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The commodity currency puzzle

Hilde C. Bjørnland and Håvard Hungnes
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The commodity currency puzzle

Hilde C. Bjørnland and Håvard Hungnes*

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Abstract
This paper addresses the purchasing power parity (PPP) puzzle for a commodity currency. In particular, we analyse the real exchange rate behaviour in Norway, which has a primary commodity (oil) that constitutes the majority of its exports. A substantial part of the literature on commodity currencies has found that, despite controlling for the effect of commodity prices, PPP does not hold in the long run. We show that once we also control for the effect of the interest rate differential in the real exchange rate relationship, the discrepancies from PPP are fully accounted for. Furthermore, with the interest rate differential included in the long run real exchange rate relationship, the real oil price plays only a minor role. Adjustment to equilibrium (half-lives) is also substantially reduced, taking no more than one year on average. Hence, contrary to earlier findings on commodity currencies, we have effectively removed the PPP puzzle.

Keywords: Exchange rate, commodity currencies, real oil price, purchasing power parity, uncovered interest parity.


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1. Introduction
Ever since Meese and Rogoff (1983) reported that a comprehensive range of exchange rate models were unable to outperform a random walk, the role of economic fundamentals in explaining exchange rate behaviour has been scrutinized. Economic theory typically predicts that the behaviour of the real exchange rate should be closely related to the behaviour of deviations from purchasing power parity (PPP). However, there seems to be a widespread agreement that substantial deviations from PPP have occurred since the abandonment of the Bretton Woods fixed exchange rate system in 1971. In particular, time series studies for this period have shown that the real exchange rate is not only very volatile in the short run; the speed of convergence to PPP in the long run is extremely slow, (see e.g. Rogoff, 1996; Froot and Rogoff, 1995, for a survey).12

The failure to find support for PPP has encouraged researchers to construct exchange rate models that investigate the role of economic fundamentals as sources of deviations from PPP. Long run deviation from PPP suggests the influence of real shocks with large permanent effects, (see Rogoff, 1996). The fact that many empirical studies do not reject the hypothesis of a unit root in the real exchange rate, also supports the argument that the variation in the real exchange rate is attributed to permanent shocks.3 However, up to now it has been difficult to successfully identify real shocks that can explain these permanent deviations from PPP.

One line of research has been to analyse the effect of different real shocks on the terms of trade and thereby the real exchange rate directly. For countries that have a substantial share of commodity exports, the direct effect of commodity price shocks on the terms of trade has been investigated, with mixed luck. In particular, Chen and Rogoff (2003) find that although the world price of commodity exports appear to have a strong influence on the real exchange rates in the countries they examine, there is still a purchasing power puzzle in the residuals; implying that there is a high degree of persistence remaining. Furthermore, Cashin et al. (2002) find that in less than 40 percent of the commodity exporting countries they examine, can they establish a long run relationship between the real exchange rates and commodity prices.

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1 The rejections are less clear-cut using panel data, see e.g. Frankel and Rose (1996) among many others. However, see O’Connell (1998) and Chortareas and Driver (2001) for critical assessments of these panel data studies. See also the recent study by Holmes (2001), who using a new panel data unit root test, finds clear evidence against PPP.
2 Studies spanning longer periods of time (say a century or more), have traditionally found more evidence in favour of PPP than the studies of the post Bretton Woods period. However, many recent studies have found persistent deviations from PPP also in the long run (see e.g. Serletis and Zimonopoulos, 1997; Cuddington and Liang, 2000; Engel, 2000; Rogoff et al., 2001).
3 This finding is also consistent with e.g. Mark (1995), who forecasts the nominal exchange rate at long horizons by predicting that it returns to a target level, which, however, is not the PPP value.
PPP has its roots in goods markets models. Hence, by analysing the effects of the real shocks on the real exchange rate directly, one effectively assumes that all deviations from PPP will work through the goods market. However, an increase in the commodity price that improves the terms of trade and force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Hence, massive movements in capital flows in response to interest rate differentials can keep the exchange rate away from PPP for long periods. A solution to the PPP puzzle for commodity currencies described above could therefore be to investigate a central parity condition for the exchange rate in capital market models, namely that of uncovered interest parity (UIP).  

Empirical evidence has generally led to a strong rejection also of the UIP condition in the Post Bretton Woods period (see e.g. McCallum, 1994; Lewis, 1995; and Engel, 1996, for recent surveys). However, as pointed out by Johansen and Juselius (1992) and MacDonald and Marsh (1997), by combining the PPP and the UIP conditions, one could more effectively capture the interactions between the nominal exchange rate, the price differential and the interest rate differential. However, whether the combination of these parity conditions will be sufficient to account for all real shocks, or whether commodity price shocks will have to be investigated separately as a source of deviations from PPP, is a separate question that will be investigated below.

This paper clarifies and calculates the concept of the long run (equilibrium) real exchange rate in a commodity exporting country, by examining different hypothesis related to persistent real exchange rate variation. In particular, we will test whether the direct effects of commodity price changes as well as the interest rate differential are possible explanations of PPP deviations. The analysis is applied to Norway. Since Norway is a dominant oil exporting country, the real oil price is the relevant commodity price to include in the analysis. Oil price shocks are of particular interest for at least two reasons. First, they have historically been very volatile, thereby causing persistent deviations from PPP. In addition they are also thought to affect the terms of trade, see Backus and Crucini (2000).

Previous studies of the determination of the real exchange rate in Norway, have generally rejected the notion of simple PPP using conventional (time series or panel data) unit root tests, see e.g. Serletis and Zimonopoulus (1997), Papell (1997) and Chortareas and Driver (2001) or by testing for PPP in multivariate studies, see e.g. Jore et al. (1998), Alexius (2001) and Bjørnland and Hungnes (2002). An

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4 A test of UIP, refers to a test of the interest rate differential as an optimal predictor of the rate of depreciation, providing the conditions of rational expectations and risk neutrality are satisfied.

5 Papell (1997), analysing a series of countries including Norway, cannot reject the unit root hypothesis in the real exchange rate for Norway at the 5% level. However, using a panel of 20 countries, he finds more evidence of PPP. However, again see footnote 1, for a reference to some critical assessments of these panel data studies.
exception is Akram (2002), who, using a multivariate cointegrating framework, find strong evidence of PPP for Norway.\footnote{However, despite the non-rejection of PPP, Akram (2002) finds the adjustment to shock to be slow, taking from two to three years for the effect of a unit shock to be reduced into half (as measured by the half-live indicator in Figure 2.4).}

Empirical tests for the UIP condition, also finds little evidence supporting this parity condition for Norway (see e.g. Holden and Vikøren, 1994; Jore et al., 1998; Nessen, 1997; and Flood and Rose, 2001).

The rest of this paper is organised as follows. In Section 2 we discuss the hypothesis of PPP and possible sources of deviations from PPP. Section 3 discusses the extent to which the Norwegian krone is petrocurrency while Section 4 identifies and estimates a multivariate econometric model for the long run real exchange rate behaviour in Norway. Section 5 investigates the speed of adjustment to long run equilibrium by calculating half-lives while Section 6 performs some sensitivity analysis. Section 7 summarises and concludes.

