

Aksoy, Yunus; Basso, Henrique S.

Working Paper

Liquidity, term spreads and monetary policy

CESifo Working Paper, No. 3988

Provided in Cooperation with:

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Aksoy, Yunus; Basso, Henrique S. (2012) : Liquidity, term spreads and monetary policy, CESifo Working Paper, No. 3988, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/67521>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Liquidity, Term Spreads and Monetary Policy

Yunus Aksoy
Henrique S. Basso

CESIFO WORKING PAPER NO. 3988
CATEGORY 7: MONETARY POLICY AND INTERNATIONAL FINANCE
NOVEMBER 2012

An electronic version of the paper may be downloaded

- *from the SSRN website:* www.SSRN.com
- *from the RePEc website:* www.RePEc.org
- *from the CESifo website:* www.CESifo-group.org/wp

Liquidity, Term Spreads and Monetary Policy

Abstract

We propose a model that delivers endogenous variations in term spreads driven primarily by banks' portfolio decision and their appetite to bear the risk of maturity transformation. We first show that fluctuations of the future profitability of banks' portfolios affect their ability to cover for any liquidity shortage and hence influence the premium they require to carry maturity risk. During a boom, profitability is increasing and thus spreads are low, while during a recession profitability is decreasing and spreads are high, in accordance with the cyclical properties of term spreads in the data. Second, we use the model to look at monetary policy and show that allowing banks to sell long-term assets to the central bank after a liquidity shock leads to a sharp decrease in long-term rates and term spreads. Third, such interventions have significant impact on long-term investment, decreasing the amplitude of output responses after a liquidity shock. The short-term rate does not need to be decreased as much and inflation turns out to be much higher than if no QE interventions were implemented. Finally, we provide macro- and microeconomic evidence for the U.S. to show the importance of expected financial business profitability in the determination of term spread fluctuations.

JEL-Code: E430, E440, E520, G200.

Keywords: yield curve, quantitative easing, financial business profits, bank portfolio.

Yunus Aksoy
Department of Economics, Mathematics
and Statistics, Birkbeck
University of London
Malet Street
UK – WC1E 7HX London
yaksoy@ems.bbk.ac.uk

Henrique S. Basso
Banco de Espana
c/Alcala, 48
Spain – 28014 Madrid
henrique.basso@bde.es

2nd October 2012 - first version June 2011

We would like to thank Federico Di Pace, Ourania Dimakou, Martin Ellison, Michael Howell, Giovanni Melina, Ron Smith, Roman Sustek and Ernesto Villanueva, and seminar participants at the University of Warwick, Bank of England and the Bank of Spain and conference participants at the RES 2012 in Cambridge, SCE 2012 in Prague and Macroprudential conference in Loughborough for helpful comments. The usual disclaimer applies.

1. Introduction

The presence of term spreads has implications on financial transactions, macroeconomic outcomes and policy design¹. The existing literature finds that the slope of the yield curve has significant predictive power in explaining US business cycle fluctuations (see, for instance, Estrella and Hardouvelis (1991) or more recently Rudebusch and Williams (2009)). This is not only linked to the fact the slope is related to the future path of short-term interest rates but also due to the changes in the term spread. Moreover, recent large scale purchase programmes adopted by the Federal Reserve Bank, the Bank of England and the European Central Bank have had significant impacts on the shape of the yield curve, increasing the importance of understanding and modeling the fluctuations in term spreads. While there is a growing literature on term spreads and macroeconomic outcomes, there is no consensus as to which are the determinants of time varying risk premia, one of the key components behind the variations in the shape of the yield curve. In this paper we propose a model that provides an explanation for endogenous variations in term spread driven primarily by changes in banks' balance sheets, their expected profitability and their appetite to bear the risk of maturity transformation.

Initial structural models that focused on the term structure of interest rates rely on the expectations hypothesis limiting the analysis to cases of either no or constant risk premium. There is evidence,² however, that the term premium is time varying; therefore these models are far from satisfactory. In a natural development from these earlier contributions, structural models were built focusing on the variability of the stochastic discount factor and its links with macroeconomic variables (macro-finance or affine term structure models). The literature is extensive. Recent work by Piazzesi and Schneider (2007) and by Rudebusch and Swanson (2008a) model risk premium as an outcome of the negative covariance between inflation and consumption growth. In this framework financial investors demand a higher risk premium as a hedge against (long-term) inflation risk³.

Although supporting the view that long run inflation risk is an important determinant of term spread fluctuations, three main empirical facts motivate the search for additional factors external to monetary policy. First, dynamics of short-run rates and inflation expectations do not explain all the variability of long-term rates, particularly in the last decade (De Graeve, Emiris, and Wouters (2009)). Second, Benati and Goodhart (2008) observe that during the 2000's the marginal predictive content of term spreads to future output increased, although monetary policy uncertainty remained low. Finally, as stressed by Gürkaynak and Wright (2010), the US treasury inflation protected securities (TIPS) forward rate dynamics have not been that different from their nominal counterparts, indicating the term premia are also influenced by real factors.

As a result, we focus on the role of financial intermediation in the determination of term spreads. We develop a DSGE model with endogenous term spreads derived from bank's portfolio choice and risk assessment of potential liquidity shortages impacting their profitability and balance sheets. The funding or banking structure of our model, which focuses on potential liquidity

¹We will use term spreads and term premia interchangeably.

²See for instance Piazzesi and Swanson (2008) and references therein.

³Other relevant contributions to the literature focus on the effects of learning about long-term inflation targets of the central bank (Kozicki and Tinsley (2005)) or on the possible segmentation of short and long-term bond markets (Vayanos and Vila (2009)) identifying other drivers of time varying risk premia.

risks, relates to the contributions⁴ of Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005). Banks hold a portfolio of equities, short and long-term lending funded by short-term borrowing, thus they bear the risk of maturity transformation. Banks' balance sheets characterized by short-term liabilities and long-term assets may be optimal due to agency problems (see for instance Diamond (1989)) and have been prevalent in the financial sector (at least) until the most recent crisis (see Brunnermeier (2009) for a discussion). We assume banks' long-term investments, which were funded by short-term liabilities, may suffer from potential liquidity shortages or that a liquidity injection is needed to maintain those assets in the banks' books, formalizing the maturity mismatch risk. Term spreads, which effectively denote the cost of hedging this potential risk, are then determined by the volatility of future short-term rates, as usual in the macro finance literature, and the additional element introduced here, the premium for bearing the maturity risk. Another relevant feature of our framework, which is not present in standard macro finance models, is that the term spread has an explicit impact on the fundamental variables, since long-term rates influence investment on capital formation.

Confirming the stylized facts we obtain counter-cyclical term spread movements. Furthermore, we observe that term premia are good predictors of future activity. On the one hand, as the economy approaches the peak of the business cycle spreads tend to be at their lowest and will tend to increase thereafter. On the other hand, as the economy approaches the bottom of the business cycle, spreads tend to be at their highest. Endogenous movements in long-term rates and term spreads are mainly driven by fluctuations in the expected profitability of banks' portfolios. Banks rely on the overall profitability of their portfolio to assess their ability to cover for any liquidity shortages. Hence, future profitability relates directly to their appetite to bear maturity risk and thus the risk premia they require to commit to provide long-term funding to firms. As output is increasing, profitability is expected to be high and hence spreads are low. During recessions, profitability is low and thus spreads tend to be high.

We also show that our model delivers considerably more volatile term spreads compared to the affine term structure models although we do not assume higher degree of risk aversion nor introduce implausibly high variance for exogenous disturbances. The main reasons for this result are (i) spread movements in our model are driven by bank profits which are more volatile than consumption (the determinant of spread movements in affine models such as Rudebusch and Swanson (2008b)) and (ii) short-term and long-term markets are segmented in our model.

Finally, we use the model to consider the impact of both conventional and unconventional monetary policies. Although we find that the base rate dynamics are implicitly influenced by movements in term spreads due to their impact on output and inflation, we do not find that responding explicitly to their movements (altering the standard (Taylor) monetary rule to include term spreads) leads to greater stabilization. We then look at the impact of unconventional monetary policy similar to the recent quantitative easing (QE) adopted in the US and the UK. Note that the channel through which QE affects the economy in our framework is distinct to the one stressed in some of the recent theoretical papers (see, among others, Gertler and Kiyotaki (2009)). In the latter the mechanism is generally through a direct replenishing of banking capital, covering for current shortages. In contrast, unconventional monetary policy in our framework aims at protecting

⁴Although we simplify the model structure excluding moral hazard problems and the potential for bank failures to focus particularly on term spreads.

banks from potential liquidity shortages in the future, increasing their willingness to carry maturity transformation risk, or in other words, reducing term spreads. We find that allowing banks to sell long-term assets to the central bank after a liquidity shock leads to a sharp decrease in long-term rates and term spreads matching the results found by several empirical studies on the recent QE policies in the US, UK and the Eurozone. Furthermore, such interventions have significant impact on long-term investment, decreasing the amplitude of output responses after a significant liquidity shock. The base rate does not need to decrease as much and inflation turns out to be higher than if no QE interventions were implemented. It is important to remark that our bank's portfolio choice mechanism does not depend on the nominal frictions introduced in the model for the purpose of analyzing conventional monetary policy.

Empirically, the importance of bank balance sheets and bank risk taking has been recently stressed by Adrian and Shin (2009, 2010), and Adrian, Shin, and Moench (2010). They show that financial intermediary balance sheets contain strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios; higher banking asset growth is related to decreasing risk premia. This link between bank's balance sheets and risk premia is indeed present in our model. As the bank expects higher profits, it increases asset holdings and consequently spreads decrease. Thus, the bank's portfolio choice in the model provides a rationale for the link between balance sheet asset growth and risk premia. While those authors focus more on leveraging and asset growth rates, we look particularly at the role of the variability of profits and the maturity transformation risk. We first document that past aggregate banking profitability is significantly associated with variations in current real output and term spreads in the U.S. after controlling for inflation and past real output. Second, using the same dataset, we show that spreads help predict future negative movements in aggregate banking profitability. Finally, we provide micro-econometric panel data evidence. We find that an increase in U.S. bank level expected financial business profitability (as measured by the expected mean forecast in earnings per share for major U.S. financial institutions) leads to a significant decline in term spreads next to variations in past real output and inflation. Thus, complementing their macro empirical evidence, we find a statistically significant link between yield spreads and changes in expected financial sector profitability, giving support to our main theoretical channel.

The paper is organized as follows. Section 2 presents a simple partial equilibrium model to highlight the relationship between banks' profitability, balance sheets and term spreads. Section 3 describes the general equilibrium model of endogenous term spreads. We start presenting our results in Section 4, focusing on the main drivers of the endogenous movements in term spreads, their potential to predict future output growth and their amplification effects. Section 5 presents the volatility in term spreads at different points of the yield curve, comparing to the data and the figures obtained by Rudebusch and Swanson (2008b). Section 6 discusses conventional and unconventional monetary policies. Macro and micro-econometric evidence on the link between financial business profitability, spreads and output in support of our channel is presented in Section 7. Finally, Section 8 concludes.

2. A Simple Model of Bank's Portfolio Choice

We start by presenting a simple partial equilibrium model of bank's portfolio decision aiming at explaining the basic link between bank's balance sheet (portfolio), their profits and the inclination of the yield curve. In the main part of the paper we embed a similar decision problem into a general equilibrium model to explore the effects of endogenous fluctuations of spreads on economic activity and monetary policy.

The simple model has three periods. At Period zero, the bank selects a portfolio of assets and holds them until maturity. While assets are in the balance sheet, the bank must fund them with short-term deposits. We assume bank's portfolio may contain three assets; namely, a long-term asset (X_L), a short-term asset (X_S) and equity (or a portfolio of the rest of risky short-term assets available in the economy), denoted by Z . The short-term asset pays out a certain return of R_S one period after the portfolio has been set. Equities also pay out in Period 1, however, their return R_Z is uncertain; we assume $R_Z \sim N(\bar{R}_Z, \sigma_Z^2)$. Long-term assets mature and pay out a certain return of R_L two periods after the portfolio decision, however, the bank might be forced to make an injection of liquidity (ρ) in its own balance sheet to keep the long-term assets that were funded with short-term borrowing in the portfolio during Period 1; we assume $\rho \sim N(\bar{\rho}, \sigma_\rho^2)$.

This liquidity injection, similar to the one used in Holmstrom and Tirole (1998), effectively implies that the bank may be exposed to cash flow shocks at Period 1, leading to an ex-post revaluation of the overall return on long-term asset holdings, replicating problems of balance sheet funding⁵. Thus, ρ can be understood as a reduced form shock to the maturity mismatch exposures in the bank's balance sheet. Finally, we denote $\rho_{Z,\rho}$ the correlation index between the two disturbances. The bank fully funds its portfolio with deposits that provide the holder with a gross return of R_D .

In order to depict the basic channel as clearly as possible, we simplify a number of features that are part of the bank's portfolio choice in our main model (Section 3). Firstly, we focus on the portfolio decision at time zero only, with short-term assets and equity remaining in the balance sheet for one period and long-term assets for two periods. Secondly, we assume deposits are in infinite supply at the equilibrium short-term rate and R_D is exogenously set and constant for the two periods the long-term assets are held⁶. These simplifications allow us to concentrate on the bank's decision of how much long-term asset to hold at time zero, the point when long-term rates, and thus term spreads, are set. The main factors affecting this decision will then be the bank's expected profitability and the liquidity risk the long-term asset holder bears.

If banks have no regard for risk, being risk neutral and/or not having to abide by any constraint on risk taking, the yield curve would be flat (this can be easily seen by setting $\sigma_B = 0$ in (7)). Hence, in order to study fluctuations in the yield curve, we assume banks care about risk in two possible ways. In the first case, also used in our general equilibrium model, we assume banks are risk averse, maximizing a constant relative risk aversion function of profits. In the second case, we assume banks are risk neutral, thus maximizing expected profits, but are subject to a value-at-risk

⁵Note that ex-post revaluations might occur if a portion of long-term assets must be sold due to lack of funding. Hence, although the liquidity injection in the model occurs on the asset side of the bank's balance sheet, it can also be understood as a reduced form liability shortage shock.

⁶Note that volatility of short-term funding costs could also generate increased maturity transformation risk. In the general equilibrium model in Section 3 short-term rates will be endogenous and thus we incorporate this risk there.

constraint. Both cases deliver a similar association between expected portfolio returns and term spread fluctuations. The bank problem, formally, is

$$\begin{aligned}
& \max_{\{X_S, X_L, Z\}} E[\Pi^B] \\
& s.t. \quad \Pi^B = \frac{(\Pi_1^B)^{1-\sigma_B}}{1-\sigma_B} + \beta \frac{(\Pi_2^B)^{1-\sigma_B}}{1-\sigma_B} \\
& \quad \Pi_1^B = (R_Z - 1)Z + (R_S - 1)X_S - \rho\Delta M - (R_D - 1)D_0 \\
& \quad \Pi_2^B = (R_L - 1)X_L - (R_D - 1)D_1 \\
& \quad D_0 = Z + X_L + X_S \\
& \quad D_1 = X_L \\
& \quad \Delta M = X_L \tag{1} \\
& \quad VaR(\Pi^B) \geq \Xi \text{ iff } \sigma_B = 0 \tag{2}
\end{aligned}$$

where ΔM denotes the portion of long-term assets that are funded by short-term borrowing, which in our simple framework, equals the total long-term asset holdings. $VaR(\Pi^B)$ is the value-at-risk of the bank's portfolio defined as the expected minimum portfolio return over the two periods within a 1% confidence interval and Ξ is the limit on that minimum return imposed on the bank when risk neutrality is assumed.

It is straightforward to see that short-term rates will be equal to the return on deposits. The key equations to determine the bank's portfolio and the return on risky assets are, therefore, given by

$$-E[(\Pi_1^B)^{-\sigma_B}(R_D - 1 + \rho)] + \beta E[(\Pi_2^B)^{-\sigma_B}(R_L - R_D)] + \mathfrak{J} \frac{\partial VaR}{\partial X_L} \varsigma = 0 \tag{3}$$

$$-E[(\Pi_1^B)^{-\sigma_B}(R_Z - R_D)] + \mathfrak{J} \frac{\partial VaR}{\partial Z} \varsigma = 0 \tag{4}$$

where \mathfrak{J} is an indicator function that takes the value of 1 when $\sigma_B = 0$ and zero otherwise⁷ and ς is the Lagrange multiplier of the value-at-risk constraint. In effect, these two equations above determine the bank's demand for equity and long-term assets. In order to obtain an equilibrium for these two markets we assume that⁸

$$X_L = \frac{\gamma_S}{R_L + E[\rho]} \tag{5}$$

$$\bar{R}_Z = \alpha_Z - \gamma_Z Z. \tag{6}$$

These assumptions imply that as the supply of long-term assets to the bank (or the investors' demand for long-term loans) decreases the long-term rate will increase and as the demand for equity increases its expected return will decrease. Once again, these assumptions are used such that the main channel between portfolio choice and spreads can be highlighted; these supply conditions will be endogenous in the general equilibrium model presented in Section 3.

⁷The VaR constraint binds since expected profit increases and VaR decreases as X_L and Z increase.

⁸The bank takes \bar{R}_Z as given while selecting Z .

Case 1 : $\sigma_B > 1$

If banks are risk averse, the equilibrium condition that determines the long-term rate is simply given by

$$E [(\Pi_1^B)^{-\sigma_B} (R_D - 1 + \rho)] = \beta E [(\Pi_2^B)^{-\sigma_B} (R_L - R_D)]. \quad (7)$$

It is easy to see that when expected profitability in Period 1 increases relative to the profitability in Period 2, due to higher expected return on equity, then the left-hand side term of (7) decreases, and in equilibrium, long-term rates will also decrease and the bank's demand for long-term assets will increase. Note that the covariance between profitability (Π_1^B) and future liquidity shortages (ρ) will also be important to determine the extent to which an increase in profitability affects the decrease in the LHS of (7). We will discuss the role of this covariance in more detail in the general equilibrium model. Also note that in this simple model we assume the deposit rate remains constant in Period 1. In the main model of the paper this will also be uncertain, affecting the term premia.

