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# The Impact of Inflation Targeting: Testing the Good Luck Hypothesis

Federico Ravenna\*

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## Abstract

Over the last twenty years the level and volatility of inflation decreased across industrial countries. The inflation behaviour can be explained by a shift in monetary policy or by a lucky period of low volatility in business cycle shocks. To test the 'luck hypothesis' we examine the inflation experience of Canada, one of the earliest and most successful adopter of an inflation targeting monetary policy. We Kalman-filter the historical structural shocks consistent with an estimated DSGE model, but allow for part of the variance in the data to be explained by a vector of non-structural random innovations. The estimated DSGE model shocks are used to build counterfactual histories. Ex-ante the estimated model predicts inflation volatility to more than halve under inflation targeting. But conditional on the shocks, we show that the luck hypothesis can explain with a high probability Canada's low inflation volatility since the early 1990s. Any inflation stabilization induced by the shift in policy is accounted for the most part by the impact on expectations. Counterfactuals built neglecting expectations would prove the inflation targeting policy irrelevant.

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# 1 Introduction

Over the last twenty years industrial countries have experienced a marked decline in the level and volatility of inflation, interest rates, and long term inflation expectations (Cecchetti and Debelle, 2006, Neumann and von Hagen, 2002). In the US the recent period of low and stable inflation also witnessed a decline in the volatility of real activity, which the literature attributes either to a change in the propagation mechanism of the economy - the most prominent explanation being a shift in the management of monetary policy - or to a reduction in the volatility of exogenous shocks.

What are the reasons behind the observed change in inflation behaviour across industrial countries? This paper examines the inflation performance of Canada, an early and successful adopter of an inflation targeting monetary policy since February 1991, and asks whether it can be explained by the 'luck hypothesis'. Accepting the luck hypothesis means that conditional on the exogenous shocks that hit the Canadian economy since 1991 the inflation behaviour would not have been significantly different under an alternative monetary policy. The results are based on a methodology that builds upon, and expands on, a vast literature on constructing historical counterfactuals using restrictions from DGSE models.

The experience of inflation targeting countries is especially suitable to assess whether good luck or good policy can account for the observed change in inflation behaviour, since the monetary authority explicitly announced - and committed to - the inflation targeting policy. Countries adopting inflation targeting have experienced lower and more stable inflation, and a decline in the volatility of a wide range of other macroeconomic indicators<sup>1</sup>. Researchers have found little cross country evidence that inflation targeting improves macroeconomic performance (Ball and Sheridan, 2005, Cecchetti and Debelle, 2006, Debelle, 1997, Levin, Natalucci and Piger, 2004). Yet cross country evidence on the impact of inflation targeting may be hard to come by because aversion to inflation variability increased across most industrial countries since the 1980s (Cecchetti and Ehrmann, 2002, Mishkin, 1999, Neumann and von Hagen, 2002).

On the contrary, in the US case the existence of a shift in monetary policy regime over the last 25 years is still widely debated. While Boivin and Giannoni (2006), Cogley and Sargent (2002, 2005) and Clarida et al. (2000) provide evidence that US monetary policy did change in the 1980s, and affected the observed behaviour of inflation, the results in a number of papers do not support the claim

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<sup>1</sup>Inflation targeting central banks have published numerous studies evaluating the impact of the monetary policy change on the national economy. For cross country comparisons, see Bernanke, Laubach, Mishkin and Posen (1999), Mishkin and Schmidt-Hebbel (2002), Neumann and Von Hagen (2002), and Truman (2003).

of a shift in the systematic behaviour of US monetary policy, or the claim that it was responsible for the observed decline in real activity volatility (Ahmed, Levin and Wilson, 2004, Bernanke and Mihov, 1998, Canova, 2006, Canova and Gambetti, 2006, Justiniano and Primiceri, 2006, Primiceri, 2005, Sims and Zha, 2006, Stock and Watson, 2003).

We investigate the impact of the shift in monetary policy by building counterfactual histories of the Canadian economy since the adoption of inflation targeting conditional on the earlier monetary policy and on a vector of shocks Kalman-filtered from data on ten aggregate variables. The shocks vector and the counterfactual history are constrained to be consistent with a maximum likelihood-estimated staggered wage and price adjustment DSGE model of the economy.

Contrary to counterfactuals built from VAR reduced-form models, as in Ahmed, Levin and Wilson (2004), Canova and Gambetti (2006), Primiceri (2005), Sims and Zha (2006), our approach allows for the policy change to affect the private sector's expectations. Since a stated goal of inflation targeting central banks has been to change inflation expectations, the VAR reduced-form approach is likely to provide a distorted picture. We show that neglecting the policy impact on expectations can bias the result from the counterfactual exercise. Our methodology is closer to the approach used, among others, by King and Rebelo (1998), Justiniano and Primiceri (2006), Rotemberg and Woodford (1998), and similar to the 'business cycle accounting' in Chari, Kehoe and McGrattan (2007).

We innovate relative to the accounting approach in several important dimension. First, we take as a starting point a DSGE model but we also allow for the possibility that the model may be too stylized to describe the data dynamics. Thus we introduce an additional vector of disturbances that identifies sources of fluctuations beyond the ones summarized in the DSGE model, and estimate the DSGE structural shocks with a Kalman filter. This approach is related to Ireland (2004) and Boivin and Giannoni (2006), and to the technique to estimate DSGE models in Sargent (1989) and McGrattan (1994). Our estimation shows that forcing the DSGE model to explain all the volatility in the data would overturn the result from the counterfactual exercise.

We show how to use our methodology to evaluate how much of the economy dynamics in the inflation targeting period is accounted for by the change in the behaviour of the interest rate (a shift in *policy*) and by the credible announcement of an inflation target (a shift in the *policy rule*) by building a counterfactual history under the assumption that the private sector did not believe the shift to an inflation targeting policy.

Finally, we provide an explicit metric to evaluate the likelihood of the luck hypothesis. To test the luck hypothesis we ask what was the probability of observing the counterfactual inflation history

conditional on the inflation targeting policy. In this way we can evaluate whether the counterfactual path would have been a more or less likely draw given the estimated variance of the shocks.

The main conclusions we reach are as follows.

With a high probability good luck can explain the low inflation volatility throughout the inflation targeting period in Canada. This result obtains despite model estimates implying the unconditional volatility of inflation to more than halve in the inflation targeting regime. We do not interpret the evidence for the luck hypothesis as implying that monetary policymaking is irrelevant. Inflation targeting simply may have not been put to test by inflationary shocks. But our result does support the good luck over good policy explanation, and - contrary to a vast literature on the impact of inflation targeting - implies that the shift in monetary policy cannot account for the historical improvement in inflation performance. While policies may be judged more or less desirable based on their unconditional properties, the historical evaluation of their performance is only meaningful if conditioned on the business cycle shocks.

The counterfactuals show that inflation targeting affected the behaviour of inflation for the largest part through the impact on expectations. Monetary policy shocks are estimated to have non-negligible variance, yet they contributed very little to inflation stabilization. Moreover, a monetary policy that did not affect private sector expectations but nevertheless stabilized inflation at its historical level would have led to a severe and prolonged recession. This result supports the claim that changes in policy regime can dramatically affect the economy dynamics by altering private agents' decision making (Sargent, 1999). At the same time, our conclusions on this issue are heavily dependent on the DSGE model adopted. We elaborate on this in Section 6.

As for the methodological contribution, our result that alternative approaches to build counterfactuals can overturn the evaluation of the luck hypothesis illustrate the importance of allowing for a vector of non-structural disturbances when filtering the historical business cycle shocks. The Kalman filtering approach we adopt allows the data to choose what is the portion of business cycle volatility that the DSGE model can explain. Our methodology to assess the impact of a change in policy against a change in the private sector *expectation* of policy can be applied to evaluate any announced change in policymakers' behaviour.

The paper is organized as follows. Section 2 compares alternative methods to build counterfactual histories. Section 3 presents the model. Section 4 explains the building of the Kalman-filtered counterfactual. Section 5 discusses the maximum likelihood estimation of the model and builds counterfactuals histories to evaluate the role of the policy rule, the role of policy shocks, and the role of

credibility on Canadian inflation performance, and to assess the luck hypothesis. The implications of alternative methodologies are discussed. Section 6 concludes.

## 2 Counterfactual Histories to Evaluate the Impact of Policy Changes

A counterfactual history is defined by two components: a series of historical shocks  $\varepsilon_t$  and a counterfactual law of motion. This section considers a simple baseline case to illustrate alternative methods to build counterfactual histories. Section 4 discusses the general case, and Appendix 7.3 provides the detailed derivation of the equations in the following.

Let a linearized DSGE model be defined by the system of stochastic difference equations:

$$0 = AE_t(Y_{t+1}) + BY_t + CY_{t-1} + Dz_t \quad (1)$$

$$z_t = Nz_{t-1} + \varepsilon_t \quad (2)$$

where  $Y_t = [y_t, x_t]'$ ,  $y_t$  is a vector of endogenous control variables,  $x_t$  is a vector of endogenous state variables,  $z_t$  is an unobservable vector of exogenous state variables,  $\varepsilon_t$  is a multivariate Gaussian stochastic process. The rational expectation equilibrium for the model can be written as:

$$Y_t = \Gamma_1 Y_{t-1} + \Gamma_2 z_t \quad (3)$$

$$z_t = Nz_{t-1} + \varepsilon_t \quad (4)$$

Assume that  $N$  is the zero matrix and that the vectors  $\varepsilon_t$  and  $Y_t$  have identical dimension. Then the equilibrium law of motion is a first order non-singular VAR process:

$$Y_t = \Gamma_1 Y_{t-1} + \Gamma_2 \varepsilon_t \quad (5)$$

The VAR approach to generate a counterfactual history starts with estimation of the matrices  $\Gamma_1$ ,  $\Gamma_2$ , and uses the reduced-form model (5) to build the historical shocks  $[\varepsilon_t]_{t=1}^T$ . Once the  $\Gamma_i$  matrices are known, to any data sample  $[Y_t]_{t=1}^T$  corresponds a unique shocks' history  $[\varepsilon_t]_{t=1}^T$ . Bernanke et al. (1999) and Mishkin and Posen (1997) use VARs estimated on data before and after the introduction of inflation targeting to measure the impact of the change in policy. A counterfactual law of motion is built by replacing the interest rate equation in the VAR estimated over the inflation targeting sample with the one estimated on the earlier period, and used to generate a counterfactual history of  $Y_t$ . Since only

the coefficients in the row of  $\Gamma_1$ ,  $\Gamma_2$  corresponding to the policy rule are changed, the policy shift is not allowed to affect the elements of  $\Gamma_1, \Gamma_2$  describing the private sector behaviour: expectations are assumed invariant to policy. More sophisticated variations of the VAR approach are used by Ahmed, Levin and Wilson (2004), Canova and Gambetti (2006), Primiceri (2005), Sims and Zha (2006) to build counterfactual histories to analyze the reduction in US business cycle volatility in recent decades<sup>2</sup>. Benati and Surico (2006) have shown that the VAR approach can have low power in discriminating between structural changes in the economy and changes in the exogenous shocks variance.

Structural methods derive the equilibrium law of motion (5) from estimation of a DSGE model, and then use the DSGE reduced-form model to construct the historical shocks  $[\varepsilon_t]_{t=1}^T$ . This approach has been adopted by various authors, among which Arias, Hansen and Ohanian (2007), Benati and Mumtaz (2006), Blankenau et al. (2001), Justiniano and Primiceri (2006), King and Rebelo (1998), Ingram, Kocherlakota and Savin (1994), Rotemberg and Woodford (1998), Stock and Watson (2003)<sup>3</sup>. Chari, Kehoe and McGrattan (2007) use a similar methodology, although the 'business cycle accounting' they propose identifies time-varying wedges in equilibrium decision-rules consistent with a whole family of models, rather than structural shocks from a uniquely identified model. When the DSGE model is the true data-generating process and the reduced form for the observable  $Y_t$  can be written as in eq. (5), the VAR approach and the structural method will result in the same shocks' history  $[\varepsilon_t]_{t=1}^T$ . But because the latter method estimates the structural coefficients in the matrices  $A, B, C, D, N$ , it is possible to build the counterfactual law of motion by solving the DSGE model conditional on the alternative policy. Therefore, all the elements of  $\Gamma_1, \Gamma_2$  are allowed to change, and expectations are model-consistent.

An important drawback of this methodology is that by construction it requires a small set of structural shocks and the model internal propagation mechanism to explain all the volatility in the data. It is an accounting approach, that does not allow for deviations from the DSGE model law of motion when building the model-consistent shocks vector. This requirement can be very taxing for the stylized models used in the DSGE literature. On the contrary, the VAR approach relies on a reduced-form model and imposes as few constraints as possible on the model describing the data. But it shuts off any change in the counterfactual dynamics of the economy that may come through a change in the

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<sup>2</sup>It is possible to build VAR counterfactuals where expectations are policy-consistent, as in Ahmed, Levin and Wilson (2004) and Primiceri (2005). But in this case there is no assurance that the VAR counterfactual accounts *exclusively* for a change in the policy rule.

<sup>3</sup>These authors use the state-space representation of the equilibrium law of motion rather than the VAR representation. The state-space representation does not require that the vector  $x_t$  be observable to obtain the historical series  $\varepsilon_t$ . Ravenna (2007) discusses conditions for a finite order VAR representation of a DSGE model equilibrium law of motion to exist.

way expectations are formed.

Our methodology extends the accounting approach by allowing for the existence of an additional vector of disturbances  $w_t$  uncorrelated with  $\varepsilon_t$ . The vector  $w_t$  identifies sources of fluctuations beyond the shocks included in the DSGE model, which are given a structural interpretation. The law of motion for the observable variables is then:

$$Y_t = \Gamma_1 Y_{t-1} + \Gamma_2 \varepsilon_t + w_t \tag{6}$$

In essence eq. (6) augments the DSGE model so that there exists a residual variance in the data that the model is not called to explain in terms of movements of the structural shocks. A similar approach is adopted in Boivin and Giannoni (2006) and Ireland (2004), and has been extensively used in the literature to allow estimation of otherwise stochastically singular models (see McGrattan, 1994, Sargent, 1989). Section 4 illustrates how the hybrid DSGE model is used to build counterfactuals which can be meaningfully compared to the historical series.

Once the vector  $w_t$  is included in the model, the structural shocks vector consistent with the observed data and the DSGE model is not unique. Its best linear unbiased estimate can be obtained using the Kalman filter. The Kalman filtering approach thus allows for part of the observable variables' variance to be explained by the vector  $w_t$  when estimating the model-consistent shocks vector, while it uses exclusively the DSGE model to build a counterfactual path with model-consistent expectations.

### 3 The General Equilibrium Model

The Canadian economy is modeled as a small open economy with nominal price rigidities, along the lines of the recent literature on monetary models of the international business cycle (see Bergin, 2003, Gali and Monacelli, 2005, Kollmann, 2001, Obstfeld and Rogoff, 2000). We adopt a small scale model where all goods are traded and firms' only input is labor. The domestic ( $H$ ) sector produces a consumption-good basket that is both consumed by domestic households and exported to the foreign ( $F$ ) sector, in exchange for a foreign-produced consumption good.

A fraction of households and domestic firms set respectively nominal wages according to the Erceg, Henderson and Levin (1999) staggered contracts mechanism, and prices as in Calvo (1983). The remaining fraction is assumed to follow a rule of thumb, so that price and wage setting is partially backward-looking. Households' preferences have a habit-persistence specification. These three features improve the performance of sticky-price models, whose failure to generate plausible degrees of output



and inflation persistence and to match the empirical correlation between real wages and output is well known (Fuhrer, 2000). Christiano, Eichenbaum and Evans (2005) and Rabanal and Rubio Ramirez (2005, 2007) provide evidence that nominal wage rigidity is at least as important as nominal price rigidity in accounting for US and Euro area business cycle fluctuations. Finally, the model allows for short-term incomplete pass-through from the foreign to the domestic price of imported goods.