2. Fundamentals and long run exchange rates

A natural starting point for discussing the relationship between exchange rates and fundamentals is the concept of PPP. Assuming no costs in international trade, then domestic prices would equal foreign prices multiplied by the exchange rate. The expression for PPP can then be written (in log-form) as

\[ v_t = p_t - p_t^*, \]

where \( p_t \) is the log of the domestic price, \( p_t^* \) is the log of the foreign price, and \( v_t \) is the log of the nominal exchange rate. However, since trade is costly, PPP will not hold continuously. It is therefore informative to define the real exchange rate as

\[ r_t = v_t - p_t + p_t^*, \]

where \( r_t \) is the real exchange rate. If PPP holds, the real exchange rate is stationary and fluctuates around a fixed value (its mean) in the short run. In a univariate framework, PPP can be tested by simply testing for whether the real exchange rate is stationary or not. Alternatively, PPP can be cast in a multivariate framework by applying cointegration methods. Note that since we will use price indices in the estimation, we can only test relative PPP. However, for the macroeconomic topic of this paper, this is the relevant hypothesis to test.
The massive empirical testing of PPP has generally cast doubt on long run PPP, either by rejecting the hypothesis that PPP follows a stationary process, or by suggesting that the real exchange rate adjusts too slowly back to a long run equilibrium rate to be consistent with traditional PPP (the half time is normally found to be 3-4 years, see e.g. Rogoff, 1996).\textsuperscript{7} Instead, long run deviation from PPP suggests the influence of real factors with large permanent effects, like commodity prices, productivity differentials, fiscal policy, and other relevant variables, again see Rogoff (1996) for a survey. These factors will work through the current account, and thereby push the real exchange rate away from PPP.

For an oil producing country like Norway, the oil price may also have important effects on the real exchange rate that may lead to deviations from PPP. More specifically, a higher real oil price will increase natural wealth and raise demand. The additional demand can only be met if the relative prices change in favour of foreign goods, so that the currency experiences a real appreciation (see e.g. Corden and Neary, 1982). This may squeeze the tradable sector. To account for the possibility that the Norwegian krone may be a “petrocurrency”, which appreciates when the oil price is high and depreciates with a low oil price, we therefore include the real oil price as an additional variable in the model. With this in mind, the equilibrium exchange rate can be written as

$$r_t = -\delta op_t,$$\hspace{1cm}(2-3)

where $op_t$ is the (log of the) real oil price, and a test for $\delta > 0$ is a test whether the Norwegian krone is a “petrocurrency”.

Previous studies of the determination of commodity currencies have to a various degree found the price of commodity exports to determine the real exchange rates. In particular, Chen and Rogoff (2003) find the world price of commodity exports to appear to have a strong influence on the real exchange rates in the three open economies they examine; Australia, Canada and New Zealand. However, despite controlling for commodity price shocks, they still find a purchasing power puzzle in the residuals, implying that there is a high degree of persistence remaining. However, Cashin et al. (2002) analysing commodity exporting countries, find that for less than 40 percent of the countries they examine (i.e. 22 out of a total of 58 countries), can they establish a long run relationship between the real exchange rates and commodity prices (Norway, Canada and New Zealand being among the countries where they could not establish a significant link).

\textsuperscript{7} In a recent study, Murray and Papell (2002) also find the half-life of deviations from PPP for each of 20 countries (including Norway) to lie between 3-5 years. However, their confidence intervals are much larger than previously reported, implying in fact that univariate methods provide virtually no information regarding the size of half-life.
Thus, although a country may be a major commodity exporter, only in the minority of cases does the price of commodity exports explain the high degree of the real exchange rate variation. Hence, one needs to examine what other sources can explain the prolonged exchange rate deviations from PPP.

A different strand of literature has emphasised the massive movements in capital flows in response to interest rate differentials as a source of exchange rate variation for prolonged periods. In particular, several authors have emphasised, (see e.g. MacDonald and Marsh, 1997; Juselius and MacDonald, 2000), that the balance of payment constraint implies that any imbalances in the current account has to be financed through the capital account. Hence, shocks that force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Large movements in capital flows in response to interest rate differentials can therefore keep the exchange rate away from purchasing power for long periods. The PPP condition in the goods market will therefore be strongly related to the central parity condition in the capital market, namely that of UIP.

This can be formalised in the following way. According to the UIP condition, the interest rate differential will be an optimal predictor of the rate of depreciation, providing the conditions of rational expectations and risk neutrality are satisfied. The expected gain, \( g^e \), from investing money in Norway is given as the deviation from UIP, i.e.

\[
g^e_t = i_t^* - i_t - \Delta v^e_{t+1}, \quad (2-4)
\]

where \( \Delta v^e_{t+1} \) is the expected depreciation rate from period \( t \) to \( t+1 \), \( i_t \) is the domestic interest rate and \( i_t^* \) is the foreign interest rate.

Assume that in the long run, the current account (CA) depends on real exchange rate and the oil price see (2-3), whereas the capital account (KA) depends on the deviation from UIP, that is, deviations from the nominal interest differentials adjusted for expected exchange rate changes (see also MacDonald and Marsh, 1997). The balance of payment then implies that

\[
CA_t + KA_t = \gamma \left( v_t + p_t^* - p_t + \delta \delta p_t \right) + \lambda \left( i_t - i_t^* - \Delta v^e_{t+1} \right) = 0, \quad (2-5)
\]
where $\gamma$ captures the elasticity of net exports with respect to competitiveness and $\lambda$ represents the mobility of international capital (where $\lambda \to \infty$ so that $g^e = 0$, if capital is perfectly mobile). Assuming that in equilibrium, $\Delta v^e_{t+1} = 0$, (2-5) can be solved for the exchange rate to yield a long run equilibrium relationship

$$v_t = p_t - p_t^* - \nu(i_t - i_t^*) - \delta \rho_t,$$

(2-6)

where $\nu = \lambda / \gamma$. Equation (2-6) states that the nominal exchange rate is a function of the price level differential, the interest rate differential and the real oil price. Another way to interpret (6) is that the non-stationarity of the real exchange rate ($v_t - p_t + p_t^*$) can be removed by the non-stationarity of the interest rate differential ($i_t - i_t^*$) and the non-stationarity of the oil price ($\rho_t$).

3. Kroner as a commodity currency?

Since the collapse of the Bretton Woods agreement, Norway has pursued different exchange rate policies, where the task of maintaining stable exchange rates against a basket of currencies has been at the core. However, throughout the 1980s, there were several occasions where the exchange rate was devalued and the basket of currencies changed, whereas in the 1990s, the exchange rate has had a (managed) float for prolonged periods. In 2001, however Norway formalized an inflation-targeting regime, of which the exchange rate since then has been freely floated.

In the analysis below, we use monthly data, and the estimation period is from 1983M1 to 1999M12. The start date for estimation is set to exclude the turbulence in the international interest rate markets in the early 1980s, which would necessitate a series of intervention dummies (see the discussion in MacDonald and Marsh, 1999). Furthermore, by excluding the 1970s we also disregard the period where capital and credit markets in Norway were highly regulated.

