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Taxing a commodity with and without revenue neutrality: An exploration using a calibrated theoretical consumer equilibrium model

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QSEP

RESEARCH INSTITUTE FOR QUANTITATIVE
STUDIES IN ECONOMICS AND POPULATION

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TAXING A COMMODITY WITH AND WITHOUT REVENUE NEUTRALITY: AN
EXPLORATION USING A CALIBRATED THEORETICAL CONSUMER EQUILIBRIUM
MODEL

Frank T. Denton and Dean C. Mountain

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ABSTRACT

It has long been recognized that taxing a commodity that generates negative externalities can be used to reduce the consumption of that commodity. A variant involves the imposition of revenue neutrality but that may alter the tax rate required to meet a consumption reduction target. We explore the relationships among the commodity tax rate, the demand and supply elasticities, and the revenue offsets by calibrating a theoretical consumer equilibrium model and then recalibrating it with alternative parameter configurations. For each configuration we simulate equilibrium for three policy scenarios: no neutrality, neutrality achieved by subsidizing other commodities, and neutrality achieved by income transfer.

Key Words: Consumer Market Equilibrium; Commodity Taxation; Revenue Neutrality

JEL Classification: H23, D11, D58

INTRODUCTION

The use of a tax instrument to discourage the consumption of a commodity that generates negative externalities has a long history in the economics literature. The externalities might be in the form of health damage and higher health care costs (taxation of tobacco products), traffic congestion (toll road fees), or environmental degradation (taxation according to carbon content), for examples. A variant on the straightforward taxation of a designated commodity involves the imposition of a revenue neutrality constraint – a requirement for an offset somewhere else in the tax system so as to leave public revenue unchanged. But a revenue neutrality constraint alters the situation because it changes the rate of taxation necessary to achieve any given consumption reduction target. Just how much it changes that rate depends heavily on the demand and supply elasticities of the targeted commodity and the cross-elasticities with other commodities in the consumer budget. We explore the relationships among the tax rate, the elasticities, and the effects of revenue neutrality by calibrating a theoretical model of consumer market equilibrium and then recalibrating the model so as to span a wide range of parameter configurations. For each parameter configuration we solve the model assuming alternative policy scenarios.

The idea that negative externalities can be internalized by taxing the commodity that generates them and that the purchase price of the offending commodity can thus be brought into line with its real marginal social costs, and welfare thereby enhanced, goes back to Pigou (1920, 1928). This idea and its extension to incorporate revenue neutrality have become important issues in the literature on environmental economics and policy in recent decades. There has been much discussion too in that literature of the possibility of a “double dividend” – a lessening of environmental pollution coupled with a reduction of other welfare distortionary taxes to satisfy the neutrality constraint. (See Goulder, 1995, Sandmo, 2009, for historical reviews and discussion.) Our present paper has nothing to contribute on the welfare side of these issues. Rather we offer an exploration and comparison of the rates of taxation that would be required to meet a particular objective with and without revenue neutrality when the

parameters of the consumer market can vary over a wide range. To put it differently, we take the position of a hypothetical policy maker who is considering the use of commodity taxation (whether based on welfare considerations or ignorant of them) in order to achieve a specified reduction of the consumption of an externality-producing consumer good and who wishes to know how the specified rate of taxation might vary, how it might be affected by revenue neutrality, and what might be the implications of neutrality for tax cuts elsewhere in the revenue system.

The model is theoretical and generic. It represents no particular country. (For examples of country-specific econometric/simulation studies see Symons, Proops, and Gay, 1994, for the U.K., Nichèle and Robin, 1995, for France, Brämlund and Nordström, 2004, for Sweden, West and Williams, 2004, for the U.S.) It envisages two broad classes of consumer goods – or simply two commodities, as we shall call them. One of the commodities generates negative externalities, and is the target of government policy; the other does not. The model incorporates demand and supply equations for the two markets, a system of taxation that includes an income tax and two ad valorem commodity taxes, a tax revenue constraint, and a number of identities required to close the system. It is nonlinear but can be solved by an iterative procedure. We calibrate the model in what we think is a reasonable fashion by defining an initial equilibrium state for the system, with initial parameter values, and later by choosing alternative initial states with alternative values, for comparative experimentation.

We consider three policies in the simulation experiments: a tax on the target commodity without revenue neutrality, a tax with revenue neutrality achieved by a subsidy on the other commodity, and a tax with revenue neutrality achieved by an income transfer (or income tax reduction). We explore the implications of these policies under alternative specifications of parameter values and initial conditions, including alternative expenditure shares, demand price elasticities, expenditure elasticities, and supply elasticities. When the policies are implemented they lead to new equilibrium states and the alternative states can be compared with the initial one, and with each other.