### 3.1 Consumption Good Aggregates

Assume a continuum of infinitely lived households, indexed by  $j \in [0, 1]$ . Domestic households can purchase a basket of differentiated home- and foreign-produced goods. The consumption aggregate  $C_t$  combines the domestic ( $C_H$ ) and foreign ( $C_F$ ) goods basket in the same proportions as household  $j$  would choose:

$$C_t^j = [(1 - \gamma)^{\frac{1}{\rho}} (C_{H,t}^j)^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} (C_{F,t}^j)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}} \quad (7)$$

where  $0 \leq \gamma \leq 1$  is the share of the foreign good and  $\rho > 0$  is the elasticity of substitution between domestic and foreign goods. The domestic good  $H$  and the foreign good  $F$  are Dixit-Stiglitz aggregates defined over a continuum of differentiated goods indexed by  $i \in [0, 1]$  with elasticity of substitution  $\vartheta$ . Households allocate their expenditure optimally across goods.  $P_t$ ,  $P_{H,t}$  and  $P_{F,t}$  indicate the price indices for the aggregate, domestic and foreign good consumption basket.

### 3.2 Firms

The home production sector is made up of a continuum of firms indexed by  $i \in [0, 1]$ . Domestic firms produce goods by combining labor services supplied by households. Firms regard each household  $j$ 's labor supply  $N_t^j$  as an imperfect substitute for the labor offered by other households. A CES labor aggregator  $N_t = (\int_0^1 N_t^{j \frac{\phi}{\phi-1}} dj)^{\frac{\phi-1}{\phi}}$  combines households' labor services in the same proportions as firms would optimally choose. The wage index  $W_t = (\int_0^1 W_t^{j^{1-\phi}} dj)^{\frac{1}{1-\phi}}$  gives the least expenditure that buys a unit of the labor index. Firm  $i$  optimal demand for each type of labor  $j$ , conditional on the total demand for labor services  $N_t(i)$ , is equal to:

$$N_t^j(i) = \left(\frac{W_t^j}{W_t}\right)^{-\phi} N_t(i) \quad (8)$$

The total cost of the differentiated labor services hired by firm  $i$  is  $\int_0^1 N_t^j(i) W_t^j dj = N_t(i) W_t$ . The firm producing good  $i$  employs a CRS technology:  $Y_t(i) = A_t N_t(i)$ , where  $A_t$  is an aggregate productivity

shock. The firms' cost minimization problem implies that when inputs quantities are chosen optimally the real marginal cost  $MC_t$  is independent of the scale of production. We adopt the hybrid pricing model in Galí and Gertler (1999). As in the time-dependent Calvo (1983) pricing model, in every period  $t$  firms adjust their prices with probability  $(1 - \theta_p)$ . A fraction  $(1 - \omega_p)$  of the price resetting firms update the price optimally, while a fraction  $\omega_p$  follows a backward-looking rule of thumb. The problem of the firm optimally setting the price at time  $t$  consists of choosing  $P_{H,t}(i)$  to maximize

$$E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \Lambda_{t,t+s} \left[ \frac{P_{H,t}(i)}{P_{H,t+s}} Y_{H,t+s}(i) - \frac{MC_{t+s}^N}{P_{H,t+s}} Y_{H,t+s}(i) \right] \quad (9)$$

subject to

$$Y_{H,t+s}(i) = \left[ \frac{P_{H,t}(i)}{P_{H,t+s}} \right]^{-\vartheta} C_{t+s}^W \quad (10)$$

$$MC_{t+s}^N = P_{H,t+s} MC_{t+s} = \frac{W_{t+s}}{MPL_{t+s}} \quad (11)$$

where  $MC^N$  is the nominal marginal cost,  $MPL$  is the marginal product of labor. In (10),  $Y_{H,t+s}(i)$  is the demand function for firm  $i$  output at time  $t + s$ , conditional on the price set  $s$  periods in advance at time  $t$ ,  $P_{H,t}(i)$ . Market clearing ensures that  $Y_{H,t}(i) = C_t^W(i) \equiv C_{H,t}(i) + C_{H,t}^*(i)$  where  $C_{H,t}^*(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\vartheta} C_{H,t}^*$  is foreign demand for good  $i$ ,  $C_{H,t}^* = \gamma^* \left[\frac{P_{H,t}}{e_t P_{F,t}^*}\right]^{-\rho^*} C_t^*$  is aggregate foreign demand and  $C_t^*$  is the exogenously given foreign aggregate demand for imports. Aggregate world demand is defined as  $C_t^W \equiv C_{H,t} + C_{H,t}^*$ . The stochastic discount factor between  $t$  and  $t + s$  is  $\beta^s \Lambda_{t,t+s}$ . The solution to the optimal pricing problem is given by:

$$P_{H,t}(i) E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \Lambda_{t,t+s} \left[ \frac{P_{H,t}(i)}{P_{H,t+s}} \right]^{1-\vartheta} C_{t+s}^W = \frac{\vartheta}{\vartheta - 1} E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \Lambda_{t,t+s} MC_{t+s}^N \left[ \frac{P_{H,t}(i)}{P_{H,t+s}} \right]^{1-\vartheta} C_{t+s}^W. \quad (12)$$

Backward looking firms update their price to the average price set in the most recent round of price adjustment,  $\bar{P}_{t-1}$ , adjusted for the lagged domestically-produced goods inflation rate  $\pi_{H,t-1}$ :

$$P_{H,t}^{RT}(i) = \bar{P}_{H,t-1} (1 + \pi_{H,t-1})$$

Conditional on the shocks vector at time  $t$ , the rule of thumb price  $P_{H,t+k}^{RT}(i)$  converges to the optimal price as  $k \rightarrow \infty$ . This hybrid model ensures that current inflation is determined partly by lagged and partly by expected inflation. At the same time, contrary to the indexation model assumed, for example,

in Christiano et al. (2005), it does not constrain all firms to change prices every period.

### 3.3 Households

Households' preferences are described by the instantaneous utility function:

$$U_t^j = \left\{ \left[ \ln(C_t^j - bC_{t-1}^j) \right] D_t - \frac{\ell N_t^{j^{1+\eta}}}{1+\eta} + \nu \left( \frac{M_t^j}{P_t} \right) \right\}$$

where  $M_t/P_t$  is real money balances,  $N_t$  is the amount of labor service supplied,  $D_t$  is an exogenous preference shock that distorts the labor-leisure decision. Hall (1997) defines this shock as a shift in "households' choice between work in the market and time spent in non-market activities". When  $b > 0$  preferences are characterized by habit persistence. We do not specify the functional form of the utility derived from money holdings  $v(\cdot)$  since we assume monetary policy is conducted through an interest rate rule, and the money demand function only determines the stock of money consistent with the policy interest rate. State-contingent claims let households fully insure consumption against the wage-resetting uncertainty, thus consumption levels are identical across households who supply different amounts of labor services. The aggregate consumption risk cannot be fully diversified, since on the international financial market the only asset traded is a riskless nominal bond.

Households maximize the expected discounted utility flow:

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t U_t^j(C_t^j, N_t^j, \frac{M_t^j}{P_t}, D_t)$$

subject to eq. (7) and the budget constraint:

$$P_t C_t^j + M_t^j + e_t v_t^* B_t^{*j} + \vec{v}_t \vec{B}_t^j \leq W_t^j N_t^j + M_{t-1}^j + e_t B_{t-1}^{*j} + B_{t-1}^j + \Pi_t^j - \tau_t \quad (13)$$

where  $e_t$  is the nominal exchange rate,  $v_t^*$  is the price of a zero-coupon riskless bond priced in foreign currency,  $B_t^*$  is the amount of foreign asset purchased,  $W_t$  is the wage rate,  $\Pi^j$  is the share of profit from the monopolistic firms rebated to the household, and  $\tau$  is a lump sum government tax. Each element of the row vector  $\vec{v}_t$  represents the price of an asset that will pay one unit of currency in a particular state of nature in period  $t+1$ . The corresponding element of  $\vec{B}_t$  represents the quantity of such claims purchased by the household.  $B_{t-1}$  indicates the value of the household portfolio of claims against domestic residents given the current state of nature.

The intra-temporal optimality condition for consumption allocation is:

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\gamma}{1-\gamma} (S_t)^{-\rho} \quad (14)$$

where  $S_t = P_{F,t}/P_{H,t}$  defines the home country terms of trade. The Euler equations for a domestic and foreign riskless nominal bond can be combined to give:

$$\begin{aligned} 0 &= E_t \left\{ MUC_{t+1} \frac{P_t}{P_{t+1}} \left[ \frac{e_{t+1}}{e_t} (1+i_t^*) - (1+i_t) \right] \right\} \\ MUC_t &= E_t \left[ \frac{D_t}{C_t - bC_{t-1}} - \beta b \frac{D_{t+1}}{C_{t+1} - bC_t} \right] \end{aligned} \quad (15)$$

where  $MUC_t$  is the marginal utility of consumption,  $(1+i_t) = v_t^{-1}$  is the gross nominal interest rate and  $(1+i_t^*) = v_t^{*-1}$  is the interest rate paid by domestic residents to borrow on the international capital market<sup>4</sup>.

Households set the nominal wage  $W^j$  in contracts which can be renegotiated with probability  $(1-\theta_w)$ . Of the households resetting the wage contract, a fraction  $(1-\omega_w)$  updates the wage optimally, while a fraction  $\omega_w$  follows a backward-looking rule of thumb. Eq. (8) and the index of aggregate employment  $N_t = \int_0^1 N_t(i) di$  give household  $j$ 's downward sloping demand function for its type of labor:

$$N_t^j = \left( \frac{W_t^j}{W_t} \right)^{-\phi} N_t \quad (16)$$

As in the staggered wage adjustment model of Erceg et al. (1999) any household  $j$  optimally resetting the wage at time  $t$  maximizes its utility functional with respect to the nominal wage  $\tilde{W}_t^j$ , subject to the sequence of budget constraints (eq. 13) and the labor demand function (eq. 16) at time  $t+s$ . Let  $MUN_{t+s}^j = \frac{\partial U_{t+s}^j}{\partial N_{t+s}^j}$ . The first order condition for household  $j$  is:

$$E_t \left\{ \sum_{s=0}^{\infty} (\theta_w^s \beta^s) \left[ \frac{\phi-1}{\phi} MUC_{t+s}^j \frac{\tilde{W}_t^j}{P_{t+s}} \right] N_{t+s}^j \right\} = -E_t \left\{ \sum_{s=0}^{\infty} (\theta_w^s \beta^s) \left[ MUN_{t+s}^j \right] N_{t+s}^j \right\} \quad (17)$$

We assume the rule of thumb adopted by a fraction  $\omega_w$  of the wage-resetting households takes into account the average nominal wage, the CPI inflation rate and the average contract duration  $\frac{1}{1-\theta_w}$ , as

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<sup>4</sup>To ensure stationarity, following Schmitt-Grohe and Uribe (2001) we assume that  $i^*$  is given by the exogenous world interest rate  $\tilde{i}^*$  plus a premium increasing in the real value of the country's stock of foreign debt:  $(1+i_t^*) = (1+\tilde{i}_t^*)g(-\tilde{B}_t)$ , where  $\tilde{B}_t = \frac{e_t B_t^*}{P_{H,t}}$  and  $g(\cdot)$  is a positive, increasing function.

in Rabanal (2001). Backward looking households update the wage to the average level prevalent across all contracts at time  $t - 1$ , adjusted for the current inflation rate  $\pi_t$  :

$$W_t^{RT^j} = W_{t-1}(1 + \pi_t)^{\frac{1}{1-\theta_w}} \quad (18)$$

For  $\omega_w \rightarrow 0$  the model converges to the Erceg et al. (1999) wage-updating mechanism. This hybrid model implies aggregate nominal wage inflation depends explicitly on current and expected CPI inflation through the indexing rule (18).

### 3.4 Import Sector

We model incomplete pass-through of imported goods prices by assuming that the foreign-produced good  $F$  is purchased by a continuum of monopolistically competitive firms in the import sector as an input for production (see Monacelli, 2005). Each firm  $z$  can costlessly differentiate the imported good  $X_F$  to produce a consumption good  $C_F(z)$  using the production technology  $Y_F(z) = X_F(z)$ , where  $X_F(z)$  denotes the amount of input imported by firm  $z$ . The nominal marginal cost of producing one unit of output is defined as  $MC_{F,t}^N(z) = e_t P_{F,t}^*$  where  $P_{F,t}^*$  is the foreign-currency price of  $X_F$ . The domestic-currency price  $P_F(z)$  is set following the Calvo (1983) pricing model with a probability of price re-optimization equal to  $(1 - \theta_F)$ . The producer faces an aggregate demand schedule given by:

$$Y_{F,t}(z) = \left[ \frac{P_{F,t}(z)}{P_{F,t}} \right]^{-\epsilon} C_{F,t}$$

where market clearing implies  $Y_{F,t}(z) = C_{F,t}(z)$ . This production structure generates deviations from the law of one price in the short run, while asymptotically the pass-through from the price of the imported good to the price of the consumption basket  $F$  is complete.

### 3.5 Government and Monetary Authority

The government budget is balanced in every period  $t$ . In the steady state the central bank follows a constant money growth rate policy. When the economy is away from the steady state, the central bank monetary policy is described by the interest rate rule:

$$\frac{(1 + \bar{i}_t)}{(1 + i^{ss})} = \left[ E_t \left( \frac{\Pi_{t+1}}{\Pi_{SS}} \right) \right]^{\omega_\pi} \left( \frac{Y_{H,t}}{Y_{H,SS}} \right)^{\omega_y} \quad (19)$$

where  $\omega_\pi, \omega_y \geq 0$  are the feedback coefficients to deviations of the expected gross CPI inflation rate  $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$  and domestic output from their steady state value. We assume the policy-maker adjusts the interest rate only gradually to the target rate  $\bar{i}_t$  :

$$(1 + i_t) = [(1 + \bar{i}_t)]^{(1-\chi)} [(1 + i_{t-1})]^\chi \varepsilon_{i,t} \quad (20)$$

where  $\chi \in [0, 1)$  is the degree of smoothing and the exogenous shock  $\varepsilon_{i,t}$  represents non-systematic movements in the monetary policy instrument.

### 3.6 Market Clearing and Aggregate Equilibrium

Market clearing in the domestic economy requires

$$Y_{H,t} = \int_0^1 A_t N_t(i) di = A_t N_t = \int_0^1 \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\vartheta} (C_{H,t} + C_{H,t}^*) di$$

or  $Y_{H,t} = (C_{H,t} + C_{H,t}^*) s_t$  where  $s_t = \int_0^1 \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\vartheta} di$ . Since all firms face identical marginal costs, any firm belonging to the fraction  $(1 - \theta_p)$  resetting the price at  $t$  chooses the same new optimal price:  $\tilde{P}_{H,t}(i) = \tilde{P}_{H,t}$  and  $P_{H,t}^{RT}(i) = P_{H,t}^{RT}$ . The CPI index evolves according to:

$$\begin{aligned} P_{H,t} &= \left[ \theta_p P_{H,t-1}^{1-\vartheta} + (1 - \theta_p) \bar{P}_{H,t}^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}} \\ \bar{P}_{H,t} &= \left[ \omega_p P_{H,t}^{RT1-\vartheta} + (1 - \omega_p) \tilde{P}_{H,t}^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}} \end{aligned}$$

Each household purchases an equal amount  $C_{H,t}^j(i)$  so that  $C_{H,t}^j(i) = C_{H,t}(i)$ . Since the marginal rate of substitution between consumption and labor will be equal across all households who can reset the wage at a given time, the wage setting equation (17) implies that all household contracting a new wage at  $t$  will choose the same new optimal wage, so that  $\tilde{W}_t^j = \tilde{W}_t$ . Using the wage index definition and  $W_t^{RTj} = W_t^{RT}$ , the aggregate wage index evolves according to:

$$W_t = \left[ \theta_w W_{t-1}^{1-\phi} + (1 - \theta_w) (1 - \omega_w) \tilde{W}_t^{1-\phi} + (1 - \theta_w) \omega_w W_t^{RT1-\phi} \right]^{\frac{1}{1-\phi}}$$

Aggregating over firms in the importing sector results in the aggregate foreign-good price index  $P_{F,t} = \left[ \theta_F P_{F,t-1}^{1-\vartheta} + (1 - \theta_F) \tilde{P}_{F,t}^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}$  where  $\tilde{P}_{F,t}$  is the price chosen by re-optimizing firms. The law of motion for the net foreign asset stock  $B_t^*$  can be derived using the households' budget constraints and the

market clearing conditions. We assume that the logarithm of the exogenous preference shock  $D_t$ , technology shock  $A_t$ , aggregate foreign demand  $C_t^*$ , and the world interest rate  $\tilde{i}_t^*$  and imports' price inflation  $P_{F,t}^*/P_{F,t-1}^*$  follow a first order autoregressive stochastic process, with random innovation  $\varepsilon_{j,t} \sim N(0, \sigma_j^2)$ . The exogenous policy shock  $\varepsilon_{i,t}$  is assumed to have no serial correlation.

