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## Persuasion by stress testing – optimal disclosure of supervisory information in the banking sector

Wolfgang Gick
(Harvard and Tufts University)

Thilo Pausch
(Deutsche Bundesbank)

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**Editorial Board:** Klaus Düllmann
Heinz Herrmann
Christoph Memmel

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
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**Abstract**

The game-theoretical analysis of this paper shows that stress tests that cover the entire banking sector (macro stress tests) can be performed by institutional supervisors to improve welfare. In a multi-receiver framework of Bayesian persuasion we show that a banking authority can create value when committing to disclose the stress-testing methodology (signal-generating process) together with the stress test result (signal). Disclosing two pieces of information is a typical procedure used in stress tests. By optimally choosing these two signals, supervisors can deliver superior information to prudent investors and enhance welfare. The paper offers a new theory to explain why stress tests are generally welfare enhancing. We also offer a treatment of the borderline case where the banking sector is hit by a crisis, in which case the supervisor will optimally disclose an uninformative signal.



*Keywords: Stress Tests, Supervisory Information, Bayesian Persuasion, Multiple Receivers, Disclosure.*

*JEL classification: D81, D83, G28.*

# Non-technical summary

The present paper applies a game-theoretical model to analyze the impact of an optimal disclosure in stress test designs that cover the entire banking sector (macro stress tests). Specifically, we inquire to which extent investors will make optimal use of information stemming from such stress tests when deciding to provide liquidity to banks, e.g. via bank deposits or by purchasing bank bonds. In addition, the paper deals with the issue on how to optimally design supervisory macro stress tests when the banking supervisor pursue the goal to ensure a macro-economically optimal level of banking activities, this by convincing investors to provide the overall optimal amount of funding to the banking sector.

In our analysis we present a multi-receiver model of a *persuasion game*. Persuasion games belong to the larger class of *cheap talk games*. In our model banking supervisors, by making use of macro stress tests, can generate information about the actual stability level of the banking sector. Banking supervisors, however, face a commitment problem when they aim at influencing investors' decisions by disclosure of stress tests results. From an investors' point of view, stress test results, which are disclosed by banking supervisors, basically do not constitute a reliable form of information about the true state of the banking system (cheap talk).

The game-theoretical analysis of the present paper, however, shows that banking supervisors will optimally design macro stress test in a way that permits them to influence the decisions of investors to increase social welfare; this by both disclosing the method of the stress tests used as well as the stress test result. We furthermore show that macro stress tests pursued by banking supervisors will always increase the informativeness of the disclosure mechanism. However, banking supervisors should generally not aim at designing macro stress tests to completely eliminate uncertainty at the investors' side as to the actual state of the banking sector. Only this feature will permit supervisors to avoid triggering some possibly extreme forms of investor behavior (such as a complete withdrawal from bank financing, or, in turn, pursuing highly risky financial decisions).

# Nicht-technische Zusammenfassung

Das vorliegende Papier analysiert mit Hilfe eines spieltheoretischen Modells die Wirkung und optimale Ausgestaltung bankaufsichtlicher Stresstests für den gesamten Bankensektor (Makro-Stresstests). Besonderes Augenmerk wird hierbei zum einen auf die Frage gelegt, ob und wie Investoren Informationen aus bankaufsichtlichen Makro-Stresstests berücksichtigen, wenn sie entscheiden, in welchem Umfang sie Banken Finanzierungsmittel, z.B. über Einlagen oder Kauf von Anleihen, zur Verfügung stellen sollen. Zum anderen betrachtet das Papier die Frage nach der optimalen Ausgestaltung von Makro-Stresstests durch die Bankenaufsicht, wenn es deren Ziel ist, einen gesamtwirtschaftlich optimalen Umfang an Bankaktivitäten durch Sicherstellung der hierfür notwendigen Finanzierung durch Investoren zu gewährleisten.

Die Analyse bedient sich einer Modellierung, die in den größeren Kreis der *Persuasion Games*, eine Untergruppe der *Cheap Talk Games*, einzuordnen ist. Bankenaufseher sind hierbei in der Lage, mittels Makro-Stresstests Informationen über den tatsächlichen Zustand eines Bankensektors zu generieren. Sollte die Bankenaufsicht jedoch versuchen, das Entscheidungsverhalten von Investoren durch Veröffentlichung von Stresstest-Ergebnissen zu beeinflussen, sieht sie sich mit einem Glaubwürdigkeitsproblem konfrontiert. Grundsätzlich könnten Investoren zunächst davon ausgehen, dass die Offenlegung von Stresstest-Ergebnissen durch die Bankenaufsicht keine verlässliche Information über den wahren Zustand des Bankensystems darstellt (Cheap Talk).

Die spieltheoretische Analyse des vorliegenden Papiers zeigt jedoch, dass Bankenaufseher durch eine gezielte Ausgestaltung von Makro-Stresstests und anschließende Offenlegung von Informationen sowohl über die Stresstest-Methode als auch über das Stresstest-Ergebnis in einer Art und Weise auf das Entscheidungsverhalten der Investoren einwirken können, die sich positiv auf die Gesamtwohlfahrt auswirkt. Darüber hinaus kann gezeigt werden, dass optimal ausgestaltete bankaufsichtliche Makro-Stresstests in jedem Fall zu einer Verbesserung der Informationslage (der Investoren) im Bankensektor führen. Allerdings sollten bankaufsichtliche Makro-Stresstests optimalerweise nicht zur vollständigen Beseitigung der Unsicherheit der Investoren über den tatsächlichen Zustand des Bankensektors führen. Nur so kann extremes Verhalten der Investoren (z.B. nahezu vollständiger Rückzug aus der Bankenfinanzierung oder Eingehen hoch riskanter Finanzierungen) vermieden werden.

# Contents

# Persuasion by Stress Testing – Optimal Disclosure of Supervisory Information in the Banking Sector[1]

## 1 Introduction

It is common knowledge that banks improve the allocation of capital in an economy and in so doing, increase social welfare. Since Diamond and Dybvig (1983), scholars have systematically researched how such processes work and what the options are for a banking system to provide adequate forms of liquidity to borrowers. One of the most prominent forms of liquidity provision occurs through maturity transformation, by which banks convert securities with short maturities into the long-term forms of liquidity that borrowers frequently request.

It lies in the very nature of such transformation processes that they come with a risk: creating liquidity may cause financial fragility, as the banking system may become unstable when investors do not have the necessary knowledge about borrowers,[2] and this instability may spread throughout the whole banking system.[3]

In this context, it is easy to see why Jaime Caruana, the General Manager of the Bank for International Settlements, has emphasized in particular that

> *"[...] strengthened, transparent disclosure is good for markets, because it helps investors make more informed decisions."*[4]

Transparency is valuable as it improves the information accessible to investors, and this increased transparency reduces uncertainty, leading to better risk-adjusted behavior on the investors' side.

Stress tests represent a very prominent form of information disclosure pursued by institutional supervisors. The European Banking Authority, the Securities and Exchange Commission (SEC) in the U.S., as well as national supervisory authorities

[1]We thank participants at the Seminar of the Deutsche Bundesbank in April 2012, the EARIE 2012 in Rome, and the 2012 meeting of the German Finance Association, our discussants Mikhel Tombak and Christoph Schmidhammer, as well as Günther Franke and Eva Schliephake for comments. All remaining errors are ours.

[2]See e.g. Freixas and Rochet (2008), as well as Diamond and Rajan (2001).

[3]See Allen and Gale (2000).

[4]Caruana, J. (2011), p. 2.

in the U.K. and in Japan, respectively, have designed and performed a series of new stress tests over the past few years. Strikingly, the number of countries *publishing* bank stress tests has increased from 0 to 40 over the past decade.[5] An unprecedented fact is the frequency by which such macroeconomic stress tests are being performed, which undoubtedly will shatter the commonly-perceived underestimation of their impact, and one may safely assume that larger and more centralized supervisory authorities will prefer to make more use of such tests rather than less.

Two new papers support this view, namely that enhanced transparency through disclosing stress test results is beneficial to financial stability. First, Goldstein and Sapra (2012), who research the impact of disclosure on ex-ante incentives of banks, are strongly in favor of a public disclosure of stress-test results. The authors believe "that – at least from a financial stability perspective – the benefits of disclosing stress test results are undeniable."[6]

Second, an empirical paper by Horvath and Vasco (2012) shows that the degree of transparency concerning financial stability has significantly increased since 2000, which underpins the general viewpoint concerning the value of transparency. More interestingly, they find that greater transparency is beneficial during typical financial periods financial stress is low. Yet, Horvath and Vasco (2012) also detect that greater transparency will *increase* financial stress when financial systems are undergoing a time of severe distress, in which situations more transparency may lead to adverse effects.[7] This result is addressed in our paper in a borderline case where the banking sector is completely vulnerable. In this special case, disclosure optimally remains uninformative.

In this light, striving for more transparency has undoubtedly become a central issue for a sector that many authors have described as the epitome of an "opaque" industry.[8] That stress tests will, in general, *reduce* the inherent degree of opacity in the banking sector, has been emphasized by Peristiani et al. (2010) in their study. While they see banks as "neither black boxes nor open books," the argument that stress tests will generally permit to reduce bank opaqueness can also be found in recent research done by Petrella and Resti (2011) and their analysis of stress tests performed by the European Banking Authority. Significant market reactions reveal that stress tests do help investors make better decisions.

A clarification is in order. While empirical studies show that stress tests leads to a reduction of bank opaqueness, *opacity* is also an issue in central bank's communication and monetary policy. Opacity, in this context, seems to remain a generally accepted fact to which economists have not added much new insight. For example, Stein (1989), in an early application of Crawford and Sobel's (1982) seminal article

[5]See Horvath and Vasco (2012).

