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## Why crises happen: Nonstationary macroeconomics

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# Why crises happen — nonstationary macroeconomics

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## Abstract

A Real Business Cycle model of the UK is developed to account for the behaviour of UK non-stationary macro data. The model is tested by the method of indirect inference, bootstrapping the errors to generate 95% confidence limits for a VECM representation of the data; we find the model can explain the behaviour of main variables (GDP, real exchange rate, real interest rate) but not that of detailed GDP components. We use the model to explain how ‘crisis’ and ‘euphoria’ are endemic in capitalist behaviour due to nonstationarity; and we draw some policy lessons.

JEL Classification : E32, F31, F41

Key Words : Nonstationarity, Productivity, Real Business Cycle, Bootstrap, Indirect Inference, banking crisis, banking regulation

Macroeconomic data are generally non-stationary, i.e. a part at least of their movement each quarter is random. This feature is responsible for the considerable uncertainty surrounding the economy's long-term future. Models of the economy have reacted to this feature by abstracting from it and using some technique for extracting the trend from the data so as to render it stationary. Tests of these models have generally been done on such stationarised data. However the techniques (such as the Band Pass and the Hodrick-Prescott filters) are not based on the theories used in these models; instead they are based on statistical properties of the data and so extract from the data information that could well have a bearing on the models' fit. It would seem that to test models convincingly one should use the original data in full. That is one aim of this paper. We propose a testing procedure that utilises the original data.

The very uncertainty implied by non-stationary data suggests another way in which a model using this original data could shed light on the economy in an important way that those assuming stationary data do not: they could explain the large deviations from steady time trends that economies experience from time to time, whether long-running booms or 'crises'. The recent Great Recession is an example of the latter that is fresh in all our minds: in it the OECD economies suffered a severe drop in activity that was impossible to forecast and may also not be reversed, in the sense that output seems set to resume its previous growth rate but not recover to its previous trend level. This description has the hallmarks of non-stationarity where random changes in GDP growth lead to permanent changes in the level of GDP. Our suggestion is that in 'normal times' such random changes either do not get repeated or partially reverse themselves, but that times of unusual boom or crisis are marked by 'runs' of several repeated changes in the same direction.

Models of crisis have been proposed before. Thus in response to currency crises (such as the Mexican and the Asian crises) work was done on open economy models in which a shift of expectations about the economy's future would trigger a run on the currency; thus these 'currency crisis' models invoked expectations shocks, based on game-theoretic models of commitment and reneging — e.g. Obstfeld (1996), Burnside et al (2004). Related to these models are 'sudden stop' models where a country faces a collateral constraint and a shock can force it to stop borrowing by pushing it up against this constraint — e.g. Benigno et al (2009). In response to the banking crisis of 2007–9 recent work has also built models with a banking sector which may generate crisis through either a shock to the economy which destroys collateral or a shock to the banking sector which destroys credit availability — e.g. Goodhart et al (2009). In addition to these models, which fall into the category of micro-founded rational expectations (DSGE) models, the recent crisis has encouraged other models in which agents' behaviour is not based on rational expectations but for example on heuristic rules of thumb or behavioural assumptions — e.g. de Grauwe (2009) and Kirman (2009). In this paper we propose a different approach based, within a DSGE model, on non-stationarity as briefly explained above. In this model crisis arises from random same-direction sequences of non-stationary shocks to productivity. In the model here there is no banking; thus it is a model of crisis rather than specifically of banking crisis.

Thus we have here a Real Business Cycle model in which the key innovation is that non-stationary productivity behaviour produces periods of strong sustained growth and also periods of 'crisis'. We regard this as a description of 'capitalism' at work — crisis being an inevitable ingredient in the process. We therefore reject the idea that crises can be avoided by for example regulatory policy. While our model contains no banking sector, and therefore there are no 'banking crises' in it, it would not be difficult to add intermediation and then to the extent that intermediaries had built up credit, their losses in the crisis period would then deplete banking sector assets — adding a banking crisis to the original productivity crisis. However, in our model the originator of any crisis remains productivity; and the model structure is a sufficient propagator to account for the economy's fluctuations without appealing to other elements. Our aim is thus to create understanding of the power of this basic mechanism; this is not of course to deny that there could be other contributory factors such as banking and money but merely to focus on what we see as the key causal mechanism.

A further innovation in this paper is that we offer a full empirical test of the model's explanatory power, something that these previous contributions have not done but would in our view need to do if they are to be contenders to explain macroeconomic events inclusive of crises. We make use of available theory to calibrate the model and we test its empirical performance for a particular economy (the UK) via the method of indirect inference implemented on unfiltered, generally non-stationary, data. We use the original data and develop tests of the null hypothesis of the model based on this data using a Vector Error Correction Model (VECM) as the 'auxiliary model' within indirect inference. This method of testing is still fairly new but the idea behind it is familiar from a large literature testing models by comparing their simulated behaviour with that of the data in respect of particular relationships such as moments and cross-moments. The distinguishing feature of indirect inference is that this comparison

is based on classical statistical inference and so normal significance tests can be used to evaluate the model. The idea is to generate the sampling distribution of the data implied by the model and to check whether the actual data lies within it at some chosen confidence level. In order to find this sampling distribution one needs to find the error processes implied by the model, and generate from them the random behaviour of which they are capable in repeated samples. For this we use the bootstrap since generally we are dealing with small samples to which asymptotic tests do not apply accurately.

So the aim of this paper is to develop and test a model of non-stationary economic behaviour in the hope of shedding light on causes of the turbulence that from time to time unpredictably grips the economy.

In what follows we start with the model and our empirical tests of it on UK data (sections 1 and 2). We then go on to discuss how its behaviour sheds light on booms and crises in a way that is entirely consistent with rational maximising agents with rational expectations (section 3). We end (section 4) with the conclusions including a brief account of the policy implications.

## 1 The Model

Consider a home economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment; all variables are in per capita terms. It coexists with another, foreign, economy (the rest of the world) in which equivalent choices are made; however because this other country is assumed to be large relative to the home economy we treat its income as unaffected by developments in the home economy. We assume that there are no market imperfections. At the beginning of each period  $t$ , the representative agent chooses (a) the commodity bundle necessary for consumption, (b) the total amount of leisure that it would like to enjoy, and (c) the total amount of factor inputs necessary to carry out production. All of these choices are constrained by the fixed amount of time available and the aggregate resource constraint that agents face. During period  $t$ , the model economy is influenced by various random shocks.

In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption,  $C_t$  in the utility function below, is composite per capita consumption, made up of agents consumption of domestic goods,  $C_t^d$  and their consumption of imported goods,  $C_t^f$ . We treat the consumption bundle as the numeraire so that all prices are expressed relative to the general price level,  $P_t$ . The composite consumption utility index can be represented as an Armington (1969) aggregator of the form

$$C_t = \left[ \omega (C_t^d)^{-\sigma} + (1 - \omega) \varsigma_t (C_t^f)^{-\sigma} \right]^{\left(\frac{-1}{\sigma}\right)} \quad (1)$$

where  $\omega$  is the weight of home goods in the consumption function,  $\sigma$ , the elasticity of substitution is equal to  $\frac{1}{1+\sigma}$  and  $\varsigma_t$  is a preference error.

The consumer maximises this composite utility index, given that an amount  $\widetilde{C}_t$  has been chosen for total expenditure, with respect to its components,  $C_t^d$  and  $C_t^f$  subject to  $\widetilde{C}_t = p_t^d C_t^d + Q_t C_t^f$  where  $p_t^d$  is the domestic price level relative to the general price level and  $Q_t$ <sup>1</sup> is the foreign price level in domestic currency relative to the general price level (the real exchange rate). The resulting expression for the home demand for foreign goods is

$$\frac{C_t^f}{C_t} = [(1 - \omega)\varsigma_t]^\sigma (Q_t)^{-\sigma} \quad (2)$$

We also note that:

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<sup>1</sup>we form the Lagrangean  $L = \left[ \omega (C_t^d)^{-\sigma} + (1 - \omega) \left( C_t^f \right)^{-\sigma} \right]^{\left(\frac{-1}{\sigma}\right)} + \mu \left( \widetilde{C}_t - \frac{P_t^d}{P_t} C_t^d - \frac{P_t^f}{P_t} C_t^f \right)$ . Thus  $\frac{\partial L}{\partial C_t} = \mu$ ; also at its maximum with the constraint binding  $L = \widetilde{C}_t$  so that  $\frac{\partial L}{\partial C_t} = 1$ . Thus  $\mu = 1$  - the change in the utility index from a one unit rise in consumption is unity. Substituting this into the first order condition  $0 = \frac{\partial L}{\partial C_t^f}$  yields equation (2).  $0 = \frac{\partial L}{\partial C_t^d}$  gives the equivalent equation:  $\frac{C_t^d}{C_t} = \omega^\sigma (p_t^d)^{-\sigma}$  where  $p_t^d = \frac{P_t^d}{P_t}$ . Divide (1) through by  $C_t$  to obtain  $1 = \left[ \omega \left( \frac{C_t^d}{C_t} \right)^{-\sigma} + (1 - \omega) \left( \frac{C_t^f}{C_t} \right)^{-\sigma} \right]^{\left(\frac{-1}{\sigma}\right)}$ ; substituting into this for  $\frac{C_t^f}{C_t}$  and  $\frac{C_t^d}{C_t}$  from the previous two equations gives us equation (3).

$$1 = \omega^\sigma (p_t^d)^{\sigma\varrho} + [(1 - \omega)\varsigma_t]^\sigma Q_t^{\sigma\varrho} \quad (3)$$

