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# Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery

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# Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery\*

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## Abstract

We study the trading dynamics in an asset market where the quality of assets is private information of the owner and finding a counterparty takes time. When trading of a financial asset ceases in equilibrium as a response to an adverse shock to asset quality, a large player can resurrect the market by purchasing bad assets which involves financial losses. The equilibrium response to such a policy is intricate as it creates an announcement effect: a mere announcement of intervening at a later point in time can cause markets to function again. This effect leads to a gradual recovery in trading volume, with asset prices converging non-monotonically to their normal values. The optimal policy is to intervene immediately at a minimal scale when markets are deemed important and losses are small. As losses increase and the importance of the market declines, the optimal intervention is delayed and it can be desirable to rely more on the announcement effect by increasing the size of the intervention. Search frictions are important for all these results. They compound adverse selection, making a market more fragile with respect to a classic lemons problem. They dampen the announcement effect and cause the optimal policy to be more aggressive, leading to an earlier intervention at a larger scale.

Keywords: Trading Dynamics, Adverse Selection, Search, Intervention in Asset Markets, Announcement Effect

JEL Classification: G1, E6

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# 1 Introduction

During the financial crisis of 2007 to 2009, there was a stunning difference in how asset markets were affected according to their infrastructure. Markets with centralized trading functioned rather well. To the contrary, in over-the-counter markets – where trading takes place on a decentralized and ad-hoc basis and where assets are less standardized and arguably more opaque – trading came to a halt. Most prominently, collateralized debt obligations, asset backed securities and commercial paper were traded only sporadically or not at all (see [Gorton and Metrick, 2010](#)). This prompted massive government intervention in the form of short-term liquidity provision and long-term asset purchases which led to a (partial) resurrection of trading in these markets over time.

Markets for structured debt financing experienced a sudden, unexpected deterioration of the average quality of assets<sup>1</sup> and some observers have pointed to adverse selection as the source for these markets not functioning properly. However, the question arises whether the peculiar features of over-the-counter trading – private information about the quality of assets and search for a counterparty – made those markets more prone to a market freeze.<sup>2</sup> If so, are these features important to understand the role and design of a government intervention in these markets? In this paper, we find that search frictions in asset markets compound adverse selection, making the market more fragile to unexpected shocks to the quality of assets. These trading frictions also determine the response of the market to a government intervention and thus influence the optimal design of a policy aimed at resurrecting trading in those markets.

To derive these insights, it is natural to combine two existing strands of literature on market microstructure which have emphasized private information and trading frictions separately. Starting with [Kyle \(1985\)](#) and [Glosten and Milgrom \(1989\)](#), models of traders that are privately informed about the asset quality have been used widely to shed light on pricing and transaction costs in financial markets. More recently, a new approach to study price and trading dynamics has spawned from the work by [Duffie, Garleanu and Pedersen \(2005\)](#) that uses random search to model over-the-counter trading, but does not take into account that assets might be opaque.<sup>3</sup>

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<sup>1</sup>[Moody's Investor Service \(2010\)](#) reports large spikes in impairment probabilities for structured debt products across all ratings and products for 2007 to 2009).

<sup>2</sup>When assets are opaque, private information can arise when owners of assets have an advantage or a higher incentive to acquire information about the quality of assets relative to potential buyers (see [Dang, Gorton and Holmström, 2009](#)).

<sup>3</sup>Another related literature uses random matching models with private information to study monetary

Our paper shows that shocks to asset quality lead to a market freeze due to a lemons problem à la [Akerlof \(1970\)](#) when traders have to search for a counterparty. Trading, however, can be restored, if the government reduces the adverse selection problem ([Mankiw, 1986](#)). We capture this idea by introducing a large player that acts as a one-time, non-competitive market-maker buying a sufficient amount of lemons permanently in response to the market breakdown. This distinguishes our paper from the work on for-profit dealers in over-the-counter markets. In this literature, market-makers alleviate temporary, market-wide liquidity shocks by holding inventories (see for example [Weill, 2007](#); [Lagos and Rocheteau, 2009](#); [Lagos, Rocheteau and Weill, 2011](#)), whereas in our work, liquidity in the market is endogenous and collapses in response to adverse selection. The intervention has then to remove assets permanently to reduce the lemons problem which always causes losses and thus requires deep pockets – hence, the interpretation of the large player as the government. Similarly, in the market-making literature government intervention in the form of asset purchases can be rationalized whenever the expectation of negative profits prevents private market-making in response to a liquidity shock.

Since our set-up is dynamic, we can study the equilibrium dynamics from a market freeze to a recovery created by a public intervention.<sup>4</sup> The intervention is characterized by the amount of lemons bought, the price paid and its timing. The time dimension is the most interesting one as it causes an *announcement effect*: merely announcing to intervene at a later point in time can cause the market to recover prior to the actual intervention. The optimal intervention trades off the costs of the intervention with the gains from a market recovery. If the market is important, it is best to ensure that it continuously functions with an immediate intervention. As the importance of the market declines, it becomes optimal to delay and to increase the size of the intervention. The announcement effect then causes a gradual recovery in trading volume before the intervention, with market prices reacting non-monotonically, first decreasing before the intervention and then recovering to their long-term value.

In our stylized set-up, traders randomly meet to trade an asset. There are two types of assets, good assets that pay a dividend and lemons which for simplicity never pay any dividend.

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exchange (for example [Williamson and Wright, 1994](#); [Lester, Postlewaite and Wright, 2008](#); [Li and Rocheteau, 2011](#), among others).

<sup>4</sup>In independent work, [Tirole \(2011\)](#) uses a static framework that analyzes a similar government policy. Hence, it can neither address the optimal timing of the intervention nor the interaction of this decision with the quantity and price of the intervention. Recently, [Guerreri and Shimer \(2011\)](#) have extended this idea to dynamic asset markets where trading is competitive. They do not, however, analyze the design of the policy in detail.

What makes trading difficult is adverse selection: only the current owner of the asset can observe its quality, while the potential buyer only learns the quality after he has bought the asset. To create an incentive to trade assets, we assume that traders are hit randomly with a valuation shock. Upon buying a good asset, the buyer starts out with a high valuation, but over time he will switch to a low valuation and want to resell the asset. In equilibrium, while lemons are always up for sale, good assets will only be sold if their owners suffer a low valuation shock. Allowing buyers to make a take-it-or-leave-it offer results in the optimal offer being a pooling contract.<sup>5</sup>

The dynamics of trading are non-trivial and driven by search frictions. To understand them, we can look at the evolution of buyers' trade surplus over time which depends on two distinct effects, one that is backward-looking and another one that is forward-looking. First, a buyer's expected trade surplus depends on an endogenous stochastic process that governs how many good assets are for sale – or the average quality of assets – in the market. It is driven by the inflow of sellers that have received a low valuation shock. But it also depends on the speed at which transactions take place in the market which is governed by how often traders meet and the willingness of buyers to make an offer. We call this effect *quality effect*, since past trading behavior is summarized in the current quality of assets for sale. Second, a buyer's expected trade surplus also depends on how easy it is to resell the asset in the future. This matters for two reasons. Having a good asset, there is value in selling it again when the owner receives a valuation shock. But more importantly, if a buyer obtains a lemon in a trade, he would like to sell it as quickly as possible, since it does not yield a dividend. We call this effect *strategic complementarity*.<sup>6</sup>

These two effects determine how search frictions influence the fragility of the asset market and its trading dynamics in response to an intervention. First, due to the strategic complementarity, equilibria with and without trade can co-exist. When search frictions are high, buyers are reluctant to take on a position since they expect that it will be difficult to find a counterparty to undo it in future. As a result, it becomes harder to sustain trading in

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<sup>5</sup>This distinguishes us from [Guerreri, Shimer and Wright \(2010\)](#) that use competitive search to obtain a separating equilibrium in asset markets with adverse selection. [Chang \(2011\)](#) builds on this work to show that liquidity in the form of endogenous market tightness is disturbed downwards in equilibrium when there is a lemons problem for trading assets. Other papers with dynamic adverse selection also arrive at a pooling equilibrium, but by requiring that transactions have to take place at a single price (see for example [Eisfeldt, 2004](#); [Kurlat, 2010](#)).

<sup>6</sup>[Garleanu \(2009\)](#) points out that this complementarity can be important for understanding trade size and portfolio choice in asset markets. For simplicity, we abstract from such considerations which could strengthen this effect.

equilibrium for a given quality of assets. Also, smaller shocks to quality can lead to a market freeze. Conditional on having a market freeze, search frictions also determine the impact of an intervention on trading through the announcement effect. A market with high search frictions recovers later because the quality effect kicks in slower and the difficulty of finding a counterparty reduces the *absolute* magnitude of the strategic complementarity.<sup>7</sup> This implies that the design of the optimal policy has to take into account the fact that market recovery is slower when search frictions are large. To resurrect such a market, the large player has to intervene more aggressively by purchasing assets at an earlier time and a higher price.

There are many papers emerging that either dwell on the role of asymmetric information or on a strategic complementarity (and, hence, multiple equilibria) to generate a market freeze in the context of financial markets. Our paper is unique as it combines both elements and analyzes the trading dynamics in response to a policy that chooses the optimal timing and scale of an intervention designed to resurrect the market.<sup>8</sup> However, we do not investigate how information is revealed through trading in the market place. [Lester and Camargo \(2010\)](#) is an interesting contribution, since it studies, when there is asymmetric information, how quickly a market can sort out assets with different qualities and clears in the absence of an intervention. In our paper, the lemons problem does not diminish over time, making an intervention necessary for a recovery. In the tradition of sequential bargaining as a foundation of trading frictions in asset markets, [Zhu \(2010\)](#) generates endogenously adverse selection in a sequential search model when sellers visit multiple buyers and infer the quality of the assets from the frequency of their meetings. Finally, [Duffie, Malamud and Manso \(2009\)](#) investigate the incentives to search for information which could be applied to a situation of trading assets when there is asymmetric information.

The paper proceeds as follows. Section 2 introduces our model. In Section 3, we analyze how an unanticipated, permanent negative shock to the asset quality can lead to a market freeze. Section 4 studies how an intervention can resurrect the market and characterizes the price and trading dynamics. Section 5 analyzes the optimal design of intervention. Section 6 calibrates the model and performs various numerical analyses. We conclude with a brief discussion.

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<sup>7</sup>In a *relative* sense, an intervention is more effective with high search frictions, as it brings the market closer to its normal time activity after an initially delayed response.

<sup>8</sup>[Bolton, Santos and Scheinkman \(2011\)](#) is an exception in that the timing of the intervention matters. An early intervention can prevent a market freeze, but it can also preclude the supply of private liquidity in the secondary market for assets.

## 2 The Environment

We employ a basic model of asset pricing under search frictions and introduce adverse selection. Time is continuous. There is a measure of  $1 + S$  traders that trade  $S$  assets. These assets are of two types. A fraction  $\pi$  of the assets yields a dividend  $\delta$  (good assets), whereas the rest does not yield a dividend (lemons). The return on these assets is private information for the owner of the asset; i.e., only the trader who owns the asset can observe its return, but not other traders.

Traders are risk-neutral and discount time at a rate  $r$ . We assume that each investor can either hold one unit of an asset or no asset.<sup>9</sup> A trader who owns a good asset is subject to a random preference shock that can reduce his valuation from  $\delta$  to  $\delta - x > 0$ . Conditional on holding a good asset, the preference shock arrives according to a Poisson process with rate  $\kappa \in \mathbb{R}_+$ . Once a trader experiences this shock, his valuation of the asset will remain low until the asset is sold. This captures the idea that some traders who own an asset might have a need for selling it – or in other words, have a need for liquidity. The higher  $\kappa$ , the more likely an investor will face such needs. Traders therefore go through a different trading status depending on their asset holdings and their valuation of the asset. There are four different stages that occur sequentially: (i) buyers ( $b$ ) do not own an asset; (ii) owners ( $o$ ) have a good asset and a high valuation; (iii) traders ( $\ell$ ) who own a lemon; and (iv) sellers ( $s$ ) who have a good assets, but have experienced a transition to low valuation. We denote the measure of traders of different types at time  $t$  as  $\mu_b(t)$ ,  $\mu_o(t)$ ,  $\mu_\ell(t)$  and  $\mu_s(t)$  respectively.

There is no centralized market mechanism to trade assets. Instead, traders with an asset and buyers are matched according to a technology given by a matching function  $M(t) = \lambda \mu_b(t) [\mu_o(t) + \mu_s(t) + \mu_\ell(t)]$ , where  $M(t)$  is the total number of matches, and  $\lambda$  is a parameter capturing the matching rate.<sup>10</sup> We assume throughout that in pairwise meetings the buyer makes a take-it-or-leave it offer to the seller to buy one unit of the asset at price  $p(t)$ <sup>11</sup> and that traders cannot dispose of an asset to become a buyer again.<sup>12</sup>

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<sup>9</sup>This is a restriction on total asset holdings. While assets are indivisible, traders can still use lotteries and employ mixed strategies to trade assets.

<sup>10</sup>The interpretation is that traders are matched according to a Poisson process with a fixed arrival rate. As a result, matches with traders seeking the opposite side of a trade occur at a rate  $\lambda$  which is proportional to the measure of traders in that group.

<sup>11</sup>This is a simplifying assumption merely to avoid the issue of formulating a bargaining procedure in the presence of imperfect information.

<sup>12</sup>By restricting the number of assets relative to measure of traders in the economy, we can easily dispense with this assumption (see Appendix B).



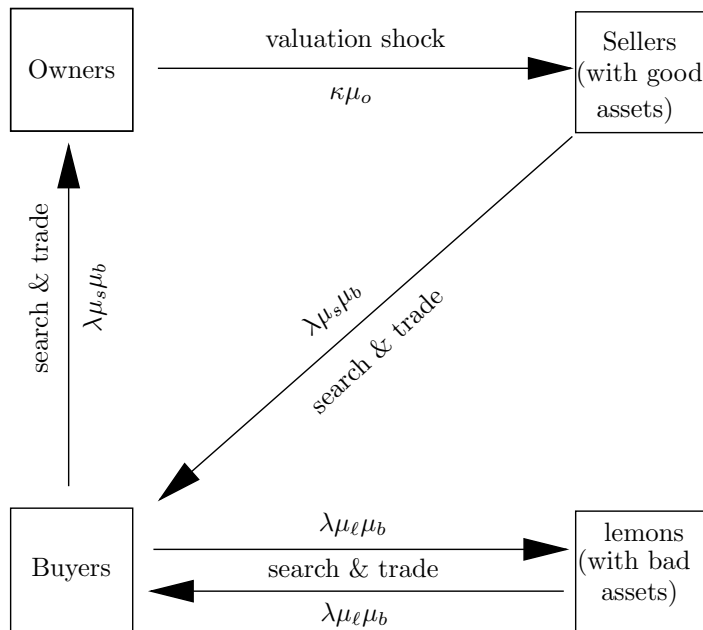


Figure 1: Flow Diagram

We can then describe the economy by a flow diagram as shown in Figure 1. A buyer becomes an owner by buying a good asset (with probability  $\lambda\mu_s$ ) or a lemon by buying a bad asset (with probability  $\lambda\mu_\ell$ ). He turns from an owner into a seller when receiving a negative preference shock (with probability  $\kappa$ ). Finally, if there is trade, good sellers and lemons sell their assets and become buyers (with probability  $\lambda\mu_b$ ). If not, they remain in their respective states. A classic adverse selection problem arises here, because lemons will choose in equilibrium to transit immediately from buying to selling the asset, while owners have first to experience a preference shock.

### 3 Market Freeze

#### 3.1 Pooling Equilibria

We first solve for steady state equilibria where buyers offer to purchase an asset at a price that pools sellers of good assets and lemons.<sup>13</sup> To allow for mixed strategy equilibria, a

<sup>13</sup>We show in Appendix C that a pooling contract dominates any other contract including a separating one with lotteries.

buyer makes a take-it-or-leave-it offer with probability  $\gamma(t)$  if in a meeting with another trader at time  $t$ . When making his offer, a buyer needs to take into account whether their price induces sellers with good assets to accept the offer. Denoting the first random time a seller gets an offer from a buyer by  $\tau$ , we obtain for the seller's value function

$$v_s(t) = E_t \left[ \int_t^\tau e^{-r(s-t)} (\delta - x) ds + e^{-r(\tau-t)} \max\{p(\tau) + v_b(t), v_s(\tau)\} \right]. \quad (1)$$

The first expression on the right-hand side is the flow value from owning the security. The second term gives the discounted value of meeting a buyer at random time  $\tau > t$  and receiving an offer. In such a meeting, the seller either accepts the offer or rejects it. If he rejects, he stays a seller ( $v_s(\tau)$ ). When accepting the offer, the seller will receive the price  $p(t)$  and become a buyer with value  $v_b(t)$ . Differentiating this expression with respect to time  $t$  and rearranging yields the following differential equation

$$rv_s(t) = (\delta - x) + \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_b(t) - v_s(t), 0\} + \dot{v}_s(t). \quad (2)$$

We can derive similar value functions for the other types of traders denoted by  $v_o(t)$  for owners,  $v_\ell(t)$  for lemons and  $v_b(t)$  for buyers. Notice that there is no gains from trading between owners and buyers, as they have the same valuation of a good asset. We thus have

$$rv_o(t) = \delta + \kappa(v_s(t) - v_o(t)) + \dot{v}_o(t) \quad (3)$$

$$rv_\ell(t) = \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_b(t) - v_\ell(t), 0\} + \dot{v}_\ell(t) \quad (4)$$

$$rv_b(t) = \gamma(t)\lambda(\mu_s(t) + \mu_\ell(t)) \max\{\max_{p(t)} \tilde{\pi}(p)v_o + (1 - \tilde{\pi}(p))v_\ell(t) - p(t) - v_b(t), 0\} + \dot{v}_b(t). \quad (5)$$

An owner enjoys the full value of the dividend flow until he receives a liquidity shock and turns into a seller which occurs with probability  $\kappa$ . Sellers of lemons – which we will simply call lemons from now on – are always on the market as they hold asset which do not yield a dividend. Upon selling the asset at price  $p$ , they become buyers again. The value function of a buyer takes into account that he can choose not to buy the asset in a meeting. If he makes an offer, the buyer will choose a price that maximizes his expected pay-off given the composition of traders that are willing to sell. This is reflected in the probability of obtaining a good asset which is a function of the price he offers ( $\tilde{\pi}(p)$ ).

