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# The dynamics of real exchange rates – A reconsideration\*

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## Abstract

While it is widely agreed that Purchasing Power Parity (PPP) holds as a long-run concept the specific dynamic driving the process is largely build upon a priori economic belief rather than a thorough statistical modeling procedure. The two prevailing time series models, i.e. the exponential smooth transition autoregressive (ESTAR) model and the Markov switching autoregressive (MSAR) model, are both able to support the PPP as a long-run concept. However, the dynamic behavior of real exchange rates implied by these two models is very different and leads to different economic interpretations. In this paper we approach this problem by offering a bootstrap based testing procedure to discriminate between these two rival models. We further study the small sample performance of the test.

In an application we analyze several major real exchange rates to shed light on the question which model best describes these processes. This allows us to draw a conclusion about the driving forces of real exchange rates.

*JEL-Numbers:* C12, C15, C22, C52, F31

*Keywords:* Nonlinearities · Markov switching · Smooth transition · Specification testing · Real exchange rates

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# 1 Introduction

An ongoing debate about the behavior of real exchange rates suggests that Purchasing Power Parity (PPP) holds as a long-run concept (see e.g. Edison and Klovland (1987), MacDonald (1998) or Taylor et al. (2001)). Econometrically speaking PPP states that real exchange rates fluctuate finitely around an equilibrium, i.e. a time stable mean, and are thus weakly stationary. However, stationarity of real exchange rates does not say anything about the detailed dynamics driving them. Modeling real exchange rates by linear stationary models did not lead to convincing results as standard unit root tests could not reject the null of a random walk in the linear framework and thus could not confirm PPP (see e.g. Adler and Lehmann (1983), Meese and Rogoff (1983), Meese and Rogoff (1988) or Caporale et al. (2003)). In a theoretical framework Sercu et al. (1995) suggest among others that under transaction costs the adjustment process towards PPP is nonlinear (see also Sarno (2005) and the discussion in Taylor et al. (2001, p. 1018)). Empirical evidence that a nonlinear adjustment mechanism could solve the PPP puzzle is recently provided by Lo (2008) and Norman (2010). Therefore, recently non-linear models came into the focus of economists. The two prevailing approaches to model the dynamic of nonlinear adjustment towards PPP are exponential smooth transition autoregressive (ESTAR) models (see e.g. Michael et al. (1997), Taylor et al. (2001) and Kilian and Taylor (2003)) and Markov switching autoregressive (MSAR) models (see e.g. Bergman and Hansson (2005) and Kanas (2006)).<sup>1</sup> Both approaches imply under certain conditions that PPP may hold as a long run concept. However, the nonlinear dynamics driving the respective processes are very distinct. The dynamic of an ESTAR process is driven by lagged values of the endogenous, and therefore observable, variable while an unobservable Markov process governs the dynamic in the MSAR case. This implies different economic intuition. The ESTAR approach is based on the idea that international trade only begins if the price differences between countries exceed a certain level which is determined by the costs of trading such as transportation costs, taxes and many others. As long as the price differences are smaller than this level no trading takes place and therefore real exchange rates fluctuate freely and behave like a random walk. As soon as the price differences exceeds this level international trading starts and real exchange rates are pulled back to a long-run equilibrium. This economic view of real exchange rates is expressed by a unit root regime near the equilibrium and a stabilizing regime

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<sup>1</sup>An exception is Lahtinen (2006) who uses a symmetric, second order logistic STAR model. However, second order logistic STAR models are designed to closely resemble ESTAR models and the estimation results imply a dynamic very similar to a switching regression.

away from the equilibrium in the ESTAR model. This approach has been applied to real exchange rates by Taylor et al. (2001), Kapetanios et al. (2003) or Rapach and Wohar (2006) among others. On the other hand the MSAR view supports the overshooting or bubble theory of Dornbusch (1976) on real exchange rates. It emphasizes the point, that real exchange rates are not just driven by a long-run behavior but that there are dominating short-run forces which are stabilized by some long-run adjustment. These short-run forces can be explained by short-run speculation or other exogenous shocks such as political regime changes, wars or economic crisis. Taking the short-run speculation on real exchange rates as an example, these could be strong enough to create bubbles and therefore, in econometric terms, can create exploding regimes in the MSAR models. A second stabilizing regime in the MSAR model determines the overall stationarity of the model and therefore the long-run convergence towards PPP. MSAR models are able to capture such occasional, sudden and large exchange rate changes caused by unobservable speculation (see e.g. van Norden (1996)). Regarding the MSAR approach there are two possibilities of modeling this economic behavior. One possibility is to implement the Markov switching within the autoregressive parameter and thus have a model with two autoregressive regimes, one of them being explosive and the other stabilizing. This model was applied among others by Kanas (2006). Stationarity conditions for this model are given in Francq and Zakoian (2001). A second option is to implement the Markov switching in the mean rather than in the autoregressive parameter and thus obtain a switching mean model. This approach was motivated by Engel and Hamilton (1990) and Hamilton (1993) who argue that switches in the mean are more appropriate for modeling the dynamics of financial data. Bergman and Hansson (2005) apply this approach to real exchange rates.

The application of either approach has so far been motivated by an a priori economic belief about the behavior of real exchange rates rather than a formal statistical model selection procedure.

In this paper we suggest for the first time a formal model selection procedure based on a parametric bootstrap to discriminate between ESTAR and MSAR models. We study the finite sample behavior of the proposed method and analyze several major real exchange rates to shed some light on the question which model and by that which theory best describes the dynamic behavior of real exchange rates.