## 4 Counterfactual Histories: Methodology

Write the linearized DSGE model equilibrium law of motion specified in eqs. (3) and (4) as:

$$\xi_{t+1} = F\xi_t + v_{t+1} \quad (21)$$

$$q_t = H'\xi_t \quad (22)$$

$${}^{n+m \times 1}\xi_t = \begin{bmatrix} {}^{n \times 1}\xi_t^1 \\ {}^{m \times 1}\xi_t^2 \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ z_t \end{bmatrix} \quad (23)$$

$${}^{n+m \times 1}v_t = \begin{bmatrix} {}^{n \times 1}0 \\ {}^{m \times 1}\varepsilon_t \end{bmatrix} \quad (24)$$

$${}^{n+m \times n+m}F = \begin{bmatrix} {}^{n \times n}F^{11} & {}^{n \times m}F^{12} \\ {}^{m \times n}0 & {}^{m \times m}F^{22} \end{bmatrix} \quad (25)$$

$${}^{r \times n+m}H' = \begin{bmatrix} {}^{r \times n}H'^1 & {}^{r \times m}H'^2 \end{bmatrix} \quad (26)$$

$$E(v_t v_t') = {}^{n+m \times n+m}Q = \begin{bmatrix} {}^{n \times n}0 & {}^{n \times m}0 \\ {}^{m \times n}0 & {}^{m \times m}\Sigma \end{bmatrix} \quad (27)$$

$$E(v_t v_\tau') = [0] \quad \text{for } \tau \neq t \quad (28)$$

$q_t$  is an  $r \times 1$  vector of observable variables, which may include elements of both the endogenous state vector  $x_t$  and control vector  $y_t$ .  $\varepsilon_t$  is a multivariate Gaussian stochastic process with covariance matrix  $\Sigma$  and unconditional expectation  $E(\varepsilon_t) = 0$ . Capital letters denote matrices. The index in the upper-left corner indicates the size of a matrix. Appendix 7.3 shows how to obtain from eqs. (21) and (22) the VAR representations (5) and (6) discussed in section 2.

The law of motion in eqs. (21), (22) can be inverted to obtain the vectors  $\xi_t^1$  and  $\xi_t^2$  conditional on a vector of observable variables  $q_t$ . Given an initial condition  $\xi_0$  and matrices  $F$ ,  $H'$  the state vector

$\xi_t$  can be calculated from the recursion:

$$\xi_t^1 = F^{11}\xi_{t-1}^1 + F^{12}\xi_{t-1}^2 \quad (29)$$

$$\xi_t^2 = (H'^2)^{-1}[q_t - H'^1\xi_t^1] \quad (30)$$

For  $H'^2$  to be invertible only  $m$  out of the  $r$  variables in the vector  $q_t$  must be included.  $\xi_t^2$  is the historical shocks vector consistent with the DSGE model and the observation vectors  $[q_1 \dots q_t]$ . Eq. (21) can be used to back out the corresponding series of innovations  $\varepsilon_t$ . We label this procedure the 'accounting approach'. Its mechanics imply that under the historical policy the model-simulated path for the observables conditional on the historical vector  $\xi_t^2$  is identical to the data. Chari, Kehoe and McGrattan (2007) use the accounting approach to measure how much of the business cycle fluctuations can be explained by each shock during historical episodes<sup>5</sup>.

The accounting approach constrains the structural shocks to explain all of the variance observed in the data. We relax this assumption by assuming that the observable variables are described by the vector  $q_t^o$ :

$$q_t^o = q_t + w_t \quad (31)$$

where  $w_t$  is an  $r \times 1$  Gaussian vector stochastic process with covariance matrix  $R$ :

$$E(w_t w_t') = R ; E(w_t w_\tau') = [0] \text{ for } \tau \neq t ; E(w_t w_\tau') = [0] \forall \tau \quad (32)$$

In the econometric literature  $w_t$  is assumed to represent a measurement error vector. It can be interpreted as summarizing the volatility in  $q_t^o$  which is not explained by the DSGE model. The introduction of the unobservable vector of disturbances  $w_t$  implies that there exists more than one vector of shocks  $\xi_t^2$  consistent with the DSGE model and given observation vector  $[q_1^o \dots q_t^o]$ . For the system defined by eqs. (21), (22) and (31), and given the assumptions in equations (23) to (28), (32) the *Kalman filtered estimate* of the vector  $\xi_t$  is defined by the recursion (Hamilton, 1994):

$$\begin{aligned} \xi_{t|t} &= \xi_{t|t-1} + P_{t|t-1} H (H' P_{t|t-1} H + R)^{-1} (q_t^o - H' \xi_{t|t-1}) \\ \xi_{t+1|t} &= F \xi_{t|t} \end{aligned}$$

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<sup>5</sup>Chari, Kehoe and McGrattan's (2007) 'business cycle accounting' identifies time-varying wedges in the equilibrium decision-rules, rather than structural shocks, since the covariance matrix  $\Sigma$  is not restricted to be diagonal. The authors show that a large class of models is equivalent to a prototype model with time-varying wedges.



$$\begin{aligned}
P_{t|t} &= P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1} \\
P_{t+1|t} &= FP_{t|t}F' + Q
\end{aligned}$$

where  $\xi_{t+1|t}$  is the linear projection of  $\xi_{t+1}$  on the sample  $[q_1^o \dots q_t^o]$  and a constant and  $P_{t+1|t}$  is the mean squared error (MSE) associated with the forecast  $\xi_{t+1|t}$ :

$$P_{t+1|t} \equiv E[(\xi_{t+1} - \xi_{t+1|t})(\xi_{t+1} - \xi_{t+1|t})']$$

The *Kalman smoothed estimate*  $\xi_{t|T} \equiv \hat{E}(\xi_t | q_1^o \dots q_T^o)$  of the vector  $\xi_t$  is based on the full sample of observable variables. It is defined by the recursion:

$$\begin{aligned}
\xi_{t|T} &= \xi_{t|t} + J_t(\xi_{t+1|T} - \xi_{t+1|t}) \\
J_t &= P_{t|t}F'P_{t+1|t}^{-1} \\
P_{t|T} &= P_{t|t} + J_t(P_{t+1|T} - P_{t+1|t})J_t'
\end{aligned}$$

Any combination of observable variables could be used to build the Kalman smoothed shocks vector  $[\xi_{t|T}^2]_{t=1}^T$ . While all would be model-consistent, the estimate  $[\xi_{t|T}^2]_{t=1}^T$  would differ, unless the model perfectly describes the data. Notice that the addition of the vector  $w_t$  implies the estimation of  $[\xi_{t|T}^2]_{t=1}^T$  is not constrained to use only as many observables as the number of structural shocks in the DSGE model.

A counterfactual history  $[\bar{q}]_{t=1}^T$  is built by simulating the model in eqs. (21) and (22) conditional on a counterfactual law of motion  $\bar{F}^{11}, \bar{F}^{12}, \bar{H}'$  and on the estimate  $[\xi_{t|T}^2]_{t=1}^T$ . A change in the behaviour of the private sector or the government will not affect the matrix  $\bar{F}^{22}$ , describing a shocks' law of motion that is exogenous to the economy by construction. Because the model includes the vector of disturbances  $w_t$ , simulation of the historical equilibrium law of motion  $F, H'$  conditional on the filtered structural shocks in  $\varepsilon_t$  will not generate the historical data series. Part of the variance in the observable  $q_t^o$  is accounted for by  $w_t$ , while the vector  $\varepsilon_t$  (and the implied  $\xi_t^2$  vector) only explains the variance of the unobservable  $q_t$ . Therefore any counterfactual history  $\bar{q}_t$  by construction generates a counterfactual path for  $q_t$  rather than  $q_t^o$ . The law of iterated projections implies that the Kalman smoothed estimate  $q_{t|T} \equiv \hat{E}(q_t | q_1^o \dots q_T^o)$  is equal to  $H'\xi_{t|T}$ . The vector  $q_{t|T}$  can be compared to the

counterfactual  $\bar{q}_t$  conditional on  $\xi_{t+1|T}^2$ , which is computed as:

$$\bar{\xi}_{t+1} \equiv \begin{bmatrix} \bar{\xi}_{t+1}^1 \\ \bar{\xi}_{t+1}^2 \end{bmatrix} = \begin{bmatrix} \bar{F}^{11} & \bar{F}^{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{\xi}_t^1 \\ \xi_{t+1|T}^2 + \tilde{\xi}_{t+1}^2 \end{bmatrix} \quad (33)$$

$$\bar{q}_t = \bar{H}'_t \bar{\xi}_t \quad (34)$$

where  $\bar{F}^{11}$ ,  $\bar{F}^{12}$ ,  $\bar{H}'$  describe the counterfactual law of motion and  $\bar{\xi}_t$  describes the counterfactual path of the state variables. For  $\tilde{\xi}_t^2 = 0 \forall t$  the system in eqs. (33) and (34) simulates the economy dynamics when only the law of motion is changed. In some instances it is useful for some of the  $\tilde{\xi}_t^2$  components to be nonzero in order to build counterfactual histories conditional on alternative shocks series. Note that by construction this method uses *only* the DSGE model in building a counterfactual history, since all the variance in  $q_{t|T}$  and  $\bar{q}_t$  is explained by the model structural shocks. The vector  $w_t$  enters in the estimation of these shocks.

## 5 The Impact of Inflation Targeting

The Kalman filtering approach is employed to evaluate the impact of inflation targeting on the Canadian economy. All counterfactual histories are built simulating the system in eqs. (33) and (34) but rely on alternative assumptions on  $\bar{F}^{11}$ ,  $\bar{F}^{12}$ ,  $\bar{H}'$  and  $\bar{\xi}_t^2$ . Estimation of the Kalman-filtered shocks proceeds as follows. First we estimate the monetary policy rule coefficients over the inflation targeting sample and over the earlier 1971-1990 sample using a GMM estimator. Then we use the Kalman filter to evaluate the likelihood function of the model in eqs. (21), (22) and (31) conditional on the GMM estimates over the inflation targeting sample. The maximum likelihood estimates are used to filter the historical shocks  $[\xi_{t|T}^2]_{t=1}^T$  and to compute counterfactuals. The two-stage estimation procedure for the behavioral parameters avoids the difficulty of estimating the system imposing a structural break in the policy coefficients at an exogenously given date (as in Adolfson et al., 2007). At the same time it permits estimation of the policy rule with the same econometric procedure over the two subsamples, since a number of time series used in the ML estimation are only available starting in 1984 while the GMM estimator allows the use of a larger data set. A detailed description of the data set is contained in Appendices 7.1 and 7.2.

## 5.1 Estimation

### 5.1.1 Monetary Policy Rule

Combining equations (19), (20) and taking a loglinear approximation around the steady state we obtain the monetary policy rule equation:

$$i_t = (1 - \chi)[\omega_\pi E_t \pi_{t+1} + \omega_y y_t] + \chi i_{t-1} + \varepsilon_{i,t} \quad (35)$$

Policy feedback rules as eq. (35) have been shown to provide a good description of monetary policy in many OECD countries, including inflation targeting countries (Clarida et al., 1998). In the DSGE literature, such rules are often described as inflation-targeting instrument rules, and in some models they correspond to the instrument rule implementing the optimal policy (McCallum and Nelson, 2005).

Canadian monetary policy in the period before and after the adoption of inflation targeting is estimated on quarterly data using a two-stage GMM estimator with heteroskedasticity and autocorrelation consistent covariance matrix. The orthogonality condition exploited in the GMM estimation is  $E[i_t - (1 - \chi)\alpha - (1 - \chi)\omega_\pi \pi_{t+1} - (1 - \chi)\omega_y y_t - \chi i_{t-1} | u_t] = 0$  where  $\alpha$  is a constant and  $u_t$  is the set of instruments. The instruments set is similar to the one used by Clarida et al. (1998), and includes: four lags of the interest rate, inflation rate, money supply growth rate and three lags of the output gap. Including lagged values of the real exchange rate among the regressors did not significantly modify the results.

The inflation targeting sample runs from 1992:1 to 2005:1. Inflation targeting was formally adopted in the first quarter of 1991, and the first target was set to reduce CPI inflation in the 1 to 3 percent range by the end of 1995, although inflation averaged 2% already in the first year. The announcement of the central bank's commitment to inflation targeting was carefully planned to attract public attention and shared the support of the government, enhancing its credibility. Before the announcement of specific inflation targets, the Bank of Canada had embarked in a three-year campaign to promote price stability as the long term objective of monetary policy, though it made little headway against the momentum in inflation expectations that had built up (Ragan, 2005). In the fourth quarter of 1990 inflation was still at 4.2%. The pre-inflation targeting sample on which eq. (35) is estimated runs from 1971:1 (following the eight year period ending in 1970 when Canada pegged its exchange rate to the US dollar) to 1990:4. Table 1 reports the estimates, showing that in the inflation targeting period the weight on expected inflation increased from 1.09 to 2.16, while the weight on the output gap

measure dropped from 1.21 to 0.27. The estimate of the coefficient  $\chi$  on lagged interest rate remained very high across subsamples. All estimated coefficients are significant at least at the 5% confidence level.

### 5.1.2 Maximum Likelihood Estimation

The literature on estimation of small open economy models has seen an increasing number of contributions in recent years, using a variety of estimators (see Adolfson et al., 2007, for an application of Bayesian methods). We adopt a maximum likelihood estimator as in Bergin (2003), Bouakez (2005) and Ireland (2004). The object of the estimation is the state-space representation of the loglinear approximation to the DSGE model law of motion, as specified in eqs. (21), (22) and (31). The likelihood function for the sample  $[q_t^o]_{t=1}^T$  can be constructed using the Kalman filter as in Hamilton (1994), and depends nonlinearly on the structural parameters. The Sims (1999) algorithm is used to search for values of the DSGE model parameters and of the matrix  $R$  that will maximize the likelihood function.

