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Is there a way out of the trap in an open economy?

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OPTIMAL FISCAL AND MONETARY POLICIES IN A RECESSION: IS THERE A WAY OUT OF THE TRAP IN AN OPEN ECONOMY?*

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ABSTRACT: The paper addresses the classical question how to lead the economy from a recession back to full employment in an open economy. In a recession caused by an adverse demand shock, optimal policies crucially depend on whether the income multiplier related to a policy is bigger than unity. This holds for investment, and so the optimal real interest rate is explicitly solved from the condition defined by Keynes (1936), i.e., it is that rate at which the elasticity of employment is zero. As to expansionary fiscal policies, however, the above multiplier condition is not uniformly satisfied and some controversial policy conclusions emerge. Secondly, it is shown that the optimal monetary policy requires a tax on foreign capital outflows, combined with inflationary finance. However, the optimal investment activity can also be reached by retaining international financial integration and channelling subsidies to investing firms.

Key words: Interest rate, recession, fiscal and monetary policies, investment

JEL Codes: E 12, F 41


Asiasanat: Korko, taantuma, finanssi- ja rahapolitiikka, investoinnit
1. Introduction

In international economics a standard result is that the welfare maximising level of the real interest rate for a small country is the foreign rate, i.e., there should be free capital flows and no regulation of the domestic financial markets - analogously as in foreign trade - which justifies the broad line of policy advocated by prominent international organisations to liberalise and integrate national financial markets world-wide and which was, e.g., completed in the EMU process in Europe.

Keynes (1936, chapter 17) discussed, in the spirit of Wicksell, the concept of a neutral interest rate. In contrast to his ‘Treatise on Money’ he now considered that the neutral rate is a function of employment and he rather defined that rate as the optimum rate of interest at which the elasticity of employment is zero, i.e., there is full employment. In a depression with deficient demand and unemployment the social marginal productivity of the flow of new investment expenditure is higher than the private return, which would on surface justify encouraging and even subsidising investment and restoring full employment. However, it is not self-evident that this is a welfare improving solution, as investment has to add more resources for consumption evaluated in an intertemporal context than it absorbs them later on via increased foreign indebtedness.

In this sense a more basic approach to monetary policies under a depression should be taken than is currently standard in the New-Keynesian analysis of monetary policymaking, where the usual loss function of policy-making includes deviations of output from full employment level as one component, see e.g. the survey by Clarida et al. (1999). The proper goal of stabilization should be derived as an integral part of consideration of optimal policies. The important criterion for the case of activism in policies to hold is that the short-run Keynesian income multiplier related to a policy is greater than unity. This is shown to be the case in our model with respect to an increase in investment outlays, and so the intertemporal welfare maximising optimal real rate under short-run unemployment can really be solved explicitly from the definition given by Keynes.

The aim of this paper is two-fold: first, we derive the optimal interest rate and optimal fiscal policies in a recession, caused by deficient demand or an adverse productivity shock. Secondly, we evaluate, by what means the optimal situation can be achieved in practice. The above multiplier criterion is not, however, generally satisfied by all kind of government expenditure so that expansionary fiscal policies are not uniformly in place, irrespective of the recession. Recently, fiscal policies and Keynesian multipliers have been analysed under imperfect competition in the goods market and competitive labour markets, see e.g. Heijdra and van der Ploeg (1996) and Hejdra and Lighart (1997). These analyses shed new insight into the channels through which fiscal policies operate, but they have been carried out in the context of a closed economy. In an open economy, the case of imperfect competition is more complex, as it can take place both in the home and export markets. Therefore we have chosen, in order to be able to present the case of optimal policies under a depression in a clear-cut way, to retain the standard small open economy case with perfect competition. However, our analysis of the economy is much more articulated than in the bulk of the recent macroeconomic literature, as we explicitly consider the optimal behaviour of both firms in their investment demand and in the labour market in a recession, and the households in their in-
tertemporal consumption-saving behaviour. Thereby we try to contribute to the recent research related to the revival of the interest shown towards the old issue of liquidity trap, see, e.g., Krugman (1998), Buitner and Panigirtzoglou (1999), Svensson (2000) and articles in *Journal of Money, Credit, and Banking*, November 2000.

It was already suggested by Kouri (1983) that under a Keynesian unemployment with wage-price rigidities, it may be a second best policy to divert the domestic financial market from the foreign and to set the domestic rate of interest lower than that abroad. But he did not derive, what this social optimum is exactly, and whether and how it could be implemented by macroeconomic policies, both issues tackled here. In an open economy, the implementation of the optimal monetary policy is intimately linked to the integration of the domestic economy with the world capital market as the separation of the domestic real rate from the foreign necessarily requires government intervention in our framework. In our model, the inflation rate is in effect the policy tool of the domestic central bank. However, it turns out to be completely futile, as it cannot lower the real interest rate. Therefore disintegration of the domestic financial market from the foreign is the solution to optimal monetary policies, in line with policies obeyed during the Keynesian era of macroeconomic policies. Under an adverse supply shock, in contrast, it is optimal to maintain free international capital flows.

On the other hand, we show that the optimal situation in the economy can also be achieved through eliminating the tax on foreign investment, i.e., by maintaining free capital mobility and without endangering price stability, but only imposing the derived optimal subsidy on investment. So, the idea of subsidising investment expenditure also emerges here as in Krugman (1998).

The structure of the paper is as follows. In Section 2 the basic intertemporal open economy model of consumption and investment is presented. Section 3 discusses the economy in a depression after the economy has been hit by a recessionary shock. The crucial Keynesian income multipliers with respect to government expenditure and investment policies are derived in Section 4. Section 5 solves the social optimum for the interest rate and government fiscal policy in a depression. Section 6 considers, whether, or not, and how the optimum policy can be implemented and Section 7 concludes briefly.

2. The model

We build the following two-period macroeconomic model, which is minimal and also sufficient to be able to discuss welfare and to derive the optimal macroeconomic policies in a dynamic context. We explicitly separate the optimising behaviour by firms, households and the government in a recession. The second period is as usual in the Keynesian macroeconomic model “the Future” (see Leijonhufvud 1968). The depression and the choice of optimal policies take place in the first period. In the second period there is either full employment or, in any case, output then is determined by supply.

