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# Structural Vector Autoregressive Models and Monetary Policy Analysis

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## Abstract

In this paper, the structural vector autoregressive (SVAR) model is used to analyze short-run and contemporaneous relationships between monetary aggregates and other macroeconomic variables. This requires imposing restrictions on the correlation structure of the VAR residuals. Different approaches can be followed to serve this task. One approach is to use the Cholesky decomposition together with the assumption of a recursive structure of the contemporaneous relationships between the variables. Another approach uses the information given by the history of the variables (generalized impulse response functions). A third possibility is to adopt restrictions from economic theory.

The purpose of this paper is to investigate the implications of the latter technique in a simple monetary framework for both Germany and the Euro area. VAR/VECM residuals are interpreted as deviations of the variables from their conditional expected values, which are also analyzed in a broad set of theoretical monetary models. The model used here builds upon the framework proposed by McCallum (1989) with an extension in order to consider the role of private banks in the money supply process. The implications of this model for impulse response analysis are discussed, and impulse responses for different models are calculated and compared to each other.

*Keywords:* Impulse response analysis, IS-LM-AS model, monetary policy, structural vector autoregressive (SVAR) models.

*JEL classification:* C32, E52.

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Until 1998, monetary policy in Germany was conducted by the Deutsche Bundesbank. The main objective of the Bundesbank, which was given by § 3 Bundesbankgesetz, was to guarantee a stable currency and a well functioning monetary system (circulation of money, supply of credit). To make this task, the Bundesbank adopted the conception of controlling a monetary aggregate in 1975. The theoretical foundations of this conception are formed by the quantity theory of money and the assumption of a stable money demand function. Since stage three of the Economic and Monetary Union (EMU) has started in January 1999, the European System of Central Banks (ESCB) is in charge of the single monetary policy in the euro area.<sup>1</sup> The primary objective of the ESCB is price stability (defined in Article 105 of the Treaty establishing the European Community<sup>2</sup>), and the ESCB has adopted a two pillar strategy to achieve its objective. These two pillars are "a prominent role for money, as signalled by the announcement of a reference value for the growth of a broad monetary aggregate; and a broadly based assessment of the outlook for future price developments and the risks to price stability in the euro area as a whole." (ECB, 1999b, p. 46) It is important to notice that the reference value for the growth rate of a monetary aggregate is not a monetary target. It "acts as an analytical and presentational tool which constitutes an important benchmark for assessing risks to price stability." (ECB, 2000, p. 41)

The purpose of this paper is to compare the role of monetary aggregates in the monetary policy conception of the Bundesbank and in the monetary policy strategy of the ESCB and to investigate their implications for the empirical analysis of monetary policy effects. The development of a framework for monetary policy analysis incorporates a trade-off between the consideration of theoretic models in modern monetary economics and the empirical applicability. Realistic and sophisticated models for monetary economies have often no unique time series implications that can be used in the empirical analysis of monetary policy. Therefore, it is tried to find a framework that is on the one hand compatible with the implications of standard theoretical models, and, on the other hand is simple enough to allow for an empirical implementation. Expressed in the words of McCallum (1989, p. 201), such a model should be "(i) reasonably simple to work with, (ii) generally compatible with the basic principles of economic theory, (iii) adequately consistent with essential empirical regularities concerning macroeconomic fluctuations in actual economies."

The plan for the paper is as follows. First, in section 2, the monetary policy strategies of the Bundesbank and the ESCB are compared. Then, in section 3, a model of a monetary economy in which the money stock is determined by the interactions of the central bank, private banks, and the public is presented. In section 4, the econometric methodology is explained. The empirical implementation and results are discussed in section 5, and section 6 concludes.

## 2 Monetary Policy in Germany and in the Euro Area

### 2.1 Price Stability as Objective of Monetary Policy

Both institutions, the Bundesbank and the ESCB, are in charge of maintaining price stability.<sup>3</sup> As mentioned above, the objective of price stability is given by law. However, the Bundesbank and the

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<sup>1</sup> Stage three started with eleven participating countries (EU-11: Belgium, Germany, Spain, France, Ireland, Italy, Luxembourg, the Netherlands, Austria, Portugal, and Finland), Greece entered the third stage of EMU on 1 January 2001 (EU-12).

<sup>2</sup> The treaty is available via internet, for example under <http://europa.eu.int>.

<sup>3</sup> If the Bundesbank and its monetary policy is referred to in this paper it is always meant the monetary policy of the Bundesbank until 1998.

ESCB can decide how to define and how to achieve price stability. The arguments for price stability as the objective of the central bank are reviewed for example in Bofinger et al. (1996) and in Issing (2000). First, it is well known that inflation reduces welfare. If the rate of inflation is correctly anticipated, the welfare costs of inflation originate from suboptimal money holdings (shoe leather costs), from the necessity of price adjustments (menu costs), and from distortional effects of the tax system. If the rate of inflation is not correctly anticipated, further costs of inflation occur. Wages and nominal interest rates are not correctly adjusted such that the allocation efficiency is harmed, and the income and wealth distributions are affected. Second, no convincing or at least no unambiguous arguments in favor of a positive inflation rate exist. For example, the positive effect of inflation on growth due to a substitution effect from money holdings to holdings of real capital proposed by Tobin can neither be shown analytically nor empirically. High and variable inflation rates are included in long-term interest rates together with a risk premium and increase therefore investment costs. In models that follow Sidrauski (money-in-the-utility-function models) and in cash-in-advance models, output does not depend on the rate of inflation.<sup>4</sup> In these models money is superneutral, that is, the growth rate of money does not affect the real equilibrium.<sup>5</sup> If price and/or wage rigidities are added to these models it is typically found that real output is affected by unanticipated inflation but not by anticipated inflation. Therefore, monetary policy cannot systematically increase or stabilize real output.

Taking together these two lines of arguments, that inflation reduces welfare and that inflation does not systematically affect real output, gives a theoretical foundation for the choice of price stability as *an* objective of monetary policy. Combining these arguments with the conceptual work of Tinbergen<sup>6</sup> about policy objectives and instruments leads to the choice of price stability as *the single or primary* objective of monetary policy.

The objective of price stability can be defined in two ways: either as a price level or as an inflation target. If the price level is targeted, deviations must be compensated in subsequent periods. This adds policy induced fluctuations in inflation and output to the economy but stabilizes the long-run expectations of the price level. In practice, price stability is defined as follows: until 1984 the Bundesbank has assumed an unavoidable inflation based on the conviction that inflation has to be reduced slowly in order to avoid increasing unemployment. From 1984 on, a “normative” increase in the cost of living index of 2% per annum has been interpreted as in line with price stability. The ECB defines price stability as a year-to-year increase in the harmonized index of consumer prices of below 2%.

## 2.2 Implementation of Monetary Policy in Germany

The monetary policy conception of the Bundesbank has been based on a two-stage approach.<sup>7</sup> Because the time lags between the occurrence of a monetary policy impulse and the observation of reactions of the relevant macroeconomic variables, especially prices, are long and variable, the Bundesbank has controlled an intermediate target since 1975.<sup>8</sup> The intermediate target has

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<sup>4</sup> Both money-in-the-utility-function models and cash-in-advance models are extensions of Walrasian general equilibrium models with completely flexible prices. They are broadly discussed for example in Walsh (1998).

<sup>5</sup> Superneutrality of money is for example defined in this way in Blanchard and Fischer (1989, Chapter 4).

<sup>6</sup> Following Tinbergen, policymakers can achieve the desired levels of targets if the number of independent instruments is not lower than the number of targets. For a brief and illuminating overview see Sachs and Larrain (1993, Ch. 19).

<sup>7</sup> This section follows mainly Deutsche Bundesbank (1995). A description of the conduct of monetary policy in Germany is also given in Brand (2001).

<sup>8</sup> See for example Friedman (1990) or Walsh (1998) for the theoretical foundations of choosing an intermediate target.

been central bank money (a virtual money stock variable consisting of currency in circulation and required reserves at constant required reserve rates of 1974) until 1987 and M3 (currency in circulation, sight deposits, time deposits and savings deposits<sup>9</sup>) from 1988 to 1998. According to the Bundesbank, M3 has been chosen as intermediate target because it exhibits a stable relation to the future price level (leading indicator property of money for prices) and is sufficiently controllable. While the leading indicator property is widely accepted in the literature it is sometimes questioned if the Bundesbank has been able to control M3. The target has been met in thirteen of twenty-four years of monetary targeting in Germany, see Holtemöller (2001a). The desired growth rate of M3 has been derived from the quantity equation that equates money stock times velocity of circulation and real output times price level. Given the normative price level growth described in the previous section, the desired money growth rate has been calculated by inserting forecasts of real potential output growth and changes in velocity. The use of *potential* output (and not *expected* real output) has been founded upon the medium-term and stability oriented nature of monetary targeting.

How has the Bundesbank controlled the money stock? The Bundesbank itself has described the control process as quantity setting (Deutsche Bundesbank, 1995, p. 94) using its monopoly on issuing currency and required reserve policy. Assuming a more or less stable relationship between the monetary base and the money stock, the Bundesbank has calculated at the beginning of each month the quantity of base money that is compatible with the monetary target. Further, the Bundesbank conducted monthly forecasts of the demand for base money of private banks. The gap between normative and actual demand for base money has then tried to be closed by influencing the money market conditions by means of interest rates (Lombard rate, discount rate, and an interest rate for open market operations) and so-called compensating or fine-tuning instruments (foreign exchange swaps, quick tenders and others). This type of monetary policy can be called reserve-oriented: the reserves needed by the banking system to fulfill the reserve requirement are controlled – not completely but to a considerable extent – by the Bundesbank.

### 2.3 *Implementation of Monetary Policy in the Euro Area*

Since January 1999 the single monetary policy in the euro area is conducted by the ESCB. In order to achieve its given primary objective of price stability, the ESCB has developed a stability-oriented monetary policy strategy that consists of a quantitative definition of price stability, and a two-pillar strategy used to achieve price stability.<sup>10</sup> The first pillar assigns a prominent role to money. A reference value for the growth of M3 is calculated (in a similar way to that one described in the previous section) and published. Within the second pillar the non-monetary risks for price stability are analyzed. These are for example the conditions and developments on goods and labor markets. The ESCB stresses that both pillars are not targets themselves. The only target or objective is price stability. Both the analysis of the growth of money stock and the evaluation of other risks to price stability are informational tools helping the governing council of the ECB to reach decisions on monetary policy actions. To the best of my knowledge, a document describing how the information is weighted is not available. Therefore, it is not clear how the ECB translates the information into monetary policy actions, and it is difficult to explain what guides the ECB in using its operational instruments. It can only be said which set of instruments

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<sup>9</sup> Time deposits and savings deposits are only included if they satisfy certain requirements, see Deutsche Bundesbank (1995).

<sup>10</sup> The main references for this section are European Central Bank (1999b) and European Central Bank (2000). Further information can be found in a recent book by Issing et al. (2001) and on the internet homepage of the European Central Bank: [www.ecb.int](http://www.ecb.int).

the ECB uses to manage liquidity and steer interest rates. This set consists of open market operations, standing facilities, and the minimum reserve requirement. Open market operations can be divided into four groups: main refinancing operations (weekly frequency, maturity of two weeks), longer-term refinancing operations (monthly frequency, maturity of three months), fine-tuning operations (reverse transactions, outright transactions, foreign exchange swaps, collection of fixed-term deposits), and structural operations. Standard facilities (marginal lending facility and deposit facility) can be used by credit institutions that are obliged to hold required reserves, and the minimum reserve requirement creates the necessity of central bank lending; in opposition to the practice in Germany until 1998, the Eurosystem pays interest on required reserves.

## 2.4 Comparison of Bundesbank Conception and ESCB Strategy

It has become clear from the previous sections that the monetary policy conception of the Bundesbank and the stability-oriented monetary policy strategy of the Eurosystem are quite different. The Bundesbank has used a theoretically well founded two-stage approach by first determining the value of the intermediate target (growth rate of money) that is compatible with the final objective (price stability), and subsequently taking actions to achieve this specific value (or range) of the intermediate target. It should be noticed that it has been questioned whether the Bundesbank really pursued a policy of monetary targeting or whether it has only used monetary targeting as a communication strategy.<sup>12</sup> However, the Bundesbank has always made clear that it has been accountable for deviations from the monetary target and that deviations are only accepted if there are important economic reasons for a deviation. Issing (1997, p. 71), who was member of the board of the Bundesbank from 1990-1998, even states that deviations from the target have been mainly “deliberate monetary policy decisions”.

The Eurosystem does not make use of the two-stage approach. It puts the focus directly on its ultimate objective price stability and uses the development of the money stock as one piece of information in its decision process, and the single monetary policy cannot be characterized as a policy of controlling the quantity of a monetary aggregate. Instead, the Eurosystem controls the conditions on the money market, especially the main refinancing operations minimum bid rate which is intensively observed by the public.

# 3 A Theoretical Framework for the Analysis of Monetary Policy

## 3.1 A Basic Model of the Economy

A model type that is widely used in the applied macroeconomic literature is the IS-LM-AS framework. It consists in general of three equations: an aggregate demand function, a money demand function and an aggregate supply function. This model has been criticized intensively, especially

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<sup>11</sup> The operational framework of the Eurosystem is described in European Central Bank (1998) and European Central Bank (1999a).

<sup>12</sup> Two examples are Mishkin (1999, p. 588): “Monetary targeting in Germany and Switzerland should instead be seen primarily as a method of communicating the strategy of monetary policy that focuses on long-run considerations and the control of inflation.”, and Svensson (2000, p. 2): the “Bundesbank has actually been an inflation targeter in deeds and a monetary targeter in words only.” See also the literature cited there. It has also been supposed in the literature that the Bundesbank followed a Taylor (1993)-type interest rate rule in order to control inflation, see for example Clarida and Gertler (1996) and Brüggemann (2001). Von Hagen (1999) stresses that a monetary policy strategy is more important for the clarification of the role of monetary policy within the economy than for guiding actual monetary policy decisions.

from the proponents of real business cycle theory. As Romer (1996) points out, the main disadvantages of the IS-LM-AS framework as an outcome of traditional Keynesian theories of the business cycle are consequences of building the model on assumptions about the relations of aggregated variables instead of deriving them from microeconomic principles. Without specifying preferences, nothing can be said about welfare effects, important effects can be overseen, and the Lucas critique applies. The advantages of the IS-LM-AS approach are its simplicity, and its robustness. Additionally, it is argued by Walsh (1998, p. 204 ff) that the IS-LM-AS is not “starting from curves” because it can be interpreted as a special case of a modified money-in-the-utility-function model that is based on first principles. The three necessary modifications are (1) the assumption of a constant capital stock, (2) the assumption of an inelastic labor supply, and (3) the assumption of one-period nominal wage contracts. These assumptions are restrictive but seem plausible if the main interest of the analysis is investigating short-run dynamics. Therefore, the IS-LM-AS framework satisfies McCallum’s criteria (i) and (ii) mentioned earlier. Criterion (iii) – the empirical quality – is analyzed in section 5.

The exact specification of the three model equations is not unique, and different specifications have been used for different purposes. The specification used here follows McCallum (1981). Basic hypotheses underlying the IS-LM-AS model are (Modigliani and Papademos, 1990): (1) the money stock under consideration is a narrow one that is used mainly for transaction purposes, (2) the monetary authority can control the money stock up to a random component, (3) non-monetary assets (financial and real) are assumed to be close substitutes, (4) wealth and capital accumulation have no impact on the supply and demand of goods and financial assets, and (5) government and fiscal effects are ignored.

