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**Indicators of inequality and poverty**

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Research Paper No. 2004/25

Indicators of Inequality and Poverty

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Abstract

This essay aims at a broad, main-stream account of the literature on inequality and poverty measurement in the space of income and, additionally, deals with measures of disparity and deprivation in the more expanded domain of capabilities and functionings. In addition to an introductory and a concluding part, the paper has four sections. The first of these, on measurement of income inequality, deals with preliminary concepts and definitions; a visual representation of inequality (the Lorenz curve); real-valued indices of inequality; properties of inequality indices; some specific inequality measures; and the relationship between Lorenz, welfare, and inequality orderings. The second section, on poverty, deals with the identification and aggregation exercises; properties of poverty indices; some specific poverty measures; the problem of plurality and unambiguous rankings; poverty measures and anti-poverty policy; and other issues in the measurement of poverty. The third section considers aspects of both congruence and conflict in the relationship amongst poverty, inequality, and welfare. The final substantive section advances the rationale for a more comprehensive assessment of human well-being than is afforded by the income perspective, it briefly reviews measurement concerns relating to generalized indices of deprivation and disparity, and it discusses the data and policy implications of the more expansive view of well-being adopted in the section.

Keywords: inequality, disparity, poverty, deprivation, measurement, income, capability, functioning, well-being

JEL classification: D63, I31, I32
Acknowledgements

This paper, by its nature, is not an original piece of work: it is constructed around the writings of a number of scholars in the fields of inequality and poverty measurement, of whom specific mention may be made of S. Anand, A. B. Atkinson, J. E. Foster, N. C. Kakwani, R. Kanbur, A. Sen, and A. F. Shorrocks. It also draws heavily on the author’s ‘Introduction’ in Subramanian (1997). The paper has benefited from discussions with, and comments by, Mark McGillivray. Thanks are due to A. Arivazhagan for help with the graphics. Adam Swallow has edited the typescript with an almost frighteningly minute eye to detail. Taina Iduooze has performed a truly heroic job of chasing up, and filling in, the gaps in the bibliographical list. The author’s most considerable debt is to James Foster for his detailed comments on the paper—comments which he himself inadequately describes as ‘[a] series of smallish critiques [and] general peevish questions’: as it happens, the critiques were seldom smallish and the questions were uniformly peevish, both of which facts have helped greatly in improving the quality of the product. All errors and deficiencies are solely the author’s.
1 Introduction

In this paper we shall take a rapid overview of certain salient issues in the measurement of inequality and poverty—two major sources of social ‘illfare’. While the intention is to assign each of the two topics the same importance, this may not be reflected in the final outcome (which seems somewhat to favour ‘poverty’ over ‘inequality’) when judged according to the amount of space allocated to the two topics: the space-allocation pattern, it must be clarified, is largely a function of the expository demands—as they have seemed to present themselves—confronting the author. Both subjects are immensely vast, and built on foundations of considerable philosophical and conceptual import which, however, we shall have little space to review here. The best one can do is to point the interested reader toward Sen’s (1981a, 1992) assessments of the conceptual underpinnings of the notions of poverty and inequality respectively. Our own concerns here will be restricted to issues in measurement, and that, principally though certainly not exclusively, in the space of incomes. This is not so much because of any underlying view to the effect that income is the most relevant dimension in which to measure inequality and poverty, as because this essay is primarily a survey of the literature; and the literature on inequality and poverty has, as it happens, concerned itself very largely with measurement in the income dimension.

In discussing inequality and poverty measures as well-being (or rather ‘ill-being’) indicators, a two-fold approach will be adopted. First, holding constant the dimension (say income) in which measurement is undertaken, the indicator will be varied, so as to furnish an idea of the greater or lesser adequacy with which alternative indicators capture features of aggregate well-being. This would call for a description of the more significantly desirable properties of inequality and poverty indices, and an assessment of which indices satisfy what properties. Second, holding the indicator fixed, the dimension in which measurement is undertaken will be varied, so as to furnish an idea of the greater or lesser appropriateness of alternative ‘spaces’ in which to assess aggregate well-being. This would call for a consideration of measurement in non-income dimensions—such as in the space of capabilities and functionings—in addition to the income dimension. Simply as a guide to unambiguous usage, the terms ‘inequality’ and ‘poverty’ will, in this essay, be generally reserved for the space of incomes, while the terms ‘disparity’ and ‘deprivation’ will be employed for more inclusive spaces.

2 Inequality measurement

2.1 Preliminary concepts

For specificity, we shall throughout work with the domain of incomes as the one in which inequality will be assessed. A fundamental unit of consideration will be an income vector. An income vector \( x \) is a list of \( n \) non-negative incomes \((x_1, \ldots, x_i, \ldots, x_n)\), where \( x_i \) \((i = 1, \ldots, n)\) stands for the income of person \( i \) in a community of \( n \) individuals. The set of individuals whose incomes are represented in the vector \( x \) will be designated by \( N(x) \); \( n(x) \) will stand for the dimensionality of \( x \); and \( \mu(x) \equiv (1/n(x))\sum_{i=1}^{n(x)}x_i \) will stand for the mean of the incomes in \( x \). What we have just discussed is a discrete income distribution. On occasion, it is helpful to work with a continuous distribution: here, \( x \) will stand for a random variable signifying income; \( f(x) \) is the density function of \( x \) (that is, the proportion of the population with income \( x \)); \( F(x) \) is the cumulative density function of \( x \) (that is, the proportion of the population with incomes not exceeding \( x \)); and \( F_1(x) \) is the first-moment distribution function of \( x \) (that is, the share in total income of units with incomes not exceeding \( x \)). For a clear statement of concepts and definitions, the reader should consult Kakwani (1980a).

2.2 A visual representation of inequality: the Lorenz curve

One of the clearest ways of obtaining a visual picture of inequality in the distribution of incomes is to plot the Lorenz curve (due to Lorenz, 1905) in the unit square. The Lorenz curve is just the plot of the first moment distribution function \( F_1(x) \) against the cumulative density function \( F(x) \). If income is perfectly equally divided, then it is clear that the Lorenz curve will coincide with the diagonal of the unit square. But a typical, unequal distribution will be represented by a Lorenz curve that lies below the diagonal, and is convex in shape. The more unequal a distribution, the further away from the diagonal the Lorenz curve will lie. For the case of complete concentration, the Lorenz curve will be described by the two equal sides of the right-angled triangle of which the diagonal of the unit square is the hypotenuse.

Given a discrete, non-decreasingly ordered \( n \)-vector of incomes \( x = (x_1, \ldots, x_i, \ldots, x_n) \), the Lorenz curve—noting that \( \mu(x) \) is the mean of \( x \)—can be derived as a plot of the following points: \((0,0); (1/n, x_1/\mu); (2/n, (x_1 + x_2)/\mu); \ldots; ((n-1)/n, (x_1 + x_2 + \ldots + x_{n-1})/\mu); (1,1)\): these points, connected by straight lines, will then yield a ‘piece-wise linear’ Lorenz curve. In Figure 1 I have drawn the Lorenz curves for three hypothetical, unequal distributions \( x, y, \) and \( z \). I shall now define the binary relation of Lorenz dominance, designated by \( L \). Given any two distributions \( x \) and \( y \), we shall say \( x \) Lorenz-dominates \( y \), written \( xL y \), if and only if the Lorenz curve for \( x \) lies somewhere inside, and nowhere outside, the Lorenz curve for \( y \). In the discrete case, for two equidimensional non-decreasingly ordered \( n \)-vectors of income \( x \) and \( y \), we shall say \( xL y \) holds whenever \( \sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} y_i \) for every \( i = 1, \ldots, n \) and there exists at least one \( i \) for which \( \sum_{j=1}^{i} x_j > \sum_{j=1}^{i} y_j \). If \( x \) Lorenz-dominates \( y \), then it seems reasonable to conclude that \( x \) has unambiguously less inequality than \( y \). Can any two distributions always be ranked according to the Lorenz-dominance relation? No. Note from Figure 1 that while it is certainly true that \( xL y \) and \( xL z \), we can neither say \( yL z \) nor \( zL y \): the relation \( L \) cannot rank intersecting Lorenz curves.
What are the properties of the relation $L$? $L$ is *irreflexive*, that is, there is no $x$ such that $xLx$ holds; it is *asymmetric*, that is, for all $x$, $y$, $xLy$ implies $\neg yLx$; it is *transitive*, that is, for all $x$, $y$, and $z$, if $xLy$ and $yLz$, then $xLz$; but $L$, as we have seen, is *not complete*, in the sense that given any distinct $x$ and $y$, it is not necessarily true that either $xLy$ or $yLx$ should hold. A binary relation like $L$, which is irreflexive, asymmetric, transitive and incomplete, is called a *strict partial ordering*. The point to note is that the ordering is partial: this may well be how it should be, but nevertheless, if we were to provisionally view the completeness of a binary relation to be a desired virtue, then the question arises: how do we secure the possibility of obtaining an inequality ordering over all distributions of income that we may be interested in? This question is addressed in the following sub-section.

