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Inflation Targeting Strategies in Small Open Economies*

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Abstract
According to recent literature on monetary policy, there are two different interpretations of inflation targeting; (1) an instrument rule that responds to a measure of inflation (forecast) deviations from target and (2) a discretionary optimizing strategy towards minimizing the inflation deviations from its target. This paper compares these strategies with some simple rules for monetary policy. In particular, attention is given to the strategies’ impact on the traded and non-traded sectors of the economy. Our conclusions suggest that there are considerable advantages in committing to a specific interest rate rule instead of letting the central bank discretionarily decide on the inflation targeting policy. The paper also provides evidence that the Taylor rule may work reasonably well in an open economy setting and gives only partial support for the Ball (1998) critique. It also discusses the structural conditions for successful targeting of inflation.

Keywords: Monetary policy, inflation targeting, simple rules, small open economy.

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1. Introduction

A large number of papers have recently been addressing monetary policy rules and their impact on important macroeconomic variables as aggregate production, inflation and the exchange rate\(^1\). Little research, however, has been carried out in assessing how different monetary policy rules may affect the short term resource allocation between different types of industries\(^2\). If stability is considered important due to the limitation of adjustment costs caused by monetary policy and its response to shocks, aggregate output stability will not suffice as a measurement of these costs if it is achieved at the expense of large sectoral fluctuations. This could happen if sectoral instability cancels out at the aggregate level. Furthermore, the argument for focusing on sectoral stability instead of aggregate stability is reinforced if adjustment costs differ across the sectors. If a given change in output in one of the sectors is achieved at a higher cost than a change in other sectors, it makes sense to stabilize the first sector to a larger degree than the others.

There might be reasons to believe that monetary policy affects the sectors differently depending on the their degree of exposure to the different monetary transmission mechanisms. A fundamental difference may be between that of the traded and the non-traded sectors. While the traded sector is considerably exposed to the exchange rate channels of monetary policy, the non-traded sector is probably more exposed to the interest rate channel. This means that monetary policy is likely to affect these sectors very differently and be a source of sectoral fluctuations. If keeping adjustment costs low is a central goal of monetary policy, the conduct of monetary policy ought to give attention towards achieving sectoral stability.

Due to the latest business cycle peak in the United Kingdom, the Bank of England, an inflation targeting CB since 1992, has pursued a contractionary policy since 1997 which has resulted in high real interest rates that has arguably caused the real exchange rate to appreciate considerably. The manufacturing industry, which counts for approximately 25\% of the UK economy, has been severely hit by this contractionary policy\(^3\). The Reserve Bank of New Zealand, practising inflation targeting since 1989, faced some of the same problems in 1992. Then a strongly appreciating currency caused difficulties for the monetary policy makers wanting to raise interest rates in order to head off increasing domestic inflationary pressure. The claim is therefore that inflation targeting may cause stronger fluctuations in traded sector output than other systems of organizing monetary policy. This paper investigates this claim and asks what sort of conditions may support it.

There is, however, some disagreement between economists on the specific characteristics of inflation targeting. Two interpretations of targeting have been offered in the literature. According to the first, promoted by

\(^1\)See, e.g., Svensson (forthcoming), Batini and Haldane (1998), McCallum and Nelson (1998) and Bryant et al. (1993).

\(^2\)But see Leitemo and Røisland (1999), Røisland and Torvik (1999) and Chapple (1994) for some exceptions.

\(^3\)The Bank of England Inflation Report (November 1998, p.22) comments:

"Manufacturing output fell by 0.6\% in August and by 0.4\% in September, leaving the level 0.1\% lower in 1998 Q3 than in Q2. Survey evidence points to a significant deterioration in business sentiment and output prospects in the manufacturing sector. The October BCC survey showed a further decline in both home and export deliveries, with the former becoming negative for the first time since 1992 and the latter reaching their lowest level since the survey began in 1989. The October quarterly CBI Industrial Trends survey reported that the balance of firms expecting output to increase over the next four months was the most negative since January 1991"
Svensson (forthcomingc), targeting means using all available information optimally in setting the instrument of monetary policy in order to keep the targeted variable close to a prescribed path. Inflation targeting is thus a discretionary optimizing framework for monetary policy. According to the second view, promoted by Batini and Haldane (1998), Haldane (1998) and McCallum and Nelson (1998), targeting means responding to a disequilibrium measurement of the targeted variable. Hence, inflation forecast targeting would mean that the instrument of policy, usually the short interest rate, responds to the forecast of inflation at a given horizon. This forecast can either be based upon a specific policy reaction function for the interest rate or some other policy assumptions.

