Gareis, Johannes; Mayer, Eric

Working Paper
Euler equations and money market interest rates: The role of monetary and risk premium shocks

Würzburg Economic Papers, No. 89

Provided in Cooperation with:
University of Würzburg, Chair for Monetary Policy and International Economics

Suggested Citation: Gareis, Johannes; Mayer, Eric (2012) : Euler equations and money market interest rates: The role of monetary and risk premium shocks, Würzburg Economic Papers, No. 89, Univ., Dep. of Economics, Chair VWL 1, Würzburg

This Version is available at:
http://hdl.handle.net/10419/65863

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes. You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Euler Equations and Money Market Interest Rates: The Role of Monetary and Risk Premium Shocks

Johannes Gareis\(^{(a)}\) and Eric Mayer\(^{(a)}\)

\(^{(a)}\)University of Würzburg
August 2012
**Postal Address**

Johannes Gareis  
University of Würzburg, Department of Economics, Sanderring 2,  
97070 Würzburg, Germany  
Email: johannes.gareis@uni-wuerzburg.de
Euler Equations and Money Market Interest Rates: 
The Role of Monetary and Risk Premium Shocks

Johannes Gareis and Eric Mayer*

August 5, 2012

Abstract

This paper challenges the view that the observed negative correlation between the Federal Funds rate and the interest rate implied by consumption Euler equations is systematically linked to monetary policy. By using a Monte Carlo experiment, we show that stochastic risk premium disturbances have the capability to drive a wedge between the interest rate targeted by the central bank and the implied Euler equation interest rate such that the correlation between actual and implied rates is negative.

Keywords  Euler Interest Rate, Monetary Policy, Risk Premium Shocks
JEL codes E10, E43, E44, E52

*University of Wuerzburg, Department of Economics, Sanderring 2, 97070 Wuerzburg, Germany
Corresponding author: Johannes Gareis; E-mail: johannes.gareis@uni-wuerzburg.de, Tel.: +49-(0)931-3183730.
1 Introduction

The limited performance of consumption Euler equations is well known. Prominent failures are documented in the literature on the equity premium puzzle, the fact that the average excess return of stocks is too high to be consistent with standard models, as well as in the literature on the risk-free rate puzzle, the fact that observed money market rates are too low. In a recent paper, Canzoneri et al. (2007) present another failure of consumption Euler equations. By using a novel approach, they challenge the view that the money market interest rate targeted by the central bank is equal to the rate implied by a Euler equation, as is commonly assumed in standard new Keynesian (NK) macroeconomic models. Canzoneri et al. (2007) use U.S. data and derive conditional moments of consumption and inflation from an estimated vector autoregression (VAR). This information and actual observations of consumption and inflation are then used to compute interest rates implied by consumption Euler equations obtained from alternative specifications of preferences. By comparing Euler equation rates with observed money market rates, two important results stand out. First, the behavior of implied rates differs significantly from the Federal Funds rate. In particular, real interest rates implied by Euler equations are strongly negatively correlated with the observed money market rate. Second, Canzoneri et al. (2007) report that the spread between the two rates is systematically linked to monetary policy. Standard regression analysis and impulse response functions show that the Federal Funds rate and the Euler equation rate move in opposite directions following a monetary policy tightening.

The purpose of this paper is to explore the link between the correlation between implied and actual interest rates and the stance of monetary policy. As explained by Canzoneri et al. (2007), the fact that the two rates do not coincide is intuitive if the representative household has standard, additively separable CRRA preferences. Empirical studies show that consumption responds in a hump-shaped fashion to a monetary contraction (see Christiano et al., 2005). That is, in the quarters following a monetary contraction interest rates and consumption growth are negatively correlated. Standard preferences, however, imply that consumption growth and interest rates are positively correlated. Consequently, using a standard Euler equation to compute implied interest rates results in a negative correlation between actual and implied interest rates.