DEVELOPMENT OF THE MODEL

Assume, for simplicity, a population of consumers that is homogeneous in respect of income and all other relevant characteristics. (Heterogeneity introduces some interesting issues but those are for another day.) Consumers consume two classes of goods, which for convenience we label commodity A and commodity B. Consumption of commodity A generates negative externalities and their reduction is the aim of government policy. Commodity B represents all other commodities in the consumer budget.

Demand for the two commodities is represented by an Almost Ideal Demand System (AIDS; Deaton and Muellbauer, 1980) which (because of the homogeneity of consumers) holds at the aggregate level:

$$w_a = \alpha_a + \gamma_{aa} \ln(p_a/p_b) + \beta_a \ln(X/P) \quad w_b = \alpha_b + \gamma_{ba} \ln(p_a/p_b) + \beta_b \ln(X/P)$$

$$\ln P = \alpha_a \ln(p_a/p_b) + \ln p_b + \frac{1}{2} \gamma_{aa} (\ln(p_a/p_b))^2$$

where w stands for income share (with $w_a + w_b = 1$), p stands for purchaser price, X stands for aggregate consumer expenditure, and P is the price deflator required by the AIDS model. In the background is a common utility function, a 2×2 matrix of compensated price elasticities (ϵ_{ij} , $i, j = a, b$), and a pair of income elasticities (η_a , η_b). The elasticities are constrained by symmetry and homogeneity in the underlying demand system, so that

$$\epsilon_{ij} = -\epsilon_{ji}, \quad \epsilon_{ji} = \epsilon_{ij}(w_j/w_i), \quad \text{and} \quad \eta_j = (1 - w_i \eta_i)/w_j \quad (i, j = a, b).$$

Given the expenditure shares, the matrix of price elasticities has one degree of freedom and is thus uniquely determined by specifying one of its elements, say ϵ_{aa} . The elasticities translate into the γ and β parameters as follows:

$$\gamma_{ij} = (\epsilon_{ij} - w_j + \delta_{ij})w_i, \quad \beta_i = (\eta_i - 1)w_i \quad (i, j = a, b)$$

where $\delta_{ij} = 1$ for $i = j$, 0 otherwise. (The translation respects the requirements

$\alpha_a + \alpha_b = 1$, $\gamma_{aa} + \gamma_{ba} = 0$, and $\beta_a + \beta_b = 0$.) The expenditure shares are converted to quantities demanded by $Qd_a = w_a X / p_a$ and $Qd_b = w_b X / p_b$. We note that an Almost Ideal Demand System was used in all four of the country-specific econometric/simulation studies cited earlier. Three of the studies used AIDS; one (Brämlund and Nordström) used QUAIDS, the quadratic extension of AIDS .

The supply sides of the A and B markets are represented by equations of the form $\ln Qs_a = \xi_a + \lambda_a \ln(q_a/q_b)$ and $\ln Qs_b = \xi_b + \lambda_b \ln(q_b/q_a)$ where q_a and q_b are supplier prices, which may differ from purchaser prices. Letting supplies depend on relative prices allows individual firms to shift the A/B composition of what they sell in response to changing relative profit incentives, if they operate in both markets. Alternatively, it allows for firms that sell only one commodity to enter or leave a market, again depending on relative profit incentives. Equilibrium in the supply/demand system is established when both markets clear:

$$Qs_a = Qd_a = Q_a, \quad Qs_b = Qd_b = Q_b.$$

The total income of consumers is set at some level Z . The government levies an income tax at a rate r and consumers save at a rate ϕ . Total consumer expenditure is then given by $X = (1 - \phi)(1 - r)Z$. The saving rate is fixed but the income tax rate is one of the three instruments available to the government for policy implementation. The other instruments are t_a and t_b , commodity taxes on A and B. Purchaser and supplier prices are thus related by $p_a = (1 + t_a)q_a$ and $p_b = (1 + t_b)q_b$. It is convenient for later purposes to think of a reduction of a commodity tax as equivalent to a subsidy. Similarly, it is convenient to think of a reduction of the income tax rate as equivalent to an income transfer. Total tax revenue generated is calculated as $R = t_a q_a Q_a + t_b q_b Q_b + rZ$. Under revenue neutrality R is fixed.