There are two aims for this paper. The first aim is to assess and contrast the two different interpretations of inflation targeting and their impact on volatility of key macroeconomic variables, with a special focus on the reliance upon sectoral fluctuations for achieving aggregate stability. The second aim is to evaluate some simple interest rate rules, like those of Taylor (1993) and Ball (1998), by comparing them to the two inflation targeting frameworks sketched above. We also study conditions that influence the performance of the inflation targeting strategies in a small, open economy.

Section 2 presents a quarterly, two-sectoral model of a small, open economy in an environment of perfect capital mobility. Production in the non-traded sector is determined by equilibrium income in the long run but may fluctuate around its steady state due to intertemporal substitution in demand. The small country assumption is invoked for the traded sector where producers face perfectly elastic demand for their goods on the international market. They also face important adjustment costs in production which create incentives for the producers in forming and reacting to forward-looking expectations of prices and costs. There is sufficiently mobility of labour between sectors to have the same average wage in each sector. Wages are set in accordance with overlapping wage contracts bargaining setup in which the parties care about the development in relative real wages between the contracts. Non-traded prices are set as markup on average wages. Section 3 presents the principles of the different monetary policy frameworks and their implications for monetary policy and interest rate setting. In section 4, numerical evaluation of the policy rules is carried out using the model presented in section 2 and structural conditions for successful inflation targeting is discussed. Finally, section 5 concludes.

2. The model

The model is an extended version of the one presented in Batini and Haldane (1998) (henceforth denoted by BH). They develop a rational expectations model calibrated to match the stylized facts of the monetary policy transmission mechanism incorporated in a larger forecasting model at the Bank of England. The BH-model is based upon the Fuhrer and Moore (1995) and Fuhrer (1997) specification of overlapping nominal wage contracts but extended along the lines of Blake and Westaway (1996) in order to allow for economic structure that is specific to the open economy. The main difference between the BH and our model is that we treat the traded and non-traded sectors separately in order to address issues raised in the introduction. More precisely, the differences can be summed upon in four points:

- Our model incorporates a forward looking tradeable sector where producers rationally predict future
product real prices and adjust accordingly;

- In addition to pressure in the labour market, wage determination is influenced by the real product wage in the tradeable sector reflecting, e.g., a stronger bargaining position of the labour unions in this sector;
- Demand for non-traded goods is partly determined by the long real interest rate;
- The sluggish adjustment of import prices to exchange rate changes are captured in an equilibrium correction mechanism.

2.1. Description

All variables, except interest rates, are measured in logs. Furthermore, all variables are measured in deviations from their (possibly time-varying) long run equilibrium values\(^4\) which are assumed to be independent of monetary policy\(^5\). Rational expectations formed at time \(t\) of a variable \(x\) at time \(t+1\) is denoted by \(x_{t+1|t}\).

The representative firm in the traded sector is forward-looking and determines next period production based upon information available in the current period according to the following production supply function

\[
y_{t+1}^T = \rho_T y_t^T + \beta \sum_{s=0}^{\infty} \delta^s (p_{t+1+s}^T - w_{t+1+s}) + u_{t+1}^T
\]  

(2.1)

where \(\delta\) is the rate at which the traded sector discounts future profit, \(p^T\) is the traded goods price in domestic currency units and \(w\) is the average wage cost per unit produced. The product real price, \(p^T - w\), can be seen as a proxy for profitability in this sector. One possible interpretation of (2.1) is that expected low future profitability means that the least cost-efficient manufacturers will decide to reduce or even stop production for some time until the option for continuing production seems more attractive. The forward-looking structure of (2.1) could also reflect the incentives to tune production and inventories in order to limit the expected adjustment costs in the production and investment processes\(^6\). Furthermore, this production setup creates inertia in the production process by letting (i) past production level be an important determinant for future production reflecting that sharp production changes may be costly and (ii) production is decided one quarter in advance reflecting a planning horizon. By taking expectations in (2.1) and using the lead operator\(^7\), expected production can be written as

\[
y_{t+1|t}^T = \rho_T y_t^T + \frac{\beta (p_{t+1|t}^T - w_{t+1|t})}{(1 - \delta F)}
\]

which can be rewritten as \((1 - \rho_T L)(1 - \delta F)y_{t+1|t} = \beta (p_{t+1|t} - w_{t+1|t})\). Combined with the fact that production is predetermined one period in advance, we arrive at the final form of the process describing traded sector output,

\[
y_{t+1} = \frac{\rho_T}{1 + \delta \rho_T} y_t^T + \frac{\delta}{1 + \delta \rho_T} y_{t+2}^T + \frac{\beta}{1 + \delta \rho_T} (p_{t+1|t} - w_{t+1|t}) + u_{t+1}^T
\]  

(2.2)

which is a convenient expression as it omits the infinite sum of product real prices.