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1 As a separate issue, it is also recognised that in a two-sector economy, the real rate of interest faced by the domestic consumers may differ from the foreign as the relative price between the nontraded and traded goods in their expenditure bundle may change in a different way in the home country than abroad, see e.g. Dornbusch (1983).
There is one domestic production sector, the output of which is both exported and used in all categories of domestic demand. There is also another good, which is imported, but used only in consumption. The economy is a price taker and so the price ratio between the domestic and foreign good is fixed in the world markets, and therefore omitted in the following. Let $Q_i$ be real output, and $C_i$, $I_i$, $G_i$, $X_i$, and $H_i$ be consumption, investment, public expenditure, exports and imports in period $i$, respectively, in terms of domestic output. The income accounting identity is,

$$Q_i = C_i + I_i + G_i + X_i - H_i.$$  

We omit investment in the second period, because the flow of new investment will be included in the capital stock only after a period. The national intertemporal budget constraint on foreign borrowing gives the net present value $R$ of the resources available for present and future private consumption,

$$R = C_1 + (1 + r^*)^{-1}C_2 = (Q_1 - I_1 - G_1) + (1 + r^*)^{-1}(Q_2 - G_2 + (1 - \delta)^2K_0 + (1 - \delta)I_1),$$

where $r^*$ is the international real rate of interest, given to the small economy and being positive, and $\delta$ is the rate of depreciation of the capital stock and $K_0$ the inherited capital stock existing in the beginning of period one. The last item on the right hand side, the remaining capital stock, is included in the budget constraint for consumption, as the “world ends” after the second period. The export and import good are separated from each other below.

Let $Q_i = A_iF(K_{i-1}, L_i)$, where $F$ is the production function with constant returns to scale and the normal (Inada) properties for production in period $i$, using $K_{i-1}$, which is the capital stock at the end of the previous period and $L_i$, the current flow of labour input. We assume that the total factor productivity $A_2$ is bigger than $A_1$ so that, with a given same level of full employment in both periods, the optimal capital stock is larger in period 2 (i.e., at the end of period 1) than in period 1 (i.e., at the end of period 0), so that there are positive investments in period 1 in the benchmark full employment situation.

The flow of investment $I_1$ by the firms in the first period is financed by borrowing from the domestic banks at the nominal money rate of interest $i$, the equivalent real rate being $r$. The government can also channel a subsidy to the investing firms of the magnitude $aI_1$. The value $V_2$ of the firms’ capital investment, carried out in period 1, to their owners, in units of domestic output, is equal to the flow of profit $\pi_2$ in period 2, added to the value of the firms’ capital less debts remaining at the end of the second period,

$$V_2 = (1 - \delta)((1 - \delta)K_0 + I_1) - P_1(1 - a)I_1/P_2 + \pi_2, \quad \text{where}$$

$$\pi_2 = A_2F((1 - \delta)K_0 + I_1, L_2) - (W_2/P_2)L_2 - iP_1(1 - a)I_1/P_2$$

$W_i$ is the nominal wage rate and $P_i$ the price level of the domestic production in period $i$. In the first period, the firms decide on how much labour to demand, given the product wage ($W_i/P_i$) and the inherited capital stock $K_0$, and how much to invest in new capital to be available for production in period 2. In the second period, the firms only make the

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2 In the second period the arbitrage immediately sets the real market price of the firms’ capital in terms of output to unity. Note that bank profits are equal to zero by definition.
employment decision. This is not the whole story in a Keynesian depression, but we shall consider this modification in more details below in Section 3. Using the definition of the real interest rate, \(1+r = P_1(1+i)/P_2\), the firms’ maximisation of profit \(\pi_1\) in the first period and \(V_2\) in the second, produces the familiar optimal conditions for their production and investment optimum,

\[
(4a) \quad A_1F_2 = W_1/P_1, \\
(4b) \quad A_2F_1 = r + \delta - a(1+r) \quad \text{and} \\
(4c) \quad A_2F_2 = W_2/P_2.
\]

These conditions can be solved recursively so that (4a) determines, with a given initial capital stock \(K_0\) and the product wage, the notional labour demand by the firms and thereby their profit maximising output \(Q_1\). In the benchmark case, we assume this to be the level where there is full employment. Assuming that period 2 is supply determined, and full employment prevails, then (4b) gives the optimal capital stock in period 2 with this same level of employment and the consequent investment \(I_1\) in period 1. Finally, (4c) determines the real wage, given \(K_1\), consistent with full employment, i.e., there is complete real wage flexibility in period 2, although not in period 1, see below. From (4b), using (4a), we can derive the flow of new investment to be

\[
(5) \quad I_1 = \max(0, I^*) \quad \text{where} \quad I^* = I(r, K_0, W_1/P_1), I_1 < 0, I_2 > 0 \text{ and } I_3 \leq 0.
\]

Constant returns to scale, perfect competition and free access to the goods market then imply, using (4a) and (4b), that in equilibrium \(V_2 = 0\) for an additional unit of investment \(I_1\).

There are \(N\) identical households in the economy. We assume that they all own an identical share of the firms’ equity. They are either employed, and they work full time, the amount of time units \(h\) then being scaled to unity, or they are unemployed and work no hours. The number of employed households in period \(i\) is \(L_i\) and unemployed \(U_i = N - L_i\). As has been said above, the second period, the long run, is one of full employment, and so \(U_2 = 0\). Each household \(j\) supplies inelastically one unit of labour to the firms. The household sector has the following instantaneous utility function in period \(i\),

\[
(6) \quad U_i = U(C_i, G_i) = U(C_{ij} + \alpha G_i), U_1 > 0, U_{11} < 0.
\]

\(\alpha\) is the constant marginal rate of substitution between private and public consumption in private welfare, and \(0 < \alpha < 1\), analogously as concluded to be the most relevant range for this parameter by Heijdra and Lighart (1997). The private consumption \(C\) consists of the bundle of domestically produced goods \(Q\) and imported goods \(H\).

The consumption-saving decisions are made after the firms have made their employment decisions, i.e., the aggregate demand for labour \(L_i\) is determined first. Each household faces the same probability \(1-L_i/N\) of being unemployed. If a household is unemployed, it gets an unemployment benefit, which is in real terms equal to \(b\). There is a constraint in the (unionised) labour market, in effect reducing the flexibility of real wage adjustment. We assume that the benefit is fixed so high that the pre-recession

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3 If there is no elasticity of substitution between capital and labour, then \(I_3 = 0\)
product wage is effectively prevented from falling in a recession. This constraint has relevance at least in the European labour markets.