Hence we can obtain the logarithmic approximation:

$$\log p_t^d = - \left( \frac{1 - \omega}{\omega} \right)^\sigma \log(Q_t) - \frac{1}{\varrho} \left( \frac{1 - \omega}{\omega} \right)^\sigma \log \varsigma_t + \text{constant} \quad (4)$$

In a stochastic environment a consumer is expected to maximise expected utility subject to the budget constraint. Each agent's preferences are given by

$$U = \text{Max} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right], \quad 0 < \beta < 1 \quad (5)$$

where  $\beta$  is the discount factor,  $C_t$  is consumption in period 't',  $L_t$  is the amount of leisure time consumed in period 't' and  $E_0$  is the mathematical expectations operator. Specifically, we assume a time-separable utility function of the form

$$U(C_t, 1 - N_t) = \theta_0 (1 - \rho_0)^{-1} \gamma_t C_t^{(1 - \rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} \zeta_t (1 - N_t)^{(1 - \rho_2)} \quad (6)$$

where  $0 < \theta_0 < 1$ , and  $\rho_0, \rho_2 > 0$  are the substitution parameters; and  $\gamma_t, \zeta_t$  are preference errors. This sort of functional form is common in the literature for example McCallum and Nelson (1999a). Total endowment of time is normalised to unity so that

$$N_t + L_t = 1 \text{ or } L_t = 1 - N_t \quad (7)$$

Furthermore for convenience in the logarithmic transformations we assume that approximately  $L = N$  on average.

The representative agent's budget constraint is

$$C_t + \frac{b_{t+1}}{1 + r_t} + \frac{Q_t b_{t+1}^f}{(1 + r_t^f)} + p_t S_t^p = (v_t) N_t - T_t + b_t + Q_t b_t^f + (p_t + d_t) S_t^p \quad (8)$$

where  $p_t$  denotes the real present value of shares (in the economy's firms which they own),  $v_t = \frac{W_t}{P_t}$  is the real consumer wage ( $w_t$ , the producer real wage, is the wage relative to the domestic goods price level; so  $v_t = w_t p_t^d$ ). Households are taxed by a lump-sum transfer,  $T_t$ ; marginal tax rates are not included in the model explicitly and appear implicitly in the error term of the labour supply equation,  $\zeta_t$ .  $b_t^f$  denotes foreign bonds,  $b_t$  domestic bonds,  $S_t^p$  demand for domestic shares and  $Q_t = \frac{P_t^f}{P_t}$  is the real exchange rate.

In a stochastic environment the representative agent maximizes the expected discounted stream of utility subject to the budget constraint. The first order conditions with respect to  $C_t$ ,  $N_t$ ,  $b_t$ ,  $b_t^f$  and  $S_t^p$  are (where  $\lambda_t$  is the Lagrangean multiplier on the budget constraint):

$$\theta_0 \gamma_t C_t^{-\rho_0} = \lambda_t \quad (9)$$

$$(1 - \theta_0) \zeta_t (1 - N_t)^{-\rho_2} = \lambda_t (1 - \tau_t) v_t \quad (10)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} \quad (11)$$

$$\frac{\lambda_t Q_t}{(1 + r_t^f)} = \beta E_t \lambda_{t+1} Q_{t+1} \quad (12)$$

$$\lambda_t p_t = \beta E_t \lambda_{t+1} (p_{t+1} + d_{t+1}) \quad (13)$$

Substituting equation (11) in (9) yields :

$$(1 + r_t) = \left( \frac{1}{\beta} \right) E_t \left( \frac{\gamma_t}{\gamma_{t+1}} \right) \left( \frac{C_t}{C_{t+1}} \right)^{-\rho_0} \quad (14)$$

Now substituting (9) and (11) in (10) yields

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} v_t}{(1 - \theta_0) \zeta_t} \right\}^{\frac{-1}{\rho_2}} \quad (15)$$

Substituting out for  $v_t = w_t p_t^d$  and using (4) equation (15) becomes

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[ (1 - \tau_t) \exp \left( \log w_t - \left( \frac{1-\omega}{\omega} \right)^\sigma \left( \log Q_t + \frac{1}{\varrho} \log \zeta_t \right) \right) \right]}{(1 - \theta_0) \zeta_t} \right\}^{\frac{-1}{\rho_2}} \quad (16)$$

Substituting (11) in (13) yields

$$p_t = \left( \frac{p_{t+1} + d_{t+1}}{(1 + r_t)} \right) \quad (17)$$

Using the arbitrage condition and by forward substitution the above yields

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1 + r_t)^i} \quad (18)$$

i.e. the present value of a share is discounted future dividends.

To derive the uncovered interest parity condition in real terms, equation (11) is substituted into (12)

$$\left( \frac{1 + r_t}{1 + r_t^f} \right) = E_t \frac{Q_{t+1}}{Q_t} \quad (19)$$

In logs this yields

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t} \quad (20)$$

Thus the real interest rate differential is equal to the expected change in the real exchange rate. Financial markets are otherwise not integrated and are incomplete, though assuming completeness makes no difference to the model's solution in this non-stationary world<sup>2</sup>.

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<sup>2</sup>It turns out that under non-stationary shocks the model solution is the same under complete contingent asset contracts.

Consider contingent assets paying 1 consumption unit in specified states of the world (for example when  $y_{t+T} = \bar{y}$ )? Here we write the price of this asset, P, as

$$P_t = \frac{\beta^T u'_{t+T}(y_{t+T}=\bar{y}) \text{prob}(y_{t+T}=\bar{y})}{u'_t} \quad (1)$$

One, the first, problem here is to define this probability. Since GDP has an infinite variance at T as T tends to infinity, we define the probability for a finite T. Such an asset will not be valued anyway for an 'infinite T' since as T tends to infinity  $\beta^T$  tends to zero. In practice therefore an asset paying off in 'infinite' time is not interesting to a household. For earlier finite periods however  $\beta^T$  is non-zero and the probabilities can be defined so that the asset is valued.

Now introduce a foreign country and allow trading of these contingent assets. We now let  $y$  stand for the vector of states in both countries. The foreign country's equivalent asset paying one unit of foreign consumption at T would be

$$P_{Ft} = \frac{\beta^T u'_{Ft+T}(y_{t+T}=\bar{y}) \text{prob}(y_{t+T}=\bar{y})}{u'_{Ft}} \quad (2)$$

Now the price a home resident would pay for this foreign asset would be

$$P_{Ft} = \frac{\beta^T u'_{t+T} Q_{t+T}(y_{t+T}=\bar{y}) \text{prob}(y_{t+T}=\bar{y})}{u'_t Q_t} \quad (3)$$

while the price a foreigner would pay for the home asset would be

$$P_t = \frac{\beta^T u'_{Ft+T} / Q_{t+T}(y_{t+T}=\bar{y}) \text{prob}(y_{t+T}=\bar{y})}{u'_{Ft} / Q_t} \quad (4)$$

By equating these two values paid for each asset by home and foreign residents we obtain the Uncovered Parity contingent asset condition:

$$1 = \frac{u'_{t+T} Q_{t+T} / u'_{Ft+T}(y_{t+T}=\bar{y})}{u'_t Q_t / u'_{Ft}} \quad (5) \text{ or}$$

$$\ln u'_{t+T} - \ln u'_t = (\ln u'_{Ft+T} - \ln u'_{Ft}) + \ln Q_{t+T} - \ln Q_t \text{ for any state of the world at T} \quad (6)$$

This ties together movements in consumption over time in the two countries with the movement in the real exchange rate. Notice that under stationary shocks the probability of the future state at  $t + T$  could be defined independently of what  $t$  is, provided  $T$  is large enough so that the effects of any shocks originating at  $t$  have died away. This allowed Chari et al (2001) to fix  $t$  at some arbitrary initial date 0 and rewrite the condition

$$\ln u'_T = \ln u'_{FT} + \ln Q_T + \ln u'_0 - \ln u'_{F0} - \ln Q_0 \quad (7)$$

We may then normalise the initial values at zero for convenience to obtain

$$\ln u'_T = \ln u'_{FT} - \ln Q_T \quad (8)$$

However under non-stationary shocks such detachment of the condition from  $t$  is impossible because the state at  $t + T$  depends crucially on the state at  $t$ : the shocks at  $t$  are permanent and therefore alter the state at  $t + T$ .

## 1.1 The Government

The government finances its expenditure,  $G_t$ , by collecting taxes on labour income,  $\tau_t$ . Also, it issues debt, bonds ( $b_t$ ) each period which pays a return next period.

The government budget constraint is:

$$G_t + b_t = T_t + \frac{b_{t+1}}{1 + r_t} \quad (21)$$

where  $b_t$  is real bonds

## 1.2 The Representative Firm

Firms rent labour and buy capital inputs, transforming them into output according to a production technology. They sell consumption goods to households and government and capital goods to other firms. The technology available to the economy is described by a constant-returns-to-scale production function:

$$Y_t = Z_t N_t^\alpha K_t^{1-\alpha} \quad (23)$$

where  $0 \leq \alpha \leq 1$ ,  $Y_t$  is aggregate output per capita,  $K_t$  is capital carried over from previous period ( $t - 1$ ), and  $Z_t$  reflects the state of technology.

It is assumed that  $f(N, K)$  is smooth and concave and it satisfies Inada-type conditions i.e. the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity.

$$\begin{aligned} \lim_{K \rightarrow 0} (F_K) &= \lim_{N \rightarrow 0} (F_N) = \infty \\ \lim_{K \rightarrow \infty} (F_K) &= \lim_{N \rightarrow \infty} (F_N) = 0 \end{aligned} \quad (24)$$

The capital stock evolves according to:

$$K_t = I_t + (1 - \delta) K_{t-1} \quad (25)$$

where  $\delta$  is the depreciation rate and  $I_t$  is gross investment.