\(^4\)Their unconditional expectations.

\(^5\)See Holden (1998) for an interesting perspective on how monetary policy regimes may affect the long run equilibria.

\(^6\)Equation 2.1 allows us to discriminate between reactions to temporary and permanent changes. This would not be possible within a purely backward-looking model.

\(^7\)The lead operator, \(F\), is defined as \(Fx_{s|t} \equiv x_{s+1|t}\)
Now turning to the traded output price, which is determined in the international market,

\[ p_t^T = s_t + p_t^* \]
\[ = s_t + p_t^* - p_t + p_t \]
\[ = e_t + p_t \] \hspace{1cm} (2.3)

where \( e_t = s_t + p_t^* - p_t \) is the real exchange rate; \( p_t^* \) is the price of foreign goods measured in foreign currency units and \( s_t \) is the nominal exchange rate.

The non-traded sector adjusts production to satisfy demand which in the long run is determined by equilibrium income. Due to intertemporal substitution in consumption, production can deviate from its long run trend. This gap is affected by monetary policy through a linear combination of the long and the short real interest rate denoted by \( R_t \) and \( r_t \) respectively. Persistence in output is an important stylized fact which can be due to habit formations. Persistence is conventionally modelled by adding a lagged output variable which lets the interest rate channel have a gradual effect upon non-traded demand. We assume that the elasticity of substitution is sufficiently low between non-traded and traded goods to ignore any relative price effects upon non-traded demand. The non-traded output deviation from trend can then be written as

\[ y_t^{N+1} = \rho_N y_t^N - \alpha(\omega R_t + (1 - \omega) r_t) + \xi_t^{N+1} \] \hspace{1cm} (2.4)

Aggregate output deviation from steady state is linearized around the equilibrium output in each sector, i.e.,

\[ y_t = \eta y_t^T + (1 - \eta) y_t^N \] \hspace{1cm} (2.5)

We adopt the wage setting idea of Fuhrer and Moore (1995) and Fuhrer (1997). This wage formation process was originally developed for the closed economy and this paper employs the extension offered by Blake and Westaway (1996) in order to take open economy considerations into account. Nominal wages are set for two periods every other period in an overlapping setting in such a way that the period real contract wage is a compromise between the contract real wage negotiated last period and the expected contract real wage next period. In addition, pressure in the labour market and profitability in the traded sector influences wage formation, i.e.,

\[ x_t - p_t^e = (1 - \phi) \left( v_t + \frac{1}{2} \gamma y_t - \frac{1}{2} \mu (w_t - p_t^T) \right) + \phi \left( v_{t+1|t} + \frac{1}{2} \gamma y_{t+1|t} - \frac{1}{2} \mu (w_{t+1|t} - p_{t+1|t}^T) \right) + \tilde{w}_t \]

where \( x_t \) is the nominal contract wage, \( w_t \) is the (average) wage rate,

\[ w_t = .5(x_t + x_{t-1}) \] \hspace{1cm} (2.6)

and \( v_t \) is an index of real contract wages prevailing at time \( t \),

\[ v_t = \frac{1}{2} (x_t - p_t^e) + \frac{1}{2} (x_{t-1} - p_{t-1}^e) \]

\[ \hspace{1cm} \text{8 Real price effects would reduce the asymmetrical impact of monetary policy on the two sectors. Thus our assumption is more likely to overestimate than underestimate sectoral fluctuations caused by the different monetary policy strategies.} \]

\[ \hspace{1cm} \text{9 Kolsrud and Nymoen (1998) and Holden and Nymoen (1998) study the wage process in several small, open economies and find support for profitability in the manufacturing sector being one of the key determinants of wages in these countries.} \]
By substituting out for \( v_t \) in the wage setting equation we get the following illustrative relationship,

\[
x_t - p_t^c = (1 - \phi)(x_{t-1} - p_c^{t-1}) + \phi(x_{t+1|t} - p_{c+1|t}) +
(1 - \phi)\gamma y_t + \phi \gamma y_{t+1|t} - (1 - \phi)\mu(w - p^T_t) +
-\phi \mu(w - p^T_{t+1|t}) + u^w_t
\]

(2.7)

where \( u_t^C = \frac{1}{2} \tilde{u}_t^C \).