[6]Goldstein and Sapra (2012), p. 2.

[7]We also thank Günther Franke for a similar argument.

[8]Morgan (2000), Haggard and Howe (2007).

on strategic information transmission to central bank disclosure policies, has argued that in equilibrium, disclosure between a public authority and an uninformed investor should optimally remain coarse and limited.

Morris and Shin (2000, 2002) have argued that public information is a double-edged instrument. The private information that actors possess may in some situations render public information detrimental for a central bank's policy. Disclosing all available public (and noisy) information may then lead to suboptimal decision making, as receivers will put too much weight on the noise in the public information. James and Lawler (2011) confirm the aspect that disclosing too much information will have adverse effects and decrease welfare.[9] However, it should be added that Morris and Shin's (2002) work has actually opened up an extended discussion, in which their model has been interpreted as arguing in favor of more transparency, as Svensson (2006) and Morris, Shin and Tong (2006) have shown in two replies.

Our explanation is entirely different in its approach. We offer a Bayesian persuasion game with multiple receivers to explain why macro stress tests are performed. We believe that opacity, while at the center of explanations concerning central bank policies over the past decades, cannot account for the specific nature of stress tests. Based on the disclosure process that makes use of two pieces of information, we propose a new theory that contrasts with earlier work. Macroeconomic stress tests, the way they are perceived by financial markets and the "dialogue about financial stability vulnerabilities"[10] require a new understanding that this paper is set out to deliver.

What we know is that a supervisory agency both discloses the stress test *design, and the result*. It is this property that carries a new viewpoint. Why informed supervisors are willing to disclose multiple pieces of information has, to our best knowledge, not been analyzed at all in the context of financial and supervisory disclosure. We show that a public banking authority can generally "persuade" investors over a wide range of (prior) beliefs to take actions toward a socially optimal trade-off between individual risk bearing and the provision of liquidity to the banking sector. By making optimal use of two informational components (the result and the signal-generating process) stress tests will influence a continuum of prudent investors toward an optimal and risk-adjusted provision of liquidity to the economy, and, in doing so, they increase welfare.

The theoretical framework that we develop belongs to a new class of disclosure games, called Bayesian persuasion games. While still belonging to the larger strand of literature on cheap talk in the tradition of Crawford and Sobel (1982), persuasion games lift this literature onto a new level, permitting new insights into disclosure processes between an informed sender and uninformed receivers. Bayesian

[9]This, again, contrasts with the findings of Horvath and Vasco (2012).

[10]Borio et al., 2011, p. 18.

persuasion games assume that the sender can, to some extent, *control the communication environment of the receiver(s)*. When there is more than just one signal – that is, when the sender can use the signal-generating process as it takes place in stress-testing procedures to deliver *additional* information to the receiver – the game reveals properties that are much different than those found in standard cheap-talk games. Since he knows that the receivers will update their prior beliefs by using Bayes' rule, the sender will be willing to perform such a test. This specific feature permits the sender to *commit to disclose*, even when the test results are not known to him *ex ante*. The capability of the sender to commit to such a procedure now eliminates a well-known drawback of cheap talk games, namely that messages are arbitrary for a wide range of beliefs, and therefore create strategic complexity.[11] Consequently, the players always face the problem of coordinating on a common language. In persuasion games, this problem is eliminated by the fact that now the sender creates a "meaning" of messages. Because of this property, persuasion games now exhibit commitment power, and the sender no longer needs to *best respond to any strategy of the receiver(s).*[12]

This specific literature on persuasion games[13] has been laid out in one seminal article: Kamenica and Gentzkow (2011) (KG hereafter), and this in a setting with one sender and one receiver. KG show for a wide range of parameters that the sender is strictly better off disclosing *both* pieces of information.

In an application different from ours, and with an extension toward a (discrete) multi-receiver setting, Wang (2012) analyzes voting rules in setups with public and private disclosure. Our paper differs from hers in several ways. We do not analyze majority voting; in our model, every investor's behavior is influenced by the supervisor. Moreover, we limit our analysis to public signals, that is, we exclude the option of the sender to privately disclose different signals to a subset of receivers.

The paper is organized as follows. Section 2 introduces the model, defining the timing of the game, players' preferences and the supervisor's utility function, the investor decisions, and the value of disclosure, treating the issue of Bayesian Plausibility in a continuous multi-receiver setting. Section 3 determines the supervisor's problem, deriving conditions for optimal disclosure, studying the benchmark of uninformative disclosure, and it expands on some welfare implications. Section 4 concludes. The proofs to all lemmas are given in the appendix.

[11]See Sobel, 2010.

[12]Sobel (2010) in his overview shows furthermore that even under arbitrary choices of messages, coordination failures disappear.

[13]For a different treatment see Chakraborty and Harbaugh (2011) and their multidimensional cheap-talk model without commitment and large biases.

# 2 Model

## 2.1 Primitives

Consider a setting with two types of agents: a single banking supervisor (sender) and a (infinitely) large number of investors (receivers). The supervisor (S) owns a technology (a stress testing mechanism) that provides him with reliable information about the true status of the banking sector under his control. While information acquisition is costly, these costs are fixed and do not depend on the information revealed about the banking sector. We assume a binary state space of the supervisor's information: either the banking sector is firm / sound ($F$) or vulnerable ($V$), meaning that some adverse situation or crisis hurts the banking sector either only marginally or heavily. We formally summarize this binary state space by $\Theta = \{F, V\}$ where $\theta$ denotes a realization of a certain stress testing exercise. The prior (objective) probability distribution over $\Theta$ is $Pr(V) = p$ and $Pr(F) = 1 - p$.

The investors (R) do not know the state of the banking sector, but they hold prior beliefs ($b$) about the probability that the banking sector is vulnerable. These priors may be considered to arise from experiences over past periods, or they may be based on the evaluation of other institutions such as rating agencies. Prior beliefs are, hence, considered to be heterogenous among investors.

We assume that the total number of investors is infinitely large and can be normalized to one. Further, let $g(b)$ denote the continuous function which represents the distribution of prior beliefs over all investors. The corresponding continuous function $G(b)$ denotes the cumulative distribution function of prior beliefs as well as the number of investors having prior beliefs of, at most, $b$.

Based on their individual beliefs, investors make their investment decisions and choose an action in the action space. They decide on their behavior in the banking sector. The action space of investors is assumed to be binary as well: investors may either act prudently ($P$) or riskily ($R$). Prudent behavior means that the investor under consideration believes that the banking sector is vulnerable and is, therefore, willing to provide funding to banks (e.g. by depositing money with banks or by buying bank bonds) only to a minimum amount which is backed by some deposit insurance mechanism or through State guarantees. All investors whose individual beliefs are *beyond* some threshold probability $b_T$ behave this way. Risky behavior, in contrast, refers to decisions that result in investors providing much more funds to the banking sector than the benchmark would minimally suggest. This latter behavior occurs when investors choose individual beliefs below the threshold probability, $b_T$. The binary action space will be denoted by $A = \{P, R\}$ in the following, and $a$ denotes a certain realization. The threshold probability $b_T$ then defines when investors prefer to switch from risky to prudent behavior because they believe that the probability that the banking sector is vulnerable is too high.

What essentially follows from these assumptions is that they imply an overall distribution of beliefs, which now determines the total number of investors who behave prudently or riskily, respectively. Specifically, $G(b_T) = Pr(b \leq b_T)$ is the total number of investors who prefer a risky strategy whereas $1 - G(b_T) = Pr(b > b_T)$ is the total number of prudent investors given their prior beliefs.

## 2.2 Timing

The timing of the game is as follows. First, the supervisor chooses an information disclosure mechanism $\pi$ to induce investors to act in the supervisor's interest. The information disclosure mechanism consists of two elements: a signal from a binary realization space of the supervisor's stress tests results $D \in \{f, v\}$, plus a related family of conditional distributions $\{\pi|\cdot\}_{\theta \in \Theta}$ over $D$. The conditional distributions follow the design, result, and accuracy of stress tests. In mathematical terms, the design of a stress test determines the probabilities of the result $d \in D$, given that the true state of the banking sector is $\theta \in \Theta$. That is, we assume that the supervisor does not cheat on investors about the stress test result. Rather, the stress-test result will be always reported truthfully. Yet the supervisor can design the stress test in such a way that an incorrect result becomes more or less likely.

Let $\pi(v|V)$ ($\pi(f|V)$) and $\pi(f|F)$ ($\pi(v|F)$) denote the probabilities that the stress test generates correct (incorrect) results. The stress test design determines the following probabilities:

$$\begin{array}{c} \pi(v|V) \\ \pi(f|V) = 1 - \pi(v|V) \end{array} \quad \text{and} \quad \begin{array}{c} \pi(f|F) \\ \pi(v|F) = 1 - \pi(f|F) \end{array}.$$

In the next step, the supervisor carries out a stress testing exercise, observes (privately) stress test result $d \in D$ and reports this result together with the full information about the stress test's design, i.e. $\pi(\cdot)$, to investors.

The investors, thereafter, observe both the supervisor's choice of the stress test mechanism and the stress test realizations to decide on either a prudent or a risky action to provide funding to the banking sector.

As already mentioned in the introduction, information disclosure in our model as well as in KG does not only mean to send out a single message as in standard models of cheap talk. Instead, disclosure includes information about the signal-generating process *and* about the obtained signal as the supervisor in our model reveals information to the public about the stress test design (including underlying assumption, information regarding the data analysis, and so on) *and* the outcome of the stress test.

