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DEPARTMENT OF ECONOMICS

Working Paper

Imposing a balance of payment constraint on the Kaldorian model of cumulative causation

By

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Working Paper 2011-28



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Imposing a balance of payment constraint on the Kaldorian model of cumulative causation

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Abstract

We combine two strands of Post Keynesian growth theory by imposing a balance of payments constraint on a Kaldorian cumulative causation model. The effects of external and internal shocks, and the degree to which cumulative causation comes into play depends on the exchange rate and capital account regimes. Exports act as the only exogenous drivers of growth only under a regime of fixed exchange rates and in the absence of relative price effects. Under flexible exchange rates, by contrast, it is internal demand that serves as the only exogenous driver of of growth. Moreover, regardless of the type of shock, the presence of cumulative causation does not boost growth, although it may render growth more sustainable.

JEL classification: E12, F43, F32, O40

Key words: Cumulative causation, balance of payments constraint, export-led growth, capital account openness, exchange rate regime.

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1 Introduction

The Post Keynesian tradition has generated two families of models that emphasize exports as a source of growth: the Balance of Payments-Constrained growth (BPCG) model, as originally enunciated by Thirlwall (1979), and the model of cumulative causation (CC) along Kaldorian lines as presented, for example, by Setterfield and Cornwall (2002). Demand-side constraints on growth underpin both these models. The CC model builds on Verdoorn's idea that aggregate labor productivity is a positive function of the size of the economy (or, more specifically, of the manufacturing sector). This view, which carries obvious reverberations of the Smithian notion of productivity depending on the extent of the market, can be justified by the presence of economies of scale, learning, or other externalities. An exogenous increase in an autonomous source of demand growth, say exports, leads via increased demand to higher output growth, which then, results in accelerated productivity growth. Greater productivity, in turn, translates into lower labor costs and, in an imperfect substitutes framework where relative price changes affect exports, increased competitiveness, setting into motion a virtuous cumulative process yielding higher output growth. Conversely, a negative shock to aggregate demand sets into motion a mirror image vicious cycle of falling demand, output, and productivity.

The real world, of course, is marked by the relative rarity of sustained vicious or virtuous cycles. What inherent mechanism constrains a cycle once it is underway? Setterfield and Cornwall (2002) provide one possible answer; the existence of a unique solution to the steady state CC growth rate requires that the cumulative causation not be too large. A sufficiently high cumulative causation effect will, for intuitively obvious reasons, result in a growth rate that either explodes to infinity (during a virtuous cycle) or collapses to zero (during a vicious one).

A more concrete factor constraining cycles is likely to be the external account. The CC model in its canonical form does not incorporate the increase in imports that is likely to accompany export and output growth. What if imports grow consistently faster than exports? By neglecting this consideration, the CC framework ignores important differences between the effects on growth of external versus internal shocks, and leaves one unable to explore one of Kaldor's key ideas, i.e., that exports are the only truly exogenous drivers of growth.

The BPCG model, by contrast, privileges the balance of payments constraint on growth. Indeed, in his paper that originally presented the BPCG model, Thirlwall explicitly stresses the role of the external account in limiting the duration of cycles. In Thirlwall's words:

If a country gets into balance of payments difficulties as it expands demand, before the short term capacity growth rate is reached, then demand must be curtailed; supply is never fully utilised; investment is discouraged; technological progress is slowed down, and a country's goods compared to foreign goods become less desirable

so worsening the balance of payments still further, and so on. A vicious circle is started. By contrast, if a country is able to expand demand up to the level of existing productive capacity, without balance of payments difficulties arising, the pressure of demand upon capacity may well raise the capacity growth rate (Thirlwall, 1979)[p. 46].

The BPCG model, however, does not articulate a specific mechanism through which the capacity growth rate may be lifted. More importantly from our perspective, modelers in this tradition have downplayed the role of relative price changes, which, in turn, neutralizes the channel through which the Kaldorian/Verdoorn cumulative causation is supposed to influence growth. In addition, the lack of relative price effects also implies a small country assumption that is inconsistent with the demand-side focus of the model.¹

The balance of payments consists of the current account, the private capital (or financial) account, and official reserve transactions. The CC framework neglects not only the trade balance (or more broadly, the current account), but also the capital account. The BPCG model, on the other hand, originally neglected the capital account, although later contributions have partially filled this gap. The presence of capital flows can loosen or tighten the external account constraint over time. Studies such as Thirlwall and Hussain (1982) have addressed the issue by assuming an exogenous growth rate of capital flows. assumption essentially imposes a current account balance restriction. Alternatively, one could assume that net capital flows elastically accommodate current account imbalances. Given that the overall balance of payments, ignoring official reserve transactions, has to be zero by definition, however, means that accommodating capital flows render the idea of an external constraint meaning-The more interesting (and plausible) case in a world with large international capital flows is one where the capital account is considered to be at least partially endogenous, i.e., capital movements respond to changes in the economic environment under consideration. In this sense, most treatments of the BPCG constraint are incomplete insofar as explicit mechanisms for the overall balance of payments (and not just the current account balance) constraint to be satisfied are left out. It is useful to think about the possible adjustment mechanisms that give economic meaning to an accounting identity. International profit rate differences typically provide one such plausible mechanism.²

The complementarities noted above between the BPCG and CC frameworks suggest that the two families of models could be usefully combined to incorporate growth enhancing internal or external changes while recognizing the crucial

¹See Section 3.2.3 and Razmi (2010).

²Regarding the external constraint, yet another possibility is to assume that the current account is a constant proportion of GDP, perhaps because international investor sentiment keeps a lid on this ratio (see McCombie and Roberts (2002) for a discussion of this issue). This is similar in essence to the assumption of exogenously given capital flows, and considering that output responds to changes in both the domestic and external markets, one still needs to assign an independent mechanism to the latter adjustment when the actual trade balance to GDP ratio deviates from this ceiling.

role of the balance of payments constraint. This paper attempts such a synthesis. We use a standard specification to endogenize capital flows, in the process introducing distributional considerations. Furthermore, we endogenize and explicitly treat the evolution of other components of aggregate demand. This enables us to explore the effects of external versus internal stimuli under different exchange rate regimes and varying degrees of capital account openness and relative price effects. It also allows us to analyze the interactions between capital account openness, relative price elasticities, and cumulative causation. Most interestingly, perhaps, it allows us to investigate the conditions under which exports are the only truly exogenous source of growth.