The rest of the paper is organized as follows: In section 2 we describe the competing models for real exchange rates which we aim to distinguish. In section 3 we discuss briefly some specifics when testing non-nested models. In section 4 we describe the

bootstrap procedure in more detail and investigate the finite sample properties of this test in section 4.1. Section 5 applies the model selection procedure to empirical data before section 6 concludes.

## 2 Competing models for real exchange rates

The general ESTAR model is given by two autoregressive regimes connected by a smooth exponential transition function  $\mathcal{G}(\cdot; \gamma, c) : \mathbb{R} \rightarrow [0, 1]$ . This function governs the transition between the two regimes in a smooth way. Alternatively, an ESTAR model can also be interpreted as a continuum of regimes which is passed through by the process. In general, univariate ESTAR( $p$ ) models,  $p \geq 1$  and  $d \leq p$ , are given by

$$y_t = [\Psi w_t] \times [1 - \mathcal{G}(y_{t-d}; \gamma, c)] + [\Theta w_t] \times \mathcal{G}(y_{t-d}; \gamma, c) + \varepsilon_t \quad (1)$$

$$= [\Psi w_t] + [\Phi w_t] \times \mathcal{G}(y_{t-d}; \gamma, c) + \varepsilon_t, \quad t \geq 1, \quad (2)$$

with  $\varepsilon_t \stackrel{iid}{\sim} (0, \sigma^2)$ .

The parameter vectors  $\Psi$  and  $\Theta$  as well as  $w_t$  are given by  $\Psi = (\psi_0, \psi_1, \dots, \psi_p)$ ,  $\Theta = (\vartheta_0, \vartheta_1, \dots, \vartheta_p)$ , and  $w_t = (1, y_{t-1}, \dots, y_{t-p})'$ . For the alternative parametrization in (2) we have  $\Phi = (\varphi_0, \varphi_1, \dots, \varphi_p) = (\psi_0 - \vartheta_0, \psi_1 - \vartheta_1, \dots, \psi_p - \vartheta_p)$ , i.e. the second regime realizes as sum of  $\Psi$  and  $\Phi$ .

For an ESTAR model the transition function  $\mathcal{G}(\cdot)$  is given by

$$\mathcal{G}(\cdot; \gamma, c) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}; \quad \gamma > 0. \quad (3)$$

Due to the symmetry of  $\mathcal{G}(\cdot)$  this functional form for the transition function is popular for modeling real exchange rates or real interest rates (see e.g. Taylor et al. (2001), Kapetanios et al. (2003) or Rapach and Wohar (2006)). Surveys of the broad field of nonlinear time series models in general and STAR models in particular are given by Potter (1999) and van Dijk et al. (2002); see also Teräsvirta (1994). The most frequently used special case of the general ESTAR model in (2) is the ESTAR(1) model

$$y_t = \psi y_{t-1} + \phi y_{t-1} \left\{ 1 - \exp(-\gamma(y_{t-1} - c)^2) \right\} + \varepsilon_t. \quad (4)$$

Moreover, the additional restriction  $c = 0$  is also frequently imposed (e.g. Kapetanios et al. (2003)). To model the globally stationary but nonlinear behavior of real exchange rates one unit root regime is also commonly imposed (see e.g. Taylor et al. (2001))

and Kapetanios et al. (2003)). This leads to the model

$$y_t = y_{t-1} + \phi y_{t-1} \left\{ 1 - \exp\left(-\gamma(y_{t-1} - c)^2\right) \right\} + \varepsilon_t, \quad (5)$$

with  $-2 < \phi < 0$  to ensure global stationarity. Kapetanios et al. (2003) show that the model in (5) although locally non-stationary is globally stationary as long as  $|\psi + \phi| < 1$ . Estimation of these models either by nonlinear least squares or maximum likelihood techniques is treated by Klimko and Nelson (1978) and Tjøstheim (1986) respectively. A standard MSAR model based on the work of Hamilton (1989) reads (see also Hamilton and Raj (2002) for an introduction)

$$y_t = \mu_{s_t} + \phi_{1,s_t} y_{t-1} + \dots + \phi_{p,s_t} y_{t-p} + \varepsilon_t. \quad (6)$$

The values of the autoregressive parameters  $\phi_{1,s_t}, \dots, \phi_{p,s_t}$  and the mean  $\mu_{s_t}$  and thus the regime switching is govern by an unobservable Markov chain

$$\mathcal{P}(s_t = j | s_{t-1} = i, s_{t-2} = k, \dots, y_{t-1}, y_{t-2}, \dots) = \mathcal{P}(s_t = j | s_{t-1} = i) = p_{ij}.$$

Extensions to this basic framework are possible, see e.g. Hamilton and Raj (2002) and the papers cited therein.

Although such a general model is possible to use in principle the models usually found in applied work are more restrictive and have only a few parameters that change depending on the regime. In the real exchange rate work in particular the most frequently used model is a first order, or at least low order, autoregression with a Markov switching mean (see e.g. Engel and Hamilton (1990), Bergman and Hansson (2005) or Kanas (2006)). This model is empirically justified by Hegwood and Papell (1998) and Montañés (1997) who find reversion to a mean which is subject to structural breaks.

