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A BEHAVIORAL AND WELFARE ANALYSIS

OF PROGRESSIVE FOREST TAXATION***

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ABSTRACT: We use the Hartman rotation model to study behavioral and social welfare effects of forest tax progression. The following new results are shown for harvest and timber taxes. First, a tax-revenue neutral increase in the timber tax rate, compensated by a higher tax exemption, will shorten the optimal private rotation age. A sufficient condition for this to hold for the yield and unit taxes is that the marginal valuation of amenities is non-decreasing with the age of forest stand. Second, for the socially optimal forest taxation, if society can use the neutral site productivity tax to collect tax revenue, the proportional forest tax is enough to internalize the externality caused by private harvesting. Finally, even though site productivity tax is not available, the tax structure should be designed so that tax exemption is neutral implying that the optimal corrective forest taxes remain unchanged.

Key words: rotation, amenity services, forest taxation, tax progression.

JEL classification: Q23, H20.

1. Introduction

Forest taxation literature has conventionally focused on the behavioral and welfare effects of proportional forest taxes (e.g. Johansson and Löfgren 1985, Gamponia and Mendelsohn 1987, Koskela and Ollikainen 2001b). Less attention has been devoted to the question whether forest taxes should be non-proportional. It is well-known that even though private landowners value amenity services from forests stands the privately optimal rotation may differ from the socially optimal rotation if non-landowner citizens have free access to amenity services of private forests. Hence, one can ask whether progressive taxation is needed as a means of adjusting the privately optimal rotation age towards social optimum.

Countries usually apply forest tax systems, which differ in details from the idealized tax systems analyzed in the forest economics literature. A typical feature is the possibility of making tax exemptions for some costs and items. For instance, in Finland, Sweden and Norway the government taxes harvest revenue and at the same time allows for some tax exemptions, which are either of lump sum type or based on actual costs. Similar features can be found also in the forest tax system of some states in U.S.¹ These tax exemptions induce progression in the tax system in the sense that the average tax rate increases with the tax base. Such features of progression raise several questions: first, how do they affect the harvesting behavior of private landowners; second, if government wants to keep the tax revenue from forestry constant, how does a switch between the tax rate and tax exemption change harvesting; and third, are tax exemptions desirable from a social welfare point of view.

Previous research has neglected these questions. The purpose of our paper is to examine, for the first time in the literature, the behavioral and social welfare effects of progressive forest taxation in the Hartman framework, where forests provide harvest revenue and amenity services.² We also incorporate public finance aspect into the analysis by assuming that

¹ See Boyd (1996) for a presentation of the U.S. forest tax system and Grayson (1993) for a description of forest tax systems in the European countries.

² The only paper, where the effects of nonlinear taxes on forest rotations in the Faustmann model are studied, is Mendelsohn (1993), but his analysis does not suit for the Hartman framework, where, unlike in the Faustmann model, forest stands produce also amenity services and private and social rotation age usually differ. This is because the comparative statics of forest taxes differs from those of the Faustmann model when the private landowners follow Hartman behavior. For an application of a two-period model to explore empirically the impact of nonlinear income taxes on harvesting, see Aronsson (1993), and for the effects of inheritance tax exemptions on forest taxation, see Uusivuori (2000).

the government has an intertemporal budget constraint. Thus we apply the conventional optimal taxation framework, applied to forestry also in Amacher and Brazee (1997) and Koskela and Ollikainen (2001), but to study different research questions. We define tax progression by assuming that the marginal tax rate is constant and that there is a tax exemption, i.e., a threshold level below which there is no taxation and above which the marginal tax rate is charged. Due to the tax exemption the average tax rate increases with the tax base so that taxation becomes linearly progressive. We consider both property and harvest taxes. A specific *property tax* under our study is *a timber tax* that is levied on the stumpage value of growing trees. Out of the *harvest tax*es we study *a yield tax*, and *a unit tax*, which are levied on the harvest revenue and on the harvested timber volume, respectively.

We establish several new results. First, a tax-revenue neutral increase in the timber tax rate – compensated by a higher tax exemption – shortens the optimal private rotation age due to the substitution effect. A sufficient condition for this result to hold for the yield tax and the unit tax is that the marginal valuation of amenities is non-decreasing with the age of the forest stand. Second, for the socially optimal forest taxation we initially show that if the society can use the optimal site productivity tax then, in the absence of equity considerations, the proportional forest tax is enough. Under proportional taxation, a society has two tax instruments to collect the tax revenue and internalize the externality caused by private harvesting. Therefore progression is not needed. Finally, we prove that in the absence of site productivity tax the tax structure, that is, tax rate and tax exemption, should be designed so that the marginal costs of public funds is equal to one, i.e., the required tax revenue is raised in a non-distortionary fashion. This allows the targeting of the distortionary tax towards the externality in forestry.

We proceed as follows. In section 2, we analyze how forest tax parameters affect the privately optimal rotation age in the Hartman framework. The effects of tax revenue- neutral changes in forest tax progression on private rotations as well as the social desirability of the progression in forest taxation are analyzed in section 3. In section 4, we offer some concluding remarks.

2. Forest tax progression in the Hartman model

2.1 Basic Model

Assume that the representative private landowner values both the net harvest revenue and the amenity services from the forest stand. Following Hartman (1976) we postulate the following quasi-linear objective function in the absence of taxes over an infinite cycle of rotations in order to solve the single representative (steady state) rotation age

$$W = V + E , \tag{1}$$

where $V = (pf(T)e^{-rT} - c)(1 - e^{-rT})^{-1}$ and $E = (1 - e^{-rT})^{-1} \int_{0}^{T} F(s)e^{-rs} ds$ describe the net

present value of harvest revenue and of amenity services over all rotations, respectively. The notation is as follows: p is stumpage price, f(T) is the volume of the stand as a function of its age T and c denotes the regeneration cost. Finally, F(s) describes the valuation of amenities provided by the stand of age s. The first-order and second-order conditions of the Hartman model, as well as the relationship between the Hartman and Faustmann rotation ages, are well-known (see e.g. Hartman 1976, or Koskela and Ollikainen 2001a).