The aggregate real disposable income $Y_1$ of the households in the first period is

$$Y_1 = Q_1 + bU_1 - T_1,$$

where $T_i$ is the aggregate lump-sum taxes collected by the government in period $i$ in real terms. In the first period the households save (or dissave) $S_1 = Y_1 - C_1$ of their current income in real terms. These funds they invest into the interest bearing asset, i.e., deposits, supplied by the banks again at the money rate of interest $i$, the corresponding real rate being $r$, and into an increase in their nominal money holdings $M_2 - M_1$. The investments into foreign securities, being initially free, are in the recession either prohibited by exchange regulations or effectively eliminated by a tax on capital outflows or by a subsidy on capital inflows, see below Section 6.

The disposable real resources $Y_2$ of the households in the second period can be written, see the Appendix for details,

$$Y_2 = Q_2 - G_2 + (1 - \delta)^2 K_0 + (1 - \delta) I_1 - (1 + r^*) \left[G_1 - Q_1 + C_1 + I_1\right].$$

The household sector intertemporal budget constraint is simply given by (8) as they consume in the second period all their disposable resources, so that $C_2 = Y_2$, which means that the aggregate household sector has the same intertemporal budget constraint as the society as a whole, i.e., (2). (8) is based on the fact that the households are farsighted and see through the government budget constraint. Similarly as in Obstfeld and Rogoff (1996, p. 537), the money stock is not at present in the intertemporal budget constraint of the private sector. But neither is the domestic real rate $r$ present in (8), even though initially it is in the intertemporal budget constraint, see the Appendix. However, this does not mean that an individual household is not affected by the current domestic real rate of interest $r$. If it raises its current consumption by $dC_1$ (as a result of lower $r$), it knows that it has to bear a rise in foreign borrowing by the government (the deficit in the current account) of the amount $(1 + r^*) dC_1 / N$ in the second period, which is negligible. So, the households internalise the effect of the domestic interest regulation on the government finances, but only in a very small part as a factor influencing their own consumption/saving decisions.

We further specify that the intertemporal consumer allocations are based on the following usual type of specification of their welfare, denoted by $VH$,

$$VH = VH(U_1, U_2) = \frac{1}{1 - \sigma} U_1^{1 - \sigma} + (1 + \theta)^{-1} \frac{1}{1 - \sigma} U_2^{1 - \sigma},$$

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4 If the households dissave, see below, their borrowing is in effect extended to them by the government at the domestic real rate $r$.

5 One alternative is to impose a rationing of the credit market and create forced savings of the households, their access to the foreign capital market being simultaneously prohibited, as was the case in many industrialised countries before the world-wide wave of financial deregulation in the 1980s, the goal previously being to favour investment and contain at the same time foreign borrowing. We do not, however, consider this arrangement in more details here.
where $\sigma$ is the intertemporal elasticity of substitution of utility and $\theta$ is the subjective rate of time preference.

We can derive the aggregate consumption equation, noting that the equity market value of the initial stock of wealth, i.e., the existing real stock of capital in the beginning of period 1, is equal to $(1-\delta)^2K_0+(1-\delta)I_1$, as there are in the initial equilibrium no pure profits related to the marginal capital investment. Now the optimisation by the consumers gives as the outcome for aggregate consumption,

$$C_t = c_1[(1+r^*)^{-1}(Q_2-(1-\alpha)G_2+(1-\delta)^2K_0+(1-\delta)I_1)+(Q_1-G_1-I_1)]-D\alpha G_t,$$

where $0 < c_1 = c_1(\theta, r) = \frac{1+\theta}{1+\frac{(1+r^*)}{1+r}^{1/\sigma}} < 1, D = (1+(1+r^*)B)^{-1}$ and $B = \frac{(1+\theta)}{(1+r^*)}^{1/\sigma}$.

It is nowadays standard to write the IS-curve simply as an Euler equation of intertemporal consumer optimisation. However, in solving the model and optimal policies we need an explicit solution to the level of consumption related to the intertemporal optimisation.

The demand for the domestically produced and imported consumption goods is simply derived in a second stage optimisation of the internal allocation of the consumption bundle to be constant shares of total consumption, as the relative price between the domestic and foreign good is fixed, see below,

$$H_t = hC_1, \text{ where } 0 < h < 1.$$

Turn then to the portfolio balance. In the domestic financial market, the financial assets are firm bonds, deposits and domestic credit, all being perfect substitutes for each other, and bearing the real rate $r$. In addition, if and as we assume here, the households have access to the similar storage technology as the firms, the savings by the consumers in the real commodity yield the real return $-\delta$.

The allocation of the financial portfolio by the households into money and interest bearing assets is determined by introducing the following demand for money equations,

$$M_t^d = kP_tQ_t f(i), f' < 0 \text{ (and } k > 0).$$

In a small open economy without a nontraded goods sector, as simplified here, the goods prices are in a standard way given from abroad in foreign currency and purchases.

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6 In the specific case where the intertemporal elasticity of substitution $\sigma$ equals unity, the price and income effects of the real interest rate on saving just cancel out each other, and the volume of saving is not affected by the real interest rate, see Blanchard and Fischer (1989) and Feldstein (1978). In this case $c_1 = (1+\theta)/(2+\theta)$.

7 In the second period, we simply can stipulate the quantity equation and omit the interest rate then as it has no relevance then, see on a similar procedure Aghion et al. (2000).
ing power parity (PPP) holds. If the nominal interest rate is fixed to be the same as abroad, see Section 6 below, then (12) determines the domestic price level and PPP determines the exchange rate e1. As in period 2 there is full employment (or anyway output then is supply constrained), the monetary expansion producing M2 determines the price level in period 2 (through the equilibrium in the money market) and so also the inflation rate \( \frac{P_2}{P_1} - 1 \). Then PPP leads to a corresponding depreciation in the exchange rate e2. Note that an expansion of M1 alone may not be enough to create inflation (and inflationary expectations). This may be a crucial weakness of the policies concerned, and we consider the possibility to overcome it below. The operation of the financial market in order to achieve the optimal interest rate is discussed below in Section 6 in more details.

If it were optimal to lower the domestic real interest rate r below the foreign \( r^* \) and if this could be achieved by some means, which are considered below in Section 6, we can say that there is in effect a tax with the rate \( t = r - r^* \) on foreign capital inflows, or a subsidy, with the rate \( t \), similarly equal to \( r - r^* \), on foreign capital inflows.