Considering the special case of an first order MSAR model with only a switching mean we have two possible ways to describe such a process

$$(y_t - \mu_{s_t}) = \phi(y_{t-1} - \mu_{s_{t-1}}) + \varepsilon_t \quad (7)$$

$$(y_t - \mu_{s_t}) = \phi(y_{t-1} - \mu_{s_t}) + \varepsilon_t. \quad (8)$$

Although the difference between these two models seems minor the dynamic patterns they produce are rather different (see Hamilton (1993)). Suppose that the process shifts from regime 1 to regime 2 at date  $t$  and stays there for  $j$  periods. Using the specification

in (8) the value  $y_{t+j}$  will be  $(\mu_2 - \mu_1)(1 + \phi + \phi^2 + \dots + \phi^j)$  units higher as a consequence from the regime switch. Thus the consequences of a shift accumulate over time. If the specification (7) is used the value  $y_{t+j}$  will be only  $(\mu_2 - \mu_1)$  units higher. The formulation in (7) is the more promising formulation for economic and financial time series according to Hamilton (1993) and thus we will consider such a model in our model selection approach.

Regarding stationarity conditions for MSAR models Francq and Zakoïan (2001) show that for the model in (7) to be globally stationary the following conditions must hold

$$(p_{11} + p_{22})\phi^2 + (1 - p_{11} - p_{22})\phi^4 < 1 \quad (9)$$

$$(p_{11} + p_{22})\phi^2 < 2. \quad (10)$$

For the model in (7) to be locally stationary the usual condition  $|\phi| < 1$  must hold.

Maximum likelihood estimation by direct, numerical maximization of the likelihood function of such models is treated in detail by Hamilton (1989) and by an iterative EM algorithm by Hamilton (1990).

### 3 Issues in testing non-nested hypotheses

Usually the comparison of different hypotheses, i.e. of competing models may be performed using standard approaches such as likelihood-ratio tests, Wald tests, Lagrange multiplier tests or the principle of Hausman (1978) (see Engle (1984) for a survey of these procedures). These principles assume that one model, the null model, can be obtained from the competing model by imposing some parameter restrictions. If this assumption is violated, i.e. none of the hypotheses is a special case of the other one, these approaches fail. For a survey of several aspects of testing non-nested hypothesis see Gourieroux and Monfort (1994).

Cox (1961, 1962) generalizes the likelihood-ratio principle to the case of non-nested models. He shows that the usual unadjusted likelihood-ratio statistic does not converge to zero if the two models are non-nested. Put more formally, let the two models for the conditional density be denoted by

$$H_f : \mathcal{F}_\theta = \{f(y_i | \mathfrak{F}_{t-1}; \theta), \theta \in \Theta \subseteq \mathbb{R}^r\}, t = 1, \dots, T \quad (11)$$

$$H_g : \mathcal{F}_\lambda = \{g(y_i | \mathfrak{F}_{t-1}; \lambda), \lambda \in \Lambda \subseteq \mathbb{R}^q\}, t = 1, \dots, T. \quad (12)$$

The process is denoted by  $y_t$  and  $\mathfrak{F}_{t-1}$  is the sigma algebra generated by  $(y_{t-1}, y_{t-2}, \dots)$ . For the sake of brevity we will suppress the dependence on  $y_t$  and  $\mathfrak{F}_{t-1}$  and simply write  $f(\theta)$  and  $g(\lambda)$  whenever possible. The likelihood-ratio statistic of Cox (1961, 1962) reads

$$T_f(\hat{\theta}, \hat{\lambda}) = [\log f(\hat{\theta}) - \log g(\hat{\lambda})] - E_f[\log f(\hat{\theta}) - \log g(\hat{\lambda})], \quad (13)$$

where  $\hat{\theta}$  and  $\hat{\lambda}$  denote the maximum likelihood estimates.  $E_f[\cdot]$  is the expectation operator evaluated with respect to the 'true' density  $f(\theta)$ . This serves as a measure of closeness between the two densities and is defined by the Kullback-Leibler information criterion (KLIC). It is defined by

$$E_f[\cdot] = \int_{\mathbb{R}} \log \frac{f(\theta)}{g(\lambda)} f(y; \theta) dy. \quad (14)$$

The quantity in (14) is then minimized by choosing  $\lambda$ .

This is can be equivalently reformulated as

$$\max_{\lambda} E_f[\log g(\lambda)]. \quad (15)$$

The solution to this problem is called the pseudo-true value of  $\lambda$  given  $\theta$ .

The main problem when using the Cox test statistic is that the measure of closeness can be derived analytically in a closed form only for very specific and simple cases such as the linear versus log-linear model (see Aneuryn-Evans and Deaton (1980)). It is however not possible in general. In cases where the exact expression of the KLIC is very complicated or even impossible to obtain, e.g. for nonlinear models, Pesaran and Pesaran (1993) and Lu et al. (2008) propose simulation methods for approximating the quantity. Regularity conditions for the applicability of Cox's test are given by White (1982b) for the *i.i.d.* case. It is basically required that consistent and asymptotically normal distributed estimators for the parameters  $\theta$  and  $\lambda$  can be obtained. This is especially important for the estimator of  $\lambda$  as we need a consistent and asymptotically normal distributed estimator which allows the likelihood function to fail to correspond to the true joint density of the observations. Such an estimator is called quasi-maximum-likelihood estimator (QMLE). Consistency and asymptotic normality results are provided by White (1982a) for the *i.i.d.* case, by White (1981) for the nonlinear case, by Bollerslev and Wooldridge (1992) for the dynamic linear case and by Gallant and White (1988) for the nonlinear dynamic case. These results ensure a wide applicability of Cox's test.