### 2.3 Real-valued indices of inequality

For every positive integer $n$, let $X_n$ to be the set of non-negative $n$-vectors of income $x$; and let $X$ be the set $\bigcup_n X_n$. If $R$ is the set of reals, then an inequality index is a mapping $I: X \to R$, such that, for every $x \in X$, $I(x)$ specifies a real number that is intended to capture the extent of inequality in the vector $x$. The (arguable) advantage with a real-valued index of inequality is that it precipitates a complete ordering over income distributions, which follows from the fact that any two real numbers are always comparable in terms of the ‘$>$’, ‘$=$’, or ‘$<$’ relationship. (One says ‘arguable’, because
the property of ‘completeness’ in a binary relation could be an over-praised attraction: as Sen has repeatedly pointed out, in a variety of contexts, it is not necessarily a virtue to force comparability on a pair of alternatives which are not inherently comparable. Further, there is always the possibility of loss of relevant information in the process of aggregation.)

How do we choose among alternative inequality indices? A means to this end resides in recognizing the usefulness of first specifying what we may think are desirable properties for such measures. The properties of inequality indices we shall review in the following sub-section have a fair measure of support from scholars working in the area. The ensuing exposition draws considerably on Shorrocks (1988) and Anand (1983).

2.4 Properties of inequality measures

If the sorts of populations we are dealing with are homogenous populations, then the *symmetry* axiom (*Axiom S*), which requires an inequality index to be invariant with respect to an interpersonal permutation of incomes, has a certain natural appeal. Second, we may wish to specify a lower bound of zero for an inequality index, and reserve this number for a distribution in which income is perfectly equally divided: this is the *normalization property* (*Axiom N*). Third, it is of the essence for an inequality measure to satisfy the *transfer property* (*Axiom T*), which is the requirement that the inequality index should register a decline in value following—other things equal—on a progressive rank-preserving transfer of income between two individuals. The next property of an inequality index is that of *continuity* (*Axiom C*), which demands that, for every positive integer n, I(\(x\)) should be continuous on \(X_n\). The preceding four properties are what Shorrocks (1988) refers to as the ‘basic’ properties of an inequality index. These do not, of course, exhaust the set of desirable features one may look for in an index. One such additional desirable feature is the so-called *replication invariance* property (*Axiom RI*), which is the requirement that the inequality index should be invariant with respect to any k-fold population replication of an income distribution. Another such feature is captured in the *scale-invariance* property (*Axiom SI*), which stipulates that the inequality index be *mean-independent*, that is, invariant with respect to any uniform scaling up or down of an income distribution. The next property is a strengthening of the transfer axiom: *transfer-sensitivity* (*Axiom TS*—see Shorrocks and Foster 1987) requires that an inequality index be more responsive to income transfers at the lower than at the upper end of an income distribution (there are alternative ways of expressing this requirement, and a fuller treatment—adapted to the context of poverty measurement—is available in section 3.3). The next two properties are concerned with the relationship between subgroup inequality and overall inequality. *Subgroup consistency* (*Axiom SC*)—see Shorrocks 1988—requires that, other things equal, an increase in any one subgroup’s inequality level should not cause overall measured inequality to decline. *Decomposability* (*Axiom D*) is the requirement that an inequality index be amenable to decomposition into two components: a *within-group* inequality component and a *between-group* inequality component. The final property we shall consider is one that can scarcely be stated in any precise or formal way. It is the requirement that the inequality index be amenable to ready interpretation in terms of its intuitive appeal. I shall call this property *Axiom E*, for ‘ease of interpretation’.

We have thus far discussed certain properties of inequality indices in very general terms. It is time now to consider specific inequality measures.
2.5 Some specific inequality indices

The literature on inequality measures differentiates between two types of measures—the so-called ethical measures, and the so-called descriptive measures. Ethical measures are those that seek to relate inequality in a distribution to the loss in social welfare arising from the presence of inequality: examples of this approach are to be found in the work of, among others, Dalton (1920), Aigner and Heins (1967), Kolm (1969), Atkinson (1970), and Blackorby and Donaldson (1978). For specificity, we shall briefly describe here the approach adopted by Atkinson (1970). Imagine that the social welfare function (SWF) is of the utilitarian type, and that each person has an (identical) increasing, strictly concave utility function, so that, given any income vector \( x \), the SWF is given by:

\[
W(x) = \sum_{i=1}^{n(x)} U(x_i),
\]

where \( U(x_i) \) defines the ith person’s utility level. Of crucial significance for an ethical inequality index is the notion of an equally distributed equivalent income, or ‘ede income’, for short. Given an income vector \( x \), the ede income \( x_{ede} \) is defined as that level of income such that, if it is equally distributed, then the resulting social welfare is exactly the same as that which obtains for the distribution \( x \).

That is, \( x_{ede} \) can be obtained as the solution to the equation

\[
\sum_{i=1}^{n(x)} U(x_{ede}) = \sum_{i=1}^{n(x)} U(x_i) \tag{1}
\]

Since the \( U(.) \) function is strictly concave, \( x_{ede} \) will be less than the mean \( \mu(x) \) of the distribution \( x \). The proportionate difference between \( \mu \) and \( x_{ede} \) then furnishes us with a measure of inequality, interpreted as the welfare-loss, in equivalent income units, occasioned by the presence of inequality.

Atkinson specialized the individual utility function to the so-called ‘constant elasticity of marginal utility’ type, given by

\[
U(x_i) = \left[ \frac{1}{1-\varepsilon} \right] x_i^{1-\varepsilon} \quad \text{for} \quad \varepsilon > 0 \quad \text{and} \quad \varepsilon \neq 1;
\]

\[
= \log x_i \quad \text{for} \quad \varepsilon = 1 \tag{2}
\]

To ensure strict concavity of \( U(.) \), \( \varepsilon \) is confined to strictly positive values, and this parameter is interpreted as measuring a degree of ‘aversion’ to inequality: as \( \varepsilon \) becomes larger and larger, the U function becomes more and more concave, so that the social welfare function \( W \) becomes more and more ‘equity-conscious’. Given (1) and (2), it is easy to see that, in the Atkinson context, the equally distributed equivalent income is given by

\[
x_{ede} = \left[ \frac{1}{n} \sum_{i=1}^{n} x_i^{(1-\varepsilon)} \right]^{1/(1-\varepsilon)} \quad \text{for} \quad \varepsilon > 0 \quad \text{and} \quad \varepsilon \neq 1;
\]

\[
= \exp\left[ \frac{1}{n} \sum_{i=1}^{n} \log x_i \right] \quad \text{for} \quad \varepsilon = 1 \tag{3}
\]

Atkinson’s ‘ethical’ measure of inequality can then be written, for any given income vector \( x \), as

\[
A(x) = \frac{\mu(x) - x_{ede}(x)}{\mu(x)} \tag{4}
\]

where \( x_{ede} \) is as given in (3).
While this approach to inequality measurement is strikingly interesting, it has its own problems, which have been most elegantly discussed by Sen (1978). Sen points out that inequality indices must measure inequality, and not the loss in social welfare occasioned by inequality: conflating the two notions could do violence to the independent descriptive role which an inequality index has.