2.2 Analytics of forest taxation in the presence of tax progression

Next we introduce non-proportional forest taxation by assuming that the tax rate is constant and there is a tax exemption, i.e., a threshold below which there is no taxation and above which the constant tax rate is charged. This means that the average tax rate increases with the tax base so that taxation is linearly progressive.³ We express both the government tax revenue function and the landowner's objective function and report the comparative static effects of the taxes and tax exemptions on rotation age. While the former is known from previous analysis (Koskela and Ollikainen 2001a), results concerning tax exemptions are new.

³ For a seminal paper about tax progression, see Musgrave and Thin (1948) and for further discussion, see Lambert (1993 Ch. 6).

A. Timber tax

We start with the timber tax, β , levied on the stumpage value of growing trees. The tax base over infinite series of rotations can be expressed as $U = (1 - e^{-rT})^{-1} \int_{0}^{T} pf(s)e^{-rs} ds$. Tax exemption is denoted by *a*, so that its present value is given by $A = a(1 - e^{-rT})^{-1}$. The government tax revenue can be written as

$$R = \frac{\beta \left[\int_{0}^{T} pf(s)e^{-rs} ds - a \right]}{(1 - e^{-rT})} = \beta (U - A) .^{4}$$
(2)

According to equation (2), the average tax rate $\frac{R}{U} = \beta(1 - \frac{A}{U})$ increases (decreases) with the tax base U when A > (<) 0. More precisely, if $\beta > 0$ and a > 0 we have a progressive taxation, while a regressive taxation in the case with ($\beta > 0, a < 0$).⁵

Under non-proportional timber taxation the objective function of the landowner can be expressed as

$$\hat{W}(\boldsymbol{\beta}, a) = V - \boldsymbol{\beta}[U - A] + E, \qquad (3)$$

where the second RHS term describes the taxation part. The first-order condition for the privately optimal rotation age is

$$\hat{W}_{T}(\beta, a) = pf'(T) - rpf(T) - rV - \beta(pf(T) - r(U - A)) + F(T) - rE = 0.$$
 (4)

⁴ Equation (2) is sometimes referred to as unmodified property tax, because β is levied on the harvest revenue generation potential of the forestland (see e.g. Englin and Klan 1990, Gamponia and Mendelsohn 1987 and Uusivuori 1997). By using the notion "timber tax", we follow here the terminology presented in Chang (1982).

⁵ If β is a subsidy ($\beta < 0$), then the average subsidy rate increases (decreases) with U when a > (<) 0 respectively. In the former case the subsidy is progressive, while in the latter case it is regressive.

The interpretation of (4) is the following: the private landowner equates the marginal benefit of delaying the harvest to age *T*, defined by $p(f'(T) - \beta f(T)) + F(T)$, to the marginal opportunity cost of delaying the harvest, defined by $r[pf(T) + (V - \beta(U - A) + E)]$. By total differentiation we get

$$T_{\beta}^{H} = -\frac{\hat{W}_{T\beta}}{\hat{W}_{TT}} < 0, \qquad (5a)$$

where $\hat{W}_{TT} < 0$ and $\hat{W}_{T\beta} = -(pf(T) - r(U - A)) < 0$. Hence, a rise in the timber tax will shorten the rotation age with (A > 0) or without (A = 0) the tax exemption. As for the impact of tax exemption we get

$$T_a^H = -\frac{\hat{W}_{Ta}}{\hat{W}_{TT}} \begin{cases} < \\ = \\ > \end{cases} 0 \quad \text{as} \quad \beta \begin{cases} > \\ = \\ < \end{cases} 0 \tag{5b}$$

where $\hat{W}_{Ta} = -\beta r(1 - e^{-rT})^{-1}$. A higher tax exemption with timber tax (subsidy) will shorten (lengthen) the rotation age thus reinforcing the effect of timber tax. The economic interpretation of this finding is the following. A rise in the tax exemption with timber tax increases the opportunity cost of waiting via the term rA. Therefore, the landowner has an incentive to shorten the rotation age.

B. Yield and unit tax

Next we study the analytics of harvest taxes. In the presence of a non-proportional yield tax on harvesting, τ , the net present value of revenue from rotations can be written as $V - \tau [Y - A]$, where $Y = pf(T)e^{-rT}(1 - e^{-rT})^{-1}$ and $A = a(1 - e^{-rT})^{-1}$ with *a* denoting the tax exemption.⁶ The objective function of the landowner can now be expressed as

⁶ If timber markets are perfectly competitive in the sense that firms and landowners are price-takers in the timber markets, then the yield tax effects are qualitatively equivalent to the effects of the unit tax. This is a well-known result in public economics literature. Proof is available from the authors upon request.

$$\hat{W}(\tau, a) = V - \tau(Y - A) + E \tag{6}$$

The first-order condition for maximizing (6) is

$$\hat{W}_{T}(\tau, a) = p(1-\tau) [f'(T) - rf(T)] - r(V - \tau(Y - A)) + F(T) - rE = 0.$$
(7)