## 4 A bootstrap based likelihood ratio test

For the likelihood-ratio approach of Cox (1961, 1962) a simulation method for the test statistic is developed by Pesaran and Pesaran (1993) and compared to extant tests in the literature by Pesaran and Pesaran (1995). Lu et al. (2008) also consider simulation methods for the Cox test statistic and additionally propose a simulated Wald-type encompassing test. However, the latter test is computationally very costly in the context of nonlinear models as it involves a double loop design similar to a double bootstrap. Coulibaly and Brorsen (1999) report the results of a simulation study in which they compare different ways of computing the Cox test statistic. Their results suggest that simulation of the whole test statistic and the use of Monte Carlo p-values instead of simulating only parts of the test statistic and relying on asymptotic critical values is the more promising approach in finite samples. These results have also been confirmed more recently by Godfrey and Santos Silva (2005) and Kapetanios and Weeks (2003). The latter authors consider non-nested testing in a time series context to distinguish between several non-nested nonlinear time series models for the conditional mean. Different methods and test statistics based on the likelihood ratio principle are explored. Similar to Coulibaly and Brorsen (1999) Kapetanios and Weeks (2003) find that a studentized but not mean-adjusted test statistic with a simple variance estimator performs best over a variety of different settings; see also Lee and Brorsen (1997) for an application to nonlinear models for the conditional variance.

Using the notation from section 3 we write the two rival but non-nested models described in section 2 as

$$H_f : \mathcal{F}_\theta = \{f(y_t | \mathfrak{F}_{t-1}; \theta), \theta \in \Theta \subseteq \mathbb{R}^r\}, t = 1, \dots, T \quad (16)$$

$$H_g : \mathcal{F}_\lambda = \{g(y_t | \mathfrak{F}_{t-1}; \lambda), \lambda \in \Lambda \subseteq \mathbb{R}^q\}, t = 1, \dots, T. \quad (17)$$

For our test the model  $\mathcal{F}_\theta$  in (16) is an ESTAR model as in (2) and the model  $\mathcal{F}_\lambda$  in (17) is a MSAR model as in (6).

To keep the notation simple we write  $f(\theta)$  and  $g(\lambda)$  for (16) and (17) respectively whenever possible. The log-likelihood functions for the models (16) and (17) can be written

as

$$l_f = \frac{1}{T} \sum_{t=1}^T \log f_t(\theta)$$

$$l_g = \frac{1}{T} \sum_{t=1}^T \log g_t(\lambda).$$

Let further denote  $\hat{\theta}$  and  $\hat{\lambda}$  the parameter values maximizing these functions. Then the log-likelihood ratio reads  $l_f(\hat{\theta}) - l_g(\hat{\lambda})$ . In order to studentize this likelihood ratio Coulibaly and Brorsen (1999) and Kapetanios and Weeks (2003) consider different estimators based on the outer-product of the scores of the models under consideration as in Berndt et al. (1974) and based on the information equality. A third alternative simply calculates

$$\hat{V}^2 = \frac{1}{T-1} \sum_{t=1}^T (d_t - \bar{d})^2, \quad (18)$$

where  $d_t = l_{f,t}(\hat{\theta}) - l_{g,t}(\hat{\lambda})$  is the likelihood ratio for the  $t$ -th observation of  $y_t$  and  $\bar{d}$  is the respective arithmetic mean. These three methods are asymptotically equivalent but Pesaran and Pesaran (1993), Coulibaly and Brorsen (1999) and Kapetanios and Weeks (2003) report superior performance of the simple variance estimator in (18). Therefore we adopt this approach in our test.

The test statistic we consider reads

$$S = \frac{\sqrt{T} \{l_f(\hat{\theta}) - l_g(\hat{\lambda})\}}{\sqrt{\hat{V}^2}}. \quad (19)$$

Note that the test statistic in (19) is not mean adjusted by an estimate of a measure of closeness of the two distributions in (16) and (17) such as the KLIC. This reduces the computational burden significantly compared to the methods of Pesaran and Pesaran (1993) and Lu et al. (2008) but renders the test statistic asymptotically non pivotal (see e.g. Pesaran and Pesaran (1993) and Godfrey (2007)).<sup>2</sup> However, as Hall and Titterington (1989) show, non pivotal statistics will have the same asymptotic accuracy, regarding size and power, as pivotal statistics. Thus we can reduce the computational burden by using the non mean adjusted statistic in (19) as we only need one bootstrap loop instead of two nested loops for computing an estimate of the KLIC (see Lee and Brorsen (1997)

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<sup>2</sup>A statistic  $\eta$  is called asymptotically pivotal if for a sequence of known constants  $\{a_n\}$  and  $\{b_n\}$ ,  $a_n + b_n\eta$  has a proper nondegenerate limiting distribution not depending on unknowns; see Hall (1992, p. 14).

for a related approach).

