 [\alpha_1 \Gamma_1 + U^R] + \mu^* (\Gamma_2 + U^R) + (\mu - \alpha_1 \gamma_1) (W_1^H - p_1) \quad (73)$$

Solving for $W_1^H - p_1$ and $U_1^H - p_1$ yields

$$W_1^H - p_1 = \Upsilon_{W_1}^I + \Xi_{W_1}^I U^R \quad (74)$$

$$U_1^H - p_1 = \Upsilon_{U_1}^I + \Xi_{U_1}^I U^R \quad (75)$$

where

$$\Upsilon_{W_1}^I = \frac{(\rho + \delta + \mu + \mu^*) (w_1 + \pi^H + \kappa + \alpha_1 \gamma_1 \Gamma_1) - (\delta - (1 - \alpha_1) \gamma_1) (w_1 - \Gamma_2 - \mu^* \Gamma_2)}{(\rho + \delta + \alpha_1 \gamma_1) \mu^* + (\rho + \gamma_1) (\rho + \delta + \mu)} \quad (76)$$

$$\Xi_{W_1}^I = \frac{(\rho + \mu + \mu^*) \gamma_1 + (\delta + \alpha_1 \gamma_1) \mu^*}{(\rho + \delta + \alpha_1 \gamma_1) \mu^* + (\rho + \gamma_1) (\rho + \delta + \mu)} \quad (77)$$

$$\Upsilon_{U_1}^I = \frac{(\rho + \delta + \mu) (w_1 + \pi^H + \kappa + \alpha_1 \gamma_1 \Gamma_1) - (\rho + \delta + \alpha_1 \gamma_1) (w_1 - \Gamma_2 - \mu^* \Gamma_2)}{(\rho + \delta + \alpha_1 \gamma_1) \mu^* + (\rho + \gamma_1) (\rho + \delta + \mu)} \quad (78)$$

$$\Xi_{U_1}^I = \frac{(\rho + \delta + \mu) \gamma_1 + (\rho + \delta + \alpha_1 \gamma_1) \mu^*}{(\rho + \delta + \alpha_1 \gamma_1) \mu^* + (\rho + \gamma_1) (\rho + \delta + \mu)} \quad (79)$$

Thus, following the same procedure as for city 1 we have

$$W_2^H - p_2 = \Upsilon_{W_2} + \Xi_{W_2} U^R \quad (80)$$

$$U_2^H - p_2 = \Upsilon_{U_2} + \Xi_{U_2} U^R \quad (81)$$

where

$$\Upsilon_{W_2} = \frac{(\rho + \delta + \mu + \mu^*) (w_2 + \pi^H + \kappa + \alpha_2 \gamma_2 \Gamma_2) - (\delta - (1 - \alpha_2) \gamma_2) (w_2 - \Gamma_2 - \mu^* \Gamma_1)}{(\rho + \gamma_2) (\rho + \delta + \mu) + (\rho + \delta + \alpha_2 \gamma_2) \mu^*} \quad (82)$$

$$\Xi_{W_2} = \frac{(\rho + \delta + \mu) \gamma_2 + (\delta + \alpha_2 \gamma_2) \mu^*}{(\rho + \gamma_2) (\rho + \delta + \mu) + (\rho + \delta + \alpha_2 \gamma_2) \mu^*} \quad (83)$$

$$\Upsilon_{U_2} = \frac{(\mu - \alpha_2 \gamma_2) (w_2 + \pi^H + \kappa + \alpha_2 \gamma_2 \Gamma_2) + (\rho + \delta + \alpha_2 \gamma_2) (\Gamma_2 + \pi^H + \mu^* \Gamma_1 + \alpha_2 \gamma_2 \Gamma_2)}{(\rho + \gamma_2) (\rho + \delta + \mu) + (\rho + \delta + \alpha_2 \gamma_2) \mu^*} \quad (84)$$

$$\Xi_{U_2} = \frac{(\rho + \delta + \mu) \gamma_2 + (\rho + \delta + \alpha_2 \gamma_2) \mu^*}{(\rho + \gamma_2) (\rho + \delta + \mu) + (\rho + \delta + \alpha_2 \gamma_2) \mu^*} \quad (85)$$

Interior Case: In this interior solution $\theta_i^{UH} < 1$, $i = 1, 2$. Using (68) and (70) this implies that

$$\gamma_1 > \gamma_1^X = \frac{\lambda R_1}{H_1 - (\lambda/\delta) N_1^{WR} - [(\delta + \mu)/\delta] (\lambda/\mu^*) R_1} \quad (86)$$

and

$$\gamma_2 > \gamma_2^Y = \frac{\lambda R_2}{H_2 - (\lambda/\delta) N_2^{WR} - [(\delta + \mu)/\delta] (\lambda/\mu^*) R_2}. \quad (87)$$

Equating (75) and (35) yields the equilibrium value of U^R as a function of City 1's matching rate

$$\rho U^R(\gamma_1; I) = w_1 + \pi^H + \kappa + \gamma_1 \alpha_1 \Gamma_1 - (\rho + \gamma_1) \Gamma_2 - \frac{(\rho + \delta + \alpha_1 \gamma_1) (w_1 - \Gamma_2)}{\rho + \delta + \mu} \quad (88)$$

Similarly equating (81) and (36) yields the equilibrium value of $U^R(\gamma_2)$:

$$\rho U^R(\gamma_2) = w_2 + \pi^H + \kappa + \gamma_2 \alpha_2 \Gamma_2 - (\rho + \gamma_2) \Gamma_1 - \frac{(\rho + \delta + \alpha_2 \gamma_2)(w_2 - \Gamma_2)}{\rho + \delta + \mu} \quad (89)$$

Equating $U^R(\gamma_1; I) = U^R(\gamma_2; I)$, yields the positive, linear relationship between γ_1 and γ_2 given by (43) where

$$\Omega^I = \frac{w_2 - w_1 + \rho(\Gamma_2 - \Gamma_1) - \frac{(\rho + \delta)(w_2 - \Gamma_2)}{\rho + \delta + \mu} + \frac{(\rho + \delta)(w_1 - \Gamma_2)}{\rho + \delta + \mu}}{\Gamma_1 - \alpha_2 \Gamma_2 + \frac{\alpha_2(w_2 - \Gamma_2)}{\rho + \delta + \mu}} \quad (90)$$

$$\Psi^I = \left(\frac{\Gamma_2 - \alpha_1 \Gamma_1 + \frac{\alpha_1(w_1 - \Gamma_2)}{\rho + \delta + \mu}}{\Gamma_1 - \alpha_2 \Gamma_2 + \frac{\alpha_2(w_2 - \Gamma_2)}{\rho + \delta + \mu}} \right) \quad (91)$$