With this intuition in mind, changing the preference specification of the representative
household seems to be a natural way of reconciling the dynamics of money market interest rates and rates implied by Euler equations. In particular, adding habit persistence to household preferences has been very successful in macroeconomic modeling. It has been proposed to strengthen the asset pricing implication of consumption-based models (see, for instance, Abel, 1990; Campbell and Cochrane, 1999), and it has proven to be highly relevant from a business cycle perspective. Most prominently, Fuhrer (2000), Christiano et al. (2005) and Smets and Wouters (2007) rely on habit persistence to explain the observed dynamics of output and consumption in response to monetary policy shocks. In light of this success, the finding in Canzoneri et al. (2007) that the implied Euler equation rate and the Federal Funds rate do not coincide across a large number of preference specifications that explicitly allow for habit formation is quite surprisingly.

In order to investigate the sources of the negative correlation between implied and actual interest rates, we make use of a Monte Carlo experiment. We assume that the model economy is defined by a fully fledged NK dynamic stochastic general equilibrium (DSGE) model. We use this model as a data-generating process and compute replications of simulated data. Then, we use the simulated data to construct implied Euler equation rates following the methodology set forth by Canzoneri et al. (2007). Based on this setup, counterfactual simulations allow us to explore the sources of the spread between implied and actual interest rates in a direct way.

We choose to use the estimated model in Smets and Wouters (2007) (henceforth, SW) as our data-generating process. We do so because of several reasons. First, the SW model has become a modern workhorse NK model for forecasting and policy analysis. It features complex dynamics with a rich set of structural shocks that aims to describe a fairly complete quantitative description of the U.S. economy. Second, the consumption Euler equation in the SW model deviates from a standard Euler equation along two dimensions. On the one hand, it features habit formation. On the other hand, it allows for non-separability between consumption and labor effort. This is relevant because Collard and Dellas (2012) find that non-separability between consumption and hours worked makes the problem identified by Canzoneri et al. (2007) less severe. The explanation for their result is straightforward: From empirical studies it is known that employment growth

---


2By applying data from G7 countries, Ahmad (2005) studies whether the finding in Canzoneri et al. (2007) is an artifact of U.S. data. His study yields correlation coefficients between implied Euler rates and money market rates that are mostly low and often negative.
declines in response to a monetary tightening. The consumption Euler equation implies that expected employment growth and real interest rates are negatively correlated.\footnote{Note that this is true if the coefficient of relative risk aversion is larger than one.} Hence, following a monetary tightening the implied Euler equation rate goes up consistently with the observed money market rate. Third, we choose to use the SW model for our Monte Carlo experiment because the model features a wedge between the money market interest rate and the interest rate implied by the consumption Euler equation. A shock to this wedge (risk premium shock) distorts the equality between the two rates and causes a change in the consumption pattern of households. Thus, given that in the data-generating process implied by the SW model the spread between Euler equation and actual interest rates is simply a statistical noise, we are able to disentangle the impact of monetary policy on the correlation between the two rates from the effect that arises from the assumption of risk premium disturbances.

In the next section, we use U.S. data to compute interest rates implied by consumption Euler equations for two sets of preferences and compare these rates to the Federal Funds rate. In Section 3, we use a Monte Carlo experiment to explore the relationship between implied and actual interest rates. Section 4 concludes.

\section{Comparing Euler equation and money market interest rates}

Here, we follow the approach in Canzoneri et al. (2007) and compute nominal and real interest rates implied by consumption Euler equations. We consider two sets of preferences. We compute implied interest rates for the specification of preferences as in Smets and Wouters (2007) and for standard, additively separable CRRA preferences. In each model, it is assumed that the representative household is infinitely lived, chooses consumption, labor effort and holdings of one-period nominal bonds in order to maximize lifetime utility subject to a budget constraint. As in Smets and Wouters (2007), the consumer’s objective function is assumed to be

\begin{equation}
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1 - \sigma_c} (C_t - H_t)^{1-\sigma_c} \right) \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1-\sigma_l} \right),
\end{equation}
where $E_t$ denotes the expectation operator at any period $t$, $C_t$ denotes consumption relative to a habit stock, $H_t$, $L_t$ is hours worked, $\sigma_c$ is the coefficient of relative risk aversion, $\sigma_l$ is the inverse elasticity of labor supply. The habit stock is external and evolves as $H_t = \lambda C_{t-1}$, where $\lambda$ governs the degree of habit formation. Smets and Wouters’ specification of consumer preferences nests the standard CRRA utility function with separability between consumption and hours worked and no habit formation. If $\sigma_c$ approaches 1 and $h = 0$, the period utility function implied by (1) approaches to a standard log utility function such that lifetime utility reads

$$E_0 \sum_{t=0}^{\infty} \beta^t \log C_t.$$  

(2)