Equation (7) has the same interpretation as equation (4): the private landowner equates the marginal benefit of delaying the harvest to age T, $p(1-\tau)f'(T) + F(T)$, to the marginal opportunity cost of delaying the harvest, $r[p(1-\tau)f(T)+V-\tau(Y-A)+E)]$. The impact of the yield tax on the private rotation age is given by

$$T_{\tau}^{H} = (-\hat{W}_{TT})^{-1}\hat{W}_{T\tau}$$
(8a)

where $\hat{W}_{T\tau} = -[p(f'(T) - rf(T)) - r(Y - A)]$. Utilizing the first-order condition (7) we get

$$T_{\tau}^{H} \begin{cases} > \\ = \\ < \end{cases} \quad 0 \quad as \ rc^{*} (1 - e^{-rT})^{-1} + F(T) - rE \begin{cases} > \\ = \\ < \end{cases} \quad 0 \quad . \tag{8b}$$

where $c^* = (ce^{-rT} - a)$. Equation (8b) is familiar from previous work (see Koskela and Ollikainen 2001a) with the exception that now also the tax exemption affects the sign of the tax effect. It still holds that if $c^* > 0$, that is, the tax exemption is not too high, then $F(T) \ge rE \Leftrightarrow F'(T) \ge 0$ provides a sufficient, but not necessary condition for the result that a rise in the yield tax lengthens private rotation age (see Koskela and Ollikainen 2001a for a more detailed discussion of the role of marginal amenity valuation).

For the effect of the yield tax exemption we get

$$T_a^H = -\frac{\hat{W}_{Ta}}{\hat{W}_{TT}} \begin{cases} < \\ = \\ > \end{cases} 0 \quad \text{as} \quad \tau \begin{cases} > \\ = \\ < \end{cases} 0 \tag{8c}$$

where $\hat{W}_{Ta} = -\tau r(1 - e^{-rT})^{-1}$. Hence, a higher tax exemption shortens the rotation age irrespective of the relationship between the regeneration costs and the marginal valuation of amenity services. Interpretation is similar as in the previous case. A rise in the tax exemption increases the opportunity cost of waiting via term rA. Hence the rotation age will become shorter.

2.3 The Slutsky decompositions of tax rate effects

Thus far we have studied the behavioral effects of the chosen forest tax rates and tax exemptions. For the purposes of further policy analysis in section 3, we next derive the Slutsky equations for the rotation effects of timber and yield taxes. Slutsky equations decompose the total effects of taxes into substitution and income effects by treating the tax exemption as a lump-sum income. When the tax rates change both the harvesting and welfare of the representative landowner changes. Therefore one can ask, what is the combination of tax rate and tax exemption changes that will keep the utility of the representative landowner constant? Derivation of the substitution effect of the tax rate provides a definite answer to this question (see Appendix 1 for the details).

The Slutsky decompositions of the total effects of taxes into substitution and income effects can be expressed as follows:

$$T_{\beta}^{Hc} = \underbrace{T_{\beta}^{H}}_{-} + \underbrace{\frac{(1 - e^{-rT})(U - A)}{\beta}}_{-} T_{a}^{H}}_{-}$$
(9a)
$$T_{\tau}^{Hc} = \underbrace{T_{\tau}^{H}}_{?} + \underbrace{\frac{(1 - e^{-rT})(Y - A)}{\tau}}_{-} T_{a}^{H}}_{-}.$$
(9b)

Equations (9a) and (9b) formalize the substitution effects of timber and yield taxes as the sum of the total tax effect and the compensated tax exemption effect, which will keep the utility of the landowner unchanged. By using comparative statics results developed earlier, the substitution effects can be calculated to be of the following form

$$T_{\beta}^{H^{c}} = \frac{pf(T)}{\hat{W}_{TT}} < 0 \tag{10a}$$

$$T_{\tau}^{H^{C}} = \frac{p[f'(T) - rf(T)]}{\hat{W}_{TT}} \begin{cases} > \\ = \\ < \end{cases} 0 \quad as \quad r(V - \tau(Y - A)) \begin{cases} < \\ = \\ > \end{cases} F(T) - rE \tag{10b}$$

The economic interpretation of (9) - (10) goes as follows.⁷ A rise in the timber tax decreases both the value of the forest stand at harvest time and the opportunity cost of harvesting, with the former effect dominating. Through this mechanism the rotation age becomes shorter. A higher timber tax also decreases the indirect utility of the landowner. The substitution effect of the timber tax results from an answer to the following question: What happens if the utility loss due to the higher timber tax is compensated by increasing the tax exemption so that the welfare of the landowner remains unchanged? We know from comparative statics that higher tax exemption will shorten the rotation and thus reinforce the negative tax rate effect. Hence the substitution effect of timber tax on the rotation is negative.

A rise in the yield tax reduces the net timber price. This tends to increase the rotation age. If the marginal valuation of amenities is a non-decreasing function of the age of the forest stand, then a higher yield tax does not make timber production less profitable relative to amenity production. Under these circumstances the total effect of the yield tax on the rotation age is positive. But if the marginal valuation of amenities decreases with the age of the forest stand, then higher yield tax makes timber production less profitable and the total effect of the yield tax on the rotation is ambiguous.⁸ As in the case of the timber tax, a higher yield tax will also decrease the indirect utility of the landowner. If the utility loss due to the higher yield tax is compensated by increasing the tax exemption so that the welfare of the landowner remains unchanged, then via this mechanism the rotation age will shorten. This runs counter to the potential positive rotation effect of the yield tax. Hence, the substitution effect of the yield tax on the rotation is a priori ambiguous for the case F'(T) > 0.

⁷ The Slutsky equation for the unit tax is qualitatively similar to that for the yield tax (see Appendix 1).

⁸ See Koskela and Ollikainen (2001a).

We summarize our findings in

Result 1. In the presence of progressive forest taxation

- (a) the substitution effect of the timber tax on the rotation age is negative,
- (b) while the substitution effect of the yield (unit) tax is generally ambiguous a priori, a non-increasing marginal amenity valuation $F'(T) \le 0$ is a sufficient, but not a necessary, condition for the negativity of this substitution effect.