Corner Y ($\theta^Y = 1$): In this case $\gamma_1 = \gamma_1^Y$ and $\gamma_2 = \gamma_2^Y$. Substituting into (68) and (69) yields the equilibrium measures of owners in each state. In this corner case (35) continues to hold, but (36) does not. Equating (75) and (35) yields

$$\rho U^R(\gamma_1^Y) = w_1 + \pi^H + \kappa + \gamma_1^Y \alpha_1 \Gamma_1 - (\rho + \gamma_1^Y) \Gamma_2 - \frac{(\rho + \delta + \alpha_1 \gamma_1^Y)(w_1 - \Gamma_2)}{\rho + \delta + \mu} \quad (92)$$

Corner X ($\theta^X = 1$): In this case $\gamma_1 = \gamma_1^X$ and $\gamma_2 = \gamma_2^X$. Substituting into (68) – (69) yields the equilibrium measures of owners in each state. In this corner case (36) continues to hold, but (35) does not. Equating (81) and (36) yields

$$\rho U^R(\gamma_2^X) = w_2 + \pi^H + \kappa + \gamma_2^X \alpha_2 \Gamma_2 - (\rho + \gamma_2^X) \Gamma_1 - \frac{(\rho + \delta + \alpha_2 \gamma_2^X)(w_2 - \Gamma_2)}{\rho + \delta + \mu} \quad (93)$$

7.2 Derivation of Case II Stationary Equilibrium

In the case, the steady-state flow of workers across states is described by

$$(\delta + \mu^* + \lambda)N_1^{WR} = \mu N_1^{UR} + \mu^* (\theta_2^{UH} N_2^{UH} + N_2^{UR}) \quad (94)$$

$$(\mu + \mu^*) N_1^{UH} = \delta N_1^{WH} + \lambda N_1^{UR} \quad (95)$$

$$(\delta + \mu^* \theta_1^{WH}) N_1^{WH} = \lambda N_1^{WR} + \mu N_1^{UH} \quad (96)$$

$$(\delta + \lambda) N_2^{WR} = \mu N_2^{UR} + \mu^* (N_1^{UR} + N_1^{UH} + N_1^{WR} + \theta_1^{WH} N_1^{WH}) \quad (97)$$

$$(\mu + \mu^* \theta_2^{UH}) N_2^{UH} = \delta N_2^{WH} + \lambda N_2^{UR} \quad (98)$$

$$\delta N_2^{WH} = \lambda N_2^{WR} + \mu N_2^{UH} \quad (99)$$

The solution to these flow equations can be expressed recursively as (66), (65), (67) and

$$N_1^{UH}(\gamma_1) = \left(\frac{\delta}{\delta + \mu + \mu^*} \right) \left[H_1 + \frac{\lambda}{\delta} R_1 - \frac{\lambda}{\delta} N_1^{WR} - \frac{\lambda}{\gamma_1} R_1 \right] \quad (100)$$

$$N_1^{WH}(\gamma_1) = \left(\frac{\mu + \mu^*}{\delta + \mu + \mu^*} \right) \left[H_1 - \left(\frac{\lambda}{\mu + \mu^*} \right) R_1 + \left(\frac{\lambda}{\mu + \mu^*} \right) N_1^{WR} - \frac{\lambda}{\gamma_1} R_1 \right] \quad (101)$$

$$N_2^{UH}(\gamma_2) = \left(\frac{\delta}{\delta + \mu} \right) \left[H_2 - \frac{\lambda}{\delta} N_2^{WR} - \frac{\lambda}{\gamma_2} R_2 \right] \quad (102)$$

$$N_2^{WH}(\gamma_2) = \left(\frac{\mu}{\delta + \mu} \right) \left[H_2 + \frac{\lambda}{\mu} N_2^{WR} - \frac{\lambda}{\gamma_2} R_2 \right] \quad (103)$$

$$\theta_1^{WH}(\gamma_1) = \frac{1}{N_1^{WH}} \left(\frac{\delta}{\delta + \mu + \mu^*} \right) \left[\left(\frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*} \right) R_1 + \frac{\lambda}{\delta} N_1^{WR} - H_1 + \frac{\lambda}{\gamma_1} R_1 \right] \quad (104)$$

Following the same procedure as in Case I, we can derive the following expression for the net values of ownership in city 1

$$W_1^H - p_1 = \Upsilon_{W_1}^{II} + \Xi_{W_1}^{II} U^R \quad (105)$$

$$U_1^H - p_1 = \Upsilon_{U_1}^{II} + \Xi_{U_1}^{II} U^R \quad (106)$$

where

$$\Upsilon_{W_1}^{II} = \frac{(\rho + \delta + \mu + \mu^*) (w_1 + \pi^H + \kappa + \mu^* \Gamma_2 + \gamma_1 \alpha_1 \Gamma_1) - (\delta - (1 - \alpha_1) \gamma_1) (w_1 - \Gamma_2)}{(\rho + \delta + \mu + \mu^*) (\rho + \mu^* + \gamma_1)} \quad (107)$$

$$\Xi_{W_1}^{II} = \frac{\mu^* + \gamma_1}{\rho + \mu^* + \gamma_1} \quad (108)$$

$$\Upsilon_{U_1}^{II} = \frac{(\rho + \delta + \mu + \mu^*) (w_1 + \pi^H + \kappa + \mu^* \Gamma_2 + \gamma_1 \alpha_1 \Gamma_1) - (\rho + \delta + \mu^* + \alpha_1 \gamma_1) (w_1 - \Gamma_2)}{(\rho + \delta + \mu + \mu^*) (\rho + \mu^* + \gamma_1)} \quad (109)$$

$$\Xi_{U_1}^{II} = \frac{\mu^* + \gamma_1}{\rho + \mu^* + \gamma_1} \quad (110)$$

The Bellman equations and hence the solution for City 2, (80) and (81), remain the same as in Case I.