THE TARGET AND THE POLICIES

The target is a reduction of the consumption of commodity A in order to reduce the associated negative externalities. If $Q_a = Q_{a0}$ in initial equilibrium the

goal is to achieve $Q_a = \bar{Q}_a < Q_{a0}$ in a new equilibrium by increasing the tax on A. Under revenue neutrality that means that the proceeds generated by the increased tax must be offset elsewhere in the tax system. We consider three alternative policies:

Policy 1: The tax on A is not revenue neutral; t_a is increased and R changes accordingly.

Policy 2: The tax on A is revenue neutral; proceeds from the increase in t_a are offset by a reduction of t_b and R is unchanged. The reduction of t_b can be thought of equivalently as a subsidy.

Policy 3: The tax on A is revenue neutral; proceeds from the increase in t_a are offset by a reduction of r and R is unchanged. The reduction of r can be thought of equivalently as an income transfer.

In Policy 2 the instrument is the tax on A and the consequence is the reduction of the tax on B necessitated by the neutrality requirement. One might think of an alternative policy under which the tax reduction (or subsidy) for B is the instrument and the increased tax on A is the necessary consequence. But for any given targeted reduction of the consumption of A (10 percent, for example) the two policies are exactly equivalent: the gain in revenue from increasing the tax on A must equal the loss of revenue from reducing the tax on B, whichever way one chooses to think about which is instrument and which is consequence.

THE COMPLETE MODEL AND ITS APPLICATION IN SIMULATION

The equations of the model are listed in Table 1. Parameters are represented by Greek symbols, variables by Latin symbols. There are 11 (independent) equations and 14 free variables. (Z , a 15th variable, is fixed at \bar{Z} and treated as a constant in all applications of the model.) The government's instruments are the three tax rates. In the initial state the instruments are assigned fixed values (indicated by placing bars over their symbols), and the model then has 11 free variables and a unique solution. When one of the three

policies is implemented the situation changes: Q_a is assigned a fixed target value, with one or two of the tax instruments then being freed up and solved for. The situations in the initial state and the three policies are described in the bottom four lines of Table 1.

All three policies stipulate $Q_a = \bar{Q}_a$. Policy 1 does not require revenue neutrality and thus needs only one free instrument, t_a . The three fixed values are now \bar{Q}_a , \bar{t}_b , and \bar{r} , and the model again has 11 free variables. Policies 2 and 3 do require revenue neutrality, so that $R = \bar{R}$, where \bar{R} is equal to government tax revenue in the initial state. Also they require two instruments. For Policy 2 the instruments are t_a and t_b ; for Policy 3 they are t_a and r .

The model is a nonlinear system of simultaneous equations. It can be solved easily and uniquely by a Gauss-Seidel iterative procedure involving successive approximations until the system converges to a new equilibrium. The procedure requires that each equation have a different variable on the left side. To that end equation (4) can be inverted to put p_a on the left, equations (6) and (7) to put q_a and q_b on the left (after exponentiation), and equation (8) to put t_a on the left. These inversions hold for all three policies, and for Policy 1 they suffice. For the other two policies R becomes fixed and equation (10) is inverted to put t_b on the left in Policy 2, or r on the left in Policy 3.

CALIBRATION OF THE MODEL

We start with what we refer to as the standard calibration. The standard calibration is the basis for a first set of experiments. It is then modified in subsequent experiments to explore the effects of alternative choices of parameter values, including initial conditions.

The standard calibration provides the initial equilibrium values for the 14 variables of the model. Purchaser prices (p_a , p_b) are normalized to 1.0 (by an implicit choice of quantity units) and commodity tax rates (t_a , t_b) are set at zero. (Setting the rates at zero initially is convenient for the interpretation of the simulation results but involves no loss of generality; we could as well have set

them at 10 percent, say.) Supplier prices (q_a, q_b) are thus equal to purchaser prices in the initial state (via equations (8) and (9)). Total consumer expenditure (X) is also normalized to 1.0. The saving rate (ϕ) is set at 5 percent and the initial income tax rate (r) at 20 percent, yielding (from equation (11)) a pre-tax level of consumer income (\bar{Z}) of 1.316. The expenditure share for commodity A (w_a) is set at 1/3, the share for commodity B (w_b) at 2/3. The quantities traded (Q_a, Q_b) then follow from equations (4) and (5), and (because of the initial normalization of prices) are also equal to 1/3 and 2/3. Total tax revenue (R) follows from equation (10), and is 0.263.