We use the following parametric bootstrap to resample the likelihood ratio statistic in (19):

- (i) Obtain the initial estimates  $\hat{\theta}$  and  $\hat{\lambda}$  from  $y_t$  and compute the test statistic  $S$  in (19).
- (ii) Generate bootstrap samples by parametric resampling from the fitted model under the null  $f(\hat{\theta})$ .  $y_t^b$  denotes the  $t$ -th observation of the  $b$ -th bootstrap sample which is dependent on  $\hat{\theta}$ , i.e.  $y_t^b(\hat{\theta})$ .
- (iii) For the  $b$ -th bootstrap sample let  $\hat{\theta}^b$  and  $\hat{\lambda}^b$  denote the parameter estimates obtained from maximizing  $l_f^b = T^{-1} \sum_{t=1}^T \log f(y_t^b(\hat{\theta})|\theta)$  and  $l_g^b = T^{-1} \sum_{t=1}^T \log g(y_t^b(\hat{\theta})|\lambda)$ . Using  $\hat{\theta}^b$  and  $\hat{\lambda}^b$  compute the bootstrap analog to  $S$  in (19):

$$S^b = \frac{\sqrt{T} \{l_f^b(\hat{\theta}^b) - l_g^b(\hat{\lambda}^b)\}}{\sqrt{\hat{V}_b^2}}.$$

- (iv) Repeat steps (ii) – (iii)  $B$  times and save the bootstrap test statistic  $S^b$ . This will give a small sample approximation of the distribution of  $S$  in (19).
- (v) Compare  $S$  from (i) with critical values obtained from the distribution of  $S^b$  to decide which model best captures the data.

This bootstrap algorithm can be easily implemented using only a single loop design and thus the computational burden is not as heavy as the simulated Wald-type test of Lu et al. (2008) which requires a double loop design.

## 4.1 Finite sample properties

For the size experiment of the bootstrap likelihood ratio test we use the following data generating process

$$y_t = 0.95y_{t-1} - 0.55y_{t-1} \left\{ 1 - \exp\left(- (y_{t-1} - 1)^2\right) \right\} + \varepsilon_t, \quad (20)$$

where  $\varepsilon_t \stackrel{iid}{\sim} N(0, 1)$ . The first regime is moderately persistent while the second is highly persistent. This feature of our data generating process is consistent with the empirical literature on ESTAR models, see e.g. Rapach and Wohar (2006), Taylor et al. (2001),

Öcal (2000) or Gatti et al. (1998). To keep the computational burden reasonable we consider  $B = 200$  bootstrap replications and  $M = 1000$  monte carlo repetitions.

Table 1 shows the size properties of the bootstrap likelihood ratio test for finite samples.

$\alpha$	$T = 100$	$T = 200$	$T = 300$	$T = 400$
1%	2.10	1.10	1.10	1.00
5%	7.30	5.50	6.00	5.30
10%	13.60	11.70	11.60	11.20

Table 1: Empirical size of bootstrap likelihood ratio test [in %].

The test shows virtually no size distortions even in small sample frequently encountered when using monthly or quarterly macro data.

For the power results we consider a Markov switching mean model which is parameterized such as to resemble empirically estimated MSAR models as in Bergman and Hansson (2005). That is a stationary but strongly persistent autoregressive regime with a switching mean which is significantly different across both regimes. The model we use reads

$$y_t = 0.85(y_{t-1} + \mu_{s_{t-1}}) + \varepsilon_t. \quad (21)$$

The mean across regimes is specified as

$$\mu_{s_t} = \begin{cases} 1, & \text{if } s_t = 1 \\ 5, & \text{if } s_t = 2. \end{cases}$$

The transition probabilities for the regimes are  $p_{11} = p_{22} = 0.9$ . This leads to two regimes with equal unconditional ergodic regime probabilities (see Hamilton (1994)) of

$$\mathcal{P}(s_t = 1) = \mathcal{P}(s_t = 2) = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} = 0.5.$$

The power results for this experiment are displayed in table 2.

$\alpha$	$T = 100$	$T = 200$	$T = 300$	$T = 400$
1%	64.70	93.10	97.80	97.80
5%	86.50	98.30	98.60	98.30
10%	94.90	98.90	98.60	98.40

Table 2: Empirical power of bootstrap likelihood ratio test [in %].

The power results are very encouraging especially for the empirically most important cases of small sample sizes. The power increases steeply with the sample size and we also recommend to preferably use the 5% and 10% level of significance rather than the restrictive 1% level. An alternative is to increase the bootstrap replications to smooth the tails of the bootstrap distribution and obtain an even better approximation to the real distribution.<sup>3</sup>

## 5 Modeling real exchange rates

The real exchange rate data we use is the data set of Smallwood (2005) to ensure comparability with the existing literature.<sup>4</sup> However, the focus of Smallwood (2005) lies on the joint testing of long memory and nonlinearity in real exchange rates. The nonlinearity found by him is then modeled as the usual ESTAR transition function and no comparison to a nonlinear alternative is considered. Hence our analysis adds to the literature by providing thorough statistical evidence of how the underlying nonlinear dynamic of real exchange rates is best described.

The data set comprises of 11 countries against the United States as domestic country. It includes Argentina, Brazil, France, Germany, Italy, Japan, Mexico, the Netherlands, Portugal, Spain, and the United Kingdom. The data starts in January 1973 and is sampled at a monthly frequency. For all countries from the Euro area the data ends in December 1998 ( $T = 312$ ). For Japan in February 2002 ( $T = 350$ ) and for the rest until March 2002 ( $T = 351$ ).