Turn then to the solution of the model. The model consists of three blocks on the real side: domestically produced goods, the labour market and the imported goods, which, however, a residual. The instantaneous, beginning-of-period financial market consists of three segments: money, domestic long asset and foreign financial long asset. The two last instruments are perfect substitutes for each other, and so the stock equilibrium in the financial market is solely determined by the demand for money function (12). Foreign currency is not held by domestic firms and households. The market for foreign exchange is a residual one in the manner explained below. We first consider the solution of the model in the pre-depression situation in period 1, where we assume that there are free capital flows, and so \( t = 0 \), and flexible exchange rates. Assume without a loss of generality that there is no foreign inflation, \( P_1^* = P_2^* \), and so \( i^* = r^* \). The portfolio balance between the domestic and foreign financial assets is given by the open interest parity for the nominal rates,

\[
(13) \quad (1 + i^*) \frac{e_2}{e_1} = 1 + i.
\]

The purchasing power parity produces the condition,

\[
(14) \quad e_* \frac{P_2^*}{P_1} = P_1^*.
\]

By inserting this into (13), we get in the standard way the real interest rate parity, \( r = r^* \). The firms’ short-run demand for labour is given by (4a) to be a function of the inherited capital stock and the real wage. The wage setting behaviour was assumed above to be such that the real wage is initially, in the pre-recession situation flexible in the sense that the economy is in full employment. The level of the supply of goods \( Q_1 \) is then given by the production function. The flow of investment is given by (4b) and consumption by (11), which requires also the solution of output in period 2. As mentioned above, we simply assume that then “in the long run”, in period 2, full employment prevails and output is given by the capital stock \( K_1 = (1-\delta)K_0 + I_1 \) and that level \( N \) of the labour input. Given the exogenous government expenditure, net exports are the only remaining variable to be determined in the real side. As imports are given by (11), exports are then finally solved as a residual from the balance constraint (1). This is the case under full em-
ployment. In the short run, under unemployment (see Section 3), the volume of exports $X$ in (1) is determined through foreign demand, which is kept fixed. This is not very realistic, but is made here in order to keep the model as manageable. Namely, there cannot be unemployment in the prototype of a small open economy, if there is an infinitely elastic demand for the home goods by foreigners at the current price level. Note also that the demand for investment goods is solely satisfied by domestic production.

The monetary side of the model operates simply so that, given the money supplies $M_1$ and $M_2$ and the levels of production $Q_1$ and $Q_2$, the equilibria (12) in the money and financial markets and the conditions (14) solve for the domestic interest rate $i$, the price levels $P_1$ and $P_2$ and the exchange rates $e_1$ and $e_2$. Under full employment there is complete neutrality with respect to monetary policy and also the fiscal policy. Monetary policy has only a one-to-one effect on inflation and depreciation of the exchange rate, nothing else. And on the other hand, from a utilitarian welfare point of view, there is neither any need to deviate from the real interest rate parity, $r = r^*$, see below Section 5.

3. The economy in a depression

As a step towards considering optimal policies in a recession, let us first consider how the firms behave in such a situation. Imagine that the economy concerned, for one reason or another, slides into a depression in period 1. The reason for this recession can be a supply (productivity) shock or a demand shock. Initially, in the pre-depression full employment situation, the real rate of interest in the home economy (being equal to the foreign, $r = r^*$) is further equal to the rate of return $F_1 = r^* + \delta$ on the existing stock of capital $K_0$ (see (4b)). The shock is a temporary one occurring only in period 1. In the case of a supply shock, denoted by $s$, we have

\[
(15) \quad Q_i = Q_i^S = (1 - s)A_i F(K_0, N), \text{where } 0 \leq s \leq 1.
\]

The production decision by the firms given by (4) is modified accordingly. The shock does not lead to unemployment irrespective of the fall in the demand for labour if the labour market were flexible enough. The first best policy would be to put the labour market to adjust efficiently so that unemployment is eliminated in the first period. This possibility is assumed to be excluded here due to an institutional constraint. Note that as a result of the fall in capital productivity in (15), the investment expenditure of the firms will anyway go down, even though employment does not fall as the rate of return $F_1$ is now lowered below $r^*$. But under wage rigidity, the labour demand falls as well, and the rate of return on capital falls even more as capital and labour are cooperative factors of production. As the rate of return on capital $F_K$ is now less than the initial level $r^*$, and as the model does not include any adjustment costs related to investment, the firms cease to make any new (gross) investment, $I_1 = 0$ in the recession.

So, we note that a temporary supply shock has a permanent effect on economy through capital accumulation, even though there is full employment. We consider optimal policy under a supply shock in Section 5.

\* All the way we assume that the existing physical capital is internationally immobile and so there is a nonnegative flow of investment in the economy considered.
In the case of an adverse demand shock we have initially

\[ Q_1 = Q_1^D = A_1 F(K_0, N) - gs, \]

where \( s > 0 \) is the demand shock and \( g \) is the Keynesian income multiplier (see Section 4) multiplying the fall in output to be either bigger or smaller than the initial shock. This is not the end of the story, as the smaller output will be produced with a smaller number of workers. Let us now make a more detailed analysis of what happens in a demand recession, see Figure 1.

**Figure 1. The firms in a depression**

Initially the firms are in point A with full employment. Due to the recession their demand shifts down, and they would like to produce the smaller output in point D, given the initial cost of capital \( r^* \) and the rigid real wage. However, they cannot get rid of the inherited capital stock \( K_0 \), and as they have to bear the earlier capital cost, they shift to produce the smaller output in point B, as it is optimal for them to shed labour and employ the initial capital in full, if this is technically possible, i.e., if there is ex post substitutability between capital and labour.

As the labour they employ is smaller than in the initial equilibrium, also the marginal productivity \( A_1 F_1 \) of capital is smaller than initially, but, on the other hand, the marginal productivity of labour exceeds in point B the current sticky real wage rate. However, the profitability of the firms is in point B less than initially or in point D (where the firms
would be in the break-even situation), even though B is the best short-run situation for them. On the other hand the firms are not, either, in point B willing to carry out any new investment and substitute capital for labour, with a given level of output. This can be seen by evaluating the change in their costs of production TC in point B with respect to a larger capital stock, which is \(\frac{dTC}{dK} = r^* + \delta + W(dL/dK) = r^* + \delta - (W_1/F_2)q(r_1 + \delta) > 0\), where \(r_1 = F_1(K_0, L_1)\).