The IS equation describing the output-interest combinations for which the planned and actual expenditures on output are equal is

$$y_t = b_0 + b_1 (i_t - (E_{t-1}p_{t+1} - p_t)) + \xi_t, \quad (3.1)$$

where  $y_t$  and  $p_t$  are logarithms of real output and price level,  $i_t$  is the nominal interest rate,  $b_1 < 0$ , and  $E_{t-1}$  denotes the expected value given the information set of  $t - 1$ . The specification of the information set is important, see footnote 13. The real interest rate  $i_t - (E_{t-1}p_{t+1} - p_t)$  has a negative impact on planned expenditures because it negatively affects planned consumption and investment. The LM relation describes output-interest combinations for which money supply and money demand are equal at a given price level. It is specified as

$$m_t - p_t = c_0 + c_1 i_t + c_2 y_t + \eta_t, \quad (3.2)$$

where  $m_t$  is logarithmic money stock,  $c_1 < 0$ , and  $c_2 > 0$ . The negative impact of the interest rate on money demand expresses the opportunity costs of holding money, and the positive sign of  $c_2$  is explained by the transaction motive of money holdings. The AS relation can be interpreted as an expectations augmented Phillips curve or as a Lucas supply curve stating that actual output is increasing if inflation is higher than expected or if positive supply shocks occur:

$$y_t = a_0 + a_1 (p_t - E_{t-1}p_{t+1}) + a_2 y_{t-1} + \zeta_t, \quad (3.3)$$

where  $a_1 > 0$ , and  $0 < a_2 < 1$ . The disturbance terms  $\xi_t$ ,  $\eta_t$ , and  $\zeta_t$  are shocks that are independent of past values of all variables. They are interpreted as aggregate demand shock, monetary shock, and aggregate supply shock, respectively.

The model can be closed by specifying a monetary policy rule. This rule or reaction function links the (intermediate) target variable of the monetary authority to the state of the economy. The choice of this target variable is discussed in section 3.3. Now, the model can be solved, and reduced form

Expressions for the endogenous variables  $p, y, v, m$  can be calculated, for example by using the undetermined coefficients procedure, see McCallum (1981) and McCallum (1989).

The main implication of this type of model is long-run neutrality of money.<sup>13</sup> Monetary policy can affect the real equilibrium only in the short-run by surprising the private agents such that the actual inflation rate deviates from the expected inflation rate.

### 3.2 Money Creation and Financial Structure

In the traditional IS-LM-AS model it is assumed that the money stock is set exogenously by the monetary authority. As discussed in Holtemöller (2001a), this assumption is not a priori reasonable for developed monetary economies like Germany or the Euro area where not narrow money but a broad monetary aggregate (M3) is usually considered in monetary policy analysis. Therefore, it is now discussed how the model of the previous section has to be modified in order to consider a more realistic money supply process.

The money multiplier approach makes clear that three different groups of agents participate in the money supply process. These are private households and firms, private banks, and the central bank. In this section, the role of private banks in the money supply process is discussed.<sup>14</sup> This makes assumptions about the behavior of banks necessary. The framework of the following analysis is the industrial organization approach to banking theory like it is also used in Vanhoose (1985) and in Holtemöller (2001a).<sup>15</sup> Banks are treated as profit-maximizing firms that accept deposits and originate loans. Additionally, banks buy and sell government securities, such that the profit function of a private bank  $n$  is:

$$\Pi_t^n = G(X_t^n) + i_t^L L_t^n + i_t^S S_t^n - i_t^D D_t^n - C(X_t^n, L_t^n, S_t^n, D_t^n), \quad n = 1, \dots, N, \quad (3.4)$$

where  $i_t^L, i_t^S, i_t^D$  are interest rates on loans ( $L$ ), securities ( $S$ ), and deposits ( $D$ ), respectively, and  $X_t^n$  are excess reserves with revenue function  $G(X_t^n)$ . This profit function is maximized subject to the balance sheet restriction

$$X_t^n + L_t^n + S_t^n = (1 - r)D_t^n, \quad (3.5)$$

where  $r$  is the required reserve rate, and banks hold required reserves  $rD_t^n$  as well as excess reserves  $X_t^n$ . Of course, the operating cost function  $C(\cdot)$  has to satisfy usual regularity assumptions to guarantee the existence and uniqueness of a profit maximum. The market structure is assumed to be as follows: banks are Cournot-oligopolists on the markets for loans and deposits. They select the quantities of loans and deposits, and the respective interest rates are determined by inverse loan demand and deposit demand functions of the public. Following Vanhoose (1985) the inverse demand function for deposits by the public is defined as

$$i_t^D = f_1 (i_t^S)^{f_2} D_t^{f_3}, \quad f_1, f_2, f_3 > 0, \quad D_t = \sum_{n=1}^N D_t^n, \quad (3.6)$$

where  $f_i$  are constants. The market for securities is competitive. The Lagrangian of the maximization problem is

$$\begin{aligned} \mathcal{L} = & G(X_t^n) + i_t^L L_t^n + i_t^S S_t^n - i_t^D D_t^n - C(X_t^n, L_t^n, S_t^n, D_t^n) \\ & + \lambda ((1 - r)D_t^n - X_t^n - L_t^n - S_t^n), \end{aligned} \quad (3.7)$$

<sup>13</sup> If expectations in equation (3.1) are described by  $E_t p_{t+1}$  instead of  $E_{t-1} p_{t+1}$ , output depends on the systematic part of monetary policy, and money is not neutral in the long run, see McCallum (1989, chapter 11.2).

<sup>14</sup> For an overview of this topic see for example Modigliani and Papademos (1990).

<sup>15</sup> This approach is based on the work of Klein (1971) and Monti (1972). An overview can be found in Freixas and Rochet (1997).



$$\frac{\partial \mathcal{L}}{\partial X_t^n} = \frac{\partial G(\cdot)}{\partial X_t^n} - \frac{\partial C(\cdot)}{\partial X_t^n} - \lambda \stackrel{!}{=} 0 \quad (3.8)$$

$$\frac{\partial \mathcal{L}}{\partial L_t^n} = \frac{\partial i_t^L}{\partial L_t^n} + i_t^L - \frac{\partial C(\cdot)}{\partial L_t^n} - \lambda \stackrel{!}{=} 0 \quad (3.9)$$

$$\frac{\partial \mathcal{L}}{\partial S_t^n} = i_t^S - \frac{\partial C(\cdot)}{\partial S_t^n} - \lambda \stackrel{!}{=} 0 \quad (3.10)$$

$$\frac{\partial \mathcal{L}}{\partial D_t^n} = -\frac{\partial i_t^D}{\partial D_t^n} D_t^n - i_t^D - \frac{\partial C(\cdot)}{\partial D_t^n} + \lambda(1-r) \stackrel{!}{=} 0 \quad (3.11)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (1-r)D_t^n - X_t^n - L_t^n - S_t^n \stackrel{!}{=} 0 \quad (3.12)$$

Using  $f_3 = \frac{\partial i_t^D}{\partial D_t^n} : \frac{\partial D_t}{\partial D_t^n}$  it follows from (3.10) and (3.11) that

$$i_t^D \left( 1 + \frac{D_t^n}{D_t} f_3 \right) + \frac{\partial C(\cdot)}{\partial D_t^n} = (1-r) \left( i_t^S - \frac{\partial C(\cdot)}{\partial S_t^n} \right). \quad (3.13)$$

From this expression, the interest rate paid on deposits can be derived by summing over all banks:

$$i_t^D = \theta \left( (1-r) \left( i_t^S - \frac{\partial C(\cdot)}{\partial S_t^n} \right) - \frac{\partial C(\cdot)}{\partial D_t^n} \right), \quad \theta = \frac{N}{N + f_3}. \quad (3.14)$$

Assuming constant marginal operating costs for deposits and securities, a constant number of banks  $N$ , and a constant inverse deposit demand elasticity makes (3.14) a reduced form expression for the interest rate on deposits. The total quantity of deposits can be calculated by inserting (3.14) into the inverse deposit demand function (3.6) and solving for  $D_t$ :

$$D_t = \left( \frac{i_t^D (i_t^S)^{-f_1}}{f_2} \right)^{\frac{1}{f_3}}, \quad (3.15)$$

or

$$D_t = D_t(i_t^S, r, \dots). \quad (3.16)$$

Now, the total quantity of reserves  $H_t$  (required and excess) is

$$H_t = rD_t + X_t, \quad (3.17)$$

and from (3.8) and (3.10) it can be seen that  $X_t^n$  is a function of  $i_t^S$ . Therefore, a log-linear approximation of the quantity of deposits is

$$d_t = \nu_0 + \nu_1 i_t^S + \nu_2 h_t, \quad \nu_1, \nu_2 > 0 \quad (3.18)$$

where  $d_t$  and  $h_t$  are the logarithms of  $D_t$  and  $H_t$ .

The financial structure developed in this section can now be added to the basic model of the previous section. The model of the economy consists now of the IS relationship<sup>16</sup> (3.1), the AS relationship (3.3), the modified LM relationship

$$m_t - p_t = c_0 + c_1(i_t^S - i_t^D) + c_2 y_t + \eta_t, \quad (3.19)$$

the interest rate equation (3.14), and the deposits-reserve relation (3.18). The opportunity costs of holding money are now measured by the interest rate spread, and it is assumed that the quantity of money equals the quantity of deposits ( $m_t = d_t$ ). That is, currency in circulation is ignored in this model. Like the basic model of section 3.1, the model is completed with a monetary policy rule, and can then be solved for the endogenous variables  $i_t^D$ ,  $i_t^S$ ,  $m_t$ ,  $p_t$ ,  $y_t$ , and  $h_t$ . The choice of the monetary policy rule is discussed in the next section.

<sup>16</sup> In opposition to the basic version of the model, there are now two interest rates:  $i_t^S$  and  $i_t^D$ . It is supposed in the following that the relevant interest rate in the IS relation is  $i_t^S$ .

In section 2.2, the monetary policy of the Bundesbank is described as reserve-oriented. This type of policy is characterized by

$$h_t = \frac{1}{\nu_2} (m_t^* - \nu_0 - \nu_1 E_{t-1}[i_t^S]). \quad (3.20)$$

First, the desired level of the money stock  $m_t^*$  is determined, and subsequently, the corresponding quantity of reserves to be implemented is calculated from (3.20). This is done one period before the implementation such that the interest rate on securities in period  $t$  has to be replaced by its expectation. Equation (3.20) is a monetary policy rule closing the model.

The monetary policy of the ECB cannot be described by such a reserve-oriented rule but may be described by an interest rate rule. Though the ECB does not intend to control a monetary aggregate it is assumed as a first approximation that the desired interest rate level is chosen such that the expected money stock is equal to the money stock  $m_t^*$  that is compatible with the reference value for money growth. This interest rate level can be calculated by inserting (3.14) into (3.19) and solving for  $i_t^S$ :

$$i_t^S = \frac{m_t^* - c_0 - E_{t-1}[p_t] - c_2 E_{t-1}[y_t] - c_1 \theta \left[ (1-r) \frac{\partial C(\cdot)}{\partial S_t} + \frac{\partial C(\cdot)}{\partial D_t} \right]}{c_1 [1 - (1-r)\theta]}. \quad (3.21)$$

A possible modification of this interest rate rule is to calculate the desired interest rate such that a certain price level is expected. Additionally, it would be more realistic to assume that the ECB tries to steer the money market interest rate  $i^D$  and not the interest rate on securities  $i^S$ . Additionally, in section 4.3 it will become clear that the model is not well specified with the monetary policy rule (3.21). In order to consider these problems, assume that the interest rate set by the central bank is a function of the money stock and the price level. Such a rule can be interpreted as a stylized “two pillar” monetary policy rule and is used in the empirical application. The exact specification is given in section 4.3.

## 4 Structural Vector Autoregressive Models as Tool for Monetary Policy Analysis

### 4.1 Statistical Analysis of Structural Vector Autoregressive Models

The empirical analysis of the impact of monetary policy on macroeconomic variables is conducted by using vector autoregressive models. This is a tool that is widely used for this purpose. In its basic form, a vector autoregressive model of order  $k$  is described by<sup>17</sup>

$$x_t = \mu_t + \sum_{i=1}^k A_i x_{t-i} + u_t, \quad (4.1)$$

where  $x_t = (x_{1t}, x_{2t}, \dots, x_{pt})'$  is a  $(p \times 1)$  vector of endogenous variables,  $u_t \sim N(0, \Sigma_u)$  is a  $p$ -dimensional n.i.d. error process with mean vector 0 and covariance matrix  $\Sigma_u$ ,  $\mu_t$  contains deterministic terms (which are ignored in the following) like a constant, a linear time trend and/or

<sup>17</sup> The econometric analysis of VAR models is discussed for example in Lütkepohl (1993), Lütkepohl (2001), and Hamilton (1994).

dummy variables. The coefficient matrices  $A_i$  and the covariance matrix  $\Sigma_u$  can be estimated using the ordinary least squares technique, and the optimal lag length  $k$  can be determined by comparing information criteria like Akaike Information Criterion (AIC), Hannan-Quinn Criterion (HQ) or Schwarz Criterion (SC). Once the parameters of the model have been estimated, the structural information of the model can be summarized in different ways. One possibility is the inspection of the implied impulse response functions measuring the impact of single innovations on the endogenous variables. Forecast error impulse responses  $\Phi_i$  are calculated from the moving average representation of the VAR:<sup>18</sup>

$$x_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i}. \quad (4.2)$$

The underlying assumption that innovations in the different equations are uncorrelated (that  $\Sigma_u$  is diagonal) is in general not compatible with the observed data and with the theoretical background. The contemporaneous relationships between the variables can be included into the model by transforming the VAR model (4.1) into the structural vector autoregressive (SVAR) model:<sup>19</sup>

$$A_0 x_t = \sum_{i=1}^k A_i^* x_{t-i} + A_0 u_t, \quad (4.3)$$

where  $A_i^* = A_0 A_i$ ,  $i = 1, \dots, k$ . A usual way of identifying the instantaneous relationships is to assume a recursive causal structure. That is, the first variable  $x_{1t}$  is only influenced by innovations in the first equation and lagged variables; the second variable is affected by innovations in the first equation, by innovations in the second equation, and lagged variables, and so on. In this case,  $A_0 = P^{-1}$ , and  $\Sigma_u = PP'$ , where  $P$  is a triangular matrix calculated using the Cholesky decomposition. Of course, the implications of the model do now depend on the ordering of the variables in  $x_t$ . A more general model nesting the recursive structure is the following one. Equation (4.1) can also be written as

$$A(L)x_t = u_t, \quad A(L) = I_p - A_1 L - \dots - A_k L^k, \quad (4.4)$$

where  $L$  denotes the Lag operator  $L^i x_t = x_{t-i}$ , and  $I_p$  is a  $p$ -dimensional identity matrix. The structure is now imposed by multiplying (4.4) with  $A = A_0$  and assuming that  $Au_t = Be_t$  such that

$$AA(L)x_t = Au_t, \quad Au_t = Be_t, \quad e_t \sim N(0, I_p). \quad (4.5)$$

This model type is called AB-model by Amisano and Giannini (1997). These authors also provide the following estimation technique. The parameters of the model are estimated in two steps: first, the reduced form (4.4) is estimated in the usual way by OLS; second the  $2p^2$  coefficients in  $A$  and  $B$  are determined by imposing at least  $p(p+1)/2$  (non-linear) restrictions, and estimating the remaining (at most  $2p^2 - p(p+1)/2$ ) free elements by maximization of the log-likelihood function, which is

$$\begin{aligned} \mathcal{L}(A, B) &= c + \frac{T}{2} \log(|A|^2) - \frac{T}{2} \log(|B|^2) - \frac{T}{2} \text{tr} \left( A' B'^{-1} B^{-1} A \widehat{\Sigma}_u \right) \\ &= c + \frac{T}{2} \log(|K|^2) - \frac{T}{2} \text{tr} \left( K' K \widehat{\Sigma}_u \right), \end{aligned} \quad (4.6)$$

where  $T$  is the sample size,  $K = B^{-1}A$ ,  $c = -(Tp/2) \ln(2\pi)$  and  $\widehat{\Sigma}_u$  is a consistent estimator of  $\Sigma_u$ . The restrictions that have to be imposed to achieve local identification can be summarized as

<sup>18</sup> Such a representation exists if the process is stationary. The case of non-stationarity is ignored for the present but is discussed later.