The non-traded output price is set as a markup on the average wage, i.e.,

\[
p_t = w_t
\]

(2.8)

The consumer price is a weighted combination of imported prices and non-traded goods prices

\[
p_t^C = (1 - \psi)p_t + \psi p_t^IM
\]

(2.9)

Imported goods prices are assumed to adjust to the traded goods prices in an equilibrium correcting framework,

\[
p_{t+1}^IM = p_t^IM + c(p_t^T - p_t^IM)
\]

which can be expressed in terms of differences by using \( \pi_t^IM = p_t^IM - p_{t-1}^IM \),

\[
\pi_{t+1}^IM = \pi_t^IM + c(p_t^T - p_{t-1}^T - \pi_t^IM) + u_{t+1}^IM
\]

(2.10)

where we also have allowed for a stochastic shock to imported price inflation.

The (quarterly) uncovered real interest parity condition is invoked, i.e.,

\[
e_t = e_{t+1|t} - .25(r_t - r_t^*)
\]

(2.11)

where \( r_t^* \) is the foreign short real interest rate. We allow, however, for the possibility of some imperfections in the capital market by treating \( r_t^* \) as including any risk premium. The short real domestic interest rate is defined according to the Fisher equation,

\[
r_t = i_t - 4(p_{t+1|t} - p_t)
\]

(2.12)

and the foreign real interest rate is modelled as an AR(1) process,

\[
r_{t+1}^* = \rho_t^* r_t^* + u_{t+1}^r
\]

(2.13)

We follow Svensson (forthcomingc) in assuming that the long real interest rate is determined according to the expectational hypothesis:

\[
R_t = \frac{1}{T} \sum_{s=t}^{t+T} r_s|t
\]

\[
\approx \frac{1}{T} \sum_{s=t}^{\infty} r_s|t
\]
where the long real interest rate is set as the average of future short interest rates. We approximate\(^{10}\) the long real interest rate for reasons that will soon become clear. Since the foreign real interest rate is modelled as an AR(1) process, the foreign long interest rate would approximately be

\[
R_t^* \approx \frac{1}{T} \frac{r_t^*}{1 - \rho_r}
\]

(2.14)

By iterating on (2.11), assuming that the real exchange rate converges to its equilibrium level \(\lim_{s \to \infty} e^{t+s}|t = 0\), we get that

\[
e_t = 0.25 \left[ \sum_{s=t}^{\infty} r^t_{t+s}|t - \sum_{s=t}^{\infty} r^t_{t+s}|t \right]
\]

and by combining these expression, we can write the long interest rate as a function of the foreign equivalent and the real exchange rate.

\[
R_t = R_t^* - \frac{4}{T} e_t
\]

(2.15)

The above model leaves the short nominal interest rate as an exogenous policy variable. In the following sections, we evaluate several strategies for endogenously modelling the interest rate. These interest rate equations will have the common property of being linear functions of the state variables in our model.

### 2.2. Calibration

The model is calibrated to match some characteristics of the UK economy. The coefficients of the model are close to those found in Batini and Haldane (1998) which again were chosen to let their model match the impulse responses of the forecasting model of the Bank of England. Some discretion is exercised in the choice of parameter values since the BH model differs somewhat from ours. Some coefficients have been estimated. Others have just been set to plausible values.

Persistence in output is considered to be high and the benchmark values are \(\rho_T = 0.85\) and \(\rho_N = 0.75\). Both are close to the persistence value of \(\rho = .8\) in the one-sectoral model of BH. The high degree of persistence in the traded sector output reflects a slower response to contemporary conditions due to, e.g., high capital intensity or high amount of adjustment costs. The real interest rate impact elasticity on non-traded sector is set to \(\alpha = 0.125\) close to the value in BH. The long interest rate weight in the interest rate index is somewhat arbitrarily set to \(\omega = .7\). The impact elasticity of production in the traded sector with respect to an expected, one period change in the real exchange rate, is set to \(\beta = .4\). Together with a quarterly discount rate of \(\delta = .5\) of future profit in this sector, the impact elasticity of an expected, permanent change in the product real wage is \(\beta \frac{1}{1-\delta} = .8\).\(^{11}\) Traded sector share of output is set to \(\eta = .25\) and its share in the CPI index is \(\psi = .2\) in accordance with BH. The degree of forward-lookingness in the wage process is set to \(\phi = .2\) which makes the inflation rates more persistent than in the original setup of the model in Fuhrer and Moore (1995). The

\(^{10}\)The discrepancy will depend on the rate of convergence of the short real interest in the model. A quick convergence means that the discrepancies will be small and unimportant. Thus the approximation will improve with the effectiveness of policy. Inspection of the impulse response functions due to the different policy rules confirms that the approximation error is negligible.