The DSGE model parameters are estimated over the 1991:1 to 2007:2 sample using the observed data vector for 10 variables:

$$q_t^o = [Y_{H,t}, \pi_t, i_t, S_t, e_t/e_{t-1}, C_{H,t}^*, \tilde{i}_t^*, \xi_t, N_t, MC_t]^o$$

where the  $o$  superscript indicates that the observation available for variable  $x_{j,t}$  is given by  $x_{j,t}^o = x_{j,t} + w_{j,t}$ . The output variable  $Y_{H,t}$  is measured by the sum of constant-price consumption and net exports, consistent with the model definition of output that includes only domestic and foreign consumption of the home produced good:  $Y_{H,t} = C_{H,t} + C_{H,t}^*$ . Canadian real GDP over the same sample is about 20% less volatile, and shows a very similar pattern of correlations with the other macroeconomic aggregates. The inflation measure  $\pi_t$  is the seasonally adjusted CPI net of indirect taxes. The interest rate  $i_t$  is the 3-month T-bill rate, although nearly identical results are obtained with the overnight rate. The terms of trade series  $S_t$  is the ratio of the Laspeyres index for import and export prices. Nominal exchange rate depreciation  $e_t/e_{t-1}$  is obtained from the Canadian/US dollar exchange rate (trade with the US accounts for about 80% of total Canadian international trade). The exports series  $C_{H,t}^*$  is given by the Canadian constant-price export index for goods and services. The foreign interest rate  $\tilde{i}_t^*$  is the US 3-month T-Bill quarterly yield. Nominal wage inflation  $\xi_t$  is measured by the average hourly earnings for the non-farm sector, including government but excluding the defense and not-for-profit sectors. The series for total labor hours  $N_t$  is built using employment and average weekly hours of the

same non-farm sector. The real marginal cost  $MC_t$  is obtained from the unit labor cost series for the business sector discounted by the appropriate price deflator. All series are measured as log-deviations from the steady state, except for  $Y_{H,t}$ ,  $C_{H,t}^*$ ,  $N_t$  and  $MC_t$  which are logged and Hodrick-Prescott filtered.<sup>6</sup>

The covariance matrix  $R$  for the shocks vector:

$$w_t = [w_{Y_{H,t}}, w_{\pi_t}, w_{i_t}, w_{S_t}, w_{e_t/e_{t-1}}, w_{C_{H,t}^*}, w_{\tilde{i}_t^*}, w_{\xi_t}, w_{N_t}, w_{MC_t}]$$

is assumed to be diagonal. Ireland (2004) suggests a more flexible specification, allowing for a VAR structure of the  $w_t$  components. We experimented with this specification, but the ML estimates of the VAR coefficients and cross-correlations were for the large part not significant, while the robustness of the search algorithm to the initial starting value decreased considerably. The covariance matrix of the structural shocks innovations  $\Sigma$  is assumed diagonal, except for the submatrix describing the covariances for the foreign shocks  $C_t^*$ ,  $\tilde{i}_t^*$  and  $P_{F,t}^*/P_{F,t-1}^*$ . The non-zero correlation across these shocks allows the model to generate a richer dynamics for the foreign sector, and at the same time is a more parsimonious specification than a full blown VAR system.

The variance of the  $w_t$  disturbance for  $\tilde{i}_t^*$  cannot be separately identified from the variance of the shock itself, and we constrain the corresponding element in the  $R$  matrix to be zero. We also constrain to zero the variance of the  $w_t$  disturbance for  $i_t$  which cannot be separately identified from the variance of the policy shock  $\varepsilon_{i,t}$ . The parameters  $\eta$  and  $\phi$  are not identified. To estimate  $\eta$ , the inverse of the steady state labor supply elasticity, the value of the steady state wage markup  $\phi/(\phi - 1)$  is assumed equal to 10%, implying  $\phi = 11$ . Amato and Laubach (1999) estimate a sticky price/sticky wage model of the US economy and find a value of  $\phi$  equal to 8.5 with a standard deviation of 6.1. Finally, the price elasticity of world demand for the home-produced good  $\rho^*$  is set to zero. We use the model's steady state restrictions to set prior to estimation some additional parameters for which the data contain only limited information. The quarterly discount factor  $\beta$  is set to 0.99, which implies a steady state real world interest rate of 4 percent. The foreign good share  $\gamma$  is equal to the steady state ratio between imports and domestic output, and is set to 0.35, the average Canadian import/output ratio over the inflation targeting sample. The elasticity of substitution between domestic goods  $\vartheta$  is set equal to 6, so that the markup in a flexible-price steady state is 20% (Gali and Monacelli, 2005).

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<sup>6</sup>We chose to filter also the series for  $MC_t$  since it is characterized by a strong downward trend throughout the inflation targeting period that the DSGE model is not built to explain.

**Parameter Estimates** ML estimates and standard errors for 30 parameters are reported in Table 2. Most of the parameters are estimated with a low level of uncertainty, including the elements of the covariance matrix  $R$ . The estimate of the standard deviations for the  $w_t$  vector components is of the same order of magnitude as the volatility estimate for some of the structural shocks in  $\xi_t^2$ . This suggests there are important features in the data that the DSGE model propagation mechanism cannot account for. The standard deviation of the preference shock innovation is  $\sigma_d = 5.68$ . This value is large compared to the other disturbances, but it is not unusual for models with nominal rigidities to require volatile shocks to fit aggregate data. Rabanal and Rubio Ramirez (2005) estimation of a sticky price/sticky wage model on US data yields a standard deviation of the taste shock twice as large ( $\sigma_d = 11.88$ ). Del Negro et al. (2006) estimate a preference shock standard deviation of 40.54 within a large-scale New Keynesian model of the US economy. Hall's (1997) empirical decomposition shows that most of the volatility in U.S. labor hours can be attributed to a preference shift between market and non-market activities, and is consistent with the empirical evidence in Eichenbaum, Hansen and Singleton (1988) on the comovements of real wages, consumption and work effort.

The estimates for the price and wage setting parameters deserve an extended comment. The estimates for  $\theta_w$  imply an average wage contract duration of five quarters, with about two thirds of the contracts being reset at the optimal wage. The average length of wage settlements in Canada over the 1991-2000 period is 29.77 months (Longworth, 2002), but this figure does not include contracts in the non-unionized sector, and worker's unionization in Canada has been declining (Dodge, 2003). Additionally, the estimate for  $\theta_w$  implies a measure for the interval between wage adjustment, rather than the interval between contract renegotiations.

The estimate for  $\theta_F$  implies an average price duration for imported good of 3.75 quarters. The estimate of the price setting probability  $\theta_p$  for the domestic production sector implies an implausibly long average price duration of 16 quarters. In addition, a fraction  $\omega_p$  close to 0.9 of the prices are adjusted according to rule of thumb behaviour. When estimating DSGE model on aggregate US postwar data, other authors have obtained estimates in the 0.85 – 0.9 range for  $\theta_p$  and in the 0.82 – 0.96 range for the degree of indexation to past prices among the non-reoptimizing firms (Eichenbaum and Fischer, 2004, Rabanal and Rubio-Ramirez, 2005, Smets and Wouters, 2005). A comparison with the full-indexation price updating mechanism in Chistiano et al. (2005) is useful to illustrate the implications of the ML estimated parameters. These authors assume that a fraction of the firms can reset the price to the optimal level, while the remaining fraction indexes to lagged CPI inflation. Given the timing

assumptions, the loglinear equilibrium condition for inflation is:

$$\begin{aligned}\pi_{H,t} &= \gamma_b^* \pi_{H,t-1} + \gamma_f^* E_{t-1} \pi_{H,t+1} + \frac{(1 - \beta \theta_p)(1 - \theta_p)}{(1 + \beta) \theta_p} E_{t-1} m c_t \\ \gamma_b^* &= \frac{1}{1 + \beta} \quad ; \quad \gamma_f^* = \frac{\beta}{1 + \beta}\end{aligned}\tag{36}$$

where lower case letters indicate log-deviations from the steady state. Since any plausible parameterization for  $\beta$  implies a value close to 1 to be consistent with estimates of the steady state real interest rate, the Christiano et al. (2005) inflation equation necessarily assigns nearly identical weights  $\gamma_b^*$  and  $\gamma_f^*$  to lagged and expected inflation, and a weight whose value is close to 1/2. The Gali and Gertler (1999) rule-of-thumb price resetting mechanism we adopt results in the loglinear equilibrium condition:

$$\begin{aligned}\pi_{H,t} &= \gamma_b \pi_{H,t-1} + \gamma_f E_t \pi_{H,t+1} + \frac{(1 - \omega)(1 - \theta_p)(1 - \beta \theta_p)}{\Psi} m c_t \\ \gamma_b &= \frac{\omega}{\Psi} \quad ; \quad \gamma_f = \frac{\beta \theta_p}{\Psi} \quad ; \quad \Psi = \theta_p + \omega[1 - \theta_p(1 - \beta)]\end{aligned}\tag{37}$$

The estimates in table 2 imply  $\gamma_b = 0.49$  and  $\gamma_f = 0.51$ , that is coefficients on lagged and expected inflation that are virtually identical to the ones that appear in the price indexation model (36). Therefore both price updating mechanisms generate similar reduced form dynamics for inflation. But to be consistent with the reduced form favored by the data, *both* the rule of thumb model and the price indexation model imply an average price duration inconsistent with the microeconomic evidence, too long for the first one (16 quarters), too short for the second one (1 quarter). We interpret this result as evidence that neither of the two price resetting mechanism can be considered fully structural.

### Likelihood Ratio Tests

We use the likelihood ratio to compare the fit of the estimated model to a state-space model where none of the DSGE restrictions is imposed. This is a strategy used by Bergin (2003) to evaluate the performance of the estimated model against an unrestricted model which takes the same reduced form, and therefore nests the DSGE model.

Estimation of the unrestricted model involves finding a parameterization for the matrices  $F$ ,  $H$ ,  $Q$ ,  $R$  that maximizes the likelihood function. Since none of the DSGE model cross-equation restrictions is imposed, the unconstrained model is bound to have a higher likelihood than the constrained DSGE model. The likelihood ratio test evaluates the statistical significance of the improvement in likelihood obtained by removing the DSGE model restrictions. The test accounts for the degrees of

freedom, equal to the number of entries in  $F$ ,  $H$  and in the lower triangular portion of the covariance matrices  $Q$ ,  $R$ , minus the number of estimated parameters in the restricted model. Results are reported in table 2. The test cannot reject the DSGE model restrictions, the p-value of the null hypothesis being equal to 0.23.

The result that the unconstrained model does not perform significantly better compared to the hybrid DSGE model is in part to be attributed to the degrees of freedom allowed by the additional vector of disturbances  $w_t$ . The relevance of the  $w_t$  vector can be evaluated comparing the estimated DSGE model to a model where the variance of (some components of)  $w_t$  is set to zero. Since the model has only six structural shocks, at least four components of  $w_t$  must have non-zero variance to avoid stochastic singularity when estimating the model over the 10-variables data set. Table 2 shows the likelihood ratio test result when the non-zero variance components of  $w_t$  are  $[w_{S_t}, w_{e_t/e_{t-1}}, w_{\xi_t}, w_{N_t}]$ . The zero-variance restrictions on the matrix  $R$  can be rejected at any significance level. Any other choice of zero-variance restriction results in an even lower likelihood.

The likelihood ratio can also be used to evaluate the role of nominal rigidities in the model performance. Table 2 provides the likelihood ratio for the restricted model where domestic wages and prices are reset optimally in every period (the parameters  $\theta_p$ ,  $\theta_w$ ,  $\omega_p$ ,  $\omega_w$  are restricted to be equal to zero) and the benchmark hybrid model. The restrictions are rejected, implying that nominal rigidities play a statistically significant role in fitting the data. Finally, we test separately for the significance of the nominal rigidity in the foreign good import sector. This test is meant to discriminate between the producer and consumer currency-pricing hypothesis for the imported good. A value of  $\theta_F = 0$  implies instantaneous, complete pass-through of foreign price movements. This restriction can be rejected at any level of significance, as shown in the last line of table 2.

### 5.1.3 Structural Shocks Estimate

The Kalman-smoothed estimate of the structural shocks over the 1991:1-2007:2 sample is in figure 1, together with  $\pm 1$  root mean square error (RMSE) bands. The foreign interest rate shock  $\tilde{i}_t^*$  has no RMSE band since is included among the observables. We assume no prior information is available at time  $t = 0$ , and the recursion starts with  $\xi_{1|0} = E(\xi_1)$  and  $P_{1|0} = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\}$ .

The technology disturbance is on average negative until 1994, then takes both negative and positive values throughout the remaining sample. This behaviour is not at odds with the strong Canadian output expansion in the 1990s. It is consistent with the Hodrick-Prescott filtered series for output used in the estimation, and with output gap estimates published by the Bank of Canada in its



Monetary Policy Report, which imply an increase in both output and potential output in this period. Up to 1995 the data also imply a large and persistently negative preference shock, contributing to the severity of the recession in the early 1990s. The monetary policy shock is small over all the sample. The estimates indicate that policy shocks were more volatile in the early years of inflation targeting, while in the 2000 to 2002 period the monetary authority set lower interest rates than warranted by the policy rule.

Figure 2 plots the historical data together with the smoothed estimate  $q_{t|T}$  for the variables  $Y_{H,t}, i_t, S_t, C_{H,t}^*$ <sup>7</sup>. The  $\pm 1$  RMSE error bands for  $q_{t|T}$  are computed from the MSE:

$$MSE_{q_{t|T}} \equiv E\{[q_t - q_{t|T}][q_t - q_{t|T}]'\} = H'P_{t|T}H + R$$

## 5.2 The Luck Hypothesis and the Cost of Inflation Targeting

The first thought experiment compares the smoothed estimate of the historical annual CPI inflation  $\pi_{t|T}$  with the counterfactual path  $\bar{\pi}_t$  under the hypothesis that inflation targeting had not been adopted in 1991 (figure 3). It assumes  $\bar{\xi}_t^2 = \xi_{t|T}^2$  and that  $\bar{F}^{11}, \bar{F}^{12}, \bar{H}'$  are computed conditional on the estimated pre-inflation targeting policy rule.

Table 4 reports statistics to compare the two sample paths. The second row of table 4 shows that inflation targeting did not lower the realized volatility of inflation in the 1991 - 2007 period. What is remarkable is that while the volatility of inflation is virtually identical under the two policies *conditional* on the Kalman-smoothed shocks, the unconditional volatility under the counterfactual policy is more than 60% higher than in the inflation targeting regime. In other words, the historical shock vector was such that the policy rule had no impact on inflation volatility, even if ex-ante we would expect a change in inflation volatility. This result could be the outcome of inflation being very inertial, given the estimates for  $\theta_p$  and  $\omega_p$  in the price-adjustment equation. The large share of rule-of-thumb firms has only a small impact on the result. In an economy with no rule-of-thumb firms (implying  $\gamma_b = 0$  and  $\gamma_f = 0.99$  in eq. 37) the counterfactual policy would have implied an increase in inflation volatility of only 8% (table 4). It is legitimate to ask whether, given the Kalman-filtered shocks vector  $\xi_{t|T}^2$ , the DSGE model is able to generate higher inflation volatility at all. This is indeed the case. Table 4 includes the result from the counterfactual experiment conducted in an economy with a higher steady-state labor supply elasticity  $1/\eta$ , increasing from 0.38 to 1. In this economy the sample volatility

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<sup>7</sup>If the estimate for  $q_t$  is computed from the Kalman filtered series  $\xi_{t|t-1}$ , rather than the Kalman smoothed series  $\xi_{t|T}$ , the estimate  $q_{t|t-1}$  has the interpretation of a forecast and is equal to the forecast  $q_{t|t-1}^o$  since  $E_{t-1}(w_t) = 0$ .

would increase more than twofold relative to the inflation targeting policy case<sup>8</sup>.

Even if it did not affect significantly the volatility, the inflation targeting policy may still have had an important impact on the sample level of inflation, contributing to keeping it low. The last two rows of table 4 illustrate this point. Over the whole sample, inflation would have been on average 40% higher under the counterfactual policy. The level increase is even larger for the period following 2003. Since the average level of inflation is low, a 40% increase in inflation translates in an average inflation differential between the counterfactual and historical level of only 0.43 percentage points.

Overall, it appears that monetary policy was largely irrelevant for inflation volatility, and helped to keep the inflation level lower after 2003. To assess the luck hypothesis, the appropriate question to ask is how likely a path of inflation as observed in the counterfactual history would have been under inflation targeting. To this end, we use the DSGE model to evaluate the probability of observing a given range of (smoothed) inflation values in any time  $t$  conditional on inflation targeting. Figure 3 plots the inflation unconditional 0.84 standard deviations bound above the mean computed from simulating the DSGE model under inflation targeting. Since the vector  $q_t$  has a multivariate normal distribution, in the ergodic state 80% of the  $\pi_t$  realizations will fall below the 0.82 standard deviations bound. The counterfactual inflation path lies above the 0.82 standard deviations bound in 15 periods over the sample. Yet in a long enough sample, inflation realizations as large as desired will occur even conditional on inflation targeting. Table 5 reports the results of a Monte Carlo exercise computing the probability of observing a given number  $n$  of inflation realizations above the 80% probability bound for the inflation targeting policy-maker over a sample as long as the 1991 to 2007 period. Would chance alone allow the economy to experience the performance of the non-inflation targeting policy - as many as 15 inflation observations above the 80% probability bound over 66 quarters? This event would not be unlikely at all: it would occur with a probability of 0.38. Even to generate the 9 consecutive inflation observation above the 0.82 standard deviations bound, as occur in the counterfactual experiment between 2000:1 and 2002:1, the inflation-targeting central bank would not need much bad luck: such an event would be observed in 35% of the samples.