Interior Solution: In this case $\theta_1^{WH}(\gamma_1) < 1$. Using (101) and (104), this implies that

$$\gamma_1 > \gamma_1^Z = \frac{\lambda R_1}{H_1 - (\lambda/\mu^*) R_1}. \quad (111)$$

We also require that $\theta_1^{WH}(\gamma_1) > 0$, which implies an upper bound on γ_1 that is equivalent to γ_1^X . Finally $N_2^{UH} \geq Y$ implies a lower bound on γ_2 which is the same as γ_2^Y .

Equating (46) and (105) yields

$$\rho U^R(\gamma_1; II) = w_1 + \pi^H + \kappa + \gamma_1 \alpha_1 \Gamma_1 - (\rho + \gamma_1) \Gamma_2 - \frac{(\delta - (1 - \alpha_1)\gamma_1)(w_1 - z)}{\rho + \delta + \mu + \mu^*} \quad (112)$$

For City 2, $U^R(\gamma_2)$ is the same as in Case I and is given by (89). Equating $U^R(\gamma_1; II) = U^R(\gamma_2)$, yields another positive, linear relationship between γ_1 and γ_2 that must pertain in this equilibrium given by (49) where

$$\Omega^{II} = \frac{w_2 - w_1 + \rho(\Gamma_2 - \Gamma_1) - \frac{(\rho + \delta)(w_2 - z)}{\rho + \delta + \mu} + \frac{\delta(w_1 - z)}{\rho + \delta + \mu + \mu^*}}{\Gamma_1 - \alpha_2 \Gamma_2 + \frac{\alpha_2(w_2 - z)}{\rho + \delta + \mu}} \quad (113)$$

$$\Psi^{II} = \left(\frac{\Gamma_2 - \alpha_1 \Gamma_1 - \frac{(1 - \alpha_1)(w_1 - z)}{\rho + \delta + \mu + \mu^*}}{\Gamma_1 - \alpha_2 \Gamma_2 + \frac{\alpha_2(w_2 - z)}{\rho + \delta + \mu}} \right). \quad (114)$$

It is straightforward to show that $\Omega^{II} < \Omega^I$ and $\Psi^{II} < \Psi^I$.

Corner Z ($\theta^Z = 1$): In this case $\gamma_1 = \gamma_1^Z$ and $\gamma_2 = \gamma_2^Z$. Substituting into (100) - (103) yields the equilibrium measures of owners in each state. In this corner case (36) continues to hold, but (35) does not. Equating (81) and (36) yields

$$\rho U^R(\gamma_2^Z) = w_2 + \pi^H + \kappa + \gamma_2^Z \alpha_2 \Gamma_2 - (\rho + \gamma_2^Z) \Gamma_1 - \frac{(\rho + \delta + \alpha_2 \gamma_2^Z)(w_2 - z)}{\rho + \delta + \mu} \quad (115)$$

7.3 Proofs of Main Propositions:

Proof of Proposition 1: We must show that the following inequalities hold in each case:

$$U_1^H - p_1 > U^R \quad (116)$$

$$W_1^H - p_1 > W_1^R \quad (117)$$

$$U_2^H - p_2 > U^R \quad (118)$$

$$W_2^H - p_2 > W_2^R \quad (119)$$

Case I:

- Inequality (116) holds in the interior and corner Y since $U_1^H - p_1 = W_2^R > U^R$.

- In corner X, using (75), we can express (116) as

$$\Upsilon_{U_1}^I(\gamma_1^X) + \Xi_{U_1}^I(\gamma_1^X)U^R(\gamma_1^X) > U^R(\gamma_1^X).$$

Using (78) and (79) and re-arranging we can express this as

$$w_1 + \pi^H + \kappa - (\rho + \delta)\Gamma_1 - \frac{(\rho + \delta + \alpha_1\gamma_1)}{\rho + \delta + \mu}\mu^*(\Gamma_1 - \Gamma_2) > \rho U^R(\gamma_1^X)$$

Using (22) and (23) this can be re-written as

$$w_1 + \pi^H + \kappa - \delta(W_1^R - U^R) + \frac{(\rho + \delta + \alpha_1\gamma_1^X)}{\rho + \delta + \mu}\mu^*(W_2^R - W_1^R) > \rho W_1^R.$$

But $\rho W_1^R = w_1 + \pi^R - r_1 + \kappa - \delta(W_1^R - U^R) + \mu^*(W_2^R - W_1^R)$ and so

$$r_1 > \pi^R - \pi^H - \left(\frac{\mu - \alpha_1\gamma_1^X}{\rho + \delta + \mu}\right)\mu^*(\Gamma_2 - \Gamma_1).$$

Thus, a sufficient condition for this to hold is that $\mu > \alpha_1\gamma_1^X$, since $r_1 \geq 0$ in equilibrium.

- Inequality (117) must hold in all sub-cases since $W_1^H - p_1 > W_2^R > W_1^R$.
- Inequality (118) must be true in the interior sub-case and corner X since $W_1^R > U^R$. In corner Y, we need that

$$\Upsilon_{U_2}(\gamma_2^Y) + \Xi(\gamma_2^Y)U^R > U^R \tag{120}$$

Using (84) and (85) this can be expressed as

$$w_2 + \pi^H + \kappa - (\rho + \delta)\Gamma_2 > \rho U^R$$

Since using (23), this can be re-written as

$$w_2 + \pi^H + \kappa - \delta(W_2^R - U^R) > \rho W_2^R$$

But $\rho W_2^R = w_2 + \pi^R - r_2 + \kappa - \delta(W_2^R - U^R)$ and so we require that $r_2 > \pi^R - \pi^H$. Since $\pi^H \geq \pi^R$ this must be true since the rental rate must be positive in equilibrium.