We find that, with a fixed exchange rate, the efficacy of both internal and external stimuli depends on the extent of capital account openness. By ensuring that growth is accompanied by trade surpluses, strong cumulative causation effects promote the sustainability of growth. When capital accounts are completely closed (so that trade is balanced), again *both* internal and external real stimuli contribute to demand growth. It is only in the absence of relative price effects, that is, in the presence of elasticity pessimism of a specific kind that exports come into their own as the *only* exogenous source of growth.

The presence of a floating exchange rate, on the other hand, yields dramatically different outcomes. With the capital account tied down by the international profit rate differential, a real external shock has no effect on the growth rate, and cumulative causation does not come into play. A positive real internal shock, on the other hand, can accelerate growth. Thus, contrary to Kaldor's hypothesis, domestic demand now becomes the only truly exogenous driver of growth. These results are not underpinned by any assumed automatic adjustment of the exchange rate driven entirely by external imbalances. Rather, they follow from the fact that, unlike mark-up adjustments, any exchange rate change affects both the goods market and the balance of payments in equal measure.

The main conclusion traditionally derived from the BPCG model is the crucial role of relative income elasticities of demand for a country's exports and imports. Combining the BPCG and CC models allows us to identify another crucial variable, the presence of cumulative causation via the Verdoorn coefficient. Countries specializing in sectors that deliver strong cumulative causation may experience higher and more sustained growth. Since cumulative causation works mainly through competitiveness enhancing relative price effects, this puts the focus back on international *price* elasticities.

Blecker (2009) is perhaps the paper closest in spirit to our contribution. However, we focus on a different set of issues including capital account and exchange rate regimes. Moreover, we endogenize capital flows and the evolution of components of domestic spending. Capital flows are neither perfectly accommodative (to trade imbalances) nor, in the general case, an exogenously given constant proportion of exports or GDP. Instead these respond to profit signals and are determined in a macroeconomic equilibrium along with the path of output and capital growth. Finally, we explicitly incorporate accumulation and capacity growth.

The next section develops the model. Sections ?? and 4 explore the compar-

ative statics of internal and external shocks under fixed and floating exchange rates, respectively. Section 5 concludes.

2 Combining the CC and BPCG Models

Consider an economy where production is defined by a constant coefficients/Leontief production function.

$$Y = \min\{qL, K\} \tag{1}$$

where Y and K denote the levels of real output and capital stock, respectively, while q is the (constant) marginal product of labor. The economy produces an imperfect substitute for the foreign good. Given the medium run nature of our set-up, and in order to focus on the capacity growth rate, we assume that firms are able to maintain their "desired" level of capacity utilization. This, however, does not eliminate the possibility of labor unemployment. Using circumflexes to denote growth rates, the growth rate of output can be written

$$\hat{Y} = \hat{K} \tag{2}$$

Prices are set as a mark-up over unit labor costs. Using τ to denote the mark-up factor and W to denote the nominal wage,

$$P = \frac{\tau W}{q} \tag{3}$$

Assuming, in line with the traditional Kaleckian specification that the nominal wage grows at an exogenously given rate which, to simplify notation, we assume to equal zero,

$$\hat{P} = \hat{\tau} - \hat{q}$$

Next, we need to define some behavioral functions for the components of aggregate demand. Consumption C is the sum of consumption out of wages and profits. For simplicity, and in order to capture the stylized fact that owners of capital, on average, save a higher proportion of their income than workers, we make the classical/Kaleckian assumption that workers do not save.³ Thus,

$$C = \frac{WL}{P} + (1-s)\frac{R}{P}$$

Making use of equations (1) and (3),

$$C = \left\lceil \frac{\tau - s(\tau - 1)}{\tau} \right\rceil Y \tag{4}$$

Or, in growth rate form,

 $^{^3}$ Relaxing this assumption will not qualitatively change our results as long as workers have a lower marginal/average propensity to consume.

$$\hat{C} = \hat{Y} - S\hat{\tau} - S(\tau - 1)\hat{s} \tag{5}$$

where $0 < S = \frac{s}{\tau - s(\tau - 1)} < 1$. Thus, consumption growth is a positive function of output growth and a negative function of saving and mark-up growth, the latter, of course, owing to the assumption that only owners of capital save.

Investment is defined in a standard manner as a function of the profit rate r and domestic demand.

$$I = f(r, Y) = f\left(\frac{\tau - 1}{\tau}, Y\right); \ f_1, f_2 > 0$$
 (6)

Or, in growth rate form,

$$\hat{I} = i_{\tau}\hat{\tau} + i_{Y}\hat{Y} \tag{7}$$

where $i_{\tau}=f_1/f\tau$ and $0\leq i_Y=f_2Y/f\leq 1$ are assumed to be constant for simplicity.

Exports and imports are defined using standard imperfect substitutes specifications. The growth rate of exports \hat{X} depends positively on world income growth and relative price changes. The growth rate of imports \hat{M} is a positive function of domestic income growth and relative price changes.

$$\hat{X} = \eta_X \hat{Y}^* + \sigma_X (\hat{E} + \hat{P}^* - \hat{P}) \tag{8}$$

$$\hat{M} = \eta_M \hat{Y} - \sigma_M (\hat{E} + \hat{P}^* - \hat{P}) \tag{9}$$

where E is the nominal exchange rate, while P^* represents the foreign price level. The symbols η_i and σ_i (i = X, M) represent the relevant income and price elasticities.

Domestic assets too are assumed to be imperfect substitutes for foreign assets. International capital flows KA are thus a function of profitability relative to the rest of the world.

$$KA = k(r - r^*)$$

Since the country is assumed to be small in international asset markets, the international profit rate is taken as a given constant. Again, translating into growth rate form,

$$\widehat{KA} = \phi \hat{\tau} \tag{10}$$

Cumulative causation can be introduced via a Kaldor/Verdoorn specification for productivity growth.

$$\hat{q} = \psi \hat{Y} \tag{11}$$

where ψ is the Verdoorn coefficient.