The rest of this sub-section will be devoted to a discussion of certain widely used descriptive measures of inequality, which are essentially statistical measures of dispersion, not explicitly motivated by a desire to link inequality to the welfare losses arising from the former. In what follows, we present the expressions for five, fairly commonly used, descriptive indices. Given any income vector \( \mathbf{x} \), the Variance of Incomes (\( V \)), the Variance of Log Incomes (\( V_L \)), the Squared Coefficient of Variation (S), Theil’s (1967) ‘Entropy’ Index (T), and the Gini Coefficient of Inequality (G) are given, respectively, by

\[
V(\mathbf{x}) = \frac{1}{n(\mathbf{x})} \sum (x_i - \mu(\mathbf{x}))^2 
\]

\[
V_L(\mathbf{x}) = \frac{1}{n(\mathbf{x})} \sum (\log x_i - \log \mu(\mathbf{x}))^2 
\]

[Note: The version of \( V_L \) presented above is the one employed by Sen (1973): the mean income which figures on the right hand side of (6) is the arithmetic mean rather than the geometric mean, which latter—strictly—is the quantity customarily employed in the ‘varlog’ measure].

\[
S(\mathbf{x}) = \frac{1}{n(\mathbf{x})} \sum \left( \frac{(x_i - \mu(\mathbf{x}))/\mu(\mathbf{x})}{\mu(\mathbf{x})} \right)^2 = V(\mathbf{x})/\mu^2(\mathbf{x}) 
\]

\[
T(\mathbf{x}) = \frac{1}{n(\mathbf{x})} \sum x_i/\mu \log(x_i/\mu) 
\]

and

\[
G(\mathbf{x}) = \frac{n(\mathbf{x}) + 1}{n(\mathbf{x})} - \frac{2}{n^2(\mathbf{x})\mu(\mathbf{x})} \sum [n(\mathbf{x}) + 1 - i]x_i 
\]

where the individuals have been indexed in non-decreasing order of their incomes, namely \( x_i \leq x_{i+1}, i = 1, \ldots, n - 1 \).

Two major problems with the indices \( V \) and \( V_L \) respectively, which tend to disqualify them from further serious consideration, are that the one violates Axiom SI and the other violates Axiom T (on the latter problem, see Sen 1973; the problem also arises with the ‘canonical’ version of the ‘varlog’ measure, where the mean income employed is the geometric mean, on which see Foster and Ok 1999). This effectively narrows down the field to the set of indices S, T, and G. A quick way of reviewing these three descriptive measures of inequality would be to consider them all together, and in relation to each other, in terms of the properties they satisfy. In the summary chart labelled Table 1, a ‘+’ stands for fulfilment of a property, and a ‘−’ stands for its violation. Where property E (ease of interpretation) is concerned, ‘H’ stands for ‘high’, ‘M’ for ‘medium’, and ‘L’ for low: it must be emphasized that the evaluation according to this particular criterion is, inevitably, infected by the author’s subjectivism.
Table 1 Some descriptive inequality measures and their properties

<table>
<thead>
<tr>
<th>Axioms</th>
<th>Squared coefficient of variation (S)</th>
<th>Theil’s index (T)</th>
<th>Gini index (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Normalization</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Transfer</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Continuity</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Replication invariance</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Scale invariance</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Transfer sensitivity</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Subgroup consistency</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Decomposability</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Ease of interpretation</td>
<td>M</td>
<td>L</td>
<td>H</td>
</tr>
</tbody>
</table>

Notes: ‘+’: fulfilment of a given property; ‘–’: violation of the property; and ‘H’, ‘M’ and ‘L’ stand, respectively, for a score of ‘high’, ‘medium’ and ‘low’.

The three inequality indices considered above have their respective merits and demerits. The squared coefficient of variation fails transfer sensitivity, unlike Theil’s T, but it scores a little better with respect to ease of interpretation. Theil’s index satisfies virtually all the desirable properties of an inequality index, except that getting an immediate intuitive handle on it—in terms of perceiving a transparent connection between the notions of inequality and entropy—is not the easiest of things (Sen 1973). Gini is not transfer sensitive; and it also disappointingly violates subgroup consistency (and therefore decomposability—see, among others, Cowell 1984). Where it does score high is in terms of its intuitive appeal: it lends itself to alternative, attractively neat interpretations—in terms of the pithy formula (conveyed forcefully to the author in personal communication by James Foster) of ‘the expected distance between two randomly drawn incomes over twice the mean’, in terms of a straightforward measure of dispersion (via the relative mean difference), in terms of welfare interpretations (via Rawls and Borda), and in terms of its link with the Lorenz curve (Gini is just twice the area enclosed by the Lorenz curve and the diagonal of the unit square)—which are reviewed in Sen (1973). Ultimately, our choice of an inequality measure must be guided by our larger intent and purpose. If, for instance, we are simply interested in ranking distributions, Gini, by virtue of the ease with which its meaning can be intuited, is a useful measure to employ. On the other hand, if we are interested in qualitative or quantitative assessments of the contributions of different subgroups to overall inequality, then passing the test of subgroup consistency becomes important: indeed, as Shorrocks (1988) has shown, the choice then gets whittled down to the class of inequality indices called Generalized Entropy Measures, of which the squared coefficient of variation and Theil’s coefficient are special cases.
2.6 Connections amongst different orderings

It is useful, at this stage, to take stock of the ground we have covered, and to register any links there might be between different approaches to the problem of inequality measurement that we have examined. The reader will note that a good deal of what we have reviewed is concerned with three distinct sorts of orderings of income distributions: the Lorenz partial order, welfare orderings, and orderings by real-valued inequality indices. It is interesting to ask what connections, if any, exist between these types of orderings.

It may be recalled first that at the basis of the ‘ethical approach’ to inequality measurement espoused by Atkinson is the postulation of an SWF given by \( W = \sum U(x_i) \), where the \( U(.) \) function is required to be increasing and strictly concave. There are any number of \( U(.) \) functions which satisfy the stated requirements. The question therefore arises: under what conditions can two distributions \( x \) and \( y \) sharing the same mean income and the same population size be ranked without particularizing any further the form of the \( U(.) \) function? Atkinson (1970) has presented a remarkable equivalence theorem which accords a central place to the Lorenz partial ordering \( L \). What his theorem states is that given any two distributions \( x \) and \( y \) with the same mean income and population size, if \( xLy \), then any social welfare function \( W = \sum U(x_i) \) for which the \( U(.) \) function is increasing and strictly concave will rank \( x \) above \( y \); and conversely, if welfare from \( x \) is judged to be greater than welfare from \( y \) according to any social welfare function \( W = \sum U(x_i) \) for which the \( U(.) \) function is increasing and strictly concave, then it will be the case that \( xLy \) holds (see Fields and Fei 1978, and Foster 1985). An additional interesting result is the following one. Let \( I^* \) be the set of all real valued inequality indices which satisfy the properties of symmetry, normalization, continuity and transfer. Then the following can be asserted. Given any two distributions \( x \) and \( y \) with the same population sizes and mean incomes, if \( xLy \), then we can be sure that \( I(x) < I(y) \) for every inequality index \( I \) that belongs to the set of indices \( I^* \). Given the earlier result, it also then follows that if \( W(x) > W(y) \) for any social welfare function \( W \) which is a sum of identical increasing and strictly concave individual utility functions, then \( I(x) < I(y) \) as long as the inequality index \( I \) belongs to the set of indices \( I^* \). More general results, involving a larger class of welfare functions through a dilution of the restrictions on their form, and extensions to comparisons of distributions with variable means and populations, are available in the literature. (The interested reader is referred to, among others, Rothschild and Stiglitz 1973, Dasgupta, Sen, and Starrett 1973, Sen 1973, Anand 1983, and Foster and Shorrocks1988c).

3 Poverty measurement

3.1 A two-fold exercise

Measuring poverty in the space of incomes entails a two-fold exercise, the first of which is identification, and the second, aggregation. Identification calls for the specification of a distinguished positive level of income \( z \), called the poverty line, such that those with incomes less than \( z \) are certified to be poor. The word ‘those’ in the preceding line avoids an explicit engagement with the important issue of the appropriate unit of consideration when it comes to specifying the income recipient: for example, should
one be concerned with individuals or households? The household is a more ‘natural’
unit to consider, but it also raises difficult questions of how to make adjustments to the
poverty line in order to allow for variations in household size and composition (see
Blackwood and Lynch 1994). For specificity, we shall here assume that the income
recipient units are individuals. Turning next to the aggregation exercise, this calls for
combining information on the distribution of incomes and the poverty line in order to
arrive at a real-valued index of poverty. Given our earlier definition of the set \( \mathbf{X} \) (section
2.3), and letting \( \mathbf{T} \) stand for the set of positive real numbers, a poverty index is a
mapping \( P: \mathbf{X} \times \mathbf{T} \rightarrow \mathbb{R} \), such that, for every permissible combination of income vector
\( \mathbf{x} \) and positive poverty line \( z \) in its domain, \( P \) specifies a unique real number which is
intended to capture the extent of poverty associated with the regime \((\mathbf{x};z)\).