The expenditure equation parameters are arrived at by specifying a set of expenditure elasticities and compensated price elasticities. As noted earlier, the 2 x 2 matrix of price elasticities is constrained by symmetry and homogeneity in the underlying demand system so that it has only one degree of freedom; we generate the full matrix by specifying a value for the compensated own-price elasticity of commodity A (ϵ_{aa}). Our standard value is -0.5. The expenditure elasticity for A (η_a) is set at 1.0 and the elasticity for B (η_b) is derived accordingly. The γ_{aa} and β_a coefficients of equation (1) then follow from the relations noted previously and α_a is set so that the chosen initial value of w_a satisfies the equation. The corresponding coefficients for commodity B follow also, but the adding-up requirement means that they are implicit rather than explicit in equation (2). We note, for emphasis, that the elasticity values that we choose are the initial equilibrium values. The actual values will change (implicitly) as the model moves away from its initial equilibrium. The ϵ and η values serve only as a convenient basis for setting the γ and β coefficients, which are then assumed not to change within a given experiment.

The supply equations, (6) and (7), can be specified by choosing values for the supply elasticity parameters (λ_a, λ_b) and then setting the scale parameters (ξ_a, ξ_b) so that the equations are consistent with the chosen values of quantities and prices. That is what we do in experiments with alternative supply elasticities. However, for the standard calibration we do something different. We assume, as our standard (or benchmark) case, conditions of perfect competition in both markets. That is to say, we think of λ_a and λ_b as tending to infinity, and with the

equations rearranged to put prices on the left, that implies that q_a and q_b tend toward fixed values (the supply curves become horizontal in both markets). With q_a and q_b fixed, the supply equations are irrelevant and can be dropped. That then is the initial state that we posit: variable demand prices but fixed supply prices. To be clear though, the elimination of supply price variability is just our standard assumption; variable supply prices play an important role in the alternative experiments and the conclusions that we draw from them.

The standard values of the variables that define the initial state of the model are provided in Table 2 and the standard parameter values (both specified and derived) are provided in the Appendix.

SIMULATIONS

Simulation results for the three policies are shown in Table 2, under standard parameter assumptions. The target is the same for all three policies: a 10 percent reduction of the consumption of commodity A. Only the choice of instruments varies. All variables that are subject to change under one or more of the policies are shown in the table. The results represent a new state of equilibrium and can be compared with the initial state.

Table 3 presents additional simulation results, based now on alternative parameter specifications. The tax rates required to meet the 10 percent reduction target are shown for sixteen alternatives: four choices for the commodity A initial expenditure share, five for its initial own-price elasticity, three for its initial expenditure elasticity, and five for the A and B supply elasticities. The parameter changes are made one at a time, with all other independent parameters retaining their standard values. (Parameters that are not independent are changed in a consistent way. The share of commodity B is equal to one minus the share of commodity A, for example; all of the own-price and cross-price demand elasticities are determined once the own-price elasticity of A is set.) For convenience in making comparisons the standard-value results from Table 2 are

repeated in Table 3 for each type of parameter, along with the alternative values. They are identified by asterisks.

WHAT DO THE SIMULATIONS TELL US?

The first thing they tell us is that revenue neutrality can make a difference to the tax rate required to bring about a significant reduction in the consumption of the target commodity. The second is that the way in which neutrality is implemented can be important. Neither of these results is a surprise, in a qualitative sense; both are well known, in general, in the literature. What are of interest are the sizes of the differences among alternative policies and parameter configurations. Under our standard assumptions (Table 2) a 13.4 tax on commodity A is required to reduce its consumption by 10 percent under Policy 1, which involves no neutrality restriction. The rate increases only modestly, to 14.9 percent, under the revenue neutral Policy 2, which requires the tax on A to be offset by a subsidy on B. Under Policy 3 though, with the offset in the form of an income transfer, the tax on A increases to 22 percent. In effect, consumers are reimbursed for their additional costs in purchasing commodity A, and then left free to add to their consumption of A by spending some of the reimbursement on it. The parameters of the system can make a big difference of course. However the observation that revenue neutrality achieved through an income transfer requires a higher commodity tax than revenue neutrality achieved through a subsidy holds generally. The tax on A is higher under Policy 3 than under Policy 2 in all of the simulations – slightly higher in some cases, greatly higher in others. It holds with standard parameters, in Table 2, and in all cases in Table 3, where alternative parameter configurations are introduced.