We now have the components required to construct our system. In order to derive the goods market equilibrium condition, consider the standard macroeconomic identity in the absence of government spending or taxes, and expressed in terms of the domestic good:

$$Y = C + I + X - \frac{EP^*}{P}M$$

Or, in growth rate form,

$$\hat{Y} = \lambda_C \hat{C} + \lambda_I \hat{I} + \lambda_X \hat{X} - \lambda_M (\hat{M} + \hat{E} + \hat{P}^* - \hat{P}) \tag{12}$$

where λ_i (i=C,I,X,M) denotes the share of each aggregate demand component in income. Substituting equations (2), (5), (7), (8), (9), and (11) into equation (12), and assuming initially balanced trade for simplicity (so that $\lambda_X = \lambda_M$ and $\lambda_I = 1 - \lambda_C$), yields the equilibrium condition for goods market equilibrium, or the IS condition:

IS:
$$\hat{K} - \lambda_C \{\hat{K} - S[\hat{\tau} + (\tau - 1)\hat{s}]\} - (1 - \lambda_C)(i_Y \hat{K} + i_\tau \hat{\tau}) - \lambda_X [\eta_X \hat{Y}^* - \eta_M \hat{K} + \sigma(\hat{E} + \hat{P}^* - \hat{\tau} + \psi \hat{K})] = 0$$
 (13)

where $\sigma = \sigma_X + \sigma_M - 1 > 0$, assuming satisfaction of the Marshall-Lerner condition. In the absence of official reserve transactions, the balance of payments condition can be written, using equations (2), (8), (9), (10) and (11), in the following form:

BP:
$$\eta_X \hat{Y}^* - \eta_M \hat{K} + \sigma(\hat{E} - \hat{\tau} + \psi \hat{K}) + \lambda_K \phi \hat{\tau} = 0$$
 (14)

where $\lambda_K = KA/X$. Equations (13) and (14) form our system of two equations. The foreign variables \hat{Y}^* and \hat{P}^* are taken as exogenous and equal to zero.

In order to understand the workings of the system, it may be helpful to consider the role of the adjusting variables with the help of partials.

$$IS_{\hat{K}} = (1 - \lambda_C)(1 - i_Y) + \lambda_X(\eta_M - \sigma\psi)$$
 (15a)

$$BP_{\hat{K}} = -(\eta_M - \sigma\psi) \tag{15b}$$

With fixed exchange rates, relative prices adjust through the mark-up.

$$IS_{\hat{\tau}} = \lambda_C S - (1 - \lambda_C)i_{\tau} + \lambda_X \sigma \tag{15c}$$

$$BP_{\hat{\tau}} = \lambda_K \phi - \sigma \tag{15d}$$

With floating exchange rates, the nominal exchange rate takes up the burden of relative price adjustment.

$$IS_{\hat{E}} = -\lambda_X \sigma < 0 \tag{16a}$$

$$BP_{\hat{E}} = \sigma > 0 \tag{16b}$$

In line with the traditional Keynesian stability condition, we assume that faster growth of capital (and hence output) creates an excess supply of goods and services. Thus, $IS_{\hat{K}} > 0$.⁴ As far as the balance of payments is concerned, faster growth of capital has offsetting effects. On the one hand, imports increase. On the other hand, increased labor productivity through the Verdoorn effect leads to greater competitiveness and, hence, more exports. Thus, the sign of $BP_{\hat{K}}$ depends on whether $\eta_M \geq \sigma \psi$, that is, whether the Verdoorn effect on competitiveness dominates or is dominated by the income effect on imports.

The effect of relative price increases via the markup rate on the goods and services market are ambiguous. The redistribution of income from workers to owners of capital reduces consumption and exports, creating excess supply. The boost to profits, however, generates demand for investment goods. Thus, if demand is investment-led, $IS_{\hat{\tau}} > 0$, otherwise $IS_{\hat{\tau}} < 0$. As we will see below, this distinction does not play much of a role in our analysis.

Positive mark-up growth attracts capital inflows from the rest of the world, while affecting external competitiveness, and hence the trade balance, negatively. The overall effect on the balance of payments depends on the degree of capital account openness and the extent to which relative price changes matter. With a completely open capital account (so that $\phi \longrightarrow \infty$), $BP_{\hat{\tau}} > 0$. With a completely closed capital account (so that $\phi = 0$), $BP_{\hat{\tau}} < 0$.

The effect of a nominal devaluation on the goods market is to create excess demand. The effect on the balance of payments is to create a surplus via positive net exports. The partials with respect to \hat{E} are, therefore, unambiguously signed.

Before we turn to exploring the effects of internal and external shocks in the next section, notice that, if we ignore the goods and services sector, we are left with equation (14). Further, allowing the mark-up factor to become exogenous and ignoring relative price effects (or assuming elasticity pessimism, which we, following Blecker (2009), define as the condition that $\sigma=0$), yields the BPCG solution, $\hat{K}=\hat{Y}=\eta_X/\eta_M$. Capital account flows are now exogenous and do not promote or inhibit external adjustment. Reintroducing relative price changes makes cumulative causation relevant again, with the qualitative effect of an exogenous shock now depending on whether or not it is adequately strong so that increased output growth creates a trade surplus.

If, alternatively, we ignore the balance of payments constraint, then we are left with equation (13). Assigning the mark-up factor to the list of exogenous variables, and ignoring imports (so that $\eta_M = \sigma_M = 0$) yields the CC model of Setterfield and Cornwall (2002).⁵

⁴Note that, in principle, this condition could be violated if either one or both of the following conditions are met: (i) $i_Y > 1$, and (ii) $\eta_M < \sigma \psi$; in other words, if the response of investment to demand growth is really strong and/or if cumulative causation is strong relative to the income elasticity of demand for imports.

 $^{^5}$ Or rather a modified version of that model since Setterfield and Cornwall do not explicitly model the components of domestic absorption.

The next two sections explore the effectiveness of internal and external shocks in influencing output growth. We term a shock an exogenous driver of growth if it has a non-zero impact on the equilibrium growth rate. Table 1 summarizes the results.

3 External and Internal Shocks With a Fixed Exchange Rate

We begin this section with the analysis of a fixed exchange rate regime with a completely open capital account before moving to the case of one with a completely closed capital account. The latter case can be interpreted as the longer-run version since continuous trade imbalances, especially deficits, are generally not sustainable over longer periods owing to debt, inflation, or distribution-related issues. Throughout we implicitly assume for simplicity that domestic capital goods are perfect substitutes for domestic assets up to a zero (or constant) risk premium so that, in the presence of completely open capital accounts, both yield similar risk-adjusted returns.