The corresponding Euler equations to (1) and (2) are

$$\exp \left( \frac{\sigma_c - 1}{\sigma_c} L_t^{1-\sigma_c} \right) \left( C_t - \lambda C_{t-1} \right)^{\sigma_c} = \beta E_t \left( \frac{\exp \left( \frac{\sigma_c - 1}{\sigma_c} L_{t+1}^{1-\sigma_c} \right) R_t \epsilon_t^b}{\left( C_{t+1} - \lambda C_t \right)^{\sigma_c} \Pi_{t+1}} \right)$$  

(3)

and

$$1 \left( C_t \right)^{\sigma_c} = \beta E_t \left( \frac{1}{C_{t+1}} \frac{R_t \epsilon_t^b}{\Pi_{t+1}} \right),$$  

(4)

where $R_t$ is the gross nominal interest rate controlled by the central bank, $\Pi_t$ is the gross inflation rate and $\epsilon_t^b$ is a risk premium shock that represents a wedge between $R_t$ and the return on bonds held by households. The shock is assumed to follow an AR(1) process in logs. Following the analysis in Canzoneri et al. (2007), we abstract from the shock term when we compute implied Euler equation interest rates.

Log-linearizing (3) around the steady state balanced growth path of the model yields the following dynamics of nominal, respectively, real interest rates\(^4\)

$$r_t = (1/c_3) (c_1 c_{t-1} - c_t + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1})) + E_t \pi_{t+1}$$  

(5)

and

$$rr_t = (1/c_3) (c_1 c_{t-1} - c_t + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1})),$$  

(6)

where $c_1 = \lambda / \gamma$, $c_2 = \frac{(\sigma_c - 1) (W^b L^*/C^*)}{\sigma_c (1+\lambda/\gamma)}$, $c_3 = \frac{1 - \lambda / \gamma}{\sigma_c (1+\lambda/\gamma)}$, and $\gamma$ is the steady state growth rate. The log-linear dynamics of nominal and real interest rates implied by (4) are given

---

\(^4\)A lower case letter denotes the log-linear deviation of the corresponding upper case letter from the balanced growth path and starred variables refer to steady state values (see Smets and Wouters, 2007, for details). Note that Canzoneri et al. (2007) compute implied interest rates under the assumption of conditional lognormality. As they have already pointed out, the assumption of lognormality results in Euler equations that differ from those derived by log-linearization only by a constant.
In order to compute implied interest rates from (5)-(8), we need to calibrate the model parameters as well as to draw information on the conditional, one-quarter ahead forecasts of consumption, inflation and labor effort. As for the former, we use the posterior mean estimates reported in Smets and Wouters (2007). For the conditional first-order moments, we follow Canzoneri et al. (2007) and assume that the dynamics of consumption, employment and inflation can be captured in a VAR defined as

$$Z_t = A_0 + A_1 Z_{t-1} + \ldots + A_p Z_{t-p} + u_t,$$

where $u_t$ is a vector of iid-normal error terms. The variables in the VAR are the log of per capita real consumption expenditures on nondurable goods and services, the inflation rate, a measure of hours worked, the log CRB price index, the log of per capita real disposable income, the log of per capita real nonconsumption GDP and the Federal Funds rate.\(^5\)

In figure 1, we present the results by comparing the time series of the Federal Funds rate and the rates implied by the two sets of consumption Euler equations. Table 1 summarizes the properties of actual and implied interest rates by reporting second-order moments. As the figure illustrates, implied Euler equation rates behave significantly different compared to actual interest rates. Remarkably, the real interest rate implied by standard preferences (SP) is found to be negatively correlated with the observed money market rate.\(^6\)

As in Collard and Dellas (2012), we do not find a negative correlation when preferences feature habits and non-separability between consumption and labor effort.