Whether the expenditure share of commodity A is large or small makes a difference under Policies 1 and 2: the smaller the share, the higher the tax. But the tax rate under Policy 3 is almost insensitive to the share. When the share (in the initial state) is 10 percent the Policy 1 and 2 tax rates are 19 and 20 percent, the Policy 3 rate 22 percent. When the share is 50 percent the rate drops to 11 percent under Policies 1 and 2, the Policy 3 rate only to 21 percent. At the same

time, the size of the income transfer required to achieve neutrality under Policy 3 varies notably, from 1.5 percent of consumer income in the first case to 7.2 in the second.

The degree of price elasticity on the demand side of the A market matters greatly, as one would expect. In the limiting case of zero price elasticity there is obviously no tax rate that would achieve the 10 percent reduction target. At a more realistic level, an initial own-price elasticity of -0.2 would require a tax on A of 21 percent under Policy 1, 35 percent under Policy 2, and 54 percent under Policy 3. This highlights the difficulties in using a tax instrument to reduce the demand for a price-inelastic commodity, and again the importance of the way in which revenue neutrality is implemented. The tax rate required falls sharply as the elasticity increases. Just how elastic is the price elasticity is obviously a critical consideration in any policy decision. On the other hand, expenditure elasticity has relatively little effect, as the results in Table 3 show.

Supply elasticity matters too. Again the limiting (if unrealistic) case of zero elasticity would make target attainment impossible. Our standard case is perfect competition (infinite elasticity) in both markets. Replacing that with lower elasticity values in the A market increases the tax rate sharply. At an elasticity of 0.5 the required commodity tax ranges from 40 percent (Policy 1) to about 66 percent (Policy 3); reducing the elasticity to 0.2 increases the rates even further, to very high levels (from 92 percent in Policy 1 to 152 percent in Policy 3, with the transfer payment in the latter case equal to 20 percent of consumer income). Setting the supply elasticity in the B market to lower levels has relatively little effect, on the other hand.

CONCLUDING OBSERVATIONS

The theoretical model we have used is built around an AIDS demand system and a log-linear supply system. Other specifications are possible and would no doubt change the numerical calculations. However we doubt that the general patterns of tax rate sensitivity and the effects of imposing revenue

neutrality would be overturned. In particular we think that the finding of possibly very large differential effects of achieving neutrality by income transfer rather than non-target commodity taxation are likely robust.

A larger initial market share for a target commodity will require a lower tax rate on that commodity, other things equal. Lower price elasticity on either the demand side or the supply side of the target commodity market will require a higher tax rate - a much higher rate if the demand or supply is highly inelastic, or if substitution possibilities are severely limited. Differences in expenditure elasticity will have little effect. These results hold whether or not there is revenue neutrality, but neutrality may enhance the effects, and possibly greatly enhance them. The results of the simulations are equilibrium results and one might speculate on the difference between short-run and long-run equilibria. It might be that lower elasticities in the short run would give way to higher elasticities in the long run, with corresponding implications for the rates of taxation.

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APPENDIX: STANDARD PARAMETER VALUES

Compensated price elasticities

$$\epsilon_{aa} = -0.500 \quad \epsilon_{ab} = 0.500 \quad \epsilon_{ba} = 0.250 \quad \epsilon_{bb} = -0.250$$

Expenditure elasticities

$$\eta_a = 1.000 \quad \eta_b = 1.000$$

Derived coefficients of demand equations

$$\gamma_{aa} = 0.056 \quad \gamma_{ab} = -0.056 \quad \gamma_{ba} = -0.056 \quad \gamma_{bb} = 0.056$$

$$\alpha_a = 0.333 \quad \alpha_b = 0.667 \quad \beta_a = 0.000 \quad \beta_b = 0.000$$

Supply elasticities

$$\lambda_a = \infty \quad \lambda_b = \infty$$

Saving rate

$$\phi = 0.050$$

Table 1: Equations of the Model under Alternative Restrictions

Equations

$$w_a = \alpha_a + \gamma_{aa} \ln(p_a/p_b) + \beta_a \ln(X/P) \quad (1)$$

$$w_b = 1 - w_a \quad (2)$$

$$P = \alpha_a \ln(p_a/p_b) + \ln p_b + \frac{1}{2} \gamma_a (\ln(p_a/p_b))^2 \quad (3)$$