As mentioned earlier, with a fixed exchange rate \hat{K} and $\hat{\tau}$ become the adjusting variables. Figure 1 helps illustrate the discussion graphically in $\hat{K} - \hat{\tau}$ space. The respective slope expressions can be written as follows:

$$\begin{split} \left. \frac{d\hat{\tau}}{d\hat{K}} \right|_{IS} &= -\frac{IS_{\hat{K}}}{IS_{\hat{\tau}}} = -\frac{(1 - \lambda_C)(1 - i_Y) + \lambda_X(\eta_M - \sigma\psi)}{\lambda_C S - (1 - \lambda_C)i_\tau + \lambda_X \sigma} \\ &\left. \frac{d\hat{\tau}}{d\hat{K}} \right|_{BP} = -\frac{BP_{\hat{K}}}{BP_{\hat{\tau}}} = \frac{(\eta_M - \sigma\psi)}{\lambda_K \phi - \sigma} \end{split}$$

An increase in output growth creates, given the Keynesian stability condition, excess supply of goods and services. The mark-up rate must rise if demand is investment-led, and fall otherwise. The IS curve will slope upward or downward accordingly.

Turning to the BP curve, consider first the case where the capital account is perfectly open $(\phi \longrightarrow \infty)$. Regardless of the degree of cumulative causation, an infinitesimally small change in mark-up is required to counter any imbalances created by changed output growth, rendering the BP curve horizontal. If the capital account is perfectly closed $(\phi = 0)$, on the other hand, then the slope depends on the strength of the Verdoorn coefficient relative to the elasticity of demand for imports. Strong cumulative causation means that an increase in output growth creates a BP surplus, so that the mark-up rate must increase to remove the surplus via reduced competitiveness. A positive slope results. Conversely, weak cumulative causation generates a negative slope.

3.1 Completely Open Capital Account

In the first case, we assume that capital is internationally mobile and that domestic assets are perfect substitutes for foreign assets. Thus, $\phi \to \infty$ and

 $r=r^*$. With the capital account completely open, an infinitesimally small adjustment in the mark-up growth rate is adequate to remove any external disequilibria. Essentially there is no independent balance of payments condition any longer and aggregate supply adjusts solely in response to disequilibria in the goods and services market.

3.1.1 Faster global income growth

Consider first the effect of a positive external shock in the form of an increase in world income/demand growth. The trade surplus created due to increased export growth is offset by an infinitesimally small decline in the mark-up and the ensuing outflow of financial capital. Accumulation and output growth increase in response to the aggregate demand created by the trade surplus. Mathematically,

$$\frac{d\hat{K}}{d\hat{Y}^*} = \frac{\eta_X \lambda_X}{(1 - \lambda_C)(1 - i_Y) + \lambda_X (\eta_M - \sigma \psi)} > 0$$
$$\frac{d\hat{\tau}}{d\hat{Y}^*} = 0$$

The Verdoorn coefficient ψ magnifies the impact of the external shock through cumulative causation. A trade surplus is created at the new steady state since the initial increase in exports is only partially crowded out by imports. Denoting the trade balance by TB,

$$\begin{split} \frac{d(\widehat{TB})}{d\hat{Y}^*} &= \eta_X - (\eta_M - \sigma \psi) \frac{d\hat{K}}{d\hat{Y}^*} \\ &= \frac{\eta_X((1 - \lambda_C)(1 - i_Y))}{(1 - \lambda_C)(1 - i_Y) + \lambda_X(\eta_M - \sigma \psi)} > 0 \end{split}$$

Figure 1 graphically illustrates this case.

3.1.2 Reduced Saving Rate

Next suppose fiscal authorities pursue incentives to lower domestic savings. Such a step creates excess demand for goods and services, raising the growth rate of capital and output. Any trade surplus or deficit created (depending on whether $\eta_M \geq \sigma \psi$), is offset by a negligibly small change in $\hat{\tau}$. The effect is qualitatively the same as illustrated in Figure 1. Mathematically,

$$\frac{d\hat{K}}{d\hat{s}} = -\frac{\lambda_C S(\tau - 1)}{(1 - \lambda_C)(1 - i_Y) + \lambda_X (\eta_M - \sigma \psi)} < 0$$
$$\frac{d\hat{\tau}}{d\hat{s}} = 0$$

$$\frac{d(\widehat{TB})}{d\hat{s}} = -(\eta_M - \sigma\psi)\frac{d\hat{K}}{d\hat{s}} \lessgtr 0$$

Again, the Verdoorn coefficient magnifies the impact of the external shock through cumulative causation.

Thus, in the case of perfect mobility, both positive external and internal shocks to aggregate demand increase the growth rate with negligible distributional effects. The other components of demand adjust endogenously in response to the shock. There is nothing inherently special about a positive export shock, and the relative magnitudes of changes in response to the two kinds of shocks depends on parameters such as the income elasticity of world demand for exports, the initial mark-up factor, and the income share of consumption and exports. In each case, cumulative causation enhances the effect of the initial stimulus. However, if the shock is internal, cumulative causation plays an additional important role. Recall that, in the case of a positive external shock, an unambiguous trade surplus is created. An internal shock, however, creates a trade surplus if cumulative causation is sufficiently strong (i.e., $\eta_M < \sigma \psi$), and a deficit otherwise. Thus, to the extent that the accompanying external debt trajectory makes trade surpluses more sustainable than deficits, the degree of cumulative causation plays a role in determining the sustainability of the higher growth rate over longer-run horizons.

3.2 Completely Closed Capital Account

A completely closed capital account (i.e., $\phi=0$) amounts to imposing a trade balance constraint. Capital flows now being unable to share the burden, distributional changes come into the forefront, with changes in $\hat{\tau}$ influencing international competitiveness. A first look at equations (15b) and (15c) may suggest that there are four possible scenarios: $\eta_M \leq \sigma \psi$ and $\lambda_C S - (1 - \lambda_C) i_\tau + \lambda_X \sigma \leq 0$. As explained earlier, the former sign depends on the strength of the cumulative causation effect while the latter depends on whether demand growth is investment-led or not. As shown in the Appendix, however, the existence of a unique steady state equilibrium requires the more stringent condition that $\lambda_C S - (1 - \lambda_C) i_\tau < 0$. Moreover, as long as this condition is satisfied, the results are qualitatively unaffected by whether demand growth is investment-led or otherwise. We, therefore, limit our discussion of external and internal shocks here to the two scenarios governed by the first condition, i.e., $\eta_M \leq \sigma \psi$.