<sup>19</sup> SVAR models are discussed for example in Amisano and Giannini (1997), Breitung (2000), and Hamilton (1994).

$$\left. \begin{array}{l} R_a \text{vec } A = d_a \\ R_b \text{vec } B = d_b \end{array} \right\} \quad \text{or} \quad \begin{bmatrix} R_a & 0 \\ 0 & R_b \end{bmatrix} \begin{bmatrix} \text{vec } A \\ \text{vec } B \end{bmatrix} = \begin{bmatrix} d_a \\ d_b \end{bmatrix}, \quad (4.7)$$

where  $\text{vec}$  is an operator that stacks the columns of a matrix into a single column vector,  $R_a$  and  $R_b$  are  $(r_a \times p^2)$  full row rank matrices,  $d_a$  and  $d_b$  are  $(r_a \times 1)$  and  $(r_b \times 1)$  vectors, and  $r_a$  and  $r_b$  denote the number of restrictions on  $A$  and  $B$ , respectively. The restrictions can also be parameterized as

$$\left. \begin{array}{l} \text{vec } A = S_a \gamma_a + s_a \\ \text{vec } B = S_b \gamma_b + s_b \end{array} \right\} \quad \text{or} \quad \begin{bmatrix} \text{vec } A \\ \text{vec } B \end{bmatrix} = \begin{bmatrix} S_a & 0 \\ 0 & S_b \end{bmatrix} \begin{bmatrix} \gamma_a \\ \gamma_b \end{bmatrix} + \begin{bmatrix} s_a \\ s_b \end{bmatrix}, \quad (4.8)$$

where  $S_a$  and  $S_b$  are  $(p^2 \times \ell_a)$  and  $(p^2 \times \ell_b)$  full column rank matrices,  $s_a$  and  $s_b$  are  $(p^2 \times 1)$  vectors, and  $\ell_a = p^2 - r_a$  and  $\ell_b = p^2 - r_b$  are the numbers of freely estimated coefficients of  $A$  and  $B$ , respectively.

The log-likelihood function has to be maximized using a numerical iterative procedure, see Appendix B.

## 4.2 Impulse Response Functions of SVARs

Following Amisano and Giannini (1997), impulse responses for structural VARs can be calculated by

$$\Theta_0 = K^{-1} = (B^{-1}A)^{-1} = A^{-1}B, \quad \Theta_i = JM^i J' \Theta_0, \quad i = 1, 2, \dots \quad (4.9)$$

where

$$M = \begin{bmatrix} A_1 & A_2 & \cdots & A_{k-1} & A_k \\ I_p & 0 & \cdots & 0 & 0 \\ 0 & I_p & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & I_p & 0 \end{bmatrix}, \quad J = [I_p \quad 0 \quad \cdots \quad 0],$$

$M$  is a  $(pk \times pk)$  matrix, and  $J$  is a  $(p \times pk)$  matrix, or by

$$\Theta_i = \Phi_i \cdot \Theta_0, \quad \Phi_0 = I_p, \quad \Phi_i = \sum_{j=1}^i \Phi_{i-j} A_j, \quad i = 1, 2, \dots, \quad (4.10)$$

where  $A_i = 0$  for  $i > k$ . The impulse responses are estimated by replacing the theoretical variables with the respective estimators. In the following, the same notation for theoretical impulse responses and estimated impulse responses is used. The estimated impulse responses that are stored up to horizon  $H$  in  $\text{vec} [\Theta_0, \Theta_1, \dots, \Theta_H]$  are unbiased and asymptotically normally independently distributed with covariance matrix  $\frac{1}{T} \Sigma_{\Theta}(H)$ . Some preliminary calculations are necessary to compute  $\Sigma_{\Theta}(H)$ :

$$\Sigma_K = [I_p \otimes B^{-1} \mid -(A'B'^{-1}) \otimes B^{-1}] \Sigma_{ab} \begin{bmatrix} I_p \otimes B'^{-1} \\ -(B^{-1}A) \otimes B'^{-1} \end{bmatrix} \quad (4.11)$$

$$\Sigma_{\Theta}(0) = \Sigma_{K^{-1}} = (K'^{-1} \otimes K^{-1}) \Sigma_K (K^{-1} \otimes K'^{-1}) \quad (4.12)$$

The matrix  $\Sigma_{\Theta}(H)$  consists of  $(H+1)(H+1)$  blocks of dimension  $p^2 \times p^2$ , denoted by  $\Sigma_{\Theta}(H)_{ij}$ ,  $i = 0, \dots, H$ ,  $j = 0, \dots, H$ . The covariance matrix of  $\text{vec } \Theta_i$  is given by the block  $\Sigma_{\Theta}(H)_{ii}$ . The blocks are defined as

$$\Sigma_{\Theta}(H)_{ij} = G_i \Sigma_{\text{vec } [A_1, \dots, A_p]} G_j' + [I_p \otimes (JM^i J')] \Sigma_{\Theta}(0) [I_p \otimes (JM^j J')]', \quad (4.13)$$

$i, j = 1, \dots, H$ , where

$$G_0 = 0, \quad G_i = \sum_{n=0}^{i-1} ([\Theta_0' J (M')^{i-1-n}] \otimes [JM^n J']), \quad i > 0. \quad (4.14)$$

The economic model discussed above and the SVAR methodology can be combined in the following way. First, the case of a reserve-oriented monetary policy is considered. If the model equations are used to calculate the deviations of the actual realizations from their expected values, the solution of the model is given by the system<sup>20</sup>

$$\underbrace{\begin{bmatrix} -b_1 & 1 & -b_1 & 0 \\ -a_1 & 1 & 0 & 0 \\ -1 & -c_2 & -c_1[1 - (1 - r)]\theta & 1 \\ 0 & 0 & -\nu_1 & 1 \end{bmatrix}}_A \begin{bmatrix} \hat{p}_t \\ \hat{y}_t \\ \hat{i}_t^s \\ \hat{m}_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix}, \quad (4.15)$$

where a hat above the endogenous variables symbolizes the expectation error  $\hat{x}_t = x_t - E_{t-1}[x_t]$ , and  $\hat{m}_t = m_t - m_t^*$ . The error terms  $e_{it}$  can now be interpreted as structural shocks:  $e_{1t}$  and  $e_{2t}$  are real shocks,  $e_{3t}$  is a money demand shock (MD) and  $e_{4t}$  is a money supply shock (MS). In the empirical application (section 5) it is assumed that  $c_1[1 - (1 - r)]\theta$  is constant. This is clearly not the case, especially for the required reserve rate  $r$ . However, the analysis in Holtemöller (2001a) has shown that the impact of the required reserve rate is only important if the relation between the monetary base and the money stock is considered. The monetary base is part of the theoretical model used here, but is not part of the reduced form (4.15). This justifies to some extent the constancy assumption.

In case of an interest rate rule, the solution is given by the system

$$\underbrace{\begin{bmatrix} -b_1 & 1 & 0 \\ -a_1 & 1 & 0 \\ -1 & -c_2 & 1 \end{bmatrix}}_A \begin{bmatrix} \hat{p}_t \\ \hat{y}_t \\ \hat{m}_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}. \quad (4.16)$$

By assuming that the expected values of the endogenous variables,  $E_{t-1}[x_t]$  can be calculated from a VAR representation, the VAR residuals  $u_t$  can be interpreted as deviations of the variables from the respective expected values:

$$u_t = \hat{x}_t = x_t - E_{t-1}[x_t]. \quad (4.17)$$

The SVAR framework provides the econometric methodology to model the relations between these deviations empirically. The structure given by the matrix  $A$  of the respective model can be estimated, and the implied impulse responses of different models can be compared. For the validity of this approach it does not matter whether the variables in the model are stationary or not. The contemporaneous relations that are considered in the SVAR model are relations between expectation deviations in the theoretical model, and relations between VAR residuals in the empirical model.<sup>21</sup> Both are stationary by definition, regardless of the order of integration of the variables themselves.<sup>22</sup>

The structure given by (4.16), however, is not identified. The first and second equation cannot be distinguished from each other empirically. Therefore, the stylized “two-pillar” monetary policy rule

$$i_t = \gamma_1 p_t + \gamma_2 m_t, \quad \gamma_1, \gamma_2 > 0. \quad (4.18)$$

<sup>20</sup> See Vanhoose (1985).

<sup>21</sup> The approach of specifying a structural model for VAR residuals is also followed by Bernanke and Mihov (1998), for example, in order to identify monetary policy shocks in the U.S.

<sup>22</sup> The effect of integration on estimation and calculation of impulse responses is discussed in section 5.2.

$$\underbrace{\begin{bmatrix} 1 & -1/a_1 & 0 & 0 \\ -b_1 & 1 & -b_1 & 0 \\ -\gamma_1 & 0 & 1 & -\gamma_2 \\ -1 & -c_2 & -c_1 & 1 \end{bmatrix}}_A \begin{bmatrix} \hat{p}_t \\ \hat{y}_t \\ \hat{i}_t \\ \hat{m}_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix}, \quad (4.19)$$

where the third equation is the monetary policy rule and the fourth equation is the money demand function, such that  $e_{3t}$  can be interpreted as monetary policy shock (MP) and  $e_{4t}$  as money demand shock (MD). This identification scheme is very similar to the reserve-oriented monetary policy model. It is less restrictive because the coefficient of the price level in the monetary policy equation is not restricted to zero like it is the case in the reserve-oriented monetary policy model.

In the empirical analysis of sections 5.2 and 5.3, an additional identification scheme that has been suggested by Leeper et al. (1996) is applied as a benchmark. This identification scheme is given by

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & a_{43} & 1 \end{bmatrix}. \quad (4.20)$$

The first and second row represent the behavior of the private sector, and it is assumed that the financial variables  $i$  and  $m$  have no contemporaneous effect on  $p$  and  $y$ . The two private sector shocks that are identified by this structure are called  $P_1$  shock and  $P_2$  shock later on. The third row can be interpreted as money demand relation<sup>23</sup>, and innovations to the fourth equation represent monetary policy shocks. This set of identifying restrictions is not based on an explicit model but is rationalized by economic arguments by Leeper et al. (1996).

## 5 Empirical Application of the Monetary Policy Analysis Framework

### 5.1 Puzzles in the Applied SVAR Literature

The SVAR approach has been used in a large number of empirical studies. However, in most of them the recursive identification scheme is applied.<sup>24</sup> Special interest has been put on the identification of monetary policy shocks and the responses of macroeconomic variables to monetary policy shocks.<sup>25</sup> Within the SVAR approach, monetary policy shocks are calculated by assuming that the AB-model (4.5) describes the relationships between the estimated VAR residuals and the underlying economic shocks. The recursive identification scheme is a special case of the AB-model with  $A = I_p$  and  $B = P$ , where  $PP' = \Sigma_u$  and  $P$  is lower triangular.<sup>26</sup> Using this identification scheme, many authors have found impulse responses that differ from intuitive economic interpretations. This is especially the case when the analyzed system consists only of the four variables price level, real output, interest rate, and money stock. Examples are the price puzzle (a positive

<sup>23</sup> Another normalization that restricts  $a_{34}$  to one and not  $a_{33}$  would make this more evident.

<sup>24</sup> There are also studies in which identification is achieved by imposing long-run and short-run restrictions in the way introduced by Blanchard and Quah (1989). This approach is not considered here.

<sup>25</sup> A review of the recent literature on this topic can be found in Christiano et al. (1999).

<sup>26</sup> An equivalent specification is  $A = P^{-1}$  and  $B$  diagonal.

response of prices on a positive innovation in the interest rate), and the liquidity puzzle (a non-negative response of the interest rate on a positive innovation in the money stock). In other words, there is a liquidity puzzle if no liquidity effect is observed. The liquidity effect is originally defined in a setup in which monetary policy is described by changes in the money stock. In such a setup, the initial decline of the interest rate after an increase in the money stock is called liquidity effect, see for example Dornbusch and Fischer (1994, p. 493 f.). In the following, the notion liquidity puzzle is used in a broader sense. In addition to the case in which there is no liquidity effect in the way just described, the case in which the interest rate is the monetary policy variable and the money stock does not respond negatively on positive interest rate innovations is also called liquidity puzzle. It should be noticed that the liquidity effect is only a short-run effect. As time goes on, this effect is possibly overcompensated by income and Fisher effect.<sup>27</sup>

In larger systems of open economies, two additional puzzles sometimes occur: the exchange rate puzzle (a depreciation of the domestic currency as response on a positive innovation in the domestic interest rate), and the forward discount bias puzzle (appreciation as response to a positive interest rate differential between domestic and foreign interest rates).<sup>28</sup> It has been tried to solve these puzzles in different ways. Suppose that the central bank increases the interest rate if it expects a higher level of prices in the future than is compatible with its price stability objective. If actual innovations in the price level are a leading indicator for future prices then positive innovations in the price level are responded by an increase in the interest rate. By extending the information set and including a leading indicator for future prices, the puzzle can possibly be solved. Often, commodity prices, exchange rates, or an import price index are considered as leading indicators for the aggregate price level. Sims (1992) shows that the positive response of prices to innovations in the interest rate in VAR models of monetary policy effects in France, Germany, Japan, the United Kingdom, and the United States tends to become smaller when commodity prices and exchange rates are included. However, the price puzzle does not disappear in all cases. For the U.S., it is possible to avoid the price and liquidity puzzles if commodity prices and additional monetary aggregates like the monetary base are added to the system. This approach is taken in Leeper et al. (1996) and Christiano et al. (1999). The augmentation makes a separation of different sources of shocks possible. While the augmentation approach still relies on a recursive identification scheme, Leeper et al. (1996) (U.S.) and Kim and Roubini (2000) (non-U.S. G-7 countries) find non-recursive identification schemes that are successful in avoiding puzzles. Aggregated Euro area data is analyzed inter alia in Monticelli and Tristani (1999) and Brand and Cassola (2000). While Monticelli and Tristani (1999) identify structural shocks by imposing an open economy IS-LM-AS model, Brand and Cassola (2000) use “a theoretically neutral way of deriving impulse responses” (p. 15) by calculating generalized impulse responses as suggested in Pesaran and Shin (1998). The open economy model of Monticelli and Tristani (1999) gives German monetary policy an anchor role within the European exchange rate mechanism (ERM). The response of inflation to monetary policy shocks is not significantly different from zero and exchange rate effects are very small. The liquidity puzzle cannot occur because the money stock is not considered in this model. In the system of Brand and Cassola (2000), there is no liquidity puzzle but the price puzzle still occurs (the response of the inflation rate on an increase in the short-term or the long-term interest rate is positive). Impulse responses in German monetary systems are addressed in Lütkepohl and Wolters (1999) and Benkwitz et al. (2001), for example. In both studies, an import price index is added without always solving the puzzles.

In the following, it is shown in a simulation example that the recursive identification scheme may

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<sup>27</sup> These effects are also explained in Dornbusch and Fischer (1994, p. 493 f.).

<sup>28</sup> Price and liquidity puzzles are discussed inter alia in Leeper et al. (1996) and Strongin (1995) for the U.S. and in Kim (1999) for the G-7 countries. Kim and Roubini (2000) analyze also the exchange rate and the forward discount bias puzzles for non-U.S. G-7 countries.

be misleading in practice. Consider the price puzzle and let the vector of endogenous variables be  $x_t = (p_t, y_t, i_t, m_t)$ . Assume that the true model of the economy is an IS-LM-AS model with a stylized “two-pillar” monetary policy rule as described by (4.19). The innovations in the third equation can be interpreted as monetary policy shocks, and the response of the price level in period  $t + i$  on a monetary policy shock in period  $t$  is given by  $\theta_{i,13}$ , that is the entry in the first row and third column of the impulse response matrix  $\Theta_i$ . The calculation of  $\Theta_i$  is explained in section 4.2. Assume that  $B = I_p$  and denote the entries of the inverse of  $A$  by  $a^{jk}$ . Recalling that  $\Theta_0 = A^{-1}B$ , the contemporaneous response of the price level to a monetary policy shock in the structural model is  $\theta_{0,13}^{AB} = a^{13}$ . In the case of recursive identification, this response is  $\theta_{0,13}^P = 0$ . For  $i \geq 1$ , the impulse responses are calculated by  $\Theta_i = \Phi_i \Theta_0$ , where  $\Phi_i$  are the coefficient matrices of the “non-structured” MA-representation given in (4.10). The  $\Phi_i$ -matrices are the same for the structural model and the recursive model and their entries are denoted by  $\varphi_{i,jk}$ . The responses of the price level on a monetary policy shock in the structural model can now be written as

$$\theta_{i,13}^{AB} = a^{13}\varphi_{i,11} + a^{23}\varphi_{i,12} + a^{33}\varphi_{i,13} + a^{43}\varphi_{i,14} \quad (5.1)$$

while the corresponding response in the recursive identification case is

$$\theta_{i,13}^P = p_{33}\varphi_{i,13} + p_{43}\varphi_{i,14}, \quad (5.2)$$

where  $p_{jk}$  are the entries of  $P$ , and  $p_{13} = p_{23} = 0$  by definition.