Due to the various exchange rate systems pursued throughout the sample, we model the exchange rate in Norway against the bilateral German DM. We do this partly because Germany is the most important trading partner, and partly because many other of Norway's trading partners indirectly have stabilised their currencies to the German Mark (due to European Monetary System and the Maastricht treaty). However, in the analysis below, we will test for robustness with regard to a basket of trading partners, where the weights of the different countries in the basket are allowed to vary over the years.
The variables used are graphed in Figure 1. The figure graphs the monthly German DM nominal and real exchange rate in the upper frame; the 3-month interest rate in Norway and Germany in the middle frame; and the real oil price (relative to the Norwegian CPI) in the lower frame.

Figure 1: Bilateral exchange rate and the interest rates in Norway and Germany

![Figure 1: Bilateral exchange rate and the interest rates in Norway and Germany](image)

1) Nominal exchange rate (v), real exchange rate (r), Norwegian interest rate (i), German interest rate (i*), real oil price (op).

Figure 1 (upper frame) shows the real exchange rate, which seems to have drifted upwards for prolonged periods. According to the unit-root tests (see Appendix A), we can not reject the null hypothesis that the real exchange rate is non-stationary. Non-stationarity implies that the real exchange rate does not fluctuate around a fixed value, but instead wanders rather wildly. This suggests that the real exchange rate has a non-stationary stochastic trend. Hence, one cannot use the PPP measure to find a base period in which the real exchange rate is believed to be in equilibrium. From the lower frame of Figure 1, decreases/increases in the oil price seem to coincide with periods of depreciation/appreciations in exchange rate. However, to what extent the changes in the oil price can explain the non-stationary of the real exchange rate or whether the interest rate differential (middle frame) is the main driving force, has to be addressed carefully by using an econometric analysis.
To shed some light on this issue, we start by establishing the degree of influence of oil for the real exchange rate in Norway. More precisely, we follow Chen and Rogoff (2003) and test for cointegration between the real exchange rate and the price of oil. We model the whole system jointly within a full information maximum likelihood (FIML) framework, see Johansen (1988). We first define the stochastic vector process as $z_t = (r_t, op_t)'$, where $r$ is the real exchange rate and $op$ is the (real) oil price. This process can be parameterised as a vector equilibrium correction model (VEqCM)

$$
\Delta z_t = \mu + \sum_{j=1}^{k-1} \Gamma_j \Delta z_{t-j} + \Pi z_{t-1} + \Psi D_t + u_t
$$

where $u_t \sim NID(0, \Sigma)$. $\mu$ is a vector of constants and $D_t$ is a vector of deterministic variables. The null hypothesis of $r$ cointegrating vectors can then be formulated as

$$
H_0 : \Pi = \alpha \beta',
$$

where $\alpha$ and $\beta$ are $2 \times r$ matrices of rank $r$, ($r<2$), and $\beta' z_t$ comprises $r$ cointegrated I(0) relations, and $\alpha$ contains the loading parameters.

### Table 1: Cointegrating rank test

<table>
<thead>
<tr>
<th>$H_0$: rank=r</th>
<th>eigenvalue $\lambda_i$</th>
<th>$\lambda$-max$^a$</th>
<th>$\lambda$-max adj.</th>
<th>$\lambda$-trace$^b$</th>
<th>$\lambda$-trace adj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>0.052</td>
<td>10.94 [0.16]</td>
<td>10.73 [0.17]</td>
<td>13.22 [0.11]</td>
<td>12.96 [0.12]</td>
</tr>
<tr>
<td>$r\leq1$</td>
<td>0.011</td>
<td>2.27 [0.13]</td>
<td>2.23 [0.14]</td>
<td>2.27 [0.13]</td>
<td>2.23 [0.14]</td>
</tr>
</tbody>
</table>

$^a$Estimation period: 1983m1 - 1999m12. The cointegrating rank test is based on two lags (i.e. $k=2$). However, including additional lags, or adding dummies to account for extreme oil price increases/decreases, do not change the main results.

The results of the cointegrating rank test are reported in Table 1.² Neither the hypothesis of rank equal to one (or less) nor the hypothesis of rank equal to zero are rejected. Furthermore, both the value of the highest eigenvalue and the significance values indicate there are no cointegrating relationships. Hence, there is no long-run relationship between the Norwegian real exchange rate and its most important commodity (oil). These results are in line with Cashin et al. (2002), which found that the non-stationary of the real exchange rate in Norway could not be explained by the non-stationary of the real oil price. On the other hand, Haldane (1997) has argued that a substantial share of the real exchange rate appreciation in Norway in the 1980s and 1990s are due to oil price increases. To
proceed any further in understanding which of the views are correct, we need to include the full set of variables identified above. Only then can the non-stationary of the real exchange rate be appropriately accounted for.

4. The long run exchange rate relationship

The variables used in the full econometric analysis are again; the nominal exchange rate in Norway measured against German DM ($v$), home ($p$) and foreign ($p^*$) prices measured by CPI, home ($i$) and foreign ($i^*$) interest rates and the real oil price ($op$), (see Appendix A for a further description of data and their sources); $z_t = \left( v_t, p_t, p_t^*, i_t, i_t^*, op_t \right)$. The sample is again monthly; 1983M1 to 1999M12. Some extreme observations are nevertheless remaining in the system, mainly due to changes in the exchange rate regimes not accounted for by the model, but also due to two periods of extreme oil price fluctuations; 1986 and 1990-1991. These observations are best represented with dummies (see Appendix A for the list of dummy variables). In addition a trend, $t$, (restricted to lie in the cointegrating space) is included.

In the model specified above, we assumed that all the variables are integrated of first order, I(1). ADF test results show that for neither of the variables can we reject the hypothesis of I(1) in favour of the (trend) stationary alternative. However, we can, for all variables, reject the hypothesis that they are integrated of order two, I(2) (see Tables A-1, A-2).

To decide on the lag length in the VAR model, a battery of lag reduction tests were applied; suggesting a lag order of two. Estimating a VAR-model with two lags, a trend and a set of dummies (including centred seasonal dummies), there is no evidence of serial correlation (see Table A-3 in Appendix A for diagnostic tests). The normality hypothesis is rejected, essentially due to excess kurtosis. However, since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis, we do not find the rejections of normality serious (see Juselius and MacDonald, 2000). Table A-3 also reveals some problems with ARCH for the exchange rate and heteroscedasticity for both the exchange rate and the oil price. Again cointegration test are not very sensitive to these violations, so we ignore them in the following analysis.

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8 The empirical estimations are conducted using PcGive 9.10/PcFIML 9.10 and PcGive 10.3, see Doornik and Hendry (1996,1997).