Upon acquiring a lemon, a buyer will immediately try to sell it again since it offers no dividend flow. To the contrary, when acquiring a good asset, he has the highest valuation of the asset and will sell it only after receiving a preference shock that lowers his valuation which occurs with frequency  $\kappa$ . This implies that the measure of different types of traders

evolves according to the following flow equations

$$\dot{\mu}_b(t) = -\gamma(t)(\mu_s(t) + \mu_\ell(t)) + \gamma(t)(\mu_s(t) + \mu_\ell(t)) = 0 \quad (6)$$

$$\dot{\mu}_o(t) = -\kappa\mu_o(t) + \gamma(t)\lambda\mu_s(t) \quad (7)$$

$$\dot{\mu}_s(t) = \kappa\mu_o(t) - \gamma(t)\lambda\mu_s(t) \quad (8)$$

$$\dot{\mu}_\ell(t) = -\gamma(t)\lambda\mu_\ell(t) + \gamma(t)\lambda\mu_\ell(t) = 0. \quad (9)$$

Due to the trading structure, the number of buyers stays constant and is normalized to  $\mu_b(t) = 1$ . Similarly, all lemons are constantly for sale and, hence,  $\mu_\ell(t) = (1 - \pi)S$ .

For a buyer to induce a seller to accept his take-it-or-leave-it offer, he needs to offer a price that compensates the seller for switching to become a buyer, or  $p(t) \geq v_s(t) - v_b(t)$ . Since lemons do not derive any flow utility from their asset, we have that  $v_s(t) \geq v_\ell(t)$  and, consequently, they will accept the buyer's offer whenever sellers do. For the buyer, the probability of buying a good asset is thus given by

$$\tilde{\pi}(t) = \begin{cases} \frac{\mu_s(t)}{\mu_s(t) + \mu_\ell(t)} & \text{if } p(t) \geq v_s(t) - v_b(t) \\ 0 & \text{if } p(t) < v_s(t) - v_b(t). \end{cases} \quad (10)$$

This formulates the basic adverse selection problem. While lemons are always for sale, good assets go on sale only if their current owner experiences a preference shock. As a consequence, there are fewer good assets for sale than in the population. Also, if the buyer offers a price that is too low, good sellers will reject the offer and he will acquire a lemon for sure. Any offer by the buyer will thus be given by  $p(t) = v_s(t) - v_b(t)$ . For the further analysis, it is convenient to define the buyer's expected surplus from buying the asset

$$\Gamma(t) = \tilde{\pi}(p)v_o + (1 - \tilde{\pi}(p))v_\ell(t) - v_s(t), \quad (11)$$

where we have taken into account that any offer will set  $p(t) = v_s(t) - v_b(t)$ . This yields the following definition of equilibrium.

**Definition 1.** *An equilibrium is given by measurable functions  $\gamma(t)$  and  $\tilde{\pi}(t)$  such that*

1. *for all  $t$ , the strategy  $\gamma(t)$  is optimal taking as given  $\gamma(\tau)$  for all  $\tau > t$ ; i.e.,*

$$\gamma(t) = \begin{cases} 0 & \text{if } \Gamma(t) < 0 \\ \in [0, 1] & \text{if } \Gamma(t) = 0 \\ 1 & \text{if } \Gamma(t) > 0. \end{cases} \quad (12)$$

2. *The function  $\tilde{\pi}(t)$  is generated by  $\gamma(t)$  and the law of motion for  $\mu_s(t)$ .*

### 3.2 Steady State Equilibria

In steady state, the measure of traders of different types are given by

$$\mu_o + \mu_s = \pi S \quad (13)$$

$$\kappa \mu_o = \gamma \lambda \mu_s. \quad (14)$$

The first equation is just an accounting identity for good assets. Solving we obtain

$$\mu_s = \frac{\kappa}{\gamma \lambda + \kappa} S \pi \quad (15)$$

$$\mu_o = \frac{\gamma \lambda}{\gamma \lambda + \kappa} S \pi. \quad (16)$$

With pooling, the probability of obtaining a good asset  $\tilde{\pi}$  is thus given by

$$\tilde{\pi} = \begin{cases} \frac{\kappa \pi}{\kappa + (1 - \pi) \gamma \lambda} & \text{if } p \geq v_s - v_b \\ 0 & \text{if } p < v_s - v_b. \end{cases} \quad (17)$$

From the value functions, it follows then immediately that

$$v_s = \frac{\delta - x}{r} \quad (18)$$

$$v_o = \frac{1}{r + \kappa} (\delta + \kappa v_s) \quad (19)$$

$$v_\ell = \frac{\gamma \lambda}{\gamma \lambda + r} v_s. \quad (20)$$

Only the value of lemons depends on the trading strategy  $\gamma$  of buyers. Due to the pooling equilibrium, they can still extract some surplus from buyers despite the take-it-or-leave-it-offer; in other words, if  $\gamma > 0$ ,  $v_\ell > 0$  reflects an informational rent for lemons. To characterize steady state equilibria, we only need to consider the optimal strategy of buyers. Buyers trade if and only if they have a positive expected surplus from trading

$$\Gamma(t) = \tilde{\pi} v_o + (1 - \tilde{\pi}) v_\ell - v_s \geq 0. \quad (21)$$

Rewriting this condition by using the value functions, we obtain

$$\tilde{\pi} \left[ \frac{\delta}{\delta - x} r + \kappa + \lambda \gamma \left( \frac{\delta}{\delta - x} - 1 \right) \right] \geq r + \kappa. \quad (22)$$

This allows us to determine two thresholds for the asset quality, below which a no trade equilibrium exists ( $\underline{\pi}$ ) and above which a trade equilibrium exists ( $\bar{\pi}$ ). First, set  $\gamma = 1$  and define  $\delta/(\delta - x) = \xi$ . Then, using the expression for  $\tilde{\pi}$  in order to get trade we need that

$$\pi \geq \frac{(\kappa + \lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \lambda(\xi \kappa + r)} \equiv \bar{\pi}. \quad (23)$$

Similarly, we get no trade ( $\gamma = 0$ ) if

$$\pi \leq \frac{r + \kappa}{\xi r + \kappa} \equiv \underline{\pi}. \quad (24)$$

Comparing the two thresholds, we obtain that  $\underline{\pi} \geq \bar{\pi}$  if and only if  $\kappa \geq r$ . Finally, for any given  $\pi$  in between these thresholds, the indifference condition requires

$$\pi = \frac{(\kappa + \gamma\lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \gamma\lambda(\xi\kappa + r)} \quad (25)$$

Differentiating we get (up to a constant)

$$\frac{\partial \pi}{\partial \gamma} = (\xi - 1)(r + \kappa)\lambda\kappa(r - \kappa), \quad (26)$$

which depends on  $r$  relative to  $\kappa$ . In particular,  $\pi$  increases with  $\gamma$  if and only if  $r > \kappa$ . This gives the following result.

**Proposition 2.** *For any given  $\pi \in (0, 1)$ , a steady state equilibrium exists.*

*If  $\pi \geq \bar{\pi}$ , we have that  $\gamma = 1$  is a steady state equilibrium in pure strategies, i.e. all buyers trade.*

*If  $\pi \leq \underline{\pi}$ , we have that  $\gamma = 0$  is a steady state equilibrium in pure strategies, i.e. buyers do not trade.*

*If  $\kappa < r$ , the steady state equilibrium is unique, with the equilibrium for  $\pi \in (\underline{\pi}, \bar{\pi})$  being in mixed strategies.*

*If  $\kappa > r$ , for  $\pi \in (\bar{\pi}, \underline{\pi})$ , there are three steady state equilibria including a mixed strategy one.*

Figure 2 depicts steady state equilibria. When the average quality of the assets  $\pi$  is too low, there cannot be any trading in equilibrium – a situation which we call *market freeze*. This is associated with welfare losses as good assets cannot be allocated between traders that have different valuations for the asset. Similarly, for high average quality  $\pi$ , trade ( $\gamma = 1$ ) is the unique equilibrium. For intermediate values of  $\pi$ , there can be multiple equilibria with partial trade ( $\gamma \in (0, 1)$ ).

### 3.3 The Role of Search Frictions

Search frictions as captured by the parameter  $\lambda$  of the matching function play a key role in determining how severe the adverse selection problem becomes for any given average quality

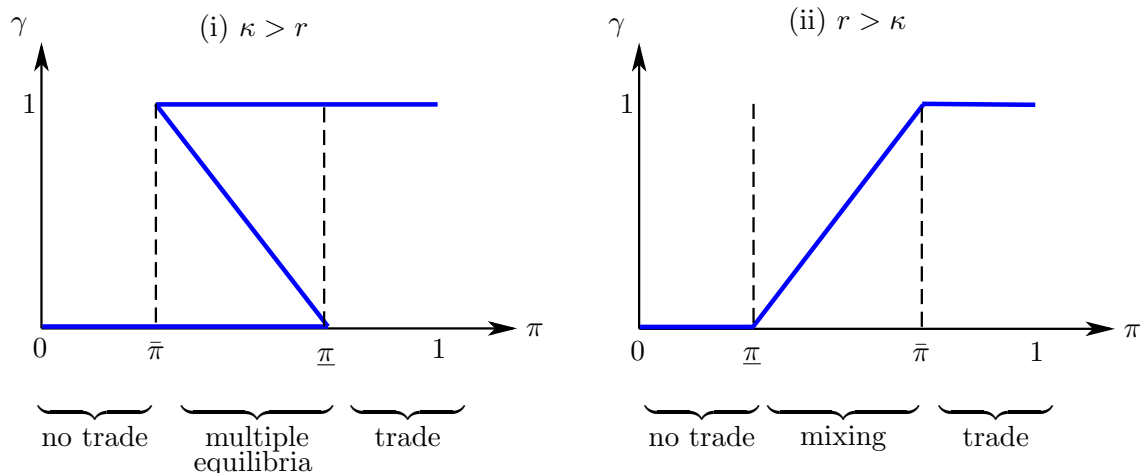


Figure 2: Steady State Equilibria

$\pi$ . This can be best understood by slightly rewriting the expected surplus of buyers to obtain

$$\frac{\Gamma}{(1 - \tilde{\pi})v_s} = \frac{\tilde{\pi}}{1 - \tilde{\pi}} \left( \frac{v_o - v_s}{v_s} \right) + \left( \frac{v_\ell - v_s}{v_s} \right) \quad (27)$$

$$= \left( \frac{\pi}{1 - \pi} \right) \left( \frac{\kappa}{\kappa + \lambda\gamma} \right) \left( \frac{1 - \pi}{\underline{\pi}} \right) - \frac{r}{r + \lambda\gamma}. \quad (28)$$

The first term captures a *quality effect* and describes how the average quality of assets for sale affects the trade surplus. If the trading volume as proxied by  $\lambda\gamma$  is large, there are relatively few good assets for sale at any point in time. This lowers the expected quality of the asset purchased by a buyer and, hence, his expected surplus.

The second term is independent of the average quality and captures a (dynamic) *strategic complementarity*. When a buyer decides to purchase an asset, it matters how easy it is to turn around a lemon in the future. If future buyers are less willing to trade, the asset market becomes less liquid giving the current buyer a lower incentive to trade. Conversely, if future buyers are more willing to purchase assets, it becomes easier for a buyer to turn around a lemon in the market increasing  $v_\ell$ . These two effects work in opposite directions and can cause trading even to decline when the quality of the asset increases as shown in Figure 2.

In general, as search frictions become larger – i.e.,  $\lambda$  decreases – the difference between a trade ( $\gamma = 1$ ) and a no-trade equilibrium ( $\gamma = 0$ ) vanishes as shown in Figure 3. Equation 28 implies that, in order to get trade in every meeting in equilibrium, we need

$$\left( \frac{\pi}{1 - \pi} \right) \left( \frac{1 - \pi}{\underline{\pi}} \right) \geq \frac{r\kappa + r\lambda}{r\kappa + \kappa\lambda}. \quad (29)$$

For  $\lambda \rightarrow 0$ , the strategic complementarity and thus the multiplicity of equilibrium disappears. The quality effect however increases. Since there are few trades, the selling pressure is large

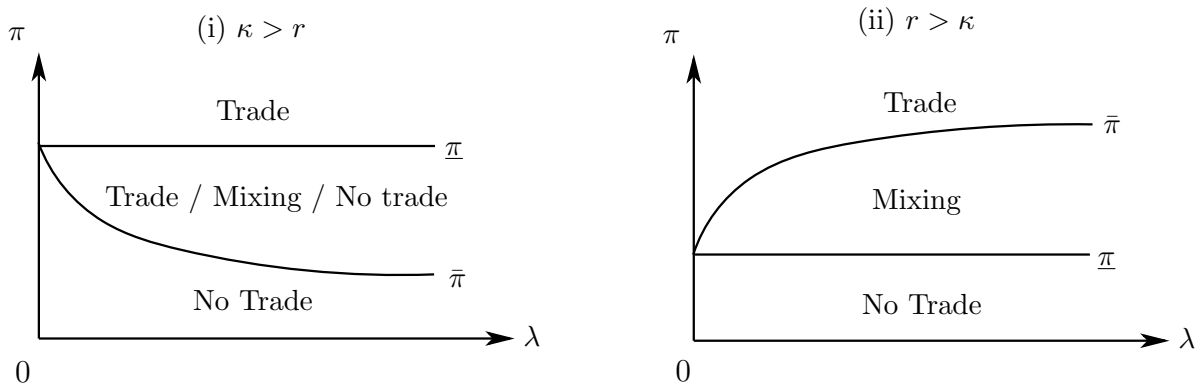


Figure 3: Role of Search Frictions

in the market: there are many good assets for sale at any point in time. However, when purchasing a lemon it is hard to sell it again. In this sense,  $\underline{\pi}$  measures the maximum quality effect.

When trading becomes frictionless ( $\lambda \rightarrow \infty$ ), we have that the strategic complementarity becomes large, but at the same time, the quality effect completely disappears. This is due to the fact that the number of good assets for sale relative to bad assets converges to 0. In the limit, we have

$$\left(\frac{\pi}{1-\pi}\right) \left(\frac{1-\pi}{\underline{\pi}}\right) \geq \frac{r}{\kappa}. \quad (30)$$

which defines an asymptote for  $\bar{\pi}$ , which is the minimum value of  $\pi$  satisfying this inequality.

When  $\kappa > r$ , the complementarity dominates and it becomes easier to support trading in equilibrium – at the expense of multiple equilibria. It is in this sense that search frictions compound the adverse selection problem. As a consequence, markets with frictional trading are more fragile when the quality of assets  $\pi$  declines as we show next.

### 3.4 Unanticipated Quality Shocks

We now consider a market freeze induced by an unexpected, permanent shock to the asset quality. Suppose we are in a steady state equilibrium with trade, but that a certain fraction of good assets turns into lemons independent of the holder of the asset being an owner or a seller. We assume that the average quality of the asset drops unexpectedly at  $t = 0$  to a level  $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\} \equiv \pi_{\min}$ . There cannot be continuous trade, since the surplus function  $\Gamma$  is strictly negative at  $t = 0$ . Furthermore, if there is no trade, the quality of assets for

sale increases over time, but can never cross the threshold  $\underline{\pi}$  where making an offer is a strictly dominant strategy for a buyer, i.e.  $\tilde{\pi}(t) \rightarrow \pi(0) < \underline{\pi}$ . Hence, there is an equilibrium where the economy converges to the no trade steady state with no trade along the transition. Finally, if the quality drops sufficiently, this no trade equilibrium is unique.

**Proposition 3.** *Suppose the economy is in a steady state with trade, but there is a shock to quality  $\pi(0) < \pi_{\min}$ . Then there cannot be an equilibrium with continuous trade, but there exists an equilibrium with no trade for any  $t$  that converges to the steady state with no trade. This equilibrium is unique, if  $\pi(0) < \left(\frac{r}{r+(1-\underline{\pi})\lambda}\right)\underline{\pi}$ .*

*Proof.* See Appendix A. □

This implies that for a large enough shock to the asset quality the market will instantaneously move from an equilibrium with trading to one without – our definition of a market freeze. Note that even a small shock to  $\pi$  can permanently freeze the market when  $\kappa > r$  as shown in Figure 2. We turn next to the question whether an intervention in the form of permanently buying assets can resurrect trading and how trading will respond to such an intervention.

## 4 Trading and Price Dynamics with Intervention

### 4.1 Intervention

We study now an intervention by a large (strategic) player – called market-maker-of-last-resort (MMLR) – in response to an unanticipated quality shock that causes the market to freeze.<sup>14</sup> We assume throughout that  $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\} \equiv \pi_{\min}$ , so that there is a unique steady state equilibrium of no trade and that without an intervention the economy converges to this equilibrium with no trade along the transition. The MMLR purchases bad assets which increases the average quality of the assets that are for sale after the intervention. This action will also influence trading behavior before the intervention takes place. As buyers anticipate the market to recover in response to the asset purchase, they can have an incentive to start buying assets before the intervention – a situation we call *announcement effect*.

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<sup>14</sup>In Appendix D, we discuss how a MMLR can use a different policy – a guaranteed price floor – to respond to a self-fulfilling freeze by eliminating equilibria with less trade when multiple equilibria co-exist.



More formally, an intervention is defined by an announcement at time  $t = 0$  to permanently purchase an amount of  $Q$  of lemons for a price  $P$  at some time  $T \geq 0$ .<sup>15</sup> We assume further that the MMLR does not have information on the quality of an asset, but knows the average quality  $\pi(0)$  of assets after the unanticipated shock has occurred and trading has ceased on the market. The MMLR can also commit to the policy, and meeting the MMLR is frictionless; i.e. at time  $T$  every trader with a lemon has an equal chance to trade with the MMLR. Finally, we assume that sellers of lemons that trade with the MMLR permanently exit the economy.<sup>16</sup> This leads to the following restriction on policies.

**Definition 4.** *An intervention  $(T, Q, P)$  is feasible if*

1.  $Q \in [S \left(1 - \frac{\pi(0)}{\bar{\pi}}\right), (1 - \pi(0))S]$
2.  $P \in [v_\ell(T), v_s]$ .