Case 2 : $\sigma_B = 0$

In both Case 1 and in the general equilibrium model we assume banks are risk averse. This delivers a straightforward link between expected profits and term premia; banks require lower premia in periods in which profits are expected to be high. However, we only require that banks care about some measure of risk in order to establish a link between profitability and risk premium. As such, in this case we assume that banks are risk neutral but are subject to a value-at-risk constraint. Our main interest is to verify how term spreads move as the overall profitability (expected returns) of the bank's portfolio varies. In doing so we aim at establishing a link between bank's appetite to accumulate long-term assets, incurring the risk of maturity transformation, the equilibrium long-term rates and the expected performance of bank's investments. The details of the solution of the model in this case is given in the appendix with the discussion provided here.

As α_Z (constant term that controls expected return on equity, see (6)) increases, the return on equity, holding demand for Z constant, rises. That implies that if banks were to hold the same portfolio, their VaR (expected minimum return) would increase above the constraint. That would allow banks to increase the demand for both equity and long-term assets, increasing expected profit, until the constraint becomes binding again (akin to an income effect). Additionally, banks could increase demand for equity and decrease the demand for long-term assets, as equity became the relatively better asset (substitution effect)⁹. As long as there is a gain in asset diversification in the bank's portfolio (or $co_{Z,\rho} < 0$), then the income effect dominates¹⁰ and the demand for long-term assets will increase as expected return on equity, or profitability, increases. Consequently, as portfolio returns increase, long-term rates decrease, leading to a flatter yield curve. As the expected profitability of banks increase they are willing to charge a lower premium to bear the risk of maturity transformation, increasing their balance sheet position until reaching the constraint on expected minimum return (or maximum expected losses). The additional expected profits can then

⁹Shin (2009) provides a similar mechanism of the effect of VaR on bank's portfolio decision, although he focuses on the overall effect of reducing default probability on asset growth while we look into the cross-asset or portfolio balancing effect.

¹⁰The strength of the substitution effect is directly related to the covariance between the asset returns. As the covariance decreases so does the substitution effect and more likely it is that the demand for long-term assets will increase with α_Z .

be used to cover potential liquidity injections needed to maintain long-term assets in the balance sheet.

Therefore, in both Case 1 and Case 2, during periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, charging a lower risk premium (flatter yield curve or smaller difference between long and short-term rates). Higher profits on investments allow banks to cover for potential liquidity shortages.

3. General Equilibrium Model

In the previous section we presented a partial equilibrium model of bank's portfolio choice, establishing a link between bank's appetite to bear maturity transformation risk, its balance sheet holdings and its expected profitability from portfolio investments. During periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, increasing long-term asset holdings and charging lower risk premia. In this section, we generalize and extend our simple portfolio choice model. We endogenize term spreads embedding banking sector's portfolio choice into a DSGE framework and study its effects on economic activity and monetary policy.

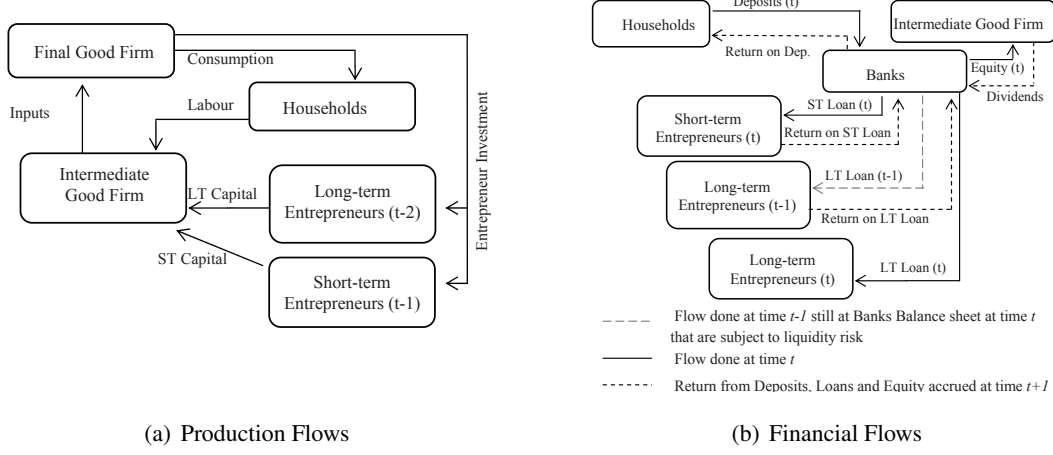
The model economy is populated by a continuum $i \in [0, 1]$ of intermediate good firms, a final good producer, a continuum of households, banks, entrepreneurs and the central bank. Entrepreneurs borrow funds from a bank and transform consumption goods into capital. There are two types of entrepreneurs, one with access to a short-run investment project and one with a long-run investment opportunity available. This introduces a segmentation of short and long-term funding requirements, similar to the one stressed by Vayanos and Vila (2009). Firm i hires labour from households, produces a differentiated input using labour and the current capital stock, and at the end of the period sells the inputs to the final good firm and buys new capital from entrepreneurs. The final good firm combines all inputs to produce consumption goods that are then sold to households and entrepreneurs. We assume households (workers) receive the profits from banks and entrepreneurs, which are all of unit mass. Thus, only households consume. An equivalent alternative would be to follow a model structure similar to Gertler and Kiyotaki (2009), where a family is split into banks and consumers but consumption is done at the family level.

The bank receives deposits from households, provides loans to both entrepreneur types and buys equity from the intermediate firms. Note that long-term loans are issued at every period but do last for two periods, thus banks' balance sheets will contain three loan agreements. These are: a short-term loan, a long-term loan and another long-term loan issued in the previous period. Finally, we assume that during the current period the bank might need to make a liquidity injection into its balance sheet to keep long-term assets that are funded with short-term borrowing in its portfolio. Figure 1 shows the production and financial flows of the model.

3.1. Households

The household maximizes its expected discounted lifetime utility given by

Figure 1: Model Structure



$$\max_{C_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0, 1) \quad \sigma, \eta, \chi > 0$$

where C_t denotes the household's total consumption and H_t denotes the composite labour index. The curvature parameters σ, η are strictly positive. β is the discount factor. The household faces the following budget constraint

$$C_t + \frac{D_t}{P_t} \leq \frac{W_t H_t}{P_t} + \frac{R_{t-1, CB} D_{t-1}}{P_t} + \frac{\tilde{\Pi}_t}{P_t}$$

where W_t is the wage index and $R_{t, CB}$ is the rate of return on deposits D_t . The central bank sets $R_{t, CB}$ directly according to a monetary policy rule to be specified. We assume the only asset available to the "worker" is a deposit made directly to the financial intermediary¹¹, thus only banks invest in equities issued by the intermediate good firms and lend to entrepreneurs. Although not modeled here, one reason for that would be the existence of higher household-firm agency costs relative to bank-firm agency costs¹².

Finally, $\tilde{\Pi}_t = \tilde{\Pi}_t^{ES} + \tilde{\Pi}_t^{EL} + \Pi_t^B P_t$ is the sum of the nominal profits, realized at period t , for entrepreneurs with short-term projects, with long-term projects and the bank, respectively, which is passed on to the household.

3.1.1. Optimal Wage Setting

Households supply a continuum of labour types $j \in [0, 1]$. The composite labour index H_t is then given by

¹¹One may assume workers make deposits on banks owned by other households.

¹²Household-bank agency costs are not modelled. That would be an interesting extension, given that potential changes to the premium households require to fund banks could also affect the bank's appetite to bear maturity transformation risk.

$$H_t = \left[\int_0^1 H_{j,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

From the subsequent firms minimization problem we have that the demand for each labour type and the wage index are given by

$$H_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\varepsilon_w} H_t,$$

$$W_t = \left[\int_0^1 W_{j,t}^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}}.$$

Households, when allowed (Calvo scheme with parameter ω_w), set wages $W_{j,t}$ to maximize expected utility subject to the budget constraint and the labour demand equation. The main reason to include both price and wage rigidity is to ensure firm's real profits are pro-cyclical after a productivity shock (see Carlstrom and Fuerst (2007)).

3.2. Entrepreneurs

Entrepreneurs are responsible for capital formation. We assume a set of mass unit of entrepreneurs has a short-term investment opportunity available, at each period. Another set of mass unit of entrepreneurs has a long-term (two periods) investment opportunity available, at each period. Thus, there are always three mass units of active entrepreneurs in the economy.

Short-term entrepreneurs borrow funds from the bank ($X_{S,t}$), buy consumption goods and transform these into capital next period using the following production function

$$yk_{t+1}^S = \gamma_S \ln(1 + X_{S,t}).$$

The capital produced is then sold to the intermediate good firms. The profits of these entrepreneurs are given by

$$\tilde{\Pi}_{t+1}^{ES} = P_{t+1} q_{t+1}^S \gamma_S \ln(1 + X_{S,t}) - R_{t,S} P_t X_{S,t},$$

where $R_{t,S}$ is the gross interest rate on short-term borrowing and q_t^S is the price of short-term capital denominated in consumption goods. Short-term entrepreneurs select $X_{S,t}$ to maximize expected profits.

Long-term entrepreneurs also borrow from the bank ($X_{L,t}$), buy consumption goods and transform these into capital after two periods with the following production function

$$yk_{t+2}^L = \gamma_L \ln(1 + X_{L,t}),$$

where $\gamma_L > \gamma_S$. The capital produced is then sold to the intermediate good firms. Long-term entrepreneurs profits are given by

$$\tilde{\Pi}_{t+2}^{EL} = P_{t+2} q_{t+2}^L \gamma_L \ln(1 + X_{L,t}) - R_{t,L} P_t X_{L,t},$$

where $R_{t,L}$ is the gross interest rate on long-term borrowing and q_t^L is the price of long-term capital denominated in consumption goods. Long-term entrepreneurs select $X_{L,t}$ to maximize expected profits.

The production function for short and long-term capital output (yk^m) take the form $\gamma_m \ln(1 + X_{m,t})$ for $m = \{S, L\}$, since we need (i) capital production to have decreasing returns (concave function) such that movements in borrowing rates influence the marginal propensity to invest¹³; (ii) long-term capital investment to be more productive than short-term capital due to the potential liquidity problem banks may face on long-term asset holdings (thus, $\gamma_L > \gamma_S$); and (iii) one unit of consumption invested to return more than one unit of capital goods, thus we normalize the production function to be the log of one plus the amount invested. That way, each unit of consumption good invested ($X_{m,t}$) is turned into one unit of capital plus an increment, which decreases as the amount invested increases, and whose overall size depends on the parameter γ_m (this interpretation holds as long as $X_{m,t}$ is small and γ_m close to one, which will be the case in our calibration).

3.3. Banks

At every period t , a bank, representing all financial business in the economy, acquires three types of nominal assets: a short-term debt ($P_t X_{S,t}$), a long-term debt ($P_t X_{L,t}$) and equity (Z_t). Furthermore, the bank has a long-term asset it carries over from last period ($P_{t-1} X_{L,t-1}$). The bank funds these investments with short-term deposits (D_t) from households¹⁴. Diamond (1989, 1991, 2004), amongst others, provide agency-based explanations for the usage of short-term debt as the optimal funding instrument for banks, theoretically supporting the balance sheet structure employed here. Equities are acquired from the intermediate good producers. The investment in equity made at time t , Z_t , pays off a gross dividend at Period $t + 1$, denoted by DIV_{t+1} (see detail below). Short-term entrepreneurs pay back the loan made at time t in Period $t + 1$, providing a return to the bank of $(R_{S,t} - 1)P_t X_{S,t}$. Long-term entrepreneurs pay back the loan made at time $t - 1$ in Period $t + 1$, providing a return to the bank of $(R_{L,t-1} - 1)P_{t-1} X_{L,t-1}$ where $R_{L,t-1}$ is the nominal long-term rate set at time $t - 1$. Finally, long-term asset holdings, acquired at time t , may require a liquidity injection at time $t + 1$ of $\rho_{t+1} P_{t+1}$ per unit of the long-term assets that is funded by short-term borrowing in the balance sheet (which in our framework is simply equal to $X_{L,t}$). We assume that the liquidity shock is given by $\rho_t = (1 - \rho_L)\bar{\rho} + \rho_L \rho_{t-1} + e_L$, where e_L is *i.i.d.* normal.

As in our simple model ρ_t is understood as a reduced form shock to the maturity mismatch exposures in the bank's balance sheet (originating from either the liability or asset side). It is important to dissociate ρ_t with an aggregate shock that increases the default rate in the economy¹⁵. A change in ρ_t occurs due to funding or balance sheet problems and is thought to be unrelated to the fundamental value of long-term asset holdings. Acharya, Gale, and Yorulmazer (2010), for instance, present a model highlighting how a funding problem might occur even when the fundamental value of asset holdings remains high.

Note that although it is natural to think of $\rho_t > 0$, or potential liquidity shortages, we could also have liquidity gains with $\rho_t < 0$, for instance due to an increase in the prospective gains from securitization of long-term assets in the banks' balance sheets. In this case, instead of liquidity

¹³This assumption allows us to have the original Keynesian investment equation dependant on the interest rate.

¹⁴In this paper we do not consider banking capital requirements.

¹⁵Although such a shock might be a trigger for the potential liquidity problems we model through a change in ρ_t .

shortages, the banking sector is characterized by excess liquidity, which could give rise to an inverted yield curve with long-term rates lower than short-term rates. That might have been the case in the UK during the few years preceding the 2007 crisis. An interesting extension of the model left for future research is to make ρ_t endogenous based on the potential for securitization vis-a-vis the expected need for liquid funds/provisions or shortage of funding.

Bank's real profits at Period $t + 1$ are given by

$$\Pi_{t+1}^B = \frac{1}{P_{t+1}} (DIV_{t+1} + (R_{L,t-1} - 1)P_{t-1}X_{L,t-1} + (R_{S,t} - 1)P_t X_{S,t} - D_t(R_{t,CB} - 1) - \rho_{t+1}X_{L,t}P_{t+1}).$$

We assume that the bank is risk averse. The only risk involved in the banking business in our model is the maturity transformation risk. Banks commit to lend money to long-term investment opportunities having to acquire funds next period to re-finance this balance sheet commitment plus any additional liquidity injection needed. Risk aversion here implies that banks do not only care about the return on short and long-term assets, requiring them simply to pay the same expected return on average. The bank will weigh these returns according to the expected profitability of the entire portfolio, requiring higher premium to bear risk when overall profitability is low but accepting lower risk compensation when overall returns are high. Effectively, the bank will care about the covariance between the returns of each asset and the returns of the overall portfolio.

Note that even if banks were risk neutral, the limits on Value-at-Risk (VaR) banks normally abide to, would effectively imply that overall profitability of assets would influence banks' required premium to bear maturity risk through the VaR constraint (as discussed in Section 2). Hence, the assumption that banks are risk averse reflects that some measure of overall riskiness and expected profitability affect their long-term rate setting decision or the premium they require for bearing maturity transformation risk. The bank's profit maximization problem is given by¹⁶

$$\begin{aligned} \max_{\{X_{S,t}, X_{L,t}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Pi_t^{B^{1-\sigma_B}}}{1 - \sigma_B} \\ \text{s.t.} \quad & D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1}, \end{aligned}$$

where σ_B controls the degree of risk aversion.

Based on the bank's demand for $\{X_{S,t}, X_{L,t}\}$ and the respective supply of assets from entrepreneurs we obtain the equilibrium values for short-term and long-term interest rates $\{R_{S,t}, R_{L,t}\}$. Given the potential for liquidity shocks or the risk of maturity transformation, the bank will require a premium to hold long-term positions, or $R_{L,t} > R_{S,t}$. If the bank were to hedge this maturity risk they could go long on short-term assets (for instance short-term government bonds) and go short on long-term assets (for instance long-term government bonds). The overall cost of such a hedging portfolio would be the difference between short-term and long-term government yields or the term spread¹⁷. Thus, assume that the bank is considering whether to increase its holdings of long-term

¹⁶We exclude Z_t from the set of choice variables in the maximization since, as equity is the best asset in the portfolio, paying a gross return higher than the short-term rate, banks always demand the total amount of equity supplied by intermediate firms.

¹⁷This occurs since our model does not include default or credit risk. If that were to be the case we could decompose the bank's overall risk by the term spread exposure plus the credit exposure, which could be hedged by credit default derivatives or default insurance.

assets which are funded by short-term borrowing. At the margin, this additional position could be offset by buying the hedging portfolio such that no additional return is gained nor risk undertaken. If the bank decides to hold that additional position unhedged, it must receive the premium adequate to the risk undertaken. As a result, at the margin, the difference $R_{L,t} - R_{S,t}$ is directly related to the term premium in the government bond market. As such, we define the term spread (annual rate in percentage points, thus the 400) between long and short-term rate in the same fashion as the macro-affine literature

$${}_t p_t = \frac{1}{2} ((R_{L,t} - 1) - (R_{t,CB} - 1) - (R_{t+1,CB} - 1)) 400. \quad (8)$$

3.4. Firms

The final good representative firm combines a continuum of intermediate inputs $i \in [0, 1]$ with the following production function

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

As standard this implies a demand function given by

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t, \quad (9)$$

where the aggregate price level is

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

The intermediate sector is constituted of a continuum of firms $i \in [0, 1]$ producing differentiated inputs with the following constant returns to scale production function

$$y_{i,t} = A_t K_{i,t}^S \alpha \zeta K_{i,t}^L \alpha (1-\zeta) H_{i,t}^{1-\alpha}, \quad (10)$$

where A_t denotes the productivity level at time t (which follows the standard AR1 process $\log(A_t) = \rho_A \log(A_{t-1}) + e_A$, with e_A *i.i.d.*), $K_{i,t}^S$ is the capital stock originated from short-term projects, $K_{i,t}^L$ is the capital stock originated from long-term projects, and $H_{i,t}$ is the household composite labour used in production. Each firm hires labour and invests in both stocks of capital. Implicit here is the assumption that short-term and long-term capital are not perfect substitutes, which reflects the fact that long-term projects might have a distinct technological enhancement compared to capital based on short-run investments.

To characterize the problem of intermediate firms, we split their decision into a pricing decision given their real marginal cost and the production decision to minimize costs. Following the standard Calvo pricing scheme (ω), firm i , when allowed, sets prices $P_{i,t}$ according to

$$\max_{P_{i,t}} E_t \left\{ \sum_{s=0}^{\infty} P_{t+s} Q_{t,t+s} \omega^s Y_{i,t+s} \left[\frac{P_{i,t}}{P_{t+s}} - \Lambda_{t+s,i} \right] \right\},$$

subject to the demand function (9), where $Q_{t,t+s}$ is the economy's stochastic discount factor, defined in the next section and $\Lambda_{t+s,i}$ is the firm's i real marginal cost at time $t+s$. To obtain the real marginal cost, we need to solve the firm's intertemporal cost minimization problem. That is

$$\min_{K_{i,t+1}^S, K_{i,t+1}^L, H_{i,t}} E_t \left\{ \sum_{t=0}^{\infty} Q_{0,t} (W_t H_{i,t} + P_t q_t^S I_{i,t}^S + P_t q_t^L I_{i,t}^L) \right\},$$

subject to the production function (10) and investment equation $I_{i,t}^m = K_{i,t+1}^m - (1-\delta)K_{i,t}^m$ for $m = \{S, L\}$ ¹⁸, where δ is the depreciation rate.