## 2.3 Preferences

Investors' preferences strongly depend on making correct investment decisions. For simplicity, let these utilities take values of 1 or 0: each investor receives a utility of 1 if he makes the correct decision, $U_R(P,V) = U_R(R,F)$. Otherwise, his utility remains $U_R(P,F) = U_R(R,V) = 0$.[14]

The general aim of banking supervisors is the promotion of stability of the banking sector (cf. BCBS, 2011, para 1). In this regard there are, however, two aspects that a supervisor needs to take into account. First, while acknowledging the economic role of banks in providing financing to private households, firms, and governments supervisors are also aware that the banking sector requires funding. The banking literature, in turn, has shown that the structure of banks' balance sheets, i.e. using short-term liabilities to fund long-term assets, makes individual banks as well as the banking sector as a whole vulnerable to shocks in the financial or economic environment.[15] Second, the supervisor is aware of the fact that investors suffer in terms of utility when their investment decisions turn out to be incorrect. It is crucial for an understanding that both aspects are interrelated: information disclosed by the supervisor may cause investor reactions that affect bank funding and in this way the stability of the banking sector.

A supervisor's utility function has therefore to consider both aspects. First, the supervisor is aware of the fact that herding behavior of investors - either too many investors acting prudently or too many investors acting riskily - may negatively impact the banking sector as a whole. With too many prudent investors banks may face severe funding problems which may in the end make the system even more vulnerable. In turn, with too many investors acting riskily, banks may find themselves in a situation of excess liquidity which may trigger excessive risk taking of banks in order to profitably invest available funds. But excessive risk taking will also aggravate the vulnerability of the banking sector.

As a result, given the objective probability $p$ of the banking sector being vulnerable, we assume that there exists a certain number of prudent investors $|P|^{max}$ that maximizes the supervisor's utility. That is, for a given $p$ at $|P|^{max}$ there is an optimal balance between providing funds to banks and investors' preference for correct investment decisions. A higher number of prudent investors, on the one hand, may increase the danger of aggravating the vulnerability of the banking sector by a shortage of bank funding. On the other hand, fewer prudent investors may also increase the danger of aggravating the banking sector's vulnerability due excessive

[14]Although each single investor makes his own decision we do not use an index for a certain investor. This is only for notational convenience and does not affect results because investors do not differ regarding utility functions.

[15]See, eg, Diamond and Dybvig (1983), Diamond (1997), Diamond and Rajan (2001), and Allen and Gale (2000).

risk taking of banks.

In addition, when the supervisor designs the disclosure mechanism (including the stress test design), the final outcome of the stress test and therefore the investors' responses to information disclosure, will remain uncertain. The supervisor can only *form expectations about investor behavior.* Therefore we assume in the following that the supervisor's utility function is invariant to the stress test outcome. Furthermore the utility function is considered hill-shaped, reaching a maximum at $|P|^{max}$ and being zero at the extremes when either all investors act prudently or all investors act riskily.

Put in more formal terms, let $U_S(|P|)$ denote the supervisor's utility as a function of the number $|P|$ of prudent investors, i.e. investors who choose $a = P$.[16] The supervisor's utility becomes zero when either all investors act riskily or all investors act prudently. Moreover, there exists a number $|P|^{max} \in (0; 1)$ where the supervisor's utility reaches a maximum $U_S^{max}$.[17] In sum, we assume the following continuously differentiable hill-shaped curve representing the supervisor's utility:

$$U_S = U_S(|P|) \text{ with } U_S'(|P|) > (<) 0 \;\forall\; |P| < (>) |P|^{max} \text{ and } U_S'(|P|^{max}) = 0. \quad (1)$$

## 2.4 Investor beliefs

### 2.4.1 Bayesian updating

By choosing the appropriate stress test design, the supervisors define the framework which shows investors - connected to the commitment assumption - how to update prior beliefs using Bayes' rule when the stress test shows a specific outcome. As in KG, Wang (2011) and Sobel (2010), the supervisor is able to influence investor behavior in a certain way.

#### Digression: The Single-Receiver Case (following KG)

*Before introducing our multi-receiver model, it seems worthwhile to briefly illustrate the standard persuasion game between a single investor (receiver), who, con-*

[16] Due to our assumptions, $|P|$ is found by inserting the threshold probability $b_T$ in the cumulative distribution function of investor beliefs in a given situation. For instance, in the case of a non-informative disclosure mechanism we have $|P| = 1 - G(b_T)$. In the case of effective information disclosure we have $|P| = 1 - \hat{G}_v(b_T)$ and $|P| = 1 - \hat{G}_f(b_T)$ when the supervisor discloses signal $d = v$ and $d = f$, respectively.

[17] We do not explicitly consider $p$ to be an argument of $U_S(\cdot)$ as we earlier assumed that $p$ is given exogenously. That is, given the (objective) probability $p$ there exists a socially optimal number of prudent investors that maximizes the supervisor's utility. Note that $|P|^{max}$ may change with the exogenously given probability $p$.

*ditional on observing that the banking sector is sound (state $F$), holds prior beliefs $\hat{x} = \pi(v|F)$ and $\hat{y} = \pi(f|F)$. In KG, the sender maximizes the overall probability to convince the receiver to change his or her mind about investing:*

$$\max p\hat{x} + (1-p)\hat{y}$$

*s.t.* $\hat{y} \geq \frac{1}{2}$.

*The informed sender will optimally choose a pair of $\hat{x}$ and $\hat{y}$ such that the receiver, upon observing this pair, will update his beliefs toward the following posterior probabilities:*

$$\mu(V|f) = \frac{(1-\hat{y})(1-p)}{(1-\hat{x})p + (1-\hat{y})(1-p)} \leq \frac{1}{2}$$

*and*

$$\mu(V|v) = \frac{\hat{y}(1-p)}{p\hat{x} + \hat{y}(1-p)} \geq \frac{1}{2}$$

*As a specific situation emerging in the one-receiver KG case, optimality requires that the first Bayes' rule has a zero numerator and updating is impossible. Still, the posterior probability is derived from the second Bayes' rule, solving optimally*[18]

$$p\hat{x} + (1-p)\hat{y} = 2(1-p)\hat{y}.$$

■

What differs in our model from those of KG and Wang (2011) is that each single investor updates his own beliefs based on individual priors. That is, because we consider a large number of investors with heterogenous prior beliefs, the magnitude (but not the direction) of Bayesian updating also differs among investors. Formally this may be written as follows. First, - as in KG (2011), Wang (2011) and Sobel (2010) - let $\mu_b(\theta|d)$ denote the posterior belief of an investor with individual prior realization $b$ that the true state of the banking sector is $\theta$ when the supervisor discloses $d$ and applies a stress test design $\{\pi|\cdot\}_{\theta\in\Theta}$. In particular the posteriors for any prior belief $b$ are:

$$\begin{array}{lcl} \mu_b(V|v) = \frac{\pi(v|V)\cdot b}{\pi(v|V)b+\pi(v|F)(1-b)} & & \mu_b(F|f) = \frac{\pi(f|F)\cdot(1-b)}{\pi(f|V)b+\pi(f|F)(1-b)} \\ & \text{and} & \\ \mu_b(F|v) = \frac{\pi(v|F)\cdot(1-b)}{\pi(v|V)b+\pi(v|F)(1-b)} & & \mu_b(V|f) = \frac{\pi(f|V)\cdot b}{\pi(f|V)b+\pi(f|F)(1-b)} \end{array} \quad (2)$$

[18]See a detailed description for the multi-receiver case in the next subsection on Bayesian updating with many receivers.

when the supervisor discloses $d = v$ and $d = f$, respectively. As a consequence, Bayesian updating will affect the cumulative distribution function of investors' beliefs. The following subsection describes how the stress test design $\{\pi|\cdot\}_{\theta\in\Theta}$ and the stress test outcome $d$ jointly affect investor beliefs.

### 2.4.2 Distribution of investors' posterior beliefs and uncertainty

Since the stress test design $\{\pi|\cdot\}_{\theta\in\Theta}$ is public information and does *not* depend on the prior belief of any single investor, the outcome of the stress tests will affect beliefs of *all* investors to the same direction. Consider the borderline case in which a stress test is completely uninformative. That is, it must be true that $\mu_b(V|v) = \mu_b(V|f)$ and $\mu_b(F|v) = \mu_b(F|f)$ for any $b$ and $\pi(v|V) = \pi(f|V) = \pi(v|F) = \pi(f|F) = \frac{1}{2}$.[19]

Now, making the stress test informative - i.e. setting $\pi(v|V) > \frac{1}{2}$ and/or $\pi(f|F) > \frac{1}{2}$ - works as follows:[20]

$$\pi(v|V) > \tfrac{1}{2} \;\Rightarrow\; \mu_b(V|v) > \mu_b(V|f) \text{ and } \mu_b(F|f) > \mu_b(F|v)\ \forall\ b$$

where the last part follows from $\pi(f|V) = 1 - \pi(v|V)$ and the definition of posterior beliefs (2) above. Moreover, perfectly analogously and using the same reasoning we now observe the following:

$$\pi(f|F) > \tfrac{1}{2} \;\Rightarrow\; \mu_b(F|f) > \mu_b(F|v) \text{ and } \mu_b(V|v) > \mu_b(V|f)\ \forall\ b.$$

In a next step, let $x \in [0, \frac{1}{2}]$ and $y \in [0, \frac{1}{2}]$ denote the level of precision – ie the amount by which $\pi(v|V)$ and $\pi(f|F)$ exceed $\frac{1}{2}$ – of the signal $d = v$ and $d = f$, respectively.