The first condition restricts an intervention to achieve full trading in steady state. Purchasing lemons raises the average quality of assets. In steady state, there will be trade as long as

$$\frac{\pi(0)S}{S - Q} \geq \bar{\pi}. \quad (31)$$

We denote that threshold as  $Q_{\min}$  and it yields an average quality of assets that are for sale in steady state equal to

$$\tilde{\pi} = \frac{\kappa \bar{\pi}}{\kappa + (1 - \bar{\pi})\lambda} = \frac{r \underline{\pi}}{r + (1 - \underline{\pi})\lambda}. \quad (32)$$

The second condition implies that only lemons have an incentive to sell the asset to the MMLR at the time of the intervention  $T$ . The price  $P$  cannot be too high as otherwise the intervention would attract also good sellers. Also, lemons need a price that is high enough to compensate them for the opportunity cost of remaining a seller with value  $v_\ell(T)$ .

The MMLR can provide an additional value through the intervention by increasing the price and the quantity of assets purchased. We call this the *option value* of the intervention and

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<sup>15</sup>We rule out purchasing good assets. This assumption is innocuous, if we assume that either the MMLR does not enjoy the dividend flow from good assets – or sufficiently less so than the traders.

<sup>16</sup>This keeps the number of buyers constant at  $\mu_b = 1$ . As we discuss further below, we could also assume that these sellers become buyers thereby increasing the total number of buyers. The intervention would then become more powerful as it permanently increases liquidity in the market by changing the relative market tightness in steady-state equilibrium via an increase in the number of buyers  $\mu_b$ .

denote it by  $V_I$ . To assess this option value, we look at the value of acquiring a lemon just an instant before the intervention

$$\begin{aligned}
v_\ell(T^-) &= \frac{Q}{S(1-\pi(0))}P(T) + \left(1 - \frac{Q}{S(1-\pi(0))}\right)v_\ell(T) \\
&= \frac{Q}{S(1-\pi(0))}(P(T) - v_\ell(T)) + v_\ell(T) \\
&= V_I + v_\ell(T).
\end{aligned} \tag{33}$$

The first term of this expression is the expected transfer of a trade with the MMLR at  $T$  provided by the option value. If it is positive, the value function of a lemon has a downward jump at  $T$ . The second term captures the case that, conditional on not being able to sell to the MMLR, the lemon remains in the market. With an intervention at  $T$ , a fraction  $Q/[S(1-\pi(0))]$  of lemons have an option to sell their bad assets to the MMLR at a price  $P(T)$ . At price  $P(T) = v_\ell(T)$ , they are just indifferent between selling to the MMLR or remaining in the market. If the price is strictly higher, they strictly prefer to sell to the MMLR as they receive an additional transfer. The option value  $V_I$  is thus the difference in expected value they obtain from the intervention relative to a minimum one. We have

$$0 \leq V_I \leq v_s - v_\ell(T). \tag{34}$$

Interestingly, given the minimum price, changing the size of the intervention  $P(T) = v_\ell(T)$  does not increase the option value of the intervention. The reason is that the chance to transact with the MMLR or afterwards in the market are perfect substitutes from the perspective of an individual lemon. Only when the MMLR increases the price above  $v_\ell(T)$  do lemons obtain an additional transfer through the intervention.

The surplus function  $\Gamma(t)$  has thus a jump at  $T$  for two reason. The intervention increases discretely the average quality  $\tilde{\pi}$ , but the value of a lemon  $v_\ell(T)$  also jumps down at that time, since a positive option value disappears for future lemon holders. More generally, the surplus function depends only on the dynamics of  $\tilde{\pi}(t)$  and  $v_\ell(t)$  over time, since buyers extract all the rents from sellers with good assets, causing both  $v_s$  and  $v_o$  to be constant. The value function of a lemon for any  $t < T$  is given by

$$v_\ell(t) = E_t \left[ e^{-r(\tau_m - t)} v_s \mathbf{1}_{\{\tau_m < T\}} + e^{-r(T-t)} (v_\ell(T) + V_I) \mathbf{1}_{\{\tau_m \geq T\}} \right], \tag{35}$$

where  $\tau_m$  is the random time for the next trade opportunity where buyers are willing to buy an asset. Solving this expression for a given trading strategy  $\gamma(t)$ , we obtain

$$\begin{aligned}
v_\ell(t) &= \lambda v_s \int_t^T \gamma(s) e^{\int_t^s -(r+\lambda\gamma(\nu))d\nu} ds + v_\ell(T^-) e^{\int_t^T -(r+\lambda\gamma(s))ds} \\
&= \lambda v_s \int_t^T \gamma(s) e^{\int_t^s -(r+\lambda\gamma(\nu))d\nu} ds + \left( \frac{\lambda}{\lambda+r} v_s + V_I \right) e^{\int_t^T -(r+\lambda\gamma(s))ds}.
\end{aligned} \tag{36}$$

This shows that the option value  $V_I$  of an intervention positively influences market trading before the intervention through its effects on  $v_\ell(t)$ . However, the option value is discounted by the rate of time preference  $r$  and by the chance of selling a lemon prior to the intervention on the market as expressed by the additional discount factor  $\lambda\gamma(t)$ .

We first characterize now the trading dynamics after the intervention. Our strategy is to work backwards. This is possible, since any feasible intervention is consistent with equilibrium with full trading *after* the intervention, independent of trading behavior before the intervention. All that matters here is that the minimum scale of the intervention  $Q_{\min}$  increases the average quality of assets sufficiently. We then characterize the equilibrium structure before the intervention which depends on the announcement effect and give some bounds for this effect. Finally, we derive implications for transaction prices in equilibrium when there is an intervention.

## 4.2 Trading Dynamics

### 4.2.1 Recovery After the Intervention

A minimum intervention,  $Q_{\min}$ , raises the average quality of assets just enough above the critical level so that there exists a steady state equilibrium with trade. But any feasible intervention also raises the average quality of the assets above this critical level along the time path after the intervention at  $T$ . Importantly, this result is independent of trading behavior before the intervention. We denote the average quality of assets for sale right after the intervention at  $T$  by  $\tilde{\pi}(T)$  and its long-run steady state level by  $\tilde{\pi}^{SS}(Q)$ .

**Lemma 5.** *Consider any intervention  $Q \in [Q_{\min}, (1 - \pi(0))S]$  at time  $T$ . Then  $\tilde{\pi}(t) \geq \tilde{\pi} = \frac{r\pi}{r+(1-\pi)\lambda}$  for all  $[T, \infty)$ . If there is full trade with  $\gamma(t) = 1$  for all  $t \in [T, \infty)$ , the average quality of assets for sale  $\tilde{\pi}(t)$  decreases monotonically to  $\tilde{\pi}^{SS}(Q) \geq \tilde{\pi}$  on  $[T, \infty)$ .*

*Proof.* See Appendix A. □

The intuition is as follows. First, the floor for the average quality of asset that are for sale is given by  $\tilde{\pi}(0)$ . This corresponds to a situation where there is continuous trading and the measure of sellers with good assets remains constant. When there is no trading at any point in time, the average quality increases because more and more owners become sellers over time due to preference shocks – or in other words, selling pressure builds up over time.

Hence, it must be the case that  $\mu_s(t) \geq \mu_s(0)$  for all  $t < T$ . Second, the MMLR removes only lemons from the market which causes a discrete jump in the average quality at time  $T$  that is sufficient to raise the floor for the average asset quality to at least  $\tilde{\pi}$  which is the steady state level of the average quality of assets that are for sale when there is trading. With trading, any built-up selling pressure will dissipate over time and the average quality of assets for sale needs to converge to this floor in the long-run.

Based on this result, it follows that continuous trade after the intervention is indeed an equilibrium. One needs to simply verify that buyers have an incentive to buy for any  $t \in [T, \infty)$  given that there is trade at any later stage which causes the value of a lemon to be constant at its steady state value. This is indeed the case as the quality of assets a buyer purchases is sufficiently high and will decline over time. Hence, there is no reason to postpone a purchase. As a result, the economy will converge monotonically to the new steady state with trade.

**Proposition 6.** *Continuous trade after the intervention is an equilibrium.*

*Proof.* When trading, the buyer's take-it-or-leave-it-offer is given by  $p(t) = v_s - v_b(t)$ . Hence, he will trade ( $\gamma(t) = 1$ ) only if

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s \geq 0.$$

Suppose that  $\gamma(t') = 1$  for all  $t' > t$ . Then,  $v_\ell(t) = \frac{\lambda}{\lambda+r}v_s$  which is time independent. Thus, rewriting the inequality above we obtain that  $\gamma(t) = 1$  if and only if

$$\tilde{\pi}(t) \geq \frac{v_s - v_\ell}{v_o - v_s} = \frac{r\pi}{r + (1 - \pi)\lambda} = \tilde{\pi}.$$

The result then follows from the previous lemma. □

#### 4.2.2 Recovery Before the Intervention

The intervention at time  $T$  can induce an announcement effect so that trading in the market starts before the actual intervention. This can be understood best by looking at how the intervention influences the two components associated with the surplus function from trading,  $\Gamma(t)$ . The quality effect is backward looking. When there is no trading, selling pressure builds up in the market (i.e.  $\mu_s(t)$  increases) which in turn increases the quality of assets that are for sale  $\tilde{\pi}(t)$ . This increases the willingness of buyers to purchase an asset today. Second, there is a strategic complementarity which is forward looking. The possibility of selling a

lemon in the future influences the willingness to purchase an asset today. Hence, anticipating the intervention which resurrects trading after  $T$ , buyers will start making offers as soon as the quality for assets improves sufficiently. In equilibrium, the trading volume has of course to be consistent with the decrease in the quality of assets when selling pressure diminishes which yields the following trading dynamics.

**Proposition 7.** *All equilibria before  $T$  can be characterized by two breaking points  $\tau_1(T) \geq 0$  and  $\tau_2(T) \in [\tau_1, T)$  such that*

- (i) *there is no trade ( $\gamma(t) = 0$ ) in the interval  $[0, \tau_1)$ ,*
- (ii) *there is partial trade ( $\gamma(t) \in (0, 1)$ ) in the interval  $[\tau_1, \tau_2)$ ,*
- (iii) *there is trade ( $\gamma(t) = 1$ ) in the interval  $[\tau_2, T)$ .*

*Proof.* See Appendix A. □

There are two breaking points. The first one,  $\tau_1(T)$ , determines when some trading – *partial trade* – takes place in the market again. With partial trade, buyers do not receive an expected surplus as  $\Gamma(t) = 0$ . While welfare improves as some good assets get reallocated, lemons extract all the rents from buyers. At a later point in time, at  $\tau_2(T)$ , the market fully recovers, a situation we refer to simply as *trade*. Whenever  $\tau_1(T) < T$ , there is an announcement effect from committing at time 0 to an intervention at a later date. The intervention itself of course influences these breaking points. Delaying the intervention reduces the strategic complementarity from a market recovery, but at the same time allows selling pressure to build up which increases the average quality of assets for buyers. Increasing the size of the intervention adds value to holding a lemon making it more attractive for buyers to make an offer.

### 4.2.3 Announcement Effect

We give now some bounds on the announcement effect, since we cannot derive analytically how changes in the intervention influence this effect. First, unless the option value is positive, there can only be partial trade prior to the intervention. This implies that there is a fixed costs associated with creating an announcement effect. Furthermore, if the initial quality drop is sufficiently large, there will be no announcement effect at all, since the quality effect is not strong enough to overcome the initial drop in quality. Second, for a large enough

option value, it is always possible to delay the intervention while still maintaining trade continuously in the market. The reason is that the MMLR can compensate a buyer fully for acquiring a lemon with the maximum option value ( $V_I = \frac{r}{r+\lambda}v_s$ ). This is sufficient to make the buyer's surplus positive provided the intervention is not delayed for too long.

**Proposition 8.** *Suppose  $V_I = 0$ . There cannot be trade with  $\gamma(t) = 1$  for any interval before  $T$  (i.e.  $\tau_2(T) = T$ ). Furthermore, if  $\pi(0) \leq \underline{\pi} \left( \frac{r}{r+\lambda(1-\underline{\pi})} \right)$ , the unique equilibrium before the intervention is no trade for any  $T$  (i.e.  $\tau_1(T) = T$ ).*

Furthermore, there exists an equilibrium with continuous trade, if and only if

$$\tilde{\pi}(0)v_o + (1 - \tilde{\pi}(0)) \left( \frac{\lambda}{\lambda + r}v_s + V_I e^{-(r+\lambda)T} \right) - v_s \geq 0. \quad (37)$$

*Proof.* See Appendix A. □

We discuss next how search frictions change the critical time  $T$  of the intervention so that an announcement effect starts to arise. When there is no announcement effect – i.e., no trade in  $[0, T)$  – the surplus function satisfies with  $V_I = 0$

$$\Gamma(T) = \left( \frac{\pi(0)}{1 - \pi(0)} \right) \left( \frac{1 - \underline{\pi}}{\underline{\pi}} \right) v_s \left( 1 - \frac{\lambda}{\lambda + \kappa} e^{-\kappa T} \right) - \frac{r}{r + \lambda} v_s < 0. \quad (38)$$

The quality effect depends negatively on  $\lambda$ , when there is no trade. When search frictions become less important, the average quality of assets for sale is lower to begin with. At the same time, the complementarity is stronger, as it is easier to turn around a lemon. Totally differentiating this expression we obtain

$$\frac{dT}{d\lambda} = \frac{1}{\lambda(\kappa + \lambda)} \left[ 1 - \left( \frac{\kappa + \lambda}{r + \lambda} \right)^2 \frac{1}{A} e^{\kappa T} \right] \quad (39)$$

where

$$A = \left( \frac{\pi(0)}{1 - \pi(0)} \right) \left( \frac{1 - \underline{\pi}}{\underline{\pi}} \right) < 1, \quad (40)$$

since  $\pi(0) < \underline{\pi}$ . When  $\kappa > r$ , this expression is always negative. The intuition is that the strategic complementarity is more important than the quality effect. As  $\lambda$  increases, the intervention becomes more powerful and the critical time  $T$  for an announcement effect to occur drops. For  $r > \kappa$ , the quality effect dominates and how search frictions influence the announcement effects depends on the size of the shock  $\pi(0)$ .

### 4.3 Price Dynamics

Due to the take-it-or-leave-it offer, market prices in equilibrium with trading are given by

$$p(t) = v_s - v_b(t) \tag{41}$$

and, hence, are inversely related to the value function of the buyer  $v_b(t)$ , which is a continuous function. Their dynamics depend on the trading behavior over time. Once trading starts again, there is a positive jump from zero to a price that is below the steady state price. Interestingly, prices then behave non-monotonically: they first decline with partial trade, before recovering to their steady-state level.

**Proposition 9.** *Given an intervention at  $T$ , prices  $p(t)$*

- (i) are zero when there is no trade and continuous on  $[\tau_1, \infty)$ ,*
- (ii) decline at rate  $r$  with partial trade in the interval  $[\tau_1, \tau_2]$ ,*
- (iii) decline at a rate lower than  $r$  or increase with full trade in the interval  $(\tau_2, T]$ ,*
- (iv) and increase monotonically to the steady state price after the intervention with a positive, discrete jump in their growth rate at  $T$ .*

*Proof.* See Appendix A. □

With trade after the intervention, the quality of the assets  $\tilde{\pi}(t)$  drops over time, implying less surplus. Hence, sellers will require a higher price, as becoming a buyer is less attractive. Before the intervention at  $T$ , there is an additional second effect. The intervention discretely increases the expected surplus from trading which is discounted by traders at the rate of time preference. When there is trading before the intervention, these two effects work against each other giving rise to the non-monotonicity in prices. With partial trade, the expected surplus remains at zero, so that the buyer has to simply compensate the seller less as one approaches the intervention.

We can quite easily compare the price range of an intervention with the market price. The MMLR can pay a higher price for assets than the transaction price in the market at the time of the intervention. That occurs, when he chooses to pay the full price for the asset and the surplus for buyers is strictly positive at the time of the intervention, i.e.  $P(T) =$

$v_s > p(T) = v_s - v_b(T)$ . This of course is an artefact of our assumption that lemons have to leave the market permanently upon selling to the MMLR. At the minimum price  $P_{\min}$ , the MMLR pays a lower price when search friction increase, as lemons have a lower expected value when sold in the market.

## 5 Optimal Intervention

### 5.1 The Cost of Intervention

In order to study the optimal intervention, we need to adopt a social welfare function for the MMLR that takes into account the costs of the intervention against the benefits of the market allocating assets among traders with different valuations. Our welfare function is akin to one that is commonly used in the public finance literature on regulation and given by

$$\int_{t=0}^{\infty} \left( (\mu_o(t) + \mu_s(t))\delta - \mu_s(t)x \right) e^{-rt} dt - \theta \int_{t=0}^{\infty} P(T)Q(T)e^{-rt} dt. \quad (42)$$

The first term describes the surplus from allocating good assets to traders with high valuation and captures the benefits from intervening to resurrect the market. The second term expresses the costs of financing the intervention. The costs are a direct transfer to traders in the market and due to linear utility are zero sum. Hence, we introduce a parameter  $\theta \in (0, \infty)$  which expresses the social costs of intervening as being proportional to the costs.<sup>17</sup>

Recall that the option value is given by

$$V_I = \frac{Q}{S(1 - \pi(0))} (P - v_\ell). \quad (43)$$

We can thus represent the minimum costs associated with an intervention at time  $T$  and option value  $V_I$  by

$$C(V_I) = \begin{cases} V_I S(1 - \pi(0)) + Q_{\min} v_\ell & \text{if } 0 \leq V_I \leq \frac{Q_{\min}}{S(1 - \pi(0))} \frac{r}{r + \lambda} v_s \equiv \hat{V}_I \\ \frac{r + \lambda}{r} V_I S(1 - \pi(0)) & \text{if } \frac{Q_{\min}}{S(1 - \pi(0))} \frac{r}{r + \lambda} v_s \equiv \hat{V}_I \leq V_I \leq \frac{r}{r + \lambda} v_s. \end{cases} \quad (44)$$

The cost function is time-invariant, piece-wise linear in  $V_I$  and strictly convex around a kink that occurs at the policy that sets  $P = v_s$  and  $Q = Q_{\min}$ . The reason is that in order to

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<sup>17</sup>One can interpret these costs as the distortions from having to tax the economy to provide this transfer to traders. Note that this set-up implies immediately that there is no role for the MMLR of buying lemons when the market is functioning. There is a (social) cost of financing the intervention, but no benefit as the intervention does not affect the allocation of good assets in steady state.



achieve any given  $V_I$ , increasing the quantity  $Q$  involves a deadweight cost. The MMLR just provides a transfer of utility to current lemons otherwise provided for by future buyers. However, after the MMLR pays a price equal to the good asset, he needs to increase the quantity to achieve a higher option value  $V_I$ .<sup>18</sup> The net present value of costs is simply given by  $C(V_I)e^{-rT}$  if the intervention takes place at  $T$ . For the purpose of characterizing optimal policies, we can thus represent any feasible intervention simply by  $(T, V_I)$ .