Finally, dividends¹⁹, which are paid one period after production takes place, are given by

$$DIV_{i,t+1} = P_{i,t} Y_{i,t} - W_t H_{i,t} - P_t (q_t^S I_{i,t}^S + q_t^L I_{i,t}^L) + P_t (q_t^S K_{i,t+1}^S + q_t^L K_{i,t+1}^L) - Z_t,$$

where $Z_t = P_{t-1} (q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)$. The first three terms comprise the profits (flow) and the last two the capital gains (due to changes in amount and price of capital held in the firm). We assume equities are bought (or evaluated) by banks at the beginning of time t . Thus, the value of the firm at time t , denoted Z_t , is equal to the value of its capital holdings at the beginning of Period t before production and investment in (new) capital takes place. The two types of capital used in production at time t are given by K_t^S and K_t^L .

3.5. Market Clearing Conditions

The capital market clearing conditions are given by

$$I_t^S = yk_t^S = \gamma_S \ln(1 + X_{S,t-1}), \text{ and} \quad (11)$$

$$I_t^L = yk_t^L = \gamma_L \ln(1 + X_{L,t-2}). \quad (12)$$

The good market clearing condition, or the aggregate demand, is given by

$$Y_t = C_t + X_{S,t} + X_{L,t} + \rho_t X_{L,t-1}. \quad (13)$$

Furthermore, capital and labour markets across firms are aggregated such that

$$K_t^m = \int_0^1 K_{i,t}^m di \text{ for } m = S, L \text{ and } H_t = \int_0^1 H_{j,t} dj.$$

The credit market clearing condition is

$$\frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{Z_t}{P_t} + \frac{X_{L,t-1}}{\pi_t},$$

where $\frac{Z_t}{P_t} = \frac{(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)}{\pi_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$.

¹⁸Note that the demand for each type of labour stated in the household wage setting problem can be obtained by minimizing the total cost of labour $\int_0^1 W_{j,t} H_{i,j,t} dj$ subject to the labour composite index.

¹⁹Dividends here are in fact profits plus capital gains and represent the gross return on equity.

3.6. Equilibrium and Calibration

The equilibrium of the economy is defined as the Lagrange multiplier $\{\Lambda_t\}$, the allocation set $\{C_t, H_t, K_{t+1}^S, K_{t+1}^L, X_{S,t}, X_{L,t}, Y_t, D_t, I_t^S, I_t^L, DIV_t, \Pi_t^B\}$, and the vector of prices $\{P_{i,t}, \pi_t, w_t, w_{j,t}, R_{L,t}, R_{t,CB}, q_t^S, q_t^L, R_{s,t}, tp_t\}$ such that the household, the final good firm, intermediate firms, entrepreneurs and the bank maximization problems are solved, and the market clearing conditions hold.

Details of the equations that determine the recursive equilibrium and the steady state of the economy are shown in Appendix C. Before discussing the results we quickly present the main parameter values used for the benchmark version of our model. As standard we set the goods market mark-up to 20%, thus $\varepsilon = 6$. The labour market mark-up²⁰ is set to 7.5% or $\varepsilon^w = 14$. We set the discount factor $\beta = 0.99$; the intertemporal elasticity of substitution in consumption $\sigma = 1$; and the Frisch elasticity of labour supply $\eta = 1$. The Calvo price and wage parameters²¹ are $\omega = 0.5$ and $\omega^w = 0.6$. The depreciation rate is set to $\delta = 0.05$, the share of capital in production to $\alpha = 0.36$, and the share of short run capital to $\zeta = 0.4$. That ensures that at the steady state the share of long-term loans in total loans is 60%. Fan, Titman, and Twite (2010) report that the debt maturity ratio, (that is, long-term interest bearing debt over total debt) is about 80% in the US, 60% in the UK, 55% in Germany and 40% in Japan during the period 1991-2006. They found that the median long-term debt ratio across 39 different countries is estimated to be around 60%. We set the degree of risk aversion of banks to $\sigma^B = 1$, which is the same as the one for the household.

The steady state long-term rate is given by $R_L = \frac{1}{\beta^2} + \frac{\rho}{\beta}$ and thus depends on the liquidity shortage at steady state ($\bar{\rho}$). We set $\bar{\rho} = 0.0025$, such that the 10 year term premium is roughly 100 basis points matching the US data (Rudebusch and Swanson (2008b)). We initially assume that the central bank follows a simple Taylor Rule with inflation parameter $\varepsilon_\pi = 2.5$ and output gap parameter $\varepsilon_Y = 0.125$. Note that higher values of ε_Y and lower values of ε_π easily lead to indeterminacy issues in models with cost channels (see Aksoy, Basso, and Coto-Martinez (2009)). Finally, we set the persistence of the productivity process $\rho^A = 0.8$ and the persistence of liquidity shocks $\rho^L = 0.8$, while setting the standard deviation of their respective *i.i.d* disturbances to $v_a = 0.01$ and $v_l = 0.0001$. The model is solved to a third order approximation using Dynare++ (without centralization).

4. Term Spreads and Economic Activity

In this section we analyze the mechanism that drives term spread fluctuations in our model, the effect of these on economic activity and the link between spread movements and future output growth.

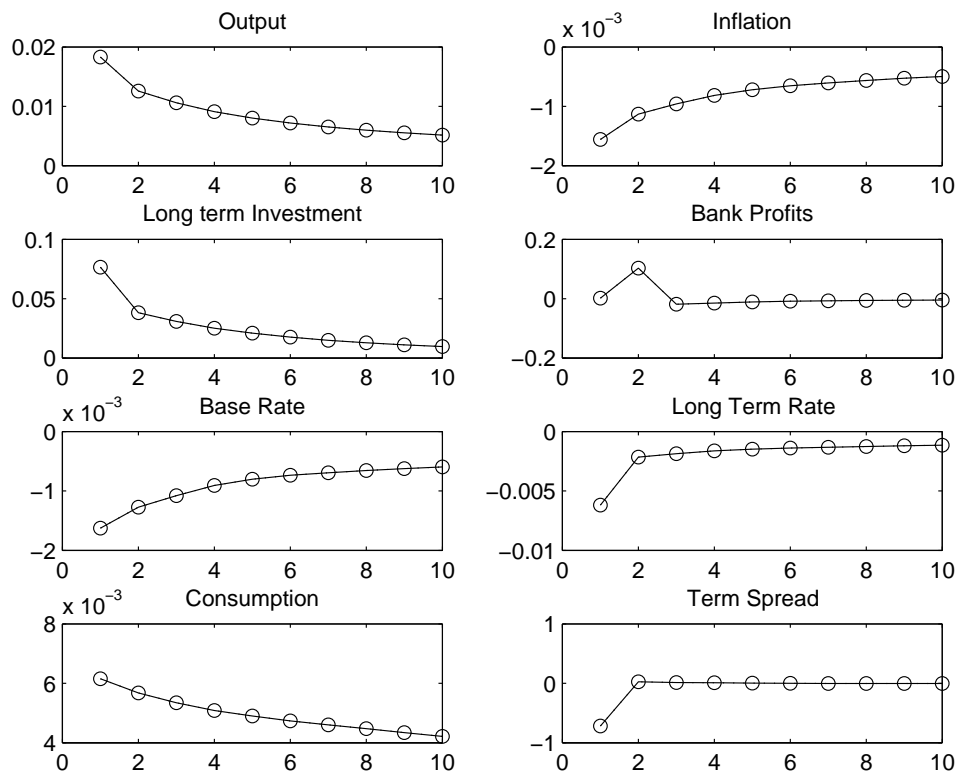
²⁰While some contributions to the DSGE literature set $\varepsilon^w = 21$ others set $\varepsilon^w = 2$. Our results are unchanged when we vary ε^w within this range.

²¹These are a bit smaller than the ones obtained in DSGE-based Bayesian estimations. However, all these studies have assumed wage and price indexation decreasing the effect of nominal rigidity on economic activity, while here for simplicity we do not. Our results are unchanged when lower degrees of price and wage rigidity are assumed.

4.1. Endogenous Term Spreads

We start by focusing on the benchmark model and the fluctuation of term spreads after a positive productivity shock. Figure 2 shows impulse responses of the main variables of interest. For all variables the percentage deviation from steady state is shown except for term spreads movements where the change in the percentage rate is reported (thus a 0.5 deviation implies a 50 basis point change in term spreads).

Figure 2: Benchmark - Productivity Shocks



As expected, a one standard deviation positive productivity shock leads to an increase in output, a decrease in inflation, and higher consumption and long-term investment. While both long-term and the base rate decline, the drop in the long rate is about 3 times larger than the base rate's deviation leading to a fall in term spreads.

The movements in spreads is mainly due to the response of bank profits after the shock. The bank sets a term premium according to the potential costs of liquidity shortage they may face in the future. In periods of higher profits, the bank is less likely to suffer balance sheet problems in the event that liquidity injections are needed, since cash flows from profits can be used to cover for these injections. Therefore, bearing maturity risk in these states becomes relatively cheaper such that long-term rates fall, resulting in lower term spreads. In states where profits are expected to

decrease the opposite occurs. Note that even though the bank is still subject to liquidity shortages on long-term investment funding in the future, these shortages may occur when banks have higher profits, and hence, we observe an expansion of long-term credit supply today. Consequently, after observing a positive productivity shock in Period 1, banks set term spreads low since they expect profits to be high in Period 2, allowing them to potentially use these high profit flows to offset the liquidity injections needed.

Our empirical results, that are presented later in Section 7, highlight that relationship between spreads and financial sector profitability. Higher term spreads indicate higher probability of observing decreasing profitability. The model also generates a negative link between bank's asset holdings and term premia. Adrian, Shin, and Moench (2010) present empirical evidence of this relationship across various asset classes. Augmenting our model to include different assets in the bank's portfolio would allow us to further explore the links between financial intermediation and the macroeconomy, providing additional theoretical support for their findings. In the benchmark specification an expected increase in profits of around 6% leads to a term spread adjustment of around 60 basis points. The standard deviation (quarterly) of U.S. Financial Business Profits used later in our empirical analysis is around 8% and the standard deviation of term spreads, depending on the maturity, ranges between 50 to 75 basis points. Thus, the magnitudes of the movements in the model are roughly in line to those in the data. In Section 5 we compare the volatility of term spreads in the data and in our model economy in detail.

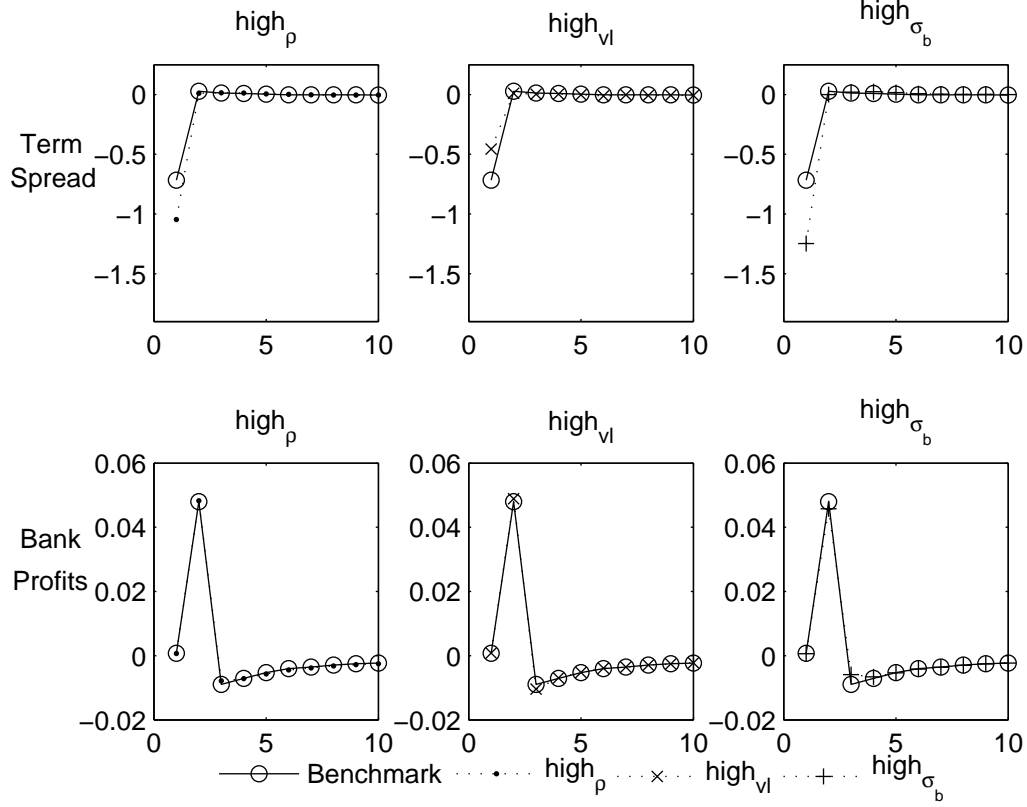
In order to further investigate the drivers of the endogenous movements in term spreads after productivity shocks we run three variants of our model and compare them to the benchmark case. In the first variant we set a higher steady state liquidity shortage ($\bar{\rho} = 0.0175$), and denote it *high ρ* . In the second we set a high variance of liquidity shock ($v_l = 0.04$), denoting it *high v_l* and finally, in the third, we assume that the bank is more risk-averse by setting ($\sigma_B = 2$) and denoting it *high σ_B* . Figure 3 shows the results.

Firstly, note that, in all three cases bank profits move in a very similar way after a positive productivity shock. In order to distinguish these three cases we need to uncover how the same response in profits leads to different dynamics in term spreads. In the first variant, we observe that spread movements are amplified under higher steady state liquidity shortages. The main intuition for this result is as follows. At the steady state, the bank sets long-term rates higher than short-term rates to offset potential liquidity shortages and hence, the higher $\bar{\rho}$ or the higher the average need for liquidity injection, the higher will the steady state long-term rates be. Given that long-term rates are relatively high, an equivalent increase in bank profits (when compared to the benchmark case) induces a stronger adjustment in long-term rates, which in turn implies long-term rates falling by a greater amount than under the benchmark case.

The opposite occurs when the variance of the liquidity shock is high. The bank is willing to bear more maturity risk in periods of high profits since they know that high profits can be used to offset liquidity shortages. However, the more volatile are these shortages, the less certain the bank will be that high profits will be enough to offset them. Therefore, an equivalent movement in bank profits leads to smoother movements in long-term rates and term spreads after a positive productivity shock.

Finally, the third variant illustrates that the term spreads will be more responsive to productivity shocks as the degree of bank risk aversion increases. σ_B effectively determines how fluctuations of bank profits influence the bank's long-term rates decision. When $\sigma_B \rightarrow 0$, the bank will

Figure 3: Fluctuations in Term Spreads after Productivity Shock



set long-term rates to be a discounted sum of short-term rates and term spreads will be constant. This mechanism is the same as the one explored in the macro-finance literature where Epstein-Zin preferences are used to increase risk aversion in order to match volatility of risk/term premia (see Rudebusch and Swanson (2008a)).

The key equilibrium condition (see Appendix C for details) that determines the long-term rate, and consequently the term spread, comes from the bank's portfolio decision. The bank will set long-term assets holdings (X_L) such that

$$E_t \left[\Pi_{t+1}^B \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] = \beta E_t \left[\frac{1}{\pi_{t+1} \pi_{t+2}} \Pi_{t+2}^B \left(R_{L,t} - R_{t+1,CB} \right) \right].$$

One can approximate this condition (the derivation is shown in Appendix D) to illustrate, formally, the effect of changes in bank profits on term spread fluctuations.

$$\begin{aligned} E_t \left[\hat{i} \hat{p} + 0.5 (\hat{i} \hat{p})^2 \right] &= E_t \left[\sigma_B (\widehat{\Pi}_{t+2}^B - \widehat{\Pi}_{t+1}^B) + 0.5 \sigma_B \left((\widehat{\Pi}_{t+2}^B)^2 - (\widehat{\Pi}_{t+1}^B)^2 \right) + \widehat{\pi}_{t+2} - 0.5 (\widehat{\pi}_{t+2})^2 + \frac{\bar{\rho}}{\Gamma} (\widehat{\rho}_{t+1} + 0.5 (\widehat{\rho}_{t+1})^2) + CovTerms \right] \\ CovTerms &= -\sigma_B \widehat{\Pi}_{t+1}^B \frac{\bar{\rho}}{\Gamma} \widehat{\rho}_{t+1} - \sigma_B \widehat{\Pi}_{t+1}^B \frac{1}{\beta \Gamma} \widehat{R}_{CB,t} + \sigma_B \widehat{\Pi}_{t+1}^B \widehat{\pi}_{t+1} - \left(\frac{1}{\beta \Gamma} \widehat{R}_{CB,t} + \frac{\bar{\rho}}{\Gamma} \widehat{\rho}_{t+1} \right) \widehat{\pi}_{t+1} \\ &\quad + \sigma_B \widehat{\Pi}_{t+2}^B \left(\frac{1}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta \Gamma} \widehat{R}_{CB,t+1} - \widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} \right) + \left(\frac{1}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta \Gamma} \widehat{R}_{CB,t+1} \right) (\widehat{\pi}_{t+1} + \widehat{\pi}_{t+2}) - \widehat{\pi}_{t+2} \widehat{\pi}_{t+1}. \end{aligned}$$

As easily verified, setting $\sigma_B = 0$, or assuming bank's utility is linear on profits, eliminates all the effects of movements of profits on term spread decisions. Looking at the covariance terms we observe that the higher the covariance between profits and the liquidity shortage (first term), the lower term spreads will be, with the strength of the effect being positively associated with σ_B and $\bar{\rho}$. Hence, as we increase $\bar{\rho}$ movements in spreads, given expected changes in profits, are amplified. Finally, the increase in the variance of the liquidity shock v_l has two opposing effects. Firstly, it tends to raise spreads since $E(\widehat{\rho}_{t+1})^2$ has a positive impact on spreads. Secondly, it becomes a stronger driver of the expected covariance between profits and ρ_{t+1} . The latter implies that the expected positive movement in profits due to the productivity shock will have little effect on the covariance term and as such one of the key drivers of the endogenous movements of spreads loses its significance. As a result of this effect on the covariance, higher volatility of liquidity shocks dampens the impact of productivity shocks on term spreads.