9 A lag length of two may be regarded as short. However, increasing the lag length marginally to three or four lags, does not really change our results.
The cointegration tests are presented in Table A-4. The table reveals clear evidence of one cointegration vector. We therefore test different hypotheses on the cointegrating vector \( \left( z_t', \tau \right) = (v, p, p^*, i, i^*, \Delta p, \Delta p^*, \text{op}_t, \ell). \) The hypotheses that are of interest to test are the following: \(^{10}\)

I Test of no trend
\[ \beta' = (1, *, *, *, *, 0) \]

II Test of significance of the oil price in the long run relationship
\[ \beta' = (1, *, *, *, 0, 0) \]

III Test of stationarity of (relative) PPP
\[ \beta' = (1, -1, 1, 0, 0, 0) \]

IV Test of stationarity of the interest rate differential (implied by the UIP condition)
\[ \beta' = (0, 0, 0, a, -a, 0, 0) \]

V Test of PPP augmented by interest rate differential and oil price
\[ \beta' = (1, -1, 1, a, -a, *, 0) \]

The first hypothesis considers whether the trend can be omitted from the cointegrating space. If that is the case, we will continue the inference on the cointegrating space without incorporating the trend. We thereafter test for the significance of the oil price, stationarity of PPP and the interest rate differential, \(^{11}\) and finally a combination of these hypotheses.

In the second half of Table A-4, different tests on \( \alpha \) and \( \beta \) are conducted under the restriction of there being one cointegration vector. Assuming the rank to be one, we start by testing on \( \beta \), that is, we test the different (cointegrating) hypotheses reported above. First, the hypothesis of no trend (hypothesis I) can (just) be accepted, so we restrict it to be zero in the remaining analysis. The zero restriction on the oil price (hypotheses II) is rejected, using a significance level of 5 per cent. Further, we can reject the hypothesis of pure PPP (hypothesis IIIa) and a stationary interest rate differential (based on pure UIP) (hypothesis IVa). However, neither of these two hypotheses can be rejected when the rest of parameters on the cointegrating vector are left unrestricted (hypotheses IIIb, c and IVb), implying that

\(^{10}\) Hypotheses II-V assume that we can restrict the trend in the cointegrating space to be zero. If not accepted, the restriction will not be incorporated in the subsequent tests.

\(^{11}\) Note that although we use nominal interest rates in the cointegration analysis, the test results will remain unchanged if we instead had used real interest rates (\( r_i \)). To see the latter, note the following: If \( i \) and \( i^* \) were replaced with \( r_i = i - \Delta p \) and \( r_i^* = i^* - \Delta p^* \) respectively in the cointegration part (i.e. if \( \beta' \tilde{z} \) were replaced with \( \beta' \tilde{z}^\prime \), where \( \tilde{z} = (v, p, p^*, i - \Delta p, i^* - \Delta p^*, \text{op}) \), all estimated parameters would be unchanged, except some in the coefficient matrix \( \Gamma_1 \). Since we do not impose restrictions on \( \Gamma_1 \), this will not influence any of the results or tests.
PPP and UIP should be combined with the oil price. This is confirmed when testing hypothesis V, where we identify a cointegration vector with PPP augmented with the interest rate differential and the real oil price. The coefficients of this vector have the expected signs. In particular, it implies a negative relationship between the (real) value of Norwegian kroner measured against German DM and the real oil price in the long run, so that a higher oil price appreciates the real exchange rate. Further if the Norwegian interest rate is high relatively to the German interest rate, the equilibrium real exchange rate must appreciate.

Testing zero restrictions on \( \alpha \) implies testing for weak exogeneity. A variable is weakly exogenous with respect to the long run parameters if it is not adjusting to the long run equilibrium. Testing for weak exogeneity shows that we can reject the null hypothesis of weak exogeneity for the exchange rate and Norwegian prices. For the other variables, the null hypothesis of weak exogeneity cannot be rejected when tested individually, although weak exogeneity of German interest rates are close to being rejected. Testing restrictions on \( \alpha \) and \( \beta \) jointly (hypothesis V & H), only German prices, the Norwegian interest rate and the oil price turns out to be weakly exogenous. Equation (4-1) reports the estimated long run exchange rate relationship (standard errors in parenthesis)

\[
v = p - p^* - 1.166(i - i^*) - 0.083 \text{op}.
\]

Equation (4-1) implies that although PPP is not by itself a stationary process, the real exchange rate cointegrates with the interest rate differential and oil price, so the residuals from the estimation are stationary. Hence, the long-run interactions between the goods and capital markets cannot be ignored. However, the fact that the oil price also enters explicitly into the cointegrating relationship, may emphasise that any long run effect that the oil prices may have on the exchange rate, is not captured by the interest rate differential. The coefficient on the oil price suggests that a 10 per cent change in the oil price implies approximately 1 per cent change in the exchange rate. The coefficient on the interest rate differential is close to one, and implies that a one percentage point increase in the interest rate differential will lead to a one per cent appreciation in the exchange rate.

How do these results compare to previous studies of the determination of the real exchange rate in Norway? As suggested above, many empirical studies have rejected PPP using conventional unit root tests or by testing for PPP in multivariate studies. An exception is Akram (2002), who, using a multivariate cointegrating framework, finds strong evidence of PPP for Norway. However, the core

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12 Note that the hypothesis of augmented PPP is only accepted when we allow for a trend. However, when all the relevant hypotheses are tested jointly, the hypotheses are accepted independent on the inclusion of trend or not.
model does not consider the interest rate differential among its variables, and as will be discussed below, adjustment to equilibrium is not particularly fast. Naug (2003), on the other hand, emphasises the role of the interest rate differential for exchange rate adjustments for the period 1999-2003. Furthermore, Bjørnland and Hungnes (2005) show that in a forecasting perspective, only a model that augments PPP with the interest rate differential in the long run, can beat a random walk.

5. Half-lives; Removal of the PPP puzzle

Having identified the long run relationship between the real exchange rate, the interest rate differential and real oil prices, we need to assess the speed of adjustment around equilibrium. A useful indicator of the speed of adjustment is the so-called half-live indicator. More specifically, assuming a system is hit by a shock, a half-live indicator expresses the time it takes for the process to correct for half of the magnitude of the shock. In a univariate process estimated with one lag, the half-live can be expressed by the adjustment parameter. If for instance the univariate process is expressed as \( \Delta x_t = \rho x_{t-1} + u_t \), the half-life is given by \( \log(1/2)/\log(1 + \rho) \). The corresponding expression for a multivariate process with one lag and one cointegrating vector is equal to \( \log(1/2)/\log(1 + \beta' \alpha) \). However, our system is estimated using two lags, so applying the formula above would give wrong estimates. In particular, had we ignored the second lag, the formula above would yield an estimate for the half-live of a shock to the cointegrating relationship at 18.5 months. However, in the multivariate case with more than one lag, the half-live indicator will depend on the type of shock hitting the system (i.e. in which error is shocked). Half-lives can therefore instead be found by applying impulse responses of the effect of shocks on the cointegrating vector in the system.