- Inequality (119) must be true if (118) holds because

$$\begin{aligned} W_2^H - p_2 - W_2^R &> U_2^H - p_2 - U^R \\ W_2^H - U_2^H &> W_2^R - U^R = \Gamma_2 \end{aligned}$$

To see this note that in the interior and corner X ($U_2^H - p_2 = W_1^R$) we have

$$W_2^H - U_2^H = \frac{w_2 - z}{\rho + \delta + \mu} > \Gamma_2$$

In corner Y ($U_2^H - p_2 < W_1^R$) we have

$$\begin{aligned}
W_2^H - U_2^H &= \frac{w_2 - z}{\rho + \delta + \mu} - \frac{\mu^* (W_1^R + p_2 - U_2^H)}{\rho + \delta + \mu} \\
&= \Gamma_2 + \frac{\mu^* \Gamma_1}{\rho + \delta + \mu} - \frac{\mu^* (\Gamma_1 + U^R + p_2 - U_2^H)}{\rho + \delta + \mu} \\
&= \Gamma_2 + \frac{\mu^* (U_2^H - p_2 - U^R)}{\rho + \delta + \mu} > \Gamma_2
\end{aligned}$$

Case II: First observe that in this case it is always true that

$$W_1^H - W_1^R = U_1^H - U^R \quad (121)$$

This follows from subtracting (38) from (47).

- In the interior sub-case, (117) must be true since $W_1^H - p_1 = W_2^R > W_1^R$. From (121) it follows that (116) must also hold in this case.
- In corner Z, using (106), we can express (116) as

$$\Upsilon_{U_1}^H(\gamma_1^Z) + \Xi_{U_1}^H(\gamma_1^Z)U^R(\gamma_1^Z) > U^R(\gamma_1^Z)$$

Using (109) and (110) this can be expressed as

$$w_1 + \pi^H + \kappa - (\rho + \delta)\Gamma_1 + \mu^* (\Gamma_2 - \Gamma_1) > \rho U^R$$

Using (22) and (23) and re-arranging, this can be written as

$$w_1 + \pi^H + \kappa - \delta (W_1^R - U^R) + \mu^* (W_2^R - W_1^R) > \rho W_1^R$$

But $\rho W_1^R = w_1 + \pi^R - r_1 + \kappa - \delta (W_1^R - U^R) + \mu^* (W_2^R - W_1^R)$ and so the condition becomes $r_1 > \pi^R - \pi^H$, which must be true in equilibrium. From (121) it follows that (117) must also hold in this case.

- Inequality (118) must be true in all cases since $W_1^R > U^R$, and (119) follows by the same reasoning as for Case I.

Proof of Proposition 2:

Existence: We require that there is sufficient rental housing at each location to ensure that unemployed renters are the marginal renters overall: $R_1 > N_1^{WR}$ and $R_2 > N_2^{WR}$. Using (65)

and (66) it is straightforward to show that a sufficient condition for this is the ratio of rental housing in each city lies between two bounds:

$$\frac{\delta + \lambda}{\mu^* + \lambda} > \frac{R_1}{R_2} > \frac{\mu^* + \lambda - \mu^* \frac{\mu}{\delta + \mu + \lambda}}{\delta + \lambda + \mu^* \left(1 + \frac{\mu^* + \lambda}{\delta + \mu + \lambda}\right)}. \quad (122)$$

Note that provided that $\delta > \mu^*$, the upper bound must exceed 1 and the lower bound must be less than 1.

Existence of case I requires that $\gamma_1^X < \gamma_1^Y$. That is

$$R_1 + R_2 + \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 + \frac{\lambda}{\delta} N_2^{WR} + H_1 - 1 < H_1 - \frac{\lambda}{\delta} N_1^{WR} - \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_1 \quad (123)$$

which can be re-written as

$$\left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_1 + \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_2 + \frac{\lambda}{\delta} N_1^{WR} + \frac{\lambda}{\delta} N_2^{WR} < 1 \quad (124)$$

That is, the total population must be sufficiently large in comparison with the stock of rental housing.

Existence of case II requires that $\gamma_1^Z < \min[\gamma_1^X, \gamma_1^Y]$. Note from (111) that it must be true that $\gamma_1^Z < \gamma_1^X$. Hence, (124) is a sufficient condition for both cases to exist. If (124) does not hold, case II may still exist if $\gamma_1^Z < \gamma_1^Y$. That is

$$R_1 + R_2 + \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 + \frac{\lambda}{\delta} N_2^{WR} + H_1 - N < H_1 - \frac{\lambda}{\mu^*} R_1$$

which can be written as

$$\left(1 + \frac{\lambda}{\mu^*}\right) R_1 + \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_2 + \frac{\lambda}{\delta} N_2^{WR} < 1 \quad (125)$$

We also require that $r_1 > 0$ and $r_2 > 0$.

Uniqueness: First note that in all cases, the function $U^R(\gamma; \cdot)$ is monotonically decreasing in γ . That is:

$$\rho \frac{dU^R(\gamma_1; I)}{d\gamma_1} = -\frac{\alpha_1(w_1 - z)}{\rho + \delta + \mu} - (\Gamma_2 - \alpha_1\Gamma_1) < 0 \quad (126)$$

$$\rho \frac{dU^R(\gamma_1; II)}{d\gamma_1} = \alpha_1\Gamma_1 - \Gamma_2 + \frac{(1 - \alpha_1)(w_1 - z)}{\rho + \delta + \mu + \mu^*} = \Gamma_1 - \Gamma_2 < 0 \quad (127)$$

$$\rho \frac{dU^R(\gamma_2)}{d\gamma_2} = -\frac{\alpha_2(w_2 - z)}{\rho + \delta + \mu} + \alpha_2\Gamma_2 - \Gamma_1 = -\frac{\alpha_2\mu^*}{\rho + \delta + \mu}\Gamma_1 - \Gamma_1 < 0 \quad (128)$$