## 5.2 Optimal Timing – Continuously Functioning Markets

When analyzing the decision for the MMLR to delay, we look at the special case how to best ensure that markets function continuously, i.e. how to ensure that there is trade at all  $t$  in response to an unanticipated shock  $\pi(0)$ . There are two options available: the MMLR could either intervene immediately and raise the quality of assets for sale sufficiently; or he could delay the intervention for some time, but increase the option value  $V_I$ . In either case, the economy jumps immediately to a new steady state at  $t = 0$ , with the number of good assets for sale being constant over time at  $\mu_s(t) = \frac{\kappa}{\kappa + \lambda} S\pi(0)$  and the amount of lemons decreasing at the time of the intervention.

The optimal policy for the MMLR is simply to minimize the net present value of costs. In order to delay, the net present value of the option value has to be sufficiently high to render the surplus function positive and, hence, the option value has to jump up discretely according to

$$V_I^{\min} = \frac{\tilde{\pi}(0)}{1 - \tilde{\pi}(0)}(v_s - v_o) + \frac{r}{r + \lambda}v_s > 0. \quad (45)$$

Hence, delaying involves a fixed cost. Moreover, for a policy to ensure continuous trading, it needs to increase  $V_I$  sufficiently when delaying the intervention. Totally differentiating condition (37), we obtain

$$\frac{dT}{dV_I} = \frac{1}{(r + \lambda)V_I} > 0. \quad (46)$$

Traders discount the option value  $V_I$  by more than the rate of time preference. They take into account that trading opportunities arrive in the market with flow  $\lambda$ . When selling the lemon to a buyer, they give up the additional transfer offered by the MMLR. Hence, the effective discount rate of an intervention is  $r + \lambda > r$ . The fixed cost combined with the increased discount rate for the option value leads to increasing costs when delaying the intervention.

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<sup>18</sup>This is somewhat an artefact of the MMLR not being able to sell back any additional amount of lemons  $Q - Q_{\min}$  to the market immediately after the intervention at a price equal to  $v_\ell - \epsilon$  (see Appendix E).

**Proposition 10.** *To ensure continuous markets ( $\gamma(t) = 1$  for all  $t$ ), it is optimal to intervene immediately, or  $T^* = 0$  and  $V_I^* = 0$ .*

*Proof.* The problem for the MMLR is to minimize the costs of any intervention that ensures continuous trade for all  $t$ . Taking into account the fixed cost given by  $P = v_s$ , the cost function is given by

$$C(V_I)e^{-rT(V_I)} = \left(\frac{\lambda + r}{r}\right) V_I S(1 - \pi(0))e^{-rT(V_I)}.$$

Differentiating with respect to  $V_I$ , we obtain

$$A \left[ 1 + V_I \frac{\partial T}{\partial V_I} (-r) \right]$$

for the derivative where  $A = S(1 - \pi(0)) \left(\frac{r + \lambda}{r}\right) e^{-rT(V_I)}$ . To ensure continuous trade, we need to require that

$$\frac{\partial T}{\partial V_I} = \frac{1}{(r + \lambda)V_I}.$$

This implies that

$$\frac{\partial C(V_I)e^{-rT(V_I)}}{\partial V_I} = A \left[ 1 - \frac{r}{r + \lambda} \right] > 0$$

Hence, it is never optimal to increase  $V_I$  and, hence, never optimal to delay the intervention in order to achieve continuously functioning markets.  $\square$

It is instructive to stress that the minimum jump in the option value is given by

$$V_I^{\min} = \hat{V}_I = \frac{Q_{\min}}{S(1 - \pi(0))} \left( \frac{r}{r + \lambda} v_s \right), \quad (47)$$

the point at which the kink occurs in the cost function which is associated with the highest price of the intervention  $P = v_s$ . The intuition is as follows. Suppose the intervention takes place at  $T = 0$  where the MMLR buys  $Q_{\min}$  at  $P = v_\ell$ . Then, the MMLR transacts with already existing lemons and he needs to pay a price that just makes them indifferent to stay a lemon and wait for a trade later in the market. Consider now  $T = \epsilon$  small, but positive. Then, the MMLR needs to convince a buyer to purchase a lemon in the market just before the intervention at  $T^-$ , even though the quality of the asset he purchases is still  $\tilde{\pi}(0)$ . He can do so, by offering an additional option value that compensates the buyer for taking on the increased risk of purchasing a lemon. The compensation required is exactly purchasing  $Q_{\min}$  lemons at the price the buyer paid for in the market. But this price is  $p = v_s - v_b(t) = v_s$  for a minimum intervention. Hence,  $P = v_s$ . Of course, as  $\epsilon > 0$ , discounting requires a larger

intervention than  $Q_{\min}$  to compensate buyers that purchased an asset at  $T = 0$ . Letting  $\epsilon \rightarrow 0$ , however, we obtain exactly the fixed cost being at the kink of the cost function. Hence, the MMLR can only delay if he pays the fair market price  $P = v_s$  for the good asset. Once the MMLR pays the fair price for lemons, any increases in the quantity  $Q$  are very costly making a delay of the intervention not optimal.<sup>19</sup>

### 5.3 Optimal Size of the Intervention – A Bang-Bang Result

The reasoning of the previous section can be used to evaluate optimal policies in terms of  $V_I$  more generally. To do so, we look at how certain marginal policy changes in a neighborhood of some initial policy  $(T, V_I)$  change the net present value of costs. The idea for the marginal policy change is to keep the net present value of the option value constant – like in expression (46) –, but for an arbitrary, fixed trading probability  $\gamma(T^-)$  just before the intervention at  $T$ . It turns out that the cost-minimizing policy among these policies sets  $V_I = \hat{V}_I$ .

**Lemma 11.** *Let  $\gamma \in (0, 1]$  and change the policy  $(T, V_I)$  according to  $dT/dV_I = \frac{1}{r+\lambda\gamma} \frac{1}{V_I}$  for all  $(T, V_I)$  with  $T > 0$  and  $V_I > 0$ . For such changes, the net present value of costs is minimized at  $\hat{V}_I$ .*

*Proof.* See Appendix A. □

We establish now that it is never optimal to increase the quantity of assets to be purchased. In other words,  $Q^* = Q_{\min}$ . This result can be understood as follows. Suppose it were optimal for the MMLR to buy more lemons. Then, it must be the case that  $V_I > \hat{V}_I$ . He could alternatively reduce the amount purchased and intervene earlier. Consider then a marginal policy change as in Lemma 11. This change will reduce the net present value of costs. But it will also leave the incentives to trade before the new, earlier intervention date unchanged. This implies that the MMLR increases welfare by lowering  $V_I$  towards  $\hat{V}$  – or equivalently, by setting  $Q^* = Q_{\min}$ .

---

<sup>19</sup>When there is a lag between observing the market freeze and being able to intervene, this result implies that the MMLR can still achieve continuous trade by announcing an intervention immediately. This would, however, require a larger than the minimal intervention at price  $P = v_s$ . Interestingly, the larger the search friction becomes – i.e. the smaller  $\lambda$  –, the more the MMLR could delay, since the market discounts the transfer provided by  $V_I$  less.

By similar reasoning, we obtain a slightly weaker result for the optimal price  $P^*$ . Consider a situation where there is full trade before the intervention, i.e.  $\gamma(T^-) = 1$ . Then, by Proposition 8, we know that the MMLR needs to provide a strictly positive option value  $V_I$ . This time, he could delay the intervention and increase  $V_I$  according to a marginal policy change that leaves  $\gamma(t) = 1$  unchanged for all  $t > T$ . As this lowers the net present value of costs, welfare again increases. However, due to the fixed cost of incurring a positive option value, it could be still better to intervene earlier, but at the minimum price  $P$ . This leads to the “bang-bang” result that an optimal intervention either offers the highest or the lowest price, while always purchasing the minimum amount of assets.

**Proposition 12.** *Any optimal policy with intervention ( $T^* < \infty$ ) features  $Q^* = Q_{\min}$ . Furthermore, if  $\tau_2(T^*) < T^*$ , the optimal policy either features  $P = v_\ell$  or  $P = v_s$ .*

*Proof.* See Appendix A. □

The relative weight between costs and benefits of an intervention,  $\theta$ , determines the optimal timing decision. If the social costs are small, but positive, it is optimal to intervene immediately. Conversely, if they become large, it is best not to intervene and let the market be frozen. A formal statement and proof of this result are relegated to Appendix F. Given these results, one would expect that for intermediate values of  $\theta$ , it is optimal to have a delayed intervention. When the intervention is delayed, equilibria exhibit generically partial trading before the intervention. Hence, welfare cannot be computed analytically anymore, as our linear differential equations have time-dependent coefficients. We will thus resort to numerically solve for the optimal policy – especially the optimal timing of the intervention – in a calibrated version of our economy.

## 6 Numerical Analysis

### 6.1 Parameter Values

We calibrate our economy to capture the market for longer-term structured finance products such as asset-backed securities (ABS) or collateralized debt obligations (CDO). We then interpret private information concerning the quality  $\pi$  of the assets as reflecting the opaqueness associated with the tranches of these assets. In our benchmark, assets are of

very high quality with an average of  $\pi = 0.99$  being good assets. This is consistent with Aaa rated corporate debt which historically has a default rate of 1.09% and 2.38% for 10 and 20 year maturities respectively and a recovery rate of about 50% (see [Moody's Investor Service, 2000](#)). Similar impairment probabilities were associated with Aaa rated tranches of structured debt products before 2007 (see [Moody's Investor Service, 2010](#)). In accordance with [Duffie, Garleanu and Pedersen \(2007\)](#) we set the annual interest rate  $r$  to 5% and the fraction of investors holding an asset to  $S/(1 + S) = 0.8$ .

We then select two key parameters in our analysis, the degree of search frictions  $\lambda$  and the arrival rate of a liquidity shock  $\kappa$ , to match annual turnover rates for debt products. A search intensity of  $\lambda = 100$  for the benchmark implies that a seller expects to contact a buyer and to sell the asset if there is trade once every  $250/100 = 2.5$  trading days. We set  $\kappa$  to 1 so that an asset holder remains an owner for an average of one year. With the proportion of lemons being 1% we obtain a turnover rate of

$$\frac{\lambda(\mu_s + \mu_\ell)}{S} = \lambda \left( \frac{\kappa\pi}{\lambda + \kappa} + 1 - \pi \right) = 1.98, \quad (48)$$

so that assets change hands about twice in a year on average. This is not inconsistent, but at the upper end of the turnover rate reported by the literature.<sup>20</sup> Later on, we conduct a robustness check with respect to the parameter  $\kappa$ , which reflects the need of turning around an asset in the secondary market. We view shorter and medium-term funding markets (e.g. commercial paper) as markets where traders face more frequent liquidity needs (higher  $\kappa$ ).

Table 1: **Benchmark Parameter Values**

$\lambda$	$\kappa$	$S$	$\delta$	$x$	$r$	$\pi$
100	1	4	1	0.035	0.05	0.99

Table 1 gives the values of the exogenous parameters and Table 2 describes the resulting steady-state equilibrium. Note that with our parameters, trades in good assets vs. lemons are approximately 4 to 1. As in [Duffie, Garleanu and Pedersen \(2007\)](#), we have normalized  $\delta = 1$ . The valuation shock is chosen to match the spread of highly rated structure finance products over the risk-free rate  $r$ . The yield is  $1/17.4578 = 5.728\%$  or 73 pbs above the risk-free rate.<sup>21</sup> As shown in Table 2, due to search frictions, a small fraction of good

<sup>20</sup>[Bao, Pan and Wang \(2008\)](#) give turnover rates between one and two years for corporate bonds, while [Goldstein, Hotchkiss and Sirri \(2007\)](#) report a slightly lower annual rate in the range of 0.8-1.2 (see also [Edwards, Harris and Piwowar, 2007](#)). Data for structured products are not readily available.

<sup>21</sup>This is the total spread in [Krishnamurthy and Vissing-Jorgenson \(2010\)](#) for corporate debt of the highest quality. [Gorton and Metrick \(2010\)](#) report a range of 50 to 100 bps on highly rated ABS before 2007.

assets –  $\mu_s/(S\pi) = 0.98\%$  of the total – is misallocated to investors with a liquidity shock. Also, since  $\kappa > r$ , the strategic complementarity dominates the quality effect, implying that  $\bar{\pi} < \underline{\pi}$ . However, the steady state equilibrium falls in the range of multiple equilibria so that it matters whether a trader can resell the asset when receiving a liquidity shock.

Table 2: **Benchmark Steady-state Equilibrium**

$\mu_b$	$\mu_o$	$\mu_s$	$\mu_\ell$	$p$	$\tilde{\pi}$	$\bar{\pi}$	$\underline{\pi}$
1	3.9208	0.0392	0.04	17.4578	0.4950	0.9669	0.9983

Figure 4 shows the equilibrium outcome for different combinations of the asset quality  $\pi$  and the search friction  $\lambda$  in steady state. The red marks indicate the baseline calibration. As shown, a negative shock to asset quality can cause a steady state equilibrium without trade. If the shock is sufficiently large, the unique equilibrium path features no trade after the shock. This outcome is more likely in a market with higher search frictions.

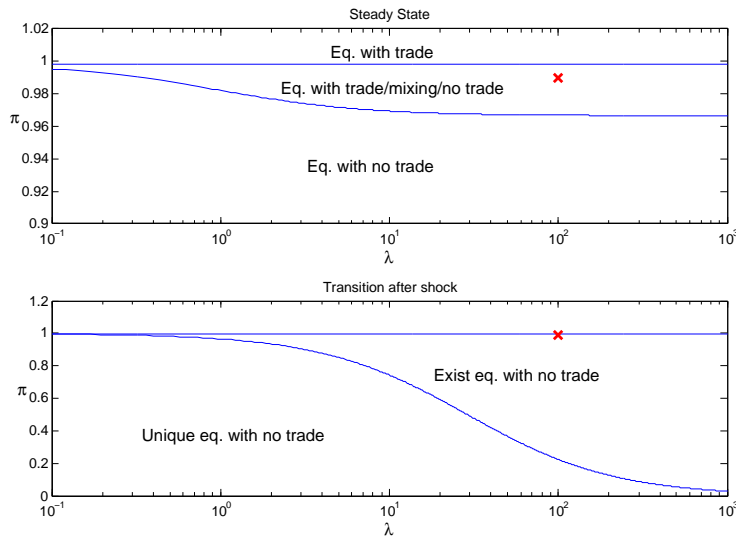


Figure 4: Steady-state Equilibrium and Transition after Shock

## 6.2 Trading and Price Dynamics

We now consider a negative quality shock at  $t = 0$  such that 10% of the assets turn from good to lemons, i.e.,  $\pi(0) = 0.89 < \bar{\pi}$ .<sup>22</sup> In order to resurrect the market, the MMLR needs to purchase at least an amount  $Q_{\min} = 0.3182$  which is about 8% of the total asset supply and 72% of lemons. We first compute the equilibrium trading response  $\gamma(t)$  and the associated market price  $p(t)$  for three different values of  $\lambda$ , when there is a minimum intervention at time  $T = 0.25$  (see Figure 5).

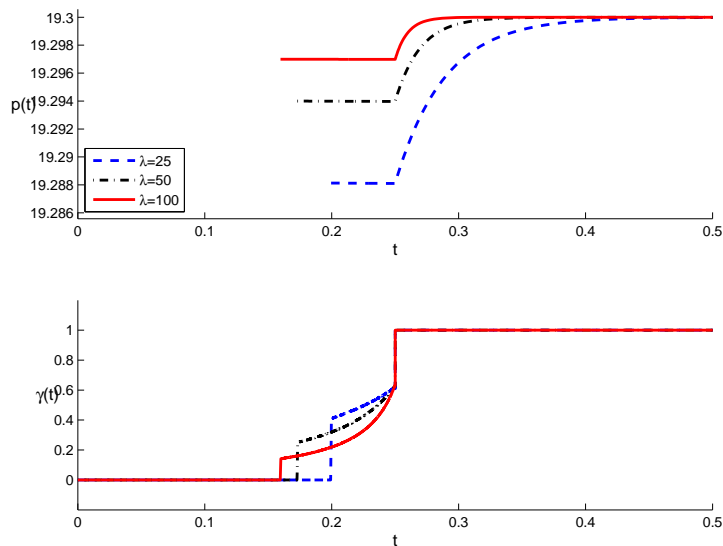


Figure 5: Equilibrium Prices and Trading Dynamics ( $T = 0.25$ ,  $V_I = 0$ ) – Impact of Search Friction

The price jumps up when the market starts to recover at  $\tau_1$ , and drops slightly over time at the rate of  $r$  as trading activity increases. After the intervention time  $T$ , full trade is restored with the price increasing monotonically towards the steady-state level. With smaller search frictions (higher  $\lambda$ ), the market recovers earlier. But with partial trading the fraction of buyers making offers,  $\gamma(t)$ , is also decreasing in  $\lambda$ . The reason is that with less search frictions there cannot be too much trading, as otherwise the quality of assets for sale would drop too fast in order to maintain a mixing equilibrium. Furthermore, a higher  $\lambda$  tends to increase the asset price and speed up its convergence.

<sup>22</sup>Such a shock falls within the range experienced in the financial crisis of 2007-09 where impairment rates on structured finance products with Aaa and Aa ratings jumped to a range of 5-20% depending on the product (see [Moody's Investor Service, 2010](#)).