\(^{11}\)In the BH model, the impact elasticity of the real exchange rate is \(-.2\) and the long run elasticity of \(-1\). Our choice of coefficients would produce corresponding elasticities of \(-.4\) and \(-2.66\) if the change is perceived to be transitory, and \(-.8\) and \(-5.25\) if the change is perceived to be permanent. Given that the traded sector accounts for 25% of the economy, these responses seem reasonable.
period real wages response to output is set to $\gamma = .2$ similar to the value used in BH. Nymoen (1999) reports estimates of the annual real wage response to a measure of producer real wages for the Nordic countries in the range of $-0.14$ to $-0.26$. Furthermore, Baardsen et al. (forthcoming) estimate that wages error correct to the equilibrium level of consumer real wages by a factor of $-0.156$ quarterly. We assume that wages responds to the traded product real wages somewhat more moderately, and set $(-\mu) = -0.12$. However, we do extensive sensitivity analysis with respect to this parameter later in the paper. The average time to maturity for long term loans is set to $T = 40$ quarters.

The foreign real interest rate including any risk premium, i.e., the level of the domestic real interest rate that would not produce expectations of real exchange rate changes, $\tilde{r}_t^*$, is approximated from the uncovered interest parity condition, so that

$$\tilde{r}_t^* = \tilde{r}_t + 4(\tilde{e}_t - \tilde{e}_{t+1})$$

and where the US is considered as the foreign country. The approximation to the UK short real interest rate is constructed as $\tilde{i}_t = i_t - \pi_{t+1}^C$ and the real exchange rate is approximated by $\tilde{e}_t = s_t + p_t^C - p_t^C$. The autoregressive parameter in (2.13), $\rho^x_r$ was then estimated by regressing the proxy $\tilde{r}_t^*$ on its one-period lagged value which gave a result of $\hat{\rho}^x_r = .19$ and $\hat{\sigma}^x_r = .21$. By including several more (insignificant) lagged terms in the regression, the sum of the coefficients remained close to .19 and the standard deviation of the residuals was almost unchanged. Our AR(1) approximation thus seems reasonable.

To get estimates of the variance-covariance to the shocks, we employed a quarterly VAR(4) with log of industrial production, log of non-traded production (GDP minus industrial production), CPI inflation, imported price inflation, industrial wage inflation, change in log of the real effective exchange rate (relative wage cost per unit), 3 month UK interest rate, log of industrial production in industrialized countries and the change in CPI in these countries. A trend and seasonal dummies were also included. The data was extracted from the IMF database and the estimation period was 1976.1-1996.2. The residuals from the industrial production, non-traded production, import price inflation, wage inflation equations were then combined with the residuals from the risk premium equation. The variance-covariance matrix was then computed$^{12}$.

<table>
<thead>
<tr>
<th>$\tilde{r}^*$</th>
<th>$u^N$</th>
<th>$u^T$</th>
<th>$u^W$</th>
<th>$u^{IM}$</th>
</tr>
</thead>
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<tr>
<td>0.049730</td>
<td>0.000036</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000295</td>
<td>0.000032 0.000087</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000524</td>
<td>0.000016 0.000044 0.000076</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000666</td>
<td>-0.000016 -0.000032 0.000599</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.000375</td>
<td>0.000021</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: The variance-covariance matrix

$^{12}$To arrive at the exact distribution of the unobservable shock to the real contract wage, $u^w_t$, one would have to employ a Kalman filter which results would critically depend on the validity of our model. Our approach is less ambitious. The model is in any case a stylized approximation to the real world and hence any method to extract information would yield imperfect results. We therefore approximate variance of the shock to the contract wage as being twice the variance of the wage equation in the VAR model - reflecting the simple overlapping structure of the model. In any case, the qualitatively conclusions in this paper are somewhat robust to changes in the assumption about the wage shock.
2.3. The state space form

The model in the above section can be conveniently set up in state space form, i.e.,

\[
\begin{bmatrix}
  x_{1t+1} \\
  x_{2t+1|t}
\end{bmatrix}
= \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
  x_{1t} \\
  x_{2t}
\end{bmatrix}
+ \begin{bmatrix}
  B_1 \\
  B_2
\end{bmatrix} i_t + \begin{bmatrix}
  u_{t+1} \\
  0
\end{bmatrix}
\] (2.16)

\[
= A \begin{bmatrix}
  x_{1t} \\
  x_{2t}
\end{bmatrix}
+ B i_t + \begin{bmatrix}
  u_{t+1} \\
  0
\end{bmatrix}
\] (2.17)