In a stochastic environment the firm maximizes the present discounted stream,  $V$ , of cash flows, subject to the constant-returns-to-scale production technology and quadratic adjustment costs for capital,

$$Max V = E_t \sum_{i=0}^T d_{it}^i [Y_{t+i} - K_{t+i}(r_{t+i} + \delta + \kappa_{t+i}) - (w_{t+i} + \chi_{t+i})N_{t+i} - 0.5\xi(\Delta K_{t+i})^2] \quad (26)$$

subject to the evolution of the capital stock in the economy, equation (25). Here  $r_t$  and  $w_t$  are the rental rates of capital and labour inputs used by the firm, both of which are taken as given by the firm. The terms  $\kappa_t$  and  $\chi_t$  are error terms capturing the impact of excluded tax rates and other imposts or regulations on firms' use of capital and labour respectively. The firm optimally chooses capital and labour so that marginal products are equal to the price per unit of input. The first order conditions with respect to  $K_t$  and  $N_t^d$  are as follows:

$$\xi(1 + d_{1t})K_t = \xi K_{t-1} + \xi d_{1t} E_t K_{t+1} + \frac{(1 - \alpha) Y_t}{K_t} - (r_t + \delta + \kappa_t) \quad (28)$$

$$N_t = \frac{\alpha Y_t}{w_t + \chi_t} \quad (29)$$

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We may now note that taking rational expectations at  $t$  of the condition we obtain:

$$E_t(\ln u'_{t+T} - \ln u'_t) = E_t(\ln u'_{Ft+T} - \ln u'_{Ft}) + E_t(\ln Q_{t+T} - \ln Q_t) \quad (9)$$

The lhs (by our non-contingent asset first order condition in the text — eqs 9 and 11 there) is simply  $T \ln R$ , the first term on the rhs is from the foreign equivalent  $T \ln R_F$ ; if  $r$  is the net real interest rate then  $\ln R \approx r$  so that we obtain UIP:

$$r_t = r_{Ft} + T^{-1}(E_t \ln Q_{t+T} - \ln Q_t) \quad (10)$$

What we discover is that under non-stationary shocks contingent assets do not change the rational expectations equilibrium of the model from that with merely non-contingent assets. The reason is that contingent asset values depend critically on the shocks at  $t$  and so do not as with stationary shocks produce a condition binding on the expected levels of variables independent of the date at which the expectation is formed.



### 1.3 The Foreign Sector

From equation (2) we can derive the import equation for our economy

$$\log C_t^f = \log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma \log Q_t + \sigma \log \varsigma_t \quad (34)$$

Now there exists a corresponding equation for the foreign country which is the export equation for the home economy

$$\log EX_t = \sigma^F \log (1 - \omega^F) + \log C_t^F + \sigma^F \log Q_t + \sigma^F \log \varsigma_t^F \quad (37)$$

Foreign bonds evolve over time to the balance payments according to the following equation

$$\frac{Q_t b_{t+1}^f}{(1 + r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \quad (40)$$

Finally there is good market clearing:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t \quad (41)$$

## 2 Calibration & Deterministic Simulation

The model is calibrated with the values familiar from earlier work and used in Meenagh et al (2010) — see Kydland and Prescott, (1982), Obstfeld and Rogoff (1996), Orphanides (1998), Dittmar, Gavin and Kydland (1999), McCallum and Nelson (1999a, 1999b), McCallum (2001), Rudebusch and Svensson (1999), Ball (1999) and Batini and Haldane (1999); the Appendix gives a full listing. Thus in particular the coefficient of relative risk aversion ( $\rho_0$ ) is set at 1.2 and the substitution elasticity between consumption and leisure ( $\rho_2$ ) at unity. Home bias ( $\omega, \omega^F$ ) is set high at 0.7. The substitution elasticity between home and foreign goods ( $\sigma, \sigma^F$ ) is set at 1 both for exports and for imports, thus assuming that the UK's products compete but not sensitively with foreign alternatives; this is in line with studies of the UK (see for example Minford et al., 1984).

Before testing the model stochastically against macro behaviour, we examine its implications in the face of a sustained one-off rise in productivity. Figure 1 shows the model simulation of a rise of the productivity level by 12% spread over 12 quarters and occurring at 1% per quarter (the increase in the whole new path is unanticipated in the first period and from then on fully anticipated) — in other words a three-year productivity ‘spurt’.

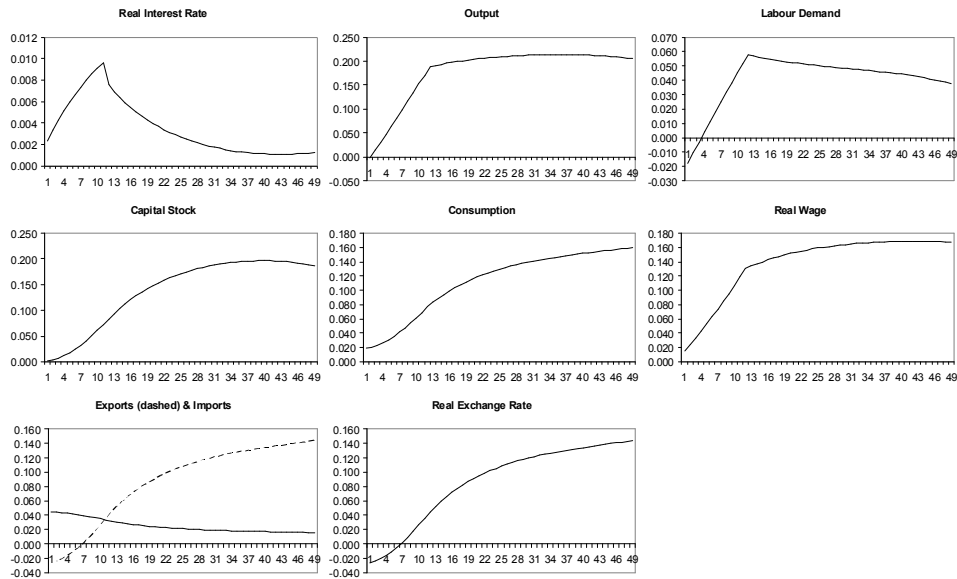


Figure 1: Plots of a 1% Productivity increase each quarter for twelve quarters

The logic behind the behaviour of the real exchange rate,  $Q$ , can be explained as follows. The productivity increase raises permanent income and also stimulates a stream of investments to raise the capital stock in line. Output however cannot be increased without increased labour supply and extra capital, which is slow to arrive. Thus the real interest rate must rise to reduce demand to the available supply while real wages rise to induce extra labour and output supply. The rising real interest rate violates Uncovered Real Interest Parity (URIP) which must be restored by a real appreciation (fall in  $Q$ ) relative to the expected future value of the real exchange rate. This appreciation is made possible by the expectation that the real exchange rate will depreciate ( $Q$  will rise) steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output,  $Q$  also moves back to equilibrium. This equilibrium however represents a real depreciation on the previous steady state (a higher  $Q$ ) since output is now higher and must be sold on world markets by lowering its price.

## 2.1 Stochastic processes

The model contains 8 stochastic processes: 7 shocks and 1 exogenous variable (world consumption). Of all these only one, the productivity shock, is treated as non-stationary and modelled as an ARIMA(1,1,0) with a constant (the drift term, hence the deterministic trend). Since it is produced as an identity from the production function it can be directly measured. This is also true of all but two of the other shocks, which can be directly ‘backed out’ of their equations since they contain no expectations terms. For the two error terms in equations containing expectations, viz consumption and the capital stock, the errors are estimated by using a robust instrumental variables estimator for the expectations due to McCallum (1976) and Wickens (1982).

Other than the productivity shock the other processes are all modelled as stationary or trend-stationary ARMA (1,0) processes plus a deterministic trend. These choices cannot be rejected by the data, when they are treated as the null; however, it turns out that at the single equation level it is not easy to distinguish the two treatments, in the sense that making the alternative the null also leads to non-rejection. Hence we have used the results from the model-testing to help determine which choices to make. The choices reported here — see Table 1 — were influenced by finding that the simulated variances of key variables explode as more processes are treated as non-stationary. (Later we report the result of even treating productivity as trend-stationary; it turns out to worsen the results substantially.)

An important implication of the deterministic components of the stochastic processes is that they generate the balanced growth path (BGP) of the model. This is integrated into our simulations so that the shock elements, be they stationary or non-stationary, are added onto this basic path. In the version of the model here these deterministic components are fixed and so is therefore the BGP; of course if we were investigating policies (such as tax) that affected growth, the BGP would respond to these, however we do not do that in this paper.

Shock	Process	$c$	$trend$	$AR(1)$
Consumer Preference	Stationary	−0.039181**		0.470434**
Productivity	Non-Stationary	0.003587**		0.022902
Labour Demand	Trend Stationary	0.263503**	−0.002141**	0.854444**
Capital	Stationary	0.086334**		0.870438**
Labour Supply	Trend Stationary	0.717576**	−0.002946**	0.962092**
Exports	Trend Stationary	−1.265935**	0.004288**	0.925119**
Imports	Trend Stationary	0.007662	0.002505**	0.836784**
Foreign Consumption	Trend Stationary	−0.685495**	0.016268**	0.964308**
Foreign Interest Rate	Stationary	0.002844		0.917345**

Note: \*\* is significant at 1%, \* is significant at 5%

Table 1: Error Processes

## 3 Model evaluation by indirect inference

Indirect inference provides a classical statistical inferential framework for judging a calibrated or already, but possibly, partially estimated model whilst maintaining the basic idea employed in the evaluation of

the early RBC models of comparing the moments generated by data simulated from the model with actual data. Using moments for the comparison is a distribution free approach. Instead, we posit a general but simple formal model (an auxiliary model) — in effect the conditional mean of the distribution of the data — and base the comparison on features of this model estimated from simulated and actual data.