3.2 Identification: absolute versus relative approaches

Should the poverty line be pitched in an absolute or a relative sense? The answer would
seem to depend on what one means by the term ‘relative’, as becomes clear from a
perusal of the Sen (1983)–Townsend (1985)–Sen (1985b) exchange. If the poverty line
is ‘relative’ in the sense of being linked to some measure of central tendency of the
income distribution, then Sen (1983) would appear to be right in the view that the
notions of poverty and relative inequality would no longer lend themselves to easy
distinction. For example, if the poverty line is pegged at, say, one-half the mean income,
then, as Sen points out, a halving of everybody’s income would leave the extent of
measured poverty unchanged, even though some individuals could be precipitated into
conditions of starvation. If, on the other hand, the poverty line is ‘relative’ in the sense
of being inter-personally or inter-regionally or inter-temporally variable, to reflect
varying resource requirements according to variable patterns of resource-needs, then
there might be a case for admitting ‘relativity’: only, it may be more meaningful to
characterize such an approach to conceptualizing poverty as ‘flexibly absolute’, rather
than as ‘relative’. Indeed, as Sen has clarified, it might be more productive to view
poverty as ‘absolute’ in the space of ‘functionings’, but ‘relative’ in the space of
resources, commodities and incomes. (On the notions of ‘absolute’ and ‘relative’
poverty in more general terms, a useful reference is Foster 1998).

One distinguished strand of the ‘absolute’ approach is the so-called ‘biological’ (see Sen
1981) conceptualization: here, the poverty line is identified in terms of the income
needed to achieve a nutritionally adequate diet. There has been very considerable
controversy on what constitutes—in terms of calories, say—a nutritionally adequate
diet: both inter- and intra-individual variations in requirements have been postulated,
with accompanying theories of ‘adjustment’ and ‘adaptation’ (see, in particular,
sorting out of the difficult issues involved, the reader is referred to Osmani (1992), and

Even when ‘relativity’ is interpreted in its ‘flexibly absolute’ sense, the identification
exercise could present potential problems, of both a conceptual and a practical nature, in
undertaking inter-temporal and cross-section poverty comparisons. Specifically: are
poverty comparisons meaningful only when the same poverty line is employed across
the board? If so, how is this view compatible with the notion that poverty in different
regions (or at different points in time for the same region) should be assessed in terms of
standards that are appropriate for these different regions (or different points in time)?
These questions acquire a particular salience in the context of cross-country poverty comparisons (see, for example, Blackwood and Lynch 1994). The difficulty may well reside in, precisely, taking an ‘either/or’ view of the problem, and it is useful, in this context, to quote Sen (1981a: 21) at some length:

There is … nothing contradictory in asserting both of the following pair of statements:

(1) There is less deprivation in community A than in community B in terms of some common standard, e.g. the notions of minimum needs prevailing in community A.

(2) There is more deprivation in community A than in community B in terms of their respective standards of minimum needs, which are a good deal higher in A than in B.

It is rather pointless to dispute which of these two senses is the ‘correct’ one, since it is quite clear that both types of questions are of interest. The important thing to note is that the two questions are quite distinct from each other.

Briefly, then, there is no uncomplicated or non-controversial route to the identification problem; but assuming that it has, somehow, been solved (usually by appeal to some consensual agreement around a ‘reasonable’ norm), the next step in poverty measurement would be constituted by the aggregation exercise, to which we now turn.

3.3 Aggregation: properties of poverty indices

As in the case of inequality measurement, we consider in what follows a set of properties of poverty indices on which the literature has produced a fair measure of agreement as to their appeal. First, the focus axiom (Axiom F), stipulates that the extent of measured poverty should, other things equal, be invariant with respect to increases in non-poor incomes. A focussed income-poverty measure, therefore, reckons well-being in terms of the condition of the poor, and not—unlike other income-based indicators of aggregate well-being—in terms of the population as a whole. Second, symmetry (Axiom S) demands that interpersonal permutations of incomes among the population should leave the value of the poverty index unchanged. Third, normalization (Axiom N) requires that, for any \((x; z) \in X \times T\), if \(x_i \geq z\) for all \(i\), then \(P(x; z) = 0\) (that is, if nobody is poor, the extent of poverty is taken to be zero). Fourth, continuity (Axiom C) is the property that the poverty index \(P\) should be continuous on the set of sub-vectors of poor incomes. Monotonicity (Axiom M) demands that, ceteris paribus, poverty should increase with a decline in any poor person’s income. Transfer (Axiom T) is the property that, other things remaining the same, a progressive rank-preserving income transfer between two poor individuals should cause poverty to decline. Transfer sensitivity requires the poverty index to be more responsive to income transfers at the lower than at the upper end of the distribution of poor incomes. There are at least two ways of capturing this requirement (see Kakwani 1984, and Foster 1984). The first—call this property transfer sensitivity-1 (Axiom TS-1)—says that a given progressive rank-preserving transfer between two poor individuals separated by a given number of individuals should cause poverty to decline by more the poorer the pair of individuals
involved in the transfer. The second—call this property transfer sensitivity-2 (Axiom TS-2)—says that a given progressive rank-preserving transfer between two poor individuals separated by a given income should cause poverty to decline by more the poorer the pair of individuals involved in the transfer. Next, a couple of invariance properties: replication invariance (Axiom RI) demands that, for all \( x, y \in X \) and \( z \in T \), if \( y = (x, \ldots, x) \) and \( n(y) = kn(x) \), where \( k \) is any positive integer, then \( P(x; z) = P(y; z) \); and scale invariance (Axiom SI) demands that, for all \((x; z) \in X \times T\), \( P(x; z) = P(px; pz) \) where \( \rho \) is any positive scalar. Subgroup consistency (Axiom SC—see Foster and Shorrocks 1991) is the property that poverty should increase with an increase in any subgroup’s poverty, other things remaining the same. Decomposability (Axiom D) is a strengthened version of subgroup consistency: it requires the poverty index to be expressible as a weighted sum of subgroup poverty levels, the weights being the relevant subgroup population shares (see Foster, Greer, and Thorbecke 1984). Finally, as in the case of inequality measurement, one could have an axiom of ready comprehensibility, or ease of interpretation (Axiom E).

The stock of desirable properties can certainly be expanded, but the more important of them have been covered in the preceding inventory. Clearly, not all poverty indices satisfy all of the axioms listed; and, on occasion, the quest for poverty indices satisfying specified sets of properties could end in the discovery of non-existence (for some impossibility theorems on poverty measures, see, among others, the articles by Kundu and Smith 1983, Donaldson and Weymark 1986, and Subramanian 2002). As in the case of inequality indices, so in the case of poverty measures, the choice of index must be guided by the appeal of the underlying axiom structure in relation to intent, motivation, purpose, and the availability of data. (For instance, when detailed data on income distributions are unavailable, and all we have are binary classifications of the population into the ‘poor’ and the ‘nonpoor’, then poverty comparisons in terms of even a ‘partial’ index—to use the terminology of Foster and Sen 1997—such as the headcount ratio are better than no comparisons: there is a case for not making the infeasibly comprehensive the enemy of the feasibly partial.) We turn now to a consideration of some of the more widely-known real-valued indices of poverty that have been advanced in the literature.

### 3.4 Aggregation: some specific poverty indices

All of the poverty indices reviewed in this section are defined on the domain \( X \times T \), that is, for every permissible combination of income distribution \( x \) and poverty line \( z \); the arguments \( x \) and \( z \) of the poverty index will simply be taken as read in much of what follows. For every combination of \( x \) and \( z \), \( Q(x; z) \) will stand for the set of all poor individuals whose incomes are represented in the vector \( x \), and \( q(x; z) \) will stand for the cardinality of \( Q(x; z) \).