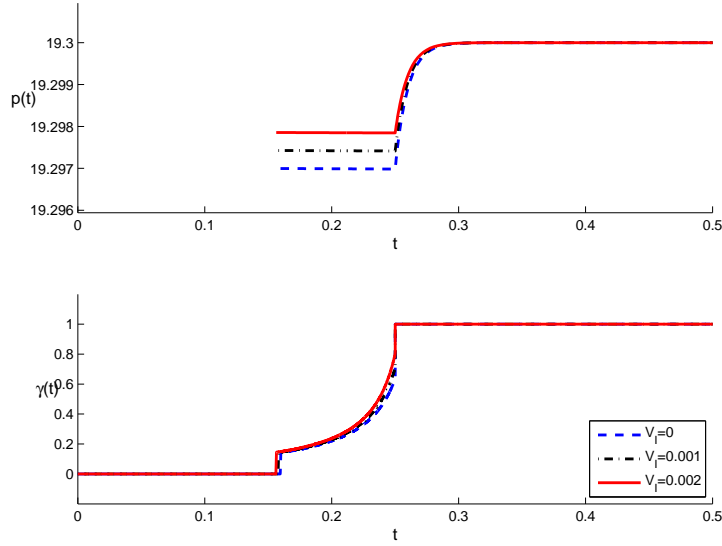


Figure 6: Equilibrium Prices and Trading Dynamics ( $T = 0.25$ ) – Impact of Option Value  $V_I$

For the benchmark case where  $\lambda = 100$ , Figure 6 examines the effects of increasing the price of the intervention  $P$  while holding  $Q$  fixed at  $Q_{\min}$ . Such an increase in  $V_I$  strengthens the strategic complementarity. Hence, it induces the market to recover earlier. It also raises trading activity before the intervention which in turn increases the market price. This is due to a faster drop in the average quality of assets that are for sale when there is more trading in the market.

### 6.3 Announcement Effect

We turn next to the announcement effect. In particular, we are interested in how the time of intervention influences the timing and intensity of market recovery. As a first pass, Figure 7 looks at the time when the market starts to respond to a minimum intervention –  $Q_{\min}$  and  $P_{\min}$  – as a function of the time of intervention  $T$ . More precisely, we plot the breaking time  $\tau_1(T)$  again for the three different levels of the search friction  $\lambda$ , with the solid line indicating our baseline case. We only plot  $\tau_1(T)$  here, since there is no full recovery ( $\tau_2(T) = T$ ) in all three cases.

Postponing the intervention delays the market recovery ( $\tau_1'(T) > 0$ ), but increases the announcement effect in the sense that the market recovers earlier relative to  $T$  ( $\tau_1'(T) \leq 1$ ). The interpretation is that it takes time for selling pressure to build up sufficiently to coun-



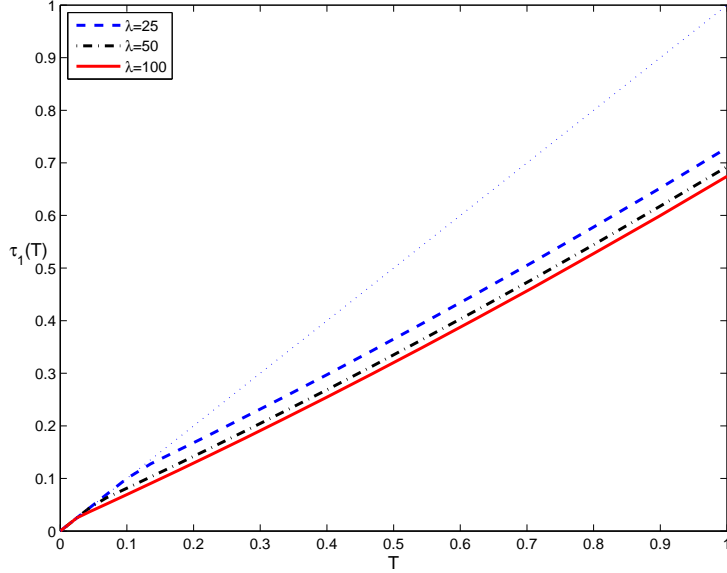


Figure 7: Announcement Effect I– Impact of Search Friction  $\lambda$  and Intervention Time  $T$

teract the lemons problem via the quality effect. For small  $T$ , however, the quality effect is too small for the announcement effect to kick in. Finally, varying  $\lambda$ , but holding the intervention time  $T$  fixed, the market recovers earlier when the search friction is lower – or,  $d\tau_1(T)/d\lambda < 0$ . The reason here is that for  $\kappa > r$  the strategic complementarity is more important than the quality effect (see our earlier discussion of equations (38) and (39)).

This however gives an incomplete picture of how effective the intervention is. As Figure 5 shows, the recovery depends not only on the time when trading starts, but also on the speed at which trading picks up again. We look therefore at two different measures of trading activity to assess the effects of an intervention. The first measure looks at the average time it takes a trader to sell an asset before the intervention  $T$ . Since in equilibrium we face a non-homogeneous Poisson process for trades, this time is given by

$$\mathcal{M}_1 = \frac{\int_0^T t \lambda \gamma(t) e^{-\int_0^t \lambda \gamma(u) du} dt}{1 - e^{-\int_0^T \lambda \gamma(u) du}}, \quad (49)$$

where the denominator reflects the probability to sell at some time before the intervention actually takes place. This measure is clearly influenced by the degree of the search friction  $\lambda$ . We therefore normalize this measure by the average time of selling an asset before  $T$  in

the steady state equilibrium with full trade.<sup>23</sup>

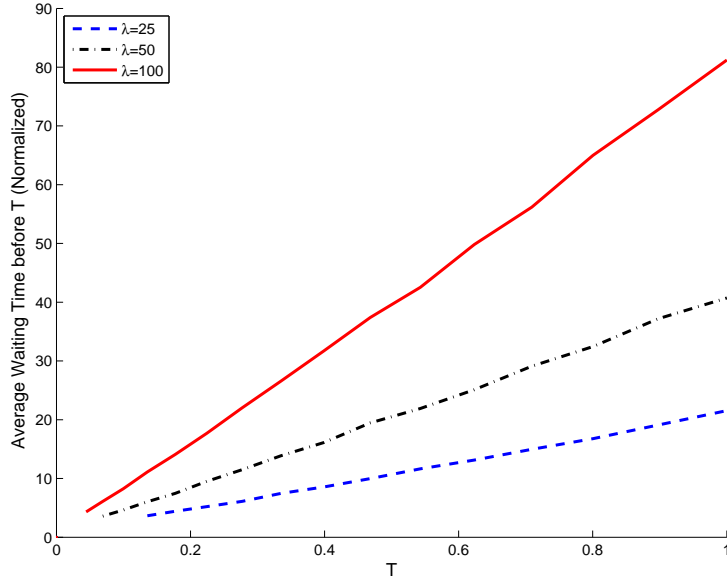


Figure 8: Announcement Effect II – Average Time of Selling an Asset before  $T$  (Normalized)

Figure 8 shows the average time it takes to sell an asset as a multiple of the one in normal times. As the intervention is postponed, the delay in selling time rises as the announcement effect increases less than proportional with the time of intervention. Most interestingly, when there are more trading frictions ( $\lambda$  decreases), the average selling time for investors is closer to normal times.

Since selling pressure builds up during a market freeze, our second measure looks at the total trading volume before  $T$  to assess how the intervention influences overall market activity. This measure is given by

$$\mathcal{M}_2 = \frac{\int_0^T \lambda \gamma(t) [\mu_s(t) + \mu_\ell(t)] dt}{\int_0^T \lambda [\mu_s^{SS} + \mu_\ell^{SS}] dt} = \frac{\int_0^T \gamma(t) [\mu_s(t) + \mu_\ell(t)] dt}{\int_0^T [\mu_s^{SS} + \mu_\ell^{SS}] dt} \quad (50)$$

where we have already normalized our measure by the steady state trading volume in normal times. Note that with continuous trade, we would again have  $\mathcal{M}_2 = 1$ . Figure 9 gives the

<sup>23</sup>Another alternative would be to look at the probability of selling the asset before  $T$

$$\frac{1 - e^{-\int_0^T \lambda \gamma(t) dt}}{1 - e^{-T\lambda}}$$

which is normalized by the corresponding probability in the normal time. However, this measure would miss the timing dimension.

trading volume associated with an intervention at time  $T$  as a percentage of normal times and shows that postponing the intervention time has a non-monotonic effect.

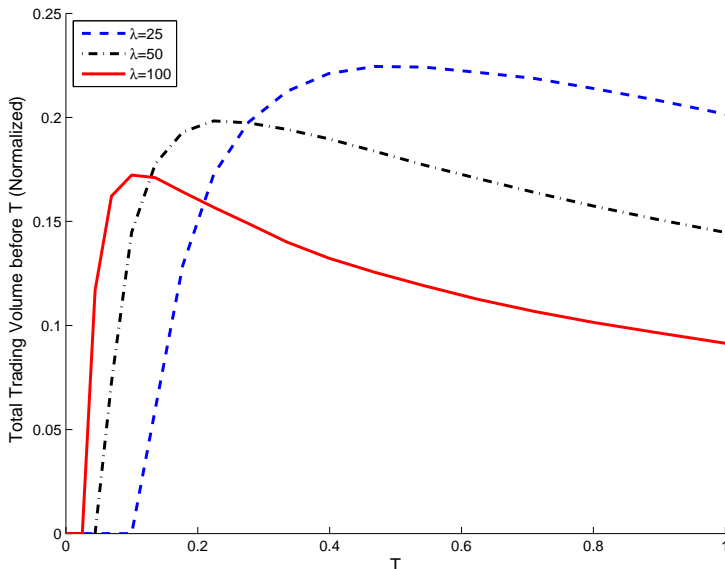


Figure 9: Announcement Effect III – Trading Volume before  $T$  (Normalized)

For a given  $\lambda$ , the trading volume relative to normal times stays zero for small  $T$ . The reason is that it takes time for quality to build up in order to create some announcement effect. Once this effect kicks in, the trading volume is increasing in  $T$  because delaying the intervention allows selling pressure to build up. The selling pressure, however, begins to dissipate once trading starts again. As the intervention is delayed further, market recovery is delayed and trading volume relative to normal times converges to 0. In markets with lower search frictions, trading volume recovers and peaks faster but at a lower overall level. The reason is that, when search frictions are small, quality improves quickly without trade, leading to a faster recovery. However, low search frictions imply that the quality would drop rapidly when there is trade, leading to a more subdued recovery of trading volume when the intervention is delayed.

Overall, search frictions determine the impact of an intervention on market activity. A market with high search frictions recovers later because the quality effect kicks in slower, and the difficulty of finding a counterparty reduces the *absolute* magnitude of the strategic complementarity. On the other hand, an intervention is more effective in a *relative* sense by bringing the market closer to its trading volume in normal times, albeit requiring a longer time to take effect.

## 6.4 Optimal Policy

We now turn to computing the optimal policy for the scenario where the quality drops by 10% leading to a market freeze. The optimal intervention depends on the (social) costs of the intervention which is captured by the parameter  $\theta$ . Figure 10 shows the optimal timing and pricing of the intervention as a function of  $\theta$  for our benchmark economy.

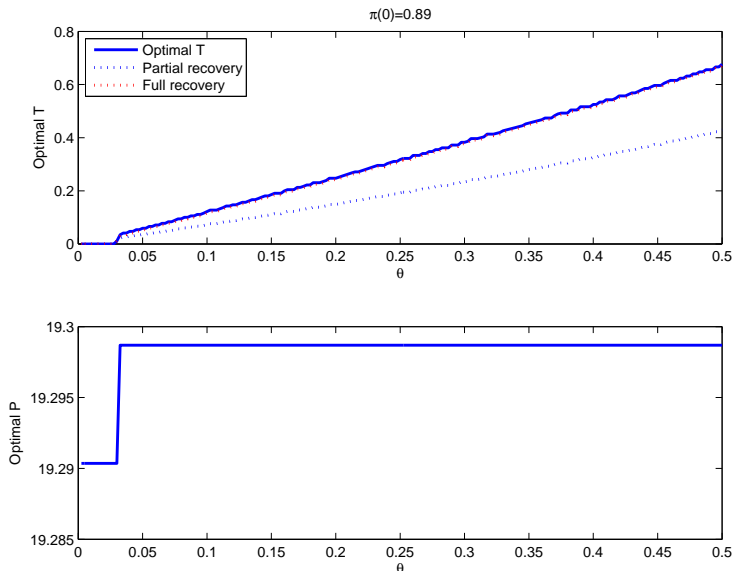


Figure 10: Optimal Intervention for the Benchmark Economy

When  $\theta$  is small, an immediate intervention is optimal which implies that there is no reason to increase the option value  $V_I$ . As the value of  $\theta$  increases, it is optimal to delay the intervention ( $T > 0$ ) more and more and provide a positive option value ( $V_I > 0$ ) in order to maximize the announcement effect. Table 3 verifies our bang-bang result: the price is always set to  $P_{\max}$  when there is sufficient delay, while the quantity of purchases remains constant at the minimum  $Q_{\min}$ .

To put these numbers into perspective, in our baseline calibration, when the deadweight loss of taxation is roughly 5%, the optimal policy is to commit to asset purchases about 3 weeks later. The market will respond by starting to recover gradually about 9 trading days after the market freezes in response to the quality shock. Finally, we address how the optimal intervention will take into account the severity of the trading frictions in the market. Table 4 reports the results for a market where trading in normal times takes 10 days ( $\lambda = 25$ ). The qualitative pattern of the optimal intervention is the same. The MMLR, however, needs to purchase a higher minimum amount of assets. Nonetheless, the optimal intervention is

Table 3: **Optimal Intervention** ( $\lambda = 100$ )

$\theta$	$T$	$P$	$Q$	$\tau_1$
0.0100	0	19.2904	0.3182	0
0.0200	0	19.2904	0.3182	0
0.0300	0.0112	19.2904	0.3182	0.0112
0.0400	0.0472	19.2987	0.3182	0.0295
0.0500	0.0593	19.2987	0.3182	0.0365
0.1000	0.1217	19.2987	0.3182	0.0729
0.1500	0.1858	19.2987	0.3182	0.1111
0.2000	0.2474	19.2987	0.3182	0.1486
0.3000	0.3828	19.2987	0.3182	0.2335
0.4000	0.5253	19.2987	0.3182	0.3266
0.5000	0.6767	19.2987	0.3182	0.4295

now more aggressive by choosing a smaller  $T$ . The reason is twofold. First, the maximal intervention price drops because the reservation value of a lemon is lower when  $\lambda$  declines. Hence, the overall cost impact from the larger asset purchase is dampened. Second, and more importantly, the announcement effect is decreasing in the trading friction for our parameter values. Hence, the MMLR has less incentives to delay and to rely on this effect in order to save costs. Quantitatively, once it becomes optimal to delay and increase the price for lemons, the difference in policy becomes very small in response to the trading friction. Hence, we draw the cautious conclusion from this analysis that ensuring market continuity is more warranted for markets that are (i) more important relative to the social costs of intervention and (ii) more riddled by trading frictions.

## 6.5 Robustness

### 6.5.1 High Liquidity Needs

In our benchmark, owners return to the market with an average frequency of once a year. We now consider a different situation where liquidity shocks hit owners of assets more often. Keeping  $\lambda = 100$  and  $\pi = 0.99$  and setting  $\kappa = 10$  implies that an owner expects to return to the market every 25 trading days resulting in an annual turnover rate of roughly 10 (see

Table 4: **Optimal Intervention Policy** ( $\lambda = 25$ )

$\theta$	$T$	$P$	$Q$	$\tau_1$
0.0100	0	19.2615	0.3215	0
0.0200	0	19.2615	0.3215	0
0.0300	0	19.2615	0.3215	0
0.0400	0	19.2615	0.3215	0
0.0500	0	19.2615	0.3215	0
0.1000	0.0360	19.2615	0.3215	0.0360
0.1500	0.1738	19.2944	0.3215	0.1140
0.2000	0.2434	19.2944	0.3215	0.1571
0.3000	0.3820	19.2944	0.3215	0.2456
0.4000	0.5269	19.2944	0.3215	0.3420
0.5000	0.6799	19.2944	0.3215	0.4479

equation (48)).

As shown in Table 5, a rise in  $\kappa$  requires a larger fraction of assets to be of good quality for trade to be a steady state equilibrium. Even though the average quality of asset  $\tilde{\pi}$  for sale improves, a buyer expects a high valuation for the dividend of the asset for a much shorter duration. Buyers are thus less willing to trade, which leads to a higher cut-off points for trade. Hence, a market freeze becomes more likely even though the quality in the market improves.

Table 5: **Steady-state masses and asset price**

$\kappa$	$\mu_b$	$\mu_o$	$\mu_s$	$\mu_\ell$	$p$	$\tilde{\pi}$	$\bar{\pi}$	$\pi$
10	1	3.6000	0.3600	0.04	17.5642	0.9000	0.9682	0.9998
1	1	3.9458	0.0158	0.0384	18.4	0.2913	0.9285	0.9904

Figure 11 illustrates how the market response to an intervention changes with  $\kappa$ . Given our parameter values, in a market with high  $\kappa$ , an intervention generates a smaller announcement effect, with later recovery and lower trading activities. Since  $\kappa$  lowers the value of being a buyer, the trading price has to be higher to induce sellers to become buyers again.<sup>24</sup> For

<sup>24</sup>A higher  $\kappa$  also reduces  $Q_{\min}$  slightly, but this change has a negligible effect.

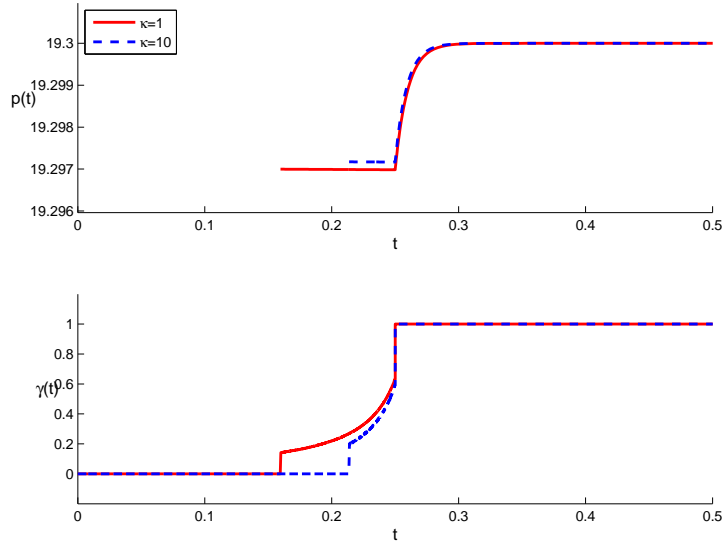


Figure 11: Equilibrium Price and Trading Dynamics ( $T = 0.25$ ,  $V_I = 0$ ) – Impact of Liquidity Needs  $\kappa$

completeness, we also report the optimal policy. Most importantly, the MMLR will be more aggressive in ensuring continuously functioning markets, as traders access the market more frequently to deal with liquidity shocks.