Indirect inference on structural models may be distinguished from indirect estimation of structural models. Indirect estimation has been widely used for some time, see Smith (1993), Gregory and Smith (1991,1993), Gouriéroux et al. (1993), Gouriéroux and Monfort (1995) and Canova (2005). In estimation the parameters of the structural model are chosen so that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from actual data. In the use of indirect inference for model evaluation the parameters of the structural model are taken as given. The aim is to compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of a structural model — which is taken as the true model of the economy, the null hypothesis — with the performance of the auxiliary model when estimated from actual data. If the structural model is correct then its predictions about the impulse responses, moments and time series properties of the data should match those based on actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates.

Le et al (2010) discuss issues that arise in the choice of a VAR as the auxiliary model and in the comparison of a DSGE model with a DSGE model — see also Canova (2005), Dave and DeJong (2007), Del Negro and Schorfheide (2004, 2006) and Del Negro et al (2007a,b) together with the comments by Christiano (2007), Gallant (2007), Sims (2007), Faust (2007) and Kilian (2007). The *a priori* structural restrictions of the DSGE model impose restrictions on the VAR; see Canova and Sala (2009) for an example of lack of identification, however DSGE models are generally over-identified via the cross-equation restrictions implied by rational expectations — see Minford and Peel (2002, pp.436-7).

A formal statement of the inferential problem is as follows. Using the notation of Canova (2005) which was designed for indirect estimation, we define  $y_t$  an  $m \times 1$  vector of observed data ( $t = 1, \dots, T$ ),  $x_t(\theta)$  an  $m \times 1$  vector of simulated time series of  $S$  observations generated from the structural macroeconomic model,  $\theta$  a  $k \times 1$  vector of the parameters of the macroeconomic model.  $x_t(\theta)$  and  $y_t$  are assumed to be stationary and ergodic. We set  $S = T$  since we require that the actual data sample be regarded as a potential replication from the population of bootstrapped samples. The auxiliary model is  $f[y_t, \alpha]$ ; an example is the  $VAR(p)$   $y_t = \sum_{i=1}^p A_i y_{t-i} + \eta_t$  where  $\alpha$  is a vector comprising elements of the  $A_i$  and of the covariance matrix of  $y_t$ . On the null hypothesis  $H_0: \theta = \theta_0$ , the stated values of  $\theta$  whether obtained by calibration or estimation; the auxiliary model is then  $f[x_t(\theta_0), \alpha(\theta_0)] = f[y_t, \alpha]$ . We wish to test the null hypothesis through the  $q \times 1$  vector of continuous functions  $g(\alpha)$ . Such a formulation includes impulse response functions. On  $H_0$   $g(\alpha) = g[\alpha(\theta_0)]$ .

Let  $a_T$  denote the estimator of  $\alpha$  using actual data and  $a_S(\theta_0)$  the estimator of  $\alpha$  based on simulated data for  $\theta_0$ . We may therefore obtain  $g(a_T)$  and  $g[a_S(\theta_0)]$ . Using  $N$  independent sets of simulated data obtained using the bootstrap we can also define the bootstrap mean of the  $g[a_S(\theta)]$ ,  $\overline{g[a_S(\theta_0)]} = \frac{1}{N} \sum_{k=1}^N g_k[a_S(\theta_0)]$ . The Wald test statistic is based on the distribution of  $g(a_T) - \overline{g[a_S(\theta_0)]}$  where we assume that  $g(a_T) - \overline{g[a_S(\theta_0)]} \xrightarrow{P} 0$ . The resulting Wald statistic may be written as

$$WS = (g(a_T) - \overline{g[a_S(\theta_0)]})' W(\theta_0) (g(a_T) - \overline{g[a_S(\theta_0)]})$$

where  $W(\theta_0)$  is the inverse of the variance-covariance matrix of the distribution of  $g(a_T) - \overline{g[a_S(\theta_0)]}$ .

$W(\theta_0)^{-1}$  can be obtained from the asymptotic distribution of  $g(a_T) - \overline{g[a_S(\theta_0)]}$  and the asymptotic distribution of the Wald statistic would then be chi-squared. Instead, we obtain the empirical distribution of the Wald statistic by bootstrap methods based on defining  $g(\alpha)$  as a vector consisting of the VAR coefficients and the variances of the data or the VAR disturbances.

The following steps summarise our implementation of the Wald test by bootstrapping:

*Step 1: Estimate the errors of the economic model conditional on the observed data and  $\theta_0$ .*

Estimate the structural errors  $\varepsilon_t$  of the DSGE macroeconomic model,  $x_t(\theta_0)$ , given the stated values  $\theta_0$  and the observed data. The number of independent structural errors is taken to be less than or equal to the number of endogenous variables. The errors are not assumed to be normally distributed. Where the equations contain no expectations the errors can simply be backed out of the equation and the data. Where there are expectations estimation is required for the expectations; here we carry this out using the robust instrumental variables methods of McCallum (1976) and Wickens (1982), with the lagged endogenous data as instruments — thus effectively we use the auxiliary model  $VAR$ .

*Step 2: Derive the simulated data*

On the null hypothesis the  $\{\varepsilon_t\}_{t=1}^T$  are the structural errors. The simulated disturbances are drawn from these errors. In some DSGE models, including the model here, many of the structural errors are assumed to be generated by autoregressive processes rather than being serially independent. If they are then, under our method, we need to estimate them. We derive the simulated data by drawing the bootstrapped disturbances by time vector to preserve any simultaneity between them, and solving the resulting model using a projection method due to Minford (1984, 1986) and similar to Fair and Taylor (1983). To obtain the  $N$  bootstrapped simulations we repeat this drawing each sample independently. We set  $N = 1000$ .

*Step 3: Compute the Wald statistic*

We estimate the auxiliary model — a VAR(1) — using both the actual data and the  $N$  samples of simulated data to obtain estimates  $a_T$  and  $a_S(\theta_0)$  of the vector  $\alpha$ . The distribution of  $a_T - \bar{a}_S(\theta_0)$  and its covariance matrix  $W(\theta_0)^{-1}$  are estimated by bootstrapping  $a_S(\theta_0)$ . The bootstrapping proceeds by drawing  $N$  bootstrap samples of the structural model, and estimating the auxiliary VAR on each, thus obtaining  $N$  values of  $a_S(\theta_0)$ ; we obtain the covariance of the simulated variables directly from the bootstrap samples. The resulting set of  $a_k$  vectors ( $k = 1, \dots, N$ ) represents the sampling variation implied by the structural model from which estimates of its mean, covariance matrix and confidence bounds may be calculated directly. Thus, the estimate of  $W(\theta_0)^{-1}$  is

$$\frac{1}{N} \sum_{k=1}^N (a_k - \bar{a}_k)' (a_k - \bar{a}_k)$$

where  $\bar{a}_k = \frac{1}{N} \sum_{k=1}^N a_k$ . We then calculate the Wald statistic for the data sample; we estimate the bootstrap distribution of the Wald from the  $N$  bootstrap samples.

We note that the auxiliary model used is a VECM(1) and is for a limited number of key macro variables. By raising the lag order and increasing the number of variables, the stringency of the overall test of the model is increased. If we find that the structural model is already rejected at order 1, we do not proceed to a more stringent test based on a higher order <sup>3</sup>.

Rather than focus our tests on just the parameters of the auxiliary model or the impulse response functions, we also attach importance to the ability to match data variability, hence the inclusion here of the VECM residuals in  $\alpha$ . As highlighted in the debates over the Great Moderation and the recent banking crisis, a major macroeconomic issue also concerns the scale of real and nominal volatility. In this way our test procedure is within the traditions of RBC analysis.

Figure 2 illustrates the joint distribution for just two parameters of the auxiliary equation for two cases: assuming that the covariance matrix of the parameters is diagonal and that it is not. One can think of estimation via indirect inference as pushing the observed data point as far into the centre of the distribution as possible. The Wald test, however, takes the structural parameters as given and merely notes the position of the observed data point in the distribution.<sup>4</sup>

<sup>3</sup>This point is illustrated in Le et al (2010) for the model dealt with in that paper with the results for varying the lag order of the VAR used there on stationary data:

	Wald stat.	Mah. Dis.
VAR(1)	100	2.8
VAR(2)	100	4.55
VAR(3)	100	5.1

Notice how the normalised Mahalanobis Distance (a transform of the Wald value — see below for the full definition) gets steadily larger, indicating a steadily worsening fit, as the lag order is increased.

In fact the general representation of a stationary loglinearised DSGE model is a VARMA, which would imply that the true VAR should be of infinite order, at least if any DSGE model is the true model. However, for the same reason that we have not raised the VECM order above one, we have also not added any MA element. As DSGE models do better in meeting the challenge this could be considered.

<sup>4</sup>To understand why DSGE models will typically produce high covariances and so distributions like those in the bottom panel of Figure 2, we can give a simple example in the case where the two descriptors are the persistence of inflation and interest rates. If we recall the Fisher equation, we will see that the persistence of inflation and interest rates will be highly correlated. Thus in samples created by the DSGE model from its shocks where inflation is persistent, so will interest rates be; and similarly when the former is non-persistent so will the latter tend to be. Thus the two estimates of persistence under the null have a joint distribution that reflects this high correlation.