For the simulation, the non-zero entries of  $A$  in the model given by (4.19) are specified as follows:

$$A = \begin{bmatrix} 1 & -0.5 & 0 & 0 \\ 2.5 & 1 & 2.5 & 0 \\ -1.25 & 0 & 1 & -0.75 \\ -1 & -1 & 2 & 1 \end{bmatrix}. \quad (5.3)$$

These values are somehow arbitrary, but the signs are compatible with the theoretical model and the values are realistic in the sense that they reflect plausible economic relationships. Given that  $u_t = A^{-1}e_t$  it follows that  $\Sigma_u = A^{-1}\Sigma_e(A^{-1})'$  and

$$A^{-1} = \begin{bmatrix} 0.44 & 0.13 & -0.13 & -0.09 \\ -1.13 & 0.25 & -0.25 & -0.19 \\ 0.01 & 0.18 & 0.23 & 0.17 \\ -0.71 & 0.03 & -0.83 & 0.38 \end{bmatrix}, \quad P = \begin{bmatrix} 0.48 & 0 & 0 & 0 \\ -0.86 & 0.83 & 0 & 0 \\ -0.03 & 0.11 & 0.31 & 0 \\ -0.50 & 0.62 & -0.25 & 0.80 \end{bmatrix},$$

such that

$$\theta_{i,13}^{AB} = -0.13\varphi_{i,11} - 0.25\varphi_{i,12} + 0.23\varphi_{i,13} - 0.83\varphi_{i,14} \quad (5.4)$$

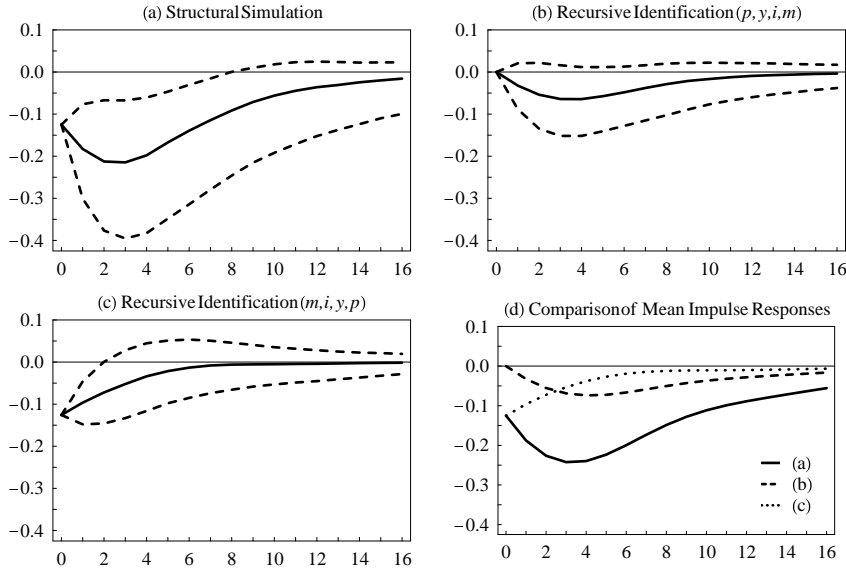
and

$$\theta_{i,13}^P = 0.31\varphi_{i,13} - 0.25\varphi_{i,14}. \quad (5.5)$$

There is a broad range of  $\Phi_i$  matrices for which  $\theta_{i,13}^{AB}$  and  $\theta_{i,13}^P$  can have different signs. The  $\Phi_i$  matrices depend on the VAR coefficient matrices  $A_i$ . Consider a VAR with lag length 1. The matrix  $A_1$  is now generated by a random number generator such that the entries are normally distributed with the following expected values and standard deviations:

$$A_1 \sim \begin{bmatrix} N(0.75, 0.125) & N(0, 0.25) & N(0, 0.25) & N(0, 0.25) \\ N(0, 0.25) & N(0.75, 0.125) & N(0, 0.25) & N(0, 0.25) \\ N(0, 0.25) & N(0, 0.25) & N(0.75, 0.125) & N(0, 0.25) \\ N(0, 0.25) & N(0, 0.25) & N(0, 0.25) & N(0.75, 0.125) \end{bmatrix},$$





Notes: The solid lines in graphs (a), (b), and (c) are the median impulse responses using the respective identification scheme. In case (a), the impulse responses are computed using the structure given by  $A$ . In cases (b) and (c), the recursive identification scheme is applied. The dashed lines are upper and lower quartiles. Graph (d) shows the mean impulse responses of cases (a), (b), and (c).

implying that the variables are autocorrelated, but there are no special assumptions about the dynamic relations between the variables. The economic structure is only given by the contemporaneous relationships expressed by  $A$ . 10,000  $A_1$ -matrices that imply a stable VAR<sup>29</sup> and negative “true” impulse responses<sup>30</sup> are computed. Then, impulse responses using the recursive identification scheme are calculated and it is counted for how many realizations of  $A_1$  the sum of the impulse responses for the periods 0 to 16 is positive ( $\sum_{i=0}^{16} \theta_{i,13}^P > 0$ ). In 27% of the considered cases, in which the theoretical structural cumulated impulse response is negative, the cumulated impulse response using the Cholesky decomposition with the ordering  $(p, y, i, m)$  is positive. One determinant of the sign of the impulse response is  $a_{1,13}$ , that is the coefficient of the one period lagged interest rate in the price level equation of the VAR. Even if this coefficient is negative, the Cholesky decomposition gives positive impulse responses in 5% of the considered cases.<sup>31</sup> Impulse responses that are calculated using a recursive identification scheme are in general not invariant to changes in the ordering of the variables. This can easily be seen when the reverse ordering is considered. In this case, the response of prices on innovations in the interest rate is measured by

$$\theta_{i,42}^{P*} = p_{22}^* \varphi_{i,42} + p_{32}^* \varphi_{i,43} + p_{42}^* \varphi_{i,44}, \quad (5.6)$$

<sup>29</sup> Only  $A_1$  matrices are considered for which all roots of  $\text{Det}[I_4 - A_1 z] = 0$  lie outside the unit circle.

<sup>30</sup> The theoretical impulse responses implied by  $A$  and  $A_1$  are calculated, and only realizations of  $A_1$  for which  $\sum_{i=0}^{16} \theta_{i,13}^{AB} < 0$  are considered. This criterion does not exclude that the impulse response is positive for single periods, but that does not harm the experiment.

<sup>31</sup> It is not clear if this coefficient is positive or negative in an economic model. It measures the average impact of interest rate changes on the price level. The interest rate can increase for different reasons. If a monetary policy action is the reason for the increase in the interest rate, a negative impact is expected. However, if the nominal interest rate rises because expected inflation is increasing, the impact could be positive. Further, a higher interest rate implies higher financing costs for firms. Therefore, the sign of  $a_{1,13}$  is not clearly given by economic theory. This justifies the random choice of the coefficient.

where the asterisk symbolizes the reverse ordering of the variables. The simulation result, however, is similar if the ordering of the variables is reversed. 29% of the realizations that imply negative “true” impulse responses lead to positive impulse responses if the Cholesky decomposition with reverse ordering  $(m, i, y, p)$  is used, and if the coefficient of the lagged interest rate in the price level equation is negative, the Cholesky decomposition gives positive impulse responses in 7% of the cases with “true” negative responses. These results are illustrated in figure 1. Part (b) of the figure shows that the upper quartiles of the impulse responses implied by the recursive identification scheme with ordering  $(p, y, i, m)$  are positive at all horizons though the cumulated true impulse response is negative. It can be seen in part (d) that in average, the sign is correct, but the impact is underestimated and the shape depends on the ordering of the variables.

## 5.2 Empirical Monetary Policy Analysis for Germany

In this section, a small monetary system for Germany is investigated. Impulse responses of macroeconomic variables to monetary policy shocks are calculated. It is shown that the kind of problem described in the previous section possibly occurs when monetary policy shocks in a small monetary system are tried to be identified using a recursive identification scheme. Like in other studies of the monetary policy transmission process and like suggested by the economic model of section 3, the  $p = 4$  endogenous variables in the SVAR analysis are the logarithmic price level ( $p$ ), the logarithmic real GDP ( $y$ ), an interest rate ( $i$ ), and the logarithmic nominal money stock M3 ( $m$ ):  $x_t = (p_t, y_t, i_t, m_t)'$ .<sup>32</sup> In the results reported here, a long-term interest rate is used.<sup>33</sup> Impulse responses that are calculated using a recursive identification scheme are compared to impulse responses that are calculated using the reserve-oriented monetary policy model, the two-pillar monetary policy rule model and the Leeper et al. (1996) identification scheme. The confidence bands for impulse responses that are used here are calculated by adding and subtracting two asymptotic standard errors.

First, in section 5.2.1, the VAR is estimated using the level representation and ordinary least squares. It is known that the considered variables are instationary and potentially cointegrated. Breitung (2000, Theorem 2.2) shows, that impulse responses can be efficiently estimated from the unrestricted VAR as well as from a vector error correction model (VECM) with correctly imposed cointegration rank. In small samples, however, as claimed by Breitung (2000, p. 75), it “may be advantageous to use a VECM representation with a proper cointegration rank to estimate the impulse responses.” Therefore, the reduced rank regression technique is applied to estimate the VECM in section 5.2.2. From the VECM representation, the level representation can be recovered, and the structural analysis can be undertaken in the same way as if the level representation was estimated directly. The results of this procedure are compared to those of the unrestricted VAR case.

### 5.2.1 Calculation of Impulse Responses from the Level Representation

*VAR Specification.* The sample period for this analysis is 1975 to 1998, and the data is quarterly (for details see A.1). For this period, the reserve-oriented model is a considerable approximation to monetary policy in Germany. However, as shown in Holtemöller (2001a), there is some empirical evidence that does not support the reserve-oriented monetary policy model. Therefore, the two-pillar monetary policy rule model is not excluded a priori.

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<sup>32</sup> The data is described in Appendix A.

<sup>33</sup> The same analysis has also been conducted using a short-term interest rate instead. The results are very similar to those reported here.

$$x_t = \mu_t + \sum_{i=1}^k A_i x_{t-i} + u_t, \quad (5.7)$$

where  $\mu_t$  comprises the following deterministic terms: constant, linear time trend, seasonal dummies, the step dummy DS902, and lags one to four of DS902.<sup>34</sup> The lag length selection criteria suggest one (SC), one (HQ), and seven (AIC) lags. Starting with the most parsimonious lag specification (SC, HQ: one), lags have been added until there seems to be no substantial autocorrelation left in the residuals which is the case for  $k = 4$ . The residual correlation matrix is

$$\begin{pmatrix} 1 & -0.09 & -0.20 & 0.20 \\ -0.09 & 1 & 0.18 & -0.09 \\ -0.20 & 0.18 & 1 & 0.07 \\ 0.20 & -0.09 & 0.07 & 1 \end{pmatrix}, \quad (5.8)$$

from which can be seen that there is still correlation between the residuals.<sup>35</sup> This remaining correlation is analyzed in the following structural analysis. All impulse responses are depicted but only the impulse responses related to monetary policy are discussed in the text. The figures have to be read as follows. The columns represent the various innovations or shocks, and in the rows, the response of a variable to the respective shocks is presented. The label  $i \rightarrow m$ , for example, means the response of the money stock on an innovation in the interest rate. In case of non-recursive identification schemes, the innovations in the respective equations can be interpreted explicitly as economic shocks, and  $MP \rightarrow p$ , for example, means the response of the price level on a monetary policy shock. As mentioned before, the confidence bands are computed by adding and subtracting two asymptotic standard errors.

*(a) Recursive Identification.* Impulse responses on innovations in  $m$  (figure 2) and  $i$  (figure 3) that are calculated using the recursive identification scheme and the ordering  $(p, y, i, m)$  are shown in column (a) of figures 2 and 3 in Appendix D.<sup>36</sup> This ordering is chosen because it can be assumed that the financial variables  $i$  and  $m$  have no contemporaneous effect on inflation and output but have an effect in later periods. This ordering is also chosen by Leeper et al. (1996). The liquidity puzzle can be observed: innovations in the money stock have a positive effect on the interest rate. An increase in the money stock has the expected positive effect on the future price level and only a small and merely significant impact on output. The price puzzle occurs: the response of prices on a positive innovation in the interest rate is positive.

*(b) Reserve-oriented Monetary Policy Model.* The identification scheme that is implied by the reserve-oriented monetary policy model is given in equation (4.15). The matrices  $A$  and  $B$  are estimated in the way described in section 4, and the results of this estimation are reported in table 2, Appendix C. The corresponding impulse responses can be seen in column (b) of figures 2 and 3.

<sup>34</sup> DS902 is zero up to the first quarter of 1990, and one afterwards. With this dummy variable the structural break in the data due to the German unification is modeled.

<sup>35</sup> The correlation between the different residuals is only weakly significant. If the residual correlation matrix was an identity matrix, the residuals would already be orthogonal. However, comparing the forecast error impulse responses (which are given by  $\Phi_i$  in equation (4.10), but are not depicted here) with the orthogonalized impulse responses shows that there is some systematic information left in the residual cross correlations. This is reflected by significant non-zero contemporaneous orthogonalized impulse responses.

<sup>36</sup> The impulse responses on innovations in  $p$  and  $y$  are not of interest in this context. They are depicted in a longer version of this paper that is available on the author's homepage.

The innovation in the interest rate equation can be interpreted as a money demand shock (MD), and the innovation in the money equation as a monetary supply shock (MS). An increase in the money stock leads to increasing future prices, and also to a contemporaneous decrease in the interest rate (this is not obvious in the figure but can be confirmed by looking at the numerical values of the impulse response and the corresponding standard error). After a short time, however, the interest rate increases. This effect can possibly be explained by the Fisher effect. Individuals expect the increase in the price level and adjust their inflation expectations. Adding the real interest rate and increasing expected inflation yields the increasing nominal interest rate. In the third column it can be seen that a price puzzle does not occur: a shock that increases the interest rate does not lead to increasing prices. This set of impulse responses is therefore more compatible with standard assumptions about reactions to innovations in money stock and interest rate than the one implied by the recursive identification scheme.

(c) *Two-pillar Monetary Policy Rule Model.* The estimated stylized two-pillar monetary policy rule is given in the third row of the  $A$ -matrix in the middle part of table 2. Innovations in the money stock have the expected impact on the interest rate, but the actual innovations in the price level enter the monetary policy rule with the wrong sign. Therefore, the impulse responses implied by this structure may not be as expected, too. They are depicted in column (c) of figures 2 and 2. The money demand shock is essentially the same as in the reserve-oriented monetary policy model, and the monetary policy shock in column (c) is essentially the negative money supply shock in column(b). Given the wrong sign and non-significance of the price level coefficient in the two-pillar monetary policy rule, the reserve-oriented monetary policy model, in which the money stock is the monetary policy variable, seems to be a reasonable approximation to monetary policy in Germany from 1975 to 1998.

(d) *Leeper et al. (1996) Identification Scheme.* For comparison purposes, the Leeper et al. (1996) identification scheme, that is described by (4.20), is also considered. The SVAR estimation results can be found in the lower part of table 2, and the corresponding impulse responses are shown in column (d) of figures 2 and 3. The money demand shock has sensible responses, there is no price puzzle. Furthermore, as the fourth column shows, an increase in the money stock has a positive impact on the future price level. However, there remains a liquidity puzzle.