Now suppose that parameters are such that there exists an interior equilibrium (as in Case I), (γ_1^*, γ_2^*) such that $U^R(\gamma_1^*) = U^R(\gamma_2^*)$. Observe that since VVII always lies to the right of VVI, there cannot also exist an interior equilibrium as in Case II. Now consider corner Y, (γ_1^Y, γ_2^Y) . Since $U^R(\gamma; I)$ is decreasing in γ

$$\begin{aligned}\gamma_1^Y > \gamma_1^* &\Rightarrow U^R(\gamma_1^Y; I) < U^R(\gamma_1^*; I) \\ \gamma_2^Y < \gamma_2^* &\Rightarrow U^R(\gamma_2^Y) > U^R(\gamma_2^*)\end{aligned}$$

and so $U^R(\gamma_2^Y) > U^R(\gamma_1^Y; I)$. If this corner case were an equilibrium then

$$W_2^R = U_1^H - p_1 \Rightarrow U^R = U^R(\gamma_1^Y; I)$$

and

$$\begin{aligned}W_1^R &> U_2^H - p_2 \\ \Gamma_1 + U^R &> \Upsilon_{U_2}(\gamma_2^Y) + \Xi(\gamma_2^Y)U^R \\ U^R &> \frac{\Upsilon_{U_2}(\gamma_2^Y) - \Gamma_1}{1 - \Xi(\gamma_2^Y)} = U^R(\gamma_2^Y)\end{aligned}$$

where $\Xi(\gamma_2^Y) < 1$. This implies that $U^R(\gamma_1^Y; I) > U^R(\gamma_2^Y)$. Hence we have a contradiction and (γ_1^Y, γ_2^Y) cannot also be an equilibrium.

Next consider the corner case (γ_1^X, γ_2^X) . Note first that

$$\begin{aligned}\gamma_1^X < \gamma_1^* &\Rightarrow U^R(\gamma_1^X; I) > U^R(\gamma_1^*; I) \\ \gamma_2^X > \gamma_2^* &\Rightarrow U^R(\gamma_2^X) < U^R(\gamma_2^*)\end{aligned}$$

and so $U^R(\gamma_2^X) < U^R(\gamma_1^X; I)$. If this corner case were an equilibrium then $W_1^R = U_2^H - p_2 \Rightarrow U^R = U^R(\gamma_2^X)$ and

$$\begin{aligned}W_2^R &> U_1^H - p_1 \\ \Gamma_2 + U^R &> \Upsilon_{U_1}^I(\gamma_1^X) + \Xi_{U_1}^I(\gamma_1^X)U^R \\ U^R &> \frac{\Upsilon_{U_1}^I(\gamma_1^X) - \Gamma_2}{1 - \Xi_{U_1}^I(\gamma_1^X)} = U^R(\gamma_1^X; I)\end{aligned}$$

where $\Xi_{U_1}^I(\gamma_1^X) < 1$. This implies that $U^R(\gamma_1^X; I) < U^R(\gamma_2^X)$. Hence we have a contradiction and (γ_1^X, γ_2^X) cannot also be an equilibrium.

Similar proofs apply to the uniqueness of interior Case II.

Proof of Proposition 3:

(1) Using (43), since $\Omega^I > 0$, a sufficient condition for $\gamma_2 > \gamma_1$ is that $\Psi^I > 1$. That is

$$\Gamma_2 - \alpha_1 \Gamma_1 + \frac{\alpha_1 (w_1 - z)}{\rho + \delta + \mu} > \Gamma_1 - \alpha_2 \Gamma_2 + \frac{\alpha_2 (w_2 - z)}{\rho + \delta + \mu}$$

Using (23) to substitute out Γ_2 on the right hand side and using (22) to substitute out $w_1 - \Gamma_2$ in the left hand side yields the condition that

$$\frac{\Gamma_2 - \Gamma_1}{\Gamma_1} > \frac{(\alpha_2 - \alpha_1) \mu^*}{\rho + \delta + \mu}$$

Since $\alpha_2 > \alpha_1$ this holds if the wage differential is sufficiently large.

(2) The home-ownership rate in city i is

$$h_i(\gamma_i) = \frac{N_i^{UH} + N_i^{WH}}{R_i + N_i^{UH} + N_i^{WH}} = \frac{H_i - \frac{\lambda}{\gamma_i} R_i}{R_i + H_i - \frac{\lambda}{\gamma_i} R_i}$$

which is increasing in γ_i . Since from (1) $\gamma_2 > \gamma_1$ the result follows.

Proof of Proposition 4: In aggregate the steady state flows into and out of the state of being an unemployed renter must satisfy

$$\begin{aligned} (\mu + \mu^* + \lambda) (N_1^{UR} + N_2^{UR}) &= \delta (N_1^{WR} + N_2^{WR}) \\ &= \delta (R_1 - N_1^{UR} + R_2 - N_2^{UR}) \end{aligned}$$

It follows that the unemployment rate amongst renters is given by

$$\nu^R = \frac{N_1^{UR} + N_2^{UR}}{R_1 + R_2} = \frac{\delta}{\delta + \mu + \mu^* + \lambda}.$$

In aggregate the steady-state flows into and out of being an unemployed owner must satisfy

$$\begin{aligned} \mu (N_1^{UH} + N_2^{UH}) + \mu^* (\theta_1^{UH} N_1^{UH} + \theta_2^{UH} N_2^{UH}) &= \delta (N_1^{WH} + N_2^{WH}) + \lambda (N_1^{UR} + N_2^{UR}) \\ &= \delta (1 - R_1 - R_2 - N_1^{UH} - N_2^{UH}) + \lambda (N_1^{UR} + N_2^{UR}) \end{aligned}$$

We can write this as

$$\begin{aligned} (\mu + \mu^* + \delta + \lambda) (N_1^{UH} + N_2^{UH}) &= \delta (1 - R_1 - R_2) + \mu^* ((1 - \theta_1^{UH}) N_1^{UH} + (1 - \theta_2^{UH}) N_2^{UH}) \\ &\quad + \lambda (N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH}) \end{aligned}$$

Dividing by $(\mu + \mu^* + \delta + \lambda)(1 - R_1 - R_2)$ yields an expression for the rate of unemployment amongst home-owners

$$\begin{aligned}\nu^H &= \frac{N_1^{UH} + N_2^{UH}}{1 - R_1 - R_2} \\ &= \frac{\delta}{\delta + \mu + \mu^* + \lambda} + \frac{\mu^* ((1 - \theta_1^{UH})N_1^{UH} + (1 - \theta_2^{UH})N_2^{UH}) + \lambda (N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH})}{(\mu + \mu^* + \delta + \lambda)(1 - R_1 - R_2)}\end{aligned}$$