In Figure 2, we suppose that the model distribution is centred around 0.5, and 0.5; and the data-based VAR produced values for their partial autocorrelations of 0.1 and 0.9 respectively for inflation and interest rates — the two VAR coefficients. We suppose too that the 95% range for each was 0–1.0 (a standard deviation of 0.25) and thus each is accepted individually. If the parameters are uncorrelated across samples, then the situation is as illustrated in the top panel. They will also be jointly accepted.

Now consider the case where there is a high positive covariance between the parameter estimates across samples, as implied by the DSGE model (with its Fisher equation). The lower panel illustrates the case for a 0.9 cross-correlation

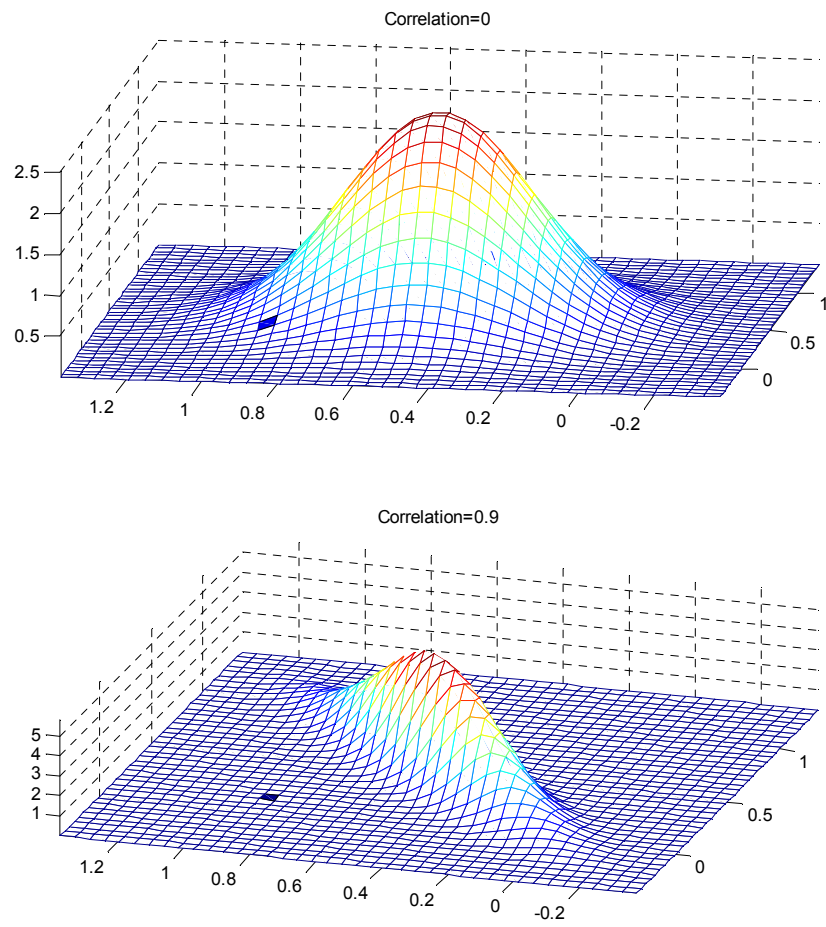


Figure 2: Bivariate Normal Distributions (0.1,0.9 shaded) with correlation of 0 and 0.9.

We refer to the Wald statistic based on the full set of variables as the Full Wald test; it checks whether the  $a$  vector lies within the DSGE model's implied joint distribution and is a test of the DSGE model's specification in a wide sense. We use the Mahalanobis Distance based on the same joint distribution, normalised as a t-statistic, as an overall measure of closeness between the model and the data. In effect, this conveys the same information as in the Wald test but is in the form of a t-value<sup>5</sup>.

We also consider a second Wald test, which we refer to as a 'Directed Wald statistic'. This focuses on more limited features of the structural model. Here we seek to know how well a particular variable or limited set of variables is modelled and we use the corresponding auxiliary equations for these variables in the VAR as the basis of our test. For example, we may wish to know how well the model can reproduce the behaviour of output and inflation by creating a Wald statistic based on the VAR equation for these two variables alone.

A Directed Wald test can also be used to determine how well the structural model captures the effects of a particular set of shocks. This requires creating the joint distribution of the IRFs for these shocks alone. For example, to determine how well the model deals with supply shocks, we construct the joint distribution of the IRFs for the supply shocks and calculate a Wald statistic for this. Even if the full model is misspecified, a Directed Wald test provides information about whether the model is well-specified enough to deal with specific aspects of economic behaviour.

In this paper we focus on testing a particular specification of a DSGE model and not on how to respecify the model should the test reject it. Rejection could, of course, be due to sampling variation in the original estimates and not because the model is otherwise incorrect. This is an issue worth following up in future work. For further discussion of estimation issues see Smith (1993), Gregory and Smith (1991,1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995), Canova (2005), Drìdi et al (2007), Hall et al (2010), and Fukac and Pagan (2010).

### 3.1 Handling non-stationary data

To use these methods on non-stationary data we need to reduce them to stationarity. This we do by assuming that the variables are cointegrated with a set of exogenous non-stationary variables, so that the residuals are stationary. We then difference the data and write the relationships as a Vector Error Correction Mechanism, as we now explain.

#### 3.1.1 The auxiliary equation

We suppose that in the class of structural models in which we are interested as potential candidates for the true model the endogenous variable vector  $y_t$  can be written in linearised form as a function of lagged  $y$ , a vector of exogenous variables  $x$ ,  $z$  and of errors  $u$ .

$$y_t = f(y_{t-1}, x_t, z_t, u_t)$$

Now we assume that  $x$  are non-stationary, I(1), variables with drift trends (which may be zero); that  $z$  are I(0) with deterministic trends (that may be zero) and that  $u$  are I(0) error processes with zero means and deterministic trends given by  $u_t = gt + a(L)v_t$ . Thus there are cointegrating relationships in the model that define the 'trend' values of  $y$  as linear functions of the 'trends' in these exogenous variables or  $A\bar{y}_t = B\bar{x}_t + C\bar{z}_t + gt$  where for example if  $\Delta x_t = a\Delta x_{t-1} + d + \epsilon_t$  then  $\bar{x}_t = x_t + \frac{\alpha}{1-\alpha}\Delta x_t + dt$ ; we note also that  $z_t = c + et + b(L)\epsilon_t$ . Hence  $\bar{y}_t = A^{-1}(B\bar{x}_t + C\bar{z}_t + gt)$ .

We now define the VECM as:

$$\Delta y_t = D\epsilon_t + E\epsilon_t + Fv_t - \Gamma(y_{t-1} - \bar{y}_{t-1})$$

We can rewrite this as a VAR in the levels of  $y$ , augmented by the arguments of  $\bar{y}$ :

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between the two parameters. The effect of the high covariance is to create a ridge in the density mountain; and the joint parameter combination of 0.1, 0.9 will be rejected even though individually the two parameters are accepted.

<sup>5</sup>The Mahalanobis Distance is the square root of the Wald value. As the square root of a chi-squared distribution, it can be converted into a t-statistic by adjusting the mean and the size. We normalise this here by ensuring that the resulting t-statistic is 1.645 at the 95% point of the distribution.

$$\begin{aligned}
y_t &= (I - \Gamma)y_{t-1} + \Gamma\bar{y}_{t-1} + \eta_t \\
&= (I - \Gamma)y_{t-1} + \Gamma A^{-1}[B(x_{t-1} + \frac{\alpha}{1-\alpha}\Delta x_{t-1} + dt) + C(c + et) + gt] + \eta_t \\
&= Hy_{t-1} + J\bar{x}_{t-1} + ht + \eta_t + cons
\end{aligned}$$

where  $\eta_t = A^{-1}[D\epsilon_t + E\varepsilon_t + Fv_t]$ . It should be noted that ‘cons’ includes dummy constants for outliers in the errors — we interpret these as effects of one-off events such as strikes.

This is our auxiliary equation in the indirect inference testing procedure. We estimate it both on the data and on the data simulated from the model bootstraps.<sup>6</sup> It allows us to test whether the model can capture the relationships in the data; we focus on the matrices  $H$ , and  $J$  and the vector  $h$ . In ongoing research we are looking at the possible generalisation of these methods to frameworks other than a VECM.

### 3.1.2 Montecarlo experiment testing the bootstrapping procedure for indirect inference

For now we report the result of a Montecarlo experiment on our methods. We treat the DSGE model as true and its error processes with their time-series parameters and innovations’ variance, skewness and kurtosis as estimated. With 1000 replications the true rejection rate at a nominal 5% confidence level is 5.7%; hence the procedure is fairly accurate. Table 2 shows the rejection rates at all the usual nominal rates. It is clear that the non-stationarity is being effectively dealt with by our VECM procedure.

Nominal Rejection Rate	Corresponding True Rejection Rate
10.0	10.1
5.0	5.7
1.0	1.6

Notes: The model used here was treated as the true model and the estimated residuals as the true residuals. 1000 samples of data were created by random draws from the innovations of these residuals, which were input into the model. The innovations were bootstrapped for each sample to find the Wald distribution for that sample and the Wald statistic calculated for that sample; the Table records how often the test at the chosen nominal rejection rate rejects.

Table 2: Montecarlo Rejection Rates

## 4 Testing the model

The numerical methods we use to solve the model are set out in the Appendix to this paper. In what follows we show how the model’s simulated behaviour matches up with that of the data.