In the next section, the robustness of these results is examined by repeating the analysis using a vector error correction model (VECM) instead of the VAR model in levels.

### 5.2.2 Calculation of Impulse Responses from the VECM Representation

*VECM Specification.* The analysis in Holtemöller (2001a) has shown that the variables considered here are not stationary but integrated of order one. To incorporate this stochastic property of the data, the following VECM is estimated:

$$\Delta x_t = \mu_t + [\Pi : \nu_1] \begin{bmatrix} x_{t-1} \\ d_{t-1} \end{bmatrix} + \sum_{i=1}^3 \Gamma_i \Delta x_{t-i} + u_t, \quad (5.9)$$

where  $d_t$  is the step dummy DS902 for the German unification, and  $\mu_t$  contains a constant, seasonal dummies, and the first differences of DS902 up to lag three. The Johansen trace test on cointegration is applied in combination with simulated critical values that consider the presence of a dummy variable that is restricted to the cointegration space. The results of the Johansen trace test are (5% critical values in brackets, the construction of these critical values is explained in

Hörmann (2001a)).  $\Pi_0 : 11.95 (56.62)$ ,  $\Pi_1 : 18.11 (55.58)$ ,  $\Pi_2 : 16.25 (19.88)$ , and  $\Pi_3 : 5.16 (5.89)$ .  $H_i$  is the hypothesis that the cointegration rank is at most  $i$ . The hypothesis is rejected for  $i = 0$  and  $i = 1$  at a significance level of 5%, such that a cointegration rank of two is applied.<sup>37</sup>

The residual correlation matrix is

$$\begin{pmatrix} 1 & -0.09 & -0.14 & 0.16 \\ -0.09 & 1 & 0.12 & -0.11 \\ -0.14 & 0.12 & 1 & 0.03 \\ 0.16 & -0.11 & 0.03 & 1 \end{pmatrix}. \quad (5.10)$$

The signs are the same as in the residual correlation matrix obtained from the estimation of the VAR in levels. The magnitudes are slightly different, but there is still contemporaneous correlation between the residuals, see also footnote 35. Now, the VAR representation is derived from the estimated VECM such that impulse responses can be calculated.

*(a) Recursive Identification.* The ordering of the variables is as before:  $(p, y, i, m)$ . The impulse responses implied by the VECM and the recursive identification scheme are plotted in column (a) of figures 4 and 5. Most of the responses are as expected. There is no liquidity effect, and the price puzzle can still be observed. It should be mentioned that the effect of an increasing money stock on the price level is now of permanent nature. The possibility of permanent impacts of shocks is introduced by considering the instationary behavior of the variables. In stable VARs, the impulse responses converge always to zero with increasing horizon.

*(b) Reserve-oriented Monetary Policy Model.* From table 3 it can be seen that the point estimates for the entries of the matrix  $A$  are very similar to those calculated from the VAR level representation. In general, this is also true for the impulse responses (column (b) in figures 4 and 5), but there are some differences. For example the price puzzle is not solved in this specification, and the impact of an increase in the interest rate on output is now permanent and not transitory. Finally, the small initial liquidity effect can also be observed here, but without the overcompensating effects that are observed in figure 2. However, the result of the model in levels that the reserve-oriented monetary policy model seems to be a valid identification scheme for Germany is supported by the structural VECM.

*(c) Two-pillar Monetary Policy Rule Model.* In case of the two-pillar monetary policy rule identification scheme, the point estimates for the entries of  $A$  – given in the middle part of table 3 – differ from the corresponding estimates in table 2. For example, the monetary policy rule is now estimated with plausible signs, and smaller standard errors. The impulse responses look also convincing, see column (c) in figures 4 and 5. Restrictive monetary policy shocks have a negative impact on the price level. However, this effect dies out after a few periods. There is also a liquidity effect in the sense that the money stock is negatively affected by the monetary policy variable in this model, namely  $i$ . However, the positive innovations in the money stock have a positive impact on the interest rate. Therefore, the reserve-oriented monetary policy model may still be regarded as the most appropriate identification scheme for the monetary transmission process in Germany.

<sup>37</sup> The cointegration relations are not reported here, and they are not used in the further analysis. Of course, it is possible to combine a detailed specification of the cointegration relations with the SVAR methodology. This is for example done in Brüggemann (2001).

(a) *Leeper et al. (1998) Identification Scheme.* Applying the Leeper et al. (1998) identification scheme in the VECM gives again a very similar picture to that in the unrestricted VAR case. The estimates for  $A$  and  $B$  can be found in the lower part of table 3 and the impulse responses are depicted in column (d) in figures 4 and 5. The impulse responses differ from the responses in case of the unrestricted VAR only in the permanence of some effects.

### 5.3 Empirical Monetary Policy Analysis for the Euro Area

The sample period for the following analysis of monetary policy effects in the Euro Area (without Greece) is 1980:1 to 1999:4. The data is quarterly and seasonally unadjusted, for details see data appendix. The vector of endogenous variables consists of the same variables as before:  $x_t = (p_t, y_t, i_t, m_t)'$ , where  $i_t$  is again a long-term interest rate. This application should be interpreted as an example for future investigation because there has been no single monetary policy in the Euro Area in the sample period. The assumption of an area wide monetary policy in this sample period has also been made by Monticelli and Tristani (1999) and Brand and Cassola (2000).

#### 5.3.1 Calculation of Impulse Responses from the Level Representation

As a VAR model for the Euro Area, equation (5.7) can be applied, but  $\mu_t$  now comprises only a constant, seasonal dummies and a linear time trend. Other dummy variables are not needed in this case because the data is adjusted for the German unification (and also other structural breaks like reclassifications or area changes) by the ECB. The lag lengths suggested by information criteria are one (SC), six (HQ), and eight (AIC). While a lag length of one does not remove all autocorrelation from the VAR residuals, there seems to be no substantial autocorrelation left after adding lags up to lag length four. Therefore, the lag length is set to  $k = 4$ . The residual correlation matrix is

$$\begin{pmatrix} 1 & 0.41 & 0.00 & 0.17 \\ 0.41 & 1 & 0.08 & 0.10 \\ 0.00 & 0.08 & 1 & 0.23 \\ 0.17 & 0.10 & 0.23 & 1 \end{pmatrix}, \quad (5.11)$$

which exhibits some differences compared to the residual correlation matrix in the German case.<sup>38</sup> It might be expected that the impulse responses also show a different picture.

*(a) Recursive Identification.* Using the recursive identification scheme, the price puzzle can also be observed in the Euro Area, see column (a) in figure 7. Additionally, it can be seen that innovations in the money stock lead to future increases in the price level, see column (a) in figure 6. The responses of the interest rate and the money stock on each other are not significant in large parts of the considered horizon, a liquidity effect cannot be observed.

*(b) Reserve-oriented Monetary Policy Model.* The estimates for  $A$  and  $B$  are given in table 4. Despite the fact that by far not all of the Euro area countries have followed a reserve-oriented monetary policy, the impulse responses for this case in column (b) of figures 6 and 7 give a more plausible picture than those implied by the recursive identification scheme. However, there is still a small positive impact of an increase in the interest rate on the price level after six quarters.

<sup>38</sup> It is again assumed that the residual correlation matrix is not an identity matrix. This assumption is in the following supported by significant contemporaneous orthogonalized impulse responses for innovation-response combinations with two different variables.

The liquidity effect can clearly be seen in the MS shock 7% impulse response: A positive innovation in the money stock leads to a contemporaneous decrease in the interest rate. This effect is (over)compensated after some quarters by the income and/or Fisher effect that have been described earlier. The response of the price level on a money supply shock is also as expected: The price level increases with a considerable time lag after a positive innovation in the money stock has occurred.

(c) *Two-pillar Monetary Policy Rule Model.* As discussed in section 4.3, this model can be interpreted as an approximation to the monetary policy strategy of the ECB. Like it is the case in the analysis for Germany, the impulse responses implied by this identification scheme are very similar to those calculated with the reserve-oriented monetary policy identification scheme. A contractive monetary policy shock expressed by a positive innovation in the interest rate has a negative impact on the price level after nine to twelve quarters, a small and transitory negative effect on real output, and a negative impact on the money stock. The price puzzle cannot be observed. However, the estimated monetary policy rule is not convincing because the coefficient of prices has the wrong sign, but is not significant, see table 4. This can possibly be explained by the fact that price stability in the euro area has not had the same importance in the sample period as it has nowadays. The positive contemporaneous effect of a positive innovation in the money stock on the interest rate is still puzzling.

(d) *Leeper et al. (1996) Identification Scheme.* For the Euro area, the Leeper et al. (1996) identification scheme seems to be not appropriate. The price puzzle occurs, and the response of the price level on a monetary policy shock, represented by column (d) of figure 6, is not plausible: an expansive monetary policy shock that increases the money stock and decreases the interest rate has a negative impact on the future price level.

### 5.3.2 Calculation of Impulse Responses from the VECM Representation

*VECM Specification.* The stochastic properties of the considered Euro area time series are different from the corresponding German time series. While nominal money is an I(1) variable in Germany, the Euro area money stock M3 is an I(2) variable.<sup>39</sup> The vector error correction model in the I(2) case can be written as:

$$\Delta^2 x_t = \mu_t + \Pi x_{t-1} + \Gamma \Delta x_{t-1} + \sum_{i=1}^2 \Psi_i \Delta^2 x_{t-i} + u_t. \quad (5.12)$$

$\mu_t$  contains again the deterministic terms. The exact specification of the deterministic terms is determined together with the cointegration indices  $r$  (number of I(0) relations) and  $s$  (number of I(1) relations) using the test procedure described in Holtemöller (2001b). The values of the test statistic  $S_{r,s}$  can be found in table 1. Following the arguments of Juselius (1998), the cointegration indices are not only determined on the basis of the test results but also on the basis of economic plausibility. At a significance level of 5%, the test procedure stops at model 2,  $r = 2$ , and  $s = 0$  implying  $p - r - s = 2$  stochastic I(2) trends. The difference between  $S_{2,0} = 35.65$  and the corresponding 5% critical value (36.12) is very small. The next test statistic in the test procedure is  $S_{2,1} = 25.00$  with a corresponding critical value of 26.00. This scenario is much more plausible from an economic point of view. The I(2) variables in this system are  $m$  and  $p$ , and it is shown

<sup>39</sup> The order of integration of Euro area money and prices is analyzed in Holtemöller (2001b). The statistical analysis of I(2) variables is also explained there.

Table 1: Testing Hypotheses about the Cointegration Indices, Euro Area System

$r$	Model	$S_{r,s}$				$Q_r$
0	2	172.13	128.97	111.55	102.48	100.31
	3.1	172.13	125.83	105.89	93.93	87.93
	3.2	170.01	123.71	103.85	92.35	87.93
1	2		96.55	67.44	56.42	53.76
	3.1		101.30	64.60	49.28	42.69
	3.2		98.21	62.19	47.75	42.69
2	2			<b>35.65</b>	<b>25.00</b>	22.93
	3.1			39.63	26.58	18.38
	3.2			36.53	24.27	18.38
3	2				13.26	8.31
	3.1				12.94	8.30
	3.2				9.00	8.30
$p - r - s$		4	3	2	1	0

Notes: Test statistic  $S_{r,s}$  for testing hypotheses about the cointegration indices and the deterministic terms as described in Holtemöller (2001b). The column Model refers to the deterministic model, see Holtemöller (2001b) and Mosconi (1998, p. 27). Centered seasonal dummies are included in all cases. A hypothesis is not rejected if the test statistic is smaller than the corresponding critical value. These cases are indicated by bold face. The critical values can be found in the Holtemöller (2001b), for example.

in Holtemöller (2001b) that these two variables cointegrate to an I(1) relation. Therefore, the cointegration indices  $r = 2$  and  $s = 1$  are chosen. Now, the VECM (5.12) is estimated using the two-step reduced rank estimation procedure.<sup>40</sup> The residual correlation matrix is

$$\begin{pmatrix} 1 & 0.45 & 0.02 & 0.16 \\ 0.45 & 1 & 0.11 & 0.09 \\ 0.02 & 0.11 & 1 & 0.23 \\ 0.16 & 0.09 & 0.23 & 1 \end{pmatrix}, \quad (5.13)$$

which is very similar to the residual correlation in the unrestricted VAR model.

(a) *Recursive Identification.* The impulse responses in this case, see column (a) of figures 8 and 9 are almost the same as in the unrestricted VAR case. That is, the price puzzle can be observed, and a liquidity effect cannot be seen. The only difference is that the responses of some variables, for example the response of real income to an innovation in the price level, diverge. This is a consequence of the I(2) specification.

(b) *Reserve-oriented Monetary Policy Model.* The estimated matrices  $A$  and  $B$  exhibit only minor differences to their counterparts in the unrestricted VAR case, see table 5. Consequently, the impulse responses look also very similar, which can be seen by comparing figures 6/7 and 8/9.

<sup>40</sup> The two-step estimation procedure proposed by Johansen (1995) is explained in Holtemöller (2001b). The I(2) VECM is estimated using the RATS application Malcolm 2.2. The following structural analysis is again performed with Mathematica using the level representation of the VECM reported by Malcolm.



(c) *Two-pillar Monetary Policy Rule Model.* This model describes the monetary transmission process equally well as in the unrestricted VAR case, see column (c) of figures 8 and 9. In particular, there is no price puzzle, the price level responds negatively to positive innovations in the interest rate after about 6 quarters. Additionally, a liquidity effect can be observed in the sense that positive innovations in the interest rate lower the money stock. The sign of the price level coefficient in the two-pillar monetary policy rule is still not compatible with the theoretical expectation, see table 5.

(d) *Leeper et al. (1996) Identification Scheme.* For the Leeper et al. (1996) identification scheme apply the same remarks as in the unrestricted VAR case. The impulse responses are not as plausible as in the two-pillar monetary policy rule model.

#### 5.4 *Summary and Critique of Empirical Results*

One finding of the empirical investigation of monetary policy shocks and the responses of macroeconomic variables on these shocks is that the choice of the identification scheme has a considerable impact on the sign and the shape of the impulse responses. Therefore, it seems important to have a plausible economic interpretation of the applied identification scheme. The recursive identification scheme has not to be excluded a priori; there may be applications in which the recursiveness is a property of the “true” model. However, in many cases a model based identification scheme is presumably preferred. The reserve-oriented model for Germany and the stylized two-pillar monetary policy model for the Euro area seem to be appropriate identification schemes in the sense that they produce impulse responses that are compatible with widely accepted assumptions about the monetary transmission process.

However, it is not clear so far, how long-run cointegration analysis and short-run structural analysis influence each other. As noticed above, the impulse responses do not depend on the way the VAR is estimated; the unrestricted VAR and the VECM produce the same impulse responses, at least asymptotically and if the cointegration restrictions are correctly specified. This can well be seen in the Euro area investigation. The results of the impulse response analysis are very similar in the unrestricted VAR case and in the VECM case. On the other hand, Benkwitz et al. (2001) show that in empirical applications not only the width of confidence bands but also the impulse responses themselves can be different when they are estimated from the unrestricted VAR model and from the restricted VECM.

In practice, it may be helpful to specify the long-run cointegration relations first, and use this information for the identification of the contemporaneous relationships. A study proceeding in this way is the analysis of a German money demand system by Lütkepohl and Wolters (1999). These authors consider the variables real money, gross national product, inflation, and interest rate spread. First, they specify a money demand cointegration relation applying a single equation error correction approach; then, they include the resulting error-correction term and lagged differences of the endogenous variables into a structural model for the first differences. The structural model allows for contemporaneous relations between the variables and is estimated using three-stage least squares. An elimination procedure is applied to eliminate insignificant regressors. The estimated structural form is then used to calculate forecast error impulse responses. This procedure has the disadvantage that a large number of restrictions stems from a sequential statistical elimination process such that the economic implications imposed by these restrictions are not easy to see. Furthermore, the single equation estimation of the cointegrating vector is only valid if the cointegration rank is one. However, this procedure takes also care of the well known problem that the number of estimated parameters in a VAR model is relatively large. The larger the number of parameters, the larger are

the variances of the estimators and the confidence bands of the impulse response functions. Within the VAR framework this problem can alternatively be addressed in the following way: the number of parameters can be reduced by imposing (zero) restrictions on some of the coefficients in the matrices  $A_i$ .<sup>41</sup> A consequence of imposing restrictions on the VAR parameters is that the likelihood function for the estimation of the structural parameters in the matrices  $A$  and  $B$  cannot be calculated like it is done in section 4.1, see for example (Hamilton, 1994, p. 331 f.). The combination of subset VAR model and SVAR model may be an interesting topic for further research.