In order to calculate half-lives, the deterministic variables in the system can be ignored. Furthermore, we need to augment the VAR model with the definitional relationship: \( \beta' z_t - \beta' \Delta z_t = \beta' z_{t-1} \). This yields

\[
\begin{bmatrix}
I_6 & 0_{6 \times 1} \\
-\beta' & 1
\end{bmatrix} \begin{bmatrix}
\Delta z_t \\
\beta' z_t
\end{bmatrix}
= \begin{bmatrix}
\Gamma_1 & \alpha
\end{bmatrix} \begin{bmatrix}
\Delta z_{t-1} \\
\beta' z_{t-1}
\end{bmatrix} + \begin{bmatrix}
u_t \\
0
\end{bmatrix}.
\]  

\( (5-1) \)

This relationship holds for all \( t \). Furthermore (assuming all future errors to be zero) the effect of a shock in period \( t \) can be expressed as (see Appendix B for further elaborations)
\[
\begin{bmatrix}
\Delta z_{t+s} \\
\beta' z_{t+s}
\end{bmatrix} = \left( I_6 \ 0_{6 \times 1} \right)^{\prime} \left( \Gamma_1 \ \alpha \right)^{\prime} \left( I_6 \ 0_{6 \times 1} \right)^{\prime} \begin{bmatrix} u_t \end{bmatrix}.
\] (5.2)

In the analysis we focus on the effect from the shocks on the cointegrating relationship \( \beta' z_t \), as we are not interested in the effect on the individual variables. All shocks are normalized to be unity in the period they take place (i.e. \( \beta' z_t = 1 \)). The shock in the error corresponding to exchange rate \( (v) \) can then be given by \( u_t = [1,0,0,0,0,0] \), and similarly for the other shocks. Note that as our system is formulated on reduced form, hence we do not give any economic interpretation of these shocks. Figure 2 shows the impulse responses on the cointegrating vector of the different shocks. The graphs are generated with Ox Professional 3.4, see Doornik (2001).

**Figure 2: Impulse responses measuring half-lives to different shocks**

For all shocks except a shock in the error corresponding to the German interest rate, half-lives are between 1 and 1½ year, which is much faster than the speed of convergence to PPP that has been found in the literature by e.g. Rogoff (1996). The half-live for a shock in the German interest rate is...
only one month. This is mostly due to the rapid adjustment in the Norwegian interest rate. When the German interest rate increases with a percentage point, the Norwegian interest rate increases by 0.45 percentage points the following month.

Figure 2 also shows that all shocks, except the shock corresponding to the German interest rate, overshoots. Cheung and Lai (2000) have argued that with such non-monotonic adjustment, much of the estimation of the half-lives are overstated in the literature. They argue that rather than calculating the half-lives after the initial shock, one should instead calculate the half-live after the responses to shocks have reached their maximum level. If we do this, half-lives are reduced even further, to an average of 9-11 months (although the shock in the error to the German interest rate still has a half-live of 1 month).

It is interesting to compare these results with those of Akram (2002), who also calculates half-lives from a cointegrating relationship for Norwegian bilateral exchange rates in a uni-variate framework (using the real exchange rate). Akram (2002) finds the half-lives of the bilateral real exchange rate versus German DM to be approximately 11 quarters (Figure 2.4), which is more than twice the time it takes the exchange rate in our model to adjust to equilibrium. Hence, by including the interest rate differential in the long run cointegrating relationship, we have effectively shown that half-lives are reduced substantially, so that in effect we have removed any purchasing power parity puzzle.

6. Robustness of the results - Trading partners

In this section we perform some alternative specifications of the model, to check the robustness of our reported results. In particular, we replace the bilateral exchange rate with a basket of trading partners, and, eventually using a different model for oil prices.

Trading partners

In the following, the German exchange rate and interest rate are replaced by the equivalent variables from a basket of trading partners, where the weights of each country in the basket are allowed to vary over time (see appendix A). Estimating a VAR with two lags, the cointegration tests indicate one cointegration vector at the 1 per cent significance level. Testing restrictions on the parameters on this cointegrating vector, we find that we can reject the long run hypotheses of pure PPP and interest rate differential. In addition, we now also find that the oil price can be excluded from the cointegration vector. Finally, we therefore identify a cointegration vector with PPP augmented with the interest rate differential. Hypotheses of weak exogeneity for the nominal exchange rate, domestic and foreign
prices are rejected at a 5 per cent level. The estimated long run exchange rate relation is reported in Equation (6-1) (standard error in parenthesis)

\[ v = p - p^* - 0.685(i - i^*) . \]

\[ (0.164) \]  

The coefficient on the interest rate differential is significant and still close to one, hence the long-run interactions between the goods and capital markets cannot be ignored. However, oil prices do not enter the cointegrating relationship explicitly, emphasising that any long run effect that the oil prices may have on the exchange rate, is already captured by the interest rate differential. One reason for this could be the fact that the different trading partners respond differently to oil price changes, thereby on average cancelling out any effects. In particular, they may respond asymmetrically to oil price increases and decreases, thereby confounding the true responses (see also Akram, 2004). This is investigated further below.

**Oil price and asymmetry**

Above we have assumed there are symmetric and linear effects of the oil price on the exchange rate (or between any of the variables). However, Mork et al. (1994) have shown that there are important asymmetries between the effects of oil price increases and decreases on the US economy. More recently, Hooker (1996) has argued that the relationship between the macroeconomy and oil has decreased dramatically in the US since 1973. However, contrary to the results obtained by Mork et al. (1994), Hooker finds that re-specifying the VAR according to asymmetry theories does not restore the oil macroeconomic relationship.

Hamilton (1996), on the other hand, argues that as most of the increases in the price of oil since 1986 have followed immediately after even larger decreases, they are corrections to the previous decline rather than increases from a stable environment. To correctly measure the effect of oil price increases on the macroeconomy, he suggests that one should compare the price of oil with where it has been over the previous year, rather than with where it was the previous quarter (or month) alone. By constructing what he refers to as the net oil price (the maximum value of the oil price observed during the preceding year), Hamilton (1996) shows that the historical correlation between oil price shocks and the macroeconomy remains.

Below we use a methodology similar to that described in Hamilton (1996) and replace the oil price with a net oil price. We continue using the model where the exchange rate is measured against the trading partners. Assuming one cointegrating vector, the results still suggest that we can identify a
long run relationship between the real exchange rate and the interest rate differential. However, there are now some more evidence of a second cointegrating vector, and restricting both in a valid way, we can identify a long run relationship between the exchange rate and the oil price (and a trend), in addition to our originally identified vector (the oil price does not enter significantly there). However, the effect of the oil price is small, but it enters with the right sign and similar in magnitude to that identified for Germany. Overall then, allowing for a non-linear relationship between the exchange rate and the oil price, there seems to be somewhat more evidence that the oil price will affect the nominal exchange rate in the long run (see also Akram, 2004, for a non-linear analysis of the effects of the changes in the oil price).

7. Conclusions

This paper addresses the purchasing power parity puzzle for commodity currencies. A substantial part of the literature on commodity currencies has found that despite controlling for the effect of commodity export prices on the floating real exchange rate, PPP does not hold in the long run. We argue that the main reason for this puzzle is that by focusing on deviations from PPP alone, one ignores the link to the capital account. In particular, an increase in the commodity price that improves the terms of trade and force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Hence, massive movements in capital flows in response to interest rate differentials can keep the exchange rate away from purchasing power parity for long periods. A solution to the PPP puzzle for commodity currencies could therefore be to investigate a central parity condition for the exchange rate in capital market models, namely that of uncovered interest parity, in conjunction with PPP.