For real exchange rates Taylor et al. (2001) propose a formulation of the ESTAR model which reads

$$(y_t - \mu) = \psi(y_{t-1} - \mu) + \phi(y_{t-1} - \mu) \left\{ 1 - \exp\left(-\gamma(y_{t-1} - \mu)^2\right) \right\} + \varepsilon_t. \quad (22)$$

The location parameter  $\mu$  models the long-run equilibrium around which the real exchange rate fluctuates. If we are in the extreme case of  $y_{t-1} = \mu$  the process behaves like a random walk and is pulled back into the stationary region once the deviation becomes too large and  $y_{t-1} \neq \mu$ . The parameter  $\gamma$  governs the speed of the mean reversion. Thus Taylor et al. (2001) assume that  $\mu$  is the long-run mean around which the process fluctuates.

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<sup>3</sup>In another set of simulations we specified the data generating process such as to resemble empirical estimates of Bergman and Hansson (2005). These simulations yielded equally satisfactory results. However, they are unreported to save space but can be obtained from the authors on request.

<sup>4</sup>The data is available at the article's website at: <http://www.bepress.com/snnde/vol9/iss2/art7/>.

tuates. We de-mean the data prior to testing and thus the long-run mean is  $\mu = 0$  and we obtain a simplified model

$$y_t = \psi y_{t-1} + \phi y_{t-1} \left\{ 1 - \exp(-\gamma y_{t-1}^2) \right\} + \varepsilon_t. \quad (23)$$

Note that this only reduces the computational burden of estimating a nonlinear model and does not affect our results. Additionally this is economically reasonable as the log real exchange rate is equal to zero if PPP holds.

We test this model against an MSAR(1) model with a switching mean as in (7) using  $B = 500$  bootstrap repetitions.<sup>5</sup>

The results of the tests for the individual countries are displayed in table 3. The column 'Critical value' shows the critical value at the 5% level of significance from the bootstrap distribution. Note that the column 'Test statistic' displays negative values because we minimized the negative log-likelihood function.

Country	Test statistic	Critical value	Suggested model
Argentina	-0.499	1.230	MSAR
Brazil	-0.727	-0.244	MSAR
France	-4.302	-17.758	ESTAR
Germany	-4.136	-17.324	ESTAR
Italy	-0.162	-3.815	ESTAR
Japan	-5.084	-0.981	MSAR
Mexico	-0.611	-0.215	MSAR
Netherlands	-4.480	-18.496	ESTAR
Portugal	-3.123	-16.744	ESTAR
Spain	-0.748	-5.806	ESTAR
United Kingdom	-0.210	-3.039	ESTAR

Table 3: Empirical test results for real exchange rates.

We can roughly classify the countries into three categories: The western European countries such as France and Germany, Latin American countries such as Argentina and Brazil and as another western-oriented major industrial nation Japan.

The test decisions were obtained based on unrestricted models. This is in contrast to

<sup>5</sup>We confirmed by simulation that the procedure from section 4 maintains its good size and power properties in this special situation. Again, the results are unreported to save space but can be obtained from the authors on request.

Taylor et al. (2001) who restricted the ESTAR model a priori to have a unit root in the first regime and a white noise in the second regime. However, it turned out that the first regime was consistently estimated as a unit root and the second regime very close to a white noise process. Therefore we performed a likelihood ratio test of the null that  $\psi = 1$  and  $\phi = -1$ . This test supports evidence that the restricted model yields equally good results. As in Taylor et al. (2001) we could not reject the hypothesis that the restricted model are an equally reasonable description of the data. We subsequently estimated the restricted model. The estimation results and the test results are displayed in table 4.

Country	$\hat{\gamma}_{restricted}$	LR test	p-value
France	0.1083	1.245	0.5367
Germany	0.0905	1.233	0.5400
Italy	0.2381	0.006	0.9968
Netherlands	0.1012	1.582	0.4533
Portugal	0.0771	0.399	0.8193
Spain	0.1034	0.645	0.7243
United Kingdom	0.3261	0.3811	0.8265

Table 4: Estimation results ESTAR.

The results are comparable with the results of Rapach and Wohar (2006) given that we use an extended data set. The conclusion is similar that we find evidence of slight nonlinearity in the data but the speed of adjustment towards an equilibrium is slower, i.e. the interval in which fluctuation around PPP is wider.

The estimation results for the MSAR models are displayed in table 5.

Country	$\hat{\mu}^{(1)}$	$\hat{\mu}^{(2)}$	$\hat{\phi}$	$\hat{p}_{11}$	$\hat{p}_{22}$	$\hat{\sigma}^2$
Argentina	0.090	0.733	0.582	0.983	0.976	0.018
Brazil	-0.126	0.150	-0.940	0.983	0.975	0.092
Japan	-0.392	0.005	0.588	0.990	0.989	0.109
Mexico	-0.191	0.123	-0.785	0.982	0.972	0.108

Table 5: Estimation results MSAR.