4.2. Yield Spreads and Output Growth

As reported by Rudebusch, Sack, and Swanson (2007), the Congressional Budget Office output gap and the 10Y term premium during the period 1960 and 2005 seem to be negatively related (see Figure 6 in their paper). Hence, the counter-cyclical movement of spreads obtained in our model after both a productivity and a monetary shock (not reported here) matches the overall characteristic of the US data. As stressed by Gürkaynak and Wright (2010), term structure models should generate a high slope of the yield curve at the beginning of recoveries from recessions and a flat yield curve during booms, feature which is related to the predictive power of yield spreads. Hamilton and Kim (2002) conclude that lower term premiums predict slower GDP growth, although this effect appears to be strong only in the short-run, while Wright (2006) shows that lower term premium raises the odds of a recession.

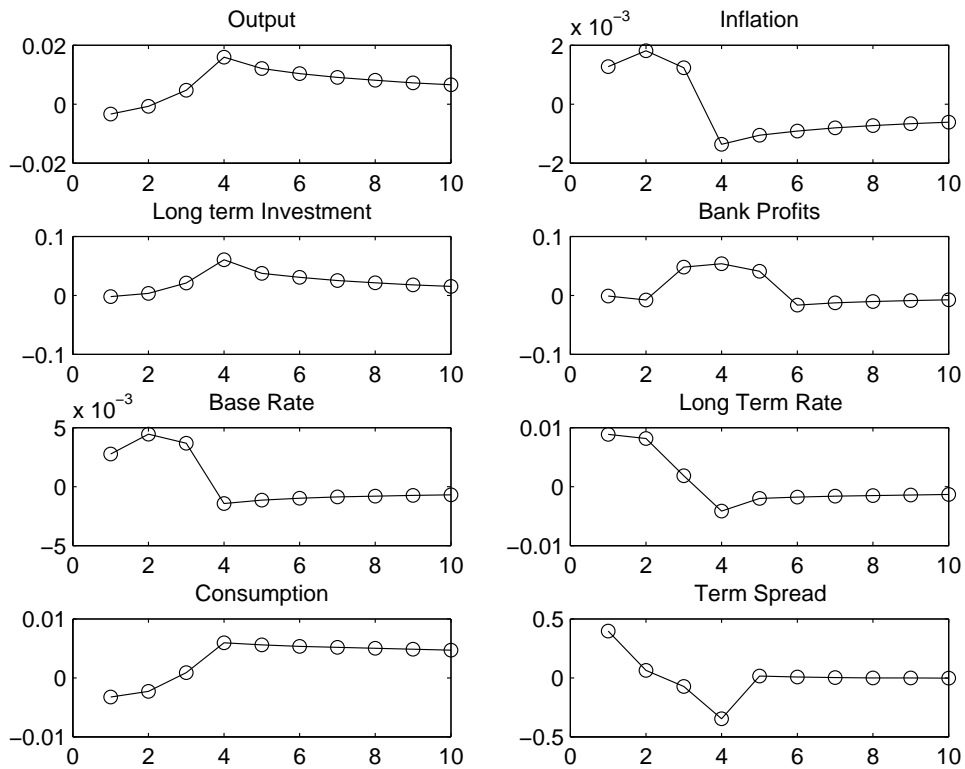
In order to verify if the dynamics of term spreads in our model is consistent with this feature we study the impact of a three period²² anticipated technology shock (Figure 4). That way, based on the information at time t , the bank forms an expectation of future growth and profits which will affect long-term rates and thus term spreads. These then feed back to the economy influencing long-term investment and output.

We observe that output and long-term investment increase from t until $t + 3$ (time of the realization of the productivity shock). Therefore, if one is regressing output gains $\widehat{y}_{t+3} - \widehat{y}_t$ on $\widehat{t}p_t$ (a variant of Hamilton and Kim (2002) estimation), getting a positive parameter estimate must imply that $\widehat{t}p_t > 0$, which is what we obtain. The main driver of this result is the future path of bank profits. Bank profits will initially decrease, making it more costly to bear maturity risk, and hence long-term rates and spreads increase in Period t . Spreads are at their highest when output is at its lowest and expected to increase in the future. As we approach $t + 3$, bank profits will be increasing and spreads decreasing given that the premium the bank charges to bear maturity risk is lower when their portfolio are expected to have higher returns. That leads to increasing long-term capital investment and consumption. At $t + 3$, when productivity is at its peak, output is at its highest and spreads at their lowest point; bank profits are then expected to decrease so that spreads start to increase thereafter. Therefore, as observed in the data, high sloped yield curve indicates future output is increasing while a flat yield curve indicates that output is at its peak.

²²At time 1 (t) agents learn there will be a productivity shock at time 4 ($t+3$).

Rudebusch, Sack, and Swanson (2007) refer to a potential contradiction while discussing the intuition behind the results of regressions of output growth on the level of term spreads (as the one performed in their paper and in Hamilton and Kim (2002)). They point out that under a standard Keynesian view, low term spreads should result in higher investment and thus higher output in the future, not lower as the level regressions suggest. They in fact confirm this view in the data by estimating output differences $\hat{y}_{t+1} - \hat{y}_t$ on spread differences $\hat{t}p_t - \hat{t}p_{t-1}$, obtaining the expected negative parameter estimate. As opposed to the standard models in the macro-finance literature where the yield curve is build based on the stochastic discount factor, the term premium here has a direct effect on long-term investment and output and thus this Keynesian mechanism is in place. As a result of that, our model also confirms the prediction that decreasing spreads leads to higher output. However, at the time the anticipated shock is known, Period 1, bank profits are expected to remain low for the next two periods, forcing the bank to initially charge more for long-term commitments. Thus, term spreads are high but decreasing.

Figure 4: Anticipated Productivity Shock



Note that Adrian, Estrella, and Shin (2010) assess empirically the link between bank, spreads and output growth. They propose that lower term spreads, holding riskiness constant, leads to

lower net interest margins, which in turn leads to lower banking asset growth and hence, lower output. Effectively, this mechanism would occur for movements of spreads that are exogenous to the bank's balance sheet decision proposed here, which is based on riskiness. We can obtain a similar link using our model if we alter γ_L (the parameter that controls entrepreneurs demand for loans given the long-term interest rate - see equation (31)). As γ_L decreases (exogenously), spreads and bank's long-term asset holdings decrease, leading to lower output. Nonetheless, in the framework presented here, endogenous fluctuations of spreads are intrinsically linked to the riskiness of bank's portfolio or asset holdings, preventing us from fully analyzing changes in spreads holding riskiness constant, as their mechanism suggests.

4.3. Term Spreads and Macroeconomic Dynamics

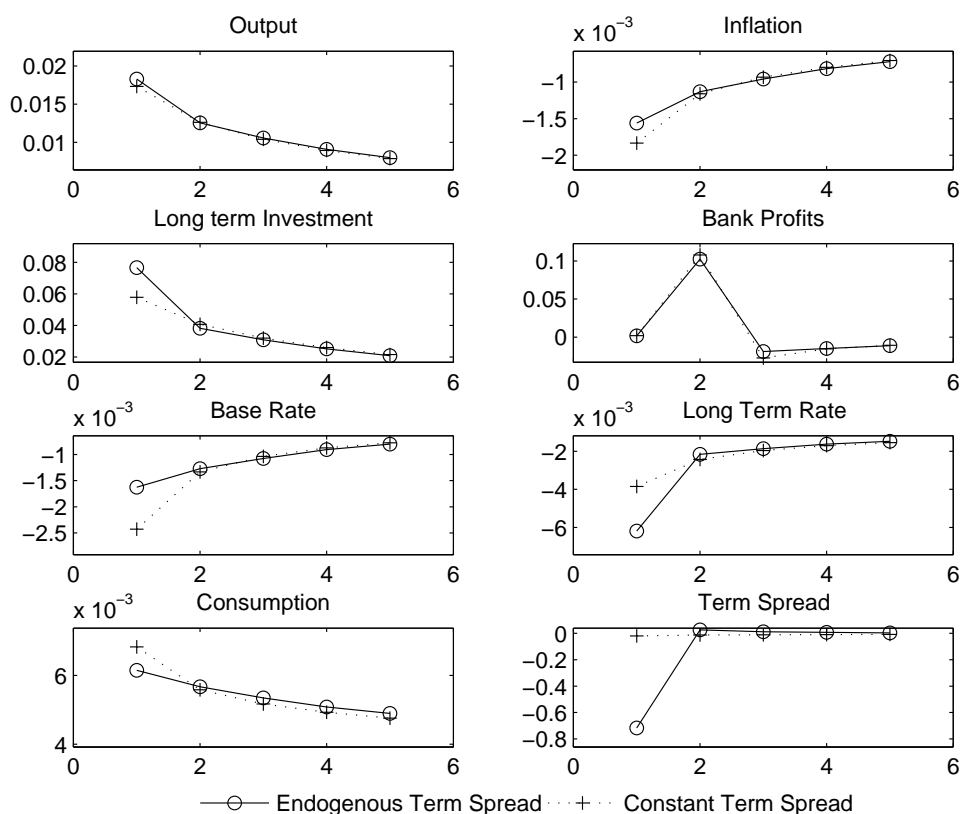
As we mentioned earlier, in the DSGE asset-pricing models, which are now standard in structural models of yield curve literature (e.g. Rudebusch and Swanson (2008b)), current output is determined by the expected path of short-term rates, and hence, term premia or long-term rates have no effect on economic activity. In our model long-term rates are determined by the bank, being a function of expected future short-term rates and potential liquidity shortages in bank's balance sheets, and most importantly, they affect firm's long-term capital investment decisions. That way, we can assess a channel through which term premia fluctuations affect economic activity. The effect of these endogenous fluctuations of term spreads on output can be highlighted by looking at the impulse responses after a productivity shock with $\sigma_B = 0$ (constant spreads), plotted against the benchmark case. Figure 5 shows the results.

Countercyclical term spread movements lead to an amplification of output responses after a productivity shock, although the change in output seems fairly small as compared to the movements observed in spreads and long-term rates. The main reason for that is the shift in the composition of output. While, after a sharp decrease in long-term rates, long-term investment increases significantly relative to the case with constant spreads, the response of consumption follows the opposite pattern. On the one hand, given that long-term rates have decreased significantly, the base rate set by the central bank does not need to fall as much. As a result, the demand channel (Euler equation) is dampened and consumption and inflation do not move as much as in the constant spread case. On the other hand, when spreads are constant, the central bank moves the base rate more aggressively, pushing consumption up and leading to higher inflation deviations.

If part of the consumption is financed by long-term borrowing (durable consumption) then the endogenous movements in term spreads would have a much stronger effect on output given that both consumption and investment would expand further relative to the constant spreads case. Moreover, monetary policy would be forced to be considerably less aggressive to control output and inflation volatility.

Finally, in all the analysis so far we kept ρ_t constant, discussing the impact of potential liquidity shortages in the future on the bank's portfolio decision. In Section 6.2. we will use the model to replicate a feature of the recent crisis when banks faced significant liquidity shortages, shocking ρ_t positively, to study the impact of different policies interventions. However, there could be periods of increased liquidity in the banking sector when ρ_t actually decreases, leading to lower long-term rates and narrowing term spreads and consequently expanding economic activity. That could be a potential description of the US and UK economies during the period 2003

Figure 5: Endogenous vs Constant Term Spreads



- 2007 where long-term rates fell significantly (the yield curve in the UK actually inverted). De Graeve, Emiris, and Wouters (2009) decompose this fall in long-term rates in the US and show that its main drivers were declining term spreads. During the same period we observed a boom in securitization or development of structured finance activities. These activities actually meant that long-term commitment/assets could be re-packaged and sold; increasing profits through fees or advantageous balance sheet operations. Effectively, banks found themselves operating in a market in which bearing maturity transformation was relatively cheap, or ρ was significantly smaller or even negative, bringing term spreads down.

Benati and Goodhart (2008) show that during the post-war period the marginal predictive content of spreads increased during periods where current (and future) monetary policy regimes were uncertain. They conjecture that the additional predictive power of the yield curve was due to higher long-term rates, which reflected this uncertainty, depressing output. They also observe that during the early 2000's the marginal predictive content of spreads also increased while monetary regimes had been successfully established. They then conjecture that external forces (to monetary policy) were holding down long-term yields, relating the predictive content of spreads to the real yield curve which reflects structural conditions of the economy. The fluctuations of parameter ρ

given the conditions in the banking sector can be one potential avenue to explain some of these external factors.

5. Yield Curve and Volatility

In our basic model, long-term funding is made for two periods while short-term funding is for one period. In order to compare the derived dynamics of term spreads movements in the model to the data, we ought to consider longer maturities. However, one of the main challenges is that, due to the liquidity premium, the expectations hypothesis does not hold. (See Dewachter, Iania, and Lyrio (2011) for new evidence on the rejection of the expectation hypothesis in term structure models.)

In order to obtain the zero coupon rate for a three period bond we need the zero coupon rate for two periods $R_{L,t}$ and the forward rate between periods $t+2$ and $t+3$, denominated ${}_{t+2}f_{t+3}$. There are two possible ways, one is to use $R_{S,t+2}$, or the short-term rate of Period $t+2$, such that it is equivalent to invest in a three period zero bond or first investing in a two period bond and then in a one period asset. However, that implies that the liquidity risk will only affect the investment from Period 1 to Period 2. The alternative is to obtain what would be the premium from investing in long-term maturity assets from $t+2$ till $t+3$. That is setting ${}_{t+2}f_{t+3} = \frac{R_{L,t+1}}{R_{S,t+1}}$ (note that the rate $R_{L,t+1}$ is for holding a long-term asset from Period $t+1$ until $t+3$). We will set forward rates following the latter and hence ${}_{t+n}f_{t+n+1} = \frac{R_{L,t+n-1}}{R_{S,t+n-1}}$, such that the future liquidity risk impacts the entire yield curve. We will therefore build the yield curve obtaining the forward rates using the forward looking long-term rates against the short-term rates at each period they are set. Let the price of a n quarters zero coupon bond be $E_t[p_t^{(n)}]$. Let the risk neutral and liquidity risk free zero coupon bond be $E_t[\hat{p}_t^{(n)}]$. The pricing of each of these assets is given by

$$E_t[p_t^{(n)}] = \exp\left(- (R_{L,t} - 1) - \sum_{i=2}^{n-1} ({}_{t+i}f_{t+i+1} - 1)\right), \text{ and} \quad (14)$$

$$E_t[\hat{p}_t^{(n)}] = \exp\left(- \sum_{i=0}^{n-1} (R_{S,t+i} - 1)\right). \quad (15)$$

The term premium will therefore be

$${}_t p_t^{(n)} = \frac{1}{n} \left(\ln(\hat{p}_t^{(n)}) - \ln(p_t^{(n)}) \right) = (R_{L,t} - 1) + \sum_{i=2}^{n-1} ({}_{t+i}f_{t+i+1} - 1) - \sum_{i=0}^{n-1} (R_{S,t+i} - 1).$$

Note that the standard measure used in the macro-finance literature (see Rudebusch and Swanson (2008b)) is effectively given by

$$\tilde{t} p_t^{(n)} = \sum_{i=0}^{n-1} \left(\frac{1}{m_{t+i}} - 1 \right) - \sum_{i=0}^{n-1} (R_{S,t+i} - 1),$$

where m_t is the stochastic discount factor obtained from the household's Euler equation. As Rudebusch and Swanson (2008b) stress this measure invariably generates term spreads movements that do not match the data unless one assumes shocks with high standard deviations, which in turn worsens the model's ability to match standard business cycle facts.

We start by comparing the volatility of the 1 and 2 years term spreads of this standard measure and the one in our model. We then compare the 5 years term spreads obtained in our model against the data (collected from Kim and Wright (2005)) and the volatility obtained by the baseline model²³ of Rudebusch and Swanson (2008b) (denoted $RS_{Baseline}$). We simulate our model without liquidity shocks, thus all variability in spreads are due to endogenous movements after productivity and monetary policy shocks. In Table 3 we provide the results of two simulations, one with the benchmark parameters and one with a higher bank risk aversion parameter ($\sigma_B = 3$).

Table 1: Volatility of Term Premium*

Banking Channel against Macro-Finance				
	Term Premium - tp	MF Premium - $\tilde{t}p$		
1Y	0.3866	0.00042		
2Y	0.1883	0.0053		
Banking Channel against the Data				
	Data (1990 - 2011)	Benchmark	High σ_B	$RS_{Baseline}$
5Y	0.657	0.0746	0.1933	0.0013

*0.5 denotes 50 basis points.

Firstly, as clearly seen the banking balance sheet/profitability channel explored in our model is able to generate considerably more volatile term spreads than the standard macro-finance measure. This occurs despite the fact that we kept the standard deviation of shocks to be in the order of 1% and the risk aversion parameter (benchmark) to be equal to 1, lower than the one used by Rudebusch and Swanson (2008b). Bank profits are considerably more volatile than consumption, delivering greater volatility of term spreads for the same degree of risk aversion and variance of exogenous shocks. Finally, when we compare our benchmark case with the data we come closer than the standard macro-finance model.

One important feature of the data, which is not replicated in our benchmark model, is the fact that both the mean and the standard deviation of term spreads tend to increase with maturity. In our model, while the mean increases, the standard deviation decreases. This is because we introduce only two securities in the bank's balance sheet, a one period and a two periods security, hence a movement in spreads today affect the short end of the curve more heavily than the long end. Financial businesses would have a variety of securities with different maturities and risk profiles, including assets bearing long-term inflation risks. Extending the model to consider a richer bank portfolio and including long-term inflation risks may be, therefore, fruitful areas for further research.

²³We use their code to calculate the 5Y term spread volatility since they only report the 10Y point.