The analysis is applied to the real exchange rate behaviour in Norway, which has a primary commodity (oil) that constitutes the majority of its export. We show that despite controlling for the effect of the commodity export price on the real exchange rate, PPP does not hold in the long run. However, when we also allow the interest rate differential to enter the relationship, the real exchange rate is effectively made stationary. The long run relationship is consistent with a synthesis of PPP and UIP. We show that once the interest rate differential is allowed to matter, the real oil price plays only a minor role in the long run real exchange rate relationship, although the sign of the effect is as expected. In particular, the Norwegian currency can be characterised as a petro-currency, which appreciates when the oil price increases and depreciates when the oil price falls. We further show that adjustment to shocks from the equilibrium relationship is fast, taking no more than one year on average. Hence, contrary to earlier findings on commodity currencies, we have effectively removed the purchasing power parity puzzle.
References


Bjørnland, H. C. and H. Hungnes (2005), The importance of interest rates for forecasting the exchange rate. Forthcoming in *Journal of Forecasting*.


Appendix A

The data set

Data sources
All the time series, except those for the oil price and German CPI, are taken from the International Financial Statistics (IFS) from International Monetary Fund (IMF). The nominal oil price is taken from Norges Bank and German CPI is taken from OECD.

Exchange rate
The exchange rate is taken from line rf in IFS from IMF. This line reports the average amount of home currency per dollar in the respective month. For Germany, the exchange rate for West Germany is used before the reunification. For the exchange rate against Norway's trading partners, the reciprocal of the nominal effective exchange rate neu is used, with the weights given in Table A-1 in Bjørnland and Hungnes (2002).

Interest rate
The interest rates are the money market interest rate (line 60b). This is the monthly average of the day-to-day interbank rate. Due to huge fluctuation in this rate for Norway, we have used the three-month interbank rate instead (line 60zb). The interest rate for Norway's trading partners is calculated by a weighted arithmetic average, using the weights reported in Table A-1 in Bjørnland and Hungnes (2002).

Prices
The consumer price indices are taken from line 64, except for the German consumer price index that is taken from OECD (CPALLTO1.IXOB.M). The consumer and producer price indices for Norway's trading partners are calculated as a weighed geometric average (again, weights reported in Table A-1 in Bjørnland and Hungnes, 2002).

Real oil price
We use the real oil price in Norwegian kroner. The real oil price is calculated by multiplying the nominal oil price in dollar with the krone-dollar exchange rate and then dividing by the Norwegian consumer price index. The source of the nominal oil price in dollar is Norges Bank (line M2001712), and it is the monthly average of daily oil prices.
The dummies used in the analysis are

\[ D_{G,t} = \{d_{84m7}, d_{84m9}, d_{86m5}, d_{86}, d_{9091}, d_{9293}, D_{float}, d_{97m1}, d_{ger91}, d_{ger93}\} \]

- \(d_{84m7}, d_{84m9}\) and \(d_{86m5}\) are impulse dummies that are unity in one month and zero otherwise, included to take account of different changes in the Norwegian exchange rate regime. In July 1984, the basket of currencies that Norway was holding the krone fixed against changed, from being calculated as an arithmetic average to a geometric average (involving a devaluation of 2 per cent). In September 1984 it was decided temporarily to hold the exchange rate index 2 per cent higher than the centre of the exchange band (involving a devaluation of 2 per cent). In May 1986 the value of the Norwegian krone was depreciated with about 10 per cent, due to the fall in the oil price and increased labour cost in production.

- \(d_{86}\) and \(d_{9091}\) capture extreme fluctuations in the oil price. \(d_{86}\) equals 1 from December 1985 to April 1986, and picks up the extreme decrease in the oil price in this period. \(d_{9091}\) equals 1 in August 1990 and -1 in January and February 1991 and captures the fluctuations in the oil price due to the Gulf war.

- \(d_{9293}\) equals 1 in September and November 1992, -1 in October 1992 and January and February 1993, and zero otherwise. This dummy controls for the speculations against the ERM system in the second half of 1992 (see Figure 2).

- \(D_{float}\) is a step dummy that equals 1 from December 1992, zero otherwise. It picks up the change to a floating exchange rate regime.

- \(d_{97m1}\) controls for the appreciation pressure against the Norwegian krone in January 1997. This dummy equals unity in January 1997 and zero otherwise.

- \(d_{ger91}\) equals 1 in July and October 1991, and controls for fluctuations in the German CPI and interest rate. (Probably repercussions of the reunion in 1990.)

- \(d_{ger93}\) equals 1 in January 1993, and zero otherwise, and is probably picking up after-effects of the speculations against ERM in the second half of 1992.

- When modelling the exchange rate against Norway’s trading partners, the dummy \(d_{tp92}\) is included (instead of \(d_{ger91}\) and \(d_{ger93}\)). This dummy equals 1 in August and September 1992, -1 in October and November 1992 and zero otherwise. This dummy is included to control for speculations against the ERM system.
Unit root test

Table A-1: Unit root tests

<table>
<thead>
<tr>
<th></th>
<th>Norway</th>
<th>Trading Partners</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without trend. Critical values: 5%=2.88, 1%=3.46.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>1.84</td>
<td>1.71</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>2.39</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>$i$</td>
<td>1.07</td>
<td>0.23</td>
<td>0.55</td>
</tr>
<tr>
<td>$i-i^*$</td>
<td>1.87</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>With trend. Critical values: 5%=3.43, 1%=4.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>1.93</td>
<td>0.06</td>
<td>0.89</td>
</tr>
<tr>
<td>$op$</td>
<td>2.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-2: Unit root tests

<table>
<thead>
<tr>
<th></th>
<th>Norway</th>
<th>Trading Partners</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without trend. Critical values: 5%=2.88, 1%=3.46.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>8.29**</td>
<td>7.26**</td>
<td></td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>8.93**</td>
<td>7.80**</td>
<td></td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>10.4**</td>
<td>9.66**</td>
<td>8.53**</td>
</tr>
<tr>
<td>$\Delta (i-i^*)$</td>
<td>10.6**</td>
<td>9.97**</td>
<td></td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>5.82**</td>
<td>5.59**</td>
<td>7.99**</td>
</tr>
<tr>
<td>$\Delta op$</td>
<td>9.04**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(**) indicates rejection at the 1 per cent significance level.