3.2.1 Faster global income growth

An increase in world demand for domestic exports results on impact in a trade surplus and excess demand for goods and services. As shown earlier in the case of a completely open capital account, the resulting rise in output and hence import growth rate only partially crowds out the trade surplus when cumulative causation is relatively weak ($\eta_M > \sigma \psi$) so that increased growth exerts negative pressure on the trade balance. In the case where cumulative

causation is relatively strong $(\eta_M < \sigma \psi)$, the rise in output growth further magnifies the initial trade surplus created on impact. In either case, a rise in the mark-up factor is required to remove the trade surplus through reduced international competitiveness. Mathematically,

$$\frac{d\hat{K}}{d\hat{Y}^*} = \frac{\eta_X}{-\frac{\sigma(1-\lambda_C)(1-i_Y)}{[\lambda_C S - (1-\lambda_C)i_T]} + (\eta_M - \sigma\psi)} > 0$$

$$\frac{d\hat{\tau}}{d\hat{Y}^*} = -\frac{\eta_X \frac{((1-\lambda_C)(1-i_Y))}{[\lambda_C S - (1-\lambda_C)i_\tau]}}{-\frac{\sigma(1-\lambda_C)(1-i_Y)}{[\lambda_C S - (1-\lambda_C)i_\tau]} + \left(\eta_M - \sigma\psi\right)} > 0$$

Figure 2 illustrates this case. The effect on output and capital growth is unambiguously positive. The only difference between the two cases ($\eta_M \leq \sigma \psi$), therefore, lies in the extent to which distributional changes share the burden of adjustment. With weak cumulative causation effects $\eta_M > \sigma \psi$, so that the trade surplus and excess demand for goods created is small, as is the change in output and mark-up growth required to transition to the new steady state. Adequately strong cumulative causation $\eta_M < \sigma \psi$, on the other hand, results in a large trade surplus and greater excess demand for goods and services, boosting both the impact on output growth and the extent of distributional changes required to balance trade.

3.2.2 Reduced Saving Rate

A negative change in the saving rate has no direct impact on the trade balance but creates excess demand for goods and services. The growth rate of output and capital unambiguously increase as a consequence. The effect on the mark up rate depends on the strength of the Verdoorn/cumulative causation effect. A trade surplus accompanies the rise in output growth when cumulative causation is adequately strong, requiring a rise in the mark-up to balance trade via reduced competitiveness. Conversely, weak cumulative causation means that the trade deficit resulting from more rapid growth requires a rise in competitiveness via a reduced mark-up. Mathematically,

$$\frac{d\hat{K}}{d\hat{s}} = \frac{\frac{\sigma\lambda_C S(\tau - 1)}{[\lambda_C S - (1 - \lambda_C)i_\tau]}}{-\sigma \frac{((1 - \lambda_C)(1 - i_Y))}{[\lambda_C S - (1 - \lambda_C)i_\tau]} + (\eta_M - \sigma\psi)} < 0$$

⁶ It can be shown that the denominator of the expressions above is positive. To see this, note that since $\lambda_C S - (1 - \lambda_C)i_{\tau} < 0$, the term $\frac{(1 - \lambda_C)(1 - i_Y)}{\lambda_C S - (1 - \lambda_C)i_{\tau}}$ is negative. Thus, the denominator is always positive when $\eta_M > \sigma \psi$. Furthermore, for the case where $\eta_M < \sigma \psi$, recall the Keynesian stability condition, $((1 - \lambda_C)(1 - i_Y)) + \lambda_X(\eta_M - \sigma \psi)$. Thus, the denominator in this case is positive if $\frac{-\sigma \lambda_X}{\lambda_C S - (1 - \lambda_C)i_{\tau}} > 1$. This condition is satisfied when demand growth is not investment-led i.e., $\lambda_C S - (1 - \lambda_C)i_{\tau} + \sigma \lambda_X > 0$. As shown in the Appendix, in the alternative case where demand is investment-led, the existence of a solution requires that the slope of the IS curve be steeper. This condition translates again into a positive denominator for the expressions in the text.

$$\frac{d\hat{\tau}}{d\hat{s}} = -\frac{\frac{\lambda_C S(\tau - 1)(\eta_M - \sigma \psi)}{[\lambda_C S - (1 - \lambda_C)i_\tau]}}{-\sigma \frac{((1 - \lambda_C)(1 - i_Y))}{[\lambda_C S - (1 - \lambda_C)i_\tau]} + (\eta_M - \sigma \psi)}$$

Hence, $d\hat{\tau}/d\hat{s} \gtrsim 0$ as $\eta_M \gtrsim \sigma \psi$. Figure 3 illustrates the case when $\eta_M > \sigma \psi$. In sum, with zero capital mobility, both positive external and internal shocks to aggregate demand increase the growth rate just as in the case of perfect capital mobility. However, external account considerations now require changes in competitiveness and hence distribution. Again, insofar as the effect on output growth is concerned, there is nothing qualitatively special about a positive export shock, and the relative magnitudes of changes in response to the two kinds of shocks depends on various parameter values. In each case, cumulative causation enhances the effect of the stimulus. Also, the higher the relative price elasticities (as captured by σ), the smaller the distributional change required. The crucial role of this parameter comes to the fore in the next sub-section.

3.2.3 The role of relative price effects

Finally, to highlight the role that cumulative causation plays through relative price effects, consider the case where $\sigma = 0$ (i.e., elasticity pessimism).⁷ With zero capital mobility and cumulative causation rendered non-existent by the absence of relative price effects, there is only one value of domestic income growth that is consistent with balanced trade *unless* an exogenous non-price change loosens or tightens the external constraint. An external shock in the form of increased global growth does just that. Mathematically,

$$\begin{split} \frac{d\hat{K}}{d\hat{Y}^*} &= \frac{\eta_X}{\eta_M} > 0\\ \\ \frac{d\hat{\tau}}{d\hat{Y}^*} &= -\frac{\eta_X \frac{((1 - \lambda_C)(1 - i_Y))}{[\lambda_C S - (1 - \lambda_C)i_\tau]}}{\eta_M} > 0 \end{split}$$

Notice that we get the canonical BPCG solution for output growth. As seen in Figure 4, the excess demand for goods and trade surplus created by the initial shock require an increase in both the mark up factor and output growth for their removal.