We note, to start with, that as usual in such studies when a wide set of variables are entered, the model is totally rejected. For example including  $Y$ ,  $Q$ ,  $C$ ,  $K$  and  $r$  leads to a normalised Mahalanobis Distance (a t-statistic) of 7.6, massively beyond the 95% critical value of 1.645. We therefore looked for Directed Wald statistics involving smaller subsets of key variables; we wish to know if the model can replicate the behaviour of some such group, and thus define its contribution. It turns out that the model can match the behaviour of a few small subsets from among the full set. Here we show the results for the subset  $Y$ ,  $Q$  and  $r$  and a summary of the subsets that get closest to the data.

The Table below shows the results for  $Y$ ,  $Q$  and  $r$ . The Wald percentile is 95.3 and the normalised distance 1.75, approximately on the 95% confidence bound; given that our method slightly over-rejects according to the Montecarlo experiment we can treat this as a borderline non-rejection. As part of the test we included the variances of the VECM residuals; these were well outside the model’s 95% bounds individually but inside the joint bounds with other aspects of the data. The relationships include those with the lagged productivity trend (YT) and with the lagged level of net foreign assets (BF) (these being the non-stationary exogenous variables) as well as the dynamic relationships with the lagged endogenous

<sup>6</sup> A necessary condition for the stationarity of the VECM arguments is that  $y_t$  is cointegrated with the elements of  $\bar{y}_t$  both in the data and in the bootstrap simulations; we check for this and report if it is not satisfied, as this would invalidate the tests.

variables, the vector of coefficients on  $t$  and the residual variances just noted. Apart from the residual variances only one individual relationship (the partial coefficient of  $r$  on  $Y$ ) lies very slightly outside its 95% bound individually; good or bad individual performances do not necessarily imply that all the relationships will lie jointly within or outside the 95% bound as this depends crucially on the covariances between the coefficients. As we see here very poor individual residual variance fits do not prevent the model overall fitting the data-estimated VECM.

	ACTUAL	LOWER	UPPER	IN/OUT
$Y^Y$	0.921745	0.547557	0.944052	IN
$Y^Q$	0.007201	-0.067114	0.189087	IN
$Y^r$	-0.130463	-0.123371	1.428657	OUT
$Y^{eYT}$	0.076943	-0.089580	0.664761	IN
$Y^{trend}$	0.000165	-0.000033	0.002030	IN
$Y^{Bf}$	-0.000001	-0.006984	0.001702	IN
$Q^Y$	0.075864	-0.220678	0.168833	IN
$Q^Q$	0.964109	0.755039	1.030719	IN
$Q^r$	0.540718	-0.232954	1.450536	IN
$Q^{eYT}$	-0.074508	-0.269644	0.499025	IN
$Q^{trend}$	-0.000257	-0.001527	0.000447	IN
$Q^{Bf}$	0.000001	-0.007092	0.003458	IN
$r^Y$	-0.021030	-0.028810	0.034483	IN
$r^Q$	-0.005892	-0.033588	0.012599	IN
$r^r$	0.654505	0.585070	0.886870	IN
$r^{eYT}$	0.033472	-0.059237	0.054454	IN
$r^{trend}$	0.000023	-0.000245	0.000119	IN
$r^{Bf}$	0.000002	-0.000755	0.000348	IN
var( $Y$ )	0.000039	0.000356	0.002848	OUT
var( $Q$ )	0.000327	0.000434	0.003845	OUT
var( $r$ )	0.000008	0.000020	0.000037	OUT
Wald	95.2830			
Transformed M-dist	1.7484			

Table 3: VAR results

The table of subset results reveals that GDP and asset prices are well explained as we have seen but that combining these with consumption or employment leads to being rejected at 99%. Also GDP and the real exchange rate match the data when combined with either employment or consumption. Summarising one can say that the model fits the data on GDP and the two main asset prices but cannot also match the detailed behaviour of component real variables.

Thus the model passes well for a small set of key variables. That it fails for a broader set is a problem this model appears to share with much more elaborate structures, such as the Smets-Wouters/Christiano et al model, with their huge efforts to include real rigidities such as habit persistence and variable capacity utilisation, as well as Calvo nominal rigidities in both wages and prices. When these are tested on stationarised data we find that invariably the inclusion of consumption wrecks the fit; however we

Subset	Wald percentile	Transformed M-dist
GDP + asset prices (+consumption or employment)		
YQr	95.3	1.77
YQC	90.4	0.89
YQCr	99.4	4.16
YQNr	99.4	3.60
GDP + Labour market bloc		
YQw	99.9	9.55
YQN	90.4	0.85
YQNW	99.9	7.58

Table 4: Table of summary results for various variable subsets



can find a good fit to US data post-1984 for output, real interest rates and inflation taken alone. On a similar SW/CEE-style two-country model of the US and the EU, again on stationarised data, we find that it can fit output and the real exchange rate on their own but no wider set of variables.

We interpret these tests to mean that this model performs rather well in the context of model performance generally, at least in the present state of the DSGE modelling art.

#### 4.1 Could productivity be trend-stationary?

One issue we have not so far emphasised but one that is nevertheless of empirical importance concerns our choice of error specification. Many of our error processes are not unambiguously either trend-stationary or non-stationary: that is, when we test the null of trend-stationarity we cannot reject it (at say 95% confidence) but neither can we reject the null of non-stationarity. Essentially this is because the distribution of the autoregressive coefficient is different under the two nulls. Hence in entering these errors into the DSGE model we need to make a choice that cannot be made on purely statistical grounds. The way we treat this is the same way that we treat the rest of the DSGE model specification choice where we have one; we reject one versus the other on the basis of indirect inference. We chose only to make productivity non-stationary because making the other errors non-stationary induced massively excessive variability in our key macro variables. However, this leaves the question whether even productivity should be trend-stationary, rather than non-stationary. Here we test the DSGE model under the assumption of trend-stationary productivity. Our findings are that the fit to the data worsens sharply, so that the subset of key variables that can be matched shrinks to none at all: the nearest is Y,Q,r whose Wald is 98.6 and M distance 3.65, hence rejected at 95% but accepted at 99% only. All others we looked at above are rejected at the 99% level. This gives rather clear evidence that treating productivity as non-stationary was the right choice. Thus we do not pursue this alternative representation of the model further.

Subset	Wald percentile	Transformed M-dist
GDP + asset prices (+consumption or employment)		
YQr	98.6	3.65
YQC	99.7	6.40
YQCr	99.8	8.57
YQNr	100	12.09
GDP + Labour market bloc		
YQw	100	31.70
YQN	99.9	7.33
YQNW	100	24.00

Table 5: Table of summary results for various variable subsets (Productivity trend-stationary)

### 5 Simulated behaviour of the model: ‘euphoria’ and ‘crisis’

When non-stationary shocks hit this economy they produce permanent changes in income, consumption, capital stock and the real exchange rate, as well as a path to the new equilibrium. But of course each period brings a fresh set of permanent shocks so that the economy is constantly moving towards a new freshly-set equilibrium. When a combination of shocks occurs that is negative for the output equilibrium and a sequence of shocks of this type occur in the same direction, output can fall sharply in what looks rather like a crisis or ‘disaster’. We illustrate this point with a randomly chosen bootstrap simulation for the UK economy over the 200-plus quarters of the sample (from the late 1940s to the early 2000s); it is taken from a very large number of such simulations, some of which are shown in Figure 3 including the BGP trend. Inspection of these random scenarios reveals that euphoric and nasty episodes are not uncommon. In Figure 4 we show the actual residuals or shocks we found were implied by this model and the UK post-war data.