Result 1b implies that we have a negative substitution effect for the yield (and unit) tax in the Faustmann model and in the case of site-specific amenities (when F'(T) = 0).

3. Tax Reform and Welfare Analysis of Forest Tax Progression

Drawing on the developed analytics of the rotation effects of the progressive forest taxation we now move on to study two policy issues. First, we ask how a tax revenue neutral change in the progression of the timber tax and yield (or unit) tax will affect the privately optimal rotation age.⁹ Second we examine the desirability of progressive forest taxation from the social welfare point of view.

3.1 Rotation effects of tax-revenue neutral changes in timber and harvesting tax progression

In this section we search for an answer to the question of what happens to the privately optimal rotation age and amenity production (and consequently to timber supply), if the government collects a given tax revenue by changing the size of the exemption and the tax rate so that the progression will increase. The government has an exogenous tax revenue target \overline{R} , and we make the Ricardian assumption (see e.g. Romer 2001, Ch. 11) that the short run

⁹ The usefulness of this kind of incentive analysis of tax policy should be evident. It helps the governments to assess the direction and strength of reforms in forest tax systems. There is plenty of evidence of reforms of forest taxation systems in many countries, including e.g. Finland, Sweden, U.S., New Zealand and most transition economies in Eastern Europe.

government debt or surplus is not regarded as an important factor, so that all what counts is the discounted sum of the tax revenue collected from forestry. We also make a natural assumption that tax revenues are positively related to the tax rates and negatively to the tax exemption.¹⁰ This implies that any simultaneous increase in the tax rate and tax exemption makes the tax schedule more progressive in the sense that the average tax rate increases more rapidly with the tax base.

A. Timber tax

In the presence of the timber tax the tax revenue is given by $R = \beta(U - A)$. Differentiating this tax revenue function with respect to β and a by keeping it constant yields the condition $dR = R_{\beta}d\beta + R_{\alpha}da = 0$ so that

$$da_{|dR=0} = -\frac{R_{\beta}}{R_a} d\beta .$$
¹¹ (11a)

Assuming that there is a positive (negative) relationship between the tax rate (the tax exemption) and the tax revenue we have

$$R_{\beta} = \beta (U_{T} - A_{T})T_{\beta}^{H} + U - A > 0$$
(11b)

$$R_{a} = \beta (U_{T} - A_{T})T_{a}^{H} - \frac{\beta e^{rT}}{1 - e^{-rT}} < 0$$
(11c)

The compensation rule between the tax rate and the tax exemption defined by equation (11a) can be regarded as a pure change in the progressivity in the ex post sense. The total rotation effect can be expressed as

¹⁰ The dependence of the tax revenue on the tax rate is usually described by the so-called Dupuit-Laffer curve, according to which there is a positive (negative) relationship between the tax revenue an the tax rate (the tax exemption). See e.g. Fullerton (1982) for a survey of the empirical literature about the relationship between the tax rate and the tax revenue, and Malcomson (1986) for a theoretical analysis of the relationship between the tax rates and tax revenue.

¹¹ For a similar analysis in the labor market context, see Koskela and Vilmunen (1996). In forest economics, analysis of forest tax switches has traditionally focused on the incentive effects of timing of a given forest tax type (see Koskela 1989) or of a change from one forest tax type to another (see Ollikainen 1993), but the issue of progressivity has not been analyzed.

$$dT^{H} = T^{H}_{\beta}d\beta + T^{H}_{a}da .$$
⁽¹²⁾

Substituting the RHS of (11a) for da in (12) and rearranging yields

$$\frac{dT^{H}}{d\beta}\Big|_{dR=0} = R_{a}^{-1} \Big[T_{\beta}^{H} R_{a} - T_{a}^{H} R_{\beta} \Big].$$
(13)

Applying, finally, the Slutsky decomposition for the timber tax rate in equation (13) both directly and for the definition of R_{β} we obtain after some manipulation

$$\frac{dT^{H}}{d\beta}\Big|_{dR=0} = -R_{a}^{-1}\frac{\beta}{1-e^{-rT}}T_{\beta}^{H^{C}} < 0$$
(13')

Hence, we have

Result 2. A rise in the tax-revenue neutral timber tax progression, so that both the timber tax rate and the tax exemption increase, will shorten the rotation age due to the negative substitution effect of the timber tax.

A tax-revenue neutral change in the timber tax progression makes the standing timber relatively less profitable via the substitution effect, so that the rotation age shortens. The fact that the government collects the same amount of money via timber tax under a shortened private rotation age has, naturally, important implications both for the provision of amenity services and timber market. A shorter rotation age decreases amenity service production and, as is well known, implies that while timber supply increases in the short run, it decreases in the long-term run under conventional assumptions (see Clark 1976). Thus, a society wishing to lengthen the privately optimal rotation age should decrease progressivity of timber taxation.