We conclude that the inflation path under the earlier monetary policy would have had a high likelihood of being generated also by an inflation targeting monetary policy. Chance alone allowed inflation stabilization. The introduction of inflation targeting in the early 1990s may have played an

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<sup>8</sup>This result is provided to show that there exist model parameterizations which *can* imply a large impact of policy on inflation volatility conditional on the estimated sample shocks vector. It does not imply that the result on the impact of inflation targeting is extremely sensitive to the estimated parameters, since for different parameter estimates the Kalman-smoothed shocks vector would be different as well.

important role in lowering *average* inflation by anchoring long term expectations, an issue the DSGE model does not address.

The Bank of Canada success in reducing inflation was associated by some critics with a high cost in terms of unemployment (Fortin, 1996). But it is by no means clear that the 10% unemployment rate reached in the early 1990s could be avoided using a different monetary policy (Mishkin and Posen, 1997). In the same period world oil markets created inflationary pressures, while low commodity prices harmed Canadian exports. The output counterfactual (figure 3) shows the inflation targeting policy imposed a substantial cost in the period 1992-1994, when the extra output loss averaged slightly less than 0.5% of trend output per quarter.

### 5.3 The Impact of Policy Shocks

The second counterfactual examines the impact of policy shocks in Canada by simulating an economy conditional on the inflation targeting policy but setting to zero the monetary policy disturbance (figure 4). The counterfactual assumes  $\bar{F}^{11} = F^{11}, \bar{F}^{12} = F^{12}, \bar{H}' = H'$ . We set the  $j^{th}$  component of the vector  $\tilde{\xi}_t^2$ , corresponding to the policy shock  $\varepsilon_{i,t}$ , equal to  $-\xi_{t|T}^{2j}$  so that  $\bar{\xi}_t^{2j} = 0 \forall t$ . The vector composed of all the elements of  $\bar{\xi}_t^2$  except the  $j^{th}$  one,  $\bar{\xi}_t^{2-j}$ , is unchanged and equal to  $\xi_{t|T}^{2-j}$ .

The impact of policy shocks on inflation is limited over the sample, of the order of a quarter of a percentage point. Unexpected movements in monetary policy played a more important role in output dynamics. By lowering the interest rate level, they contributed to keeping output above trend over the 2002-2005 period. The improvement in output is as high as 1%. The counterfactuals show that the non-systematic component of monetary policy did not carry any weight for the poor output performance in the early 1990s.

### 5.4 The Impact of Credibility

Central banks adopting inflation targeting frameworks consider the achieved macroeconomic stability, at least in part, a result of the increased credibility of the policy commitment to inflation stabilization. Some authors (see Kuttner and Posen, 1999) argue instead that the adoption of inflation targeting simply indicates the central bank's shift towards a greater conservatism with respect to the inflation goal, with increased credibility and transparency playing a minor role for the policy's outcome.

The third counterfactual evaluates the role of the inflation targeting credibility in Canadian's inflation performance since 1991 by building a history under the assumption that the shift in monetary policy occurred but was not believed by the private sector (figure 5). It assumes  $\bar{\xi}_t^2 = \xi_{t|T}^2$ . The

matrices  $\overline{F}^{11}, \overline{F}^{12}, \overline{H}'$  are built under the hypothesis that the monetary authority adjusts interest rates according to the estimated post-1991 inflation targeting policy ( $\overline{F}^{11} = F^{11}, \overline{F}^{12} = F^{12}, \overline{H}' = H'$ ), while the private sector expectation  $\tilde{E}_t^{ps}[s_{t+1}]$  of any variable  $s_t$  are conditioned on the belief the central bank adopts the policy rule estimated for the earlier period. Thus the private sector interprets as non-systematic movements in the interest rate  $\varepsilon_{i,t}^{ps}$  what truly is the sum of a policy shock and the distance between the interest rate implied by the true and believed policy rule:

$$\begin{aligned}
i_t - \{(1 - \chi^{ps})[\omega_\pi^{ps} \tilde{E}_t^{ps} \pi_{t+1} + \omega_y^{ps} y_t] + \chi^{ps} i_{t-1}\} &= \varepsilon_{i,t}^{ps} \\
&= \varepsilon_{i,t} + (\chi^{cb} - \chi^p) i_{t-1} \\
&\quad + [(1 - \chi^{cb}) \omega_y^{cb} - (1 - \chi^{ps}) \omega_y^{ps}] y_t \\
&\quad + [(1 - \chi^{cb}) \omega_\pi^{cb} E_t^{cb} \pi_{t+1} - (1 - \chi^{ps}) \omega_\pi^{ps} \tilde{E}_t^{ps} \pi_{t+1}]
\end{aligned}$$

where the index  $ps$  indicates the private sector believes, the index  $cb$  indicates the true value of a parameter in the central bank policy rule, and we assume the central bank's expectation of inflation is model-consistent:  $E_t^{cb} \pi_{t+1} = E_t \pi_{t+1}$ . Appendix 7.4 describes the equilibrium concept adopted to solve the DSGE model when the private sector holds incorrect beliefs.

Figure 5 shows that despite the monetary authority increased aggressiveness towards inflation, the counterfactual inflation volatility is much larger than under the full-credibility historical path. Indeed a non-credible policy would achieve an inflation path very close to the one under the assumption inflation targeting had not been introduced, that is under a less inflation-averse, but fully credible policy. At the same time, the non-credible inflation targeting regime would not lead to as large a stabilization of output as the credible non-inflation targeting regime. The counterfactual shows that conditional on the shocks vector the largest part of the change in inflation dynamics after 1991 originated from the expectation channel: the private sector adjusting its behaviour given the belief that the central bank's stance against inflation has become more aggressive.

To measure the gain from the expectation channel, we build a fourth counterfactual assuming the monetary authority still follows the inflation targeting policy without being believed by the private sector - but at each time  $t$  adjusts the nominal interest rate to bring inflation to its historical level (figure 5). The counterfactual assumes the same matrices  $\overline{F}^{11}, \overline{F}^{12}, \overline{H}'$  as in the previous exercise while  $\tilde{\xi}_t^{2j}$  is computed to ensure  $\bar{\pi}_t = \pi_t$ . Appendix 7.5 provides the recursion to build the appropriate  $\tilde{\xi}_t^{2j}$ . The counterfactual history describes an economy where the central bank achieves the given historical inflation path by actually changing the path of the interest rate through surprise policy interventions,

rather than relying on its commitment to a policy rule to adjust the interest rate in response to the state of the economy.

Figure 5 shows the implied path of output. After 1996, output decreases persistently and dramatically. In the counterfactual, when faced with inflationary shocks the monetary authority raises interest rates and forces the economy into a severe recession to be able to achieve the inflation target path. Firms' expect policy to be more accommodative towards inflation than it really is, and the incorrect believes would lead firms to set persistently higher prices, *ceteris paribus*. The central bank reacts with unexpected increases in the nominal interest rate, that translate in real interest rate increases, to curb demand and the firms' inflationary behaviour. Because only 6% of the prices are updated in every period, incorrect believes are very persistent in the economy, and result in a prolonged recession.

The distance between the historical and counterfactual output represents the gain from credibility: the extra movement in output relative to the one a credible central bank, achieving the same inflation rate, would have experienced. Ex-ante, the distance can be positive or negative, depending on the combination of shocks hitting the economy. Given the Kalman-smoothed shocks, it leads to a prolonged recession in the sample.

## 5.5 Alternative Methods to Build Counterfactuals

Figure 6 compares the baseline Kalman filtering method to two alternative methodologies to build a counterfactual inflation series conditional on the pre-inflation targeting policy. The top panel plots the result from an exercise closer to the accounting approach. The shocks vector  $\xi_{t|T}^2$  is Kalman-filtered using the DSGE model estimates in table 2 but assuming the only non-zero variance components of  $w_t$  are  $[w_{S_t}, w_{e_t/e_{t-1}}, w_{\xi_t}, w_{N_t}]$ . This assumption allows using the same 10 data series to filter shocks as in the benchmark model. This fifth counterfactual is then obtained from the recursion in eqs. (29) and (30) assuming  $\bar{F}^{11}, \bar{F}^{12}, \bar{H}'$  are built conditional on the estimated pre-inflation targeting policy rule. The accounting method implies a shocks vector  $\bar{\xi}_t^2$  with a larger variance, and this in turn returns a counterfactual path for inflation which is much more volatile than the counterfactual smoothed estimate of inflation in figure 3. By construction, our methodology returns a more conservative counterfactual path since a larger portion of the data volatility is explained by the vector  $w_t$ . At the same time, since the volatility of the unobserved component  $w_{\pi_t}$  is constrained to 0, the counterfactual path must be compared to the observed historical path  $\pi_t^o$  - which has a comparatively higher variance than the smoothed estimate  $\pi_t$ . Nevertheless, the accounting method would imply a very substantial impact of the monetary policy shift on inflation dynamics. Over the whole sample, inflation volatility would

increase by 59% relative to the historical volatility. The full-fledged Kalman-smoothed counterfactual implied an increase of only 1%.

The bottom panel of figure 6 still relies on the DSGE model, but shows a counterfactual close in method to the VAR approach. The sixth counterfactual assumes  $\bar{\xi}_t^2 = \xi_{t|T}^2$ . The matrices  $\bar{F}^{11}, \bar{F}^{12}, \bar{H}'$  are built under the hypothesis that the monetary authority adjusts interest rates according to the estimated pre-1991 policy, while the private sector believes the inflation targeting policy is being implemented. The solution method for the DSGE model law of motion is the same as the one used to evaluate the role of credibility. As in the VAR approach, the monetary authority changes its policy, but this change is not allowed to affect the private sector's expectations. This methodology implies that counterfactual inflation would have followed very closely the historical path, the maximum distance being less than 0.2% in any period - and that inflation targeting played virtually no role in stabilizing inflation.

## 6 Conclusions

Industrial countries over the last two decades have experienced lower and more stable inflation - in some countries, as the US, together with increased macroeconomic stability. In countries which explicitly adopted an inflation targeting policy, this outcome is widely considered the result of the shift in monetary policy. Good luck shared across industrial countries rather than good policy among inflation targeters could also explain this outcome.

This paper considers the inflation performance of an early and successful inflation targeter, Canada, to investigate whether the recent period of inflation stability can be explained by the 'luck hypothesis'. Contrary to a vast literature on the impact of inflation targeting, our estimates show that with a high probability good luck can explain the low inflation volatility throughout the inflation targeting period in Canada. The inflation targeting policymaker would have observed the counterfactual inflation path generated by the pre-inflation targeting policy with a probability of 0.35. This is remarkable, since the counterfactual is built from an estimated DSGE model which unconditionally predicts a much lower variance of inflation under inflation targeting, implying the inflation realizations generated by the counterfactual economy are ex-ante an unlikely event under the historical monetary policy. We do not interpret the evidence for the luck hypothesis as implying that monetary policymaking is irrelevant: simply, inflation targeting may have not been put to test by inflationary shocks.

We introduce a method to evaluate the impact of policy announcements' credibility, and show

that the inflation stabilization after 1991 is accounted for the most part by the credible announcement of an inflation target (a shift in the *policy rule*) rather than by the change in the behaviour of the interest rate (a shift in *policy*). We show that a non-credible inflation targeting policy that stabilized inflation at its historical level would have generated a prolonged recession. This result supports the claim that changes in policy regime can dramatically affect the economy dynamics by altering private agents' decision making (Sargent, 1999). Additionally, we show how neglecting the policy impact on expectations can bias the result from the counterfactual exercise.

We obtain our results by Kalman-filtering the historical vector of structural disturbances from a maximum likelihood-estimated DSGE model. We innovate relative to a vast literature on constructing historical counterfactuals using restrictions from DSGE models (as in King and Rebelo, 1998, Justiniano and Primiceri, 2006, Chari, Kehoe and McGrattan, 2007) by allowing for the possibility that the model may be too stylized to describe the data dynamics. Thus we introduce an additional vector of disturbances that identifies sources of fluctuations beyond the ones identified in the DSGE model, an approach used in estimating stochastically singular models (Sargent, 1989, McGrattan 1994). Our estimation shows that forcing the DSGE model to explain all the volatility in the data would overturn the result from the counterfactual exercise.

The drawback of using a structural model to build counterfactuals is reliance on strong assumptions on the economy's shocks propagation mechanism. The DSGE model provides the lens through which the data are read to filter the historical shocks series, and the law of motion to build counterfactuals. While our hybrid model approach helps mitigating potential model mis-specification in filtering the shocks, the subsequent building of a counterfactual history wholly depends on the internal working of the DSGE model. If using a RBC model, our exercise would prove alternative monetary policies irrelevant. If using a completely backward-looking model, the role of credibility would be nil. The question of how to assess policy changes ensuring robustness to the modeling assumptions is open for future research.

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## 7 Appendix

### 7.1 Data Sources and Definitions

All data are sampled at quarterly frequency. Growth, inflation and interest rates are not annualized. Unless otherwise specified, data for Canada is obtained from the CANSIM database, and US data is obtained from the Federal Reserve Bank of St. Louis FRED database.

*Canada* Output: Constant prices chain-weighted GDP, SA. Consumption: Constant prices chain-weighted personal consumption expenditure, SA. Exports: constant prices chain-weighted exports of goods and services, SA. Imports: constant prices chain-weighted imports of goods and services, SA. Interest rate: Three-month Treasury bill tender rate, monthly average. Core inflation (CPIX): all items CPI less the eight most volatile components and the effect of indirect tax. Inflation net of indirect taxes: all items CPI excluding the effect of indirect taxes, SA. Inflation: all items CPI, SA (OECD Economic Outlook). Money supply: Bank of Canada

M2 definition. Labor productivity: output per man-hour, business sector, SA. Import prices: Laspeyeres fixed weight import unit value index, BOP basis, SA. Export prices: Laspeyeres fixed weight export unit value index, BOP basis, SA. Nominal exchange rate: Canadian Dollar to US Dollar exchange rate (IMF-IFS). Nominal wage inflation: average hourly earnings for hourly-paid employees, business sector (includes government and excludes not-for-profit and defense sectors), SA. Hours: average weekly hours for hourly-paid employees including overtime, business sector, SA. Emploment: total payroll employment, business sector, SA. Unit labor cost: business sector, SA. Business sector price index: implicit price deflator, SA.

*US* Interest rate: Three-month treasury bill secondary market rate. Output: Constant prices chain-weighted GDP non-farm business sector, SA (Bureau of Economic Analysis).

## 7.2 Policy Rule Estimation

All data are sampled at quarterly frequency. Growth, inflation and interest rates are not annualized. Over both samples, the output gap measure  $y_t$  is given by HP-filtered constant-prices GDP, which approximates well the Bank of Canada output gap measure published in the Monetary Policy Reports. The policy interest rate during the inflation targeting period is the overnight rate, but since this series is only available since 1975 we measure  $i_t$  with the 3-month Canadian T-bill rate. The two rates are very highly correlated. Since inflation targeting was adopted, the Bank of Canada uses a measure of inflation excluding the impact of indirect taxes and the eight most volatile components as an operational target, and this is the inflation rate used for the  $\pi_t$  series. In the earlier sample the measure of inflation  $\pi_t$  is built using the seasonally adjusted total CPI measure net of indirect taxes, available starting in January 1984, and the seasonally adjusted total CPI index, which starts as early as 1961.