Table 6: **Optimal Intervention Policy** ( $\kappa = 10$ )

$\theta$	$T$	$P$	$Q$	$\tau_1$
0.0100	0	19.2904	0.3231	0
0.0200	0	19.2904	0.3231	0
0.0300	0	19.2904	0.3231	0
0.0400	0	19.2904	0.3231	0
0.0500	0	19.2904	0.3231	0
0.1000	0	19.2904	0.3231	0
0.1500	0	19.2904	0.3231	0
0.2000	0.0073	19.2904	0.3231	0.0073
0.3000	0.0373	19.2986	0.3231	0.0258
0.4000	0.0531	19.2986	0.3231	0.0364
0.5000	0.0701	19.2986	0.3231	0.0484

### 6.5.2 The Liquidity Channel of Policy

We have assumed throughout that when traders sell lemons to the MMLR they exit the market forever. This assumption kept the market tightness constant in the long-run. Suppose now instead that traders who sell lemons to the MMLR can stay in the market and become buyers again after they sold to the MMLR. An intervention can then have more powerful effects, since it increases market liquidity *permanently* by raising the number of buyers in the market from 1 to  $1 + Q$ . This reinforces the strategic complementarity, since we have now for the value of a lemon after full recovery that

$$v_\ell(t) = \frac{\lambda(1+Q)}{\lambda(1+Q)+r}v_s > \frac{\lambda}{\lambda+r}, \quad (51)$$

for all  $t \geq \tau_2(T)$ . We call this additional effect of an intervention *liquidity channel*.

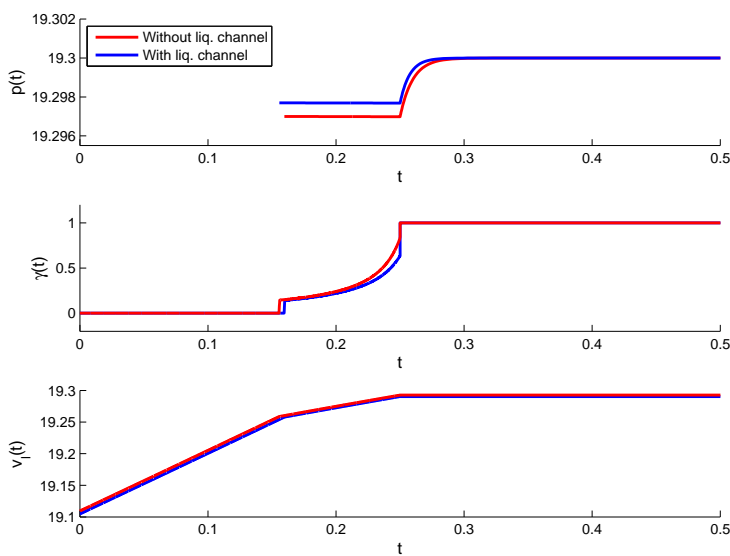


Figure 12: Equilibrium Price and Trading Dynamics – Liquidity Channel

Figure 12 compares the equilibrium trading dynamics of an intervention with and without this liquidity channel of policy. As shown, the liquidity channel quantitatively plays only a small role. The value function for lemons is almost identical for our benchmark economy, and thus the market price and trading dynamics are only slightly altered by this additional effect.<sup>25</sup>

<sup>25</sup>The liquidity channel might empirically be even less relevant, as there could be entry and exit of traders in a specific market. Also, the shock to quality could be temporary with the MMLR being able to lay off some of the assets purchased after recovery leaving market tightness unchanged in the long-run.



## 7 Discussion

We have shown in this paper that search frictions are important for understanding why some markets are more prone to be fragile when there is adverse selection. It is not surprising that a simple policy of purchasing lemons in the market by a government or central bank can alleviate the adverse selection problem. However, we have also shown that there is an announcement effect: simply announcing the intervention can induce investors to start trading again. This effect gives then rise to an interesting trade-off between between the size and the timing of the intervention which in turn depends on the degree of search frictions. Our paper provides then some guidance for policy makers when to intervene and in what form. In particular, we can rationalize intervention to ensure market continuity when a particular asset market is deemed sufficiently important. Nonetheless, our analysis has abstracted from some interesting aspects.

A stark assumption in our analysis makes the shock to the quality of the asset permanent. With a random recovery time for  $\pi(0)$  to jump back to the original level our results might change. If the initial shock is small and recovery relatively likely, the market might just function continuously on its own. Also, there might be an incentive now to delay the intervention – even if the MMLR would like to ensure continuous markets. Delaying saves costs in expected terms, even if this requires an increase in the size of the intervention  $V_I$ . Also, once the asset quality recovers, the MMLR has the option to sell back some of the assets it has acquired. This might induce it to intervene earlier as now the expected costs of transfers have decreased.

A related issue is that the type of shock might also matter for optimal policy. Suppose instead that the average quality of assets stays constant, but the market tightness temporarily increases as some potential buyers leave the market, i.e.  $\mu_b$  drops. This will lower the strategic complementarity and therefore can also lead to a market breakdown. A large player can again intervene, but there is no an important difference. He will buy lemons at a depressed price, being able to seller them later at a higher price when the market recovers. The policy then looks like market-making-of-last-resort rather than buying-of-last-resort. Since there is a potential for profit now, private market-makers could have an incentive to stabilize the market provided they have access to liquidity.

Another interesting detail of the announcement effect is that there is a time-consistency problem. Suppose for the moment that investors believe the announcement that the intervention will take place at some time  $T > 0$  and start trading. The MMLR has then an

incentive at  $T$  to surprise the market by postponing the intervention for a small interval. Even if the market would freeze for a short period, this could mean saving costs by delaying the intervention. Investors will then view such announcements as not credible. To solve this time-consistency problem, the MMLR might have to spread out purchases over time, thereby reducing the gains from delaying the intervention.

We have also not looked at another, related problem. The quality shock is exogenous in our model. Suppose, however, that investors can create new assets. Anticipating that a MMLR will resurrect the market, investors will have an incentive to create lemons. In other words, a moral hazard problem arises from intervening in the event of a market freeze. This shifts the emphasis from intervention to improving the infrastructure in certain markets such as over-the-counter markets. Our paper has unveiled two avenues for reducing the inherent fragility in these markets: improving the transparency of assets traded and increasing market liquidity. We leave a detailed analysis of the last two issues for future work.

# Appendix A Proofs

## A.1 Proof of Proposition 3

In steady state, the surplus function is positive, if and only if the steady state measure of sellers satisfies  $\mu_s \geq \frac{\kappa}{\kappa+\lambda\gamma}S\pi_{\min}$ . Hence, if at time 0, the quality drops to  $\pi(0) < \pi_{\min}$ ,  $\Gamma(0) < 0$  and there cannot be continuous trade.

If there is no trade for any  $t$ , the law of motion for good assets that are for sale is given by

$$\dot{\mu}_s(t) = -\dot{\mu}_o(t) = \kappa\mu_o(t).$$

Since the fraction of good assets drops to  $S\pi(0)$  at time  $t = 0$ , the initial condition is given by  $\mu_s(0) = \frac{\kappa}{\kappa+\lambda\gamma}S\pi(0)$ . This implies that the fraction of good assets on the market for sale at time  $t$ ,  $\tilde{\pi}(t)$ , is increasing monotonically to  $\pi(0)$ .

Since  $v_\ell(t) = 0$  for all  $t$ , we are left to verify that

$$\tilde{\pi}(t)v_o - v_s \leq 0.$$

for all  $t$ . We have that  $\tilde{\pi}(t) < \pi(0)$  for all  $t$  and  $\tilde{\pi}(t) \rightarrow \pi(0)$  as  $t \rightarrow \infty$ . Hence, there exists an equilibrium with no trade as long as

$$\pi(0)v_o - v_s \leq 0$$

or, equivalently,  $\pi(0) \leq \underline{\pi}$ .

To show uniqueness, consider the buyer's surplus if there is trade ( $\gamma(t) = 1$ ) for all  $t$ . Since  $\sup_t \tilde{\pi}(t) \leq \pi(0)$ , it suffices to show that

$$\pi(0)v_o + (1 - \pi(0))v_\ell - v_s \leq 0,$$

where  $v_\ell = \frac{\lambda}{\lambda+r}v_s$ . Hence, if

$$\pi(0) \leq \frac{v_s - v_\ell}{v_o - v_\ell} = \tilde{\pi} = \frac{\kappa\bar{\pi}}{\kappa + (1 - \bar{\pi})\lambda} = \frac{r\underline{\pi}}{r + (1 - \underline{\pi})\lambda}$$

it is a strictly dominant strategy not to buy an asset at any time  $t$ , which completes the proof.

## A.2 Proof of Lemma 5

Let  $T$  be the time of intervention by the MMLR and assume that the MMLR removes  $Q \geq Q_{\min}$  assets. We show first that  $\tilde{\pi}(t)$  declines for any interval in  $[T, \infty)$  where  $\gamma(t) = 1$ .

For this case, the law of motion is given by

$$\dot{\mu}_\ell = 0 \quad (\text{A.1})$$

$$\dot{\mu}_s(t) = \kappa\mu_o(t) - \lambda\mu_s(t) \quad (\text{A.2})$$

$$\dot{\mu}_o(t) = -\kappa\mu_o(t) + \lambda\mu_s(t). \quad (\text{A.3})$$

Therefore, the measure of sellers is given by

$$\mu_s(t) = S\pi(0)\frac{\kappa}{\kappa + \gamma\lambda} (1 - e^{-(\kappa + \gamma\lambda)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa + \gamma\lambda)(t-\tau)}$$

Again, for an interval with trade  $[\tau, \tau']$  with  $\tau \geq T$  we have

$$\tilde{\pi}(t) = \frac{S\pi(0)\frac{\kappa}{\kappa + \gamma\lambda} (1 - e^{-(\kappa + \gamma\lambda)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa + \gamma\lambda)(t-\tau)}}{S\pi(0)\frac{\kappa}{\kappa + \gamma\lambda} (1 - e^{-(\kappa + \gamma\lambda)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa + \gamma\lambda)(t-\tau)} + (1 - \pi(0))S - Q}$$

which gives up to a constant

$$\frac{\partial \tilde{\pi}(t)}{\partial t} = -(\kappa + \gamma\lambda)e^{-(\kappa + \gamma\lambda)(t-\tau)} [(1 - \pi(0))S - Q] \left( \mu_s(\tau) - S\pi(0)\frac{\kappa}{\kappa + \gamma\lambda} \right). \quad (\text{A.4})$$

If there has been continuous trade from  $t = 0$  until  $\tau$ , i.e.  $\gamma(t) = 1$  for all  $t \in [0, \tau]$ , we have that  $\mu_s(\tau) = \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa + \lambda}$ . Hence,  $\tilde{\pi}(t)$  is constant as long as  $\gamma(t) = 1$ .

If there has not been full trade at some time before  $\tau$ , i.e.  $\gamma(t) < 1$  for some  $[t_1, t_2] \subset [0, \tau]$ , it must be the case that  $\mu_s(\tau) > \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa + \lambda}$  which completes the proof of the first statement.

For the second result, notice that the average quality of assets for sale with a minimum intervention  $Q_{\min} = S(1 - \frac{\pi(0)}{\bar{\pi}})$  and at the lower bound for  $\mu_s$  is given by

$$\frac{\frac{\kappa}{\kappa + \lambda}S\pi(0)}{\frac{\kappa}{\kappa + \lambda}S\pi(0) + S(1 - \pi(0)) - Q} = \frac{\kappa\bar{\pi}}{\kappa + (1 - \lambda)\bar{\pi}} = \tilde{\pi}. \quad (\text{A.5})$$

We denote this level by  $\tilde{\pi}^{SS}(Q_{\min})$ . Any intervention  $Q > Q_{\min}$  will lead to  $\tilde{\pi}(T^+) > \tilde{\pi}^{SS}(Q_{\min})$  and by the previous lemma, we have that  $\tilde{\pi}(T^+)$  declines monotonically. Since  $\mu_s(t) \rightarrow \mu_s(0)$ , we obtain immediately that  $\tilde{\pi}(t) \rightarrow \tilde{\pi}^{SS}(Q) \geq \tilde{\pi}^{SS}(Q_{\min}) = \frac{\kappa\bar{\pi}}{\kappa + (1 - \lambda)\bar{\pi}}$  for  $t \rightarrow \infty$ .

### A.3 Proof of Proposition 7

First, we show that once the surplus function  $\Gamma(t)$  becomes positive it has to stay positive.

**Lemma A.1.** *If  $\Gamma(t_0) \geq 0$  for some  $t_0 < T$ , then  $\Gamma(t_1) \geq 0$  for all  $t_1 \in (t_0, T)$ .*

*Proof.* Suppose not. Then, there exists a  $t_1 \in (t_0, T)$  such that  $\Gamma(t_1) < 0$ . As  $\Gamma$  is continuous, this implies that there must be an interval  $(\tau_0, \tau_1) \subset (t_0, t_1)$  where there is no trade, i.e.  $\gamma(t) = 0$ . But then, over this interval, the average quality  $\tilde{\pi}(t)$  increases and we have that  $\dot{v}_\ell(t) = v_\ell(T^-)e^{-r(T-t)} > 0$ . Hence,  $\Gamma(t)$  must be strictly increasing over this interval starting out at  $\Gamma(\tau_0) = 0$ . A contradiction.  $\square$

This implies that after there was some trade in the economy ( $\gamma(t) > 0$ ), we cannot have no trade ( $\gamma(t) = 0$ ) anymore and the surplus function  $\Gamma(t)$  has to stay non-negative. We next show that with full trade ( $\gamma(t) = 1$ ) in some interval, the surplus function has to be strictly convex.

**Lemma A.2.** *If  $\gamma(t) = 1$  for some interval  $[t_0, t_1]$  with  $t_1 < T$ , then  $\Gamma(t)$  is strictly convex over this interval.*

*Proof.* We have

$$\Gamma(t) = \tilde{\pi}(t)(v_o - v_\ell(t)) + (v_\ell(t) - v_s) \quad (\text{A.6})$$

$$\dot{\Gamma}(t) = \dot{\tilde{\pi}}(t)(v_o - v_\ell(t)) + (1 - \tilde{\pi}(t))\dot{v}_\ell(t) \quad (\text{A.7})$$

$$\ddot{\Gamma}(t) = \ddot{\tilde{\pi}}(t)(v_o - v_\ell(t)) - 2\dot{\tilde{\pi}}(t)\dot{v}_\ell(t) + (1 - \tilde{\pi}(t))\ddot{v}_\ell(t). \quad (\text{A.8})$$

We will show that  $\Gamma(t)$  is strictly convex if it is positive and strictly increasing. Assuming  $\gamma(t) = 1$  for the asset quality and omitting time indexes, we have

$$\dot{\tilde{\pi}} = (1 - \tilde{\pi}) \frac{\dot{\mu}_s}{\mu_s + \mu_\ell} \quad (\text{A.9})$$

$$\ddot{\tilde{\pi}} = -\dot{\tilde{\pi}} \frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (1 - \tilde{\pi}) \frac{(\mu_s + \mu_\ell)\ddot{\mu}_s - (\dot{\mu}_s)^2}{(\mu_s + \mu_\ell)^2}. \quad (\text{A.10})$$

Since  $\gamma(t) = 1$ ,  $\dot{\tilde{\pi}}(t) < 0$ . Also, we either have continuous trade or we need to have a region with less than full trade ( $\gamma(t) < 1$ ) before. Assume w.l.o.g. that  $\Gamma(t)$  crosses zero from below at time  $t_0$ . Hence, at  $t_0$ , we need that the right-hand derivative  $\dot{\Gamma}(t_0^+) > 0$ . This can only be the case if  $\dot{v}_\ell(t_0^+) > 0$ . Also, we have that

$$v_\ell(t) = \frac{\lambda}{\lambda + r} v_s (1 - e^{-(r+\lambda)(t_1-t)}) + v_\ell(t_1) e^{-(r+\lambda)(t_1-t)}. \quad (\text{A.11})$$

Hence,  $\dot{v}_\ell(t) > 0$  for  $t \in [t_0, t_1]$  if and only if  $\dot{v}_\ell(t_0^+) > 0$ . This implies that  $v_\ell(t)$  is a strictly increasing and strictly convex function over this interval. Hence, the last two terms are positive in the expression for  $\ddot{\Gamma}(t)$ . Note that  $v_o - v_\ell(t) > 0$ . If  $\ddot{\pi}(t)$  is positive we are done.

Suppose to the contrary that  $\ddot{\pi}(t) < 0$ . As long as  $\dot{\Gamma}(t) > 0$  over the interval, it must be the case that

$$0 < v_o - v_\ell(t) < -\frac{1 - \tilde{\pi}(t)}{\dot{\tilde{\pi}}(t)} \dot{v}_\ell(t). \quad (\text{A.12})$$

Using this in the expression for  $\ddot{\Gamma}(t)$ , it suffices to show that

$$-\ddot{\tilde{\pi}}(t) \frac{1 - \tilde{\pi}(t)}{\dot{\tilde{\pi}}(t)} \dot{v}_\ell(t) - 2\dot{\tilde{\pi}}(t) \dot{v}_\ell(t) + (1 - \tilde{\pi}(t)) \ddot{v}_\ell(t) > 0 \quad (\text{A.13})$$

or that

$$-\frac{\ddot{\tilde{\pi}}(t)}{\dot{\tilde{\pi}}(t)} - 2\frac{\dot{\tilde{\pi}}(t)}{1 - \tilde{\pi}(t)} + \frac{\ddot{v}_\ell(t)}{\dot{v}_\ell(t)} > 0. \quad (\text{A.14})$$

Note that  $\ddot{v}_\ell(t) = (r + \lambda)\dot{v}_\ell(t)$ . Hence, rewriting and using the fact that  $\dot{\mu}_s = -(\kappa + \lambda)\dot{\mu}_s$ , we obtain

$$-\left[ -\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + \frac{-(\mu_s + \mu_\ell)(\kappa + \lambda) - \dot{\mu}_s}{\mu_s + \mu_\ell} \right] - 2\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (\lambda + r) > 0 \quad (\text{A.15})$$

$$(\kappa + \lambda) + (r + \lambda) > 0 \quad (\text{A.16})$$

which completes the proof.  $\square$

The proposition follows now directly as a corollary from this lemma. Without trade, it must be the case that the surplus function  $\Gamma(t)$  is increasing over time, as both  $\dot{\tilde{\pi}} > 0$  and  $\dot{v}_\ell > 0$ . Once the surplus function becomes strictly positive, it cannot decrease anymore, as it is strictly convex. Hence,  $\dot{\Gamma} > 0$  whenever  $\gamma(t) = 1$  is an equilibrium.