Table A-3: Diagnostic tests$^{a,b}$

<table>
<thead>
<tr>
<th></th>
<th>$v$</th>
<th>$p$</th>
<th>$p^*$</th>
<th>$i$</th>
<th>$i^*$</th>
<th>$op$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 1-7</td>
<td>F(7,162)</td>
<td>1.31 [0.25]</td>
<td>1.68 [0.12]</td>
<td>1.70 [0.11]</td>
<td>0.94 [0.48]</td>
<td>1.34 [0.23]</td>
</tr>
<tr>
<td>Norm</td>
<td>X2(2)</td>
<td>22.1 [0.00]**</td>
<td>34.0 [0.00]**</td>
<td>18.1 [0.00]**</td>
<td>30.7 [0.00]**</td>
<td>24.8 [0.00]**</td>
</tr>
<tr>
<td>Skewness</td>
<td></td>
<td>0.23</td>
<td>0.01</td>
<td>0.44</td>
<td>0.75</td>
<td>0.23</td>
</tr>
<tr>
<td>Exc. kurt.</td>
<td></td>
<td>1.78</td>
<td>2.28</td>
<td>1.72</td>
<td>3.02</td>
<td>1.93</td>
</tr>
<tr>
<td>ARCH 7</td>
<td>F(7,155)</td>
<td>1.67 [0.12]</td>
<td>0.27 [0.96]</td>
<td>1.46 [0.19]</td>
<td>0.72 [0.66]</td>
<td>2.37 [0.02]**</td>
</tr>
<tr>
<td>Het</td>
<td>F(26,142)</td>
<td>2.11 [0.00]**</td>
<td>1.17 [0.28]</td>
<td>1.11 [0.34]</td>
<td>1.40 [0.11]</td>
<td>1.02 [0.44]</td>
</tr>
<tr>
<td>System:</td>
<td>AR 1-7 F(252,734)=1.36 [0.00]**</td>
<td>Norm X2(12)=162.5 [0.00]<strong>, Het F(546,2085)=1.121 [0.04]</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ p = 2, $^b$ AR 1-7 is Harvey's (1981) test of residual autocorrelation up to order 7; Norm is the normality test described in Doornik and Hansen (1994); ARCH is the Engle (1982) test for autoregressive conditional heteroscedasticity up to order 7 in the residuals; and Het is a test for residual heteroscedasticity due to White (1980). (*) denotes rejection at the 5 per cent significance level while (**) indicates rejection at the 1 per cent level.
### Table A-4: Cointegrating tests

#### Cointegrating rank tests

<table>
<thead>
<tr>
<th>H0:rank=r</th>
<th>eigenvalue $\lambda_0$</th>
<th>$\lambda$-max</th>
<th>$\lambda$-max adj.</th>
<th>95%</th>
<th>$\lambda$-trace</th>
<th>$\lambda$-trace adj.</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>0.308</td>
<td>75.11**</td>
<td>70.69**</td>
<td>44.0</td>
<td>158.1**</td>
<td>148.8**</td>
<td>114.9</td>
</tr>
<tr>
<td>r≤1</td>
<td>0.171</td>
<td>38.23*</td>
<td>35.98</td>
<td>37.5</td>
<td>83.02</td>
<td>78.14</td>
<td>87.3</td>
</tr>
<tr>
<td>r≤2</td>
<td>0.133</td>
<td>29.04</td>
<td>27.33</td>
<td>31.5</td>
<td>44.79</td>
<td>42.15</td>
<td>63.0</td>
</tr>
<tr>
<td>r≤3</td>
<td>0.045</td>
<td>9.30</td>
<td>8.75</td>
<td>25.5</td>
<td>15.75</td>
<td>14.82</td>
<td>42.4</td>
</tr>
<tr>
<td>r≤4</td>
<td>0.022</td>
<td>4.55</td>
<td>4.28</td>
<td>19.0</td>
<td>6.49</td>
<td>6.07</td>
<td>25.3</td>
</tr>
<tr>
<td>r≤5</td>
<td>0.009</td>
<td>1.90</td>
<td>1.79</td>
<td>12.3</td>
<td>1.90</td>
<td>1.79</td>
<td>12.3</td>
</tr>
</tbody>
</table>

#### Unrestricted $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>p</th>
<th>$\rho^*$</th>
<th>i</th>
<th>$i^*$</th>
<th>$\alpha$, $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.046</td>
<td>0.019</td>
<td>-0.005</td>
<td>0.004</td>
<td>-0.008</td>
<td>0.117</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>-1.341</td>
<td>0.540</td>
<td>1.069</td>
<td>-1.032</td>
<td>0.072</td>
<td>2.326</td>
</tr>
</tbody>
</table>

#### Cointegrating vectors

<table>
<thead>
<tr>
<th>Number</th>
<th>Interpretation</th>
<th>Restrictions, $\beta$</th>
<th>Estimated $\beta$</th>
<th>LR prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>No trend</td>
<td>(1,<em>,</em>,<em>,</em>,*,0)</td>
<td>(1,-1.0491,1.498,1.648,-1.944,0.090,0 )</td>
<td>3.39 [0.07]</td>
</tr>
<tr>
<td>II</td>
<td>No oil/trend</td>
<td>(1,<em>,</em>,<em>,</em>,0,0)</td>
<td>(1,-1.509,2.124,2.316,-2.655,0,0 )</td>
<td>7.74 [0.02]</td>
</tr>
<tr>
<td>IIIa</td>
<td>Pure PPP</td>
<td>(1,-1,1,0,0,0,0,0)</td>
<td>(1,-1,1,1.435,-1.435,0.079,0 )</td>
<td>2.48 [0.29]</td>
</tr>
<tr>
<td>IIIb</td>
<td>Augm. PPP, no trend</td>
<td>(1,-1,1,<em>,</em>,*,0)</td>
<td>(1,-1,1,1.315,-1.596,0.105,0,717,0 )</td>
<td>8.85 [0.03]</td>
</tr>
<tr>
<td>IIIc</td>
<td>Augmented PPP</td>
<td>(1,-1,1,<em>,</em>,<em>,</em>,0)</td>
<td>(1,-1,1,1.435,-1.435,0.079,0 )</td>
<td>2.48 [0.29]</td>
</tr>
<tr>
<td>IVa</td>
<td>Interest rate differential</td>
<td>(0,0,0,1,-1,0,0,0)</td>
<td>(0,0,0,0.1,1,0,1,0,0,0 )</td>
<td>54.6 [0.00]**</td>
</tr>
<tr>
<td>IVb</td>
<td>Augm. UIP, no trend</td>
<td>(1,<em>,</em>,<em>,</em>,0,0)</td>
<td>(1,-1,0.064,1.534,1.833,1.833,0.091,1,0 )</td>
<td>7.74 [0.02]</td>
</tr>
<tr>
<td>V</td>
<td>PPP, UIP, no trend</td>
<td>(1,-1,1,<em>,</em>,<em>,</em>,*,0)</td>
<td>(1,-1,1,1.435,-1.435,0.079,0 )</td>
<td>9.33 [0.05]</td>
</tr>
</tbody>
</table>