The most commonly used index of poverty is the so-called headcount ratio, \( H \), which simply measures the proportion of the poor population in total population. The income-gap ratio, \( Y \), measures the proportionate shortfall of the average income of the poor, \( \mu^p = (1/q) \sum_{i \in Q(x)} x_i \), from the poverty line \( z \). The product of \( H \) and \( Y \), denoted by \( R \), expresses the income-gap ratio in per person terms: \( R \) is the per capita income-gap ratio. The principal virtue of indices like \( H \) and \( Y \) is that they satisfy Axiom E: the underlying meaning of both is very easy to grasp. A well-known problem with \( H \) is that it violates monotonicity; and while \( Y \) and \( R \) respect monotonicity, neither satisfies the transfer property. Sen (1976) sought to remedy this deficiency by pursuing an axiomatic
approach to the construction of a ‘distribution-sensitive’ ordinal poverty index, $P_S$, expressed as a normalized weighted sum of the income-gap ratios of the poor, the weights being the respective rank-orders in the sub-vector of poor incomes:

$$P_S = \frac{2}{(q + 1)nz}\sum_{i \in Q}(z - x_i)(q + 1 - i)$$

(10)

where the poor individuals have been ranked in non-decreasing order of their incomes, namely, $x_i \leq x_{i+1}$ for all $i \in Q \setminus \{q\}$.

Given the rank-order weighting system employed in the expression for $P_S$, it should not be surprising if the Gini index of inequality had a role to play in the poverty index. Indeed, it turns out that, for ‘large’ values of $q$, the Sen index can be asymptotically approximated to the following expression:

$$P_S = H[Y + (1 - Y)G^p]$$

(10′)

where $G^p$ is the Gini coefficient of inequality in the distribution of poor incomes. By combining information on the incidence, the depth, and the ‘severity’ of poverty, the Sen index furnishes a more comprehensive account of poverty than any of $H$, $Y$, or $R$: in particular, $P_S$ satisfies both the monotonicity and transfer axioms. Among early critiques and modifications of the Sen index are those by Takayama (1979) and Thon (1979). Takayama proposed a variant of the Sen index, involving the use of a ‘censored’ income distribution, which, unfortunately fails the monotonicity test. While the Sen index penalizes any regressive transfer among the poor which leaves the beneficiary of the transfer poor, it does not invariably punish a regressive transfer which enables the beneficiary to escape poverty. If this is seen as a shortcoming, then a way of rectifying it is to employ a weighting system on the right hand side of (10) wherein the relevant weight is the rank-order in the entire income vector rather than in the sub-vector of poor incomes; and this leads to Thon’s variant of the Sen index (see also Shorrocks 1995). Kakwani (1980b) sought a parametric generalization of Sen’s index, in a bid to meet the requirement of Axiom TS-1, which the Sen index fails.

A distinguished class of poverty indices—in which the Sen index and its variants are not included—is that constituted by the additively separable indices (see Atkinson 1987, Foster and Shorrocks 1991, and Keen 1992). Here, resort is had to a set of individual deprivation functions $\phi(x_i;z)$, with the property that $\phi(x_i;z) > 0$ for $x_i < z$, and $\phi(x_i;z) = 0$ for $x_i \geq z$; and a poverty index $P$ is additively separable if it can be written as a simple average of these deprivation functions, viz. for all $(x;z) \in X \times T$:

$$P(x;z) = (1/n)\sum_{i \in Q}\phi(x_i;z)$$

(11)

Many of the properties of poverty indices discussed earlier are implied, in the context of additively separable measures, by restrictions on the individual deprivation functions. Given a deprivation function $\phi(x;z)$, if $\phi$ is a continuous function of $x \in [0,z)$, then $P$ is continuous; if $\phi$ is a declining function of $x$ in the range $[0,z)$, then $P$ satisfies monotonicity; and if, additionally, $\phi$ is a strictly declining and convex function of $x$ for all $x \in [0,z)$, then $P$ satisfies the transfer axiom. A number of poverty indices advanced in the literature are different specializations of the function $\phi(x;z)$. The more salient of these indices are quickly reviewed in what follows.
For \( x_i < z \), if we set \( \phi(x_i;z) = \log_e(z/x_i) \) in (11), we obtain Watts’ (1968) poverty index \( P_W \); if \( \phi(x_i;z) = (1/\beta)[1 - (x_i/z)^\beta] \), \( \beta < 1 \), we obtain the Clark, Hemming, and Ulph (1981) class of indices \( P_{\text{CHU}}(\beta) \); if \( \phi(x_i;z) = 1 - (x_i/z)^\sigma \), \( \sigma \in (0,1) \), we obtain the Chakravarty (1983) set of indices \( P_{\text{C}}(\sigma) \); if \( \phi(x_i;z) = (1 - x_i/z)^\alpha \), \( \alpha \geq 0 \), we obtain the Foster, Greer, and Thorbecke (1984) \( P_{\alpha} \) family of indices; and corresponding to \( \phi(x_i;z) = e^{\gamma(1-x_i/z)} - 1 \), \( \gamma > 0 \), we obtain Zheng’s (2000) group of ‘constant distribution-sensitivity’ indices \( P_{\text{Z}}(\gamma) \).

All of the above indices (or families of indices) satisfy Axioms F, SI, RI, and D (and therefore, SC). Watts’ index satisfies Axioms M, T, and TS-2 as well. For the other families of indices, Axioms M, T, and TS-2 are satisfied for parameter values of \( \beta, \delta, \alpha, \) and \( \gamma \) which are lesser or greater than specified cut-off values for the fulfilment of the respective axioms. (For example, the \( P_{\alpha} \) indices all satisfy focus, and scale- and replication-invariance, for \( \alpha \geq 0 \); monotonicity for \( \alpha > 0 \); transfer for \( \alpha > 1 \); and TS-2 for \( \alpha > 2 \).) A major feature of all these indices is that they are decomposable—unlike the Sen index which fails even the weaker condition of subgroup consistency.

Finally, mention must be made of the so-called ‘ethical’ indices of poverty—see, among others, Blackorby and Donaldson (1980) and Hagenaars (1987)—which are similar in motivation to the Atkinson-type ‘ethical’ indices of inequality we have encountered earlier. Here, the idea is to express the poverty index as a distinguished per capita income-gap ratio \( R^* = HY^* \), where \( Y^* \) is the proportionate shortfall from the poverty line of Atkinson’s ‘equally distributed equivalent income’ \( x_{P \text{ede}} \)—computed for the distribution of poor incomes. Depending on which particular underlying ‘social evaluation function’ is favoured, one can obtain different expressions for \( x_{P \text{ede}} \), and therefore—via \( Y^* \)—for the ethical poverty index \( R^* \).

### 3.5 Plurality and ranking

Plurality can interfere with the possibility of unambiguous poverty rankings in at least two ways. First, there may be a range of plausible poverty lines, rather than a unique line, to consider, and it could happen that a given poverty index measures more poverty in distribution \( x \) than in distribution \( y \) for one poverty line \( z_1 \), but measures more poverty in \( y \) than in \( x \) for some other poverty line \( z_2 \), where both \( z_1 \) and \( z_2 \) belong to the plausible range of poverty lines. Second, with more than one poverty index figuring in the analyst’s menu, it is possible, given a poverty line \( z \), that one index pronounces that \( x \) has more poverty than \( y \), while another index pronounces that \( y \) has more poverty than \( x \). Foster and Shorrocks (1988c) and Zheng (2000), among others, have investigated the first problem, while treatments of the second problem can be found in Foster (1984), Atkinson (1987), Foster and Shorrocks (1988a, 1988b), Shorrocks (1995), Spencer and Fisher (1992), Foster and Sen (1997), and Zheng (2000). Here we will simply note, very quickly, that the prospect of obtaining unambiguous poverty rankings, in respect of both categories of problems outlined, is linked to the fulfilment of various stochastic dominance, ‘generalized Lorenz’ dominance, ‘poverty profile’ dominance, and ‘generalized poverty profile’ dominance, conditions. A generic problem for poverty measurement, using the difficulty presented for unambiguous ranking by a multiplicity of poverty indices as an example, is the following. It is true that the probability that the sorts of ‘dominance conditions’ we have mentioned will be satisfied increases as we restrict the set of poverty indices in contention through restrictions on their properties.
(such as on ‘distribution sensitivity’, as in Zheng 2000); but even as the uncertainty of obtaining consensus on rankings declines, the uncertainty regarding the ‘rightness’ of the poverty indices retained presumably increases, as one constrains the set of admissible indices. The problem has a certain analogy with the conduct of an election. If a movement toward unanimity is preferred, then a means to that end would be to confine voting rights to a smaller and smaller set of ‘like-minded’ voters—until, in the limit, all ambiguity is eliminated, assuming no ambiguity is attached to the desirability of just one vote counting, through straightforward dictatorship! Briefly, the poverty analyst must always contend with the problem that while plurality can promote ambiguity in one sense, singularity can promote ambiguity in another sense. There is no simple golden rule for the ‘right’ choice of the range of poverty indices and poverty lines that could be employed in poverty analysis.