## 7.3 VAR Representation of DSGE models

The DSGE rational expectations equilibrium for  $x_t$ ,  $z_t$ ,  $y_t$  can be written as the system (21), (22) by assuming  $q_t = y_t$ . To obtain the equilibrium law of motion in eqs. (3) and (4) write:

$$\begin{aligned}
 Y_t &= \Gamma_1 Y_{t-1} + \Gamma_2 z_t & (38) \\
 z_t &= N z_{t-1} + \varepsilon_t \\
 Y_t &= \begin{bmatrix} x_t \\ y_t \end{bmatrix} ; \Gamma_1 = \begin{bmatrix} F^{11} & 0 \\ H^{11} & 0 \end{bmatrix} ; \Gamma_2 = \begin{bmatrix} F^{12} \\ H^{12} \end{bmatrix} ; N = [F^{22}]
 \end{aligned}$$

where the vector  $Y_t' = [x_t, y_t]$  has dimension  $1 \times n + r$ . Assume the vector  $z_t$  has dimension  $m = n + r$ ,

therefore the number  $n + r$  of observable variables is equal to the number of shocks, and the matrix  $\Gamma_2$  is square (see Ravenna, 2007, for details on the non-square case). Since:

$$z_t = \Gamma_2^{-1}Y_t - \Gamma_2^{-1}\Gamma_1Y_{t-1} = N[\Gamma_2^{-1}Y_{t-1} - \Gamma_2^{-1}\Gamma_1Y_{t-2}] + \varepsilon_t$$

we obtain a VAR(2) representation for eq. (38):

$$\begin{aligned} Y_t &= (\Gamma_1 + \Gamma_2 N \Gamma_2^{-1})Y_{t-1} - (\Gamma_2 N \Gamma_2^{-1} \Gamma_1)Y_{t-2} + \Gamma_2 \varepsilon_t \\ &= \bar{\Gamma}_1 Y_{t-1} + \bar{\Gamma}_2 Y_{t-2} + \eta_t \end{aligned} \quad (39)$$

If  $N = [0]$  the VAR(2) in eq. (39) simplifies to the VAR(1) in eq. (5). Typically DSGE models have a larger number of observable variables than unobservable shocks, that is  $m < n + r$ . In this case a non-singular VAR representation for the observable variables can still be obtained by including only an  $m$ -dimensional subset of the vector  $Y_t$ . In general, for the DSGE equilibrium to have a finite-order VAR representation  $Y_t$  must include all the elements of the vector  $x_t$  (see Ravenna, 2007).

In the case where part of the variance of  $Y_t$  is explained by random shocks not included in the DSGE model we have that  $Y_t = \Gamma_1 Y_{t-1} + \Gamma_2 z_t + C w_t$  where  $w_t$  is a vector white noise. Then

$$\begin{aligned} Y_t &= (\Gamma_1 + \Gamma_2 N \Gamma_2^{-1})Y_{t-1} - (\Gamma_2 N \Gamma_2^{-1} \Gamma_1)Y_{t-2} + \Gamma_2 \varepsilon_t - \Gamma_2 N \Gamma_2^{-1} C w_{t-1} + C w_t \\ &= \bar{\Gamma}_1 Y_{t-1} + \bar{\Gamma}_2 Y_{t-2} + \eta_t + \tilde{w}_t \end{aligned} \quad (40)$$

where  $\tilde{w}_t = [C w_t - \Gamma_2 N \Gamma_2^{-1} C w_{t-1}]$ . For  $N = [0]$  and  $C = I$  eq. (40) simplifies to eq. (6).

#### 7.4 DSGE Model Solution Method under Private Sector Incorrect Beliefs

Assume a DSGE model is described by eqs. (1) and (2). Define the rational expectation equilibrium under the monetary policy rule  $L_a$  as:

$$\begin{aligned} Y_t &= \Gamma_{a1} Y_{t-1} + \Gamma_{a2} z_t \\ z_t &= N z_{t-1} + \varepsilon_t \end{aligned}$$

If the private sector expects the central bank to behave according to the policy rule  $L_b$ , expectations are consistent

with the rational expectation equilibrium defined by:

$$\begin{aligned} Y_t &= \Gamma_{b1}Y_{t-1} + \Gamma_{b2}z_t \\ z_t &= Nz_{t-1} + \varepsilon_t \end{aligned} \tag{41}$$

Let  $\tilde{E}_t^L$  indicate the expectation of a variable under the belief that the central bank follows the policy rule  $L$ . The structural model (1) for  $Y_t$  can then be written as:

$$\begin{aligned} 0 &= A\tilde{E}_t^b(Y_{t+1}) + BY_t + CY_{t-1} + Dz_t \\ &= A[\Gamma_{b1}Y_t + \Gamma_{b2}Nz_t] + BY_t + CY_{t-1} + Dz_t \end{aligned} \tag{42}$$

The model (42) can be solved for  $Y_t$ . For any policy  $L_a \neq L_b$  the solution for (42) will be different from the reduced-form law of motion (41). If the central bank's policy  $L_a$  is specified in terms of expected inflation, the DSGE model under incorrect private sector beliefs is given by:

$$0 = A[\Gamma_{b1}Y_t + \Gamma_{b2}Nz_t] + A^{cb}E_t(Y_{t+1}) + BY_t + CY_{t-1} + Dz_t \tag{43}$$

The expectation term appears because the central bank's instrument reacts to the rational expectation inflation forecast.  $A^{cb}$  includes the coefficients on the central bank's expected values of  $Y_{t+1}$  in the policy rule. Solving the model (43) for the rational expectations equilibrium yields the law of motion for  $Y_t$ :

$$Y_t = \Gamma_{c1}Y_{t-1} + \Gamma_{c2}z_t$$

which assumes the central bank knows the private sector incorrect beliefs about the policy regime  $L$ , and takes them into account when formulating its own inflation forecast.

## 7.5 Building Policy Shock Series to Achieve Historical Inflation Path

Let  $q_t^i$  define the  $i^{th}$  row of the vector  $q_t$  and  $q_t^{-i}$  the vector including all the rows of  $q_t$  except the  $i^{th}$  one, which we assume is the row corresponding to the inflation variable. Eqs. (33), (34) imply that for any counterfactual

path  $\xi_t^{1'}$  and Kalman-filtered estimate  $\xi_{t|T}^2$  :

$$\begin{aligned}
\bar{\xi}_t^1 &= \bar{F}^{11\bar{1}} \bar{\xi}_{t-1}^1 + \bar{F}^{12} \xi_{t-1|T}^2 \\
q_t^i &= \bar{H}'^{1i\bar{1}} \bar{\xi}_t^1 + \bar{H}'^{2i} \xi_{t|T}^2 \\
&= \bar{H}'^{1i\bar{1}} \bar{F}^{11\bar{1}} \bar{\xi}_{t-1}^1 + \bar{H}'^{1i\bar{1}} \bar{F}^{12} \xi_{t-1|T}^2 + \bar{H}'^{2i} \xi_{t|T}^2 \\
&= \bar{H}'^{1i\bar{1}} \bar{F}^{11\bar{1}} \bar{\xi}_{t-1}^1 + \bar{H}'^{1i\bar{1}} \bar{F}^{12} \xi_{t-1|T}^2 + \bar{H}'^{2i-j} \xi_{t|T}^{2-j} + \bar{H}'^{2ij} \xi_{t|T}^2
\end{aligned} \tag{44}$$

where  $\bar{H}'^{1i}$  ( $\bar{H}'^{2i}$ ) is the  $i^{th}$  row of the matrix  $\bar{H}'^1$  ( $\bar{H}'^2$ ),  $\bar{H}'^{2ij}$  is the  $j^{th}$  column of the matrix  $\bar{H}'^{2i}$ ,  $\bar{H}'^{2i-j}$  is a matrix including all the columns of  $\bar{H}'^{2i}$  except the  $j^{th}$ , which we assume is the column corresponding to the policy shock variable  $\xi_{t|T}^{2j}$ , and  $\xi_{t|T}^{2-j}$  is the vector composed of all the elements of  $\xi_{t|T}^2$  except the  $j^{th}$  one. By setting the variable  $\bar{q}_t^i$  equal to its historical smoothed estimate  $q_{t|T}^i$  eq. (44) can be solved recursively for the shock  $\hat{\xi}_t^{2j} = \hat{\varepsilon}_{i,t}$  :

$$\hat{\xi}_t^{2j} = \left( \bar{H}'^{2ij} \right)^{-1} \left[ q_{t|T}^i \bar{H}'^{1i} - \bar{F}^{11\bar{1}} \bar{\xi}_{t-1}^1 - \bar{H}'^{1i\bar{1}} \bar{F}^{12} \xi_{t-1|T}^2 - \bar{H}'^{2i-j} \xi_{t|T}^{2-j} \right]$$

Since it is assumed that for the inflation equation  $w_t^i = 0 \forall t$ , it follows that  $q_{t|T}^i = q_t^{0i}$  and  $\hat{\xi}_t^{2j}$  is the value of the policy shock ensuring that inflation equals its historical value in the counterfactual history where the dynamics of the economy is described by the matrices  $\bar{F}^{11}$ ,  $\bar{F}^{12}$ ,  $\bar{H}'$ .



**Table 1:** *Estimates of the Monetary Policy Rule*

	$\omega_\pi$	$\omega_y$	$\chi$	$\alpha$
1971 : 1 – 1990 : 4	1.09 (0.51)	1.21 (0.30)	0.91 (0.02)	-0.08 (0.90)
1992 : 1 – 2005 : 1	2.16 (0.72)	0.27 (0.09)	0.84 (0.03)	-0.02 (0.34)

Note: Estimate of eq. (35) on Canadian quarterly data using GMM estimator with heteroskedasticity and autocorrelation consistent covariance matrix.  $\alpha$  is a constant. Standard errors reported in brackets. The instrument set includes four lags of the interest rate, inflation rate, money supply growth rate and three lags of the HP-filtered output. See the Appendix for data sources.

**Table 2: Maximum Likelihood Estimates**

Parameter		Estimate	Standard Deviation
Inverse labor supply elasticity	$\eta$	2.65	1.82
Price elasticity of demand - foreign basket	$\rho$	2.23	0.87
Share of rule-of-thumb domestic firms	$\omega_p$	0.89	0.06
Share of rule-of-thumb households	$\omega_w$	0.32	0.12
Probability of not resetting price - domestic firms	$\theta_p$	0.94	0.04
Probability of not resetting wage	$\theta_w$	0.79	0.09
Probability of not resetting price - import sector	$\theta_F$	0.73	0.07
Habit persistence coefficient	$b$	0.94	0.03
AR(1) parameter - technology shock	$\rho_a$	0.93	0.07
AR(1) parameter - preference shock	$\rho_d$	0.82	0.11
AR(1) parameter - foreign demand shock	$\rho_{c^*}$	0.73	0.06
AR(1) parameter - foreign good inflation shock	$\rho_{\pi^*}$	0.96	0.02
AR(1) parameter - foreign interest rate shock	$\rho_{i^*}$	0.97	0.01
<b>Covariance Matrix Q</b>			
Technology shock innovation	$\sigma_a$	0.47	0.09
Preference shock innovation	$\sigma_d$	5.68	3.27
Monetary policy shock innovation	$\sigma_i$	0.15	0.01
Foreign demand shock innovation	$\sigma_{c^*}$	1.47	0.24
Foreign good inflation shock innovation	$\sigma_{\pi^*}$	0.14	0.04
Foreign interest rate shock innovation	$\sigma_{i^*}$	0.11	0.01
<b>Correlation - foreign shocks innovations</b>	$\text{corr}(c^*, i^*)$	0.56	0.11
	$\text{corr}(c^*, \pi^*)$	-0.31	0.21
	$\text{corr}(\pi^*, i^*)$	0.46	0.19
<b>Covariance Matrix R</b>			
Output	$\sigma_{Yh}$	1.14	0.16
CPI inflation	$\sigma_\pi$	0.38	0.03
Terms of trade	$\sigma_s$	2.73	0.31
Exchange rate depreciation	$\sigma_{\Delta e}$	2.32	0.26
Foreign consumption of domestic good	$\sigma_{ch^*}$	0.89	0.28
Wage inflation	$\sigma_\xi$	0.45	0.04
Labor hours	$\sigma_n$	0.81	0.22
Real marginal cost	$\sigma_{mc}$	1.02	0.16

Note: Maximum likelihood estimates for DSGE model in eqs. (21), (22) and (31). The vector  $[\sigma_{a_t}, \sigma_{d_t}, \sigma_{\varepsilon_{i,t}}, \sigma_{C_t^*}, \sigma_{\pi_{F,t}^*}, \sigma_{\tilde{i}_t^*}]$  contains the square root of the main diagonal entries of covariance matrix  $Q$  for structural shocks innovations. The vector  $[\sigma_{Y_{H,t}}, \sigma_{\pi_t}, \sigma_{S_t}, \sigma_{\Delta e_t}, \sigma_{C_{H,t}^*}, \sigma_{\xi_t}, \sigma_{N_t}, \sigma_{MC_t}]$  contains the square root of the non-zero main diagonal entries of covariance matrix  $R$  for  $w_t = [w_{Y_{H,t}}, w_{\pi_t}, w_{i_t}, w_{S_t}, w_{e_t/e_{t-1}}, w_{C_{H,t}^*}, w_{\tilde{i}_t^*}, w_{\xi_t}, w_{N_t}, w_{MC_t}]$ . Data set includes the variables  $[Y_{H,t}, \pi_t, i_t, S_t, e_t/e_{t-1}, C_{H,t}^*, \tilde{i}_t^*, \xi_t, N_t, MC_t]$ . Sample: 1991:1 - 2007:2

**Table 3: Likelihood Ratio Tests**

Model	Log likelihood value	Likelihood ratio	P value
Benchmark model	757	-	-
Unrestricted model	649	216 (202)	0.23
Restricted covariance matrix R: [ $\sigma_{yh} = 0, \sigma_p = 0, \sigma_{ch^*} = 0, \sigma_{mc} = 0$ ]	797	80 (4)	0.00
Flexible domestic producer prices and wages [ $\theta_p = 0, \theta_w = 0, \omega_p = 0, \omega_w = 0$ ]	771	28 (4)	0.00
Flexible import prices [ $\theta_F = 0$ ]	777	40 (1)	0.00

Note: Likelihood ratio tests with respect to benchmark model. Negative of log-likelihood value reported. Unrestricted model is the model in eqs. (21), (22) and (31) estimated without any restriction on the matrices  $F$ ,  $H$ ,  $Q$ ,  $R$ . Restricted covariance matrix model assumes the only non-zero variance components of  $w_t$  are [ $w_{S_t}, w_{e_t/e_{t-1}}, w_{\xi_t}, w_{N_t}$ ]. Degrees of freedom for likelihood ratio statistic reported in brackets.

**Table 4: Inflation statistics**

	Historical policy	Counterfactual policy	Ratio $\frac{\text{counterfactual}}{\text{historical}}$	Difference $(\pi_{count} - \pi_{hist})$
<b>Standard deviation</b>				
Unconditional	0.98	1.59	1.62	
Conditional	0.72	0.73	1.01	
<b>No rule-of-thumb firms</b> [ $\omega_p = 0$ ]				
Conditional standard deviation	0.75	0.80	1.08	
<b>Higher labor supply elasticity</b> [ $\eta = 1$ ]				
Conditional standard deviation	0.66	1.77	2.69	
<b>Inflation realizations</b>				
1991:1 - 2007:2			1.40	0.43
2003:2 - 2007:2			2.01	0.82

Note: Statistics for smoothed estimate of annual CPI inflation under historical and counterfactual policy rule. Counterfactual policy rule is estimated over the 1971:1 - 1990:4 sample (Table 1). Conditional standard deviation is sample estimate for the realized inflation path given the Kalman-smoothed shocks vector and theoretical expectation of inflation. Benchmark model estimates:  $\omega_p = 0.89$ ,  $\eta = 2.66$ . Inflation realization statistics report sample average ratio and difference between counterfactual and historical smoothed inflation estimate. Standard deviations reported in percentage points.

**Table 5:** *Probability of inflation realizations conditional on inflation targeting policy in the period 1991-2007*

	<i>n</i>				
	3	6	9	12	15
Probability of <i>n</i> or more realizations larger than 0.84 s.d. above the mean	0.83	0.70	0.58	0.47	0.38
Probability of 9 or more consecutive realizations larger than 0.84 s.d. above the mean					0.35

Note: Probability calculated over 25,000 simulations of 66 quarters sample with the model ergodic distribution conditional on the policy rule coefficients estimated over the inflation targeting sample (Table 1).