Rudebusch and Swanson (2008a) have extended the standard macro-finance model to include long-term inflation shocks and adopt Epstein-Zin preferences to break the link between intertemporal elasticity of substitution and coefficient of risk aversion. They are able to deliver volatile term spreads and match the dynamics of the main macroeconomic variables. Although acknowledging the importance of long-term inflation risk as an important driver of term premium - as discussed by Gürkaynak, Levin, and Swanson (2010) while covering the regime changes in the UK in the last 20 years - there is also evidence that the dynamics of short-run rates and inflation expectations do not explain all the variability of long-term rates (De Graeve, Emiris, and Wouters (2009)) or its output predictive power (Benati and Goodhart (2008)). More importantly, as stressed by Gürkaynak and Wright (2010), the US treasury inflation protected securities (TIPS) forward rate dynamics have not been that different than their nominal counterparts (see Figure 5 in their paper), indicating the term premia are also influenced by real factors. Hence, we see the bank balance sheet channel explored here as a potential complement to variations in term spreads due to inflation risk premia. In fact, given that the bank would be exposed to inflation risk while bearing maturity risk, a potential long-term inflation shock would also generate increased volatility of term spreads in our model without the need of increasing the degree of risk aversion.

6. Monetary Policy

In this section we firstly look at conventional monetary policies in the presence of endogenous term spreads, focusing on different short-term rate policy rules. We then look at unconventional policies during periods of large shocks to liquidity shortages.

6.1. Conventional

Our interest here is to provide an answer to the following question: Should the central bank directly change short-term rates given the fluctuations in term spreads? This direct adjustment of short-term rates could be relevant in our model since spreads feed back to the macroeconomic variables of the model through their effects on long-term investment. A similar question is tackled by Aksoy, Basso, and Coto-Martinez (2010) in the case of banking spreads movements. Their finding is that welfare increases when central banks explicitly take spreads into account while setting base rates.

In order to calculate welfare under different policy rules we first obtain a third order approximation solution of the main variables of the model and then approximate the unconditional mean of the expected utility function of the household using a standard Monte Carlo Method, thus

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right) &= \int_{-\infty}^{\infty} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right) \right] f(z^t) dz^t \\ &\approx \frac{1}{M} \sum_i^M \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right) \right], \end{aligned}$$

where z^t is a series of shocks, $f(z^t)$ the probability distribution of these shocks and x_i^t the realization of the endogenous variables under a specific series of shocks z_i^t for $i = 1$ to M .

We set the monetary rule to be

$$\frac{R_{t,CB}}{\bar{R}} = \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\varepsilon_{\pi}} \left(\frac{Y_t}{\bar{Y}} \right)^{\varepsilon_Y} \left(\frac{tp_t}{\bar{tp}} \right)^{\varepsilon_{tp}} \right].$$

We observe that setting ε_{tp} , the degree to which the central bank moves the base rate given fluctuations of term spreads, to be greater than 0.002 causes model indeterminacy (no matter how strongly the central bank responds to inflation). Nonetheless, for our benchmark case we observe that setting $\varepsilon_{tp} = 0.001$ leads to higher welfare. This is due to the fact that as monetary policy responds to spread fluctuations it is able to compensate for the distortion created by the liquidity shortages which is assumed to be positive at steady state. We confirm this conclusion by modifying the model assuming $\bar{\rho} = 0$, in which case the bank no longer faces liquidity shortages in long-term investments on average, or at steady state. Setting $\varepsilon_{tp} = 0.001$ under this specification does not lead to higher welfare relative to the standard monetary rule where the base rate depends only on output and inflation deviations.

Hence, we conclude that although fluctuation of term spreads are important for economic activity, and monetary policy should take these fluctuations into account, the central bank should not respond explicitly to term spread movements, unless it wants to correct for steady state distortions. However, note that fluctuations in term spreads do affect output and inflation, and thus, even when relying on a standard monetary policy rule, the central bank is taking spread movements implicitly into account (see Figure 5).

6.2. Unconventional

The Federal Reserve Bank (FED) conducted two purchase programmes of long-term Treasuries and other long-term bonds, known as QE1 in 2008-2009 and QE2 in 2010-2011. These quantitative easing policies comprised of purchase of mortgage backed securities, Treasuries and “Agencies” from the private sector. Gagnon, Raskin, Remache, and Sack (2010) analyze the effectiveness of the Large-Scale Assets Purchases conducted by the FED. They find that the purchase programme lead to reductions in long-term interest rates on a range of securities, including some securities that were not included in the purchase programme, indicating that portfolio balancing effects were in play. They argue that the reductions in interest rate primarily reflect lower risk/liquidity premiums rather than lower expectations of future short-term rates. Krishnamurthy and Vissing-Jorgensen (2011) find that these QE policies in the U.S. led to a significant decline in nominal rates on long-term safe assets (Treasuries and “Agencies”, assets which were more heavily traded by the FED) and only a small effect on less safe assets such as corporate rates and mortgage rates (assets which were less heavily influenced by FED market activity). Their results suggest that the effects of asset purchases on the duration of risk premium are small, while effects on liquidity-safety premium are substantial.

Beirne, Dalitz, Ejsing, Grothe, Manganeli, Monar, Sahel, Suec, Tapking, and Vong (2011) report on the effectiveness of the Covered Bond Purchase Programme (CBPP), which started in July 2009 for a period of 12 months in the Eurozone, and show that covered bond yields decreased by 12 basis points; the programme increased the liquidity of secondary market and that it managed to encourage lending. Joyce, Lasasosa, Stevens, and Tong (2010) report that the QE interventions in the UK led to a 100 basis point decrease in Gilt yields. Given the effects on other asset classes,

although the purchase programme has been overwhelmingly of government securities, they also stress the importance of portfolio balancing effects. Finally, Borio and Disyatat (2010) provide a survey of different forms of possible unconventional monetary policies and argue that the main balance sheet channel operates through the central bank's ability to reduce yields and ease financing constraints by altering the risk profile of private portfolios. Overall, a constant theme in these studies is the effect on long-term rates through lower term spreads being crucial for the effectiveness of the interventions and for allowing the financial market to continue funding economic activity.

Two main features of our model are particularly important in formalizing this type of interventions. First, fluctuations in term spreads are a relevant factor in determining output as long-term rates influence investment decisions, and hence, an intervention aimed at lowering long-term rates affects economic activity. Second, given that term spreads or long-term rate decisions are directly determined by fluctuations in future bank profits and changes in their balance sheet holdings, our model provides a new channel through which the effects arise. However, an important caveat, which underlines a promising path for future research, is the fact that our bank portfolios are fairly simple, with only three assets. They do not include, for instance, housing debt/mortgages, thereby restricting the analysis of some of the portfolio balancing effects mentioned.

In order to study the main effects of QE policies we first introduce two types of unconventional monetary policies and then analyze their impact after a liquidity shortage shock. The first is a simple liquidity injection (QE_t) to the bank, financed by a lump-sum tax collected from households. Liquidity injection, which is costless to the receiving bank, is set such that $QE_t = \xi_t X_{L,t-1} P_{t-1} \rho_t$, where $\xi_t = \phi_\xi \left(\frac{\bar{X}_L}{\bar{X}_S} - \frac{\bar{X}_{L,t}}{\bar{X}_{S,t}} \right) \frac{\bar{X}_S}{\bar{X}_L}$ and $\frac{\bar{X}_{L,t}}{\bar{X}_{S,t}}$ is the ratio of long to short run funding that would be in place without QE intervention. The liquidity injection is a proportion ξ_t of the current bank's long-term asset exposure, and its intensity depends on how skewed current investment funding is towards short-term relative to long-term funding. Note that this relative difference will be a direct function of future liquidity conditions.

The second unconventional policy is the existence of favourable conditions for the bank to borrow funds from the central bank using their long-term asset positions as collateral. favourable conditions in our context imply a lower rate of borrowing relative to the short-term funding currently available. The bank now decides the fraction of long-term assets (Θ_t) they want to pledge as collateral to get funds from the central bank. Effectively, at time t , the bank makes a two period investment. At Period $t + 1$ they sell a portion (Θ_t) of these assets to the central bank to get additional funds, promising to buy them back at $t + 2$ before they mature. The total cost of central bank funding is $\Theta_t X_{L,t-1} P_{t-1} (R_{t,QE} - 1) + \frac{\phi_{re}}{2} \Theta_t^2$, where $R_{t,QE}$ is the borrowing rate. The term $\frac{\phi_{re}}{2} \Theta_t^2$ is included such that the marginal cost of this type of funding is increasing as usage increases. The bank's problem now becomes

$$\begin{aligned} \max_{\{X_{S,t}, X_{L,t}, \Theta_t\}_0} & E_0 \sum_{t=0}^{\infty} \beta_t \frac{\Pi_t^{B^{1-\sigma_B}}}{1 - \sigma_B} \\ s.t. & D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1} - \Theta_t X_{L,t-1} P_{t-1} \end{aligned}$$

where

$$\begin{aligned}\Pi_{t+1}^B &= \frac{1}{P_{t+1}}(DIV_{t+1} + (R_{L,t-1} - 1)P_{t-1}X_{L,t-1} + (R_{S,t} - 1)P_t X_{S,t} - D_t(R_{S,t} - 1) - \\ &\quad - \rho_{t+1}X_{L,t}P_{t+1}) - \Theta_t X_{L,t-1}P_{t-1}(R_{QE,t} - 1) - \frac{\phi_{QE}}{2}\Theta_t^2.\end{aligned}$$

The first order conditions in this case are

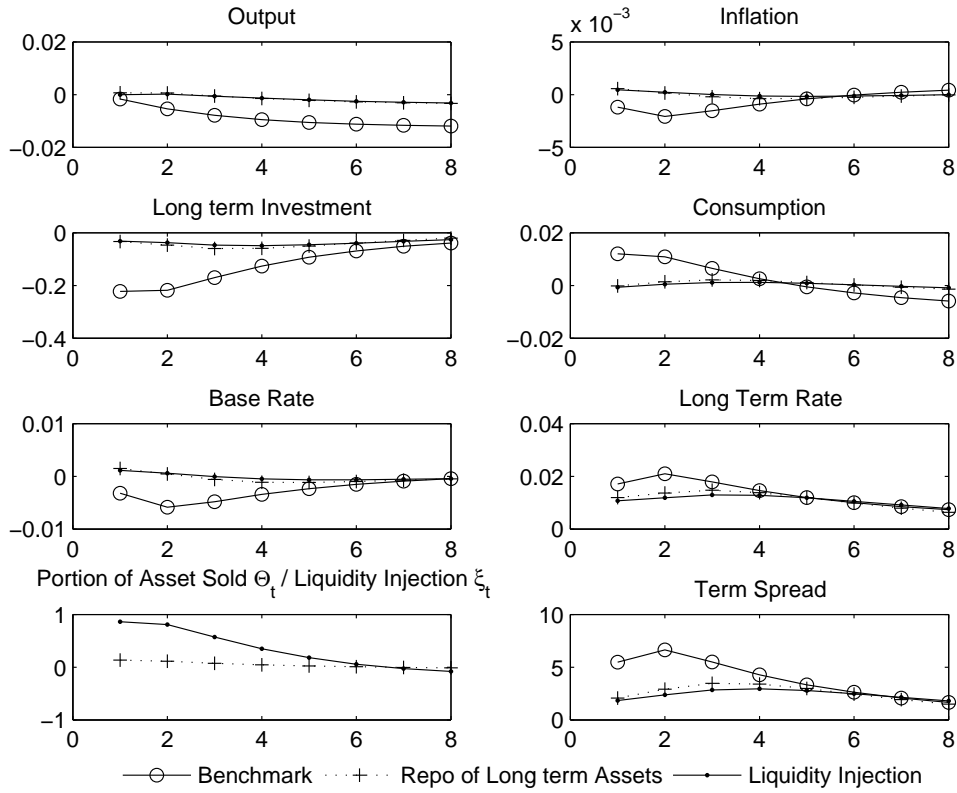
$$\begin{aligned}R_{S,t} &= R_{t,CB}, \\ E_t \left[\Pi_{t+1}^B^{-\sigma_B} \left(\frac{(R_{S,t} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] &= E_t \left[\frac{\beta \Pi_{t+2}^B^{-\sigma_B}}{\pi_{t+1} \pi_{t+2}} (R_{L,t} - R_{t+1,CB} + \Theta_{t+1}(R_{S,t+1} - R_{QE,t+1})) \right], \\ 0 &= \Pi_{t+1}^B^{-\sigma_B} \left((R_{S,t} - R_{QE,t}) \frac{X_{L,t-1}}{\pi_t \pi_{t+1}} - \phi_{QE} \Theta_t \right).\end{aligned}$$

Finally, we assume the central bank sets $R_{QE,t} = R_{S,t} \left(1 - \phi_{re} \left(\frac{\bar{X}_L}{\bar{X}_S} - \frac{\bar{X}_{L,t}}{\bar{X}_{S,t}} \right) \frac{\bar{X}_S}{\bar{X}_L} \right)$. Thus, lower the long-term funding relative to short-term without intervention, more favourable central bank funding will be.

Figure 6 illustrates the results for a liquidity shock of 0.04 with $\phi_\xi = 3$, $\phi_{QE} = 0.35$ and $\phi_{re} = 1$. That translates in the central bank covering roughly 90% of the liquidity shortage under the first intervention after the shock or, in the second case, buying roughly 10% of long-term assets from the bank's balance sheets (see the bottom left graph in Figure 6). We observe that both QE policies do have a significant impact on long-term rates and term spreads. That leads to dampened responses of long-term investment and output relative to the case where only conventional monetary policy is used. The main difference between these two interventions is that asset purchases have a stronger initial impact since they immediately free up the balance sheet of the bank which are then able to maintain long-term funding despite the liquidity shock. Liquidity injections do affect term spreads and eventually help sustain higher output; however, these require a longer period to work through the bank's long-term rate setting as it protects them from future liquidity shortages. Note that we allow base rates to move freely in both cases, hence, short-term rates do not need to decrease as much while asset purchases are conducted. An important result is that inflation turns out to be significantly higher after QE interventions, thus even if nominal rates are close to 1, QE interventions will lead to lower real rates.

One of the important debates at central banks in the UK, the US and the Eurozone is at which point to unwind the large-scale purchases. Although not completely suited to give a definite answer to such a question, we can use our model to verify the effectiveness of short-term asset purchase agreements, which sell securities back to banks after one period, and the interventions that allow banks to move long-term assets away from balance sheets for longer periods. In order to do that we modify the model such that long-term investments now require one year (4 quarters) commitments from firms and hence from banks. The appendix shows the details of the model and of each of the two QE interventions: one period asset purchases and three periods asset purchases. Figure

Figure 6: QE Policies

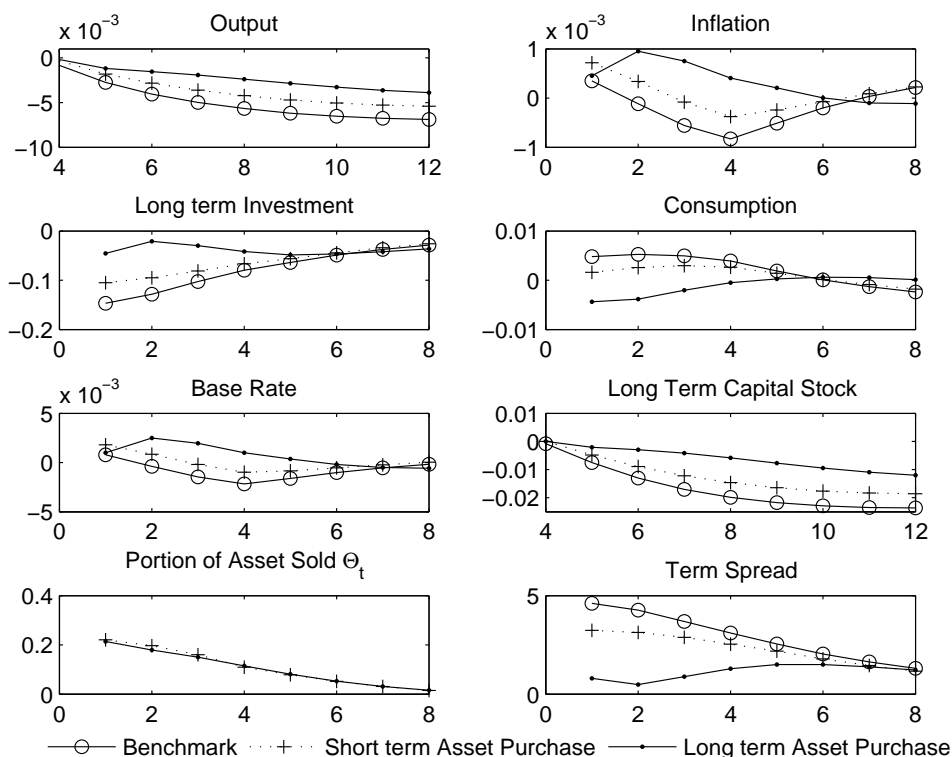


7 shows the results²⁴. We set ϕ_{QE} for each of these two interventions such that the portion of long-term assets bought by the central bank are matched (roughly 20%, see graph at the bottom left corner).

We observe that when the central bank holds assets for longer periods the same intervention in terms of assets purchases leads to lower levels of term spreads/long-term yields and to a lower decrease in long-term investment after a liquidity shock. There is a gain for the central bank to hold the securities bought in such interventions for longer periods of time since they are more effective in freeing up the bank's balance sheet, fomenting long-term funding. Obviously, these securities remain in the central bank balance sheet for longer and thus the monetary authority is taking significantly more risks than when it keeps securities for only one period. Finally, the long holding period interventions allow central banks to bring short-term rates back to their steady state levels sooner.

²⁴Impulse responses for output and long-term capital stock are shown after the fourth period since that is the point changes to long-term investment done at time $t = 1$ start having effects.

Figure 7: Short versus Long-term Asset Purchases



7. Empirical Evidence

As argued in the introduction there is strong evidence that US term spreads help predicting US real output growth (see, for instance, Rudebusch and Williams (2009)). Furthermore, Adrian, Shin, and Moench (2010) highlight the importance of financial sector variables, particularly the growth in financial intermediary asset holdings, in predicting several asset prices and risk measures. Finally, Adrian, Estrella, and Shin (2010) look at the link between term spreads, future banking asset growth and economic activity. We complement this macro empirical evidence by looking particularly at the linkages between financial sector profitability, term spreads and output growth. For this purpose we conduct three sets of empirical analysis. First we intend to highlight dynamic correlations between bank profits at the aggregate level, term spreads and real output variations. Second, we study the aggregate level expected bank profitability and evolution of term spreads. Finally, we study the link between the changes in the expected bank profitability at the bank level and evolution in term spreads.