B. Yield tax

In the presence of the yield tax the government tax revenue is given by $R = \frac{\tau \left[p \ f(T)e^{-rT} - a\right]}{1 - e^{-rT}} = \tau (Y - A).$ Differentiating this with respect to τ and a by keeping tax revenue constant yields $dR = R_{\tau}d\tau + R_{a}da = 0$, where we, again, assume a positive (negative) relationship to hold between the tax rate (the tax exemption) and the tax revenue. Hence, we have

$$da_{|dR=0} = -\frac{R_{\tau}}{R_{a}}d\tau \tag{14a}$$

where

$$R_{\tau} = \tau (Y_T - A_T) T_{\tau}^{H} + Y - A > 0$$
(14b)

$$R_{a} = \tau (Y_{T} - A_{T})T_{a}^{H} - \frac{\tau}{1 - e^{-rT}} < 0$$
(14c)

Changing the tax rate and exemption affects the privately optimal rotation age through

$$dT^{H} = T^{H}_{\tau} d\tau + T^{H}_{a} da \,. \tag{15}$$

Substituting the RHS of (14c) for da in (15) and rearranging yields

$$\frac{dT^{H}}{d\tau}\Big|_{dR=0} = R_{a}^{-1} \Big[T_{\tau}^{H} R_{a} - T_{a}^{H} R_{\tau} \Big]$$
(16)

Applying, again, the Slutsky decomposition for (16) gives,

$$\frac{dT^{H}}{d\tau}\Big|_{dR=0} = -R_{a}^{-1}\frac{\tau}{1-e^{-rT}}T_{\tau}^{H^{c}} = ?$$
(16')

The effect of a rise in the tax-revenue neutral progression in the timber tax on the private rotation age is ambiguous a priori due to indeterminacy of the substitution effect.¹² Hence, we have

¹² The same result holds for the unit tax. Proof is available upon request.

Result 3. A rise in the tax-revenue neutral yield (or unit) tax progression, so that both the yield (or unit) tax rate and tax exemption increase, will have an a priori ambiguous effect on the rotation age due to ambiguity of the substitution effect. A non-increasing marginal amenity valuation $F'(T) \leq 0$ a sufficient, but not a necessary condition, for the rotation effect to be negative.

Under the stated condition for the negative rotation effect, reflecting landowner preferences for young and not old stands in the provision of amenity services, the consequences of this policy for amenity provision and timber market are the same as in the case of Result 2.¹³ Therefore, a society valuing amenity services associated with old stands might not want to use a tax reform of higher yield (or unit) tax progression in forest taxation.

3.2 Socially Optimal Forest Tax Progression

Now we ask whether the forest tax progression is desirable from the social welfare viewpoint. The government is assumed to maximize the social welfare function by choosing tax rates and exemptions. We assume that the representative landowner and recreators value amenity services from forests. For simplicity the amenity valuation function is postulated to be the same for the representative landowner and recreators. When citizens have full access to enjoy the amenity services from private forests and there are no congestion effects associated with enjoying the amenity services of forests, we have the following social welfare function:

$$SW^{H} = V^{*}(s(i),\tau,\beta) + E^{*} + (n-1)E, \qquad (17)$$

where $V^* + E^*$ describes the indirect utility function of the representative landowner in the presence of forest taxes, *n* is the number of citizens, (*n*-1) is the number of recreators and *s*(*i*) is the site productivity tax.

¹³ In the Faustmann model Mendelsohn (1993) has shown that while a rise in the yield tax increases rotation and thereby distorts rotation age, a rise in progressivity shortens rotation and may thus produce a neutral tax design. Our analysis is thus a generalization of his result for the Hartman framework, where the private landowners are assumed to value both the net harvest revenue and the amenity services from the forest stand.

We continue assuming that there is an exogenous tax revenue target, denoted by \overline{R} . We assume that, in addition to the yield tax and the timber tax, the government has available a neutral forest tax. We have shown elsewhere (Koskela and Ollikainen 2001b) that the only neutral tax in the Hartman model is the site productivity tax *s*. In the presence of the site productivity tax, and yield and timber taxes with tax exemptions we can express the tax revenue as

$$R = \frac{\tau \left[pf(T)e^{-rT} - a \right] + \beta \left[\int_{0}^{T} pf(s)e^{-rs} ds - a \right]}{1 - e^{-rT}} + \frac{s}{r} = \tau (Y - A) + \beta (U - A) + \frac{s}{r}.$$
 (18)

A. Timber Tax Progression

In the case of the timber tax the social planner maximizes the social welfare function subject to the given tax revenue requirement $\overline{R} \leq R = \frac{s}{r} + \beta(U - A)$. The Lagrangian is now $\Omega^{H} = V^{*} + E^{*} + (n-1)E - \lambda(\overline{R} - R)$. Choosing *s* so as to maximize the Lagrangian yields $\Omega_{s}^{H} = -\frac{1}{r} + \lambda \frac{1}{r} = 0 \iff \lambda = 1$. Hence, in the presence of the site productivity tax the marginal cost of public funds is unity.¹⁴ Taking this into account we can write the optimal conditions for the timber tax β and the tax exemption *a* as follows

$$\Omega^{H}{}_{\beta}|_{s=s}^{*} = T^{H}_{\beta}[(n-1)E_{T} + \beta(U_{T} - A_{T})] = 0$$
(19a)

$$\Omega^{H}{}_{a}|_{s=s}^{*} = T^{H}_{a}[(n-1)E_{T} + \beta(U_{T} - A_{T})] = 0, \qquad (19b)$$

where $T_{\beta}^{H} \neq 0$ and $T_{a}^{H} \neq 0$. Hence, both conditions imply that $\Omega^{H}{}_{\beta}\Big|_{s=s}^{*} = 0 \Leftrightarrow [(n-1)E_{T} + \beta(U_{T} - A_{T})] = 0$. By utilizing the expressions for E_{T} , U_{T} and A_{T} and rearranging we get for the optimal timber tax

¹⁴ This results from the neutrality of site productivity tax, i.e., from that $T_s^H = 0$ (see Koskela and Ollikainen 2001a).

$$\beta^*|_{s=s^*} = \frac{-(n-1)[F(T) - rE]}{pf(T) - r(U - A)}$$
(20)

Because the denominator is always positive, the sign of the optimal timber tax depends on the sign of [F(T) - rE]. Thus, we have a timber tax (subsidy) for the case where the marginal valuation of amenities decreases (increases) with the age of the stand, i.e. when F'(T) > (<)0. For the constant marginal valuation (site-specific amenities) neither tax nor subsidy is needed. The interpretation of this result is the following. When the marginal valuation of amenities decreases (increases) with the age of the stand, the privately optimal rotation age is too long (too short) from the society's perspective. Therefore, a positive timber tax (timber subsidy), which shortens (lengthens) the rotation age, is needed.