where \( x_{1t} = \begin{bmatrix} y_t^N & y_{t-1}^N & y_t^T & y_{t-1}^T & \pi_t & \pi_{t-1} & \pi_{t-2} & \pi_{t-3} & \pi_{t-4} & \pi_{t|t-1}^m \end{bmatrix} \) is the (22x1) vector of predetermined variables, \( x_{2t} = \begin{bmatrix} \pi_t & y_{t+1|t} & e_t \end{bmatrix} \) is the vector of the forward looking variables, \( A \) is the transitional matrix and \( B \) is a vector of interest response coefficients. \( u \) is the vector of disturbance terms. Given the state space form, the solution to the model can be computed using for instance the procedure described by Klein (1998) which computes for a rule for the interest rate,

\[
i_t = F x_{1t}
\] (2.18)

the minimum state variable solution\(^{13}\) (McCallum (1983)) for the forward looking variables as function of the predetermined variables, i.e.,

\[
x_{2t} = H x_{1t}
\] (2.19)

and by substituting these expressions into (2.16) we derive the closed form solution for the predetermined variables

\[
x_{1t+1} = \left( A_{11} + A_{12} H + B_1 F \begin{bmatrix} I \\ H \end{bmatrix} \right) x_{1t} + \begin{bmatrix} u_{t+1} \\
0
\end{bmatrix}
\]

3. The monetary policy framework of inflation targeting

Inflation targeting was introduced in several countries during the 1990s. In some countries like New Zealand and Canada, inflation targeting came as a response to a need for getting inflation under tighter control. In other countries like Sweden and the United Kingdom, inflation targeting replaced nominal exchange rate targeting after the financial turmoil in 1992.

An inflation targeting framework for monetary policy is recognized by the following criteria\(^{14}\): (i) there is a publicly announced target level or band for inflation to which inflation should be kept or return to at a specified horizon. (ii) policy is transparent and explained in an inflation report issued by the monetary authority and (iii) the central bank has instrument independence in setting its monetary policy instrument. The fourth criterion is more controversial and researchers seem to disagree whether it should be:

- (iv-a) The central bank discretionarily uses all available information optimally in setting the instrument in order for inflation to stay or return to its target level

\(^{13}\)We restrict ourselves to consider the MSV solution to the model which is free of bubble and sunspot components. See McCallum (1999) for further details.

\(^{14}\)See Svensson (forthcoming).
(iv-b) The central bank adheres to a specific instrument rule that reacts in particular to deviations of (forecasted) inflation from its target level

Svensson (1998, p.1-2) notes with respect to this debate:

"[Inflation targeting means] that all relevant information is used in conducting monetary policy. It also implies that there is no explicit instrument rule, that is, the current instrument setting is not a prescribed explicit function of current information. Nevertheless, the procedure results in an endogenous reaction function, which express the instrument as a function of the relevant information. ...it will depend on...anything affecting the central bank’s conditional inflation forecast... Furthermore, the reaction function is generally not only a function of the gap between the inflation forecast....and the inflation target. In the literature, "targeting" .. are frequently associated with a particular information restriction for the reaction function, namely that the instrument must only depend on the gap between the ... target variable and the target level (and lags of this gap). I find this information restriction rather unwarranted."

McCallum and Nelson (1998, p. 36-37), on the other hand, has a different opinion:

"Svensson’s basic criticism of traditional terminology is as follows. A rule that responds to deviations of [inflation] does not constitute targeting because 'to target [inflation]', means 'using all relevant information to bring [inflation] in line with the target path'. ...And in typical cases, optimal instrument rules will entail responses to other variables in addition to [inflation]. But here 'optimal' actually means optimal with respect to one particular objective function and one particular model of the economy. But the point of a simple rule such as \[ i_t = \mu_0 + \mu_1 (\pi_t - \pi^*) + \mu i_{t-1} \] is that with \( \mu_1 > 1 - \mu_3 \) it will call for \( i_t \) adjustments that will keep \( \pi_t \) close to its target value \( \pi^* \), without being dependent upon any particular objective function or model. ..... A second reason for retaining the traditional language is that it corresponds more closely, in our judgement, to actual practice of 'inflation targeting' as represented by the central banks of Canada, New Zealand, and the United Kingdom."

We compare and contrast these two strategies. The Svensson targeting concept is as of now a discretionary optimizing framework, while the targeting concept of McCallum and Nelson is associated with inflation forecast based rules or simple rules in which the instrument of monetary policy responds to some measurement of inflation.