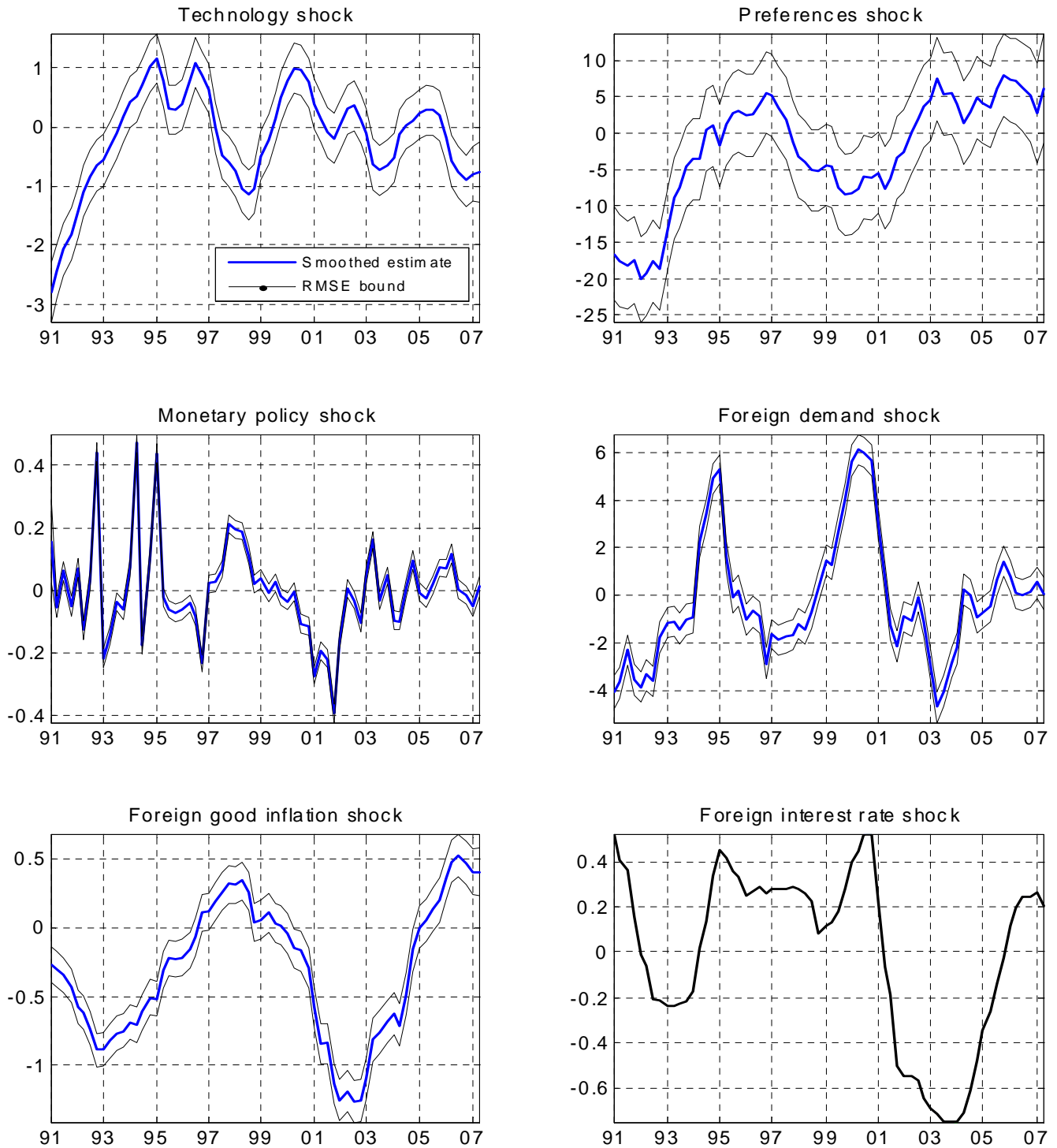


Figure 1: Kalman-smoothed structural shocks vector  $\xi_t^2$ , 1991:1 - 2007:2 sample. Law of motion obtained from DSGE model conditional on policy rule coefficients estimated over the inflation targeting sample (Table 1). Light lines plot Root Mean Square Error bands. Scaling in percent.

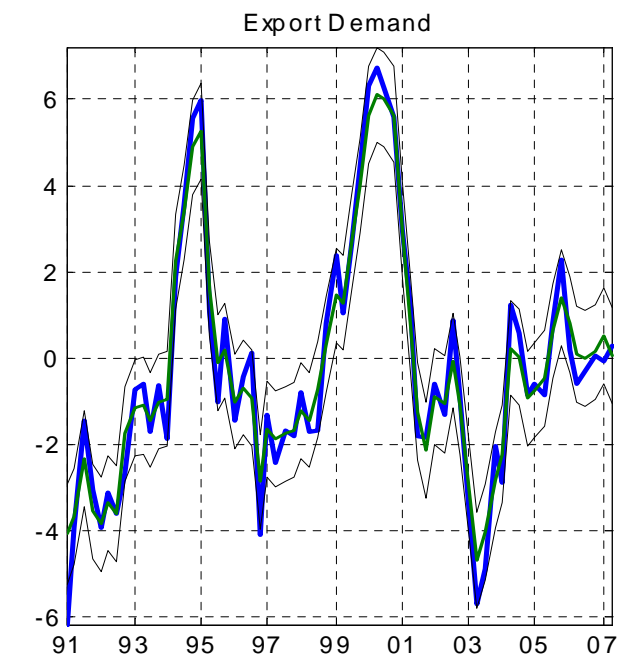
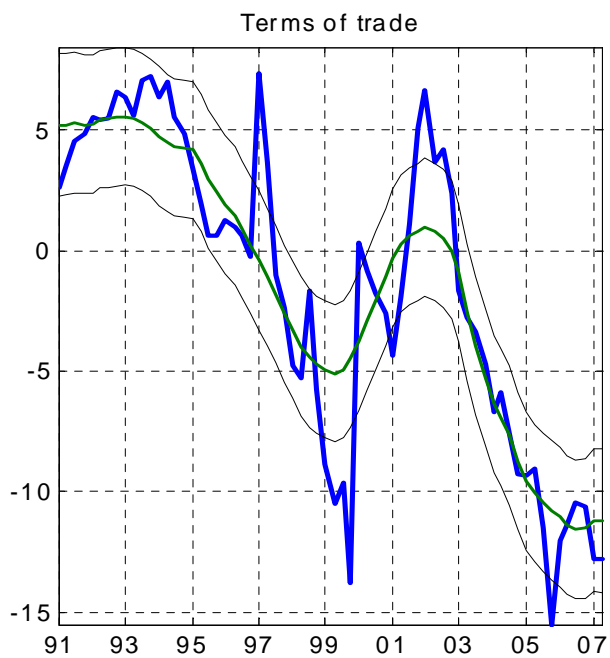
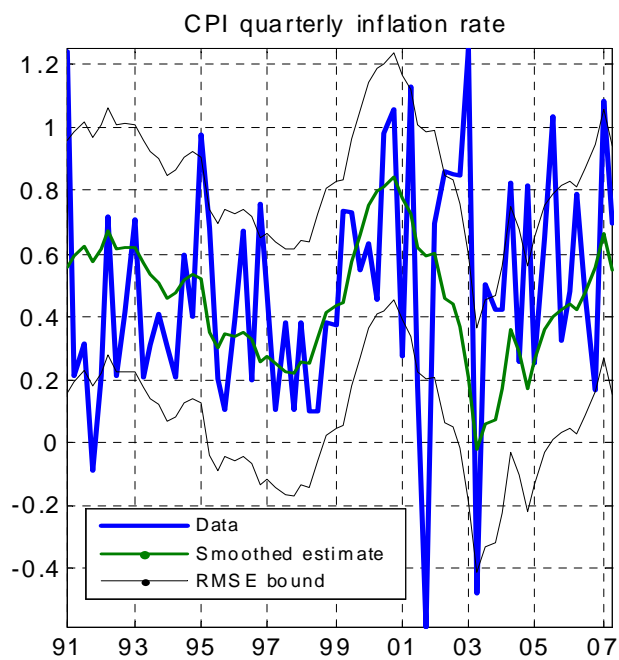
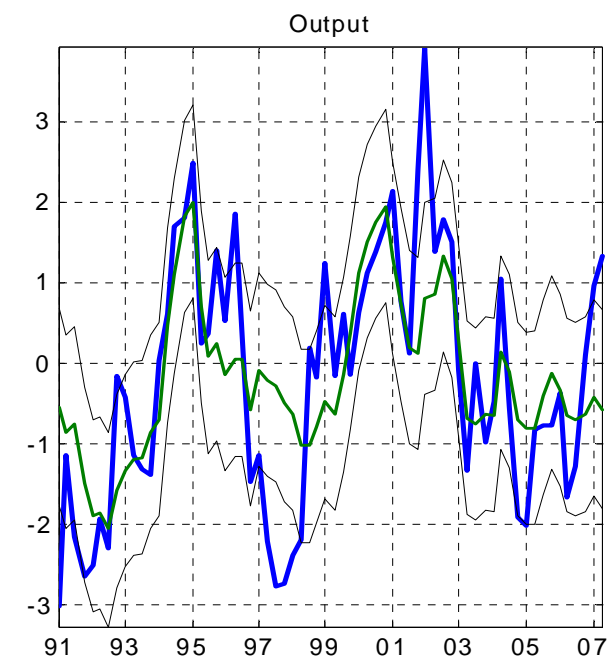


Figure 2: Historical data and Kalman-smoothed estimate. Law of motion obtained from DSGE model conditional on policy rule coefficients estimated over the inflation targeting sample (Table 1). Output is defined as  $Y_{H,t} = C_{H,t} + C_{H,t}^*$ . Light lines plot Root Mean Square Error bands. Scaling in percent.

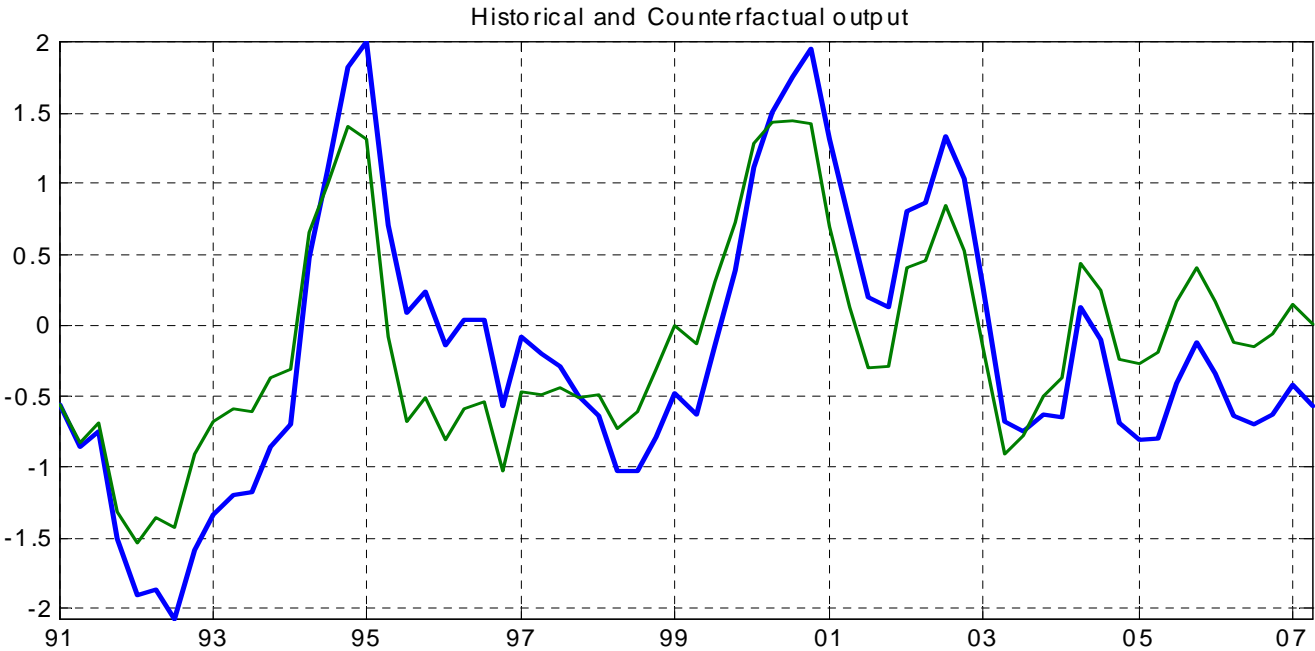
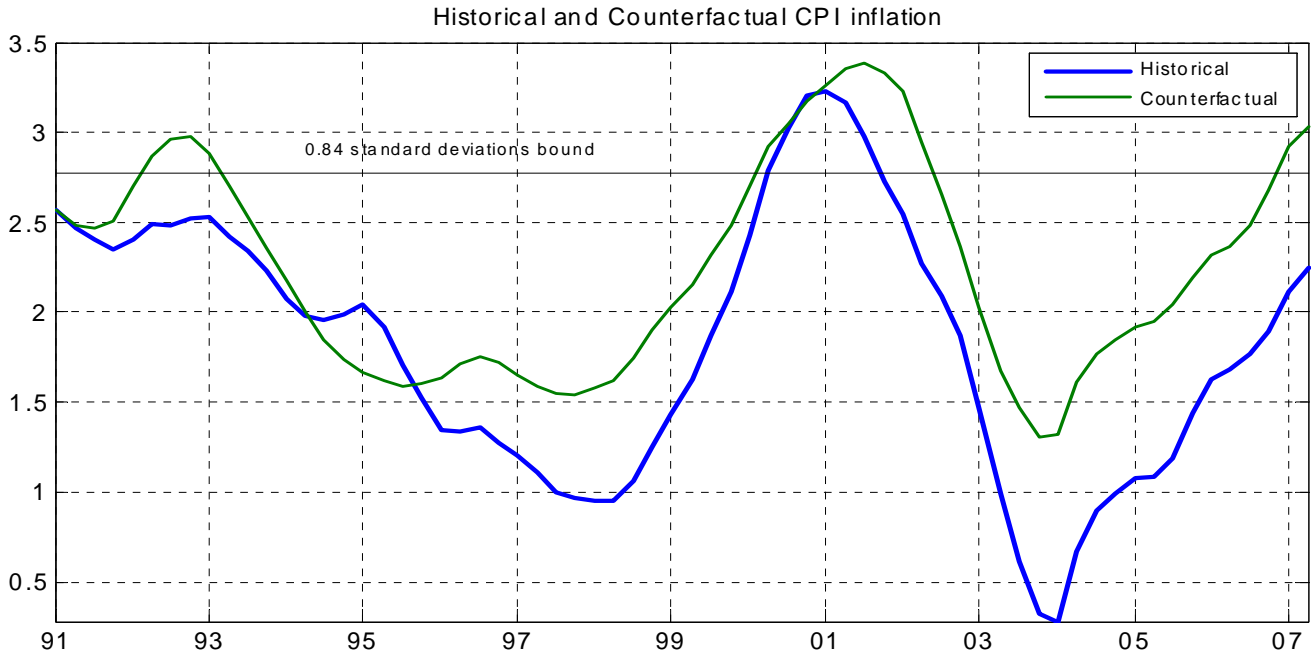


Figure 3: **The impact of inflation targeting.** Historical series vs. counterfactual under pre-inflation targeting policy. Law of motion for counterfactual path obtained from DSGE model conditional on policy rule coefficients estimated over the sample 1971:1 - 1990:4 (Table 1). The historical inflation measure is the Consumer Price Index net of indirect taxes. Inflation and output historical series are the Kalman-smoothed estimate of the data. Conditional on the inflation targeting policy, realizations of inflation larger than 0.84 standard deviations above the mean are observed with probability  $< 20\%$ .