3.6 Poverty indices and anti-poverty policy

Direct income transfers to the poor, and wage employment schemes, are two instruments for combating poverty. Given a budget of fixed size, what is an optimal pattern of income allocation to the poor, or an optimal wage, depending on which instrument is wielded? The answers would typically depend on how poverty (which is the quantity being sought to be minimized) is measured. Bourguignon and Fields (1990) and Gangopadhyay and Subramanian (1992) address the income transfer problem. For specificity, one could measure poverty by the $P_\alpha$ class of indices. It turns out that for $\alpha < 1$, the prescribed optimal allocation is one in which only the richest of the poor, close to the poverty line, are the beneficiaries; for $\alpha = 1$, the odd outcome is that any feasible transfer schedule which exhausts the budget is also optimal; and for $\alpha > 1$, one has a ‘lexicographic maximin solution’, whereby, through a sequence of progressive and income-equalizing transfers, the poorest of the poor are raised to that level of income which is compatible with exhausting the budget. In the matter of wage employment programmes (see, among others, Basu 1981, Dreze and Sen 1989, Ravallion 1991, and Gangopadhyay and Subramanian 1992), for $\alpha = 0$, the optimal wage is the poverty line income $z$; and as $\alpha$ increases, the optimal wage declines and coverage increases until, in the limit, for $\alpha$ tending to infinity, the wage is pitched as low as is compatible with creating the maximum number of jobs for which there are takers, given the size of the budget. The nature of the solution, in each case—with specific reference to the question of ‘distribution-sensitivity’—is a comment on, and serves to clarify our intuitive grasp of, the poverty index employed.

It should also be clear from the above that in all but the last case (poverty minimization through wage employment in the ‘$\alpha$ tending to infinity’ setting, which is compatible with the phenomenon of ‘self-selection’), the intended beneficiaries of the poverty redress scheme would have to be identified and targeted, through (presumably costly) ‘means testing’. This raises questions pertaining to the relative merits of ‘universal provisioning’ and means-testing (Besley 1990); the principles of targeting (Besley and Kanbur 1993); and the possibilities of ‘imperfect targeting’ in a variety of contexts (Kanbur 1987, Besley and Kanbur 1988, Ravallion and Chao 1989), all of which issues lend themselves to being addressed as part of the analytics of poverty measurement.
3.7 Some other issues in poverty measurement

Research in poverty measurement has made many advances, and clarified many sources of conceptual confusion, since the early systematic efforts initiated by Watts (1968) and Sen (1976). There are still many issues which require more sustained investigation. Simply by way of allusion, some of these issues are mentioned here. Fuzzy approaches to the measurement of poverty have been considered by, among others, Shorrocks and Subramanian (1994) and Martinetti (2000). (Indeed, inequality measurement is also open to a fuzzy approach, for two examples of which see Basu 1989, and Ok 1995.) The aggregation of deprivation assessed multi-dimensionally presents certain difficulties, for treatments of which the reader is referred to Mukherjee (2001) and Tsui (2002). A completely different variety of problem, revolving around the ‘adjustment’ of poverty indices for premature excess mortality among the poor, is considered by Kanbur and Mukherjee (2002): the larger issue revolves around the ethical and logical sustainability of divorcing the outcome of poverty changes from the processes leading to these changes. Yet another problem arises from the competing claims of the aggregate headcount A and the headcount ratio H as the appropriate means of factoring the incidence of poverty into a poverty index: two accounts of the nature and implications of the conflict are available in Chakravarty, Kanbur, and Mukherjee (2002) and Subramanian (2002). These, and other problems, may not be amenable to completely satisfactory ‘solutions’, but an identification and elaboration of these problems is arguably itself of some instructional value.

4 Inequality and poverty: links and disjunctions

The inter-connections between poverty and inequality are apparent in a number of ways. For one thing, and as we have seen, for the entire set of ‘distribution-sensitive’ poverty indices (namely, those which satisfy the transfer axiom), an increase in inequality in the distribution of poor incomes will, ceteris paribus, cause measured poverty to rise. For another, letting \( x^P_z \) stand for the vector of poor incomes given any income vector \( x \) and poverty line \( z \), it would be the case that, for all equidimensional \( x, y \in X \) and \( z \in \mathbb{R} \), if \( \mu(x) = \mu(y) \), \( q(x;z) = q(y;z) \), \( x^P_z y^P_z \), and \( xL \), then \( P(x;z) < P(y;z) \) for any poverty index \( P \) which belongs to the class \( P^* \) of indices which are focused, monotonic, transfer-prefering and scale-invariant; while if the Lorenz curves of \( x \) and \( y \) coincide, \( q(x;z) = q(y;z) \), and \( \mu(x) > \mu(y) \), then again \( P(x;z) < P(y;z) \) for any poverty index \( P \in P^* \). These considerations have led some analysts (Kakwani and Subbarao 1990 and Datt and Ravallion 1992) to seek to ‘decompose’ a poverty change into a ‘growth’ component (attributable to a change in mean income, holding the Lorenz curve fixed) and an ‘inequality’ component (attributable to a change in the distributional parameters of the Lorenz curve, holding the mean income fixed). These are instances of ‘congruence’ between poverty and inequality.

But there are clearly also cases of conflict. We have noted earlier that certain poverty indices (like Sen’s) do not invariably register an increase following on a regressive income transfer between two poor individuals when the beneficiary is thereby enabled to cross the poverty line. This has led some commentators like Pyatt (1987) to characterize Sen-type indices as being ‘badly behaved’. The underlying presumption seems to be that the SWF should be of the standard ‘two objective’ type, increasing in mean income and declining in inequality, so that a ‘three objective’ SWF, with poverty
also explicitly factored into it, is effectively reduced to what Brent (1986: 93) calls ‘the two objective version in disguise’. (For a number of other treatments of the subject of poverty, inequality and social welfare, the reader is referred to Hagenaars 1987, Vaughan 1987, and Lewis and Ulph 1988). However, Brent (op. cit.) has demonstrated plausible conditions under which a well-defined, ‘equity-conscious’ SWF can actually register an increase with an increase in the headcount ratio of poverty: an SWF which favours equality could be inimical to poverty-reduction. The ‘bad behaviour’ of poverty indices, then, is attributable to a notion that cannot simply be dismissed out of hand: namely, that the poverty line is a distinguished dividing line, such that the ability to cross it is invested with a special welfare significance. Furthermore, in situations wherein the mean income is less than the poverty line, poverty-minimization may call for a ‘man overboard’ solution to the ‘lifeboat dilemma’: equalizing incomes across the board could leave the entire population in very straitened circumstances, whereas allocating no income to a subset of the population while permitting the rest to be raised to the poverty line level of income might prove to be a harsh but pragmatic necessity (see Dasgupta and Ray 1986 and Subramanian 1989).

Briefly, then, while there are clearly many congruent linkages between poverty and inequality, the conflicts between the two must also be recognized: there may be no particular virtue to insisting on a view of social welfare in which welfare increases through inequality-reduction must necessarily always be accompanied by reductions also in poverty. Poverty and inequality are related, but distinct concepts.

5 Deprivation and disparity: towards a more inclusive approach

5.1 Income alone?

Are poverty and inequality assessed in the space solely of incomes sufficient to convey a picture of how well or badly a society is doing? Is an ‘adequate’ level of personal disposable income a sufficient guarantor of achievement in the dimensions of, say, literacy, nutrition, longevity and health? Inter- and intra-country comparisons do not invariably furnish an affirmative answer to these questions. Thus, for example, relatively income-poor countries like China or Ecuador or Costa Rica have relatively impressive records in dimensions of well-being such as literacy, life expectancy and health, while relatively income-rich countries like some of the Arab States display relatively poor performances in extra-income aspects of well-being. Similarly, the Indian state of Kerala is way ahead of a relatively (income-wise) richer state like Haryana when it comes to assessing well-being in dimensions such as fertility, expectation of life at birth, infant mortality, and literacy. These empirical findings suggest strongly the importance of going beyond the metric of income in assessing deprivation, disparity, and well-being.