The second term must be positive since $\theta_i^{UH} \leq 1$, so it follows that

$$\nu^H > \nu^R.$$

Proof of Proposition 5: From (65) and (66) we can write:

$$\alpha_2 = \frac{N_2^{WR}}{R_2} = \frac{\mu + (\mu^* + \lambda)x}{\delta + \mu + \lambda} \quad (129)$$

$$\alpha_1 = \frac{N_1^{WR}}{R_1} = \frac{\mu + (\mu^* + \lambda)/x - \mu^* \alpha_2/x}{\delta + \mu + \mu^* + \lambda} \quad (130)$$

where $x = R_1/R_2$. In order for $\alpha_2 > \alpha_1$ we require that

$$\alpha_2 > \frac{\mu + (\mu^* + \lambda)/x - \mu^* \alpha_2/x}{\delta + \mu + \mu^* + \lambda}.$$

Re-arranging and substituting for α_2 using (129) yields

$$\frac{\mu + (\mu^* + \lambda)x}{\delta + \mu + \lambda} > \frac{\mu + (\mu^* + \lambda)/x}{\delta + \mu + \lambda + \mu^*(1 + 1/x)}$$

If $x \geq 1$, this inequality must hold. It also holds for $x < 1$ provided x large enough.

Derivation of city level unemployment rate: The unemployment rate in city i is

$$\nu_i = \frac{N_i^{UH} + N_i^{UR}}{R_i + N_i^{UH} + N_i^{WH}} = \frac{R_i - N_i^{WR} + \left(\frac{\delta}{\delta + \mu}\right) \left(H_i - \frac{\lambda}{\gamma_i} R\right) - \frac{\lambda}{\delta + \mu} N_i^{WR}}{R_i + H_i - \frac{\lambda}{\gamma_i} R_i}$$

where the second equality uses (68) and (69). Dividing through by R_i and re-arranging yields

$$\nu_i = \frac{1 - \left(1 + \frac{\lambda}{\delta + \mu}\right) \alpha_i}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}} + \left(\frac{\delta}{\delta + \mu}\right) \left(\frac{\frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}\right)$$

Re-arranging and using (51) yields (52).

Proof of Proposition 6: Using (68) and the aggregate rate of unemployment is given by

$$\begin{aligned}\bar{\nu} &= N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH} \\ &= R_1 + R_2 - N_1^{WR} - N_2^{WR} + \left(\frac{\delta}{\delta + \mu}\right) \left(H_1 + H_2 - \frac{\lambda}{\gamma_1}R_1 - \frac{\lambda}{\gamma_2}R_2 - \frac{\lambda}{\delta}(N_1^{WR} + N_2^{WR})\right).\end{aligned}$$

Using (27) we can write this as

$$\bar{\nu} = R_1 + R_2 - N_1^{WR} - N_2^{WR} + \left(\frac{\delta}{\delta + \mu}\right) \left(1 - R_1 - R_2 - \frac{\lambda}{\delta}(N_1^{WR} + N_2^{WR})\right)$$

Using (65) and (66) it can be seen that

$$\begin{aligned}N_1^{WR} + N_2^{WR} &= \frac{\mu R_1 + (\mu^* + \lambda)R_2}{\delta + \mu + \mu^* + \lambda} + \left(1 - \frac{\mu^*}{\delta + \mu + \mu^* + \lambda}\right) \frac{\mu R_2 + (\mu^* + \lambda)R_1}{\delta + \mu + \lambda} \\ &= \left(\frac{\mu + \mu^* + \lambda}{\delta + \mu + \mu^* + \lambda}\right) (R_1 + R_2).\end{aligned}$$

Substituting and noting that the aggregate home-ownership rate is $\bar{h} = 1 - R_1 - R_2$

$$\bar{\nu} = \left(\frac{\delta}{\delta + \mu}\right) + \left(\frac{\delta}{\delta + \mu + \mu^* + \lambda}\right) (1 - \bar{h}) - \left(\frac{\delta}{\delta + \mu}\right) \left(1 + \frac{\lambda}{\delta} \left(\frac{\mu + \mu^* + \lambda}{\delta + \mu + \mu^* + \lambda}\right)\right) (1 - \bar{h}) \quad (131)$$

Thus home-ownership has two effects on unemployment. The first is negative and comes from the reduction in the number of unemployed renters. The second is positive and comes from the increase in the measure of unemployment owners. Re-arranging (131) yields (53).

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8 Appendix B: Supplemental Calculations

8.1 Intra-city Relocation

In each city there are six types of households, as each may be either employed or unemployed, either rent or own a house and, if they are owners, may either be matched or mismatched with their house. The measures of households in city i that are matched employed-owners, mismatched employed-owners, employed-renters, matched unemployed-owners, mismatched unemployed-owners and unemployed-renters are given by N_i^{WH} , \tilde{N}_i^{WH} , N_i^{WR} , N_i^{UH} , \tilde{N}_i^{UH} and N_i^{UR} respectively. The values associated with being in each of these states are given by W_i^H , \tilde{W}_i^H , W_i^R , U_i^H , \tilde{U}_i^H and U_i^R , respectively. We let q_i^{WR} , q_i^{UR} , q_i^{WH} and q_i^{UH} denote the prices paid for houses in City i by employed and unemployed renters and by employed and unemployed owners respectively.

As in the basic model we restrict attention to equilibria which are stationary and symmetric. We also restrict our attention to the case where the marginal homeowner in both cities is a matched unemployed owner. Within this case we impose the following restrictions and check that they hold in equilibrium:

- (1) employed renters in the low-wage city who are offered a job in the high-wage city choose to relocate, but not *vice versa*.
- (2) mismatched owners in both cities do not become renters

$$\tilde{W}_i^H - p_i > W_i^R \quad \text{and} \quad \tilde{U}_i^H - p_i > U_i^R \quad i = 1, 2.$$

- (3) All renters and mismatched owners buy houses when they get the chance.