## A.4 Proof of Proposition 8

Suppose that  $0 \leq \tau_2(T) < T$ . Then,  $\tilde{\pi}(t)$  declines over  $[\tau_2, T]$ , while  $v_\ell(t) = v_\ell(T)$ . Hence, the surplus from trade  $\Gamma(t)$  declines as well over this interval. Also, it must be the case that  $\gamma(t) < 1$  for  $t \in [0, \tau_2)$  given the initial shock  $\pi(0) < \pi_{\min}$ . Hence,  $\Gamma(t) \leq 0$  for  $[0, \tau_2)$ . Since  $\Gamma(t)$  must be continuous over  $[0, T)$ , this implies that  $\Gamma(t) < 0$  for  $(\tau_2, T]$ . A contradiction.

For the second part, note that the surplus for buyers is given by

$$\begin{aligned}
\Gamma(t) &= \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s \\
&< \pi(0)v_o + (1 - \pi(0))v_\ell(T) - v_s \\
&\leq \tilde{\pi}_{\min}v_o - (1 - \tilde{\pi}_{\min})v_\ell - v_s \\
&= 0.
\end{aligned}$$

Hence, the surplus from trading is strictly negative for any  $t_0 < T$  prior to the intervention, even if there is trade with  $\gamma(t) = 1$  for all  $T > t > t_0$ . Hence, it is a strictly dominant strategy not to trade.

For the last statement, note first that if there is trade from  $t = 0$  onward, the quality of assets for sale remains unchanged at  $\tilde{\pi}(0)$  until the intervention takes place at  $T$ . Suppose first that condition (37) is satisfied. We then have that

$$\Gamma(t) > \Gamma(0) \geq 0 \tag{A.17}$$

for all  $t \in (0, T]$ , as  $v_\ell(t) = \frac{\lambda}{\lambda+r}v_s + V_I e^{-(r+\lambda)T}$  is strictly increasing when there is trading for all  $t$ . If the condition does not hold, it is optimal to not trade for  $t$  sufficiently close to 0, even if there is trade forever afterwards.

## A.5 Proof of Proposition 9

Since there is no trade in the interval  $[0, \tau_1)$ , we set  $p(t) = 0$ . For  $[\tau_1, \tau_2)$ , there is partial trade. The expected surplus  $\Gamma(t)$  is constant at zero in this interval which implies for the value function of the buyer that

$$v_b(t) = v_b(\tau_2)e^{-r(\tau_2-t)}. \tag{A.18}$$

Since prices are given by  $p(t) = v_s - v_b(t)$ , they decrease at a rate  $r$ .

For the interval  $[\tau_2, T)$ , we have that the differential equation of buyers is given by

$$\dot{v}_b(t) - rv_b(t) = -\lambda(\mu_s(t) + \mu_\ell(T^-))\Gamma(t). \tag{A.19}$$

Rewriting, we have

$$\Gamma(t) = \frac{1}{\lambda(\mu_s(t) + \mu_\ell(T^-))} (rv_b(t) - \dot{v}_b(t)) > 0. \tag{A.20}$$

Hence,

$$r > \frac{\dot{v}_b(t)}{v_b(t)} \tag{A.21}$$

which means that the price can decline at most at rate  $r$  which is less than with partial trade.

Finally, we turn to the the interval  $[T, \infty)$ . After the intervention we have for the differential equation of buyers

$$\dot{v}_b(t) - rv_b(t) = -\lambda \left( \mu_s(t)(v_o - v_s) - \mu_\ell(T) \frac{r}{r + \lambda} v_s \right). \quad (\text{A.22})$$

Note that the right-hand side of this expression is strictly negative and continuously increasing as  $\dot{\mu}_s(t) < 0$ . Differentiating the differential equation for  $v_b(t)$ , we obtain

$$\frac{\partial \dot{v}_b(t)}{\partial t} = r\dot{v}_b(t) - \lambda \dot{\mu}_s(t)(v_o - v_s). \quad (\text{A.23})$$

If  $\dot{v}_b(t) > 0$ ,  $v_b(t)$  is strictly convex and continuity of  $v_b(t)$  in  $(T, \infty)$  would imply that it diverges. This is a contradiction since  $v_b(t)$  is bounded from above.

Hence,  $v_b(t)$  is strictly decreasing implying that prices are increasing or  $\dot{p}(t) > 0$ . Finally, denoting the steady state level of the buyers value function by  $v_b(Q)$ , we have

$$\lim_{t \rightarrow \infty} v_b(t) = \frac{\lambda}{r} \left\{ S\pi(0) \frac{\kappa}{\kappa + \lambda} + (S(1 - \pi(0)) - Q) \frac{r}{r + \lambda} v_\ell \right\} \equiv v_b(Q), \quad (\text{A.24})$$

which implies that  $\lim_{t \rightarrow \infty} p(t) = p = v_s - v_b(Q)$ .

For the last statement in the proposition, note that  $v_b(t)$  is continuous, but has a discrete jump in its derivative as  $\Gamma(T)$  jumps discretely at the time of intervention. The left- and right-hand derivative both exist at  $T$  and are given by

$$\dot{v}_b(T^-) = rv_b(T) - \lambda \mu_s(T)(v_o - v_s) - \lambda \mu_\ell(T^-) \left( -\frac{r}{r + \lambda} v_s + V_I \right) \quad (\text{A.25})$$

$$\dot{v}_b(T) = rv_b(T) - \lambda \mu_s(T)(v_o - v_s) - \lambda \mu_\ell(T) \left( -\frac{r}{r + \lambda} v_s \right). \quad (\text{A.26})$$

We thus obtain for their difference

$$\begin{aligned} \dot{v}_b(T) - \dot{v}_b(T^-) &= \lambda \left[ \mu_\ell(T^-) \left( V_I - \frac{r}{r + \lambda} v_s \right) + \mu_\ell(T) \frac{r}{r + \lambda} v_s \right] \\ &= \lambda \left[ S(1 - \pi(0)) \left( V_I - \frac{r}{r + \lambda} v_s \right) + \left( S(1 - \pi(0)) - Q \right) \frac{r}{r + \lambda} v_s \right] \\ &= \lambda \left[ S(1 - \pi(0)) V_I - Q \frac{r}{r + \lambda} v_s \right]. \end{aligned} \quad (\text{A.27})$$

Using the definition of  $V_I$ ,

$$\begin{aligned} \dot{v}_b(T) - \dot{v}_b(T^-) &= \lambda \left[ Q(P - v_\ell(T)) - Q \frac{r}{r + \lambda} v_s \right] \\ &= \lambda Q (P - v_s) \\ &\leq 0, \end{aligned} \quad (\text{A.28})$$



where the last inequality follows from  $P \leq v_s$ . Hence, at  $T$ , the derivative jumps down discretely and thus, the derivative of the price function increases discretely.

## A.6 Proof of Lemma 11

Consider the cost isoquants for any policy  $(T, V_I)$ . These isoquants are given by

$$\frac{\partial C(V_I)e^{-rT(V_I)}}{\partial V_I} = e^{-rT(V_I)} \left[ \frac{dC}{dV_I} - rC(V_I) \frac{dT}{dV_I} \right] = 0.$$

Let  $V_I > V_I^{\min}$ . Then, we have

$$\frac{1}{r} \frac{dC/dV_I}{C(V_I)} = \frac{1}{r} \frac{1}{V_I} > \frac{1}{r + \lambda\gamma} \frac{1}{V_I} = \frac{dT}{dV_I}.$$

for all  $V_I$ . Hence, the net present value of costs is increasing in the marginal policy change  $(T, V_I)$ .

Now, let  $V_I < V_I^{\min}$ . We have

$$\frac{1}{r} \frac{dC/dV_I}{C(V_I)} - \frac{dT}{dV_I} < 0$$

if and only if

$$V_I S(1 - \pi(0)) < \frac{r}{r + \lambda\gamma} (V_I S(1 - \pi(0)) + Q_{\min} v_\ell)$$

or

$$1 < \frac{r}{r + \lambda\gamma} \left( 1 + \frac{Q_{\min} v_\ell}{V_I S(1 - \pi(0))} \right) = \frac{1}{r + \lambda\gamma} \left( r + \lambda \frac{Q_{\min} \frac{r}{r + \lambda} v_s}{V_I S(1 - \pi(0))} \right).$$

Since  $V_I S(1 - \pi(0)) \leq Q_{\min} \frac{r}{r + \lambda} v_s$ , we have that the right-hand side of the last expression is bounded below by  $(r + \lambda)/(r + \lambda\gamma) > 1$  when  $\gamma \leq 1$ . Hence, the net present value of costs is decreasing in the marginal policy change  $(T, V_I)$ .

## A.7 Proof of Proposition 12

For establishing that  $Q^* = Q_{\min}$  we distinguish two cases, full ( $\tau_2(T) < T$ ) and partial recovery ( $\tau_1(T) < \tau_2(T) = T$ ) before the intervention given any policy  $(T, V_I)$ .

Consider first any policy  $(T, V_I)$  for which  $V_I > V_I^{\min}$  and  $\tau_2(T) < T$ . We construct a cheaper policy  $(T', V'_I)$  that leaves the incentives to trade unchanged at any  $t$ . Define the new time of intervention by  $T' = T - \Delta \in (\tau_2(T), T)$  and define the new size of the intervention corresponding to a marginal policy change by

$$V'_I = V_I e^{-(r + \lambda)(T - T')}. \tag{A.29}$$

Since  $\Gamma(t) > 0$  for all  $[T', \infty)$ , this leaves  $v_\ell(T'^-)$  unaffected. This implies that the old equilibrium strategy  $\gamma(t)$  is still an equilibrium for  $[0, T)$ , as both  $v_\ell(t)$  and  $\tilde{\pi}(t)$  remain unchanged leading to the same surplus function  $\Gamma(t)$  as before for all  $t \in [0, T']$ . For small enough  $\Delta > 0$ , we have  $V'_I \geq V_I^{\min}$ . Hence, by Lemma 11 the net present value of costs has decreased which implies that the original policy  $(T, V_I)$  cannot be optimal.

For a policy  $(T, V_I)$  such that  $V_I > V_I^{\min}$  and  $\tau_1(T) < \tau_2(T) = T$ , we have that at  $t \in (\tau_1(T), T)$ ,

$$\begin{aligned} v_\ell(t) &= v_\ell(T) e^{\int_t^T -(r+\lambda\gamma(s))ds} - \int_t^T e^{\int_t^s -(r+\lambda\gamma(s))ds} (-\gamma(s)) \lambda ds \\ &= v_\ell(T) e^{-(r+\lambda\bar{\gamma})(T-t)} + \frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} v_s (1 - e^{-(r+\lambda\bar{\gamma})(T-t)}) \\ &= \left( \frac{\lambda}{\lambda + r} v_s + V_I \right) e^{-(r+\lambda\bar{\gamma})(T-t)} + \frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} v_s (1 - e^{-(r+\lambda\bar{\gamma})(T-t)}), \end{aligned}$$

for some  $\bar{\gamma} \in (0, 1)$ . This allows us to define a new policy with  $T' = T - \Delta \in (\tau_1(T), T)$  and the size of intervention given by

$$\begin{aligned} V'_I &= V_I e^{-(r+\lambda\bar{\gamma})(T-T')} + \left( 1 - e^{-(r+\lambda\bar{\gamma})(T-T')} \right) \left( \frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} - \frac{\lambda}{\lambda + r} \right) v_s \\ &< V_I e^{-(r+\lambda\bar{\gamma})(T-T')}. \end{aligned}$$

For  $\Delta$  sufficiently small,  $V'_I > 0$  and this new policy saves more costs than a marginal policy change with an average  $\bar{\gamma} \in (0, 1)$ . As in the argument above, since  $v_\ell(T'^-)$  stays constant for the new policy  $(T', V'_I)$ , the old equilibrium strategies for  $[0, T')$  and  $\gamma(t) = 1$  for  $[T', \infty)$  form an equilibrium. Since the marginal policy change decreases costs and the allocation in the market has improved, the policy  $(T, V_I)$  cannot be optimal. This completes the proof of the first part of the proposition.

For optimal prices, consider a policy  $(T, V_I)$  with  $\tau_2(T) < T$ . By Proposition 8, we must have that  $V_I \in (0, V_I^{\min})$ . Define a new policy by  $T' = T + \Delta$  and  $V'_I = V_I e^{-(r+\lambda)(T-T')}$ . This defines a marginal policy change that leaves  $v_\ell(T)$  unchanged. Furthermore, for  $\Delta$  sufficiently small we have that  $\Gamma(t) > 0$  for all  $t \in [T, T')$ , since at the original policy  $\Gamma(T^-) > 0$ . Hence, the old equilibrium strategy  $\gamma$  is still an equilibrium, but the new policy is cheaper. Hence,  $(T, V_I)$  cannot be optimal.

## Appendix B Free Disposal of Lemons

We derive here a sufficient condition on the number of securities  $S$  that rules out incentives for traders to dispose of assets in steady state in order to become buyers again. Notice first that the value of sellers is larger than the value of lemons independent of the trading strategy  $\gamma$  by buyers,

$$v_s \geq v_\ell. \quad (\text{B.1})$$

Hence, it is sufficient to show that  $v_\ell \geq v_b$  which also implies that  $p > 0$  for any  $\gamma > 0$ . For  $\gamma > 0$ , we have

$$\begin{aligned} \frac{v_\ell(\gamma) - v_b(\gamma)}{v_s} &= \frac{\lambda\gamma}{\lambda\gamma + r} - \frac{1}{r}\lambda\gamma(\mu_s + \mu_\ell) \left[ \tilde{\pi} \left( \frac{1}{\underline{\pi}} - 1 \right) + (1 - \tilde{\pi}) \frac{r}{\lambda\gamma + r} \right] \\ &= \frac{\lambda\gamma}{\lambda\gamma + r} - \frac{1}{r} \left[ \mu_s \lambda\gamma \left( \frac{1}{\underline{\pi}} - 1 \right) + \mu_\ell \lambda\gamma \frac{r}{\lambda\gamma + r} \right] \\ &= \left( \frac{\lambda\gamma}{\lambda\gamma + r} \right) \left( (1 - \mu_\ell) - \mu_s \left( \frac{\lambda\gamma + r}{r} \right) \left( \frac{(\xi - 1)r}{r + \kappa} \right) \right). \end{aligned} \quad (\text{B.2})$$

Hence, the sign of this expression depends on

$$\left( 1 - (1 - \pi)S \right) - S\pi \left( \frac{\kappa(\lambda\gamma + r)}{(\kappa + \lambda\gamma)r} \right) \left( \frac{(\xi - 1)r}{r + \kappa} \right). \quad (\text{B.3})$$

This expression is increasing in  $\gamma$  whenever  $\kappa > r$  and decreasing otherwise.

Assume first that  $r > \kappa$ . Then, with  $\gamma = 0$ , the expression above is positive as long as

$$(1 + S) \geq S\pi \left( 1 + \frac{(\xi - 1)r}{r + \kappa} \right) \quad (\text{B.4})$$

or

$$(1 + S) \geq S \frac{\pi}{\underline{\pi}}. \quad (\text{B.5})$$

Similarly, for  $\kappa > r$ , set  $\gamma = 1$  and the expression is positive as long as

$$(1 + S) \geq S \frac{\pi}{\bar{\pi}}. \quad (\text{B.6})$$

This implies that a sufficient condition for traders not to dispose of any assets is given by

$$\frac{\max\{\underline{\pi}, \bar{\pi}\}}{1 - \max\{\underline{\pi}, \bar{\pi}\}} \geq S \quad (\text{B.7})$$

whenever  $\pi \geq \min\{\underline{\pi}, \bar{\pi}\}$ .

When  $\pi < \min\{\underline{\pi}, \bar{\pi}\}$ , free disposal does not matter, as there is no trade in steady state and it cannot be an optimal strategy for any trader with a lemon to dispose of his asset with positive probability so that there is trade again, since he would then have a strictly higher utility from retaining the lemon.

## Appendix C Pooling vs. Separating Contracts

We show here that offering a pooling contract is a dominant strategy for buyers. We assume that there is positive trade surplus for good assets, but non-positive trade surplus for lemons.<sup>26</sup> Denote the *net* value of a good asset to a seller as  $v_s^g$  and to a buyer as  $v_b^g > v_s^g$ . Denote the net value of a lemon to a seller as  $v_s^\ell$  and to a buyer as  $v_b^\ell \leq v_s^\ell$ . Also, we assume that  $v_s^g > v_s^\ell$ .<sup>27</sup>

Consider any contract  $(p, q)$ , where  $p$  is the price paid by the buyer and  $q$  is the probability that the seller transfers the asset. We want to show that buyers always prefer a pooling contract or in other words do not have an incentive to separate sellers by using contracts with lotteries (i.e.  $q < 1$ ).

### Case (1): Pooling contract with $(p, q)$ to trade with both types

Sellers with good asset will sell at price  $p$  if and only if

$$p + (1 - q)v_s^g \geq v_s^g. \quad (\text{C.1})$$

Hence, given  $q$ , the buyer will offer the price  $p = qv_s^g$ . The best contract for the buyer offers

$$\max_{q \in [0,1]} \tilde{\pi} q v_b^g + (1 - \tilde{\pi}) q v_b^\ell - p = q (\pi v_b^g + (1 - \pi) v_b^\ell - v_s^g). \quad (\text{C.2})$$

Since the return is linear in  $q$ , we have that no lottery will be used and the solution is either  $(p, q) = (0, 0)$  or  $(p, q) = (v_s^g, 1)$ .