7.1. Information Content of Financial Business Profitability

As a first step, we investigate whether variations in financial sector profits contain exploitable information that will help predict variations in real output beyond those already predictable by using past variations in real output, term spreads and the short-term interest rates, similar in nature to the analysis conducted by Benati and Goodhart (2008). We are also interested in investigating whether variations in past financial profits are associated with variations term spreads. We use quarterly US data covering the period 1970Q1-2007Q2. Our data consists of seasonally adjusted US real GDP expressed in billions of chained 2005 Dollars as reported by the Bureau of Economic Analysis and financial business undistributed corporate profits as reported by the Flow of Funds Statistics of the US Federal Reserve. This measure is for all financial businesses, including not only commercial banks but also for instance mutual funds and security brokers and dealers. Term spreads are computed using the US Treasury Bill rate in percent per annum and 10 year government bond rate in percent per annum as reported by the IMF/IFS.

We first investigate the additional marginal predictive content in movements of financial business profits to explain real output movements next to autoregressive components of real output, past term spreads and the T-Bill rate (which controls for the level of interest rates in the economy) and the additional marginal predictive content in movements of financial business profits to explain variations in term spreads. We focus explicitly on correlation structures as the preliminary test of statistical connectedness between variables. The reduced form/information value approach is immune to questions of causality and exogeneity issues as first advocated by Sims (1972, 1980) and Friedman and Kuttner (1992) among others.

Our first specification for real output changes is given by

$$\Delta y_t = \alpha + \sum_{i=1}^8 \beta_i^y \Delta y_{t-i} + \sum_{i=1}^8 \gamma_i^y (r_{t-i}^L - r_{t-i}^S) + \sum_{i=1}^8 \delta_i^y \Delta fbp_{t-i} + \sum_{i=1}^8 \zeta_i^y \Delta r_{t-i}^S + \sum_{i=1}^8 \tau_i^y \pi_{t-i} + \epsilon_t^y \quad (16)$$

where Δy represents the annualized changes in output, $(r^L - r^S)$ the spread between 10 year government bond (r^L) and 3-months Treasury Bill (r^S), Δfbp the annualized changes in financial business profits, π the annualized quarterly inflation rate and, finally, ϵ represents the error term. We include 8 lags for independent variables as an inverted yield curve is found to contain predictive power for recessions within a 12 to 18 months period (see for instance, Estrella and Hardouvelis (1991) and Rudebusch and Williams (2009)). We test whether the lagged coefficients of each variable are jointly significant (Wald test) and report associated χ^2 p -values.

In a second specification, we investigate whether the annualized changes in financial business profits contain additional marginal predictive content to explain variations in term spread next to autoregressive variations in past term spreads, the rate of inflation and changes in real output.²⁵

$$(r_t^L - r_t^S) = \alpha + \sum_{i=1}^8 \beta_i^{TS} \Delta y_{t-i} + \sum_{i=1}^8 \gamma_i^{TS} (r_{t-i}^L - r_{t-i}^S) + \sum_{i=1}^8 \delta_i^{TS} \Delta fbp_{t-i} + \sum_{i=1}^8 \tau_i^{TS} \pi_{t-i} + \epsilon_t^{TS} \quad (17)$$

²⁵It appears that in 2001Q1 there is a very large variation in both financial business profits and finance companies corporate profits; therefore we include a dummy variable for this particular data point to capture this effect. See the probit stability results for additional evidence of a possible structural break in 2001. We use throughout White heteroskedasticity consistent standard errors.

Table 2: Marginal Predictive Content Results- Financial Business Profits, Output Growth and Term Spreads ($\chi^2 - p$ values)

	Real Output Equation			Term Spreads Equation		
	70:1-07:2	80:1-07:2	90:1-07:2	70:1-07:2	80:1-07:2	90:1-07:2
$\Delta fb p (\sum_{i=1}^8 \delta_i)$.42	.02	.01	.02	.07	.08
$(r^L - r^S) (\sum_{i=1}^8 \gamma_i)$.75	.06	.05	.00	.00	.00

Table 2 reports the χ^2 , p -values of the joint significance of two types of estimated financial profits and term spread coefficients for the full sample 1970Q1-2007Q2 and two sub-samples 1980Q1-2007Q2, 1990Q1-2007Q2.

Firstly, in real GDP estimations that include variations in aggregate financial business profits ($\Delta fb p$), there appears to be a significant association between financial profits and real GDP after 1980's coinciding with the relaxation in financial market regulations next to the marginal predictive content in term spreads in explaining real GDP variations as suggested by Rudebusch and Williams (2009) among others. Secondly, in term spread estimations, we observe that variations in aggregate financial business profits contain marginal predictive content in explaining the term spread variations next to a large set of control variables as specified in Equation (17) in all samples we consider. We conclude that there appears to be significant dynamic correlation patterns between financial business profits, term spreads and real GDP variations in the US.

7.2. Spreads and Expected Financial Business Profitability

In the previous section we argued that both spreads and financial business profitability are closely linked and are important to predict future output growth. Furthermore, term spreads are inherently forward looking variables reflecting the future path of short-term rates plus the risk premia from holding long-term positions, and hence variation in spreads should contain information on the expected profitability. For this purpose we follow Wright (2006) closely and analyze whether spreads today are associated with future decreases in profitability.

Let the yearly financial business profits to be equal to $fb p_t^Y = \frac{\sum_{i=0}^3 fb p_{t-i}}{4}$. The change in profits at Period t is given by $\Delta fb p_t^Y = \frac{fb p_t^Y - fb p_{t-1}^Y}{fb p_{t-1}^Y}$. We then construct a binary dummy variable ($D(fb p_t^Y)$) that takes the value 1 when financial business profitability is decreasing and zero otherwise. Formally,

$$D(fb p_t^Y) = \begin{cases} 1 & \text{if } \Delta fb p_t^Y < \Delta fb p_{t-1}^Y \\ 0 & \text{otherwise} \end{cases}$$

We employ the following probit model to assess whether spreads are linked to future prof-

Table 3: Probit Estimation Results - Spreads and Future Profitability

	Level Equation $D(fb p_{t+6}^Y)$		Ratio Equation $D\left(\frac{fb p_{t+6}^Y}{y_{t+6}}\right)$	
	Coefficient	p -value	Coefficient	p -value
$(r_t^L - r_t^S)$ (γ_1)	0.21	0.011	0.177	0.037
Mc Fadden R^2	0.032		0.022	

itability, estimating for the period of 1970Q1-2007Q2²⁶

$$P(D(fb p_{t+6}^Y) = 1) = \Phi(\gamma_0 + \gamma_1 (r_t^L - r_t^S)),$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function. Wright (2006) performs a similar estimation linking term spreads with a dummy variable that takes the value of 1 when a recession occurs, concluding that low spreads are linked with a high probability of observing a recession. If γ_1 is significant then spreads are relevant in predicting the future (negative) movements in profitability. The left part of Table 3 shows the results. The estimated coefficient for the term spread is significant and positive. This suggests that high term spreads are associated with a higher probability of observing decreasing financial business profitability in the future. We also run an augmented probit model controlling for past 3 month T-Bill, inflation rates and the growth rate of output. Results remain qualitatively the same.

In order to investigate whether this result is due to the cyclical fluctuations of output we alter the binary variable to consider the ratio of profits to the GDP. Hence, the modified binary dummy variable is given by

$$D\left(\frac{fb p_t^Y}{y_t}\right) = \begin{cases} 1 & \text{if } \Delta \frac{fb p_t^Y}{y_t} < \Delta \frac{fb p_{t-1}^Y}{y_{t-1}} \\ 0 & \text{otherwise} \end{cases}$$

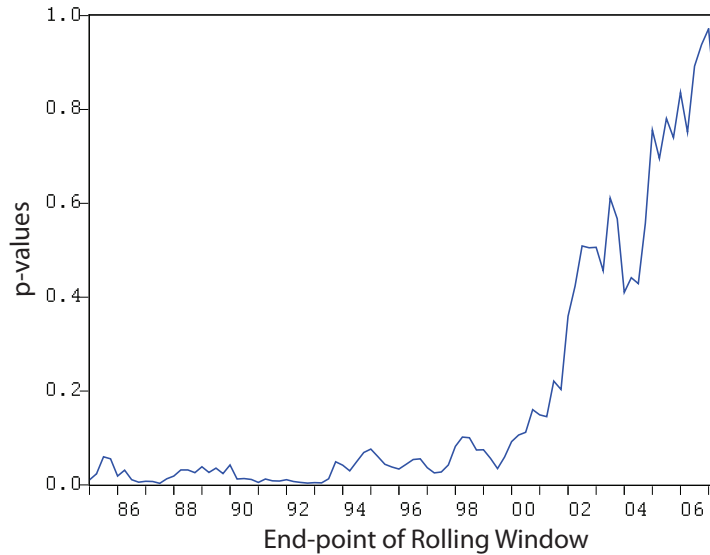
The right part of the Table 3 shows the results for the ratio probit estimation for the period of 1970Q1-2007Q2. Once again high term spreads are associated with periods where banks expect decreasing ratios of profits over GDP. Thus, the result of the level probit estimation does not seem to be driven by the cyclical link between financial business profits and the GDP.

We also provide 15 years rolling window estimates starting from 1970Q1-1984Q4 and ending in 1993Q3- 2007Q2. Figure 8 shows that the significance, as reflected in the recursive p-values for γ_1 , sharply declines when data from 2001 is included in the dataset. We conjecture that the early 2000's are associated with highly volatile stock markets, potentially affecting the profitability of financial businesses, independently of spread movements. The evidence presented here indicates that expected profitability may influence term spreads, providing support to the channel explained

²⁶We estimate $D(fb p_{t+6}^Y)$ on term spreads determined at time t , which reflect the slope of yield curve from quarter $t + 1$ to quarter $t + 40$ (3 month to 10 year points), since that binary variable depends on financial business profits from $t + 1$ until $t + 6$.

in our model. In the next subsection, we provide micro-econometric evidence related to profitability and term spreads at the financial institution level.

Figure 8: Stability of Benchmark Probit Estimation



7.3. Spreads and Expected Financial Business Profitability: Bank Level Evidence

Throughout the paper, we argued that one of the mechanisms that affects the financial sector’s risk of maturity transformation is due to changes in the expected profitability. In other words, during a boom, the financial sector expects an increase in future profits and therefore spreads should be low. Likewise during a recession the financial sector expects a decrease in future profits, and therefore spreads should be high.

In the previous subsection, we use a probit model to overcome the lack of data on expected profits, providing an evidence of the link between spreads today and future movements in profitability. In this subsection, we use forecasts of earning per share of the main financial business in the U.S. as proxy for their expected profits. Relying on this micro level data we obtain further corroborating evidence of the link between expected profitability and spreads.

We collect daily financial business data on forecast and actual earnings per share as reported by Thomson-Reuters, I/B/E/S, of financial companies in the S&P 500. We convert the resulting dataset into quarterly frequency²⁷ for the period of 1990Q1 and 2007Q2 for a set of 22 large US financial institutions for which we have longer time series data available. They are: JP-Morgan Chase & Co (JPM), Chubb Corp (CHUBB), Lincoln National Corp (LINCOLN), Marsh and McLennan Cos (MARSH), PNC Financial Services Group Inc (PNC), Suntrust Banks Inc (SUN), Torchmark Corp (TORCH), Loews Corp (LOEWS), Morgan Stanley (MS), Comerica Inc

²⁷Given that there are more than one forecast done in each quarter we average them producing a mean forecast for the quarter.

Table 4: Pooled (EGLS) Estimation Results - Spreads and Future Expected Profits

	OLS		Pooled EGLS	
	Coefficient	<i>p</i> - value	Coefficient	<i>p</i> - value
$(r_{t-1}^L - r_{t-1}^S)$ (α_{TS})	.95	.00	.95	.00
Δy_{t-1} (α_y)	-.08	.00	-.08	.00
$\Delta \pi_{t-1}$ (α_π)	-.09	.00	-.11	.00
$(\Delta E_{t-1} \Pi_{i,t})$ (α_Π)			-.12	.03
n.o: 1124; $R^2 = 0.89$; Adjusted Sample: 1990Q3 2007Q2				

(COMER), Fifth Third Bancorp (5TH), Progressive Corp-Ohio (PROG), Huntington Bancshares (HUNTING), Northern Trust Corp (NORTH), Franklin Resources Inc (FRANK), Equity Residential (EQRESID), Goldman Sachs (GOLDMAN), Prudential Financial Inc (PRUD), Apartment Inv and Mgmt (APINV), Federated Investors Inc (FED), Ameriprise Financial Inc (AMFIN) and Cincinnati Financial Corp (CINNFIN). We define the changes in the expected (mean forecast in earnings per share) profitability as

$$\Delta E_{t-1} \Pi_{i,t} = \frac{[E_{t-1}(EPS)_{i,t} - EPS_{i,t-1}]}{[E_{t-2}(EPS)_{i,t-1} - EPS_{i,t-2}]}$$

We estimate the following unbalanced fixed (cross section) effects panel data specification given by

$$(r_t^L - r_t^S) = \alpha_i + \alpha_{TS} (r_{t-1}^L - r_{t-1}^S) + \alpha_y (\Delta y_{t-1}) + \alpha_\pi (\pi_{t-1}) + \alpha_\Pi (\Delta E_{t-1} \Pi_{i,t}) + \varepsilon_{i,t}$$

While we report estimation results based on the model with cross-section weights/panel corrected standard errors and covariance, we also estimate the model with specifications that account for various patterns of correlation between the residuals (Robust Coefficient Covariances). Our results are not affected by the specification of basic variance structures. We report two sets of estimations in Table 4: first an OLS estimation without controlling for the changes in the expected banking profitability and second panel data estimation that controls for the expected banking profitability.

First, in line with the predictions of our general equilibrium model, we observe that expected increases in the bank profitability leads to a statistically significant decline in term spreads. Furthermore, the coefficient for $(\Delta E_{t-1} \Pi_{i,t})$ is economically significant. A marginal increase in the growth rate of earnings per share leads to a decline in term spreads by about 12 basis points. Although we do not report cross section fixed effects here, we observe a large degree of heterogeneity across financial institutions; fixed effects range from -.17 to .02. Second, our results are robust to changes in the sample period that includes the period of financial crisis (up to 2011Q2) and also when we control for past changes in the T-Bill rate²⁸. When we extend the sample to capture the

²⁸We estimate the model controlling to T-Bill rate changes two periods before $(r_{t-2}^S - r_{t-3}^S)$ to avoid a multicollinearity between T-Bill changes, spreads and actual returns.

financial crisis period, we find that the coefficient of growth in expected profits ($\hat{\alpha}_\Pi$) decreases to $-.06$ but is highly significant (p -value : $.002$). When we control for past variations in the T-Bill rates we find that $\hat{\alpha}_\Pi = -.09$ with a p -value of $.053$.

8. Conclusions

Term spread fluctuations have relevant implications for macroeconomic outcomes and may predict output growth. Undoubtedly, inflation expectations or more generally long-term inflation risks are important determinants of these fluctuations. However, the observation that nominal and real yield curves move together in many instances suggests that other factors are in play. We propose a model that delivers endogenous variations in term spreads driven primarily by changes in banks' portfolio decision and their appetite to bear the risk of maturity transformation. We show that fluctuations of banks' portfolio future profitability affect their ability to cover for any liquidity shortage and hence influence the premium they require to carry maturity risk. Another feature that distinguishes our framework from standard macro-affine DSGE models is that fluctuations in term spread impact economic activity since they alter investment financing costs.

While we present a model in which bank portfolios are fairly simple, we are able to match important features of the data. Our model suggests that factors external to monetary policy may contribute not only to the marginal predictive power of spreads but also to the understanding of the linkages between banks, spread movements and the macroeconomy.

Embedding this simple banking sector framework into a DSGE model allows us to analyze the interaction between these spread movements and conventional and unconventional policies. Spread movements effectively imply tighter or looser monetary conditions forcing the central bank to adjust short-term rates accordingly. Once again, spreads between different interest rates in the economy are shown to be crucial and should be explicitly included in models that analyze optimal policies. Unconventional policies are shown to have a strong impact on spread movements fomenting long-term investment and helping reduce output losses after negative liquidity shocks, matching the general view on the effects of recent asset purchases programmes. Finally, we show that asset purchases programmes that keep the assets in the central bank balance sheet for longer are more effective in offsetting a liquidity shock. These policy measures allow the central bank to restore short-term rates to steady state levels more quickly. This result supports the type of intervention decision such as the ECB to hold assets until maturity that are purchased under the CBPP programme.

We then provide both macro-econometric and micro-econometric empirical support for our theoretical claims. Employing macroeconomic data, we first document that historical information regarding *actual* financial business profitability contains significant additional marginal predictive power to explain variations in real output and term spreads next to a range of control variables. Second, we present evidence that high spreads are associated with future or *expected* decreases in aggregate bank profitability. Using micro level data, we show that *expected* increases in bank level profitability are associated with lower term spreads in a significant fashion, both statistically and economically.

The present work highlights three areas in which further research, exploring the role of bank's portfolio decisions, may be fruitful. First, increasing the complexity of banks' portfolios will pro-

vide a better understanding of this important channel, most notably, including (workers) housing investment funded by financial intermediation. That would mean term spread fluctuations would not only influence investment but also consumption, potentially amplifying the effects of spread movements, since as we observe, consumption and investment move in opposite directions compensating each other. Moreover, including other long-term asset classes may potentially allow us to study portfolio balancing effects after QE interventions. Second, making liquidity shortages endogenous based on the potential for securitization of long-term assets may be crucial to fully understand those factors behind spread movements and their marginal predictive power. Finally, final investors (after securitization) and bank sentiment or risk assessment could also be time varying affecting the linkage between long-term funding risks and economic activity.