These conditions together imply that employed home owners (matched and mismatched) are also unwilling to move from from the high wage city to the low wage one in equilibrium:

$$W_2^H - p_2 > \tilde{W}_2^H - p_2 > W_2^R > W_1^R. \quad (132)$$

The steady-state flow of workers between states is described by (9), (26) and the following

10 equations:

$$(\delta + \mu^* + \lambda)N_1^{WR} = \mu N_1^{UR} + \mu^* \left(N_2^{UR} + \tilde{N}_2^{UH} + \theta_2^{UH} N_2^{UH} \right) \quad (133)$$

$$(\mu + \psi) N_1^{UH} + \mu^* \theta_1^{UH} N_1^{UH} = \delta N_1^{WH} + \lambda \left(N_1^{UR} + \tilde{N}_1^{UH} \right) \quad (134)$$

$$(\delta + \psi) N_1^{WH} = \lambda \left(N_1^{WR} + \tilde{N}_1^{WH} \right) + \mu N_1^{UH} \quad (135)$$

$$(\mu + \mu^* + \lambda) \tilde{N}_1^{UH} = \psi N_1^{UH} + \delta \tilde{N}_1^{WH} \quad (136)$$

$$(\delta + \lambda) \tilde{N}_1^{WH} = \psi N_1^{WH} + \mu \tilde{N}_1^{UH} \quad (137)$$

$$(\delta + \lambda) N_2^{WR} = \mu N_2^{UR} + \mu^* \left(N_1^{UR} + N_1^{WR} + \tilde{N}_1^{UH} + \theta_1^{UH} N_1^{UH} \right) \quad (138)$$

$$(\mu + \psi) N_2^{UH} + \mu^* \theta_2^{UH} N_2^{UH} = \delta N_2^{WH} + \lambda \left(N_2^{UR} + \tilde{N}_2^{UH} \right) \quad (139)$$

$$(\delta + \psi) N_2^{WH} = \lambda \left(N_2^{WR} + \tilde{N}_2^{WH} \right) + \mu N_2^{UH} \quad (140)$$

$$(\mu + \mu^* + \lambda) \tilde{N}_2^{UH} = \psi N_2^{UH} + \delta \tilde{N}_2^{WH} \quad (141)$$

$$(\delta + \lambda) \tilde{N}_2^{WH} = \psi N_2^{WH} + \mu \tilde{N}_2^{UH} \quad (142)$$

The solution to this system can be expressed recursively as

$$N_2^{WR} = \frac{\mu R_2 + (\mu^* + \lambda) R_1}{\delta + \mu + \lambda} \quad (143)$$

$$N_1^{WR} = \frac{\mu R_1 + (\mu^* + \lambda) R_2 - \mu^* N_2^{WR}}{\delta + \mu + \mu^* + \lambda} \quad (144)$$

$$N_i^{UR} = R_i - N_i^{WR} \quad i = 1, 2 \quad (145)$$

$$N_i^{UH}(\gamma_i) = - \left[\gamma_i + \frac{(\lambda + \gamma_i) \psi}{\tilde{\Delta}} (\delta + \mu + \mu^* + \lambda) \right] \frac{\lambda}{\Delta_i} N_i^{WR} \quad (146)$$

$$+ \frac{1}{\Delta_i} \left(\delta + \psi - \frac{\lambda \psi}{\tilde{\Delta}} (\mu + \mu^* + \lambda) \right) (\gamma_i H_i - \lambda R_i) \quad (147)$$

$$N_i^{WH}(\gamma_i) = \left[\gamma_i + \frac{(\lambda + \gamma_i) \psi}{\tilde{\Delta}} (\delta + \mu + \lambda) \right] \frac{\lambda}{\Delta_i} N_i^{WR} + \frac{\mu}{\Delta_i} \left(1 + \frac{\lambda \psi}{\tilde{\Delta}} \right) (\gamma_i H_i - \lambda R_i) \quad (148)$$

$$\tilde{N}_i^{UH}(\gamma_i) = \frac{\psi}{\tilde{\Delta}} [(\delta + \lambda) N_i^{UH}(\gamma_i) + \delta N_i^{WH}(\gamma_i)] \quad (149)$$

$$\tilde{N}_i^{WH}(\gamma_i) = \frac{\psi}{\tilde{\Delta}} [\mu N_i^{UH}(\gamma_i) + (\mu + \mu^* + \lambda) N_i^{WH}(\gamma_i)] \quad (150)$$

$$\theta_i^{UH}(\gamma_i) = \frac{\lambda R_i - \mu^* \tilde{N}_i^{UH}(\gamma_i)}{\mu^* N_i^{UH}(\gamma_i)}, \quad (151)$$

where $\tilde{\Delta} = (\delta + \lambda)(\mu + \mu^* + \lambda) - \mu\delta$ and

$$\begin{aligned}\Delta_i &= \left[\gamma_i + \frac{(\lambda + \gamma_i)\psi}{\tilde{\Delta}}(\delta + \mu + \mu^* + \lambda) \right] \mu \left(1 + \frac{\lambda\psi}{\tilde{\Delta}} \right) \\ &\quad + \left[\delta + \psi - \frac{\lambda\psi}{\tilde{\Delta}}(\mu + \mu^* + \lambda) \right] \left[\gamma_i + \frac{(\lambda + \gamma_i)\psi}{\tilde{\Delta}}(\delta + \mu + \lambda) \right]\end{aligned}$$

The flow utilities of owners in this equilibrium are given by

$$\begin{aligned}\rho W_i^H &= w_i + \pi^H + \kappa + \delta(U_i^H - W_i^H) + \psi(\tilde{W}_i^H - W_i^H) \\ \rho U_i^H &= z + \pi^H + \kappa + \mu(W_i^H - U_i^H) + \psi(\tilde{U}_i^H - U_i^H) \\ \rho \tilde{W}_i^H &= w_i + \pi^H - \varepsilon + \kappa + \delta(\tilde{U}_i^H - \tilde{W}_i^H) \\ \rho \tilde{U}_i^H &= z + \pi^H - \varepsilon + \kappa + \mu(\tilde{W}_i^H - \tilde{U}_i^H) + \mu^*(U_i^H - \tilde{U}_i^H)\end{aligned}\tag{152}$$

$$\rho \tilde{U}_i^H = z + \pi^H - \varepsilon + \kappa + \mu(\tilde{W}_i^H - \tilde{U}_i^H) + \mu^*(U_i^H - \tilde{U}_i^H)\tag{153}$$