These observations already lead to two results. First, although the supervisor can decide about the precision of a certain signal, i.e. $d = v$ and $d = f$, basically separately, there is an interaction between both. This interaction will make signals

[19] Note that $\mu(V|v) = \mu(V|f)$ and $\mu(F|v) = \mu(F|f)$ both require

$$\frac{\pi(v|F)}{\pi(v|V)} = \frac{\pi(f|F)}{\pi(f|V)}.$$

Due to $\pi(v|V) = 1 - \pi(f|V)$ and $\pi(v|F) = 1 - \pi(f|F)$ the former condition holds if and only if $\pi(v|V) = \pi(f|V) = \pi(v|F) = \pi(f|F) = \frac{1}{2}$.

[20] While $\pi(v|V) < \frac{1}{2}$ may occur, we limit our attention to the case with $\pi(v|V) > \frac{1}{2}$ in the case of informative disclosure. The reason is twofold: first, $\pi(v|V) > \frac{1}{2}$ refers to a situation of truthful disclosure, which directly translates into investor utility. Second, $\pi(v|V) < \frac{1}{2}$ simply means that the supervisor is likely to send a signal that is exactly the opposite of the true state. Since investors will know the disclosure mechanism, this supervisory strategy will cause investors to decide inversely.

reinforce each other in terms of their precision. As a consequence, posterior beliefs are driven by the precision of both signals as well as their interaction. The second result is that the relations hold for any arbitrary prior investor belief $b$.

This gives the model the following interesting twist: when the supervisor's disclosure mechanism is informative, the public signal $d \in D$ will crucially affect the cumulative distribution function of posterior beliefs. Therefore, let $\hat{G}_v(\cdot)$ and $\hat{G}_f(\cdot)$ denote the cumulative distribution functions of posterior beliefs in the case of signals $d = v$ and $d = f$, respectively. Further, for any $b$, let us denote $\hat{b}_v(b) = \mu_b(V|v)$ and $\hat{b}_f(b) = \mu_b(V|f)$ as the respective posterior beliefs that the banking sector is vulnerable when signals $d = v$ and $d = f$ are observable. Then, for any $b$, the above arguments imply:

$$\begin{aligned} \pi(v|V) > \tfrac{1}{2} \quad &\Rightarrow \quad \hat{b}_v(b) \geq b \text{ and } \hat{b}_f(b) < b \; \forall \; b && (3) \\ &\Rightarrow \quad \hat{G}_v(b) \leq G(b) \text{ and } \hat{G}_f(b) > G(b) \; \forall \; b. && (4) \end{aligned}$$

and

$$\begin{aligned} \pi(f|F) > \tfrac{1}{2} \quad &\Rightarrow \quad \hat{b}_f(b) \leq b \text{ and } \hat{b}_v(b) > b \; \forall \; b && (5) \\ &\Rightarrow \quad \hat{G}_f(b) \geq G(b) \text{ and } \hat{G}_v(b) < G(b) \; \forall \; b. && (6) \end{aligned}$$

In words: an informative disclosure mechanism increases or decreases the beliefs of any single investor *relative to his priors*, depending on the disclosed signal – this is what (3) and (5) reveal. As a result, we have for the aggregate setting, depending on the supervisor's signal, that information disclosure now shifts mass to the left tail or to the right tail of the cumulative distribution function – see (4) and (6). We summarize these findings as follows:

**Lemma 1** *Informative disclosure by the supervisor shifts the cumulative distribution function of investors' beliefs in the sense of first-order stochastic dominance (FSD). The signal $v$ ($f$) deteriorates (improves) the cumulative distribution function of investor beliefs in the sense of first-order stochastic dominance (FSD) compared to the prior distribution.*

**Proof:** See the appendix. ■

Note in particular that the FSD shift of the cumulative distribution functions may be formally written as

$$\frac{\partial \hat{G}_v(b|x,y)}{\partial x} \leq 0 \text{ and } \frac{\partial \hat{G}_v(b|x,y)}{\partial y} < 0 \; \forall b \tag{7}$$

$$\frac{\partial \hat{G}_f(b|x,y)}{\partial x} > 0 \text{ and } \frac{\partial \hat{G}_f(b|x,y)}{\partial y} \geq 0 \; \forall b. \tag{8}$$

The FSD shift and the finding that the direction of the FSD shift depends on the stress test outcome $d$ have an important implication regarding the supervisor's decision environment: Implementing an informative stress testing mechanism *replaces the certain decision-making situation of the supervisor with an uncertain one.* Recall that the supervisor does not know the true state of the banking sector at the time when he decides on the optimal stress testing mechanism. Therefore, the supervisor faces uncertainty about the signal to be sent to investors in a later stage of the game. In the case of an uninformative stress testing mechanism this fact, however, is irrelevant for the supervisor. In this latter case, investors will decide based on their prior beliefs, which implies that the supervisor knows the number of prudent investors *with certainty.* In contrast, in a situation of informative disclosure the stress test outcome $d$ determines the direction the investors' beliefs shift and, as a result, the number of prudent investors. As the stress test outcome $d$ is unknown at the time when the stress test mechanism is designed, the supervisor creates an environment of *uncertainty* regarding the effective number of prudent investors.

To summarize: when designing an informative disclosure mechanism, the supervisor actually replaces a certain decision-making situation with an uncertain decision-making situation *(disclosure lottery).* Whether this is valuable depends on both the supervisor's utility function and the level of expected utility generated by the disclosure lottery, relative to the utility of the certain situation with uninformative disclosure. In what follows, we analyze this property in great detail.

### 2.4.3 Investor decisions

Investors, in general, have to choose an action out of their action space $A = \{P, R\}$ in order to maximize individual expected utility. Let, as a general representation, $Pr_i(V)$ and $Pr_i(F)$ denote the individual probabilities (beliefs) of an arbitrary investor $i$ that the true state of the banking system is vulnerable ($V$) or firm ($F$). Then investor $i$'s expected utility of choosing prudent behavior ($a = P$) or risky behavior ($a = R$) is

$$\mathrm{E}\left(U_R(a = P)\right) = Pr_i(V)U_R(P, V) + Pr_i(F)U_R(P, F) = Pr_i(V) \tag{9}$$

or

$$\mathrm{E}\left(U_R(a = R)\right) = Pr_i(V)U_R(R, V) + Pr_i(F)U_R(R, F) = Pr_i(F), \tag{10}$$

respectively, because of $U_R(P, V) = U_R(R, F) = 1$ and $U_R(P, F) = U_R(R, V) = 0$.

From expected utilities (9) and (10) it is easily verified that an arbitrary investor $i$ is indifferent between $a = P$ and $a = R$ if and only if $Pr_i(V) = Pr_i(F) = \frac{1}{2}$. As a consequence, any investor who believes that $Pr_i(V) \leq \frac{1}{2}$ will choose $a = R$, and any investor who thinks that $Pr_i(V) > \frac{1}{2}$ will choose $a = P$. In other words: the threshold probability $b_T$ that we mentioned earlier, which defines when investors switch from a risky to a prudent strategy is unambiguously

$$b_T = \tfrac{1}{2}.$$

Specifically, it is irrelevant in this context whether investors revert to their prior beliefs for decision making or whether they update and build posterior beliefs. The mode of decision making is unaffected by setting $Pr_i(V) = b$, $Pr_i(V) = \hat{b}_v(b)$, or $Pr_i(V) = \hat{b}_f(b)$. Therefore, it must be true that $b_T = \frac{1}{2}$ is constant, regardless of the information disclosed by the supervisor.

Supervisory information disclosure, however, may affect the *level* of expected investor utility. In the previous subsection (Lemma 1) we found that an informative disclosure mechanism causes $\hat{b}_v(b) \geq b \;\forall\; b$ when the supervisor sends a signal $d = v$, and $\hat{b}_f(b) \leq b \;\forall\; b$ when the supervisor's signal is $d = f$. That is, in the case where $d = v$, any investor's expected utility when deciding prudently increases, compared to the situation without supervisory information disclosure. Instead, when supervisor sends $d = f$, any investor's expected utility of prudent behavior will *decrease*. Therefore, on one hand, investors may prefer to adjust their investment decisions according to the signal received. The supervisor, on the other, is able to affect the investors' expected utility and their choice of actions in the desired way by applying an optimally designed disclosure mechanism.

#### 2.4.4 Bayesian Plausibility and the value of informative disclosure

The standard persuasion literature, which considers games with a single sender and a single receiver, argues that Bayesian Plausibility (BP) is the only restriction imposed on a sender's mechanism.[21] This implies that the expected posteriors must be equal to the *objective probability* of a specific situation, in an immediate and simple way, as already illustrated in the digression above.

The present model now adds a special twist to the basic setup: with an infinitely large number of investors and with heterogenous prior beliefs, we are now able to generalize BP to find an equilibrium condition. This becomes evident by taking a closer look at the formal representation of BP for the single investor (receiver) case:

$$\mu_b(V|v)Pr(v) + \mu_b(V|f)Pr(f) = p \tag{11}$$

with $Pr(v) = \pi(v|V) \cdot p + \pi(v|F) \cdot (1-p)$ and $Pr(f) = \pi(f|V) \cdot p + \pi(f|F) \cdot (1-p)$.

Although (11) is written for an arbitrary realization $b$ of investors' prior beliefs, it can be shown that (11) does not hold for all (heterogenous) $b$ simultaneously.

[21] See Sobel (2010), p. 20, and Kamenica and Gentzkow (2011), p. 2596.