An internal shock in the form of reduced domestic savings, on the other hand, fails to affect output growth. The intuition is simple; there is no *direct* effect on the external constraint in this case and the rate of output growth commensurate with external equilibrium remains unchanged. The mark-up rate soaks up, through distributional changes, any excess demand created in the goods and services market by a decline in savings. Mathematically,

⁷Notice that the small (in the sense of being a price-taker) case with σ approaching infinity does not yield a unique solution since in this case the IS and BP curves have exactly the same slope.

$$\begin{split} \frac{d\hat{K}}{d\hat{s}} &= 0\\ \\ \frac{d\hat{\tau}}{d\hat{s}} &= -\frac{\lambda_C S(\tau-1)}{[\lambda_C S - (1-\lambda_C)i_{\tau}]} > 0 \end{split}$$

Thus, now exports come into their Kaldorian own as the *only* purely exogenous driver of aggregate demand and growth. It is in the absence of capital mobility and, ironically, cumulative causation (and related relative price effects) that the economy is truly balance of payments constrained and exports become qualitatively unique in their effect on growth. Any policy that does not loosen the balance of payments constraint directly fails to register an impact on output growth. In terms of Figure 5, any policy measure that fails to shift the BP curve has no effect on the equilibrium rate of output growth.

4 A Floating Exchange Rate Regime

With the capital account determined by the (now exogenous) mark-up factor, nominal exchange rate changes now drive relative price adjustments. Thus, the relevant adjusting variables now are \hat{K} and \hat{E} . Unless there is an exogenous change in the mark-up growth rate, therefore, the degree of capital account mobility is no longer relevant. Moreover, with exogenous mark up and wage growth, nominal exchange rate changes are qualitatively different in their effects since, unlike the mark-up rate, these do not affect distribution. Their only influence, in other words, is on net exports. Consumption and investment growth are unaffected. This means that changes in the exchange rate have an exactly identical impact on the two sectors.

4.1 Faster global income growth

Accelerated growth in world demand fails to affect domestic income growth as any initial change in exports is crowded out by exchange rate appreciation. Export demand thus ceases to be an autonomous driver of output growth. Mathematically,

$$\frac{d\hat{K}}{d\hat{Y}^*} = 0$$

$$\frac{d\hat{E}}{d\hat{Y}^*} = -\frac{\eta_X}{\sigma} < 0$$

Graphically, as illustrated by Figure 6, the two schedules shift equally in the vertical direction, leaving growth unchanged. The intuition is simple. The initial external shock creates a trade surplus and an equal excess demand for goods and services. Since the nominal exchange rate affects both sectors identically through the trade balance, the change in \hat{E} required to remove imbalances

in both the sectors must be identical. This can only be true if the other adjusting variable, i.e., output growth is unchanged at the new equilibrium. The exchange rate appreciation thus completely neutralizes the initial trade surplus and excess demand for goods and services.

Notice that cumulative causation via the Verdoorn coefficient never comes into play as the economy settles at a new equilibrium with an appreciated real exchange rate.

4.2 Reduced Saving Rate

Again, an injection of domestic spending into the economy does not directly affect the balance of payments. The excess demand created in the market for goods and services results in increased domestic growth. The exchange rate appreciates if cumulative causation effects are strong enough so that growth generates external surpluses and depreciates otherwise. Thus, the only role played by cumulative causation is to increase the likelihood that growth resulting from a domestic stimulus is sustainable. The Verdoorn coefficient is irrelevant to output growth determination. Mathematically,

$$\frac{d\hat{K}}{d\hat{s}} = -\frac{\lambda_C S(\tau - 1)}{(1 - \lambda_C)(1 - i_Y)} < 0$$

$$\frac{d\hat{E}}{d\hat{s}} = -\frac{\lambda_C S(\tau - 1)(\eta_M - \sigma \psi)}{\sigma (1 - \lambda_C)(1 - i_Y)}$$

The key to understanding why a floating exchange rate renders cumulative causation irrelevant to the growth rate lies in the absence of a *direct* effect of an internal spending shock on the balance of payments. Thus, while the growth rate accelerates in response to the increase in expenditure, it can do so only to the extent that it removes the initial excess demand created. Any further effect through cumulative causation is crowded out by an exchange rate appreciation that keeps the external account balanced.⁸

The reversal of roles between the fixed and flexible exchange rate cases is quite dramatic as internal demand now appears as the only exogenous driver of growth. This result itself harks back to a large body of literature on the role of the exchange rate regime in determining the response of an economy to exogenous shocks. The extent and nature of cumulative causation effects too differ between the two exchange rate regimes. With flexible exchange rates, this role is largely limited to ensuring the sustainability of an internal stimulus.

⁸Recall that we know from equation (13) that a unit increase in output growth is required to offset increased excess demand growth of a magnitude $(1 - \lambda_C)(1 - i_Y)$ while keeping trade balanced. The increase in demand growth caused by reduced domestic saving growth is $\lambda_C S(\tau - 1)$. Thus the expression derived above for $d\hat{K}/d\hat{Y}^*$.

5 Conclusions

Kaldor's model of cumulative causation, as typically presented, has only one constraint on the system, i.e., that cumulative causation not be too strong to exclude the possibility of a unique equilibrium. This paper combines this model with the balance of payments constrained growth model by imposing an external account constraint. Simultaneous endogenous changes in the growth of output and relative prices (via distributional changes or exchange rates) now jointly ensure the maintenance of internal and external balances.

Under a fixed exchange rate regime, the effect of exogenous shocks varies with the capital account regime in place. With a completely open capital account, the entire burden of adjustment falls on output, which responds to both internal and external shocks. As in the traditional cumulative causation framework, the presence of cumulative causation has the effect of magnifying the impact of these shocks. The additional lesson that can be derived from the combination of the CC and BPCG models is that the presence of adequately strong cumulative causation effects, by ensuring that faster growth generates trade surpluses rather than deficits, makes such growth more sustainable. In the BPCG framework, it is the income elasticity of demand for exports relative to that for imports that plays this role. Thus, our framework allows us to isolate another factor, in addition to trade elasticities, that can help loosen the external constraint in a growing economy.

With a completely closed capital account, both output growth and distributional changes contribute to adjustment. In this case, which is akin to having balanced trade, distributional changes via the mark-up factor play the important role of determining the external competitiveness of the economy. Again, both external and internal shocks affect the economy, although cumulative causation now plays a crucial part in determining the direction of the distributional change. As long as relative price changes matter, however, there is nothing special about exports as far as boosting output growth is concerned.