3.1. Discretionary optimizing framework

This framework of monetary policy has been studied by Svensson (forthcominga), Svensson (forthcomingc) and Leitemo and Røisland (1999). Under this strategy, the monetary policy authorities use all available information in designing policy so as to minimize inflation deviations from a target level (which is normalized to 0) under discretionary policy setting and a model of the economy. This can be expressed in terms of a quadratic loss function of the form,

\[
L_t^{CB} = (\pi_t^C)^2 + \lambda y_t^{CB} y_t^2 + \lambda_i (i_t - i_{t-1})^2
\]

(3.1)
The central bank’s problem is then to intertemporally minimize its expected loss, i.e.,

$$\min_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} L_{t+s}^{CB} \right]$$

where $i_t$ is the policy instrument. This is equivalent to minimizing the unconditional weighted variances of the targeted variables around their target levels. $\lambda_{x}^{CB}$ is the weight given to a target variable, $x$, other than inflation. Inflation targeting is said to be strict if inflation is the dominant variable in the loss function with only small weights attached to other variables. Flexibility is attained by increasing the weights other variables carry in the loss function. The optimization procedure is outlined in the appendix.

This type of rule defines the target variables as being the arguments in the loss function of the CB. The loss function may or may not be representing some underlying social loss function. The loss function can better be viewed as the mandate given to the monetary authority by the government. It may differ from the society’s loss function in order to resolve time inconsistency issues (Barro and Gordon (1983), Svensson (1997)) or increase transparency and accountability. As the society’s loss function is expected to be rather complex, it would be difficult to monitor if the CB actually was pursuing a loss minimizing policy if it was given such a loss function. Simplicity is thus an important issue if transparency and accountability are considered important.

Interest smoothing seems to be an essential part of the monetary policy process. Several studies of monetary policy suggest that instrument smoothing comes as an optimal response to model and data uncertainty\(^1\) whereby policy makers reacts less strongly to changes in the state of the economy and the interest rate shows higher correlation with past rates. Another justification may be that society cares about smoothing interest rates due to reasons of financial stability. To simulate the operations of such a policy, we attach a significant weight to interest smoothing in the loss function for our benchmark optimizing strategy.

The discretionary optimizing procedure, which is described by Backus and Drifill (1986), differs from the overall optimizing procedure in assuming that the central bank lacks commitment mechanisms that can enforce the optimal but time-inconsistent rule. The existence of forward-looking variables, which per definition react to expectations about the future, creates opportunities for the central banks to ”cheat” on their previous announced policy (Kydland and Prescott (1977)) by reoptimizing in later stages and thereby gain both from false expectations and the ability to let policy react to the ’current’ state of the economy irrespective of earlier attempts at precommitment. Rational agents would foresee this and understand that the commitment policy does not produce a rational expectations equilibrium given the inability to commit. The discretionary optimizing strategy produces a time-consistent policy by means of dynamic programming and hence policy becomes subgame perfect.

A large literature exists on how to improve on this discretionary outcome, see e.g. Rogoff (1985), Walsh (1995) and Svensson (1997). Delegating monetary policy to a central banker with preferences that differ in some respect from the social loss function, could improve the outcome by eliminating the inflation bias. In this paper, we assume that the policy maker does not set an employment/output goal higher than the natural rate and hence there is no average (level) bias in the model. However, discretionary policies are nevertheless hampered by inflation and stabilization biases that produce more instability in the targeted variables than

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\(^{15}\)See e.g. Rudebusch (1998)
the overall optimizing policy under precommitment would give. How much it differs from the precommitment solution, will depend on the role the forward-looking variables play in the structure of the model. If the forward-looking variables affect the monetary transmission mechanism in some fundamental ways, we would expect to observe that precommitment is crucial in attaining overall optimality and hence would differ significantly from the time-consistent solution.

The discretionary optimizing strategy is hampered by a number of problems. The time-consistent solution to (3.2) implies a instrument rule - a solution to the interest rate as a function of all the state variables in the model. The solution is therefore complex as it requires a large amount of information about variables that are important to the dynamics of the model that may or may not be observable. The performance of the instrument rule is also likely to be dependent on the specific model employed in the optimization process. If the rule where employed in a different model, its performance may be significantly changed. It is therefore open to criticism about not being robust to different views of the world. Given the lack of consensus about the true model, this is indeed a serious obstacle. However, no central bank has yet announced a specific instrument rule and the operation of inflation targeting is carried out in a discretionary manner. A discretionary optimizing strategy may therefore be a valid way of modelling inflation targeting behaviour. It does, however, assume a rather optimistic view of the CB: that it can use all available information in setting the instrument optimally.