Calculating the posterior beliefs and taking into account the previous definition of precision of the supervisor's disclosure mechanism, we can rewrite (11)[22]

$$(\tfrac{1}{2}+x)\,b\frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b(x+y))}+(\tfrac{1}{2}-x)\,b\frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b(x+y))} \;=\; p. \qquad (12)$$

Recall that in the basic persuasion model of KG, Bayesian Plausibility (11) requires that posterior beliefs need to be unbiased *in expected terms*. That is, for a persuasion mechanism to work properly, the sender needs to make sure that beliefs are not distorted in a way such that receivers would suffer a welfare loss from acting according to information disclosed. Note that the left-hand side of (11) - ie $\mu_b(V|v)Pr(v)+\mu_b(V|f)Pr(f)$ - represents the (single) receiver's expected utility in the case of prudent behavior.[23] If persuasion increases (decreases) this term beyond (below) the objective probability $p$, prudent behavior becomes more (less) attractive for a (single) investor. However, due to the distortion, an investor will find himself *ex post* too often in a situation where his initial decision proves to be incorrect. The receiver then realizes $U_R(P,F)=0$ or $U_R(R,V)=0$ instead of $U_R(P,V)=1$ or $U_R(R,F)=1$ – compared to decision-making based on prior beliefs when ignoring the supervisor's information.

However - and here we differ from the literature such as Wang's (2011) voting model - *this cannot hold for every single investor when there are many investors with heterogenous prior beliefs and when the supervisor's disclosure mechanism is publicly known*, i.e. $(x,y)$ stays the same for all investors. In this situation (11) - or (12) for our particular situation - can only hold for an investor whose prior belief $b$ is exactly identical to the objective probability $p$ of a vulnerable banking sector. That is, except for this latter investor, individual prior beliefs of all other investors *will appear more or less distorted* compared to the objective probability $p$.

For instance, all investors with $b<(>)p$ believe that the probability of a vulnerable banking sector is lower (higher) than $p$. For all these "distorted" investors, an informative disclosure mechanism may correct their individual distortions to *some*

[22] Due to previous definitions we have:

$$\begin{aligned}
\mu_b(V|v) &= \frac{(\frac{1}{2}+x)\,b}{(\frac{1}{2}+x)\,b+(\frac{1}{2}-y)\,(1-b)} = \frac{(\frac{1}{2}+x)\,b}{\frac{1}{2}-(y-b(x+y))}\\
\mu_b(V|f) &= \frac{(\frac{1}{2}-x)\,b}{(\frac{1}{2}-x)\,b+(\frac{1}{2}+y)\,(1-b)} = \frac{(\frac{1}{2}-x)\,b}{\frac{1}{2}+(y-b(x+y))}\\
Pr(v) &= (\tfrac{1}{2}+x)\,p+(\tfrac{1}{2}-y)\,(1-p) = \tfrac{1}{2}-(y-p(x+y))\\
Pr(f) &= (\tfrac{1}{2}-x)\,p+(\tfrac{1}{2}+y)\,(1-p) = \tfrac{1}{2}+(y-p(x+y)).
\end{aligned}$$

[23] In section 2.4.3 it was shown that an investor's utility of a specific action $a$ is given by the (posterior) belief regarding the status of the banking sector.

degree. Put differently: an informative disclosure mechanism of the supervisor generates expected posterior beliefs that are higher (lower) than the individual priors of investors in the case of $b < p$ ($b > p$). We so state and prove

**Lemma 2** *Bayesian Plausibility in our model with multiple investors (receivers) and heterogenous investor prior beliefs requires*

$$\mu_b(V|v)Pr(v) + \mu_b(V|f)Pr(f) = p \;\Leftrightarrow\; b = p.$$

**Proof:** See the Appendix. ■

From the investors' point of view information disclosure suggests that their individual priors understate or overstate the probability of a vulnerable banking sector in the case of $b < p$ or $b > p$, respectively:

$$\left(\tfrac{1}{2}+x\right) b \frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b(x+y))} + \left(\tfrac{1}{2}-x\right) b \frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b(x+y))} \;>\; b \qquad \forall\, b < p \tag{13}$$

or

$$\left(\tfrac{1}{2}+x\right) b \frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b(x+y))} + \left(\tfrac{1}{2}-x\right) b \frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b(x+y))} \;<\; b \qquad \forall\, b > p \tag{14}$$

For these "distorted" investors, the supervisor's disclosure mechanism now corrects (part of) this distortion of prior beliefs *in a way that investor beliefs move toward the true probability of a vulnerable banking sector.* As a consequence, information disclosure helps investors to make correct investment decisions. Against this background, the value of an informative disclosure mechanism for investors becomes evident:

**Corollary 3** *A disclosure mechanism that is Bayesian plausible according to Lemma 2 raises the expected utility of investors whose prior beliefs deviate from the objective probability of a vulnerable banking sector.*

**Proof:** Note that the left-hand side of (12) represents an investor's expected utility from behaving prudently, acting according to the supervisor's information disclosure, and having prior belief $b$. Under a Bayesian plausible disclosure mechanism, then, from the proof of Lemma 2 it is immediately clear that informative disclosure by the supervisor increases the expected utility of prudent behavior for all investors who understate the probability of the banking sector being vulnerable ($b < p$). That is, informative disclosure better aligns the evaluations of latter investors with the true state of the banking system. Moreover, informative disclosure causes investors with $\mu_b(V|v) \geq \frac{1}{2} > b$ to switch from a risky to a prudent investment strategy.

The opposite effect appears to be the case with investors whose priors overstate the true vulnerability of the banking sector. Their expected utility from prudent behavior decreases under a Bayesian plausible disclosure mechanism (see proof of Lemma 2). As a result, investors with $\mu_b(V|v) < \frac{1}{2} \leq b$ switch from a prudent to a risky investment strategy.

For both situations, the proof of Lemma 2 (see Appendix B) shows that under a Bayesian-plausible disclosure mechanism the expected utility of those investors, which change their investment strategy in response to supervisory disclosure, will increase. $\blacksquare$

In other words, under a Bayesian plausible disclosure mechanism, "distorted" investors' expected utility will increase when they base their decisions on supervisory information instead of prior beliefs. This result is novel to the literature on the disclosure of supervisory information in the banking sector.

# 3 Optimal disclosure

We now turn to optimality analysis. We now ask whether the supervisor should optimally implement an informative stress testing (disclosure) mechanism. For this purpose, we first state the supervisor's optimization problem before, and then the existence of an informative disclosure mechanism, to derive some welfare implications.

## 3.1 The supervisor's problem

The supervisor's goal is to maximize his utility while taking into account all the factors that were analyzed in the previous sections. Note that the supervisor's utility function, has a unique maximum when the number of prudent investors is exactly $|P|^{max}$. Therefore, when the supervisor finds himself in a situation where the investors' prior beliefs result in[24]

$$|P|^{max} = 1 - G(\tfrac{1}{2}),$$

the supervisor will refrain from implementing any informative disclosure mechanism as this cannot increase his utility.

In the following we consider situations when prior beliefs generate numbers of prudent investors which deviate from $|P|^{max}$. In this context there are two possibilities: either we have $1 - G(\frac{1}{2}) < |P|^{max}$ (case a)) or we have $1 - G(\frac{1}{2}) > |P|^{max}$ (case

[24]Note that in section 2.4.3 we argued that the unique threshold when investors switch from a risky to a prudent investment strategy is $b_T = \frac{1}{2}$.

b)). Against this background the supervisor's objective is to design an informative disclosure mechanism – by choosing $x$ and $y$ – such that the distance between the supervisor's maximum utility and the expected utility realized by informative disclosure is minimized:

$$
\begin{aligned}
\min_{x,y}\ \Delta U_S \quad &\equiv \quad Pr(v)\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) + Pr(f)\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2}))\right)\\
\text{with} \quad & \quad Pr(v) = (\tfrac{1}{2}+x)\,p + (\tfrac{1}{2}-y)\,(1-p)\\
& \quad Pr(f) = (\tfrac{1}{2}-x)\,p + (\tfrac{1}{2}+y)\,(1-p) \qquad (15)\\
& \quad x, y \in (0, \tfrac{1}{2}).
\end{aligned}
$$

Recall that, in optimization problem (15), $U_S^{max}$ denotes the supervisor's maximum possible utility, which is achieved when $1 - G(\frac{1}{2}) = |P|^{max}$. Furthermore $1 - \hat{G}_v(\frac{1}{2})$ and $1 - \hat{G}_f(\frac{1}{2})$ denote the number of prudent investors when the supervisor sends signals $d = v$ and $d = f$, respectively. Bayesian Plausibility is implicitly considered in the cumulative distribution functions of the investors' posterior beliefs $\hat{G}_v(\cdot)$ and $\hat{G}_f(\cdot)$.[25]

## 3.2 Optimality of informative disclosure

The analysis of optimal information disclosure starts with the derivation of the first-order necessary conditions of optimization problem (15):

$$
\begin{aligned}
\frac{\partial \Delta U_S}{\partial x} \quad &= \quad p\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) - p\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2}))\right) +\\
& \quad + Pr(v)U_S'(1-\hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial x} + Pr(f)U_S'(1-\hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial x} = 0 \qquad (16)\\
\frac{\partial \Delta U_S}{\partial y} \quad &= \quad -(1-p)\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) + (1-p)\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2}))\right) +\\
& \quad + Pr(v)U_S'(1-\hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial y} + Pr(f)U_S'(1-\hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial y} = 0. \qquad (17)
\end{aligned}
$$

The following considerations, which are essential to our analysis, build on a number of insights regarding cumulative distribution functions $\hat{G}_v$ and $\hat{G}_f$ to which we refer in great detail in section 2.4.2 as well as in the Appendix. First, it should be noted that the numbers of prudent investors are determined by the cumulative distribution functions of prior and posterior beliefs and that – due to the features of these functions – the following relation always holds:

$$1 - \hat{G}_f(\tfrac{1}{2}) \leq 1 - G(\tfrac{1}{2}) \leq 1 - \hat{G}_v(\tfrac{1}{2}).$$

[25] See the Appendix for a formal proof.