It is only when elasticity pessimism precludes a role for relative price changes that exports come into their own as the *only* exogenous drivers of growth. Any external shock that boosts exports raises, in this case, the rate of output growth consistent with internal and external equilibrium. An internal shock, on the other hand, fails to have an impact on output growth precisely because it does not relax the external constraint on impact. The system now reduces to the BPCG model in its traditional form and cumulative causation does not get a chance to come into play.

The BPCG literature has widely emphasized the lesson that countries should strive to develop sectors that exhibit a high international income elasticity of demand. Under a fixed exchange rate regime, combining the BPCG and CC models allows us to identify another crucial variable. Countries that specialize in sectors that exhibit potential for cumulative causation may not only exhibit higher growth but such growth may also be less susceptible to constraints imposed by factors such as external debt and international investor confidence. Introducing cumulative causation brings international price elasticities of de-

mand back into the picture.

Under a flexible exchange rate system, the role of internal and external shocks reverses. An external shock now becomes ineffective, as does the potential presence of cumulative causation. A rise in domestic demand, on the other hand, raises the domestic growth rate, with strong cumulative causation effects lending the higher growth rate a greater degree of sustainability. Exports lose their role as exogenous drivers of growth.

We have explored the consequences of combining the CC and BPCG models by imposing a balance of payments constraint on the former. Razmi (2010) has argued that the inclusion of non-tradables tends to lower the lower the BPCG growth rate. To the extent that the non-tradable sector may be less likely to exhibit cumulative causation than the tradable industrial sector, one may argue that the inclusion of non-tradables will lower the externally constrained growth rate even further in our model. We have considered real shocks only. Implicitly we have assumed that money is endogenous with the central bank absorbing changes in money demand. Relaxing this assumption and incorporating monetary shocks could generate further interesting insights, especially in the case of floating exchange rates. Our model excludes the possibility of nominal exchange rate changes affecting demand though distribution. Incorporating imported intermediate inputs and/or endogenizing wage growth could establish such a mechanism. We leave these questions to future research.

6 Appendix

As mentioned in Section 3.2, with a closed capital account and a fixed exchange rate, we get four possible cases defined by $\eta_M \geq \sigma \psi$ and $\lambda_C S - (1 - \lambda_C)i_\tau + \sigma \lambda_X \geq 0$. The latter condition is always satisfied when demand growth is investment-led. However, even in the case where demand growth is not investment-led, the existence of a unique steady state equilibrium requires, as a sufficient condition the expression $\lambda_C S - (1 - \lambda_C)i_\tau < 0$ to be satisfied, narrowing our comparative static results to only 2 cases, i.e., $\eta_M \leq \sigma \psi$. Here we demonstrate this statement and provide some underlying intuition.

Section ?? derives the slopes of the IS and BP curves. The expressions for the horizontal and vertical intercepts are given by:

$$\begin{split} \hat{K}_{IS} &= \frac{\lambda_X [\eta_X \hat{Y}^* + \sigma(\hat{E} + \hat{P}^*)] - \lambda_C S(\tau - 1) \hat{s}}{(1 - \lambda_C)(1 - i_Y) + \lambda_X (\eta_M - \sigma\psi)} \\ \hat{K}_{BP} &= \frac{\lambda_X [\eta_X \hat{Y}^* + \sigma(\hat{E} + \hat{P}^*)]}{\lambda_X (\eta_M - \sigma\psi)} \\ \hat{\tau}_{IS} &= \frac{\lambda_X [\eta_X \hat{Y}^* + \sigma(\hat{E} + \hat{P}^*)] - \lambda_C S(\tau - 1) \hat{s}}{\lambda_C S - (1 - \lambda_C) i_\tau + \sigma\lambda_X} \\ \hat{\tau}_{BP} &= \frac{\lambda_X [\eta_X \hat{Y}^* + \sigma(\hat{E} + \hat{P}^*)]}{\sigma\lambda_X} \end{split}$$

Case 1: $\eta_M > \sigma \psi$ and $\lambda_C S - (1 - \lambda_C)i_\tau + \sigma \lambda_X < 0$

Panel (a) of Figure 8 illustrates this case. The BP curve is downward-sloping and has a positive horizontal intercept. Since demand growth is investment-led, the IS curve has a positive slope and the horizontal intercept must be less than that of the BP curve. The horizontal intercept expressions satisfy this condition.

Case 2: $\eta_M > \sigma \psi$ and $\lambda_C S - (1 - \lambda_C) i_\tau + \sigma \lambda_X > 0$

Panel (b) of Figure 8 illustrates this case. Again, the BP curve is downward-sloping and has a positive horizontal intercept. Since demand growth is not investment-led, the IS curve too has a negative slope. Its horizontal intercept must, therefore, be positive. Moreover, we know from the horizontal intercept expressions above that it is less than that of the BP curve. This latter property means that the existence of a solution requires that the IS curve be steeper. Mathematically, this reduces to the condition

$$\sigma(1 - \lambda_C)(1 - i_Y) > [\lambda_C S - (1 - \lambda_C)i_\tau](\eta_M - \sigma\psi)$$

which is unambiguously satisfied if $\lambda_C S - (1 - \lambda_C)i_{\tau} < 0$.

Case 3: $\eta_M < \sigma \psi$ and $\lambda_C S - (1 - \lambda_C)i_\tau + \sigma \lambda_X < 0$

Panel (c) of Figure 8 illustrates this case. The BP curve is now upward-sloping and has a negative horizontal intercept. Since demand growth is investment-led, the IS curve too has a positive slope and its horizontal intercept is positive. The existence of a solution requires, therefore, that the IS curve be steeper. Mathematically, this reduces to the condition

$$\sigma(1 - \lambda_C)(1 - i_Y) > [\lambda_C S - (1 - \lambda_C)i_\tau](\eta_M - \sigma\psi)$$

Case 4: $\eta_M < \sigma \psi$ and $\lambda_C S - (1 - \lambda_C)i_\tau + \sigma \lambda_X > 0$