References

- ACHARYA, V. V., D. GALE, AND T. YORULMAZER (2010): “Rollover Risk and Market Freezes,” NBER Working Papers 15674, National Bureau of Economic Research, Inc.
- ADRIAN, T., A. ESTRELLA, AND H. S. SHIN (2010): “Monetary Cycles, Financial Cycles, and the Business Cycle,” Staff Reports 421, Federal Reserve Bank of New York.
- ADRIAN, T., AND H. S. SHIN (2009): “Financial Intermediaries and Monetary Economics,” Staff Reports 398, Federal Reserve Bank of New York.
- (2010): “Liquidity and Leverage,” *Journal of Financial Intermediation*, 19(3), 418–437.
- ADRIAN, T., H. S. SHIN, AND E. MOENCH (2010): “Financial Intermediation, Asset Prices, and Macroeconomic Dynamics,” Staff Reports 422, Federal Reserve Bank of New York.
- AKSOY, Y., H. S. BASSO, AND J. COTO-MARTINEZ (2009): “Liquidity Effects and Cost Channels in Monetary Transmission,” Birkbeck Working Papers in Economics and Finance 0902, Birkbeck, School of Economics, Mathematics & Statistics.
- (2010): “Lending Relationships and Monetary Policy,” Working Paper Series 2009:18, Uppsala University, Department of Economics.
- BEIRNE, J., L. DALITZ, J. EJSING, M. GROTHE, S. MANGANELLI, F. MONAR, B. SAHEL, M. SUEC, J. TAPKING, AND T. VONG (2011): “The Impact of the Eurosystem’s Covered Bond Purchase Programme on the Primary and Secondary Markets,” Occasional Paper Series 122, European Central Bank.
- BENATI, L., AND C. GOODHART (2008): “Investigating Time-variation in the Marginal Predictive Power of the Yield Spread,” *Journal of Economic Dynamics and Control*, 32(4), 1236–1272.
- BORIO, C., AND P. DISYATAT (2010): “Unconventional Monetary Policies: An Appraisal,” *Manchester School*, 78(s1), 53–89.
- BRUNNERMEIER, M. K. (2009): “Deciphering the Liquidity and Credit Crunch 2007-2008,” *Journal of Economic Perspectives*, 23(1), 77–100.

- CARLSTROM, C. T., AND T. FUERST (2007): "Asset Prices, Nominal Rigidities, and Monetary Policy," *Review of Economic Dynamics*, 10(2), 256–275.
- DE GRAEVE, F., M. EMIRIS, AND R. WOUTERS (2009): "A Structural Decomposition of the US Yield Curve," *Journal of Monetary Economics*, 56(4), 545–559.
- DEWACHTER, H., L. IANIA, AND M. LYRIO (2011): "Information in the Yield Curve: A Macro-Finance Approach," Ibmec Working Papers 230/2011, Ibmec Working Paper, Ibmec São Paulo.
- DIAMOND, D. W. (1989): "Reputation Acquisition in Debt Markets," *Journal of Political Economy*, 97(4), 828–62.
- (1991): "Debt Maturity Structure and Liquidity Risk," *The Quarterly Journal of Economics*, 106(3), 709–37.
- DIAMOND, D. W. (2004): "Presidential Address, Committing to Commit: Short-term Debt When Enforcement Is Costly," *Journal of Finance*, 59(4), 1447–1479.
- DIAMOND, D. W., AND R. G. RAJAN (2001): "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109(2), 287–327.
- (2005): "Liquidity Shortages and Banking Crises," *Journal of Finance*, 60(2), 615–647.
- ESTRELLA, A., AND G. A. HARDOUVELIS (1991): "The Term Structure as a Predictor of Real Economic Activity," *Journal of Finance*, 46(2), 555–76.
- FAN, J. P., S. TITMAN, AND G. TWITE (2010): "An International Comparison of Capital Structure and Debt Maturity Choices," NBER Working Papers 16445, National Bureau of Economic Research, Inc.
- FRIEDMAN, B. M., AND K. N. KUTTNER (1992): "Money, Income, Prices, and Interest Rates," *American Economic Review*, 82(3), 472–92.
- GAGNON, J., M. RASKIN, J. REMACHE, AND B. SACK (2010): "Large-scale Asset Purchases by the Federal Reserve: Did They Work?," Staff Reports 441, Federal Reserve Bank of New York.
- GERTLER, M., AND N. KIYOTAKI (2009): "Financial Intermediation and Credit Policy in Business Cycle Analysis," Mimeo, NYU.
- GÜRKAYNAK, R. S., A. LEVIN, AND E. SWANSON (2010): "Does Inflation Targeting Anchor Long-Run Inflation Expectations? Evidence from the U.S., UK, and Sweden," *Journal of the European Economic Association*, 8(6), 1208–1242.
- GÜRKAYNAK, R. S., AND J. WRIGHT (2010): "Macroeconomics and the Term Structure," CEPR Discussion Papers 8018, C.E.P.R. Discussion Papers.
- HAMILTON, J. D., AND D. H. KIM (2002): "A Reexamination of the Predictability of Economic Activity Using the Yield Spread," *Journal of Money, Credit and Banking*, 34(2), 340–60.

- HOLMSTROM, B., AND J. TIROLE (1998): “Private and Public Supply of Liquidity,” *Journal of Political Economy*, 106(1), 1–40.
- JOYCE, M., A. LASAOSA, I. STEVENS, AND M. TONG (2010): “The Financial Market Impact of Quantitative Easing,” Bank of England working papers 393, Bank of England.
- KIM, D. H., AND J. H. WRIGHT (2005): “An Arbitrage-free Three-factor Term Structure Model and the Recent Behavior of Long-term Yields and Distant-horizon Forward Rates,” Finance and Economics Discussion Series 2005-33, Board of Governors of the Federal Reserve System (U.S.).
- KOZICKI, S., AND P. TINSLEY (2005): “What Do You Expect? Imperfect Policy Credibility and Tests of the Expectations Hypothesis,” *Journal of Monetary Economics*, 52(2), 421–447.
- KRISHNAMURTHY, A., AND A. VISSING-JORGENSEN (2011): “The Effects of Quantitative Easing on Interest Rates,” Unpublished manuscript, Kellogg School of Management, Northwestern University.
- PIAZZESI, M., AND M. SCHNEIDER (2007): “Equilibrium Yield Curves,” in *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pp. 389–472. National Bureau of Economic Research, Inc.
- PIAZZESI, M., AND E. T. SWANSON (2008): “Futures Prices as Risk-adjusted Forecasts of Monetary Policy,” *Journal of Monetary Economics*, 55(4), 677–691.
- RUDEBUSCH, G. D., B. P. SACK, AND E. T. SWANSON (2007): “Macroeconomic Implications of Changes in the Term Premium,” *Federal Reserve Bank of St. Louis Review*, pp. 241–270.
- RUDEBUSCH, G. D., AND E. T. SWANSON (2008a): “The Bond Premium in a DSGE Model with Long-run Real and Nominal Risks,” Research series 200810-18, National Bank of Belgium.
- (2008b): “Examining the Bond Premium Puzzle with a DSGE Model,” *Journal of Monetary Economics*, 55(Supplement 1), S111–S126.
- RUDEBUSCH, G. D., AND J. C. WILLIAMS (2009): “Forecasting Recessions: The Puzzle of the Enduring Power of the Yield Curve,” *Journal of Business & Economic Statistics*, 27(4), 492–503.
- SHIN, H. (2009): “Securitisation and Financial Stability,” *Economic Journal*, 119(536), 309–332.
- SIMS, C. A. (1972): “Money, Income, and Causality,” *American Economic Review*, 62(4), 540–52.
- (1980): “Comparison of Interwar and Postwar Business Cycles: Monetarism Reconsidered,” *American Economic Review*, 70(2), 250–57.
- SVEEN, T., AND L. WEINKE (2007): “Firm-specific Capital, Nominal Rigidities, and the Taylor Principle,” *Journal of Economic Theory*, 136(1), 729–737.

VAYANOS, D., AND J.-L. VILA (2009): "A Preferred-Habitat Model of the Term Structure of Interest Rates," NBER Working Papers 15487, National Bureau of Economic Research, Inc.

WRIGHT, J. H. (2006): "The Yield Curve and Predicting Recessions," Finance and Economics Discussion Series 2006-07, Board of Governors of the Federal Reserve System (U.S.).

A Simple Model - Case 2

Given the assumption for the two stochastic processes (R_Z, ρ) and setting $\sigma_B = 0$, total profits Π^B are also normally distributed with mean $\mu_\Pi = (\bar{R}_Z - R_D)Z + (\beta(R_L - R_D) - R_D - \bar{\rho})X_L$ and variance $\sigma_\Pi^2 = Z^2\sigma_Z^2 + \beta^2X_L^2\sigma_\rho^2 + 2co_{Z,\rho}X_LZ\sigma_Z\sigma_\rho$.

Using an approximation for the first percentile of the profit probability density function we then obtain

$$0.01 = \frac{1}{2} \left[1 + erf \left(\frac{VaR - \mu_\Pi}{\sigma_\Pi \sqrt{2}} \right) \right] \quad (18)$$

where erf is the error function. We can differentiate the equation above with respect to X_L and Z to obtain

$$\begin{aligned} \frac{\partial VaR}{\partial X_L} &= (\beta(R_L - R_D) - R_D - \bar{\rho})\sigma_\Pi\sqrt{2} \\ &\quad + [VaR - \mu_\Pi](\sigma_\Pi)^{-1}\sqrt{2}(\beta^2X_L\sigma_\rho^2 + 2co_{Z,\rho}Z\sigma_Z\sigma_\rho) \\ \frac{\partial VaR}{\partial Z} &= (\bar{R}_Z - R_D)\sigma_\Pi\sqrt{2} \\ &\quad + [VaR - \mu_\Pi](\sigma_\Pi)^{-1}\sqrt{2}(Z\sigma_Z^2 + 2co_{Z,\rho}X_L\sigma_Z\sigma_\rho) \end{aligned}$$

We then substitute these conditions into (2) - (6) and (18) to determine the equilibrium. Our main interest is to verify how term spreads (measure in basis points), defined as

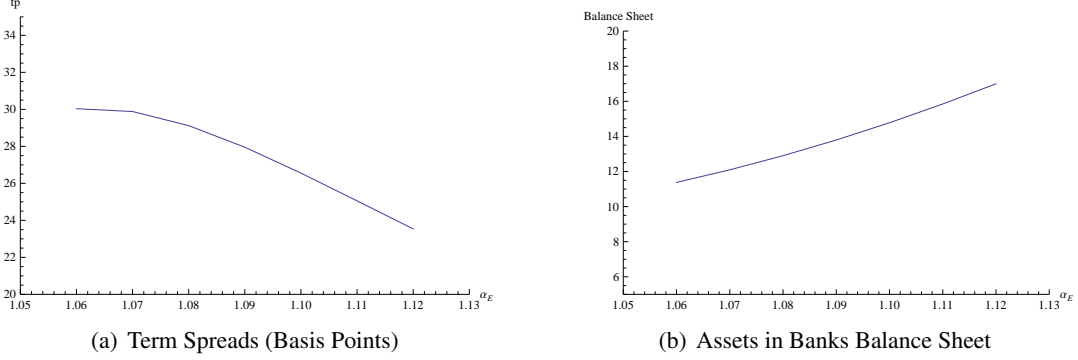
$$tp = \frac{1}{2} ((R_L - 1) - (R_D - 1) - (R_D - 1)),$$

move as the overall profitability (expected returns) of the bank's portfolio varies. In doing so we aim at establishing a link between banks' appetite to accumulate long-term assets in the balance sheet, incurring the risk of maturity transformation, the equilibrium long-term rates and the expected performance of bank investments. As such we look at the equilibrium level of term premium as α_Z (which controls expected returns on equity) varies. Figure 8 shows the results for the following parameter values $R_D = 1.01$ (base rate equal to a 4% annual), $\bar{\rho} = 0.005$ (annual spread of roughly 100 basis points), $\sigma_Z = 0.03$, $\sigma_\rho = 0.01$, $\gamma_L = 6$, $\gamma_Z = 0.0006$, $\Lambda = -0.3$ (the VaR limit implies a loss of roughly 2.5 standard deviations). The qualitative implications are unchanged when these are altered. Finally, the important parameter to determine the results is the correlation between the asset returns. We set it to -0.1 (allowing for gains of diversification). The impact on term spreads reverse when this correlation is positive, since in this instance the substitution effect will be greater than the income effect (see the discussion in the text). Note that the increase in banks' balance sheet occur since both Z and X_L holding increase, although equity holding increase more sharply.

B Data

This provides a description of the data used in the empirical study.

Figure 9: Assets and Term spreads as expected bank profits increase



- Treasury Bill Rate (Units: Percent per Annum), (Series ID: 60C..ZF) Source: International Financial Statistics/IMF
- Government Bond Yield: 10 year (Units: Percent per Annum), (Series ID: 61...ZF) Source: International Financial Statistics/IMF
- CPI All Items City Average (Units: Index Number), (Series ID: 64...ZF), Source: International Financial Statistics/IMF
- Real Gross Domestic Product, Seasonally Adjusted Annual Rate , (Series ID: GDPC96) Source: U.S. Department of Commerce: Bureau of Economic Analysis
- Financial Business; undistributed corporate profits excluding CCAdj, (FOF Code: FA796006403.Q), Source: Flow of Funds Accounts, Board of Governors of the Federal Reserve

C Equilibrium Conditions and Steady State

The household maximization routines yield the following equilibrium conditions

$$\beta E_t \left(\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = \frac{C_t^{-\sigma}}{R_{t,CB}} \quad (19)$$

and

$$w_{j,t} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_w \beta)^s \frac{\chi H_{t+s}^\eta}{C_{t+s}^{-\sigma}} H_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_w \beta)^s H_{t+s} \left(\prod_{k=1}^s \pi_{t+k} \right)^{-1} \right\}}. \quad (20)$$

This equation can be conveniently expressed in recursive form as such

$$\begin{aligned} 0 &= f_{1,t}^w \frac{\varepsilon_w}{\varepsilon_w - 1} - f_{2,t}^w w_{j,t}, \\ f_{1,t}^w &= H_t \frac{\chi H_t^\eta}{C_t^{-\sigma}} + E_t \left[\beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} f_{1,t+1}^w \right], \\ f_{2,t}^w &= H_t + E_t \left[\beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{-1} f_{2,t+1}^w \right], \end{aligned}$$

where, $w_{j,t} = W_{j,t}/P_t$ and $w_t = W_t/P_t$, is given by

$$w_t^{1-\varepsilon_w} = (1 - \omega_w) w_{j,t}^{1-\varepsilon_w} + \omega_w w_{t-1}^{\varepsilon_w - 1}. \quad (21)$$

We assume firms discount future payoffs using the household's stochastic discount factor given by

$$Q_{t,t+1} = \beta E_t \left(\frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \right) = \frac{1}{R_{CB,t}}.$$

Given that the purpose of our analysis is not to look at the effects of firm-specific capital we assume that there exists a capital market within firms. That way all firms will have the same labour-capital ratio and $\Lambda_{t,i} = \Lambda_t$ for all i , as in the case where a capital rental market is available. The net aggregate investment in (new) capital is then acquired from entrepreneurs. Note that, as shown by Sveen and Weinke (2007), the relevant difference of considering firm-specific capital is that the parameter on the marginal cost in the Phillips curve would be lower, increasing effective price stickiness. Our results are not qualitatively affected by this change.

Based on that, $p_{i,t}$ is determined by solving the price setting maximization, substituting for the stochastic discount factor and using $\Lambda_{t+s,i} = \Lambda_{t+s}$. That gives

$$p_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega \beta)^s \Lambda_{t+s} Y_{t+s} (\prod_{k=1}^s \pi_{t+k})^\varepsilon \right\}}{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega \beta)^s Y_{t+s} (\prod_{k=1}^s \pi_{t+k})^{\varepsilon-1} \right\}}. \quad (22)$$

The recursive formulation is given by

$$\begin{aligned} 0 &= f_{1,t} \frac{\varepsilon}{\varepsilon - 1} - f_{2,t} p_{i,t}, \\ f_{1,t} &= Y_t \Lambda_t + E_t \left[\beta \omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^\varepsilon f_{1,t+1} \right], \\ f_{2,t} &= Y_t + E_t \left[\beta \omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{\varepsilon-1} f_{2,t+1} \right], \end{aligned}$$

where, $p_{i,t} = P_{i,t}/P_t$ and $\pi_t = P_t/P_{t-1}$, is given by

$$1 = (1 - \omega) p_{i,t}^{1-\varepsilon} + \omega \pi_t^{\varepsilon-1}. \quad (23)$$

From the firm cost minimization problem we obtain the demand for capital and labour. After rearranging the first order conditions and substituting for the stochastic discount factor $Q_{t,t+1}$, we obtain the following equilibrium conditions²⁹

$$Y_t = A_t K_t^{\zeta\alpha} K_t^{L(1-\zeta)\alpha} H_t^{1-\alpha}, \quad (24)$$

$$\Lambda_t = \frac{w_t H_t}{Y_t(1-\alpha)}, \quad (25)$$

$$q_t^S = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \left[\Lambda_{t+1} \frac{\alpha \zeta Y_{t+1}}{K_{t+1}^S} + (1-\delta) q_{t+1}^S \right] \right\}, \text{ and} \quad (26)$$

$$q_t^L = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \left[\Lambda_{t+1} \frac{\alpha(1-\zeta) Y_{t+1}}{K_{t+1}^L} + (1-\delta) q_{t+1}^L \right] \right\}. \quad (27)$$

Using the capital aggregation conditions, investment evolves according to

$$I_t^S = K_{t+1}^S - (1-\delta) K_t^S, \text{ and} \quad (28)$$

$$I_t^L = K_{t+1}^L - (1-\delta) K_t^L. \quad (29)$$

From entrepreneurs maximization problems we obtain

$$X_{S,t} = \frac{\gamma_S E_t [q_{t+1}^S \pi_{t+1}]}{R_{S,t}} - 1, \text{ and} \quad (30)$$

$$X_{L,t} = \frac{\gamma_L E_t [q_{t+2}^L \pi_{t+1} \pi_{t+2}]}{R_{L,t}} - 1. \quad (31)$$

Finally, from the bank maximization problem we have that

$$R_{S,t} = R_{t,CB}, \text{ and} \quad (32)$$

$$E_t \left[\Pi_{t+1}^{B}^{-\sigma_B} \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] = \beta E_t \left[\frac{1}{\pi_{t+1} \pi_{t+2}} \Pi_{t+2}^{B}^{-\sigma_B} (R_{L,t} - R_{t+1,CB}) \right], \quad (33)$$

where

$$\Pi_t^B = \frac{DIV_t}{P_t} + \frac{(R_{t-2,L} - 1)}{\pi_t \pi_{t-1}} X_{L,t-2} + (R_{t-1,CB} - 1) \left(X_{S,t-1} - \frac{D_{t-1}}{P_{t-1}} \right) \frac{1}{\pi_t} - \rho_t X_{L,t-1}, \quad (34)$$

$$\begin{aligned} \frac{DIV_t}{P_t} \pi_t &= Y_{t-1} - w_{t-1} H_{t-1} - q_{t-1}^S I_{t-1}^S - q_{t-1}^L I_{t-1}^L \\ &\quad + q_{t-1}^S K_t^S + q_{t-1}^L K_t^L - \frac{q_{t-2}^S K_{t-1}^S + q_{t-2}^L K_{t-1}^L}{\pi_{t-1}}, \text{ and} \end{aligned} \quad (35)$$

$$\frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)}{\pi_t} + \frac{X_{L,t-1}}{\pi_t}. \quad (36)$$

²⁹Once again we have used the fact that marginal costs are the same across firms.