So, the firms are not satisfied with their situation in point B, where they are making a loss. This means that a Keynesian depression with an insufficient demand involves at the same time a classical underpinning in the form of a squeeze on profits. The firms are, however, in point B willing to expand their production, if there is an expansion in the effective demand for it, as the higher level of output and labour demand reduces their loss. This is a key feature behind expansionary policies in a Keynesian recession. So, irrespective of the sticky real wage, an expansion in demand leads to a higher output. The firms also start to expand their investment, if the cost of capital \(r\) is lower than \(r_1\). Optimal policies to fight unemployment and depression will be derived in Section 5.

4. The Keynesian multipliers

In Keynesian macroeconomics the income multiplier is in a key position in the following. In this model, the multiplier also depends on the intertemporal behaviour of the private sector and is therefore not the same with respect to investment and public expenditure in period 1. We consider the following five types of policies to alleviate unemployment in period 1: a temporary expansion in public expenditure, i.e., solely in \(G_1\); a permanent expansion in public expenditure, i.e., in \(G_1\) and \(G_2\) alike; and a switch in public expenditure, i.e., an increase in \(G_1\) and a cut in \(G_2\) so that the present value of total public expenditure \(G_1 + (1 + r^*)^{-1}G_2\) remains unchanged; and fourthly, an increase in private investment \(I_1\), achieved through lowering the interest rate \(r\). As the fifth policy, the government can channel subsidies to investing firms, but retain \(r\) at the level of \(r^*\), i.e., free international mobility of capital.

So, consider first the fiscal multipliers in isolation, assuming that investment expenditure is fixed. Note that all the following fiscal multipliers refer to debt financing, but are the same as those related to financing with lump-sum taxes, because of the Ricardian equivalence. We compare the multipliers to the threshold value of unity, shown to be a key dividing line in Section 5 as to optimal policies. The first period income identity (1), and taking into account (10) and (11), yields the result for the corresponding multiplier, denoted in general by the symbol \(g\), in the case of a temporary expansion in \(G_1\),

\[
\begin{align*}
  g(G_1, dG_2=0) &= \frac{dQ_1}{dG_1} = 1 - \frac{(1-h)\alpha D}{(1-c_i(1-h))} < 1, \text{ if } \alpha > 0, \\
  &< (17)
\end{align*}
\]

\[
  g(G_1, dG_2=0) = 1, \text{ if } \alpha = 0.
\]

Similarly for a permanent expansion in government expenditure we get the expression,

\[
(18) \quad g(G_1, dG_1 = dG_2) = \frac{dQ_1}{dG_1} + \frac{dQ_1}{dG_2} = 1 - \frac{(1-h)\alpha c_i(1-h)(1+r^*)^{-1} + D\alpha}{1-c_i(1-h)} < 1, \text{ if } \alpha < 1.
\]
If we take the standard benchmark closed economy case $\theta = r^*$ and $\sigma = 1$, $h = 0$ and $\alpha = 0$, i.e., there are no imported goods and the intertemporal elasticity of substitution is unity, then inserting $c_1$ from (10) into (18), we come to the special case where the multiplier in (18) is equal to zero, which means that fiscal policies are of no use in an effort to expand current output. Consumers react with a full immediate offset to the increase in public expenditure and the consequent reduction in their resources available for private consumption, caused by a future rise in taxes. However, if a part of the consumed goods are imported, as here, only a part of the cut in total consumption falls on the domestically produced goods, and therefore the multiplier in (18) is positive. But what is important in the context of optimal intertemporal policy, see Section 5, (18) is unambiguously less than unity, if $\alpha < 1$, as we assume here.

Consider then the switching of public expenditures. Now we get the multiplier,

\[
g(G_t, dG_t + (1 + r^*)^{-1}dG_2 = 0) = \frac{1 - \alpha(c_1(1-h) + D)}{1 - c_1(1-h)}.
\]

This is clearly bigger than unity, if $\alpha = 0$, and is then indeed identical with the classical static Keynesian multiplier in the open economy context. In general, for small values of $\alpha$ (19) is bigger than unity.

Consider then a rise in investment. In order for this to be meaningful, we have to assume that a lowering of the real interest rate from the level $r^*$ to $r$, can at least from some level downward spur new flow investment. And this it will do, as we shall show below. In this case, inserting the investment behaviour of the firms from (4b) into (1), and leaving the investment subsidy aside, i.e. $a = 0$ in (4b), we have

\[
g(I_1) = \frac{dQ_1}{dI_1} = \frac{1 + c_1(1-h)\frac{r - r^*}{1+r^*}}{1 - c_1(1-h)}.
\]

This is unambiguously bigger than unity, as $c_1$ is positive and $h$ is less than unity, which means that there is a simultaneous increase in the demand for domestically produced goods. The maximum of (20) is the same as the traditional Keynesian multiplier (i.e., (19) with $\alpha = 0$), reached in the case where $r = r^*$. Its minimum value is unity at the extreme absolute point where $r = -1$.

A subsidy $aI_1$ to investing firms is a transfer from the government to the owners of the firms. In this sense its financing is similar to a reduction in current taxes and an increase in them in the second period. Therefore, they are not present in the intertemporal budget constraint (8) of the households and do not cause an additional change in consumption compared to that in the investment multiplier (20). We are now ready to go over to derive the optimal policies in a depression.

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9 It is relevant to concentrate only to the case where $r \leq r^*$, see Section 5.
10 We have omitted here the interaction between fiscal and monetary policies. If the expansion of the government expenditure is able to raise investment at the same time, the fiscal policy multipliers in (17)-(19) have to be modified accordingly.
11 Note that in the imperfect competition model of a closed economy Heijdra and van der Ploeg (1996) interestingly also reach the situation where the fiscal multiplier is uniformly bigger than unity, if and as the elasticity of substitution between the various goods is bigger than unity.
5. Optimal policies in depression

Assume that the government is utilitarian, i.e., it has the same welfare function \( U = U(C_1, G_1, C_2, G_2) \) as the private consumers have in (5), but does not pay any attention to the value of leisure in workers preferences. The optimal policies are those which simply maximise the total intertemporal resources \( R \) available for the society as a whole, or equivalently for the consumers, for consumption, weighting the private and social consumption by the private price ratio \( \alpha \). In the derivation, we take into account the multipliers (17)–(20), respectively, which incorporate the constraints imposed by the private sector behaviour on policies.