Solving yields

$$W_i^H = \frac{1}{\rho} \left(\hat{w}_i + \pi^H + \kappa - \frac{\delta(\hat{w}_i - \hat{z})}{\rho + \delta + \mu} \right)\tag{154}$$

$$U_i^H = \frac{1}{\rho} \left(\hat{z} + \pi^H + \kappa + \frac{\mu(\hat{w}_i - \hat{z})}{\rho + \delta + \mu} \right)\tag{155}$$

$$\tilde{W}_i^H = W_i^H - \Omega^W \varepsilon\tag{156}$$

$$\tilde{U}_i^H = U_i^H - \Omega^U \varepsilon\tag{157}$$

where $\hat{w}_i = w_i - \psi\Omega^W \varepsilon$ and $\hat{z} = z - \psi\Omega^U \varepsilon$ and

$$\Omega^W = \frac{\rho + \delta + \mu + \mu^* + \psi}{(\rho + \delta + \mu + \mu^* + \psi)(\rho + \psi) + \delta\mu^*}\tag{158}$$

$$\Omega^U = \frac{\rho + \delta + \mu + \psi}{(\rho + \delta + \mu + \mu^* + \psi)(\rho + \psi) + \delta\mu^*}\tag{159}$$

The sale price of housing in city i satisfies

$$\begin{aligned}\rho p_i &= \gamma_i \left\{ \eta_i [\alpha_i(q_i^{WR} - p_i) + (1 - \alpha_i)(q_i^{UR} - p_i)] + (1 - \eta_i) [\zeta_i(q_i^{WH} - p_i) + (q_i^{UH} - p_i)] \right\} \\ &= \gamma_i \left\{ \eta_i [\alpha_i(W_i^H - W_i^R - p_i) + (1 - \alpha_i)(U_i^H - U^R - p_i)] + (1 - \eta_i) [\zeta_i(W_i^H - \tilde{W}_i^H) + (U_i^H - \tilde{U}_i^H)] \right\}\end{aligned}$$

Solving for p_i yields

$$p_i = \frac{\gamma_i}{\rho + \eta_i \gamma_i} \left\{ \eta_i [\alpha_i(W_i^H - W_i^R) + (1 - \alpha_i)(U_i^H - U^R)] + (1 - \eta_i) [\zeta_1 \Omega^W + (1 - \zeta_1) \Omega^U] \varepsilon \right\}\tag{160}$$

In city 1 this can be expressed as

$$p_1 = \frac{\gamma_1}{\rho + \eta_1 \gamma_1} \left\{ \eta_1 [\alpha_1 (W_1^H - \Gamma_1) + (1 - \alpha_1) U_1^H - U^R] + (1 - \eta_1) [\zeta_1 \Omega^W + (1 - \zeta_1) \Omega^U] \varepsilon \right\} \quad (161)$$

Since, in this equilibrium, matched home-owners in city 1 are indifferent between staying or leaving we have that

$$U_1^H - p_1 = \Gamma_2 + U^R$$

Substituting for p_1 and solving for U^R we get

$$\rho U^R(\gamma_1) = (\rho + \alpha_1 \eta_1 \gamma_1) U_1^H - (\rho + \eta_1 \gamma_1) \Gamma_2 - \gamma_1 \left\{ \eta_1 \alpha_1 (W_1^H - \Gamma_1) + (1 - \eta_1) [\zeta_1 \Omega^W + (1 - \zeta_1) \Omega^U] \varepsilon \right\}$$

Similarly, for city 2 we have

$$\rho U^R(\gamma_2) = (\rho + \alpha_2 \eta_2 \gamma_2) U_2^H - (\rho + \eta_2 \gamma_2) \Gamma_1 - \gamma_2 \left\{ \eta_2 \alpha_2 (W_1^H - \Gamma_2) + (1 - \eta_2) [\zeta_2 \Omega^W + (1 - \zeta_2) \Omega^U] \varepsilon \right\}$$

Equating $U^R(\gamma_1) = U^R(\gamma_2)$ yields a generalized VVI curve. Using (59) the generalized AM curve can be expressed as

$$H_1 + H_2 + R_1 + R_2 - 1 = \frac{\lambda}{\gamma_1} \left(R_1 + \tilde{N}_1^{WH}(\gamma_1) + \tilde{N}_1^{UH}(\gamma_1) \right) + \frac{\lambda}{\gamma_2} \left(R_2 + \tilde{N}_2^{WH}(\gamma_2) + \tilde{N}_2^{UH}(\gamma_2) \right).$$

The intersection of these two curves yields the equilibrium values of γ_1 and γ_2 .

8.2 Rental Vacancies

In steady state, flows of rental units out of and into being vacant must be equal in each city. For city 1 this implies that

$$\tau (R_1 - N_1^{WR} - N_1^{UR}) = (\lambda + \mu^*) (N_1^{WR} + N_1^{UR}). \quad (162)$$

Re-arranging yields (63). For city 2, employed renters turn down outside offers and some unemployed renters move to city 1 even if they have no job offer. Consequently, the steady state flow condition is given by

$$\tau (R_2 - N_2^{WR} - N_2^{UR}) = \lambda (N_2^{WR} + N_2^{UR}) + \mu^* N_2^{UR} + \phi N_2^{UR} \quad (163)$$

where ϕ denotes the endogenous rate at which unemployed renters with no job offer move from city 2 to city 1. In equilibrium

$$\phi N_2^{UR} = \mu^* (N_1^{WR} + N_1^{UR} - N_2^{UR} + \theta_1^{UH} N_1^{UH} - \theta_2^{UH} N_2^{UH}). \quad (164)$$

Recall from that $\mu^* \theta_i^{UH} N_i^{UH} = \lambda R_i$, and so

$$\mu^* N_2^{UR} + \phi N_2^{UR} = \mu^* (N_1^{WR} + N_1^{UR}) + \lambda R_1 - \lambda R_2. \quad (165)$$

Substituting into (163) we get

$$\tau (R_2 - N_2^{WR} - N_2^{UR}) = \lambda (N_2^{WR} + N_2^{UR}) + \mu^* (N_1^{WR} + N_1^{UR}) + \lambda R_1 - \lambda R_2. \quad (166)$$

Using (63) and re-arranging yields (64).