We define the term spread (annual rate in percentage points) between long and short-term rate as

$${}^t p_t = \frac{1}{2} ((R_{L,t} - 1) - (R_{t,CB} - 1) - (R_{t+1,CB} - 1)) 400. \quad (37)$$

Finally, the central bank sets monetary policy according to

$$\frac{R_{t,CB}}{\bar{R}_{CB}} = \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\varepsilon_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\varepsilon_Y} \right]. \quad (38)$$

where \bar{X} is the steady state value of X_t . This is the standard monetary rule whereby the base rate responds to deviations in inflation and output. The recursive equilibrium is determined as the solution to equations (11)-(13) and (19) - (38).

Steady State From pricing equation (normalizing prices at steady state to 1) we have that

$$\Lambda = \frac{\varepsilon - 1}{\varepsilon}.$$

From wage pricing equation we have that

$$\bar{w} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi \bar{H}^\eta}{\bar{C}^{1-\sigma}}.$$

From the firm problem we have that

$$\Lambda = \frac{\bar{w} \bar{H}}{\bar{Y}(1-\alpha)}$$

$$\bar{q}_s = \frac{\beta \Lambda \alpha \zeta \bar{Y}}{\bar{K}^S (1 - \beta(1 - \delta))}$$

$$\bar{q}_L = \frac{\beta \Lambda \alpha (1 - \zeta) \bar{Y}}{\bar{K}^L (1 - \beta(1 - \delta))}.$$

From entrepreneurs problems we have that

$$\bar{X}_S = \gamma_S \beta \bar{q}_S - 1$$

$$\bar{X}_L = \frac{\gamma_L \bar{q}_L}{R_L} - 1.$$

From the bank problem we have that

$$\left(\frac{1}{\beta} - 1 \right) + \rho = \beta \left(R_L - \frac{1}{\beta} \right) \text{ or } R_L = \frac{1}{\beta^2} + \frac{\rho}{\beta}.$$

The term spread at the steady state is given by

$$\bar{t} p = \frac{1}{2} ((R_L - 1) - (1/\beta - 1) - (1/\beta - 1)) 400.$$

Clearing conditions and investment flow equation determine that

$$\bar{Y} = \bar{C} + \bar{X}_S + (1 + \rho)\bar{X}_L$$

$$\bar{Y} = \bar{H}^{(1-\alpha)} \bar{K}_S^{\alpha\zeta} \bar{K}_L^{\alpha(1-\zeta)}$$

$$\delta\bar{K}_S = \gamma_S \ln(1 + \bar{X}_S)$$

$$\delta\bar{K}_L = \gamma_L \ln(1 + \bar{X}_L).$$

Appendix D Second Order Approximation of Long-term Rate Decision

Bank equilibrium condition is given by

$$E_t \frac{1}{\pi_{t+1}} \left[(\Pi_{t+1}^B)^{-\sigma_B} ((R_{CB,t} - 1) + \rho_{t+1}) \right] = \beta E_t \frac{1}{\pi_{t+1}\pi_{t+2}} \left[(\Pi_{t+2}^B)^{-\sigma_B} (R_{L,t} - R_{CB,t+1}) \right],$$

and at the steady state

$$\left(\frac{1}{\beta} + \bar{\rho} \right) \frac{1}{\beta} = R_L \text{ and } R_{CB} = \frac{1}{\beta}.$$

Let $W_1 = (R_{CB,t} - 1 + \rho_{t+1})$ and $W_2 = (R_{L,t} - R_{CB,t+1})$,

Approximation of Left-Hand Side (LHS)

$$\begin{aligned} & E_t \frac{1}{\pi_{t+1}} \left[(\Pi_{t+1}^B)^{-\sigma_B} ((R_{CB,t} - 1) + \rho_{t+1}) \right] = \\ & E_t \frac{1}{\pi_{t+1}} \left[(\Pi_{t+1}^B)^{-\sigma_B} W_1 \right] \approx \\ & E_t \left[-\sigma_B \widehat{\Pi_{t+1}^B} + 0.5\sigma_B^2 (\widehat{\Pi_{t+1}^B})^2 + \widehat{W}_1 + 0.5(\widehat{W}_1)^2 - \widehat{\pi_{t+1}} + 0.5(\widehat{\pi_{t+1}})^2 - \sigma_B \widehat{\Pi_{t+1}^B} \widehat{W}_1 + \sigma_B \widehat{\Pi_{t+1}^B} \widehat{\pi_{t+1}} - \widehat{W}_1 \widehat{\pi_{t+1}} \right]. \end{aligned}$$

From the definition of W_1

$$\begin{aligned} \widehat{W}_1 &= \frac{1}{\beta\Gamma} \widehat{R_{CB,t}} + \frac{\rho}{\Gamma} \widehat{\rho_{t+1}}, \text{ where } \Gamma = \left(\frac{1}{\beta} - 1 + \bar{\rho} \right), \text{ and} \\ \widehat{W}_1 + 0.5(\widehat{W}_1)^2 &= \frac{1}{\beta\Gamma} \left(\widehat{R_{CB,t}} + 0.5(\widehat{R_{CB,t}})^2 \right) + \frac{\bar{\rho}}{\Gamma} \left(\widehat{\rho_{t+1}} + 0.5(\widehat{\rho_{t+1}})^2 \right). \end{aligned}$$

Hence, LHS becomes

$$E_t \left[\begin{aligned} & -\sigma_B \widehat{\Pi_{t+1}^B} + 0.5\sigma_B^2 (\widehat{\Pi_{t+1}^B})^2 + \frac{1}{\beta\Gamma} \left(\widehat{R_{CB,t}} + 0.5(\widehat{R_{CB,t}})^2 \right) + \frac{\bar{\rho}}{\Gamma} \left(\widehat{\rho_{t+1}} + 0.5(\widehat{\rho_{t+1}})^2 \right) - \widehat{\pi_{t+1}} + 0.5(\widehat{\pi_{t+1}})^2 - \\ & -\sigma_B \widehat{\Pi_{t+1}^B} \left(\frac{1}{\beta\Gamma} \widehat{R_{CB,t}} + \frac{\bar{\rho}}{\Gamma} \widehat{\rho_{t+1}} \right) + \sigma_B \widehat{\Pi_{t+1}^B} \widehat{\pi_{t+1}} - \left(\frac{1}{\beta\Gamma} \widehat{R_{CB,t}} + \frac{\bar{\rho}}{\Gamma} \widehat{\rho_{t+1}} \right) \widehat{\pi_{t+1}}. \end{aligned} \right]$$

Approximation of Right-Hand Side (RHS)

$$\begin{aligned} & \beta E_t \frac{1}{\pi_{t+1}\pi_{t+2}} \left[(\Pi_{t+2}^B)^{-\sigma_B} (R_{L,t} - R_{CB,t+1}) \right] = \\ & \beta E_t \frac{1}{\pi_{t+1}\pi_{t+2}} \left[(\Pi_{t+2}^B)^{-\sigma_B} W_2 \right] \approx \\ & E_t \left[-\sigma_B \widehat{\Pi}_{t+2}^B + 0.5\sigma^b (\widehat{\Pi}_{t+2}^B)^2 + \widehat{W}_2 + 0.5(\widehat{W}_2)^2 - \widehat{\pi}_{t+1} + 0.5(\widehat{\pi}_{t+1})^2 - \widehat{\pi}_{t+2} + 0.5(\widehat{\pi}_{t+2})^2 \right. \\ & \left. - \sigma_B \widehat{\Pi}_{t+2}^B \widehat{W}_2 + \sigma^b \widehat{\Pi}_{t+2}^B \widehat{\pi}_{t+1} + \sigma_B \widehat{\Pi}_{t+2}^B \widehat{\pi}_{t+2} - \widehat{W}_2 \widehat{\pi}_{t+1} - \widehat{W}_2 \widehat{\pi}_{t+2} + \widehat{\pi}_{t+2} \widehat{\pi}_{t+1} \right]. \end{aligned}$$

From the definition of W_2

$$\begin{aligned} \widehat{W}_2 &= \left(\frac{\frac{1}{\beta} + \rho}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} \right) \text{ where } \Gamma = \left(\frac{1}{\beta} - 1 + \bar{\rho} \right) \text{ and} \\ \widehat{W}_2 + 0.5(\widehat{W}_2)^2 &= \frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \left(\widehat{R}_{L,t} + 0.5(\widehat{R}_{L,t})^2 \right) - \frac{1}{\beta\Gamma} \left(\widehat{R}_{CB,t+1} + 0.5(\widehat{R}_{CB,t+1})^2 \right) \end{aligned}$$

Hence, RHS becomes

$$E_t \left[\begin{aligned} & -\sigma_B \widehat{\Pi}_{t+2}^B + 0.5\sigma_B (\widehat{\Pi}_{t+2}^B)^2 + \frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \left(\widehat{R}_{L,t} + 0.5(\widehat{R}_{L,t})^2 \right) - \frac{1}{\beta\Gamma} \left(\widehat{R}_{CB,t+1} + 0.5(\widehat{R}_{CB,t+1})^2 \right) - \widehat{\pi}_{t+1} + 0.5(\widehat{\pi}_{t+1})^2 - \widehat{\pi}_{t+2} + 0.5(\widehat{\pi}_{t+2})^2 - \\ & -\sigma_B \widehat{\Pi}_{t+2}^B \left(\frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} \right) + \sigma_B \widehat{\Pi}_{t+2}^B \widehat{\pi}_{t+1} + \sigma_B \widehat{\Pi}_{t+2}^B \widehat{\pi}_{t+2} - \left(\frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} \right) \widehat{\pi}_{t+1} - \left(\frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} \right) \widehat{\pi}_{t+2} + \widehat{\pi}_{t+2} \widehat{\pi}_{t+1}. \end{aligned} \right]$$

From the definition of term premium we have that $t p = 0.5(R_{L,t} - R_{CB,t+1} - R_{CB,t} + 1)$, hence³⁰

$$\widehat{t p} + 0.5(\widehat{t p})^2 \approx \frac{\frac{1}{\beta} + \rho}{\Gamma} \left(\widehat{R}_{L,t} + 0.5(\widehat{R}_{L,t})^2 \right) - \frac{1}{\beta\Gamma} \left(\widehat{R}_{CB,t+1} + 0.5(\widehat{R}_{CB,t+1})^2 \right) - \frac{1}{\beta\Gamma} \left(\widehat{R}_{CB,t} + 0.5(\widehat{R}_{CB,t})^2 \right).$$

We can now combine the LHS and RHS to get

$$E_t [\widehat{t p} + 0.5(\widehat{t p})^2] = E_t \left[\sigma_B (\widehat{\Pi}_{t+2}^B - \widehat{\Pi}_{t+1}^B) + 0.5\sigma_B \left((\widehat{\Pi}_{t+2}^B)^2 - (\widehat{\Pi}_{t+1}^B)^2 \right) + \widehat{\pi}_{t+2} - 0.5(\widehat{\pi}_{t+2})^2 + \frac{\bar{\rho}}{\Gamma} (\widehat{\rho}_{t+1} + 0.5(\widehat{\rho}_{t+1})^2) + CovTerms \right]$$

where,

$$\begin{aligned} CovTerms &= -\sigma_B \widehat{\Pi}_{t+1}^B \left(\frac{1}{\beta\Gamma} \widehat{R}_{CB,t} + \frac{\bar{\rho}}{\Gamma} \widehat{\rho}_{t+1} \right) + \sigma_B \widehat{\Pi}_{t+1}^B \widehat{\pi}_{t+1} - \left(\frac{1}{\beta\Gamma} \widehat{R}_{CB,t} + \frac{\bar{\rho}}{\Gamma} \widehat{\rho}_{t+1} \right) \widehat{\pi}_{t+1} \\ &+ \sigma_B \widehat{\Pi}_{t+2}^B \left(\frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} - \widehat{\pi}_{t+1} - \widehat{\pi}_{t+2} \right) + \left(\frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma} \widehat{R}_{L,t} - \frac{1}{\beta\Gamma} \widehat{R}_{CB,t+1} \right) (\widehat{\pi}_{t+1} + \widehat{\pi}_{t+2}) - \widehat{\pi}_{t+2} \widehat{\pi}_{t+1}. \end{aligned}$$

³⁰Note that the approximated signed is also used here since the denominator should be $\left(\frac{1}{\beta} - 1 + \rho \right) + \beta - 1$ and not $\Gamma = \left(\frac{1}{\beta} - 1 + \rho \right)$.

E Long-term Investment with 1Y maturity

If we assume long-term investments are done at period t but mature at $t + 4$ then $X_{L,t}$ becomes

$$X_{L,t} = \frac{\gamma_L E_t [q_{t+4}^L \pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}]}{R_{L,t}} - 1.$$

And the long-term rate is set such that

$$\begin{aligned} \beta^3 E_t \left[\frac{1}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \Pi_{t+4}^B {}^{-\sigma_B} (R_{L,t} - R_{t+3,CB}) \right] &= E_t \left[\Pi_{t+1}^B {}^{-\sigma_B} \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] + \\ &\beta E_t \left[\Pi_{t+2}^B {}^{-\sigma_B} \left(\frac{(R_{t+1,CB} - 1)}{\pi_{t+1} \pi_{t+2}} + \rho_{t+2} \right) \right] + \\ &\beta^2 E_t \left[\Pi_{t+3}^B {}^{-\sigma_B} \left(\frac{(R_{t+2,CB} - 1)}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} + \rho_{t+3} \right) \right]. \end{aligned}$$

Where

$$\begin{aligned} \Pi_t^B &= div_t + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1) (X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} \\ &\quad - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}), \text{ and} \\ d_t &= X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_t} + \frac{X_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}} + z_t. \end{aligned}$$

We define the term premium (annual rate in percentage points) between long and short-term rate as

$$tp = \frac{1}{4} (R_{L,t} - R_{t,CB} - R_{t+1,CB} - R_{t+2,CB} - R_{t+3,CB} + 3) 400.$$

Finally, the good market clearing condition is

$$Y_t = C_t + X_{S,t} + X_{L,t} + \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}).$$

Short-term asset purchase agreements

We assume banks can only repo the long-term asset that is about to mature. Profits and deposits are given by

$$\begin{aligned} \Pi_t^B &= div_t + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1) (X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} \\ &\quad - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-1} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-1}^2, \text{ and} \\ d_t &= X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_t} + \frac{X_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}} + z_t - \Theta_t \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}}. \end{aligned}$$

Which implies

$$\begin{aligned}
\beta^3 E_t \left[\frac{\Pi_{t+4}^B \pi_{t+1}^{-\sigma_B} (R_{L,t} - R_{t+3,CB} + \Theta_{t+3} (R_{t+3,CB} - R_{QE,t+3}))}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \right] &= E_t \left[\Pi_{t+1}^B \pi_{t+1}^{-\sigma_B} \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] \\
&+ \beta E_t \left[\Pi_{t+2}^B \pi_{t+1}^{-\sigma_B} \left(\frac{(R_{t+1,CB} - 1)}{\pi_{t+1} \pi_{t+2}} + \rho_{t+2} \right) \right] \\
&+ \beta^2 E_t \left[\Pi_{t+3}^B \pi_{t+1}^{-\sigma_B} \left(\frac{(R_{t+2,CB} - 1)}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} + \rho_{t+3} \right) \right] \\
0 &= \Pi_{t+1}^B \pi_{t+1}^{-\sigma_B} \left((R_{CB,t} - R_{QE,t}) \frac{X_{L,t-3}}{\pi_{t+1} \pi_t \pi_{t-1} \pi_{t-2}} - \phi_{QE} \Theta_t \right).
\end{aligned}$$

Long-term asset purchase agreements

We assume banks can sell the long-term asset with the longest maturity and buy back before maturity.

Profits and deposits are given by

$$\begin{aligned}
\Pi_t^B &= \text{div}_t + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1) (X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} \\
&- \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-3} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-3}^2, \text{ and} \\
D_t &= P_t X_{S,t} + P_t X_{L,t} + P_{t-1} X_{L,t-1} + P_{t-2} X_{L,t-2} + P_{t-3} X_{L,t-3} + Z_t - \Theta_t P_{t-1} X_{L,t-1} - \Theta_{t-1} P_{t-2} X_{L,t-2} - \Theta_{t-2} P_{t-3} X_{L,t-3}.
\end{aligned}$$

Which implies

$$\begin{aligned}
\beta^3 E_t \left[\frac{\Pi_{t+4}^B \pi_{t+1}^{-\sigma_B} (R_{L,t} - R_{t+3,CB} + \Theta_{t+3} (R_{t+3,CB} - R_{QE,t+3}))}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \right] &= E_t \left[\Pi_{t+1}^B \pi_{t+1}^{-\sigma_B} \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] \\
&+ \beta E_t \left[\Pi_{t+2}^B \pi_{t+1}^{-\sigma_B} (1 - \Theta_{t+1}) \left(\frac{(R_{t+1,CB} - 1)}{\pi_{t+1} \pi_{t+2}} + \rho_{t+2} \right) \right] \\
&+ \beta^2 E_t \left[\Pi_{t+3}^B \pi_{t+1}^{-\sigma_B} (1 - \Theta_{t+1}) \left(\frac{(R_{t+2,CB} - 1)}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} + \rho_{t+3} \right) \right]
\end{aligned}$$

$$\Pi_{t+1}^B \pi_{t+1}^{-\sigma_B} \frac{(R_{CB,t} - 1) X_{L,t-1}}{\pi_t \pi_{t+1}} + \Pi_{t+2}^B \pi_{t+1}^{-\sigma_B} \frac{(R_{CB,t+1} - 1) X_{L,t-1}}{\pi_t \pi_{t+1} \pi_{t+2}} + \Pi_{t+3}^B \pi_{t+1}^{-\sigma_B} \left((R_{CB,t+3} - R_{QE,t}) \frac{X_{L,t-1}}{\pi_t \pi_{t+1} \pi_{t+2} \pi_{t+3}} - \phi_{QE} \Theta_t \right) = 0.$$