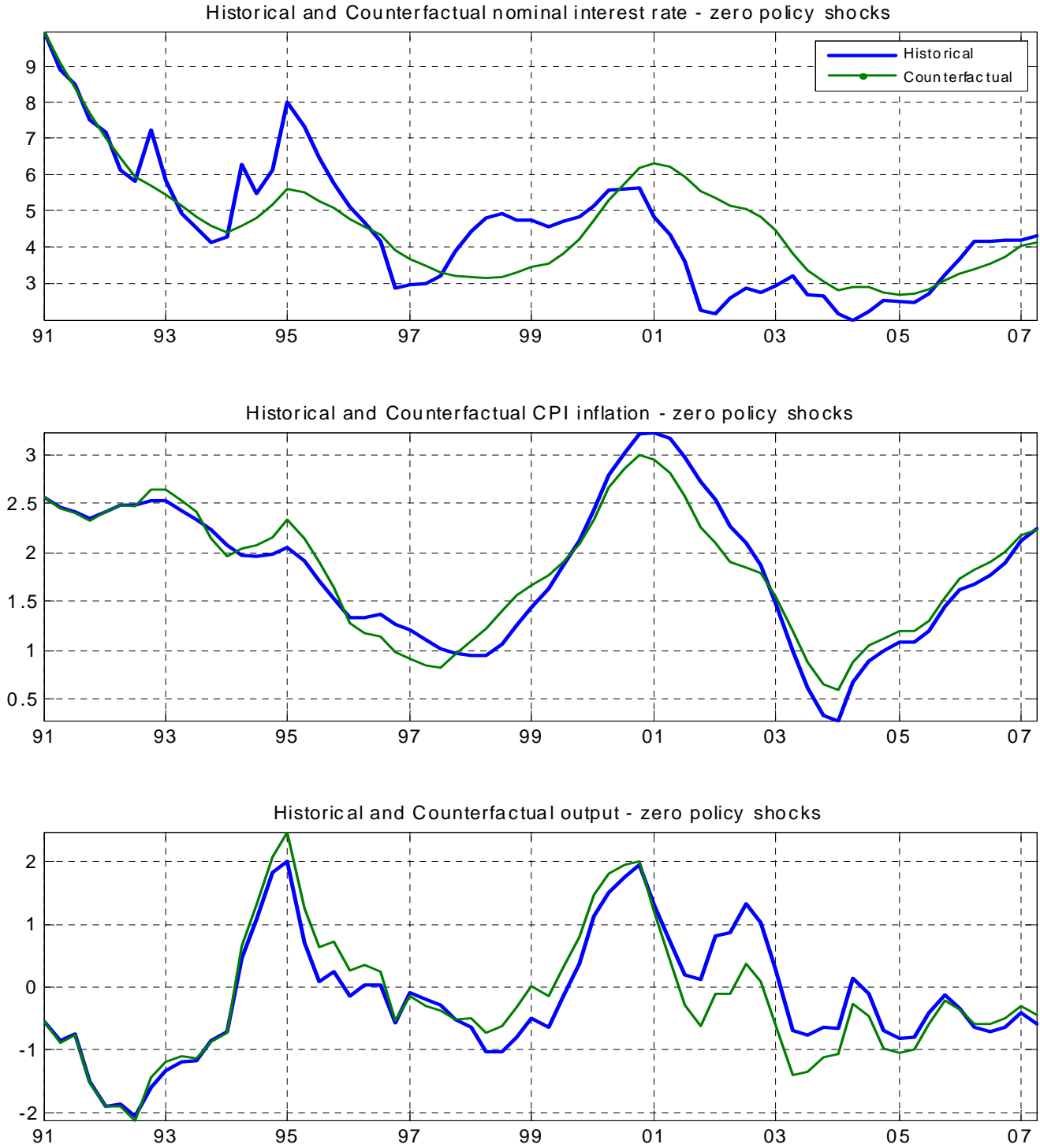


Figure 4: **The impact of policy shocks.** Historical series vs. counterfactual under inflation targeting policy conditional on policy shock  $\varepsilon_{i,t} = 0$  in every period. All other components of the shocks vector are set equal to the Kalman-smoothed estimate. Law of motion for counterfactual path obtained from DSGE model conditional on policy rule coefficients estimated over the inflation targeting sample (Table 1). Historical inflation measure is the Consumer Price Index net of indirect taxes. Inflation, output, and interest rate historical series are the Kalman-smoothed estimates of the data.

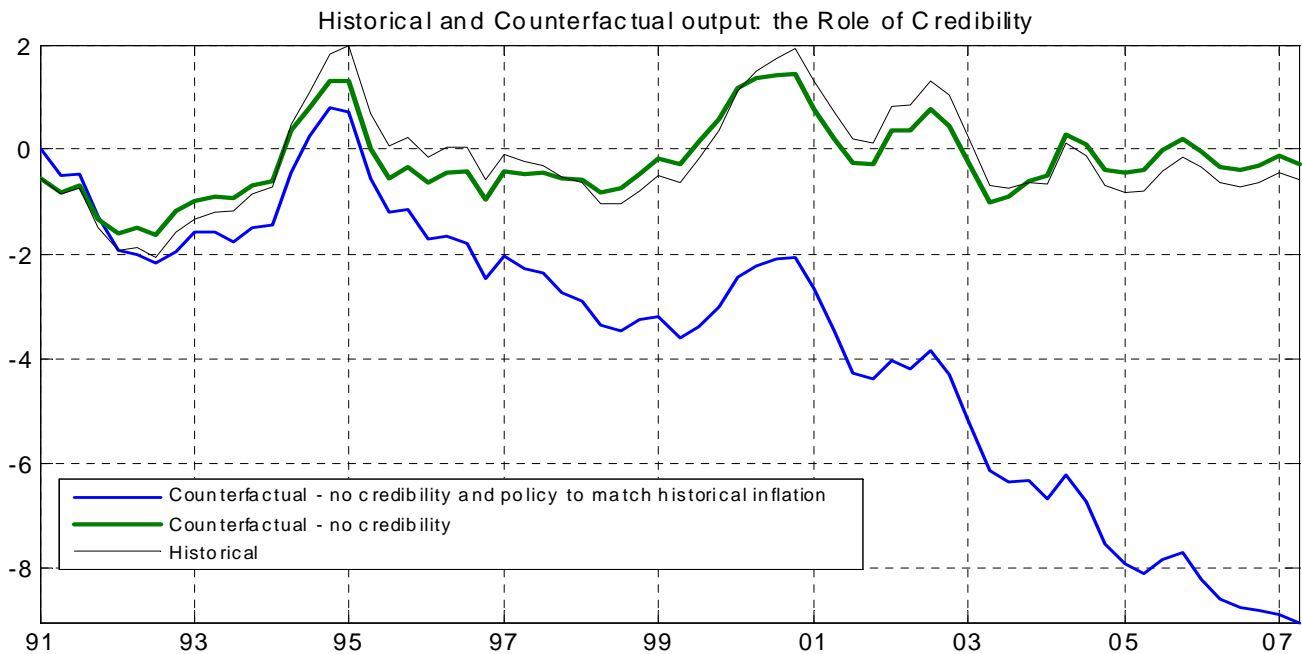
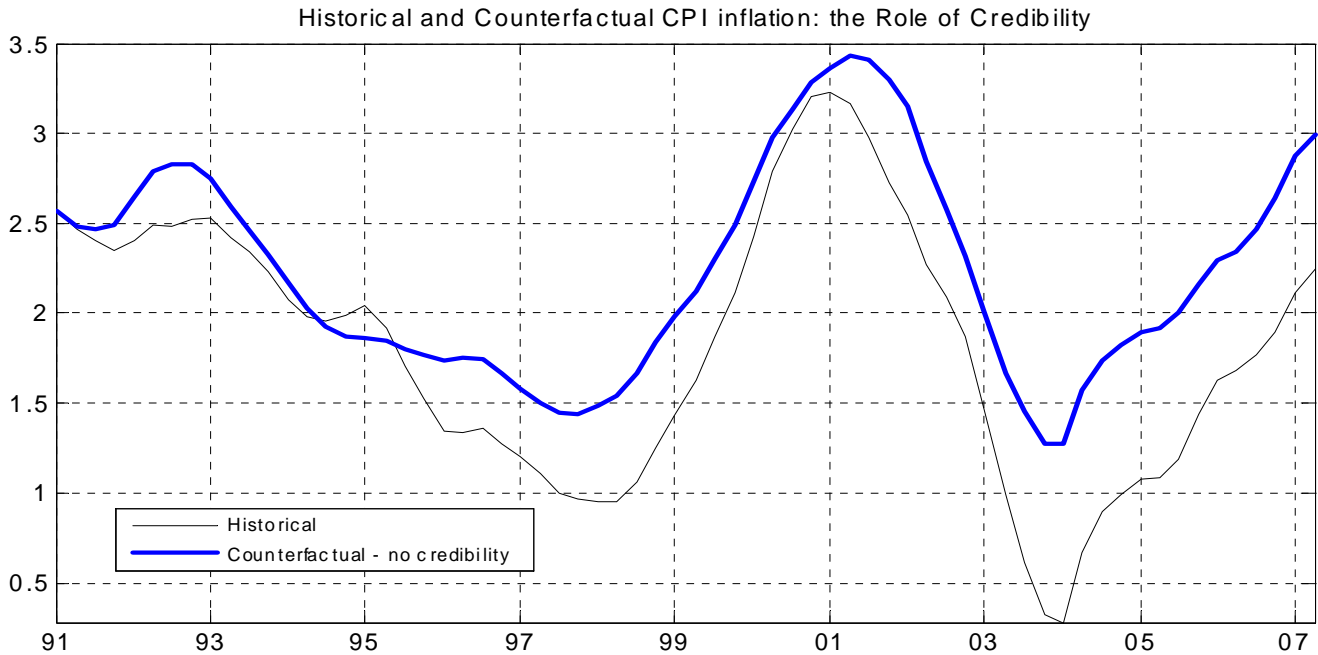


Figure 5: **The impact of credibility.** Historical series vs. counterfactual under inflation targeting policy when policy change to inflation targeting is not believed by the private sector. Law of motion for counterfactual path obtained from DSGE model conditional on policy rule coefficients estimated over the inflation targeting sample. Private sector expectations are consistent with policy rule coefficients estimated over the sample 1971:1 - 1990:4. Bottom panel plots additional counterfactual output when policy shock  $\varepsilon_{i,t}$  is set so that counterfactual inflation follows historical path.

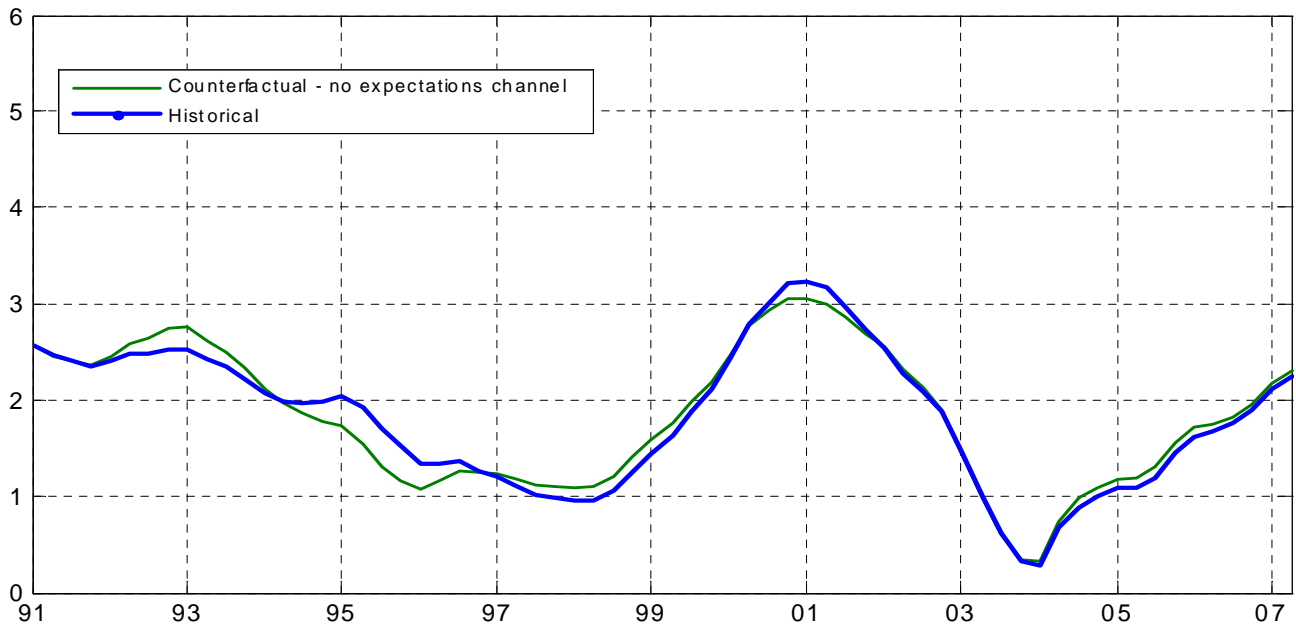
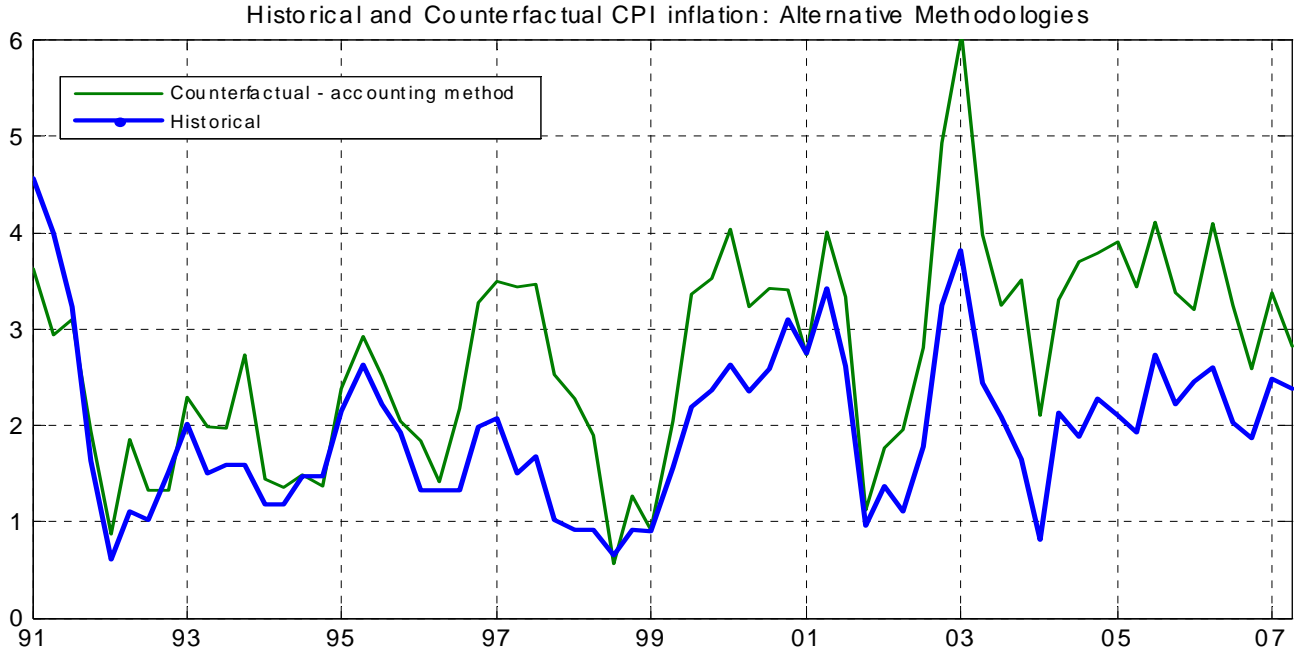


Figure 6: **Comparing methodologies to build counterfactuals.** Historical series vs. counterfactual under pre-inflation targeting policy. Accounting method series is the counterfactual obtained assuming the only non-zero variance components of  $w_t$  are  $[w_{S_t}, w_{e_t/e_{t-1}}, w_{\xi_t}, w_{N_t}]$ . No expectation channel series assumes law of motion is obtained from DSGE model conditional on policy rule coefficients estimated over the sample 1971:1 - 1990:4, while private sector expectations are consistent with policy rule coefficients estimated over the inflation targeting sample.