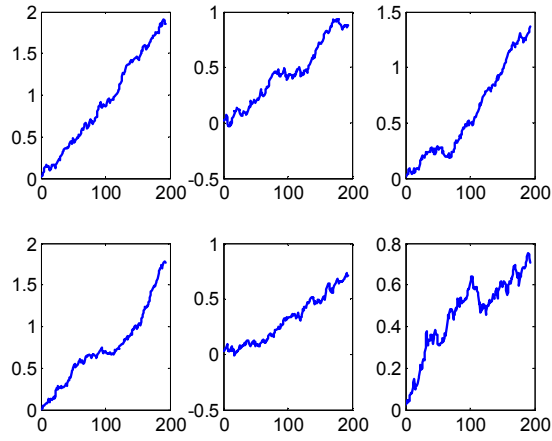


Figure 3: Output simulations with deterministic trend

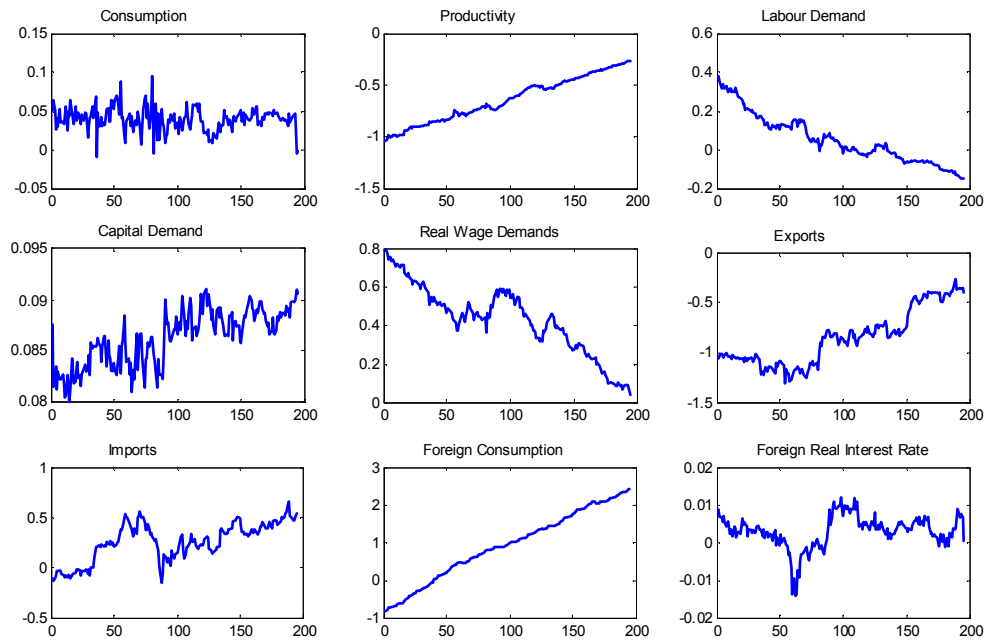


Figure 4: Shocks

We now take the particular randomly chosen simulation referred to above and examine it in some detail. In Figure 5 we show the shocks used in this particular scenario. Figure 6 shows what they give in terms of output alone including its BGP trend — we have put the post-war years along the x axis to show that one can think of this as a ‘rerun’ of the post-war period with shocks selected in a different order. We can focus on two sharp downswings here (the shaded part of the shock charts below), the one after quarter 75 (around the year 1977) and the one after quarter 155 (around the year 2000). In each case the large negative shocks to productivity dominate the situation. For example in the second after 30 or so quarters of rapid growth, it levels off and falls moderately for the next 30 quarters before then plunging for the following 20 quarters from quarter 151 — this is the period shaded on the shock charts. This precipitates a similar profile in output. Productivity dominates because it is the only non-stationary shock in the model. The other shocks contribute but because they are stationary only temporarily. The result is the collapse of output we see; notice that competitiveness varies directly with output because the more is produced the higher competitiveness has to be to sell the extra output (i.e. UK relative prices have to be lower to sell it). The units here are natural logarithms, so that these are substantial movements in output and competitiveness — maybe a bit too substantial for realism but then this is still a fairly primitive model, and the first we have built of its type. But the presence of the crisis element will, we think, remain valid as we introduce more sophistication into the model.

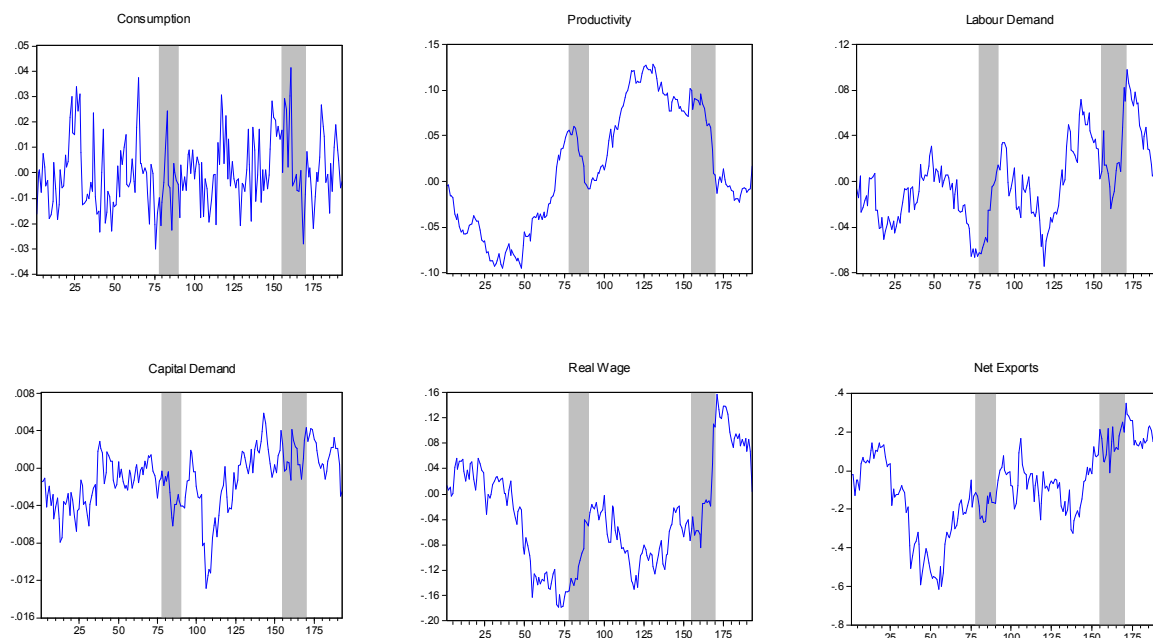


Figure 5: Shocks to Chosen Simulation

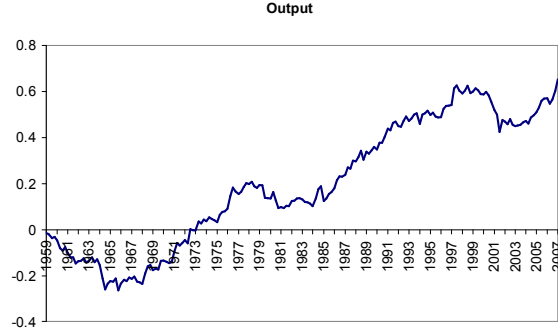


Figure 6: Output Simulation

## 6 Conclusions

The economy operates under the influence of non-stationary shocks, mainly productivity. These can be considered as a source of ‘Knightian uncertainty’. It is possible for the economy to enjoy a long period of steady strong growth when productivity growth is favourable; and then if that growth turns unfavourable — for example because of shortages of key resources as seems to have happened in the mid-2000s — it can collapse as over-investment (as seen with hindsight) takes a toll on business plans. Such a collapse is a ‘crisis’ and it will usually create a financial crisis among the intermediaries that financed the previous growth and the over-investment.

In this paper we have built a Real Business Cycle model without a banking system to model this crisis tendency which we argue comes from the behaviour of productivity, the fundamental shock driving the economy. We have argued that a banking crisis could occur additionally in consequence of a productivity-created crisis if the banks were heavily involved in lending to the affected sectors. But the crisis is severe with or without the accompanying banking crisis.

We have tested the model’s empirical performance by the method of indirect inference under which we ask whether its simulated behaviour produces relationships in the simulated data like those in the actual data. Though our data here is non-stationary our use of a VECM as the auxiliary equation appears to deal with the non-stationarity satisfactorily, according to Montecarlo experiment. We found that provided we require the model only to replicate broad macro behaviour — i.e. here that of output, real interest rates and the real exchange rate — it can meet this indirect inference test rather well.

Thus our first innovation in this paper is to return to the original RBC model with non-stationary productivity and in an open economy as a way of explaining UK macro behaviour inclusive of occasional violent movement. Our second innovation is to test this model statistically against postwar UK data using the method of indirect inference. Perhaps against professional expectation we found that this model does fit key aspects of post-war UK behaviour.

We then showed a typical simulation of the post-war period produced by randomly drawing shocks in a different order. In this simulated post-war scenario crisis periods are clearly visible — just as indeed they occurred in actual fact during the post-war period, though at different times and with differing intensities.

We argue therefore that crises of this sort, as well as the run of ‘good times’ that usually precede them, are endemic in capitalism, that is the free play of decentralised markets. Few policymakers today would wish to trade capitalism for a centralized, planned economy because there is ample evidence that in the long term capitalism delivers higher growth. But this carries some broad policy implications for them.

First, they must not over-regulate the financial system, one of the key capitalist mechanisms. A major need in regulation is for disclosure so that risks can be more accurately judged by private agents. For banks with deposit or other guarantees existing regulations already substitute for the discipline of depositor anxiety.

Second, they must stand ready to ‘firefight’ a financial crisis, with all the means that have become familiar in this crisis, where fortunately they were deployed to good effect. This process is a massive extension to the whole financial sector of Bagehot’s ‘lender of last resort’ support, necessitated by the connectedness of the sector as well as its key role in capitalism.

Third, they must realise that crises cannot be forecast and so they cannot take advance action to avoid them. Nor can they blame private agents for doing things that were rational during the pre-crisis period even if with hindsight they were later seen to be wrong. These agents all pay big enough penalties (at least if share- and bond-holders are acting effectively) anyway from their pre-crisis actions; the existence of firefighting services does not imply moral hazard as the fire burns badly enough.

In future research we hope to model the mechanisms that can trigger a banking crisis on top of the originating productivity crisis. But we hope at least in this paper to have shown how crises can be triggered by the normal operations of the economy, even when agents are acting with complete rationality.

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## 7 Appendix: Listing of the RBC Model

### Behavioural Equations

- (1) Consumption  $C_t$  ; solves for  $r_t$ :

$$\begin{aligned}(1 + r_t) &= \frac{1}{\beta} E_t \left( \frac{C_t}{C_{t+1}} \right)^{-\rho_0} \left( \frac{\gamma_t}{\gamma_{t+1}} \right) \\ \log(1 + r_t) &= r_t = -\rho_0(\log C_t - E_t \log C_{t+1}) + \log \gamma_t - E_t \log \gamma_{t+1} + c_0\end{aligned}$$

Here we use the property that for a lognormal variable  $x_t$ ,  $E_t \log x_{t+1} = \log E_t x_{t+1} - 0.5\sigma_{\log x}^2$ . Thus the constant  $c_0$  contains the covariance of  $(-\rho_0 \log C_{t+1})$  with  $(\log \gamma_{t+1})$ .