All estimated regime probabilities show a highly persistent behavior over time. All real exchange rates show a distinct mean switch between the regimes with a changing sign in each regime. An exception is Argentina, however, the difference between the mean coefficients is quite large given the range of the data. Note that our estimates

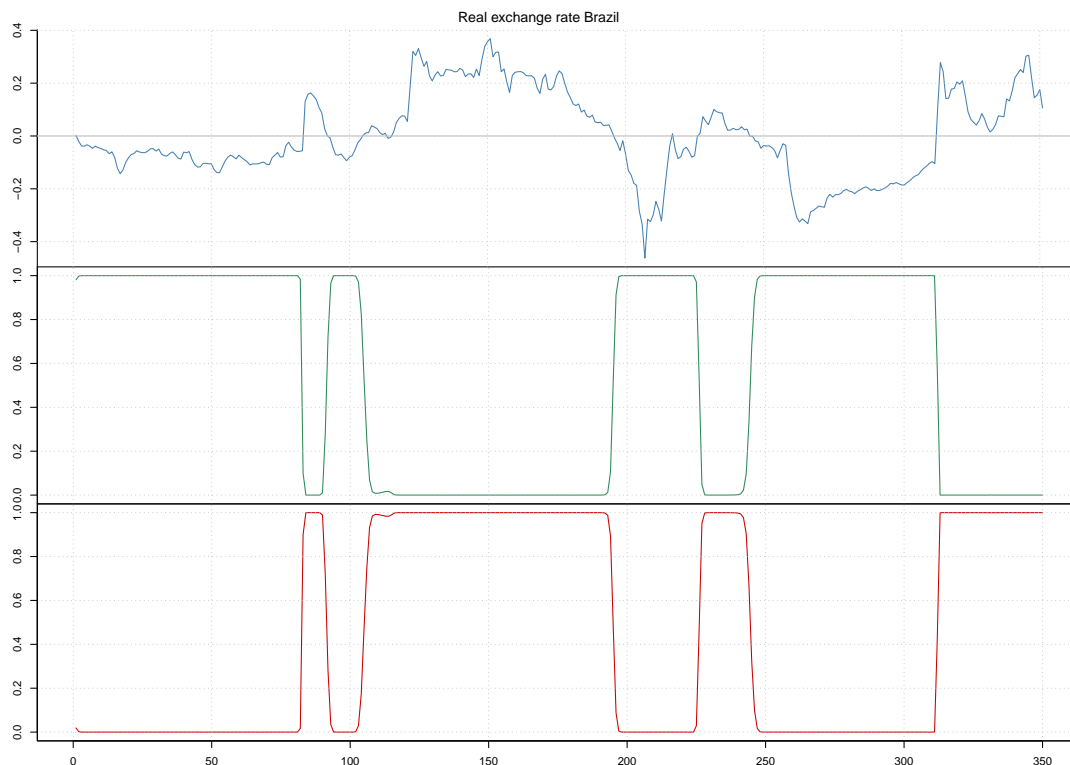


Figure 1: Real exchange rate Brazil with smoothed transition probabilities.

are smaller than those of Bergman and Hansson (2005) because we did not multiply the data with 100. Looking at the stationarity conditions for our models in (9) and (10) we immediately see that all estimated models fulfil these conditions. As an example of how the regime change dynamic behaves over time figures 1 and 2 show the real exchange rates of Brazil and Mexico along with their smoothed transition probabilities, respectively.

These groups show some interesting and distinctive results. According to the theory reviewed in section 1, adequacy of the MSAR model supports the bubble theory for real exchange rates. Looking at the Latin American countries an MSAR model seems to be more appropriate to model the dynamics of real exchange rates while the western European countries seem to be well described by an ESTAR model. For Japan, again, the MSAR specification seems to be more appropriate. The question is: What is the difference between these groups that explains the different real exchange rate dynamic? In the 1960s and 1970s many Latin American countries borrowed huge sums from international creditors to foster industrialization. With the beginning of the world recession and the first oil crisis during the first half of the 1970s the developing countries found themselves in a severe liquidity crunch. This raised interest rates in the United States



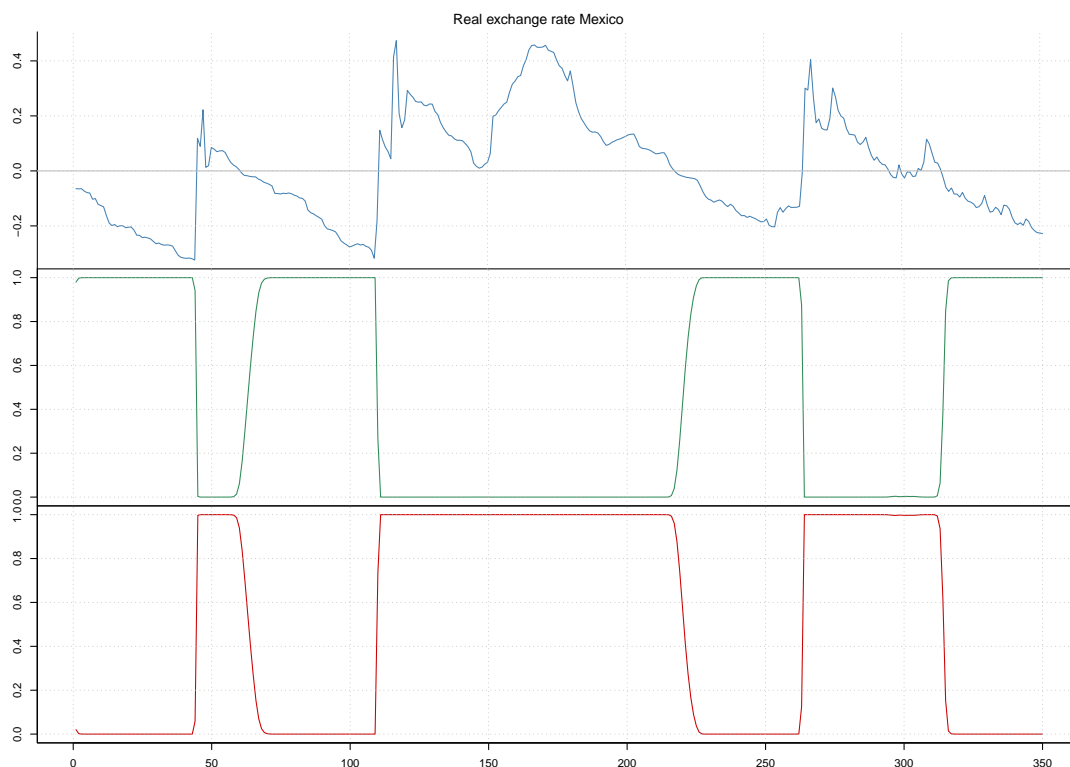


Figure 2: Real exchange rate Mexico with smoothed transition probabilities.