In general, the change in resources \( R \) for private consumption (2) is equal to

\[
\frac{dR}{dO_i} = g_{O_i} - 1 + (1 + r^*)^{-1} \left( \frac{dQ_2}{dO_i} + (1 - \delta) \frac{dI_i}{dO_i} - \frac{dG_2}{dO_i} \right),
\]

where \( O_i \) is one of the above five policy measures considered and \( g_{O_i} \) the corresponding multiplier derived above in Section 4. The total resources \( TR \) for the private welfare are then,

\[
\frac{dTR}{dO_i} = \frac{dR}{dO_i} + \alpha \left( \frac{dG_1}{dO_i} + (1 + r^*)^{-1} \frac{dG_2}{dO_i} \right).
\]

Consider first fiscal policy, an expansion in the public expenditure (multipliers (17)–(19)). As to the temporary expansion in public expenditure \( G_1 \), it is easy to see using (17) that (21a and b) is zero, if \( \alpha = 0 \), i.e., current public expenditure is irrelevant as to the welfare of the society. It expands current output with the same amount as it drives down the resources available for consumption through increased foreign borrowing. If \( \alpha > 0 \), a reduction of \( G_1 \) is in place, irrespective of the recession, as the multiplier is less than unity and both (21a) and (21b) are negative.

A permanent expansion in government expenditure is definitely counterproductive, as the multiplier in (18) is less than unity and so (21a) is negative. More resources are used up than added. Note that this result essentially depends on the fact that the second period is supply constrained, i.e., that \( G_2 \) does not have an effect on \( Q_2 \). The policy conclusion is then that irrespective of the depression, permanent government expenditure should be cut. However, there is a value for \( \alpha \), less than unity, such that (21b) is positive.

On the contrary, a switch in public expenditure, an expansion today and a restriction tomorrow is a worthwhile policy in a Keynesian depression, as its short-run multiplier (19) is bigger than unity if \( \alpha = 0 \), and (21a) and (21b) are in general positive. The practical problem with this is, of course, the difficulty to credibly commit by the government to this kind of a policy mix, but this problem is a not considered any further here.

Turn then to consider encouraging current investment through a lower real interest rate. As the multiplier (20) was derived to be bigger than unity, and the second term in brackets in (21a) is in firm equilibrium equal to \( (1+r)/(1+r^*) \geq 0 \), we come to the conclusion
that the optimum rate of interest should be lowered as low as possible, even to its absolute minimum \( r = -1 \), in order to expand investment in a Keynesian depression. However, this extreme solution is not required as the firm equilibrium stipulates that \( r \geq -\delta \), see (4). And on the other hand, if \( r \) were lower than \( -\delta \), the possibility to pursue riskless arbitrage between the credit market and the goods market opens up, and therefore the purchases of the domestic good by borrowed funds from the banks would immediately rise enough to produce the initial full employment level of output.

Before proceeding further in this direction, let us first note what the optimal policy under a supply shock, i.e. the so-called Classical unemployment would be. In this case output \( Q_1 \) is determined by supply (see (15)) and demand expansion in any category has no impact at all on it, and therefore all the multipliers \( g_{0i} \) in (17)–(20) are zero. Inserting this into (21 a,b) produces the traditional outcome, referred to in the Introduction, that it is optimal to have the same domestic real rate as is the foreign, i.e., \( r = r^* \). Similarly, to increase welfare, cuts in public expenditure are at place, if \( \alpha < 1 \). Only the switching policy related to public expenditure produces a break-even situation, as is also plausible.

Return then back to solve the optimal rate of interest under a Keynesian depression. As mentioned, it is indeed optimal from an intertemporal welfare maximisation point of view in a Keynesian depression to expand investment as much as is needed to reach full employment \( L_1 = N \). The optimal interest rate is really, as defined by Keynes, the rate at which the elasticity of employment is zero. Accordingly, the equation from which this rate can be solved is the following for the flow of investment in period 1,

\[
Q_1^D = Q_1^0 + \int_0^{l_1} g_1(I_1)dI_1 = Q_1^S(K_0, N),
\]

where \( Q_1^D \) is the aggregate demand, \( Q_1^0 \) is the base (depression) level of output and \( Q_1^S \) is the aggregate supply, based on the inherited capital stock and the fixed initial supply of labour. We assume that in the pre-depression situation \( r = r^* \).

If the rate of interest is lowered, there can also be an additional autonomous effect coming from the consumption to output. This depends on \( \sigma \), the intertemporal elasticity of substitution, and may lead to a rise in \( c_1 \) in (11), or a reduction in it, depending on whether \( \sigma \) is higher or smaller than unity. However, in the benchmark case where \( \sigma \) is unity, \( dc_1/dr = 0 \), and no additional effect will have to added to (22). Let us for simplicity stick to this case in the following.

We next make a change of the variable in the integration in (22) from \( I_1 \) to \( r \). As noted above, in the depression the current rate of return on capital is less than the initial rate \( r^* \). Therefore, for all real interest rates \( r \) between the values from \( r^* \) to \( r_1 = F_1(K_0, L_1) \), there is no flow of new investment. Taking into account of this, inserting (20) into (22), and carrying out the integration, and by keeping as an approximation the differential between \( I \) and \( r \) (i.e., \( I_r \)) as a constant in the integration, we can solve the optimal real interest rate \( r \) from the following second order equation,

\[
(r - r_1)(1 + \frac{c_1(1-h)}{1 + r^*} \frac{r + r_1}{2} - r^*) = \frac{\Delta Q_1(1-c_1(1-h))}{I_r(r = r_1)},
\]
\( \Delta Q_t = Q_t^s - Q_t^D \). The solution to (23) is then the optimum rate of interest \( r_{\text{opt}} \) defined by Keynes. We note that if there is no unemployment (\( \Delta Q_1 = 0 \)), the optimal interest rate is the same as the foreign. We have \( F_1(K_0,N) = r^* \) and then in (23) we can consider only the root producing this outcome. However, under unemployment the optimal rate is lower than \( r^* \) and \( r_1 \).

In Figure 2 we have numerically solved the optimal rate as a function of the deepness of the recession. The baseline case is that where the production function \( F \) is Cobb-Douglas (‘\( r_{\text{opt}} \) CD’ in the Figure). This assumption implies a very big reaction \( I_r \) of the investment demand with respect to lowering the rate of interest below \( r_1 \). The optimal domestic real interest rate remains throughout on the positive side and is in fact very near \( r_1 \), the current rate of return on capital in depression. Anyway, the optimal policy requires a separation of the domestic rate from that abroad. As an alternative, we have arbitrarily fixed \( I_r \) to a much smaller figure, to minus unity. In this case (‘\( r_{\text{opt}} \) alt’ in the Figure) the optimal real rate becomes very clearly negative as the depression becomes worse.