- (2) UIP condition:

$$r_t = r_t^F + E_t \log Q_{t+1} - \log Q_t + c_1$$

where  $r^F$  is the foreign real interest rate.

Note that equations (1) and (2) are combined.

- (3) Production function  $Y_t$ :

$$\begin{aligned}Y_t &= Z_t N_t^\alpha K_t^{1-\alpha} & \text{or} \\ \log Y_t &= \alpha \log N_t + (1 - \alpha) \log K_t + \log Z_t\end{aligned}$$

- (4) Demand for labour :

$$\begin{aligned}N_t &= \left( \frac{\alpha Y_t}{w_t(1 + \chi_t)} \right) & \text{or} \\ \log N_t &= c_2 + \log Y_t - \log w_t + \chi_t\end{aligned}$$

- (5) Capital :

$$\begin{aligned}\xi(1 + d_{1t})K_t &= \xi K_{t-1} + \xi d_{1t} E_t K_{t+1} + \frac{(1 - \alpha) Y_t}{K_t} - (r_t + \delta + \kappa_t) & \text{or} \\ \log K_t &= c_3 + \zeta_1 \log K_{t-1} + \zeta_2 E_t \log K_{t+1} + (1 - \zeta_1 - \zeta_2) \log Y_t - \zeta_3 r_t - \zeta_3 \kappa_t\end{aligned}$$

(6) The producer wage is derived by equating demand for labour,  $N_t$ , to the supply of labour given by the consumer's first order conditions:

$$\begin{aligned}(1 - N_t) &= \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[ \exp \left( \log w_t - \left( \frac{1-\omega}{\omega} \right)^\sigma (\log Q_t + \frac{1}{\rho} \log \varsigma_t) \right) \right]}{(1 - \theta_0) \xi_t} \right\}^{\frac{-1}{\rho_2}} & \text{or} \\ \log(1 - N_t) &= -\log N_t = c_4 + \frac{\rho_0}{\rho_2} \log C_t - \frac{1}{\rho_2} \log w_t + \frac{1}{\rho_2} \left( \frac{1-\omega}{\omega} \right)^\sigma \log Q_t \\ &\quad + \frac{1}{\rho_2} \left( \frac{1-\omega}{\omega} \right)^\sigma \log \varsigma_t + \frac{1}{\rho_2} \log \xi_t\end{aligned}$$

where  $Q_t$  is the real exchange rate,  $(1 - \omega)^\sigma$  is the weight of domestic prices in the CPI index.

- (7) Imports  $IM_t$ :

$$\log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma \log Q_t - \sigma \log \varsigma_t$$

- (8) Exports  $EX_t$ :

$$\log EX_t = \sigma^F \log(1 - \omega^F) + \log C_t^F + \sigma^F \log Q_t - \sigma^F \log \varsigma_t^F$$

### Budget constraints, market-clearing and transversality conditions:

- (9) Market-clearing condition for goods:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

where investment is :

$$I_t = K_t - (1 - \delta)K_{t-1}$$

and we assume the government expenditure share is an exogenous process. Loglinearised using mean GDP shares, this becomes

$$\log Y_t = 0.77 \log C_t + 6.15(\log K_t - \log K_{t-1}) + 0.3 \log G_t + 0.28 \log EX_t - 0.3 \log IM_t$$

(10) Evolution of  $b_t$  ; government budget constraint:

$$b_{t+1} = (1 + r_t)b_t + PD_t$$

(11) Dividends are surplus corporate cash flow :

$$\begin{aligned} d_t \bar{S}_t &= Y_t - N_t^s w_t - K_t(r_t + \delta) \\ d_t &= \frac{Y_t - N_t^s w_t - K_t(r_t + \delta)}{\bar{S}_t} \end{aligned}$$

(12) Market-clearing for shares,  $S_{t+1}^p$  :

$$S_{t+1}^p = \bar{S}_t .$$

(13) Present value of share :

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1 + r_t)^i}$$

where  $d_t$  (dividend per share),  $p_t$  (present value of shares in nominal terms).

(14) Primary deficit  $PD_t$  :

$$PD_t = G_t - T_t$$

(15) Tax process  $T_t$  designed to ensure convergence of government debt to transversality condition:

$$T_t = T_{t-1} + \gamma^G \frac{(PD_{t-1} + b_t r_t)}{Y_{t-1}}$$

(16) Evolution of foreign bonds  $b_t^f$ :

$$\frac{Q_t b_{t+1}^f}{(1 + r_t^f)} = Q_t b_t^f + EX_t - Q_t IM_t$$

(17) Evolution of household net assets  $A_{t+1}$ :

$$A_{t+1} = (1 + r_{At})A_t + Y_t - C_t - T_t - I_t$$

where  $r_{At}$  is a weighted average of the returns on the different assets.

(18) Household transversality condition as  $T \rightarrow \infty$ :

$$\Delta\left(\frac{A_T}{Y_T}\right) = 0$$

(19) Government transversality condition  $T \rightarrow \infty$ :

$$\Delta\left(\frac{b_T}{Y_T}\right) = 0$$

## Values of coefficients

Coefficient	Value — Single equation
$\alpha$	0.70
$\beta$	0.97
$\delta$	0.0125
$\rho_0$	1.20
$\theta_0$	0.50
$\gamma^G$	0.05
$\rho_2$	1.00
$\omega$	0.70
$\rho$	-0.50
$\omega^f$	0.70
$h$	0.80
$\rho_3$	-0.50
$\sigma$	1
$\sigma_F$	1
Coefficient	Value — Single equation
$\zeta_1, \zeta_2, \zeta_3$	0.5, 0.475, 0.25

### Model solution methods

The model is solved in the loglinearised form above using a projection method set out in Minford et al. (1984, 1986); it is of the same type as Fair and Taylor (1983) and has been used constantly in forecasting work, with programme developments designed to ensure that the model solution is not aborted but re-initialised in the face of common traps (such as taking logs of negative numbers); the model is solved by a variety of standard algorithms, and the number of passes or iterations is increased until full convergence is achieved, including expectations equated with forecast values (note that as this model is loglinearised, certainty equivalence holds). Terminal conditions ensure that the transversality conditions on government and households are met- equivalent to setting the current account to zero). The method of solution involves first creating a base run which for convenience is set exactly equal to the actual data over the sample. The structural residuals of each equation are either backed out from the data and the model when no expectations enter as the values necessary for this exact replication of the data; or, in equations where expectations enter, they are estimated using a robust estimator of the entering expectations as proposed by McCallum (1976) and Wickens (1982), using instrumental variables; here we use as instruments the lagged variables in univariate time-series processes for each expectational variable. The resulting structural residuals are treated as the error processes in the model and together with exogenous variable processes, produce the shocks perturbing the model. For each we estimate a low-order ARIMA process to account for its autoregressive behaviour. The resulting innovations are then bootstrapped by time vector to preserve any correlations between them. Two residuals only are treated as non-stochastic and not bootstrapped: the residual in the goods market-clearing equation (the GDP identity) and that in the uncovered interest parity (UIP) condition. In the GDP identity there must be mis-measurement of the component series: we treat these measurement errors as fixed across shocks to the true variables. In the UIP condition the residual is the risk-premium which under the assumed homoscedasticity of the shocks perturbing the model should be fixed; thus the residuals represent risk-premium variations due to perceived but according to the model non-existent movements in the shock variances. We assume that these misperceptions or mismeasurements of variances by agents are fixed across shocks perturbing the model- since, although these shocks are being generated by the true variances, agents nevertheless ignore this, therefore making these misperceptions orthogonally.

To obtain the bootstraps, shocks are drawn in an overlapping manner by time vector and input into the model base run (including the ARIMA processes for errors and exogenous variables). Thus for period 1, a vector of shocks is drawn and added into the model base run, given its initial lagged values; the model is solved for period 1 (as well as the complete future beyond) and this becomes the lagged variable vector for period 2. Then another vector of shocks is drawn after replacement for period 2 and added into this solution; the model is then solved for period 2 (and beyond) and this in turn becomes the lagged variable vector for period 3. Then the process is repeated for period 3 and following until a bootstrap simulation is created for a full sample size. Finally to find the bootstrap effect of the shocks the base run is deducted from this simulation. The result is the bootstrap sample created by the model's shocks. We generate some 1500 of such bootstraps.

We add these bootstraps to the Balanced Growth Path implied by the model and the deterministic trend terms in the exogenous variables and error processes. We find this BGP by solving for the effect of

a permanent change in each error/exogenous variable at the terminal horizon  $T$ ; we then multiply this steady-state effect by the deterministic rate of change of this variable. When this BGP is incorporated in every bootstrap we have 1500 full alternative scenarios for the economy over the sample period; these bootstrap samples are then used in estimation of the VECM auxiliary equation.

To generate the model-implied joint and individual distributions of the parameters of the VECM estimated on the data, we carry out exactly the same estimation on each bootstrap sample. This gives us 1500 sample estimates which provide the sampling distribution under the null of the model. The sampling distribution for the Wald test statistic,  $[a_T - \alpha_S]'W[a_T - \alpha_S]$ , is of principal interest. We represent this as the percentile of the distribution where the actual data-generated parameters jointly lie. We also compute the value of the square root of this, the Mahalanobis distance, which is a one-sided normal variate; we reset this so that it has the 95% value of the variate at the same point as the 95th percentile of the bootstrap distribution (which is not necessarily normal). This ‘normalised Mahalanobis Distance’ we use as a measure of the distance of the model from the data under the bootstrap distribution. Its advantage is that it is a continuous variable representation of the theoretical distribution underlying the bootstrap distribution- which is made finite by the number of bootstraps.