How does the tax exemption affect the socially optimal timber tax rate? The denominator of equation (20) reveals the relationship between timber tax β and tax exemption a. If the marginal valuation of amenities increases with the age of the stand, one should have a timber subsidy, which is negatively related to the size of tax exemption a. But if the marginal valuation of amenities decreases with age, one should have a timber tax, which is again negatively related with the size of the tax exemption a. But do we need progressive or regressive timber taxation? Comparing equations (19a) and (19b) yields the following answer:

Result 4. When the government has two targets, collecting tax revenues and correcting externality, and two instruments, a site productivity tax and a proportional timber tax, then progression is not needed from the social welfare point of view.

The economic interpretation of Result 4 is straightforward and intuitively appealing. The government has already two instruments to handle with two targets, so that the use of any third instrument (progression) becomes redundant in the presence of perfect foresight. Thus, the optimal forest tax system uses timber tax to induce the private landowner to follow the socially optimal rotation age, and the tax revenue is collected with a neutral site productivity tax.

B. Yield Tax Progression

For the yield tax the social planner maximizes the social welfare function subject to the given tax revenue requirement $\overline{R} \leq R = \frac{s}{r} + \tau(Y - A)$. The Lagrangian can be expressed as $\Omega^{H} = V^{*} + E^{*} + (n-1)E - \lambda(\overline{R} - R)$. Choosing *s*, so as to maximize the Lagrangian yields again $\lambda = 1$. If the site productivity tax is at the optimum we get for the yield tax and the tax exemption

$$\Omega^{H}_{\tau}|_{s=s}^{*} = T_{\tau}^{H}[(n-1)E_{T} + \tau(Y_{T} - A_{T})] = 0$$
(21a)

$$\Omega^{H}_{a|s=s}^{*} = T_{a}^{H} [(n-1)E_{T} + \tau(Y_{T} - A_{T})] = 0, \qquad (21b)$$

If $T_{\tau}^{H} \neq 0$ and $T_{a}^{H} \neq 0$, both conditions give a rise for $[(n-1)E_{T} + \tau(Y_{T} - A_{T})] = 0$. We can solve for the socially optimal yield tax rate

$$\tau_{|s=s^*} = \frac{-(n-1)[F(T) - rE]}{p(f'(T) - rf(T)) - r(Y - A)}$$
(22)

We have previously (see equations 8a and 8b) established that the sign of the denominator can be defined as follows:

$$p(f'(T) - rf(T)) + r(Y - A) \begin{cases} > \\ = \\ < \end{cases} 0 \quad as \quad rc^* (1 - e^{-rT})^{-1} + F(T) - rE \begin{cases} < \\ = \\ > \end{cases} 0.$$

Thus, the optimal yield tax depends on the properties of the amenity valuation, the question of whether the forest stand is a public good and the effect of the yield tax on the rotation age. When n > 1 and F'(T) > 0 then the denominator is negative, so that the optimal yield tax is positive. The same result holds to the unit tax.¹⁵ The interpretation follows along similar lines as in the case of the timber tax. When F'(T) > 0 the privately optimal rotation

¹⁵ Proof is available upon request.

age is too short from the society's perspective. Hence, raising the yield tax (or the unit tax), which lengthens the rotation age, is needed.¹⁶

What about the optimal exemption parameter? Comparing equations (21a) and (21b) gives

Result 5. When the government has two targets, tax revenues and correcting externality, and two instruments, a site productivity tax and a proportional yield tax (or unit tax), then progression is not needed from social welfare point of view.

Interpretation is the same as that of Result 4: the third instrument (tax progression) is redundant in the absence of equity considerations.

Results 4 and 5 are based on the assumption that the government has a neutral tax available. Would these results change if the government cannot use site productivity tax? It turns out that the optimal exemption and the tax rate should be designed so that that the marginal costs of public funds is equal to one, i.e., the required tax revenue is raised in a nondistortionary fashion. This allows the targeting of the corrective tax towards the internalizing the externality in forestry.

For the timber tax our conclusion can be justified as follows. The first-order conditions for the timber tax rate and tax exemption are

$$\Omega_{\beta}^{H} = 0 = V_{\beta}^{*} + (n-1)E_{\beta} + \lambda R_{\beta}$$
(23a)

$$\Omega_{a}^{H} = 0 = V_{a}^{*} + (n-1)E_{a} + \lambda R_{a}$$
(23b)

where

$$\begin{split} V_a^* &= \frac{\beta}{(1 - e^{-rT})}; \ V_\beta^* = -\frac{(U - A)(1 - e^{-rT})}{\beta} V_a^*; \ E_a = \frac{T_a^H e^{-rT}}{1 - e^{-rT}} [F(T) - rE]; \\ E_\beta &= \frac{T_\beta^H e^{-rT}}{1 - e^{-rT}} [F(T) - rE]; \ R_a = -\frac{\beta}{1 - e^{-rT}} + \frac{T_a^H e^{-rT} \beta}{1 - e^{-rT}} [pf(T) - r(U - A)]; \\ R_\beta &= (U - A) + \frac{T_\beta^H e^{-rT} \beta}{1 - e^{-rT}} [pf(T) - r(U - A)] \end{split}$$

¹⁶ Notice, however, that even though the denominator in (22) is positive implying that the yield tax has a negative effect on the rotation age according to (8b), the numerator can be negative. In this case we would have the optimal yield subsidy.