## 6 Conclusions

In this paper, the monetary policy conception in Germany from 1975 to 1998, the monetary policy strategy in the Euro area from 1999 up to now, and the respective implementation by the Bundesbank and the European Central Bank are explained. Both the Bundesbank and the Eurosystem are committed to maintain price stability. While the monetary policy of the Bundesbank from 1975 to 1998 can be described as monetary targeting, the Eurosystem has adopted a two-pillar strategy that focuses on the money stock as well as on other risks to price stability. The Eurosystem strategy should not be confused with monetary targeting. An interest rate rule seems more appropriate to describe the way how monetary policy is implemented in the Euro area.

A theoretical framework is developed that allows to incorporate the differences in monetary policies in empirical models of the monetary transmission process. This framework consists of an IS-LM-AS model extended by a regime specific monetary policy rule. It is shown how this framework can be combined with the SVAR methodology. Finally, the framework is applied to analyze monetary policy in Germany and in the Euro area empirically. For Germany a reserve-oriented monetary policy model, and for the Euro area, a stylized two-pillar monetary policy rule model work reasonably well. While the widely used recursive identification scheme (Cholesky decomposition) fails to produce impulse responses that are compatible with standard assumptions about the monetary transmission mechanism, the two model based identification schemes suggested here produce sensible impulse response sets. The main conclusion is therefore that the statistical VAR model should be augmented with economic structure in empirical applications. This can be done by cointegration analysis and by analyzing the contemporaneous relationships between the variables using the SVAR approach like illustrated here.

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<sup>41</sup> VAR models with restrictions on some of the coefficients are called subset VAR models, see for example Brüggemann and Lütkepohl (2001).

## A.1 German Data

*M3*: End of month money stock M3 (currency in use plus sight deposits of domestic non-banks at domestic banks in Germany plus time deposits for less than four years of domestic non-banks at domestic banks plus savings deposits at three months' notice of domestic non-banks at domestic banks in Germany) in billions of DM, seasonally unadjusted. Monthly data (TU0800) from the Compact Disc Deutsche Bundesbank (1998), continued with data from the monthly bulletin of the Deutsche Bundesbank, table II.2. 1975:01-1990:5 West Germany, and 1990:06-1998:12 Germany, not adjusted for German unification. Quarterly data are end of quarter stocks.

*Nominal GDP*: Gross domestic product in current prices, in billions of DM, seasonally unadjusted. Quarterly data (DQ0727) for 1975:01-1990:02, West Germany, from the Compact Disc Deutsche Bundesbank (1998), continued with data for 1990:03-1998:04, Germany, from Deutsches Institut für Wirtschaftsforschung (DIW) Berlin (from the DIW-statfinder: <http://www.diw-berlin.de>).

*Real GDP*: Gross domestic product in 1991 prices, in billions of DM, seasonally unadjusted. Quarterly data for 1975:01-1990:02, West Germany, from the Compact Disc Deutsche Bundesbank (1998) (DQ0728). Data for 1990:03-1998:04, Germany, is calculated as the ratio of nominal GDP and GDP deflator.

*GDP deflator*: Nominal GDP divided by real GDP, 1975:01-1990:02, quarterly data, West Germany, continued with GDP deflator, 1990:03-1998:04, quarterly data, Germany, from DIW Berlin (DIW-statfinder: <http://www.diw-berlin.de>).

*Long-term interest rate*: Yields on bonds outstanding issued by residents, monthly averages, fractions, monthly data (WU0017) from the Compact Disc Deutsche Bundesbank (1998), continued with data from the monthly bulletin, table VII.5. Quarterly data are the respective values of the last month in a quarter.

## A.2 Euro Area Data

The Euro area data was provided by Nuno Cassola. Further notes on the construction of the euro area aggregates (11 countries) before the introduction of the euro (back to 1980) can be found in Brand and Cassola (2000). The definition of the variables is as follows (adjusted stocks are constructed with flow statistics):

*M3*: Adjusted stock of the euro area monetary aggregate M3 in billions of euro. The quarterly data are averages of monthly data. The index of stocks (December 1998 = 100) is multiplied by the December 1998 stock of M3. The percentage change between any two dates (after October 1997) corresponds to the change in the aggregate excluding the effect of reclassifications etc. M3 comprises M1 and in addition, deposits with agreed maturity up to two years, deposits redeemable at notice up to three months (M2) and marketable instruments issued by the MFI sector (repurchase agreements, money market fund shares, money market paper and debt securities up to two years).

*Real GDP ( $Y^r$ )*: National series on real gross domestic product at market prices are added after they have been rebased to a common base year (1995) and converted to euro via the irrevocable fixed conversion rates of 31 December 1998. Adjusted for German unification. Billions of euro.

*Nominal GDP (1995)*: National series on real domestic product at market prices are added after they have been converted to euro via the irrevocable fixed conversion rates of 31 December 1998. Adjusted for German unification. Billions of euro. ESA95 data to the widest extent possible.

*GDP Deflator (GDPP)*: Nominal GDP divided by real GDP.

*Long-term interest rate*: Weighted average of euro-11 annual long-term interest rates. Weights: GDP weights at purchasing power parity exchange rates in 1995. National series are from the Bank for International Settlements, 10-year government bond yield or closest substitute. Fractions.

## *Appendix B. Numerical Maximization of the SVAR Log-Likelihood Function*

The method described in this section is a method of the Marquardt type, that is, it is an extension of the method of scoring in order to reduce numerical problems, especially in the case of a near singular information matrix. The method belongs to the class of ordinary gradient methods and can be characterized by the iteration algorithm<sup>42</sup>

$$\theta'^{n+1} = \theta'^n + h^n D^n, \quad (2.1)$$

where the variables that are freely estimated are collected in the row vector  $\theta'$ , that is in the actual case  $\theta' = [\gamma'_a \ \gamma'_b]$ . The positive scalar  $h^n$  is the step size of each iteration step, and  $D^n$  is the direction vector which is defined as the product of a weighting matrix  $W^n$  and the gradient of the objective function in row form, evaluated at  $\theta^n$ :

$$D^n = W^n \mathcal{L}_{\theta'^n}, \quad \mathcal{L}_{\theta'^n} = \left( \frac{\partial \mathcal{L}}{\partial \theta} \Big|_{\theta^n} \right)'. \quad (2.2)$$

While the Newton-Raphson method, that is derived from a second-order Taylor series expansion around  $\theta^n$ , uses the negative inverse of the Hessian

$$H = \frac{\partial^2 \mathcal{L}}{\partial \theta \partial \theta'} \quad (2.3)$$

as weighting matrix, the method of scoring replaces the Hessian with the negative of the information matrix

$$I[\theta] = -E[H], \quad (2.4)$$

such that

$$\theta'^{n+1} = \theta'^n - (-I[\theta'^n])^{-1} \mathcal{L}_{\theta'^n}. \quad (2.5)$$

The above mentioned extension is captured by

$$\theta'^{n+1} = \theta'^n - (-I[\theta'^n] - \alpha I_{\ell_a + \ell_b})^{-1} \mathcal{L}_{\theta'^n}. \quad (2.6)$$

The parameter  $\alpha$  is determined by

$$\alpha = \lambda_1 + R \|\mathcal{L}_{\theta'^n}\|, \quad (2.7)$$

where  $\lambda_1$  is the largest eigenvalue of  $H$ , and  $R$  is chosen individually depending on the goodness of the quadratic approximation of the objective function that has been used above in the Taylor

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<sup>42</sup> This follows Goldfeld and Quandt (1972) and Judge et al. (1988).

series expansion (it is increasing if the approximation is poor). The iterating rule is now (2.6) if  $\alpha > 0$  and (2.5) if  $\alpha \leq 0$ . The iteration algorithm stops if usual convergence criteria are achieved (absolute/relative increase in the log-likelihood or absolute/relative change in  $\theta$  are below a certain bound or a total number of iterations is reached).

The gradient vector and the information matrix are given in Amisano and Giannini (1997). The calculations in section 5 are done using a Mathematica implementation of this algorithm. A version of this paper with more technical details is available on the author's homepage.

## Appendix C. SVAR Estimation Results

Table 2: SVAR Results, Germany, VAR Model (5.7)

Reserve-oriented Monetary Policy Model (4.15)								
Number of iteration steps is 36. The maximized log-likelihood is 1293.76								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
IS	2.26 (0.96)	1 –	2.26 (0.96)	0 –	0.02 (0.00)	0 –	0 –	0 –
AS	–9.36 (4.48)	1 –	0 –	0 –	0 –	0.04 (0.02)	0 –	0 –
MD	–1 –	–2.92 (3.69)	11.58 (16.05)	1 –	0 –	0 –	0.07 (0.09)	0 –
MS	0 –	0 –	–0.26 (0.25)	1 –	0 –	0 –	0 –	0.01 (0.00)
Two-pillar Monetary Policy Rule Model (4.19)								
Number of iteration steps is 34. The maximized log-likelihood is 1296.06								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
AS	1 –	–0.94 (0.05)	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
IS	2.04 (4.85)	1 –	2.04 (0.85)	0 –	0 –	0.02 (0.00)	0 –	0 –
MP	1.33 (1.16)	0 –	1 –	–3.06 (2.41)	0 –	0 –	0.02 (0.02)	0 –
MD	–1 –	–2.97 (4.29)	12.25 (18.88)	1 –	0 –	0 –	0 –	0.07 (0.11)
Leeper et al. (1996) Identification Scheme (4.20)								
Number of iteration steps is 15. The maximized log-likelihood is 1297.14								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
P <sub>1</sub>	1 –	0 –	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
P <sub>2</sub>	0.30 (0.37)	1 –	0 –	0 –	0 –	0.02 (0.00)	0 –	0 –
MD	0.43 (0.18)	–0.08 (0.05)	1 –	–0.52 (0.28)	0 –	0 –	0.01 (0.00)	0 –
MP	0 –	0 –	1.29 (0.86)	1 –	0 –	0 –	0 –	0.01 (0.00)

Notes: SVAR estimation algorithm as described in Appendix B. The algorithm stops if the relative change of the log-likelihood is smaller than 0.00001. Standard errors in brackets.

Table 3: SVAR Results, Germany, VECM (5.9)

Reserve-oriented Monetary Policy Model (4.15)								
Number of iteration steps is 36. The maximized log-likelihood is 1337.43								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
IS	1.67 (0.81)	1 –	1.67 (0.81)	0 –	0.02 (0.00)	0 –	0 –	0 –
AS	–11.54 (6.88)	1 –	0 –	0 –	0 –	0.05 (0.02)	0 –	0 –
MD	–1 –	–2.63 (3.54)	11.30 (17.03)	1 –	0 –	0 –	0.06 (0.09)	0 –
MS	0 –	0 –	–0.20 (0.27)	1 –	0 –	0 –	0 –	0.01 (0.00)
Two-pillar Monetary Policy Rule Model (4.19)								
Number of iteration steps is 48. The maximized log-likelihood is 1337.27								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
AS	1 –	–7.57 (27.07)	0 –	0 –	0.10 (0.35)	0 –	0 –	0 –
IS	20.16 (22.38)	1 –	20.16 (22.38)	0 –	0 –	0.11 (0.13)	0 –	0 –
MP	–1.62 (0.46)	0 –	1 –	–0.51 (0.19)	0 –	0 –	0.01 (0.00)	0 –
MD	–1 –	0.01 (0.07)	0.27 (0.22)	1 –	0 –	0 –	0 –	0.01 (0.00)
Leeper et al. (1996) Identification Scheme (4.20)								
Number of iteration steps is 15. The maximized log-likelihood is 1339.94								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
P <sub>1</sub>	1 –	0 –	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
P <sub>2</sub>	0.33 (0.37)	1 –	0 –	0 –	0 –	0.01 (0.00)	0 –	0 –
MD	0.37 (0.23)	–0.07 (0.06)	1 –	–0.67 (0.51)	0 –	0 –	0.01 (0.00)	0 –
MP	0 –	0 –	1.57 (1.31)	1 –	0 –	0 –	0 –	0.01 (0.00)

Notes: SVAR estimation algorithm as described in Appendix B. The algorithm stops if the relative change of the log-likelihood is smaller than 0.00001. Standard errors in brackets.

Table 4: SVAR Results, Euro Area, VAR Model (5.7)

Reserve-oriented Monetary Policy Model (4.15)								
Number of iteration steps is 17. The maximized log-likelihood is 1257.51								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
IS	2.23 (1.42)	1 –	2.23 (1.42)	0 –	0.01 (0.01)	0 –	0 –	0 –
AS	–3.43 (0.82)	1 –	0 –	0 –	0 –	0.01 (0.00)	0 –	0 –
MD	–1 –	–0.61 (0.38)	2.84 (1.83)	1 –	0 –	0 –	0.01 (0.01)	0 –
MP	0 –	0 –	–0.58 (0.21)	1 –	0 –	0 –	0 –	0.00 (0.00)
Two-pillar Monetary Policy Rule Model (4.19)								
Number of iteration steps is 19. The maximized log-likelihood is 1257.89								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
AS	1 –	–0.25 (0.10)	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
IS	1.51 (1.62)	1 –	1.51 (1.62)	0 –	0 –	0.01 (0.01)	0 –	0 –
MP	0.31 (0.60)	0 –	1 –	–2.07 (1.24)	0 –	0 –	0.01 (0.00)	0 –
MD	–1 –	–0.86 (1.01)	4.69 (6.99)	1 –	0 –	0 –	0 –	0.02 (0.02)
Leeper et al. (1996) Identification Scheme (4.20)								
Number of iteration steps is 16. The maximized log-likelihood is 1257.60								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
P <sub>1</sub>	1 –	0 –	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
P <sub>2</sub>	–1.21 (0.31)	1 –	0 –	0 –	0 –	0.01 (0.00)	0 –	0 –
MD	–0.46 (1.06)	–0.08 (0.23)	1 –	2.45 (6.20)	0 –	0 –	0.01 (0.02)	0 –
MP	0 –	0 –	–2.18 (3.02)	1 –	0 –	0 –	0 –	0.01 (0.01)

Notes: SVAR estimation algorithm as described in Appendix B. The algorithm stops if the relative change of the log-likelihood is smaller than 0.00001. Standard errors in brackets.