5.2 Well-being beyond income

Much of our preceding review of the measurement of inequality and poverty has been related to a somewhat special and narrow conception of well-being. ‘Well-being’, in this view, has been largely conflated with ‘welfare’, which itself has generally been seen to be some aggregation of individual ‘utilities’, with a person’s utility being taken to
depend just on their income. In Sen (1980), and a host of related writings, we see the beginnings of a substantive engagement with the question ‘equality of what?’ Depending on what particular view of well-being we may be disposed to favour, we may choose to assess deprivation and disparity in the dimensions of income, or resources generally, or utilities, or, as in Rawls (1971), ‘primary goods’. Sen himself (1985) advances the view that the most relevant engagement with the notion of well-being obtains when one’s concern is with human capability. Deprivation, in this reckoning, is fruitfully seen as a failure of capability; and the containment of disparity is reflected in moving towards equality in the space of capabilities. The capability in question is not an abstract capability, but the capability to function—a functioning being what Sen calls ‘a state of being or doing’. In this expanded view of well-being, a concern with income (and therefore with measures of income-poverty and inequality as indicators of ‘ill-being’) is not invalidated; what is, however, called into question is an exclusive preoccupation with income-related indicators—which can result in a picture of development which is both partial and misleading. A more inclusive view of well-being such as is afforded by the capability approach underlines the need for data on, and measurement of, indicators that go beyond the income metric (Qizilbash 1996, 1997).

5.3 Generalized well-being/deprivation indicators

As part of the more expansive view of well-being which is dictated by the sorts of considerations just reviewed, there has been an increasing concentration of effort among scholars to derive and justify measures of human well-being and capability deprivations that transcend an exclusive concern with the space of incomes. Among salient contributions to this effort—with, naturally, differences in content and emphasis—are the ‘basic needs’ approach to reckoning achievement (Hicks and Streeten 1979); the concern with assessing the ‘quality of life’ (Morriss 1979, Sen 1981b); the importance attached to individuals’ ‘capability to function’ (Sen 1985a, Sen et al. 1987); and the primacy accorded to the evaluation of ‘human development’ (UNDP 1990-2002, McGillivray 1991, McGillivray and White 1993).

The Human Development Index (HDI) has, through successive annual compilations of its country-wise values by the United Nations Development Programme’s Human Development Reports (or UNDP’s HDRs, for short), become a widely-known shorthand measure of aggregate well-being. This is the subject of an entirely separate paper in the UNU-WIDER Social Development Indicators project and will therefore not be dealt with here. Additionally, the emphasis in this essay is on deprivation, as such: the HDI reckons the well-being of any given population by reference to the population as a whole, whereas, from a poverty perspective, there would be a case for measuring well-being with specific reference to the condition only of the deprived section of the population. (This would correspond to the distinction between (a) a focussed income-poverty measure and (b) other income-based measures of aggregate well-being of the population considered in its entirety.) Driven by this motivation, a number of measures of deprivation, seen in the light of capability failure, have been developed: these would include the ‘Capability Poverty Measure’ in UNDP’s HDR 1996; the ‘Human Poverty Index’ in UNDP’s HDR 1997; Mahbub ul Haq’s (1997) ‘Human Deprivation Measure’; and Majumdar and Subramanian’s (2001) ‘Capability Failure Ratio’. In the interests of specificity, and also because it is the most comprehensive deprivation measure among the indices just mentioned, it is the Human Poverty Index (HPI) which will be the focus of attention here.
5.4 The Human Poverty Index

The HPI is a *multi-dimensional* index. It measures deprivation in three dimensions—those of longevity, knowledge, and standard of living, and thus achieves a certain correspondence with the components of the HDI, with the difference that there is no specifically *income* component in the HPI. The HPI can be written as a combination of distinguished headcount ratios of failure in selected dimensions of the capability to function. Specifically, let \( \Pi_1 \) be the proportion of the population which is expected not to survive to the age of 40; let \( \Pi_2 \) be the adult illiteracy rate; and let \( \Pi_3 \) be a composite of the proportion of the population without access to health services and to safe water, and the proportion of the under-5 population which is under-nourished. Then, \( \Pi_1, \Pi_2, \) and \( \Pi_3 \) are measures of capability failure in the dimensions, respectively, of longevity, knowledge and standard of living. The HPI can be written—for a detailed treatment the reader is referred to ‘Technical Note 1. Properties of the human poverty index’ in UNDP’s *HDR 1997*—in its most general form, as a weighted average of order \( \eta, \Pi(\eta) \), which is given by

\[
\Pi(\eta) = [(w_1\Pi_1^\eta + w_2\Pi_2^\eta + w_3\Pi_3^\eta)/ (w_1 + w_2 + w_3)]^{1/\eta}
\]

(12)

where \( w_k > 0 \) (\( k = 1,2,3 \)) is a weight attached to the headcount ratio of human poverty in the \( k \)th dimension, and \( \eta \geq 1 \) is an indicator of the extent of ‘substitutability’ between the components of the HPI (with a higher value of \( \eta \) reflecting a lower degree of substitutability, and as \( \eta \) becomes indefinitely large we move toward zero substitutability, so that \( \lim_{\eta \to \infty} \Pi(\eta) = \max\{\Pi_1, \Pi_2, \Pi_3\} \)).

Certain distinguished members of the \( \Pi(\eta) \) class of indices, obtained for specified values of \( \eta \) and specified patterns of the weighting structure \( \{w_k\} \), are presented below:

\[
\Pi^*(\eta) = [(\Pi_1^\eta + \Pi_2^\eta + \Pi_3^\eta)/3]^{1/\eta}, \eta \geq 1
\]

(13)

\( \Pi^*(\eta) \) is the *ordinary mean of order \( \eta \)*, obtained by setting \( w_1 = w_2 = w_3 \).

\[
\Pi(1) = [(w_1\Pi_1 + w_2\Pi_2 + w_3\Pi_3)/(w_1 + w_2 + w_3)]
\]

(14)

\( \Pi(1) \) is the *weighted mean of order 1*, or *weighted arithmetic mean*, obtained by setting \( \eta = 1 \).

\[
\Pi^*(1) = (\Pi_1 + \Pi_2 + \Pi_3)/3
\]

(15)

\( \Pi^*(1) \) is the *simple arithmetic mean* of \( \Pi_1, \Pi_2, \) and \( \Pi_3 \), obtained by setting \( \eta = 1 \) and \( w_1 = w_2 = w_3 = 1 \). It may be noted that the measure \( \Pi^*(1) \) is *decomposable*. That is to say, if the population is partitioned into \( M \) mutually exclusive and exhaustive subgroups; if \( \Pi_{mk} \) is the headcount ratio of deprivation for the \( m \)th group (\( m = 1,\ldots,M \)) in the \( k \)th dimension (\( k = 1,2,3 \)); if \( \Pi_m^*(1) = (\Pi_{m1} + \Pi_{m2} + \Pi_{m3})/3 \) is the simple arithmetic mean version of the HPI for group \( m \) (\( m = 1,\ldots,M \)); and if \( t_m \) is the population share of group \( m \) (\( m = 1,\ldots,M \)); then, it is true that

\[
\Pi^*(1) = \sum_{m=1}^M t_m \Pi_m^*(1)
\]

(16)
Finally, the HPI, as it is computed in HDR 1997, is obtained by setting $\eta = 3$ and $w_1 = w_2 = w_3$: the resulting measure, $\Pi^*(3)$, is written as

$$\Pi^*(3) = \left[\frac{\Pi_1^3 + \Pi_2^3 + \Pi_3^3}{3}\right]^{1/3}$$  \hspace{1cm} (17)

Does a multi-dimensional human poverty index convey the same information as a uni-dimensional income-based poverty index? This, of course, in an empirical question, and the answer would depend on the precise indices one uses, the particular poverty norms one adopts, and the units of observation one considers in performing the comparison exercise. In this connection, some of the cross-country findings reported in UNDP’s HDR 1997 are instructive. Employing 1993 data for a set of 36 countries on the HPI $\Pi^*(3)$ of (5.4.6) and on the income-based headcount ratio of poverty (call it H) obtained by employing a poverty line of a dollar a day (in 1985 purchasing power parity dollars), the HDR (1997: 22) states that ‘regression analysis indicates a weak relationship between the headcount index of income poverty and HPI…’. For a subset of 41 countries for which data on both HPI (as measured by $\Pi^*(3)$) and H are available in HDR 1997, it turns out that the coefficient of rank correlation between an ordering of countries by HPI and an ordering by H is fairly strong (Spearman’s rank correlation coefficient is of the order of 0.82), but not perfect. These results suggest, at the least, that income-based measures of poverty are not necessarily completely adequate surrogates for a more expansive, capability-oriented reckoning of disparity. Additionally, the experiences of countries such as China, Costa Rica, Kenya, Peru, the Philippines, and Zimbabwe—see HDR 1997—which have displayed greater success in reducing human poverty than income-poverty point to the possibilities of enhancing achievements in the space of human functionings by routes different from those centred exclusively on income-growth and the percolation of that growth to the poor. In particular, these experiences would stress the importance of state intervention in securing relief from human poverty—an emphasis which is at some variance with a view, which is increasingly gaining currency in some quarters, that somewhat sidelines the state in favour of the market and civil society as agents for the promotion of aggregate well-being.