Second, for the partial derivatives of the cumulative distribution functions of investor beliefs setting $\gamma = \frac{1}{2}$ (see appendix) yields

$$\begin{aligned} \frac{\partial \hat{G}_v(\frac{1}{2})}{\partial x} &= -g\left(\frac{\frac{1}{2}-y}{1+x-y}\right)\frac{\frac{1}{2}-y}{[1+x-y]^2} \leq 0 \\ \frac{\partial \hat{G}_v(\frac{1}{2})}{\partial y} &= -g\left(\frac{\frac{1}{2}-y}{1+x-y}\right)\frac{\frac{1}{2}+x}{[1+x-y]^2} < 0 \end{aligned} \quad \text{and} \quad \begin{aligned} \frac{\partial \hat{G}_f(\frac{1}{2})}{\partial x} &= g\left(\frac{\frac{1}{2}+y}{1-x+y}\right)\frac{\frac{1}{2}+y}{[1-x+y]^2} > 0 \\ \frac{\partial \hat{G}_f(\frac{1}{2})}{\partial y} &= g\left(\frac{\frac{1}{2}+y}{1-x+y}\right)\frac{\frac{1}{2}-x}{[1-x+y]^2} \geq 0 \end{aligned} \tag{18}$$

with

$$\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial y} < \frac{\partial \hat{G}_v(\frac{1}{2})}{\partial x} \leq 0 \quad \text{and} \quad \frac{\partial \hat{G}_f(\frac{1}{2})}{\partial x} > \frac{\partial \hat{G}_f(\frac{1}{2})}{\partial y} \geq 0 \ . \tag{19}$$

We are now able to derive the following lemma from the first-order necessary conditions (16) and (17):

**Lemma 4** *It is never optimal for the supervisor to implement an informative disclosure mechanism that either shifts $1 - \hat{G}_v(\frac{1}{2})$ beyond $|P|^{max}$ in case a), i.e. in the case when $1 - G(\frac{1}{2}) < |P|^{max}$, or that shifts $1 - \hat{G}_f(\frac{1}{2})$ below $|P|^{max}$ in case b), i.e. in the case when $1 - G(\frac{1}{2}) > |P|^{max}$.*

In words: it is not optimal for the supervisor to disclose information in a way that in the case of a signal $d = f$ the number of prudent investors is below $|P|^{max}$, whereas in the case of the signal $d = v$ the number of prudent investors is beyond $|P|^{max}$.

**Proof:** Assume, to the contrary, that the informative disclosure mechanism causes $1 - \hat{G}_f(\frac{1}{2}) < |P|^{max}$ and $1 - \hat{G}_v(\frac{1}{2}) > |P|^{max}$. This implies $U_S'(1 - \hat{G}_f(\frac{1}{2})) > 0$ and $U_S'(1 - \hat{G}_v(\frac{1}{2})) < 0$. Then, for the first-order condition (16) to hold

$$U_S^{max} - U_S(1 - \hat{G}_v(\tfrac{1}{2})) < U_S^{max} - U_S(1 - \hat{G}_f(\tfrac{1}{2}))$$

is required to hold because of relations (19) and $Pr(v), Pr(f) > 0$.

Analogous reasoning for the other first-order condition (17), reveals that this holds only if

$$U_S^{max} - U_S(1 - \hat{G}_v(\tfrac{1}{2})) > U_S^{max} - U_S(1 - \hat{G}_f(\tfrac{1}{2})),$$

a contradiction. Therefore, it must be true that an optimal disclosure mechanism ensures $1 - \hat{G}_f(\frac{1}{2}) \leq 1 - \hat{G}_v(\frac{1}{2}) \leq |P|^{max}$ in case of $1 - G(\frac{1}{2}) < |P|^{max}$ and $|P|^{max} \leq 1 - \hat{G}_f(\frac{1}{2}) \leq 1 - \hat{G}_v(\frac{1}{2})$ in case of $1 - G(\frac{1}{2}) > |P|^{max}$. ■

Building on the above lemma, we now state our main result in the following proposition:

**Proposition 1** *In either case, i.e. in case a) as well as in case b), there exists an unique optimum $(x^*, y^*)$ with $0 < x^*, y^* < \frac{1}{2}$. In other words, in the case where*

$1 - G(\frac{1}{2}) < |P|^{max}$ *as well as in the case where* $1 - G(\frac{1}{2}) > |P|^{max}$ *there exists an informative (but not fully revealing) disclosure mechanism that minimizes the distance between the supervisor's maximum utility and the expected utility arising from informative disclosure.*

**Proof:** Consider case a) first. If the disclosure mechanism is informative, it is easy to see that by the first result above we have $1 - \hat{G}_f(\frac{1}{2}) < 1 - G(\frac{1}{2}) < 1 - \hat{G}_v(\frac{1}{2}) \leq |P|^{max}$ must be true. The corresponding utilities for the supervisor are, then, ranked $U_S(1 - \hat{G}_f(\frac{1}{2})) < U_S(1 - G(\frac{1}{2})) < U_S(1 - \hat{G}_v(\frac{1}{2})) \leq U_S^{max}$. This implies $0 < U_S^{max} - U_S(1 - \hat{G}_v(\frac{1}{2})) < U_S^{max} - U_S(1 - \hat{G}_f(\frac{1}{2}))$ and

$$\begin{aligned} p\left(U_S^{max} - U_S(1 - \hat{G}_v(\tfrac{1}{2}))\right) - p\left(U_S^{max} - U_S(1 - \hat{G}_f(\tfrac{1}{2}))\right) &< 0 \\ &\text{and} \\ -(1-p)\left(U_S^{max} - U_S(1 - \hat{G}_v(\tfrac{1}{2}))\right) + (1-p)\left(U_S^{max} - U_S(1 - \hat{G}_f(\tfrac{1}{2}))\right) &> 0 \end{aligned}$$

for the first lines of the first-order conditions (16) and (17), respectively. Therefore, the first-order condition (16) hold if and only if

$$Pr(v)U_S'(1 - \hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial x} + Pr(f)U_S'(1 - \hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial x} > 0,$$

and the first-order conditions (17) holds if and only if

$$Pr(v)U_S'(1 - \hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial y} + Pr(f)U_S'(1 - \hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial y} < 0.$$

These conditions, however, are not contradictory since relations (18) and $U_S'(1 - \hat{G}_v(\frac{1}{2})), U_S'(1 - \hat{G}_f(\frac{1}{2})) > 0$ in the present case imply

$$\begin{aligned} 0 \geq Pr(v)U_S'(1 - \hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial x} &> Pr(v)U_S'(1 - \hat{G}_v(\tfrac{1}{2}))\frac{\partial \hat{G}_v(\frac{1}{2})}{\partial y} \\ &\text{and} \\ 0 \leq Pr(f)U_S'(1 - \hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial y} &< Pr(f)U_S'(1 - \hat{G}_f(\tfrac{1}{2}))\frac{\partial \hat{G}_f(\frac{1}{2})}{\partial x}. \end{aligned}$$

For case b) the proof is analogous to the that of case a). The only difference with the previous case is that the relationships between terms are reversed and marginal utilities are negative, i.e. $U_S'(1 - \hat{G}_v(\frac{1}{2})), U_S'(1 - \hat{G}_f(\frac{1}{2})) < 0$.

Lastly, it follows immediately from Lemma 1 that the optimal disclosure mechanism cannot be fully revealing, ie $(x^*, y^*) \neq (\frac{1}{2}, \frac{1}{2})$: a fully revealing disclosure mechanism implies that $1 - \hat{G}_f(\frac{1}{2}) = 0 < |P|^{max}$ and $1 - \hat{G}_v(\frac{1}{2}) = 1 > |P|^{max}$. Using Lemma 1, it is easy to see that this cannot be optimal. ■

In the previous analysis we have excluded corner solutions, limiting $p$ to values of less then 1. Our former results are valid as long as there is no crisis going on in the banking sector. For the case when the banking sector is hurt by a systemic crisis, we can easily show that the supervisor will not anymore apply an informative disclosure mechanism. When the banking sector is hit by a crisis, the *objective probability* of a vulnerable banking sector will approach unity.

To illustrate, let $p = 1$, from which $Pr(v) = \pi(v|V)$ and $Pr(f) = \pi(f|F)$. Moreover, using equation (2) we find that $\mu_b(V|v) = 1$ and $\mu_b(V|f) = 1$ for investors' posterior beliefs when $b = p = 1$. Applying Bayesian Plausibility (equation (11) and *Lemma* 2) now requires $Pr(v) = \pi(v|V) = \frac{1}{2}$ and $Pr(f) = \pi(f|F) = \frac{1}{2}$. This implies that $Pr(v) = \pi(f|V) = \frac{1}{2}$ and $Pr(f) = \pi(v|F) = \frac{1}{2}$. The outcome in case of an ongoing banking crisis is that the supervisor's disclosure will optimally have to remain uninformative – a result perfectly in line with a number of recent observations made during the subprime crisis of 2007-2009, as well as during the sovereign crisis since 2010.[26]

### 3.3 Welfare

Our welfare implications follow immediately from Proposition 1 and Corollary 3. Recall that in Section 2 we argued that due to Bayesian Plausibility and the investors' decision-making process, investors either *gain* from information disclosure or realize *at least the same utility* as in a situation without informative disclosure. Moreover, supervisors always gain as there exists an optimal informative disclosure mechanism with $0 < x^*, y^* < \frac{1}{2}$ that minimizes the distance between the supervisor's maximum possible utility (for a given objective probability $p$) and the expected utility arising from informative disclosure, which has been shown above in section 3. In sum, total welfare increases as a consequence of the supervisor's optimal information disclosure mechanism.