Table 5: SVAR Results, Euro Area, VECM (5.12)

Reserve-oriented Monetary Policy Model (4.15)								
Number of iteration steps is 18. The maximized log-likelihood is 1298.53								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
IS	2.43 (1.72)	1 –	2.43 (1.72)	0 –	0.01 (0.01)	0 –	0 –	0 –
AS	–3.72 (0.87)	1 –	0 –	0 –	0 –	0.01 (0.00)	0 –	0 –
MD	–1 –	–0.64 (0.43)	3.28 (2.29)	1 –	0 –	0 –	0.01 (0.01)	0 –
MS	0 –	0 –	–0.54 (0.20)	1 –	0 –	0 –	0 –	0.00 (0.00)

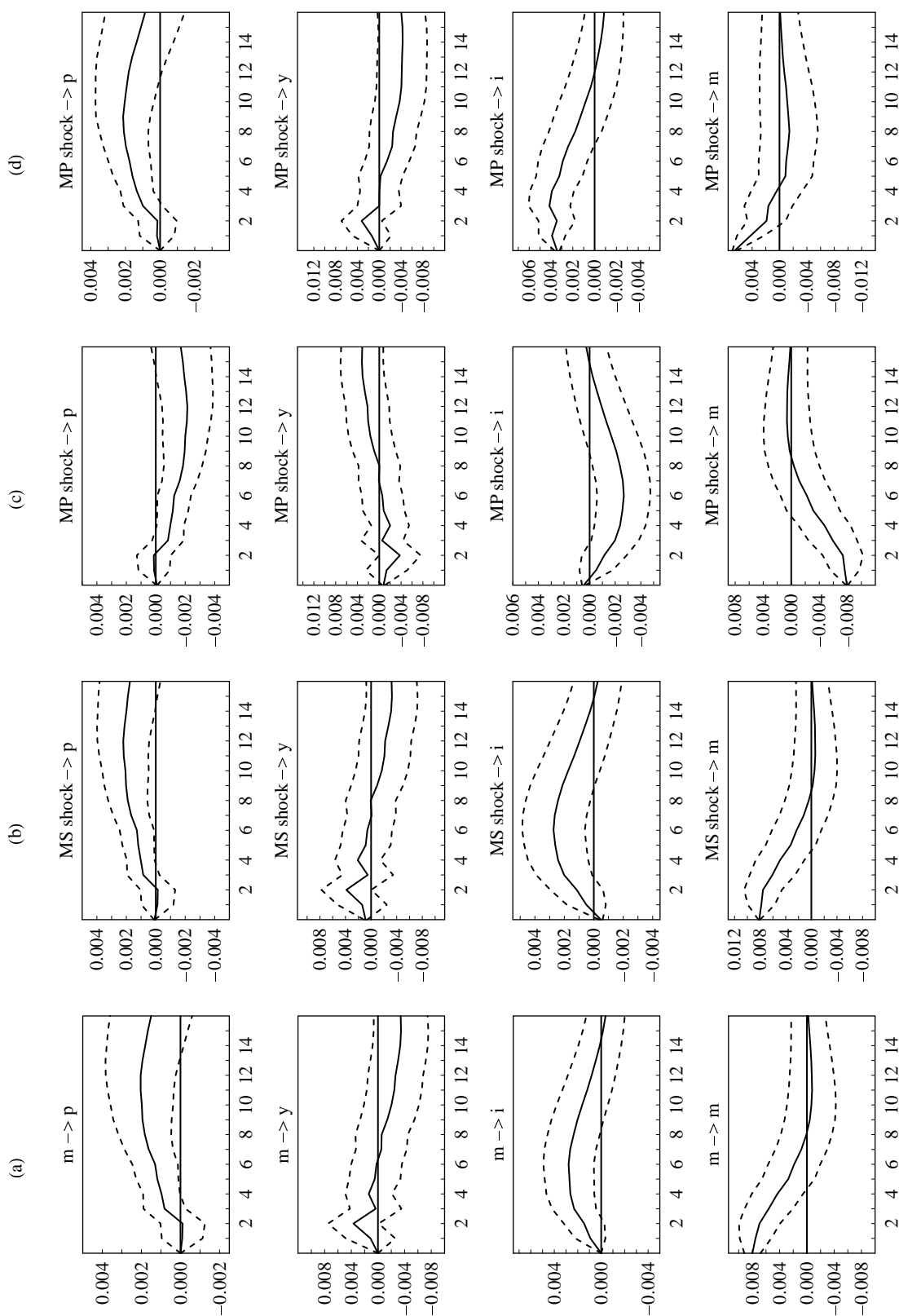
Two-pillar Monetary Policy Rule Model (4.19)								
Number of iteration steps is 20. The maximized log-likelihood is 1298.93								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
AS	1 –	–0.23 (0.09)	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
IS	1.62 (1.93)	1 –	1.62 (1.93)	0 –	0 –	0.01 (0.01)	0 –	0 –
MP	0.30 (0.62)	0 –	1 –	–2.19 (1.38)	0 –	0 –	0.01 (0.00)	0 –
MD	–1 –	–0.88 (1.13)	5.24 (8.43)	1 –	0 –	0 –	0 –	0.02 (0.02)

Leeper et al. (1996) Identification Scheme (4.20)								
Number of iteration steps is 14. The maximized log-likelihood is 1298.60								
	<i>A</i>				<i>B</i>			
	<i>p</i>	<i>y</i>	<i>i</i>	<i>m</i>	<i>e</i> <sub>1</sub>	<i>e</i> <sub>2</sub>	<i>e</i> <sub>3</sub>	<i>e</i> <sub>4</sub>
P <sub>1</sub>	1 –	0 –	0 –	0 –	0.00 (0.00)	0 –	0 –	0 –
P <sub>2</sub>	–1.43 (0.33)	1 –	0 –	0 –	0 –	0.01 (0.00)	0 –	0 –
MD	–0.32 (0.60)	–0.06 (0.14)	1 –	1.78 (3.75)	0 –	0 –	0.01 (0.01)	0 –
MP	0 –	0 –	–1.82 (2.32)	1 –	0 –	0 –	0 –	0.01 (0.00)

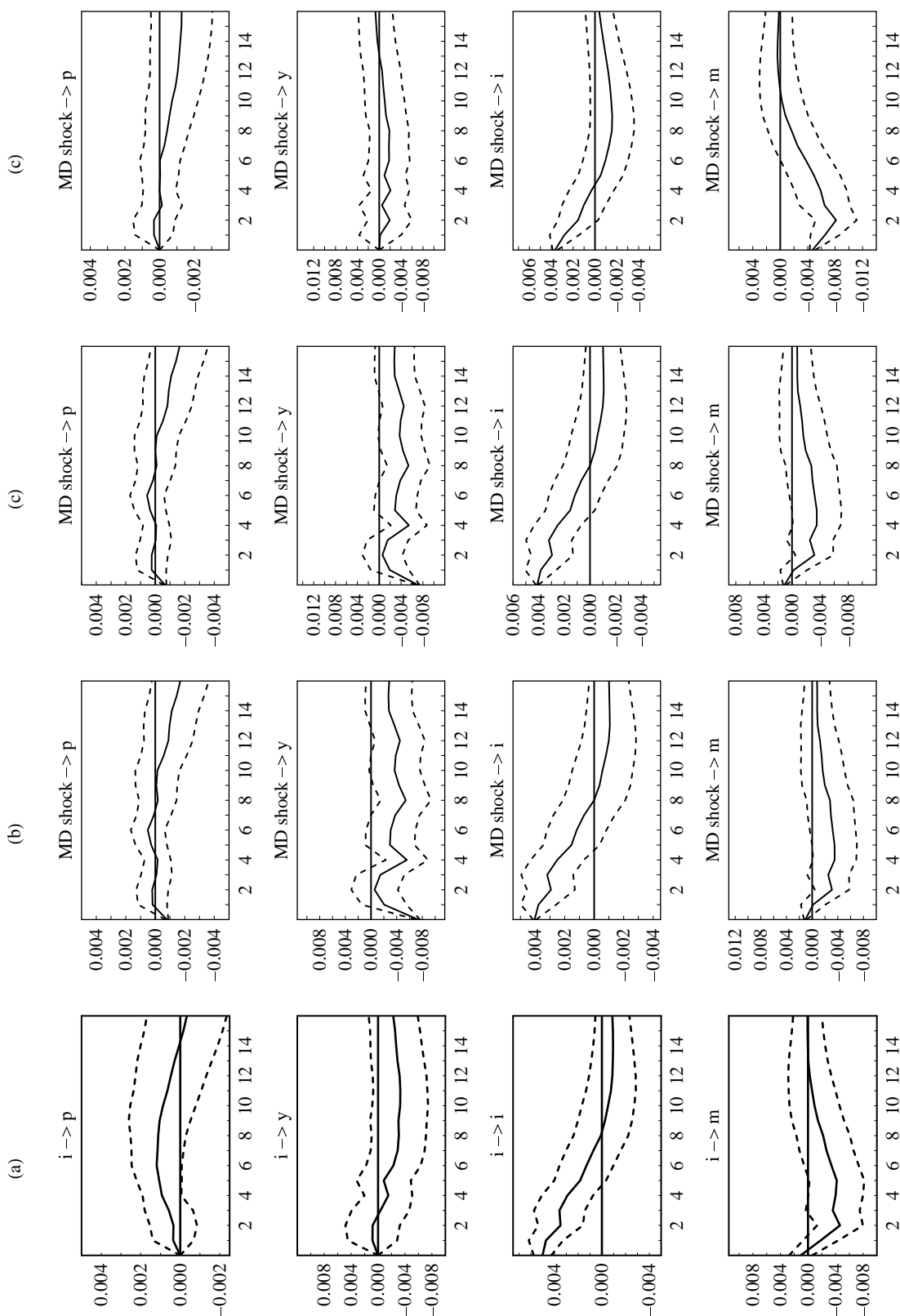
Notes: SVAR estimation algorithm as described in Appendix B. The algorithm stops if the relative change of the log-likelihood is smaller than 0.00001. Standard errors in brackets.

Figure 2: Impulse Responses, Germany, VAR Model (5.7)



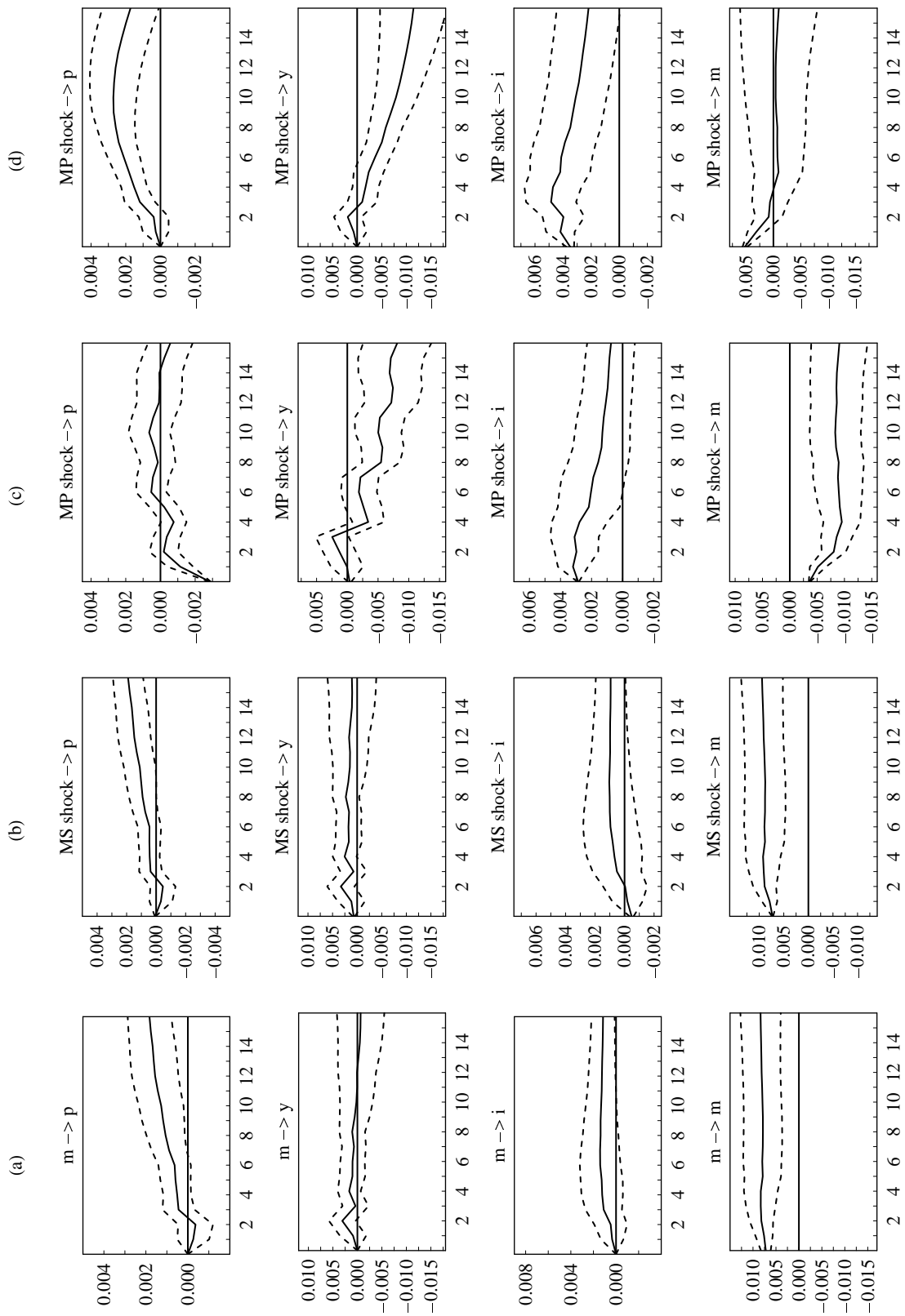
Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

Figure 3: Impulse Responses, Germany, VAR Model (5.7)



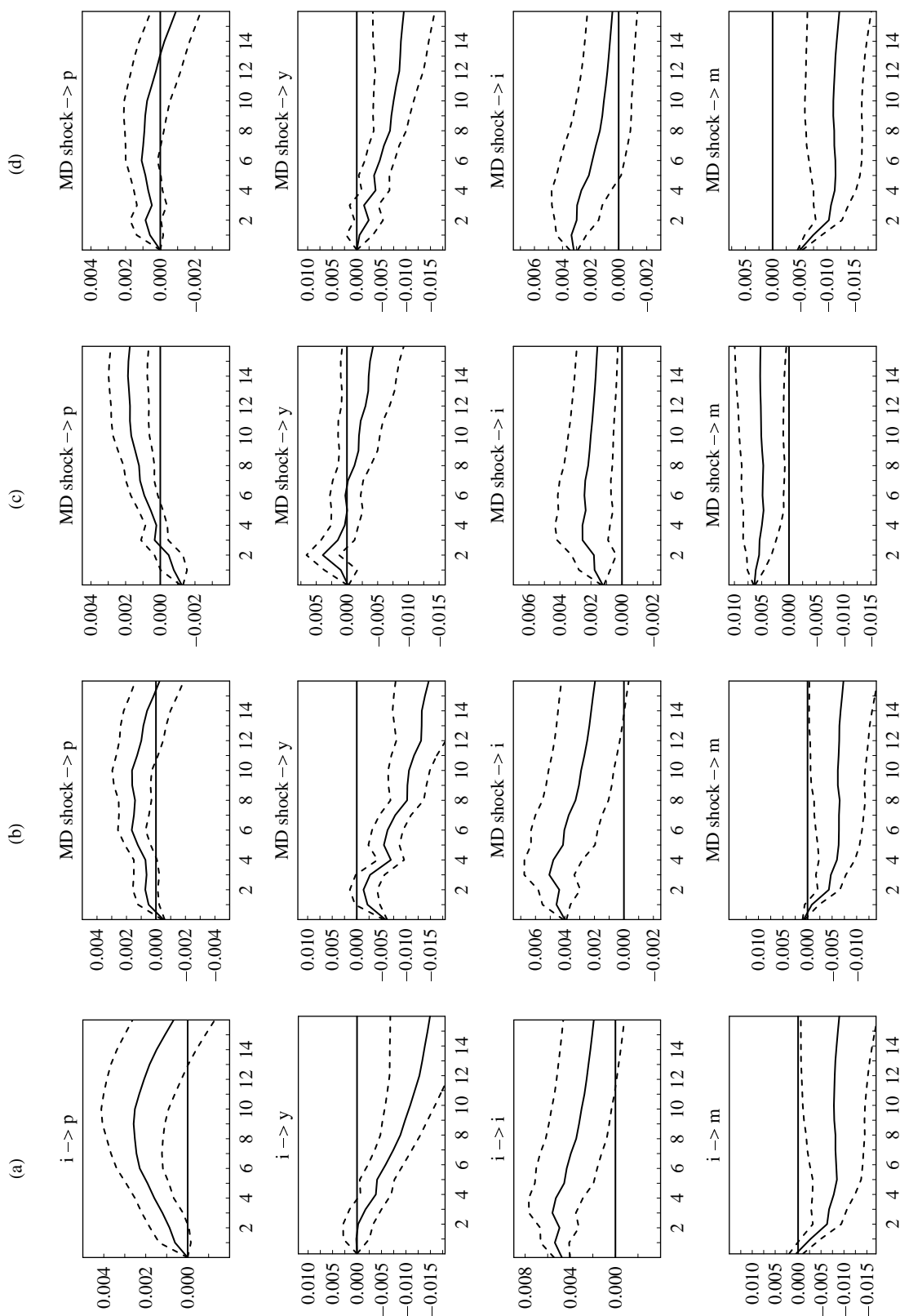
Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