$$Q_a = w_a X / p_a \quad (4)$$

$$Q_b = w_b X / p_b \quad (5)$$

$$\ln Q_a = \xi_a + \lambda_a \ln(q_a/q_b) \quad (6)$$

$$\ln Q_b = \xi_b + \lambda_b \ln(q_b/q_a) \quad (7)$$

$$p_a = (1 + t_a) q_a \quad (8)$$

$$p_b = (1 + t_b) q_b \quad (9)$$

$$R = t_a q_a Q_a + t_b q_b Q_b + r \bar{Z} \quad (10)$$

$$X = (1 - \phi)(1 - r) \bar{Z} \quad (11)$$

Restrictions

Initial state: $t_a = \bar{t}_a, t_b = \bar{t}_b, r = \bar{r}$

Policy 1: $Q_a = \bar{Q}_a, t_b = \bar{t}_b, r = \bar{r}$ (Instrument: t_a)

Policy 2: $Q_a = \bar{Q}_a, r = \bar{r}, R = \bar{R}$ (Instruments: t_a, t_b)

Policy 3: $Q_a = \bar{Q}_a, t_b = \bar{t}_b, R = \bar{R}$ (Instruments: t_a, r)

Table 2: System Status in the Initial State and under Alternative Taxation Policies Designed to Reduce the Consumption of Commodity A by 10% : Standard Parameter Specification

Variable	Initial state	Policy 1	Policy 2	Policy 3
Expenditure shares				
w_a	0.333	0.340	0.345	0.344
w_b	0.667	0.660	0.655	0.656
Total expenditure				
X	1.000	1.000	1.000	1.063
Quantities purchased				
Q_a	0.333	0.300	0.300	0.300
Q_b	0.667	0.660	0.700	0.697
Consumer prices				
p_a	1.000	1.134	1.149	1.220
p_b	1.000	1.000	0.936	1.000
P	1.000	1.043	1.003	1.070
Supplier prices				
q_a	1.000	1.000	1.000	1.000
q_b	1.000	1.000	1.000	1.000
Tax rates				
t_a	0.000	0.134	0.149	0.220
t_b	0.000	0.000	-0.064	0.000
r	0.200	0.200	0.200	0.150
Tax revenue				
R	0.263	0.304	0.263	0.263

Note: Policy 1: Tax on A, no offset. Policy 2: Tax on A offset by subsidy on B. Policy 3: Tax on A offset by income transfer.

Table 3: Values of Tax Instruments Required for a Reduction of the Consumption of Commodity A by 10% : Alternative Taxation Policies and Parameter Specifications

Parameter specification	Policy 1	Policy 2		Policy 3	
	t_a	t_a	t_b	t_a	Δr
Initial expenditure share					
$w_a = 0.100$	0.187	0.202	-0.020	0.224	-0.015
$w_a = 0.200$	0.161	0.180	-0.039	0.223	-0.031
$w_a = 0.333^*$	0.134	0.149	-0.064	0.220	-0.050
$w_a = 0.500$	0.111	0.111	-0.091	0.212	-0.072
Demand: initial own-price elasticity					
$\varepsilon_{aa} = -0.2$	0.210	0.353	-0.151	0.542	-0.124
$\varepsilon_{aa} = -0.5^*$	0.134	0.149	-0.064	0.220	-0.050
$\varepsilon_{aa} = -0.8$	0.097	0.092	-0.039	0.134	-0.031
$\varepsilon_{aa} = -1.1$	0.082	0.073	-0.031	0.106	-0.024
$\varepsilon_{aa} = -1.2$	0.071	0.061	-0.026	0.088	-0.020
Demand: initial expenditure elasticity					
$\eta_a = 0.75$	0.150	0.150	-0.064	0.224	-0.051
$\eta_a = 1.00^*$	0.134	0.149	-0.064	0.220	-0.050
$\eta_a = 1.25$	0.122	0.148	-0.063	0.216	-0.049
Supply elasticities					
$\lambda_a = \lambda_b = \infty^*$	0.134	0.149	-0.064	0.220	-0.050
$\lambda_a = 1.0, \lambda_b = \infty$	0.261	0.288	-0.107	0.427	-0.088
$\lambda_a = 0.5, \lambda_b = \infty$	0.401	0.442	-0.142	0.658	-0.121
$\lambda_a = 0.2, \lambda_b = \infty$	0.921	1.013	-0.218	1.519	-0.205
$\lambda_a = \infty, \lambda_b = 1.0$	0.136	0.143	-0.061	0.211	-0.048

Note: * indicates standard value. Specified parameter changes are made one at a time; all other (independent) parameters retain their standard values. See note to Table 2 for definitions of policies.

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