It remains now to consider how, given a deprivation index which, like $\Pi^*(1)$ in (15), is decomposable (even if not multi-dimensional), one may assess the extent of group-related disparity in the distribution of that deprivation.

5.5 Reckoning inter-group disparities in the distribution of deprivation

A decomposable real-valued index of generalized deprivation—call it D—is really a measure of central tendency: it presents the aggregate deprivation in a society, averaged over the deprivations of specific groups constituting the society. That is, suppose the population is partitioned into $M$ mutually exclusive and completely exhaustive groups, identified by the running index $m = 1, \ldots, M$; then, if $D_m$ is the deprivation level of the $m$th group (and it will be assumed that the groups are arranged in non-increasing order of deprivation, so that $D_m \geq D_{m+1}$, $m = 1, \ldots, M - 1$), and if $t_m$ is the population share of the $m$th group, $D$ can be written as: $D = \sum_{m=1}^{M} t_m D_m$. For future use, let us also define $T_m$ to be the cumulate proportion of the population with deprivation levels not exceeding that of the $m$th group, for every group $m = 1, \ldots, M$. $D$, being a simple average of group-specific deprivation levels, conceals any inequality there may be in the inter-group distribution of deprivations. Such group-related disparity is clearly an important datum.
in assessing aggregate well-being, and there is therefore a strong case for reckoning such disparity in the measurement of deprivation (see Stewart 2001). To this end, one can construct ‘adjusted’ measures of deprivation, where the adjustment takes the form of buttressing information on the average level of deprivation with information on the inter-group disparity of its distribution. Two such adjusted measures, D* and D**, are presented below.

\[ D^*(\lambda) = \left[ \sum_{m=1}^{M} t_m D_m^\lambda \right]^{1/\lambda}, \quad \lambda \in [1, \infty) \]  

(18)

and

\[ D^{**}(\delta) = \frac{1}{1/(M-1)} \sum_{m=1}^{M} \left[ (M-1-\delta m) t_m + \delta T_m \right] D_m, \quad \delta \in [0,1] \]  

(19)

\( \lambda \) and \( \delta \) in (18) and (19) respectively are parameters of ‘inter-group disparity aversion’, with the extent of aversion being an increasing function of the values of the parameters. Consider the special case in which \( M = 2 \) (where, for example, the population has been partitioned into ‘males’ and ‘females’). For \( \lambda = 1 \) [respectively, \( \delta = 0 \)], \( D^* \) [respectively, \( D^{**} \)] just collapses to \( D \); this the ‘(average) Benthamite’ rule of reckoning aggregate deprivation simply in terms of the average level of deprivation; for \( \lambda \to \infty \) [respectively, \( \delta = 1 \)], \( D^* \) [respectively, \( D^{**} \)] just collapses to \( D_1 \); this is the ‘Rawlsian’ rule of reckoning aggregate deprivation in terms of the deprivation of the worst-off group. In general, each of \( D^* \) and \( D^{**} \) is amenable to being expressed as the average level of deprivation \( D \) enhanced by a factor incorporating a measure of between-group inequality: as it happens, this inequality measure, in the case of \( D^* \), is an Atkinson-type ‘ethical’ index of inter-group disparity and, in the case of \( D^{**} \), a Gini-type ‘descriptive’ measure. \( D^* \) is essentially an adaptation of a procedure advanced by Anand and Sen (1995), and subsequently adopted by the UNDP’s Human Development Reports, for constructing a ‘gender-adjusted HDI’; and \( D^{**} \)—see Subramanian and Majumdar 2002—is a generalization of what Majumdar and Subramanian (2001) call an ‘adjusted capability failure ratio’, which the authors have computed, in an application to Indian data, for a partitioning of the population according to gender, caste and sector of residence. For a version of the ‘adjusted capability ratio’, as applied to an assessment of disparity in the cross-country distribution of deprivation, the reader is referred to Subramanian (2003).

5.6 Expanding the interpretation of well-being: orientation, policy and data

By taking a more expansive view of well-being than is afforded by a wholly income-centred approach, we have seen that the measurement emphasis also shifts from an exclusive concern with indicators of poverty and inequality to more general indicators of deprivation and disparity. From many perspectives, this is a welcome shift. For one thing, data on income or consumption expenditure, which are required for constructing indices of poverty and inequality, are not always wholly reliable. Inter-temporal comparisons of poverty and inequality, based on sample surveys, are often vitiated by changes in concepts, definitions, and reference periods of recall. Additionally, the identification problem is notoriously difficult to solve, and eliciting consensus on a poverty line is frequently a vexed business, which is customarily disposed of through a stance of philosophical resignation to the inevitability of some measure of arbitrariness and subjectivism in the measurement exercise. Further, there is always room for (endless) controversies on the ‘correct’ choice of price deflators with which to update
base-year poverty lines so that these may be expressed in current prices. All of these problems are amply reflected in, for example, the Indian literature on poverty. Despite insistence on an ‘ordinally pure’ interpretation of poverty indices, few would really agree that there is no difference between a decline, over a forty-year period, in the headcount ratio from 50 per cent to 30 per cent according to one set of poverty norms and a decline, over the same period, from 70 per cent to 65 per cent according to another set of norms. For all of these reasons, there is a strong case for being guided by generalized indicators of deprivation and disparity rather than solely by indictors of poverty and inequality. By focusing directly on the capability to function, in addition to reckoning income-based indicators, one can get a fuller picture of time-series and cross-section variations in well-being deprivations and disparities. This would call for the compilation, by official data-generating agencies, and the use, by policy-makers and researchers, of data which are richer and more extensive than a narrow preoccupation with income will allow. Indeed, both national and international agencies are increasingly turning to the compilation and use of data sets on achievements with respect to literacy, health, nutrition, longevity, fertility, and the like. From the points of view of both social explanation and collective redress, it is fruitful to address problems of—for example—child labour, women’s well-being, demographic transition, and social exclusion, by paying attention not only to achievements in the income dimension but to achievements in, say, the provision of potable water, sanitation, energy for cooking, electricity, public health care, and roads. Hence the catholic approach to measurement, in this paper and in this research project.

6 Concluding observations

In this paper, an attempt has been made to cover, however quickly, certain crucial issues in the measurement of economic poverty and inequality, as well as more generalized deprivation and disparity. We have discussed Lorenz orderings, welfare orderings, and inequality orderings; we have examined the welfare bases of inequality comparisons; we have presented axioms for both inequality and poverty measurement; we have reviewed a number of both the so-called ‘ethical’ and descriptive measures of inequality and poverty; we have attempted to evaluate these indices, and to take stock of the importance of being guided by motive and purpose in their choice for concrete applications; we have pointed to sources of ambiguity in the measurement of the phenomena under investigation; we have attempted to locate anti-poverty policy in the context of measurement issues; we have sought to elucidate the relationship between inequality and poverty within an overall framework of welfare; and we have presented a rationale for, and discussed measurement issues relating to, the assessment of deprivation and disparity in an expanded framework of human well-being which moves beyond the income dimension to a consideration of human capabilities and functionings. This, without a doubt, amounts to not much more than scratching the surface; but given the vastness of that surface, it is to be hoped that the exercise will have had something to offer to the reader who is looking for a helpful preliminary overview of the subject.
References