While habit formation and non-separability lead to a positive correlation between implied and actual real interest rates, we find that the implied Euler equation rates are extremely volatile. On the one hand, this is in line with Canzoneri et al. (2007) who find that excess volatility arises across a number of preference specifications that include habit

---

\(^5\)The inflation rate is measured as the log change in the deflator for expenditures on nondurable goods and services. The data on hours worked is constructed as in Smets and Wouters (2007). The VAR is estimated over the sample from 1966:i to 2008:ii. We use two lags. As it is common in the literature we do not detrend real aggregates (see, for instance, Canzoneri et al., 2007; Collard and Dellas, 2012).

\(^6\)We measure actual real interest rates by subtracting one-quarter ahead forecasts of inflation obtained from the VAR from the Federal Funds rate.
Figure 1: Euler equation versus Federal Funds rate

Table 1: Statistics for interest rates (% p.a.)

<table>
<thead>
<tr>
<th></th>
<th>FFr</th>
<th>Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SP</td>
</tr>
<tr>
<td>Nominal interest rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. dev</td>
<td>3.28</td>
<td>1.69</td>
</tr>
<tr>
<td>Corr(FFr, model)</td>
<td>-</td>
<td>0.53</td>
</tr>
<tr>
<td>Real interest rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. dev.</td>
<td>2.15</td>
<td>0.97</td>
</tr>
<tr>
<td>Corr(FFr, model)</td>
<td>-</td>
<td>-0.07</td>
</tr>
</tbody>
</table>
formation. Clearly, habit formation implies a desire for a smooth consumption path. Thus, by adding habit persistence to household preferences, a higher volatility of implied interest rates is needed to explain the observed volatility of U.S. consumption. On the other hand, the finding of excess volatility of implied interest rates is in contrast to Collard and Dellas (2012). They find that non-separability between consumption and labor can in principal solve for this issue. Note that our finding of excess volatility do not challenge their result as we can report that choosing \( h = 0.5 \) leads to a perfect match between the volatilities of implied and actual real interest rates, while the corresponding correlation remains at about 0.10.

3 Monte Carlo experiment

In this section, we challenge the findings in the previous section by making use of a Monte Carlo experiment. We take the estimated model in Smets and Wouters (2007), assume that it is the true data-generating process, and compute replications of simulated data. Then, for each replication, we compute implied interest rates as outlined in section 2. Counterfactual model simulations allow us to explore the relationship between implied and actual interest rates.

3.1 Baseline results

In figure 2, we plot the distribution of the correlation between implied and actual (model generated) interest rates based on 1000 replications of artificial time series of consumption, inflation, hours worked and interest rates of length equal to that of the data used in the previous section.\(^7\)

It is apparent from the figure that the standard Euler equation as well as the SW Euler equation fails to mimic the behavior of actual interest rates. For both Euler equations the correlation between implied and actual interest rates is centered around zero. While for standard preferences the correlation is highly volatile, it is tight for the specification of preferences as in the SW model because the excess volatility of interest rates implied by consumption habits mechanically ties the correlation to zero. Noteworthy, for both Euler equations, the correlation between the Federal Funds rate and the Euler equation interest

\(^7\)We simulate data from the model evaluated at its posterior mean. Hence, we rule out parameter uncertainty when we construct implied interest rates.
rate lies well within the corresponding distribution of the correlation between actual and implied interest rates based on the Monte Carlo experiment.

### 3.2 The role of monetary policy and risk premium shocks

In our experiment we can identify three sources that obviously account for the spread between actual and implied interest rates. A first source is model misspecification. Clearly, this is the case for standard preferences. A second source stems from the fact that in order to compute implied interest rates information on households’ forecasts has to be drawn. In our experiment we can easily address this issue by using the relevant information from the simulated model. By doing so, we find that the baseline results remain virtually unchanged such that we conclude that this source of divergence is of little relevance. A third source arises due to the omission of risk premium shocks as implied by (5)-(8). Clearly, if actual consumption dynamics are influenced by risk premium disturbances, implied Euler rates and actual interest rates diverge.

To explore whether the spread is systematically linked to monetary policy (see Canzoneri et al., 2007) and to investigate the role of risk premium disturbances we make use of counterfactual model simulations. Figure 3 shows the distribution of the correlation be-
between actual and implied interest rates based on replications of artificial data as a function of the variance of risk premium ($\sigma_b$) and monetary policy shocks ($\sigma_r$). With this setup at hand, we are able to disentangle the impact of monetary policy on the correlation between the two rates from the effect that arises from the assumption of risk premium disturbances.