## 4 Conclusion

The goal of this paper is to deliver a rationale for macro stress tests and to explain why public supervisors use such a design. We so shed new light on the discussion on transparency and financial stability, which is central to the current financial debate. The paper has shown that disclosing two pieces of information, namely the signal generating process together with a signal, permits institutional supervisors (or banking authorities) to persuade investors (Bayesian receivers) to act in a welfare-enhancing way. Our paper shows that to optimally design a stress test (disclosure)

[26] See Horvath and Vasco (2012).

mechanism for a continuum of investors, public supervisors will recur to a variety of stress tests and disclose their information to the public, to capture the varying performance of the underlying banking sector.

The disclosure mechanism that we offer reveals some attractive features. It shows a unique interior optimum plus other valuable properties that permit a novel explanation why disclosure processes in the banking sector are designed the way they are. Specifically, our results tell practitioners that the design of disclosure mechanisms such as macro stress tests will induce Bayesian investors to update their beliefs in a direction consistent with the banking authority. In addition, we have shown that for the borderline case of a systemic crisis, disclosure should be optimally uninformative.

While our results are robust and applicable to a wider range of real-world situations, it seems worthwhile to mention related research that this paper may open up and complement, while pointing out differences that may lead to new avenues of research. Note first that in our model, and different from Wang (2012), disclosure is always welfare enhancing. Typically, in our multi-receiver setting and different from KG, the sender will never use a value of zero for one of the parameters $x$ and $y$. That is, every signal carries additional information. With one of the two parameters set to zero, information would only be conveyed indirectly as solely the remaining parameter can change the distribution of posterior beliefs.

Our setting can also be extended to include banks as players, which may then be differentiated along a new (type-) dimension. In such a model, the supervisor would become the middleman. One possible advantage of such a setup would consist in the option to segment the investment market according to bank types and investor types, involving even more differentiated disclosure processes handled by the supervisor. Needless to say, such a treatment would come at the cost of reduced tractability. Given the already high degree of complexity that we have reached so far, we will leave this avenue, as well, for future research.

# A Proof of Lemma 1: FSD shift of investor beliefs distribution

Consider the posteriors of any investor with any prior belief $b$

$$\begin{aligned}
\hat{b}_v(b) &= \mu_b(V|v) = \frac{\left(\frac{1}{2}+x\right)b}{\left(\frac{1}{2}+x\right)b+\left(\frac{1}{2}-y\right)(1-b)} \\
\hat{b}_f(b) &= \mu_b(V|f) = \frac{\left(\frac{1}{2}-x\right)b}{\left(\frac{1}{2}-x\right)b+\left(\frac{1}{2}+y\right)(1-b)}
\end{aligned}$$

where we used the notion of signal precision as defined in section 2.4.2. Changing the precision parameters $x \in [0, \frac{1}{2}]$ and $y \in [0, \frac{1}{2}]$, the mechanism has the following general effect on posterior beliefs:

$$\begin{aligned}
\frac{\partial \hat{b}_v(b)}{\partial x} &= \frac{\left(\frac{1}{2}-y\right)(1-b)b}{\left[\left(\frac{1}{2}+x\right)b+\left(\frac{1}{2}-y\right)(1-b)\right]^2} \geq 0 \\
\frac{\partial \hat{b}_f(b)}{\partial x} &= -\frac{\left(\frac{1}{2}+y\right)(1-b)b}{\left[\left(\frac{1}{2}-x\right)b+\left(\frac{1}{2}+y\right)(1-b)\right]^2} < 0 \\
\frac{\partial \hat{b}_v(b)}{\partial y} &= \frac{\left(\frac{1}{2}+x\right)(1-b)b}{\left[\left(\frac{1}{2}+x\right)b+\left(\frac{1}{2}-y\right)(1-b)\right]^2} > 0 \\
\frac{\partial \hat{b}_f(b)}{\partial y} &= -\frac{\left(\frac{1}{2}-x\right)(1-b)b}{\left[\left(\frac{1}{2}-x\right)+\left(\frac{1}{2}+y\right)(1-b)\right]^2} \leq 0
\end{aligned}$$

where the first and the last line become equal to zero when $y=\frac{1}{2}$ and $x=\frac{1}{2}$, respectively. As a result, a higher level of $x$ implies, *ceteris paribus*, $\hat{b}_v(b) \geq b$ and $\hat{b}_f(b) < b$ for any $b$ whereas a higher level of $y$ implies, *ceteris paribus*, $\hat{b}_v(b) > b$ and $\hat{b}_f(b) \leq b$ for any $b$. That is, from a formal perspective the disclosure mechanism $(D, \{\pi|\cdot\}_{\theta\in\Theta})$ is a monotonic transformation of investor beliefs.[27]

Let us now denote $\hat{g}_v(b)$ and $\hat{g}_f(b)$ the distribution functions of investors' posterior beliefs when the supervisor sends $d=v$ and $d=f$, respectively. Given the impact of the signaling mechanism on investor beliefs above, the distribution functions of posteriors can be determined to be:

$$\hat{g}_v(b) \;:\; \hat{b}_v(b) \longmapsto g(b)\ \forall\ b \tag{20}$$

$$\hat{g}_f(b) \;:\; \hat{b}_f(b) \longmapsto g(b)\ \forall\ b. \tag{21}$$

The corresponding cumulative distribution functions are, by definition,

$$\begin{aligned}
\hat{G}_v(\gamma) &= \int_0^\gamma \hat{g}_v(\hat{b}_v(b))d\hat{b}_v(b) \\
\hat{G}_f(\gamma) &= \int_0^\gamma \hat{g}_f(\hat{b}_f(b))d\hat{b}_f(b).
\end{aligned}$$

Applying the definitions of $\hat{b}_v(b)$ and $\hat{b}_f(b)$ above allows for the calculation of these cumulative distribution functions based on the distribution of prior beliefs:

$$\hat{G}_v(\gamma) = \int_0^{\frac{\left(\frac{1}{2}-y\right)\gamma}{\left(\frac{1}{2}+x\right)(1-\gamma)+\left(\frac{1}{2}-y\right)\gamma}} g(b)db \tag{22}$$

$$\hat{G}_f(\gamma) = \int_0^{\frac{\left(\frac{1}{2}+y\right)\gamma}{\left(\frac{1}{2}-x\right)(1-\gamma)+\left(\frac{1}{2}+y\right)\gamma}} g(b)db. \tag{23}$$

[27]Note that $x$ and $y$ reinforce each other regarding the impact on investor posteriors when a certain signal is received.

Equations (20) and (21) show first that $x \in [0, \frac{1}{2}]$ and $y \in [0, \frac{1}{2}]$ represent parameters which determine the upper limit of the integrals. Therefore the cumulative distributions may be considered to be conditional on $x$ and $y$, denoted

$$\hat{G}_v(\gamma) \equiv \hat{G}_v(\gamma|x,y) \text{ and } \hat{G}_f(\gamma) \equiv \hat{G}_f(\gamma|x,y) \;\forall\; \gamma.$$

Second, the impact of $x$ and $y$ on the cumulative distributions of posteriors at any $\gamma \in [0,1]$ can be determined by calculating the partial derivatives:

$$\begin{aligned}
\frac{\partial \hat{G}_v(\gamma|x,y)}{\partial x} &= -g\left(\frac{\left(\frac{1}{2}-y\right)\gamma}{\left(\frac{1}{2}+x\right)(1-\gamma)+\left(\frac{1}{2}-y\right)\gamma}\right)\frac{\left(\frac{1}{2}-y\right)(1-\gamma)\gamma}{\left[\left(\frac{1}{2}+x\right)(1-\gamma)+\left(\frac{1}{2}-y\right)\gamma\right]^2} \leq 0\\
\frac{\partial \hat{G}_v(\gamma|x,y)}{\partial y} &= -g\left(\frac{\left(\frac{1}{2}-y\right)\gamma}{\left(\frac{1}{2}+x\right)(1-\gamma)+\left(\frac{1}{2}-y\right)\gamma}\right)\frac{\left(\frac{1}{2}+x\right)(1-\gamma)\gamma}{\left[\left(\frac{1}{2}+x\right)(1-\gamma)+\left(\frac{1}{2}-y\right)\gamma\right]^2} < 0\\
\frac{\partial \hat{G}_f(\gamma|x,y)}{\partial x} &= g\left(\frac{\left(\frac{1}{2}+y\right)\gamma}{\left(\frac{1}{2}-x\right)(1-\gamma)+\left(\frac{1}{2}+y\right)\gamma}\right)\frac{\left(\frac{1}{2}+y\right)(1-\gamma)\gamma}{\left[\left(\frac{1}{2}-x\right)(1-\gamma)+\left(\frac{1}{2}+y\right)\gamma\right]^2} > 0\\
\frac{\partial \hat{G}_f(\gamma|x,y)}{\partial y} &= g\left(\frac{\left(\frac{1}{2}+y\right)\gamma}{\left(\frac{1}{2}-x\right)(1-\gamma)+\left(\frac{1}{2}+y\right)\gamma}\right)\frac{\left(\frac{1}{2}-x\right)(1-\gamma)\gamma}{\left[\left(\frac{1}{2}-x\right)(1-\gamma)+\left(\frac{1}{2}+y\right)\gamma\right]^2} \geq 0
\end{aligned}$$

where die inequalities follow from $g(\cdot) > 0$, $x, y \in [0, \frac{1}{2}]$, and $\gamma \in [0,1]$.

# B Proof of Lemma 2: Bayesian Plausibility

Let $b_{BP}$ denote the prior belief of the investor for which (12) holds.