#### Weak exogenity

<table>
<thead>
<tr>
<th>Number</th>
<th>Variable</th>
<th>Restrictions, $\alpha$</th>
<th>LR prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$v$</td>
<td>(0,**,<em>,</em>,<em>,</em>,*)</td>
<td>6.08 [0.01]**</td>
</tr>
<tr>
<td>B</td>
<td>$p$</td>
<td>(<strong>,0,</strong>,<em>,</em>,*)</td>
<td>21.0 [0.00]**</td>
</tr>
<tr>
<td>C</td>
<td>$p^*$</td>
<td>(<strong>,*,0,</strong>,<em>,</em>)</td>
<td>2.94 [0.09]</td>
</tr>
<tr>
<td>D</td>
<td>$i$</td>
<td>(<strong>,*,0,</strong>,<em>,</em>)</td>
<td>0.41 [0.52]</td>
</tr>
<tr>
<td>E</td>
<td>$i^*$</td>
<td>(<strong>,*,0,</strong>,<em>,</em>)</td>
<td>3.75 [0.05]</td>
</tr>
<tr>
<td>F</td>
<td>$\alpha$</td>
<td>(<strong>,*,0,</strong>,<em>,</em>)</td>
<td>0.89 [0.34]</td>
</tr>
<tr>
<td>G=D+F</td>
<td>$i$ and $\alpha$</td>
<td>(<strong>,*,0,</strong>,0,0,0,0)</td>
<td>1.39 [0.50]</td>
</tr>
<tr>
<td>H=C+D+F</td>
<td>$p^*, i$ and $\alpha$</td>
<td>(<strong>,0,</strong>,0,0,0,0,0)</td>
<td>5.14 [0.16]</td>
</tr>
<tr>
<td>J=D+E+F</td>
<td>$i$, $i^*$ and $\alpha$</td>
<td>(<strong>,*,0,</strong>,<em>,</em>,<em>,</em>)</td>
<td>5.38 [0.15]</td>
</tr>
<tr>
<td>K=C+D+E+F</td>
<td>$p^<em>, i$, $i^</em>$ and $\alpha$</td>
<td>(<strong>,*,0,</strong>,0,0,0,0,0)</td>
<td>13.3 [0.01]**</td>
</tr>
</tbody>
</table>

#### Joint tests of $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>Comb.</th>
<th>Estimated $\alpha$</th>
<th>Estimated $\beta$</th>
<th>LR prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>V &amp; H</td>
<td>(-0.032,0.019,0,0,-0.012,0)</td>
<td>(1,-1,1,1.166,-1.166,0.083,0 )</td>
<td>11.6 [0.11]</td>
</tr>
<tr>
<td>V &amp; J</td>
<td>(-0.0210,0.014,0.005,0,0,0,0)</td>
<td>(1,-1,1,2.469,-2.469,0.073,0 )</td>
<td>19.0 [0.01]**</td>
</tr>
</tbody>
</table>

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**a)** The $\lambda$-max test the null hypothesis of $r$ cointegrating vectors against the alternative $r+1$, $r=0,1,2,...,n-1$, where $n$ is the number of variables in the model, see Johansen (1988). In the adjusted version of the test, the number of observations $T$ is replaced by $T-nk$, where $k$ is the number of lags, see Reimers (1992). **b)** The $\lambda$-trace test for $r=0,1,2,...,n-1$. Also here we report the adjusted version (see Reimers, 1992). **c)** The estimated coefficient for the trend is multiplied by 1000. Critical values are taken from Osterwald-Lenum (1992), and (*) denotes rejection at the 5 per cent significance level while (**) indicates rejection at the 1 per cent level.
Calculating half-lives

In order to calculate half-lives the deterministic variables in the system can be ignored. Without the deterministic variables, the system can be written as

\[ \Delta z_i = \left[ \Gamma_i, \alpha \right] \begin{bmatrix} \Delta z_{i-1} \\ \beta' z_{i-1} \end{bmatrix} + u_i, \]  

(B.1)

where

\[ \alpha' = \begin{bmatrix} -0.032 & 0.019 & 0 & 0 & -0.012 & 0 \end{bmatrix}, \]
\[ \beta' = \begin{bmatrix} 1 & -1 & 1 & 1.166 & -1.166 & 0.083 \end{bmatrix}, \]
\[ \Gamma_i = \begin{bmatrix} 0.239 & 0.216 & 0.134 & 0.415 & 0.078 & -0.000 \\ 0.046 & 0.114 & -0.088 & -0.027 & 0.178 & 0.007 \\ -0.016 & -0.259 & 0.108 & -0.049 & 0.002 & 0.005 \\ 0.022 & -0.011 & 0.212 & 0.012 & 0.453 & -0.007 \\ -0.023 & 0.065 & -0.026 & -0.035 & 0.059 & 0.001 \\ -0.208 & -5.663 & -6.170 & -1.593 & 1.827 & 0.194 \end{bmatrix}. \]

To effectively use (B.1) for calculating half-lives, we need to augment (B.1) with the definitional relationship: \( \beta' z_i - \beta' \Delta z_i = \beta' z_{i-1} \). This yields

\[ \begin{bmatrix} I_6 & 0_{6 \times 1} \\ -\beta' & 1 \end{bmatrix} \begin{bmatrix} \Delta z_i \\ \beta' z_i \end{bmatrix} = \begin{bmatrix} \Gamma_i & \alpha \end{bmatrix} \begin{bmatrix} \Delta z_{i-1} \\ \beta' z_{i-1} \end{bmatrix} + \begin{bmatrix} u_i \\ 0 \end{bmatrix}, \]  

(B.2)

or

\[ \begin{bmatrix} \Delta z_i \\ \beta' z_i \end{bmatrix} = \begin{bmatrix} I_6 & 0_{6 \times 1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_i & \alpha \end{bmatrix} \begin{bmatrix} \Delta z_{i-1} \\ \beta' z_{i-1} \end{bmatrix} + \begin{bmatrix} I_6 & 0_{6 \times 1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_i \\ 0 \end{bmatrix}. \]  

(B.3)

Since the relationship should hold for all \( t \), we can write
Furthermore, assuming that the system is in equilibrium in period $t-1$ (this is not a necessary assumption; it only simplifies the presentation because the left hand side of (B.4) then can be interpreted as the effect of the shock) and that all future errors are zero ($u_{t+i} = 0, \forall i \geq 1$), the effect on the variables in the system in period $t+s$ of a shock in period $t$ can be written as

\[
\begin{bmatrix}
\Delta z_{t+s} \\
\beta' z_{t+s}
\end{bmatrix}
= \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left( \begin{bmatrix} \Gamma_1 & \alpha \\
0_{1 \times 6} & 1
\end{bmatrix} \right)^{s+1} \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left( \begin{bmatrix} \Delta z_{t-1} \\
\beta' z_{t-1}
\end{bmatrix} + \sum_{i=0}^{s} \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left( \begin{bmatrix} \Gamma_1 & \alpha \\
0_{1 \times 6} & 1
\end{bmatrix} \right)^{i} \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left[ u_{t+i} \right] \right).
\] (B.4)

We are not interested in the effect on the whole system of the shock, but only in the effect on the cointegrating relationship. Therefore, we pre-multiply with the selection vector $[0,0,0,0,0,0,1]$. The effect on the cointegrating relationship of a shock in the errors $u_i$, i.e. after $s$ periods (here; months), is hence given by the expression

\[
\beta' z_{t+s} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left( \begin{bmatrix} \Gamma_1 & \alpha \\
0_{1 \times 6} & 1
\end{bmatrix} \right)^{s} \left( \begin{bmatrix} I_6 & 0_{6 \times 1} \\
- \beta' & 1
\end{bmatrix} \right)^{-1} \left[ u_{t+i} \right] \right).
\] (B.5)