**Figure 2. The optimal real rate of interest as a function of the percentage output gap**

![Graph showing the optimal real rate of interest as a function of the percentage output gap.]

Turn then finally to consider a subsidy by the government to the investing firms, lowering the effective cost of funds, even though the domestic real rate of interest will be retained to be the same as foreign, i.e., at \( r^* \). As to the optimal investment subsidy, we can solve from the optimal investment condition (4b),

---

12 The drop in the current rate of production is exogenous. Then we first solve for the employment demanded by the firms in a recession, given the initial stock of capital (i.e., corresponding to point B in Fig. 1), and calculate \( r_1 = AF_1 \) in this point. Then \( I_r \) is solved in this point from the marginal productivity condition of capital, given this reduced amount of labour. Finally we solve the optimal rate \( r_{\text{opt}} \) from (23). In the Figure \( r^* = 4 \) per cent, \( c_1=0.5, h=0.3 \) and Cobb Douglas form of \( F \) has the elasticity of capital being 0.4.
\[ a_{opt} = \frac{r^* - r_{opt}}{1 + r^*}, \]

which is positive and normally less than unity. In the special case when capital depreciates instantaneously, \( \delta = 1 \), \( a_{opt} \) approaches unity, if the optimal rate of interest also goes down to its absolute minimum of minus unity.

Now in the Keynesian optimum the domestic real interest rate \( r \) is lower than the rate of time preference \( \theta \). In a closed economy the growth of the capital stock requires that \( r > \theta \), as otherwise there would not be any positive savings. In the international economy we must accordingly have the condition that \( r^* > \theta \). However, in a single economy if foreign funds are available, the national condition can be violated, and that what matters is the international constraint and that is not violated in the model here. On the other hand, it may well be that the Keynesian optimal solution is dynamically inefficient, defined in the normal way, e.g., that as \( r \) may be negative, there is too much capital in the economy. This violation is due to the social external effect of investment, in excess of the private return, on output and employment present in a Keynesian model of depression, which has no role in the basic models of growth theory.

### 6. Implementation of the optimal policy

We now consider how to realise the optimal real interest rate. First, (12) gives the money rate of interest \( i \), which has to be positive. Assume that it is in the pre-depression situation the same as the foreign rate \( i^* \). As is clear from what has been said above, in order to reach the optimal rate of interest, it is necessary to separate the domestic financial market from the foreign by a tax/subsidy scheme. Write now, instead of the open interest parity in (13), between the after-tax foreign and domestic nominal interest rates, (13b)

\[ (1 + i^* - t) \frac{e_2}{e_1} = 1 + i. \]

The assumption of PPP gives, as there is no foreign inflation, that \( P_2/P_1 = e_2/e_1 \). Inserting this into (13b) and using the definition of the real interest rate produces the optimal tax on foreign financial flows, \( t = r^* - r_{opt} \). Assume that the money demand does not allow lowering of the real interest rate \( r \) to be achieved through lowering of the nominal rate \( i \), the liquidity trap case. In order to reach the optimal domestic real interest rate \( r_{opt} \) lower than \( r^* \), there must then be domestic inflation, and accordingly, an expected devaluation, put into effect by a monetary expansion in the domestic economy. We see from (13b) that, with a fixed \( t \), the expected devaluation would lead to a pari passy rise in the domestic interest rate which would be futile from the point of view of the optimal policy. *This caveat is, however, circumvented by choosing the tax \( t \) on foreign capital flows so that the optimal social real interest rate can be achieved with the initial money rate of interest \( i^* \).* Note that it may very well be the case that the optimal tax means not only a tax on the interest income, but includes also a component of the wealth tax on the foreign assets. The problem is then, how to commit to the “irresponsible inflationary financing” so that the private sector believes in this announcement, see Krugman.
The future money stock $M_2$ cannot be committed to by the central bank in period 1, it can only expand the current money stock $M_1$. Imposing the tax $t$ on foreign capital flows in period 1 is, however, an indication to the private sector of the desire by the government to push the real rate down from the level of $r^*$ and to do this by inflationary policies.

A second solution would be to give a subsidy on the domestic financial savings, i.e., to shift the tax parameter $t$ from the left to the right hand side of equation (13b). This would work in a similar way as described above and drives down the domestic real interest rate. So, anyway, both a tax on foreign capital flows or a subsidy on domestic financial savings, and inflation are necessary in order to reach the optimal solution.

There has recently been a rapid spread of interest towards analysing the potential, or the lack of it, of monetary policies under the condition of a zero lower bound for the nominal rate of interest see e.g. the papers in the special issue of *Journal of Money Credit and Banking*, November 2000, Svensson (2000), and also Oko (2001) in a more theoretical sense. The basic starting point of this renewal of interest is the persistent Japanese slump during the 1990s.

The most relevant of these papers from our point of view is Svensson (2000). In effect, he proposes a “fool proof” jump start for an open economy in a depression. The proposal consists of forcing a devaluation of the exchange rate through the pegging it to a lower level than the initial one so that the economy will be pushed to a rising target path of the price level. The initial depreciation of the currency will then be followed by a real appreciation toward the equilibrium real exchange rate which is the same as the initial one. This would result in a lowering of the real interest rate. The proposal, however, raises a few doubts. In many cases, the real depreciation has proved to be long lasting so that the key assumption of the paper that the real exchange rate is stationary, may not be well founded. This implies that the private sector is not likely to be aware of the new equilibrium exchange rate as the initial depreciation may not lead to expectations of an immediate turn around to a real appreciation, but to expectations of further depreciation. This would have the effect that the expected real interest rate is not lowered in the manner as is suggested by Svensson. There is another problem with the proposed process, namely that he suggests a floating rule after the initial depreciation so that the nominal exchange rate is depreciated by the differential between the domestic and foreign inflation rates. This would, however, prevent the key real appreciation, as the exchange rate would be stay unchanged in real terms under such a peg and would not allow a reduction in the domestic real rate of interest.