Panel (d) of Figure 8 illustrates this case. Again, the BP curve is upward-sloping and has a negative horizontal intercept. Since demand growth is not investment-led, the IS curve has a negative slope and its horizontal intercept is positive. The existence of a solution requires, therefore, that the vertical intercept of the IS curve be greater than that of the BP curve. Mathematically, this reduces to the following necessary condition:

$$\lambda_C S - (1 - \lambda_C)i_\tau < 0$$

Thus, in all cases, the existence of a solution is associated with the condition $\lambda_C S - (1 - \lambda_C)i_{\tau} < 0$. A brief discussion may help illustrate the intuition. A look at the expressions for goods market equilibrium and the balance of payments (eqs. (13) and (14)) indicates that a change in \hat{K} has proportionately a larger impact on the goods market than on the external balance. Furthermore, the same is true for $\hat{\tau}$ if $\lambda_C S - (1 - \lambda_C)i_{\tau} > 0$. With this in mind, suppose

we begin at point A on the IS curve in Figure 9. The line is assumed to have unit slope for conceptual simplicity. Next, a decline in mark up growth equal to $\Delta \hat{\tau}$ pulls us down vertically to point B. Getting back to equilibrium requires a change in output growth so that $-\Delta \hat{\tau} = \Delta \hat{K}$. That output growth has a proportionately larger impact on the goods market than on the external balance implies that the change in \hat{K} that offsets $\Delta \hat{\tau}$ has to be smaller for the goods market (i.e., BP must be flatter than IS). That mark-up growth too has a proportionately larger impact on the goods market than on the external balance implies that the change in \hat{K} that offsets $\Delta \hat{\tau}$ has to be larger for the goods market (i.e., BP must be steeper than IS). The two requirements are contradictory. By ensuring that $\hat{\tau}$ has a greater impact on the external balance than the goods market, the condition $\lambda_C S - (1 - \lambda_C)i_{\tau} < 0$ avoids this contradiction.

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Table 1: Summary Results of Thought Experiments

Exchange Rate Regime	Fixed			Floating
Relative price elasticities (σ)	>	0	0	-
Degree of KA openness (ϕ)	∞	0	0	-
Effect on \hat{Y} of:				
$\uparrow \hat{Y}^*$	+	+	+	0
$\downarrow \hat{s}$	+	+	0	+
Cumulative causation relevant for \hat{Y} ?	Yes	Yes	No	No

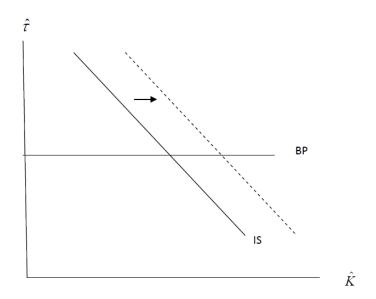


Figure 1: Fixed exchange rate: A positive external shock when the capital account is completely open

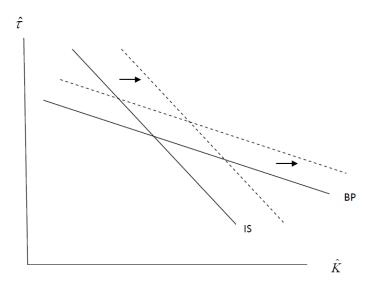


Figure 2: Fixed exchange rate: A positive external shock when the capital account is completely closed

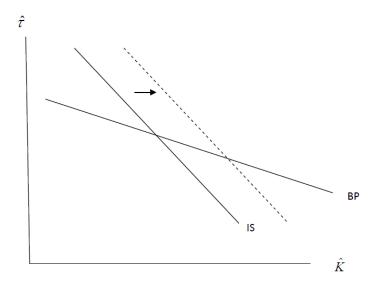


Figure 3: Fixed exchange rate: A positive internal shock when the capital account is completely closed

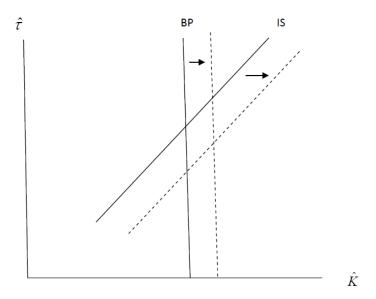


Figure 4: Fixed exchange rate: A positive external shock when the capital account is completely closed and elasticity pessimism holds

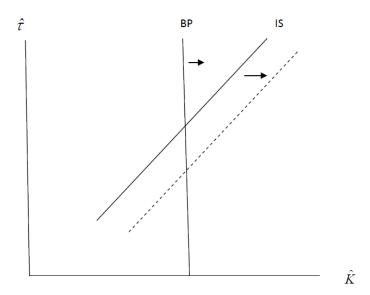


Figure 5: Fixed exchange rate: A positive internal shock when the capital account is completely closed and elasticity pessimism holds

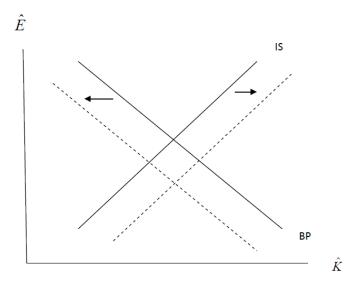


Figure 6: Floating exchange rate: A positive external shock under a flexible exchange rate regime

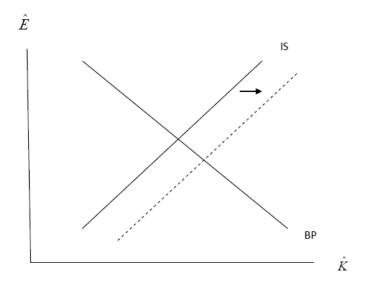


Figure 7: Floating exchange rate: A positive internal shock under a flexible exchange rate regime

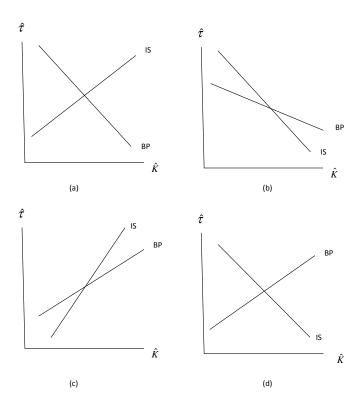


Figure 8: The four possible cases illustrated in the appendix.

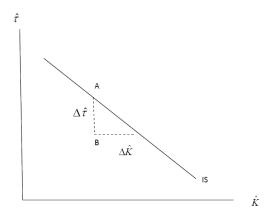


Figure 9: Illustrating the condition for the existence of an equilibrium solution