Using these partial derivatives and the Slutsky-decomposition for the timber tax we can reexpress equations (23a) and (23b) as follows

$$\Omega_{\beta}^{H} = 0 = -\frac{(U-A)(1-e^{-rT})}{\beta}\Omega_{a}^{H} + \frac{T_{\beta}^{Hc}e^{-rT}}{1-e^{-rT}}[(n-1)(F(T)-rE) + \lambda\beta H]$$
(24a)
$$\Omega_{a}^{H} = 0 = V_{a}^{*}(1-\lambda) + \frac{T_{a}^{H}e^{-rT}}{1-e^{-rT}}[(n-1)(F(T)-rE) + \lambda\beta H]$$
(24b)

where H = [pf(T) - r(U - A)].

When *a* is set at the optimum in (24b), the first term in (24a) is zero. This has two implications: At the optimum for a^* , we have $\lambda = 1$ and the optimal β can be expressed as

$$\beta^* = -\frac{(n-1)(F(T) - rE)}{[pf(T) - r(U - A)]}$$
(25)

According to (25), we have timber tax (subsidy) like in the presence of site productivity tax when (n-1)(F(T) - rE) < (>)0. In the subsidy case *a* must be negative, i.e. a lump sum tax in order to match the budget revenue constraint. Hence, in the presence of timber subsidy it is optimal to have a regressive forest tax system. While the budget revenue requirement directly implied the nature of exemption in the case of timber subsidy, we cannot say whether the optimal exemption, a^* , is a tax or subsidy in the presence of timber tax. Both options for the exemption may be compatible with the first-order conditions (24a) and (24b), which only require that $\lambda = 1$.

For the yield tax a similar analysis holds. The first-order conditions in terms of the flat yield tax rate and tax exemption can be written as

$$\Omega_{\tau}^{H} = 0 = V_{\tau}^{*} + (n-1)E_{\tau} + \lambda R_{\tau}$$
(26a)

$$\Omega_a^H = 0 = V_a^* + (n-1)E_a + \lambda R_a$$
(26b)

where

$$V_{a}^{*} = \frac{\tau}{(1 - e^{-rT})}; \quad V_{\tau}^{*} = -\frac{(Y - A)(1 - e^{-rT})}{\tau} V_{a}^{*}; \quad E_{a} = \frac{T_{a}^{H} e^{-rT}}{1 - e^{-rT}} [F(T) - rE];$$
$$E_{\tau} = \frac{T_{\tau}^{H} e^{-rT}}{1 - e^{-rT}} [F(T) - rE]; \quad R_{a} = -\frac{\tau}{1 - e^{-rT}} + \frac{T_{a}^{H} e^{-rT} \tau}{1 - e^{-rT}} [pf'(T) - rpf(T) - r(Y - A)]$$

$$R_{\tau} = (Y - A) + \frac{T_{\tau}^{H} e^{-rT} \tau}{1 - e^{-rT}} [pf'(T) - rpf(T) - r(Y - A)]$$

By using these and the Slutsky-decomposition we can re-expressed equations (26a) and (26b) as follows:

$$\Omega_{\tau}^{H} = 0 = -\frac{(Y-A)(1-e^{-rT})}{\tau}\Omega_{a}^{H} + \frac{T_{\tau}^{Hc}e^{-rT}}{1-e^{-rT}}[(n-1)(F(T)-rE) + \lambda \tau K]$$
(27a)

$$\Omega_a^H = 0 = V_a^* (1 - \lambda) + \frac{T_a^H e^{-rT}}{1 - e^{-rT}} [(n - 1)(F(T) - rE) + \lambda \tau K]$$
(27b)

where K = [pf'(T) - rpf(T) - r(Y - A)]. The first-order conditions (27a) and (27b) imply that $\lambda = 1$ and the following optimal yield tax rate

$$\tau^* = -\frac{(n-1)(F(T) - rE)}{\left[pf'(T) - rpf(T) - r(Y - A)\right]}$$
(28)

which is identical to the case with site productivity tax. Similar result holds for the unit tax.¹⁷ Hence, given the neutrality of optimal a^* we have the same result for the yield tax as we did under the site productivity tax: regressive tax system for the yield subsidy because of the government tax revenue constraint. But like in the case of timber tax a^* can be a tax or subsidy, so that the progressive tax system is also possible.

Thus, under certain conditions timber and yield tax progression might be optimal as a means to affect the rotation period. But this is an empirical question, which depends on the precise characterization of the amenity valuation and the size of government budget constraint.

4. Concluding Remarks

We used the Hartman framework to study behavioral and social welfare aspects of the progressive forest taxation, when progressivity was modeled so that the average tax rate depends on the size of the tax base. We first focused on the behavioral effects of progressive

¹⁷ Proof is available upon request.

taxes and asked how does a switch between the tax rate and tax exemption change harvesting if the government wants to keep the tax revenue from forestry constant. Then, we studied the social welfare effects of progressive timber and harvest taxes when citizens have a full access to enjoy amenity services from private forests without any congestion effects. In the welfare analysis we assumed that government has also available the site productivity tax, which is a lump-sum type tax and thus neutral in the Hartman framework.

We demonstrated three sets of new results. First, a tax-revenue neutral increase in the timber tax progression – a higher timber tax rate compensated by a higher tax exemption so as to keep the tax revenue unchanged – will shorten the privately optimal rotation age. For the yield tax, a non-increasing amenity valuation in terms of the forest stand age provides a sufficient, but not a necessary, condition for the same result. Second, for the socially optimal forest taxation, when the site productivity tax is at the optimum and in the absence of equity considerations, the proportional forest tax is enough from the viewpoint of social welfare. Hence, progressivity of forest taxation is not desirable. Third, if the government does not have a neutral tax available, the optimal exemption and the tax rate should be designed so that that the marginal costs of public funds is equal to one, i.e., the required tax revenue is raised in a non-distortionary fashion.