Figure 4: Impulse Responses, Germany, VECM (5.9)



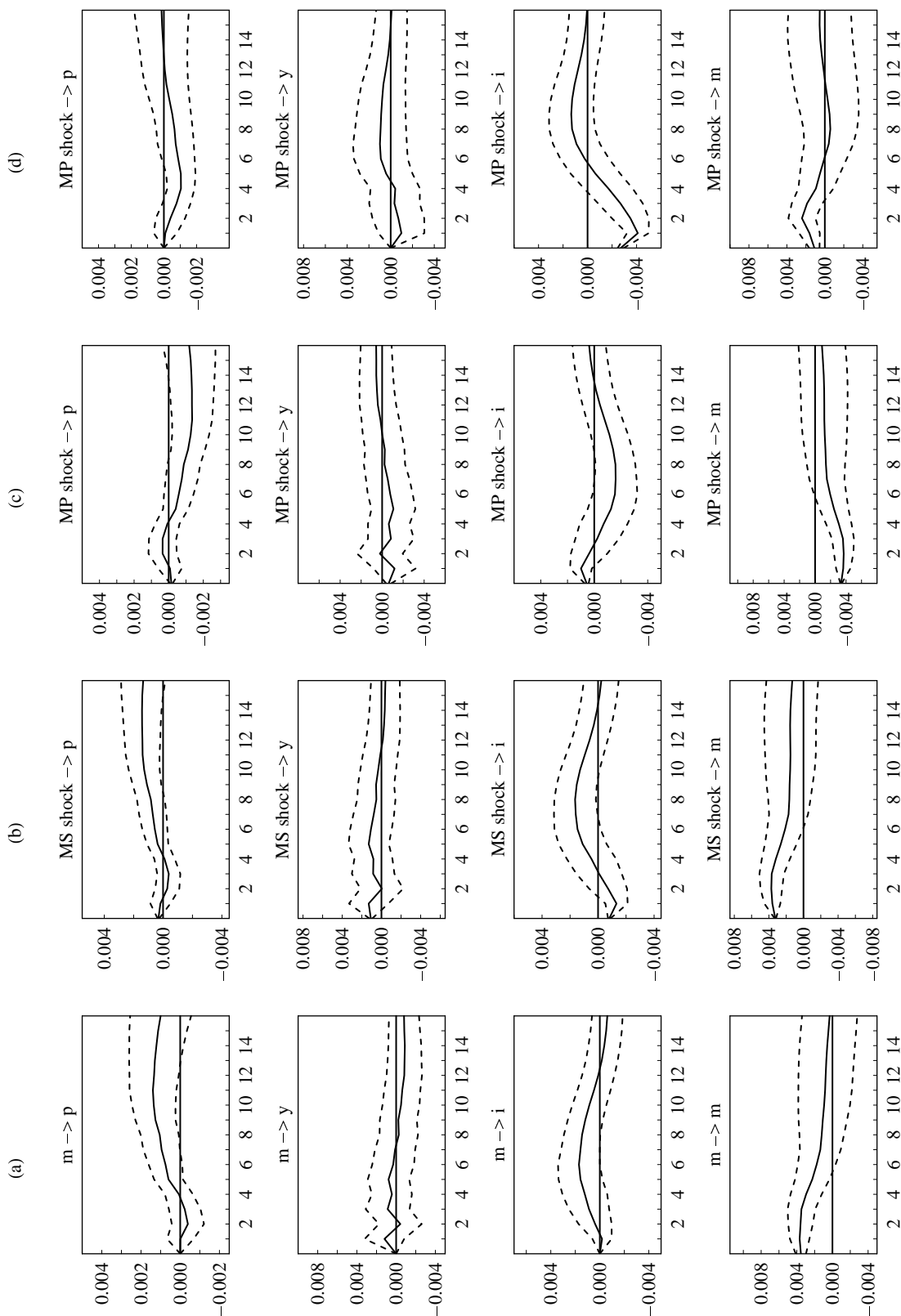
Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

Figure 5: Impulse Responses, Germany, VECM Model (5.9)



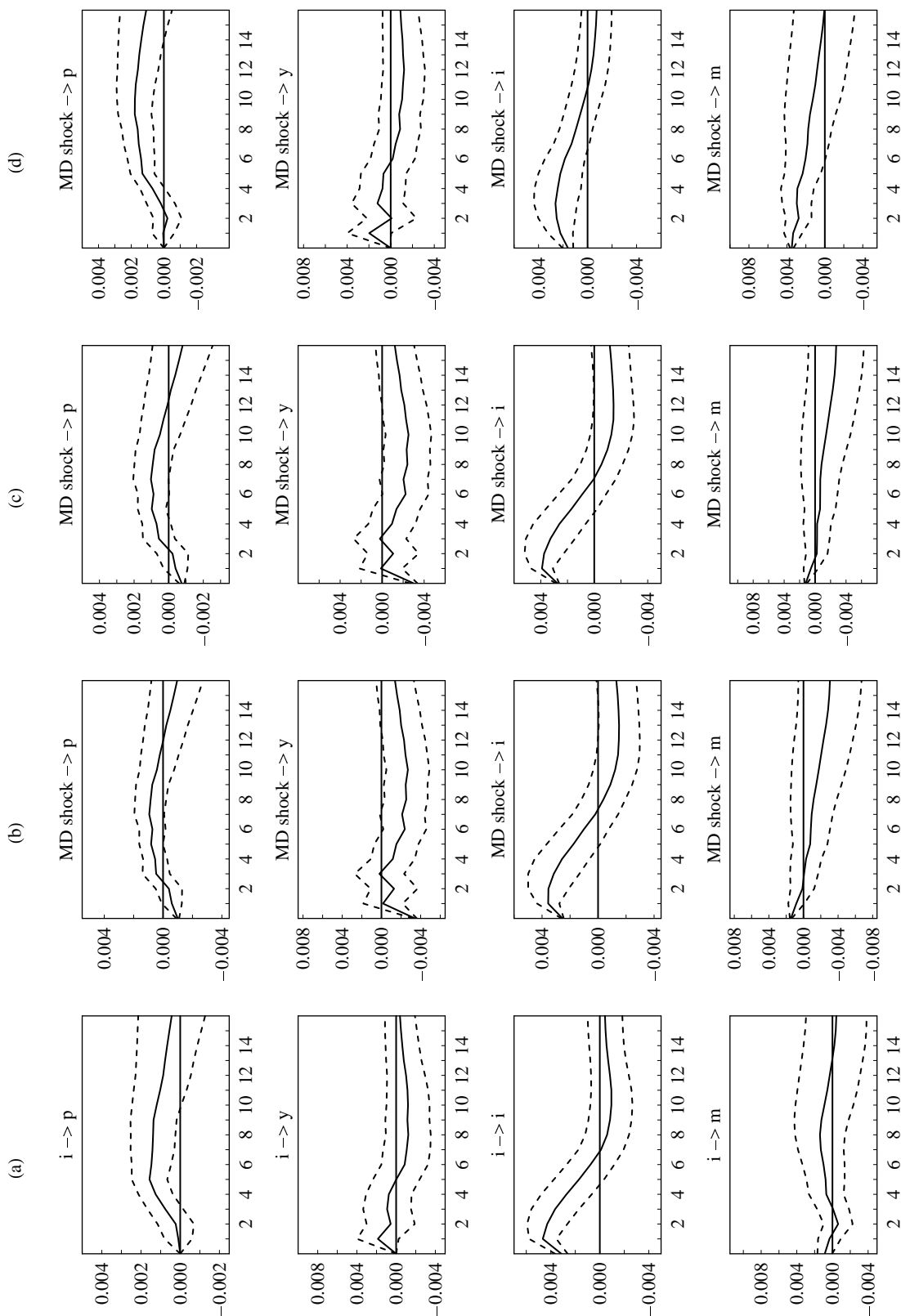
Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

Figure 6: Impulse Responses, Euro Area, VAR Model (5.7)



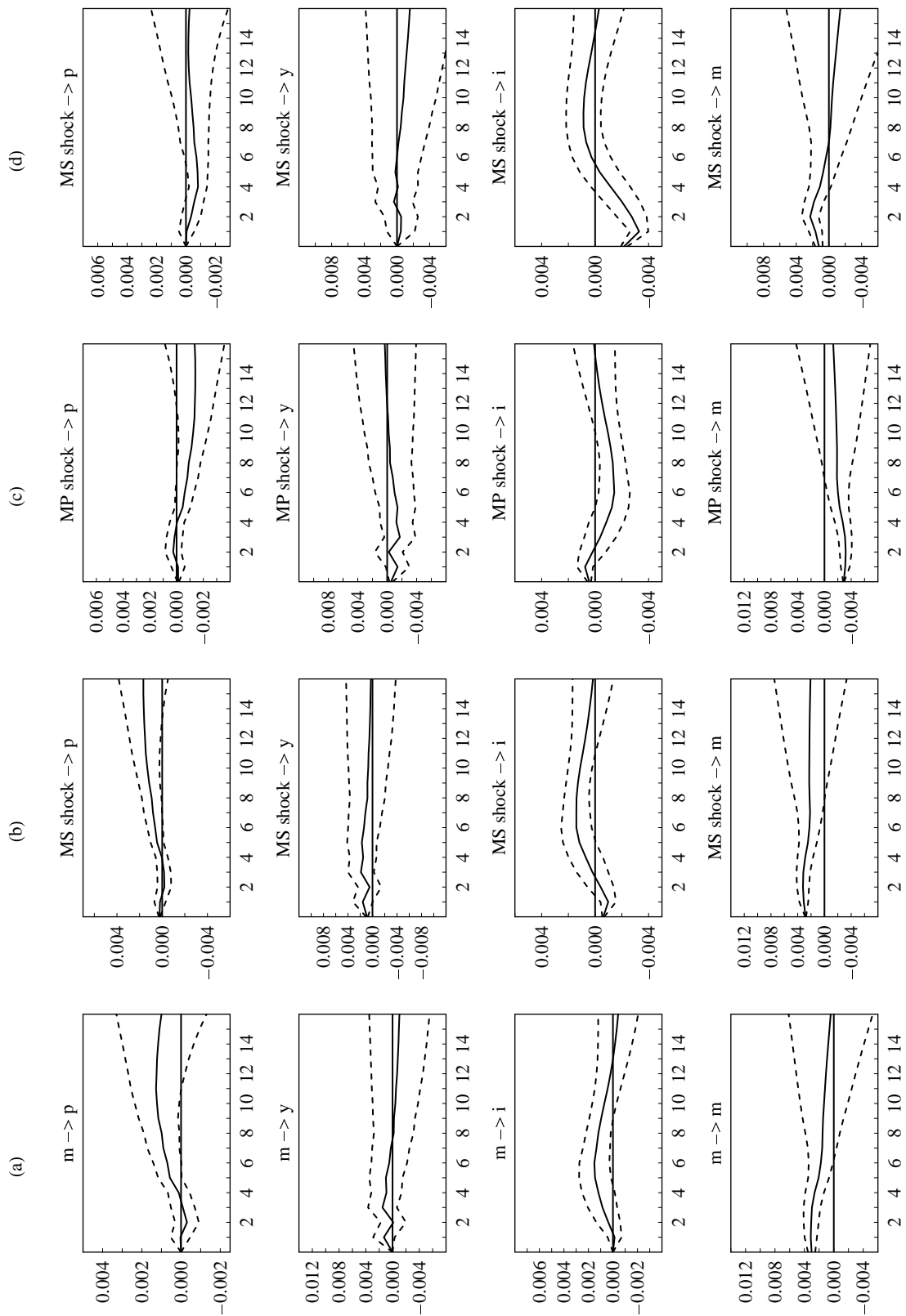
Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

Figure 7: Impulse Responses, Euro Area, VAR Model (5.7)



Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

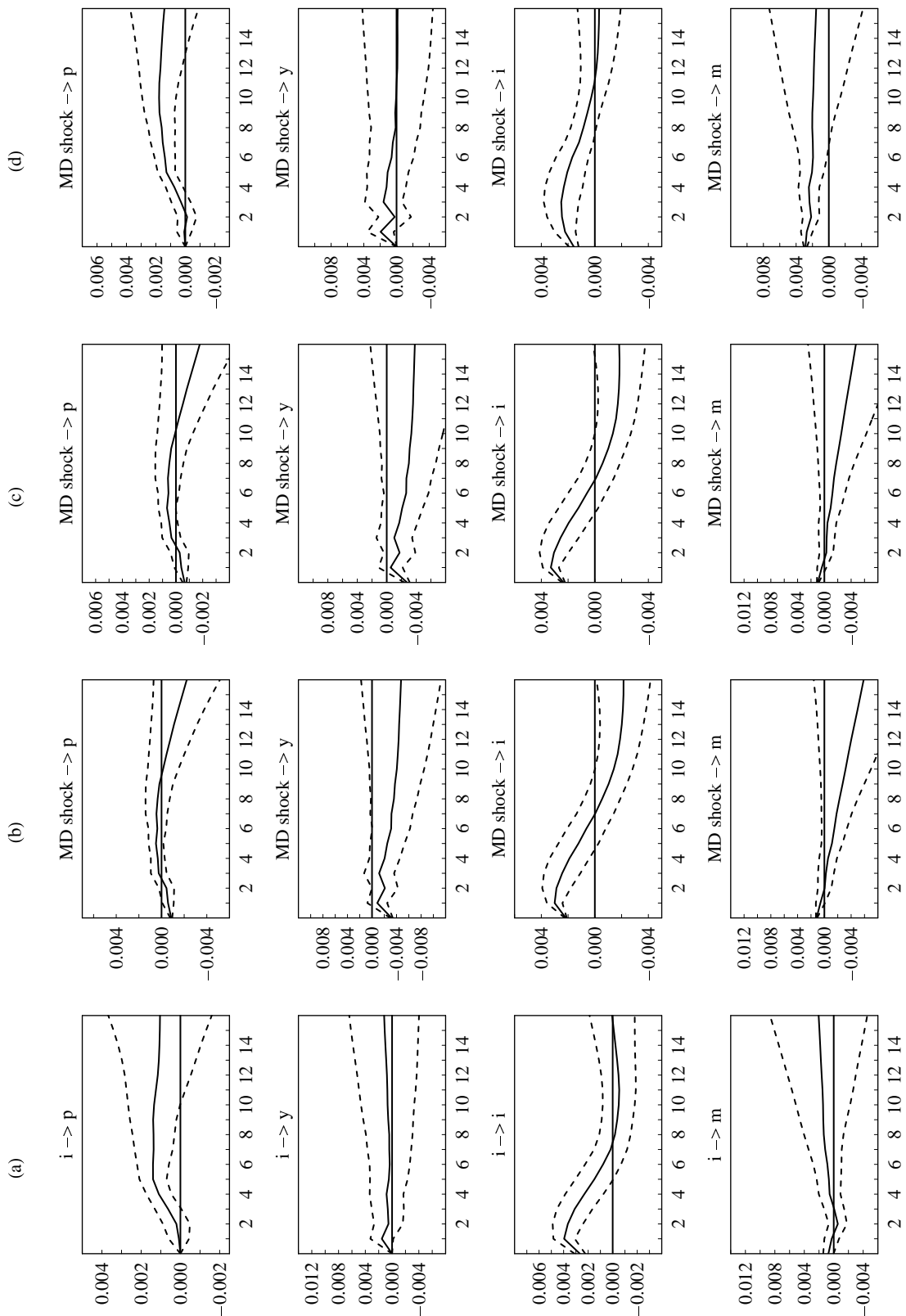
Figure 8: Impulse Responses, Euro Area, VECM (5.12)



Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).



Figure 9: Impulse Responses, Euro Area, VAR Model (5.12)



Notes: Identification schemes: (a) recursive ( $p, y, i, m$ ), (b) reserve-oriented model, (c) two-pillar monetary policy rule model, (d) Leeper et al. (1996).

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