The results are clear-cut. In the case of standard preferences, a higher variance of both monetary and risk premium shocks results in a negative correlation. This is not the case when implied interest rates are computed using the SW Euler equation. While a higher variance of monetary policy shocks induces a positive correlation, a higher variance of risk premium shocks makes it negative. In that sense, an increasing importance of monetary policy disturbances stabilizes the correlation between implied and actual interest rates.

The analysis reveals that only risk premium shocks have the capability to drive a wedge between actual and implied interest rates such that the observed correlation between the two rates is negative.

In figure 4 we gain further insights into the link between implied and actual interest rates by computing impulse response functions of implied interest rates for a risk premium as well as a monetary policy shock. For the sake of completeness, impulse responses of model variables are shown in figure 5. As can be seen, the spread between implied and actual interest rates is related to monetary policy in the case of standard preferences. Fol-
Figure 4: Implied interest rate responses for risk premium and monetary policy shock

Note: The solid lines represent impulse responses of the baseline model. The dashed lines are impulse responses to a serially uncorrelated shock.

Figure 5: Model impulse responses for risk premium and monetary policy shock

Note: The solid lines represent impulse responses of the baseline model. The dashed lines are impulse responses to a serially uncorrelated shock.
ollowing a monetary tightening, interest rates rise and consumption responds in a humped-shaped fashion. Since standard preferences imply a positive relation between interest rates and consumption growth, implied Euler equation interest rates initially drop. This is not the case when preferences exhibit habits and non-separability between consumption and labor effort. Interest rate dynamics implied by the SW Euler equation are equal to actual dynamics. In the case of the SW Euler equation, the spread between implied and actual interest rates is linked exclusively to risk premium shocks. As shown in figure 5, a negative shock to the wedge between the money market interest rate targeted by the central bank and the return on bonds held by households causes consumption, labor, inflation and interest rates to rise. Interest rates implied by the consumption Euler equation, however, drop initially. Hence, in the wake of risk premium disturbances the observed correlation between implied and actual interest rates is negative.

4 Conclusion

At first glance, the message of our analysis for the problem posed by the failure of consumption Euler equations identified by Canzoneri et al. (2007) is straightforward. Given that the model economy of our Monte Carlo experiment is true, we can conclude that risk premium shocks have the capability to drive a wedge between money market interest rates and interest rates implied by consumption Euler equations such that the observed correlation between the two rates is negative. Moreover, the fact that the spread between actual and implied interest rates is simply a statistical noise is good news for the analysis of monetary policy within the NK framework that equates the two rates. However, the analysis in our paper is not without controversy because whether the model put forward by Smets and Wouters (2007) represents the true data-generating process is subject of debate. Clearly, the model lacks a structural modeling of risk premium disturbances. Chari et al. (2009) make the point that the estimated variance of the risk premium shock seems to be implausible large compared to the variance of the Federal Funds rate. To our opinion, there is no necessary conflict between their concern and our result that risk premium shocks may resolve the evidence on implied Euler equation rates. In fact, both are simply two sides of the same coin. Nevertheless, we should note that our analysis does not rely on whether shocks to the wedge between the money market interest rate and the Euler equation rate are truly risk premium disturbances. In principal, any disturbance
term that alters households’ intertemporal optimality condition for consumption has the capability to induce a negative correlation between observed and implied Euler equation interest rates. With respect to this, the message of our paper is that more has to be done to fully reconcile observed consumption dynamics and the structural underpinning implied by the consumption Euler equation. One promising way is to implement financial factors in the form of collateral constraints along the lines of Kiyotaki and Moore (1997). In a highly influential paper, Iacoviello (2005) extends the standard NK framework to account for borrowing constraints tied to housing values and nominal debt. By using structural estimation, he finds that collateral effects are crucial to explain U.S. consumption dynamics in response to fluctuations in house prices. In a related work, Iacoviello and Neri (2010) provide evidence that housing collateral in the period from 1989:4 to 2006:4 account for 12% of the total variance in U.S. consumption growth. Given this quantitatively large effect of borrowing constraints on U.S. consumption dynamics, this friction might be a candidate to resolve the failure of consumption Euler equations implied by the analysis in Canzoneri et al. (2007).
References