Even though we demonstrated in the case of the representative landowner that in many cases tax progression is not desirable, there are still some other considerations that might lead to progressive taxation from the viewpoint of the society. An interesting topic for further research would be to explore how for instance allowing for differences between private landowners in terms of forest productivity and their welfare levels would affect the optimal design of tax structure.

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Appendix 1: Slutsky decompositions for timber, yield and unit taxes

This appendix presents the Slutsky decompositions for the timber, yield and unit taxes. Starting with the timber tax the envelope theorem implies that

(1)
$$\hat{W}_a^* = \frac{\beta}{1 - e^{-rT}} > 0$$

(2)
$$\hat{W}_{\beta}^* = -[U-A] < 0$$
,

where \hat{W}^* is the indirect utility function of the representative landowner and $A = a(1 - e^{-rt})^{-1}$. Given (1) we can invert the indirect utility function $\hat{W}^*(\beta, a) = \hat{W}^{*o}$ for the tax exemption *a* in terms of the timber tax rate, β , and the utility maximizing level \hat{W}^{*o} so as to get $a = h(\beta, \hat{W}^o)$. Substituting this for *a* in \hat{W}^{*o} gives the compensated indirect utility function

(3)
$$\hat{W}^{*c}(\beta, h(\beta, \hat{W}^{*o})) = \hat{W}^{*0}$$

This compensated utility function (see, e.g. Diamond and Yaari 1972) answers the following question: if the tax rate β is increased, how much has the tax exemption *a* to be changed so as to keep the utility of the landowner unchanged? Differentiating (3) with respect to β yields $\hat{W}_{\beta}^{*c} + \hat{W}_{a}^{*c} h_{\beta} = 0$, so that we have

(4)
$$h_{\beta} = -\frac{\hat{W}_{\beta}^{*c}}{\hat{W}_{a}^{*c}} = \frac{\int_{0}^{0} pf(s)e^{-rs}ds - a}{\beta} = \frac{(1 - e^{-rT})(U - A)}{\beta} > 0$$

According to (4) a rise in the timber tax rate requires an increase in the tax exemption to keep the indirect utility of the landowner unchanged.

The utility maximizing rotation age equals the cost minimizing solution in terms of *a* to get the same utility for given timber tax rate β and given utility \hat{W}^{*0} . This determines the compensated rotation age T^{Hc} , so that due to duality theory we have

(5)
$$T^{H}(\beta, a) = T^{H}(\beta, h(\beta, \hat{W}^{*0})) = T^{H_{c}}(\beta, \hat{W}^{*o})$$

Differentiating (5) with respect to β gives $T_{\beta}^{H} + T_{a}^{H}h_{a} = T_{\beta}^{Hc}$ and using (4) we get the Slutsky decomposition

(6)
$$T_{\beta}^{Hc} = T_{\beta}^{H} + \frac{(1 - e^{-rT})(U - A)}{\beta} T_{a}^{H} = \frac{pf(T)}{\hat{W}_{TT}} < 0$$

In the case of yield tax we can proceed analogously to get the following indirect utility effects of the tax exemption and the tax rate

(7)
$$\hat{W}_{a}^{*} = \frac{\tau}{1 - e^{-rT}} > 0$$

(8) $\hat{W}_{\tau}^{*} = -\frac{pf(T)e^{-rT} - a}{rT} = -\frac{(1 - e^{-rT})(Y - A)}{rT} < 0$

Inverting the indirect utility function for
$$a$$
 in terms of τ and \hat{W}^{*o} gives a

Inverting the indirect utility function for a in terms of τ and \hat{W}^{*o} gives $a = g(\tau, \hat{W}^{*o})$ and the compensated indirect utility function $\hat{W}^{*c}(\tau, g(\tau, \hat{W}^{*o})) = \hat{W}^{*o}$. Differentiating this with respect to τ yields

(9)
$$g_{\tau} = \frac{pf(T)e^{-rT} - a}{\tau} = \frac{(1 - e^{-rT})(Y - A)}{\tau} > 0$$

Equalizing the uncompensated rotation age $T^{H}(\tau, a)$ with the compensated rotation age $T^{Hc}(\tau, \hat{W}^{*o})$ at the utility maximizing point for a given yield tax rate τ , $T^{H}(\tau, g(\tau, \hat{W}^{*o})) = T^{Hc}(\tau, \hat{W}^{*o})$, and differentiating this with respect to τ gives the following Slutsky decomposition

(10)
$$T_{\tau}^{Hc} = T_{\tau}^{H} + \frac{(1 - e^{-rT})(Y - A)}{\tau} T_{a}^{H} = \frac{p(f'(T) - rf(T))}{\hat{W}_{TT}} = ?$$

Using the first-order condition for the privately optimal rotation age one can see that

$$T_{\tau}^{Hc} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ as } \left[r(V - \tau(Y - A) - (F(T) - rE) \right] \begin{cases} < \\ = \\ > \end{cases} 0 \end{cases}$$

By an analogous procedure we get for the Slutsky decomposition of the unit tax

(11)
$$T_t^{H^c} = \underbrace{T_t^H}_{?} + \underbrace{(1 - e^{-rT})(Y - A)}_{t} T_a^H.$$

where the substitution effect can be shown to be

(12)
$$T_t^{H^C} = \frac{[f'(T) - rf(T)]}{\hat{W}_{TT}} \begin{cases} > \\ = \\ < \end{cases} 0 \quad as \quad r(V - \tau(Y - A)) \begin{cases} < \\ = \\ > \end{cases} F(T) - rE \\ * * * * * \end{cases}$$

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