### Case (2): Separating contract $(p^g, q^g)$ and $(p^\ell, q^\ell)$

To separate and trade with the two types, the two contracts have to satisfy incentive constraints and make all sellers willing to sell. These constraints are given by

$$p^g - q^g v_s^g \geq 0 \quad (\text{C.3})$$

$$p^g - q^g v_s^g \geq p^\ell - q^\ell v_s^g \quad (\text{C.4})$$

$$p^\ell - q^\ell v_s^\ell \geq 0 \quad (\text{C.5})$$

$$p^\ell - q^\ell v_s^\ell \geq p^g - q^g v_s^\ell. \quad (\text{C.6})$$

Since there is no trade surplus from trading lemons, any equilibrium with trade must have some trade of the good asset or  $q^g > 0$ . This implies that (C.5) is never binding or that

<sup>26</sup>We could dispense of this assumption. There could then be a separating equilibrium where only lemons are traded, but not the good asset. We would interpret such a situation still as a market freeze.

<sup>27</sup>These net values are defined by  $v_s^g = v_s - v_b$ ,  $v_b^g = v_o - v_b$ ,  $v_s^\ell = v_\ell - v_b$  and  $v_b^\ell = v_\ell - v_b$ .

lemons will always be sold (see  $v_s^g > v_s^\ell$ ). Also, both (C.3) and (C.6) must be binding, otherwise it is profitable for the buyer to lower both  $p^g$  and  $p^\ell$  by some  $\varepsilon > 0$  or  $p^\ell$  by some  $\varepsilon > 0$ . Then, (C.4) and (C.6) imply

$$(q^g - q^\ell)v_s^\ell = p^g - p^\ell \geq (q^g - q^\ell)v_s^g. \quad (\text{C.7})$$

Therefore, (C.4) is satisfied if and only if  $q^\ell \geq q^g$ . We also obtain for the prices

$$p^g = q^g v_s^g \quad (\text{C.8})$$

$$p^\ell = (q^\ell - q^g)v_s^\ell + q^g v_s^g \quad (\text{C.9})$$

implying that  $p^\ell \leq p^g$ .

The buyer then solves

$$\begin{aligned} & \max_{q^\ell, q^g} \pi[q^g v_b^g - p^g] + (1 - \pi)[q^\ell v_b^\ell - p^\ell] \\ & = \max_{q^\ell, q^g} \pi[q^g v_b^g - q^g v_s^g] + (1 - \pi)[q^\ell v_b^\ell - (q^\ell - q^g)v_s^\ell - q^g v_s^g] \\ & = \max_{q^\ell, q^g} q^g (\pi[v_b^g - v_s^g] + (1 - \pi)(v_b^\ell - v_s^\ell)) + q^\ell (1 - \pi)[v_b^\ell - v_s^\ell] \end{aligned} \quad (\text{C.10})$$

subject to  $q^\ell \geq q^g$ . If  $v_b^\ell < v_s^\ell$ , it is thus optimal to set  $q^\ell = q^g$  implying that  $p^g = p^\ell$ . We then have a pooling contract. To the contrary, if  $v_b^\ell = v_s^\ell$ , the problem becomes

$$\max_{q^g} q^g (\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g) \quad (\text{C.11})$$

which gives exactly the same payoff as with a pooling contract. In particular, if  $\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g > 0$ , then  $q^g = q^\ell = 1$  and  $p^g = p^\ell = v_s^g$ . If  $\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g < 0$ , then there exist only trades with lemons which generates zero trade surplus.

### Case (3): Trade by good or bad sellers only at $(p^g, q^g)$ or $(p^\ell, q^\ell)$ , respectively

To exclude lemons, we need  $p^\ell - q^\ell v_s^\ell < 0$ , which by (C.6) implies that  $p^g - q^g v_s^\ell < 0$  contradicting (C.3). By assumption, trade of bad assets only cannot generate any positive trade surplus for the buyers.

## Appendix D Avoiding a Self-fulfilling Freeze through Guarantees

Trading on the market can also cease because of a coordination failure (see Figure 2). In such a case, there is either partial trading or no trading in steady state. We show here for steady state equilibria that guaranteeing a floor on the value of the asset can resurrect trading. The reason is that such a policy makes purchasing an asset when in a meeting a strictly dominant strategy for buyers for any steady state equilibrium. More interestingly, such a guarantee would be costless in equilibrium. Having bought a lemon, a buyer is always worse off taking the guarantee than waiting to trade his lemon to another trader in the functioning market.

To show this, let  $\kappa > r$  and  $\pi \in (\bar{\pi}, \underline{\pi})$ . Then there exists a no trade and a partial trade steady state equilibrium. Define the guarantee offered by the MMLR by the price

$$P_G = v_\ell - \epsilon - v_b(\gamma). \quad (\text{D.1})$$

In steady state, the surplus from trading for a buyer is given by

$$\tilde{\pi}(\gamma)v_o + (1 - \tilde{\pi}(\gamma)) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - p - v_b(\gamma). \quad (\text{D.2})$$

where  $v_\ell(\gamma) = \frac{\lambda\gamma}{\lambda\gamma+r}v_s < \frac{\lambda}{\lambda+r}v_s$ . The max operator expresses the fact that a buyer with a lemon has the option to receive a utility transfer equal to  $v_\ell - \epsilon$  or can wait for a trade in the market anticipating to receive an offer with probability  $\gamma$  when he meets a buyer.

Suppose first that  $\gamma \in [0, 1)$ . Recall that the market price is given by  $p = v_s - v_b(\gamma)$  if  $\gamma \in (0, 1]$ . Then, since  $\tilde{\pi}$  decreases with  $\gamma$ , we have that

$$\begin{aligned} & \tilde{\pi}(\gamma)v_o + (1 - \tilde{\pi}(\gamma)) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - p - v_b(\gamma) \\ & > \tilde{\pi}(\gamma = 1)v_o + (1 - \tilde{\pi}(\gamma = 1)) (v_\ell - \epsilon) - v_s > 0 \end{aligned} \quad (\text{D.3})$$

for  $\epsilon > 0$  sufficiently small, since  $\pi > \tilde{\pi}$ . Since the trading surplus is strictly positive independent of the future buyers' trading strategy, it is a strictly dominant strategy to set  $\gamma = 1$ . Hence, there cannot be a steady state equilibrium with  $\gamma < 1$ . For  $\gamma = 0$ , we have  $v_b(\gamma) = v_\ell(\gamma) = 0$ . The result follows then directly, since offering  $p = v_s$  yields again

$$\pi v_o + (1 - \pi) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - v_s > \tilde{\pi}(\gamma = 1)v_o + (1 - \tilde{\pi}(\gamma = 1)) (v_\ell - \epsilon) - v_s > 0. \quad (\text{D.4})$$

with  $\pi > \bar{\pi}$  and  $\epsilon$  sufficiently small.

Finally, with  $\gamma = 1$ , buyers that obtain a lemon will not take advantage of the guarantee as they can obtain a higher value by waiting to trade in the market for any positive  $\epsilon$ . Hence,

$\gamma = 1$  is the only steady state equilibrium and the guarantee is never used in equilibrium. Notice, however, that the floor that the guarantee provides depends explicitly on how much trading there is on the market as reflected by  $v_b(\gamma)$ . In the unique steady state equilibrium with trade, the guarantee gets smaller as the surplus from trading increases.

## Appendix E    Avoiding Deadweight Costs when Intervening

The MMLR incurs a deadweight cost when increasing the quantity of lemons bought. For any additional quantity  $\Delta_Q = Q - Q_{\min}$  of lemons bought at price  $P$ , only the portion  $(P - P_{\min})\Delta_Q$  provides an additional transfer to lemons, since the market functions again after the intervention and a lemon has an expected market value of  $P_{\min}$ . The MMLR could, however, avoid the deadweight cost by selling the additional quantity  $\Delta_Q$  back to buyers immediately after the intervention has taken place at a price just below  $P_{\min}$ . The market will still function continuously after the sale, since the average quality of the asset in the market remains above the threshold to sustain trading in equilibrium. Also, buyers would be willing to purchase lemons at this price, as they make non-negative profits from this transaction in expected terms. The reason is that later on they can sell the lemon again on the market. Thus, the deadweight cost is simply shifted from the MMLR to future buyers.

When selling additional lemons back to the market at price  $v_\ell$ , the cost of the intervention is then approximately given by<sup>28</sup>

$$QP - (Q - Q_{\min}) \left( \frac{\lambda}{\lambda + r} \right) v_s = Q_{\min} P_{\min} + Q \left( P - \left( \frac{\lambda}{\lambda + r} \right) v_s \right). \quad (\text{E.1})$$

The costs are thus given by a constant for the minimum intervention plus the *net cost* for providing an option value  $V_I > 0$ . In a sense, the MMLR can now recover the fix cost per additional lemon purchased, while before this was a deadweight cost providing no additional transfer to traders. Of course, the MMLR is only able to sell lemons in the market, if it cannot be observed who bought these asset from the MMLR in the first place.

Finally, the cost of providing an option value  $V_I \geq \hat{V}_I$  have now decreased making it more attractive to rely on the announcement effect and thus delaying the intervention. Indeed, the cost function is now smooth at  $\hat{V}_I$  when assuming immediate exit from any additional purchases of lemons. Also, it does not matter for the costs anymore whether the MMLR increases  $Q$  or  $P$ . They are now perfect substitutes in terms of the costs of providing any particular option value  $V_I$ .

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<sup>28</sup>The sale would occur at  $T + \epsilon$  at price  $P_{\min} - \epsilon$ . To ease the exposition, we neglect the infinitesimally small terms.



## Appendix F Social Costs of Intervention and Optimal Policy

When the social costs of intervening are positive, but sufficiently low, it is optimal to achieve continuously functioning markets. This implies immediately that the MMLR should intervene immediately in those circumstances. Conversely, when these costs get large, it is never optimal for the MMLR to intervene. This can be stated more formally as follows.

**Proposition F.1.** *Suppose  $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$ . There exists a cut-off point  $\underline{\theta} > 0$  such that it is optimal to intervene immediately for all  $\theta \in (0, \underline{\theta}]$ . Conversely, if  $\theta \rightarrow \infty$ , it is never optimal to intervene ( $T^* \rightarrow \infty$ ).*

We establish the first result by a series of lemmata. We denote by  $V(T = 0)$  and  $c(T = 0)$  the benefits and costs associated with continuous trade. Note that we have shown that the optimal policy conditional on continuous trade is given by  $T = 0$  and a minimum intervention  $P_{\min}Q_{\min}$ . We show next that never intervening cannot be optimal for small enough (social) costs of intervention.

**Lemma F.2.** *For  $\theta$  sufficiently close to  $1/2$ ,  $T = \infty$  cannot be optimal.*

*Proof.* Suppose to the contrary that  $T = \infty$  is optimal. We then have that

$$V(T = \infty) > V(T = 0) - \theta c(T = 0). \tag{F.1}$$

Since  $V(T = \infty) < V(T = 0)$ , for  $\theta$  sufficiently close to 0 this inequality is violated.  $\square$

We strengthen this result in two ways. First, we show that the optimal policy given  $\theta$  – denoted by  $\Psi(\theta)$  – converges to continuous trade as the (social) costs of intervention vanish. Then, we show that the optimal policy has the time of intervention as well as the option value go to zero. The value associated with the optimal policy is denoted by  $V(\Psi(\theta))$  and the cost by  $c(\Psi(\theta))$ .

**Lemma F.3.** *The optimal policy implies  $V(\Psi(\theta)) \rightarrow V(T = 0)$  for  $\theta \rightarrow 0$ . Furthermore, the optimal policy  $\Psi(\theta)$  satisfies  $T(\Psi(\theta)) \rightarrow 0$  and  $V_I(\Psi(\theta)) \rightarrow 0$  for  $\theta \rightarrow 0$ .*

*Proof.* Since  $\Psi(\theta)$  is the optimal policy, we have that

$$V(T = 0) - \theta c(T = 0) < V(\Psi(\theta)) - \theta c(\Psi(\theta)), \quad (\text{F.2})$$

where it must be the case that  $c(\Psi(\theta)) < c(T = 0)$ . Hence,

$$0 < V(T = 0) - V(\Psi(\theta)) < \theta [c(T = 0) - c(\Psi(\theta))], \quad (\text{F.3})$$

and the first result follows.

For the second result, suppose to the contrary that  $\liminf T(\theta) = \underline{T} > 0$  for  $\theta \rightarrow 0$ . Then, it must be the case that  $V(\Psi(\theta_n)) \rightarrow V(T = 0)$ . This implies that  $\tau_2(T) \rightarrow 0$ . As  $T(\theta_n) > \underline{T}$ , it must be the case that

$$\liminf c(\Psi(\theta_n)) > c(T = 0) + c(V_I^{\min}) > c(T = 0). \quad (\text{F.4})$$

But this implies that for  $\theta$  sufficiently close to 0,  $c(T = 0)$  is cheaper. Hence,  $\Psi(\theta)$  cannot be the optimal policy, a contradiction.

The result for  $V_I$  follows from an analogous argument. □

We complete the proof of the first part of the proposition, by showing first that the optimal policy has no announcement effect for  $\theta$  close enough to 0. We can then show that subject to having no announcement effect, it is indeed optimal to intervene immediately when  $\theta > 0$ , but small.

**Lemma F.4.** *For any sequence  $\theta_n \rightarrow 1/2$ , there exists some  $N$  such that  $\tau_1 = T(\Psi(\theta_n))$  for all  $n > N$ .*

*Proof.* Suppose that  $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$  and let  $T > 0$ . For a minimum intervention, i.e.  $V_I = 0$ , we have that

$$\Gamma(0) = \tilde{\pi}(0)(v_o - v_s) + (1 - \tilde{\pi}(0))(v_\ell(0) - v_s) < 0. \quad (\text{F.5})$$

The value of a lemon for all  $t < T$  can be at most  $v_\ell(t) < \frac{\lambda}{\lambda+r}v_s + V_I$ . For  $V_I < V_I^{\min}$ , we also have that

$$\Gamma(t) = \tilde{\pi}(t)(v_o - v_s) + (1 - \tilde{\pi}(t))(v_\ell(t) - v_s) < 0, \quad (\text{F.6})$$

for all  $t$  in some interval  $[0, \mathcal{T})$  where  $\mathcal{T} > 0$ .

Given  $\tilde{\pi}(0)$ , we can then choose  $N$  large enough such that  $T$  and  $V_I$  are sufficiently close to 0 and satisfy

$$\Gamma(T^-) < \tilde{\pi}(T^-)(v_o - v_s) + (1 - \tilde{\pi}(T^-)) \left( \frac{r}{\lambda + r} v_s + V_I \right) < 0. \quad (\text{F.7})$$

Then,  $\Gamma(t) < 0$  for all  $[0, T)$ . Hence, there is no trade before the intervention for any candidate policy that is better than  $T = 0$  and where  $\theta$  is sufficiently close to  $1/2$ .  $\square$

**Lemma F.5.** *Consider any policy such that there is no trading before the intervention ( $\tau_1 = T$ ). There exists an interval  $[0, \underline{\theta}]$  with  $\underline{\theta} > 0$  such that it is optimal to intervene immediately.*

*Proof.* Without trading before the intervention, it is never optimal to increase the price or the quantity of the intervention. For this case, it is also possible to calculate explicitly the welfare of traders subject to an intervention taking place at time  $T$ , which is given by

$$w_0 = S\pi(0) \left\{ \left( \frac{\delta - x}{r} \right) + x \frac{\lambda}{\lambda + \kappa} \left[ \left( \frac{1}{r + \kappa} \right) + e^{-rT} \left( \frac{1}{r} - \frac{1}{r + \lambda + \kappa} \right) - e^{-(r+\kappa)T} \left( \frac{1}{r + \kappa} - \frac{1}{r + \lambda + \kappa} \right) \right] \right\}. \quad (\text{F.8})$$

Total welfare is then given by

$$W(\theta) = w_0 - \theta e^{-rT} P_{\min} Q_{\min}. \quad (\text{F.9})$$

Taking a derivative with respect to  $T$  for the above expression, we obtain

$$\frac{\partial W(\theta)}{\partial T} = -e^{-rT} r (A - \theta C) + (r + \kappa) B e^{-(r+\kappa)T}, \quad (\text{F.10})$$

where

$$A = \left( \frac{1}{r} - \frac{1}{r + \lambda + \kappa} \right) x S\pi(0) \frac{\lambda}{\lambda + \kappa} \quad (\text{F.11})$$

$$B = \left( \frac{1}{r + \kappa} - \frac{1}{r + \lambda + \kappa} \right) x S\pi(0) \frac{\lambda}{\lambda + \kappa} \quad (\text{F.12})$$

$$C = P_{\min} Q_{\min}. \quad (\text{F.13})$$

The sign of  $\frac{\partial W(\theta)}{\partial T}$  is equal to the sign of

$$- [r (A - \theta C) - (r + \kappa) B e^{-\kappa T}]. \quad (\text{F.14})$$

The last term is largest for the lowest feasible  $T$ . Since  $rA > (r + \kappa)B$ , there exists  $\underline{\theta}$  such that the expression above is exactly 0 at this  $T$ . Hence, for any  $0 \leq \theta < \underline{\theta}$ ,  $\frac{\partial W(\theta)}{\partial T} < 0$  for all feasible  $T$  which completes the proof.  $\square$

Finally, we prove the second part of the proposition which is restated here as a separate lemma for the convenience of the reader.

**Lemma F.6.** *If  $\theta \rightarrow \infty$ , it is never optimal to intervene ( $T^* \rightarrow \infty$ ).*

*Proof.* Suppose by way of contradiction that  $\limsup_{\theta \rightarrow \infty} T = \bar{T} < \infty$ . This implies that

$$c(\Psi) \geq P_{\min} Q_{\min} e^{-r\bar{T}} > 0. \quad (\text{F.15})$$

We have that the benefits of any policy are bounded by  $V(T = 0)$ . Hence, for any optimal policy we have that

$$V(T = 0) - V(T = \infty) \geq V(\Psi(\theta)) - V(T = \infty) \geq \theta c(\Psi(\theta)) \geq \theta P_{\min} Q_{\min} e^{-r\bar{T}}. \quad (\text{F.16})$$

Letting  $\theta \rightarrow \infty$  violates this inequality. □

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