and Europe and caused the deterioration of the exchange rates of the Latin American countries. In the aftermath of the declaration of the national bankruptcy of Mexico in 1982 the incomes in the Latin American countries dropped, economic growth stagnated and inflation rose to levels of hyperinflation.

In Argentina the crisis was tightened by the military coup in 1976 which led to a monetarist financial liberalization. The excessive money supply increased the inflation rate to over 100% a year and increased the national foreign debt causing bank-runs and destroying business confidence. During the 1980s and early 1990s Argentina experienced periods of high inflation rates peaking in 1989 with 5000% a year which subsequently fell down to single digits by 1993 due to a more liberal economic policy. The high government spending increases the national debt during the 1990s resulting in loss of confidence of foreign investors and causing another bank-run and a subsequent economic crisis in the late 1990s and early 2000s.

In Brazil the oil crisis stopped the economic growth and caused a recession which piled up foreign debt. The inflation remained a major problem during these periods due to the exchange rate devaluation of the austerity program of the IMF and growing public deficit. The inflation peaked in 1993 to 5000% per year. In the 1990s the first post-

military-government introduced a stabilization plan ('plano real') including a monetary reform which stabilized the inflation rate subsequently on a moderate level.

The history of Mexico can be classified into three phases. In the 1970s as a response to the oil shock the inflation rose and foreign debt was piled up. From 1983 to 1988 the inflation rose to an average of 100% per year. In the 1990s Mexico ratified the North American Free Trade Agreement (NAFTA) which contributed to the economic recovery and helped to control the inflation.

All three countries are characterized by frequent and sudden economic regime changes in response to adverse economic situations which caused very volatile and high inflation rates. Despite of no formal cooperation on the political level, the economies in the countries behave very similar (see Singh (2006)). The changes could not have been anticipated and are thus better characterized by an abrupt change modeled by the MSAR model.

During the same period the western European countries started to cooperate politically and in economic policy in particular within the European Economic Community (EEC) which collaborated even closer with the transition to the European Community (EC). This led to a very similar behavior of the real exchange rates of these countries. In particular the central banks of the western European countries are independent from each other but nevertheless pursued a similar interest rate policy to control inflation on a rather low level which led to the adoption of inflation targeting during the 1990's.

In Europe the aftermath of the oil crisis has not been as severe as in the Latin American countries because the foreign national deficit was not as high. Additionally the European countries have been much more developed compared to the Latin American countries and were therefore able to deal with increasing inflation without risking hyperinflation. This is empirically investigated by Catão and Terrones (2005) who show a strong positive correlation between fiscal deficit and inflation for developing countries but not for industrialized countries.

The effect on the real exchange rate for the western European countries is thus better described by a smooth adjustment process such as postulated by the ESTAR model.

The finding that Japan is better characterized by a MSAR model although it is a highly developed country is due to the persistent deflation over the last two decades. After the oil crisis in 1973 the inflation rose. During the late 1980 a lot of speculative money has been invested in Japan causing the Japanese asset price bubble which burst in 1991 followed by a decade of zero inflation rates and even deflation in the 1990s. Since then the Japanese central bank keeps the interest rate at near zero but could not stop de-

flation tendencies. These two regimes of high inflation during the 1970s and extremely low inflation during the late 1980s and 1990s can be captured well by a MSAR model. It is however important to note that in the case of Japan this results is much less clear because standard economic paradigms such as the relation regarding interest rate and inflation rate levels do not hold in Japan.

## 6 Conclusion

Different theories about the existence of the PPP support different economic views of the in-sample dynamic driving real exchange rates. These different views result in different models frequently used in the analysis of PPP; namely ESTAR and MSAR models. As both models are able to support PPP under certain conditions the question which model to use for the analysis is usually answered upon prior economic belief rather than statistical model selection procedures. However, as the dynamics of the competing models are rather different the question which model best captures the data is important as it results in different economic theories. In this paper we propose a bootstrap based likelihood ratio test that allows us to discriminate between both classes of non-linear time series models. The bootstrap approximation of the asymptotic distribution of the test statistic allows us to obtain convincing power results for sample sizes frequently encountered in empirical studies. This is important as asymptotic test have been shown to work unsatisfactorily in small samples (see Coulibaly and Brorsen (1999) and Kapetanios and Weeks (2003)).

In an empirical application we find that the real exchange rates of countries with high inflation rates such as Argentina or Japan are best modeled using MSAR models and thus supporting the theory of bubbles in real exchange rates. Countries not suffering from high inflation such as France or Germany are better described with an ESTAR model and thus continuous adjustment towards a long-run PPP equilibrium can be concluded.

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