The analysis of our paper is, however, in a crucial sense too limited to be able to discuss the policy problem in the most relevant terms. We have here assumed purchasing power parity to prevail throughout, and therefore there is no room for a real depreciation or appreciation. But in contrast to this, we have tried to explicitly derive the optimum monetary and fiscal policies, which is not done by e.g. Svensson.

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13 Note as a separate issue that from the point of view of the labour market, this inflationary process makes the real wage adjustment a lot easier to reach full employment in period 2.

14 As evidenced by the recent crises of Mexico in 1995, Russia in 1998, and in Europe Finland and Sweden in 1991-92.
7. Conclusions

In this paper we have explicitly derived the socially optimum rate of interest, fiscal policy and consequent integration with, or isolation of the national financial market from the foreign, under various assumptions on the typology of unemployment, using an intertemporal model of an open economy. The optimal policies are to some extent in sharp conflict with each other in the Keynesian and Classical case of unemployment, which is by no means a surprise. The outcome of the Keynesian situation also crucially depends on, whether the short run income multiplier with respect to the policy measure is bigger or less than unity. The results on the optimal use of fiscal policies are quite interesting. Expansionary policies are not uniformly recommended.

It should be added that in practice the optimal interest rate depends on the expected duration of the depression; if this is short, the Keynesian policy with respect to the domestic interest rate approaches the Classical. We showed that Keynesian policies either lead to an effective isolation of the domestic economy from the foreign, or anyway to a subsidisation of the investment expenditure of the private sector.

The consequent inflation resulting from the process of driving down the real interest rate can be avoided by the policy of integrating the domestic market completely with the foreign, in combination with subsidising only domestic investment in the manner shown above. In practice, the policies in an open economy are also often severely constrained by the limited availability of foreign finance to run a larger deficit in the current account. This important complication has been overlooked in the present analysis.

There is likely to be an important negative spillover from the policies derived above of a microeconomic sort, not included as an element in a macro model like the one considered here. It has been forcefully argued by Davidson (1997) that the so called Tobin tax on foreign exchange transactions can have a large impact stemming international trade by raising the costs of hedging operations. Here taxing of foreign trade transactions is not involved, but it may be quite difficult to limit the tax only to capital transactions, and in any case hedging of trade flows and their covering operations by banks concern capital transactions as well. But this should not in principle be something surprising as it is quite common when Keynesian policies are considered. There is also an inefficiency caused by intentional financial repression, prevailing as a long-run neoclassical situation in many developing countries, see Giovannini and de Melo (1993).

Under a common currency in the EU, the above policy conclusions in a Keynesian depression are obviously hard to be realised. There can be a national tax on interest earnings on savings, but there is no domestic monetary policy, therefore the optimal policy outlined above is not feasible. It remains to give a subsidy on capital investment, and it seems to be possible to implement it at least under certain conditions – and is done currently in Europe - even though it could also be interpreted as a subsidy to domestic production and as such not to be in accordance with the free Internal Market. It is true that macroeconomic policies in the European Union, as elsewhere, have got more of the classical tone, emphasising the flexibility in the labour market under supply shocks, and under these conditions also the results in this paper on optimal policies are clear-cut favouring the present tone in policies.
References


Appendix. Derivation of the intertemporal budget constraint of the private sector

The flow of real factor income $Y_F^2$ of the households can be written in the second period,

\[(A1) \quad Y_F^2 = \pi_2 + (W_2/P_2)L_2 + iP_1[S_1 - (M_2-M_1)/P_1]/P_2 .\]

In addition to the flow of income, also the stock of wealth, net, $V_{PR}^2$, of the private sector existing at the end of period 2 is available for consumption then. This net wealth can be written with some manipulation as follows,

\[(A2) \quad V_{PR}^2 = (1-\delta)^2 K_0 + (1-\delta)I_1 - P_1(K_0+I_1)/P_2 + P_1[S_1 - (M_2-M_1)/P_1]/P_2 + P_1(K_0+I)/P_2 + M_2/P_2 ,\]

where the first three terms are the equity of the firms existing at the end of period 2 after paying the flow of profits as dividends in period 2 to their owners (see (7b)), the fourth term is the stock of government bonds, the next term is the assets owned by the banks at the end of period 2 to be handed over to their owners, i.e., households, and the last term is the claim created by the money stock on the government. Now the households’ disposable real resources in period 2 can be written,

\[(A3) \quad Y_2 = Y_F^2 + V_{PR}^2 - T_2 .\]

The banks have as their liabilities the savings of the households and as their assets the loans extended to the firms for investment. The surplus $S_1 - (M_2-M_1)/P_1 - I_1$ the banks invest in government bonds at the domestic money rate of interest $i$. The foreign assets are omitted for similar reasons as presented above in connection with the households. The rest of the budget deficit the government finances by issuing money, i.e., by borrowing from the central bank, and by borrowing from abroad at the given foreign nominal rate of interest $i^*$, the corresponding real rate of interest being $r^*$ . In the first period government borrowing is equal to the government expenditure $G_1$, used to purchase goods produced by the firms, and the unemployment benefits, less the taxes $T_1$ then. The government budget constraint in the second period gives rise to the following taxation in terms of output,

\[(A4) \quad T_2 = G_2 + iP_1[S_1 - I_1 - (M_2-M_1)/P_1]/P_2 + i^*(P_1/e_1)[G_1 + bU_1 - T_1 - (S_1 - I_1) - (S_1 - I_1)](e_2/P_2) + D_2/P_2 ,\]

where $D_2$ is the net government liabilities at the period 2, i.e.,

\[(A5) \quad D_2 = P_1[S_1 - (M_2-M_1)/P_1]/P_2 + P_1/e_1)[G_1 + bU_1 - T_1 - (S_1 - I_1) - (S_1 - I_1)](e_2/P_2) + M_2/P_2 .\]

\[\text{To be more exact, in the government budget constraint the real financing cost of foreign borrowing is } (1+i^*)(P_1/e_1)/(P_2/e_2) = (1+i^*)P_1^* / P_2^* = 1+r^*, \text{ where } i^* \text{ is the foreign nominal rate of interest, } e_i \text{ is the exchange rate, price of a unit of foreign currency in terms of home currency, and } P^* \text{ the foreign price level.} \]
The last item on the right hand side is due to the fact that the money stock is a claim on the government by the private sector. (A4) and (A5) form at the same time the intertemporal budget constraint of the government. Inserting (A5) into (A4) and further into (A3), the disposable real resources $Y_2$ of the households in the second period can be written in the form (8) above.