Consider the situation $b < b_{BP}$ first. For a given decision $(x,y)$ of the supervisor we observe $y - b(x+y) > y - b_{BP}(x+y)$ which implies $\frac{1}{2} - (y - b(x+y)) < \frac{1}{2} - (y - b_{BP}(x+y))$ and $\frac{1}{2} + (y - b(x+y)) > \frac{1}{2} + (y - b_{BP}(x+y))$. For the fraction terms in (12) we therefore find

$$\begin{aligned}
\frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b(x+y))} &> \frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b_{BP}(x+y))}\\
&\text{and}\\
\frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b(x+y))} &< \frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b_{BP}(x+y))}.
\end{aligned}$$

Since $\left(\frac{1}{2}+x\right)b \geq \left(\frac{1}{2}-x\right)b$ – and note that $b \in [0,1]$ and $x \in [0, \frac{1}{2}]$ – we have

$$\left(\tfrac{1}{2}+x\right)b\frac{\frac{1}{2}-(y-p(x+y))}{\frac{1}{2}-(y-b(x+y))}+\left(\tfrac{1}{2}-x\right)b\frac{\frac{1}{2}+(y-p(x+y))}{\frac{1}{2}+(y-b(x+y))} > b \qquad \forall\; b < b_{BP}.$$

Regarding the situation $b > b_{BP}$ the arguments are analogous but the previous relations turn in the opposite direction. That is, for a given decision $(x,y)$ we observe $y - b(x+y) < y - b_{BP}(x+y)$, $\frac{1}{2} - (y - b(x+y)) > \frac{1}{2} - (y - b_{BP}(x+y))$ and $\frac{1}{2} + (y - b(x+y)) < \frac{1}{2} + (y - b_{BP}(x+y))$. As a consequence the relations between the fraction terms in (12) are:

$$\frac{\frac{1}{2} - (y - p(x+y))}{\frac{1}{2} - (y - b(x+y))} \quad < \quad \frac{\frac{1}{2} - (y - p(x+y))}{\frac{1}{2} - (y - b_{BP}(x+y))}$$

and

$$\frac{\frac{1}{2} + (y - p(x+y))}{\frac{1}{2} + (y - b(x+y))} \quad > \quad \frac{\frac{1}{2} + (y - p(x+y))}{\frac{1}{2} + (y - b_{BP}(x+y))}.$$

With $(\frac{1}{2} + x)\, b \geq (\frac{1}{2} - x)\, b$ we finally have in the current situation:

$$(\tfrac{1}{2} + x)\, b \frac{\frac{1}{2} - (y - p(x+y))}{\frac{1}{2} - (y - b(x+y))} + (\tfrac{1}{2} - x)\, b \frac{\frac{1}{2} + (y - p(x+y))}{\frac{1}{2} + (y - b(x+y))} < b \qquad \forall\, b > b_{BP}.$$

Moreover, the above arguments actually prove that $b_{BP} = p$ is the only feasible opportunity to make persuasion work: Note that for $(x, y) = (0, 0)$ Bayesian Plausibility (12) holds for any possible $b$. This is trivial because $(x, y) = (0, 0)$ means that the disclosure mechanism is completely non-informative and investors' posteriors are equivalent to their prior beliefs.

Conversely, in the case where $(x, y) \neq (0, 0)$ it is easily verified that Bayesian Plausibility (to reach a high degree of transparency) holds if and only if

$$\frac{\frac{1}{2} - (y - p(x+y))}{\frac{1}{2} - (y - b(x+y))} = \frac{\frac{1}{2} + (y - p(x+y))}{\frac{1}{2} + (y - b(x+y))}$$

which requires $b = p$ to hold. In words: in the current context Bayesian Plausibility needs to be met only for an investor whose prior belief $b$ equals the *objective* probability for a vulnerable banking sector $p$.

# C Bayesian Plausibility in the supervisor's optimization problem

Consider the supervisor's problem in an explicit form, i.e. including the Bayesian Plausibility constraint (BP):

$$\begin{aligned}
\min_{x,y} \ \Delta U_S \quad &\equiv \quad Pr(v) \left( U_S^{max} - U_S(1 - \hat{G}_v(\tfrac{1}{2})) \right) + Pr(f) \left( U_S^{max} - U_S(1 - \hat{G}_f(\tfrac{1}{2})) \right) \\
\text{with} \quad & \quad Pr(v)\hat{b}_v(p) + Pr(f)\hat{b}_f(p) = p \quad (BP) \\
& \quad Pr(v) = (\tfrac{1}{2} + x)\, p + (\tfrac{1}{2} - y)\, (1 - p) \\
& \quad Pr(f) = (\tfrac{1}{2} - x)\, p + (\tfrac{1}{2} + y)\, (1 - p) \qquad (24) \\
& \quad x, y \in [0, \tfrac{1}{2}].
\end{aligned}$$

Starting from the corresponding Lagrangean

$$\begin{aligned}\mathcal{L} &= Pr(v)\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) + Pr(f)\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2}))\right) + \\ &\quad +\lambda\left[Pr(v)\hat{b}_v(p) + Pr(f)\hat{b}_f(p) - p\right]\end{aligned}$$

and using the Kuhn-Tucker Theorem yields the following first-order necessary conditions:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial x} &= p\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) - p\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2})\right) + \\ &\quad +Pr(v)U_S'(1-\hat{G}_v(\tfrac{1}{2}))\frac{\partial\hat{G}_v(\frac{1}{2})}{\partial x} + Pr(f)U_S'(1-\hat{G}_f(\tfrac{1}{2}))\frac{\partial\hat{G}_f(\frac{1}{2})}{\partial x} + \\ &\quad +\lambda\left[p\hat{b}_v(p) + Pr(v)\frac{\partial\hat{b}_v(p)}{\partial x} - p\hat{b}_f(p) + Pr(f)\frac{\partial\hat{b}_f(p)}{\partial x}\right] \geq 0; \\ &\quad x \geq 0\ ;\ \frac{\partial\mathcal{L}}{\partial x}x = 0 \qquad (25) \\ \frac{\partial\mathcal{L}}{\partial y} &= -(1-p)\left(U_S^{max} - U_S(1-\hat{G}_v(\tfrac{1}{2}))\right) + (1-p)\left(U_S^{max} - U_S(1-\hat{G}_f(\tfrac{1}{2})\right) + \\ &\quad +Pr(v)U_S'(1-\hat{G}_v(\tfrac{1}{2}))\frac{\partial\hat{G}_v(\frac{1}{2})}{\partial y} + Pr(f)U_S'(1-\hat{G}_f(\tfrac{1}{2}))\frac{\partial\hat{G}_f(\frac{1}{2})}{\partial y} + \\ &\quad +\lambda\left[-(1-p)\hat{b}_v(p) + Pr(v)\frac{\partial\hat{b}_v(p)}{\partial y} + (1-p)\hat{b}_f(p) + Pr(f)\frac{\partial\hat{b}_f(p)}{\partial y}\right] \geq 0; \\ &\quad y \geq 0\ ;\ \frac{\partial\mathcal{L}}{\partial y}y = 0. \qquad (26) \\ \frac{\partial\mathcal{L}}{\partial\lambda} &= Pr(v)\hat{b}_v(p) + Pr(f)\hat{b}_f(p) - p = 0. \qquad (27)\end{aligned}$$

Inspection of terms in square brackets in (25) and (26), which are the derivatives of (BP), shows that they both equal zero: using the explicit formulations of $Pr(v)$,

$Pr(f)$, $\hat{b}_v(p)$, $\hat{b}_f(p)$, $\frac{\partial \hat{b}_v(p)}{\partial x}$, $\frac{\partial \hat{b}_f(p)}{\partial x}$, $\frac{\partial \hat{b}_v(p)}{\partial y}$, and $\frac{\partial \hat{b}_f(p)}{\partial y}$ (see the proof of Lemma 1) yields

$$\begin{aligned}
& \left[p\hat{b}_v(p) + Pr(v)\frac{\partial \hat{b}_v(p)}{\partial x} - p\hat{b}_f(p) + Pr(f)\frac{\partial \hat{b}_f(p)}{\partial x}\right] = \\
= \;\; & p\left[\frac{(\frac{1}{2}+x)p + (\frac{1}{2}-y)(1-p)}{(\frac{1}{2}+x)p + (\frac{1}{2}-y)(1-p)} - \frac{(\frac{1}{2}-x)p + (\frac{1}{2}+y)(1-p)}{(\frac{1}{2}-x)p + (\frac{1}{2}+y)(1-p)}\right] = \\
= \;\; & p\left[1-1\right] = 0 \\
& \text{and} \\
& \left[-(1-p)\hat{b}_v(p) + Pr(v)\frac{\partial \hat{b}_v(p)}{\partial y} + (1-p)\hat{b}_f(p) + Pr(f)\frac{\partial \hat{b}_f(p)}{\partial y}\right] = \\
= \;\; & -(1-p)\left[\frac{(\frac{1}{2}+x)p - (\frac{1}{2}+x)p}{(\frac{1}{2}+x)p + (\frac{1}{2}-y)(1-p)} - \frac{(\frac{1}{2}-x)p - (\frac{1}{2}-x)p}{(\frac{1}{2}-x)p + (\frac{1}{2}+y)(1-p)}\right] = \\
= \;\; & -(1-p)\left[0-0\right] = 0
\end{aligned}$$

due to $p \in (0, 1)$. Including Bayesian Plausibility (BP) in the supervisor's optimization problem, hence, does not affect the relevant first-order necessary conditions for the optimum. Rather, calculations show that the probability distributions already comprise the crucial features of (BP).

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