3.2. Inflation forecast responding rules

An inflation forecast responding rule (henceforth, IFR-rules) has been proposed by among others Batini and Haldane (1998) as reflecting the operations of CBs in an inflation targeting regime. Whereas the discretionary optimizing framework only indirectly implied an instrument rule derived from the optimization process, an inflation forecast responding rule is an expression relating the interest rate explicitly to a particular restricted information set: the policy consistent inflation forecast. We study such a rule of the following form,

$$i_t = \varphi i_{t-1} + 4\pi_{t+1}^C + \chi \pi_{t+h}^C$$

where $\varphi$ measures the degree of instrument persistence, $\chi$ is the coefficient of instrument response and $h$ is the forecast horizon. The real interest rate $r_t \equiv i_t - 4\pi_{t+1}^C$ responds to the rule consistent CPI inflation forecast $h$ quarters ahead with a degree of nominal interest rate smoothing. This is, however, not a simple rule. The rational forecast of inflation is in general a function of all the state variables in the model and not restricted to a few of them\textsuperscript{16}. In this way, it is different from, e.g., the Taylor rule, which is 'model independent' in the sense that all its components can be directly observed. Furthermore, an IFR-rule requires precommitment from the policy maker to this particular rule which in general deviates from the discretionary policy presented above. This is by no means obvious. McCallum (1997a), p.1, argues, however, that "it is inappropriate to presume that central banks will, in the absence of any tangible precommitment technology, inevitably behave in a 'discretionary' fashion that implies an inflationary bias". Although commitment to any instrument rule would pose at least theoretical problems, committing to a rule that is partly based upon model judgement could show to be even more difficult. Still, we proceed by assuming the ability to precommit to some interest rate rule in the rest of the paper. Arguably, the CB may announce its instrument rule and, through the use of

\textsuperscript{16}See Rudebusch and Svensson (1998) for an explanation of how forecast rules are programmed in the state space framework.
their inflation report, explain the operation of policy according to it and thus provide full transparency. Such a commitment mechanism, would certainly be dependent on the characteristics of the rule. A badly working policy rule would most certainly have problems in gaining confidence and credibility.

The length of the forecast horizon is an important issue in monetary policy debate. It is argued that a short targeting horizon may result in the interest rate reacting to a forecast that is insensitive to the instrument due to lags in the model. A too short targeting horizon may therefore create instrument instability and a low degree of both nominal and real stability. If the horizon is too long, however, the forecast may have an unimportant effect on the interest setting. The reason is that (3.3) even with \( \chi = 0 \) may produce a low forecast of inflation at a sufficiently long horizon. The term \( \chi \pi^C_{t+h|t} \) would then be small irrespective of the value of \( \chi \). Too long a forecast horizon could then provide a too soft policy with a low degree of stability. Section 4 provides some support of these views.

### 3.3. Constant-interest-rate inflation forecast targeting

Strict inflation forecast targeting of the four-quarter inflation rate, \( \pi^C \), \( h \) periods ahead, i.e.,

\[
\bar{\pi}^C_{t+h|t} = \pi^* 
\]

does not have a unique solution unless \( h \) is equal to the shortest lag policy influences inflation or some more restrictions are placed upon policy. Under constant-interest-rate inflation forecast targeting\(^{17}\) (henceforth CII-targeting) the central bank target a forecast of inflation that is based upon the policy assumption of a constant interest rate in the targeting period, i.e.,

\[
\bar{\pi}^C_{t+h|t} = \pi^* 
\]

Equation (3.4) has a simple structure and strong implications for policy: set the interest rate exactly so as to have the forecast of inflation at the end of the targeting period, conditioned on an unchanged interest rate in the targeting period, equal to target.

Both Bank of England and Bank of Sweden publish constant-interest-rate forecasts in their respective inflation reports which arguably have been used as indicators of the stance of monetary policy with policy implications. Furthermore, Svensson (forthcomingb) argues that \( (\bar{\pi}^C_{t+h|t} - \pi^*) \) may be a good indicator of the appropriateness of policy under inflation targeting.

In order to derive the implication policy, write \( \bar{\pi}^C_t = K \begin{bmatrix} x_{1t} & x_{2t} \end{bmatrix} \) where \( K \) is appropriately defined. By using (2.16) we can then write the forecast of inflation as

\[
\bar{\pi}^C_{t+1|t} = KA \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + KB_i t
\]

and after repeated substitutions, the general \( h \)-quarter forecast can be written as

\[
\bar{\pi}^C_{t+h|t} = KA^h \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + K \left( \sum_{j=0}^{h-1} A^j B_{i+h-1-j|t} \right)
\]

\(^{17}\)Leitemo (1999) reviews this type of targeting in some more detail.
Given the policy assumption of a fixed interest rate in the targeting period, i.e., \( i_{t+j|t} = i_t \), where \( h-1 \geq j \geq 0 \), the constant-interest-rate inflation forecast can be written as

\[
\hat{\pi}^C_{t+h|t}(i_t) = KA^h \left[ \begin{array}{c} x_{1t} \\ x_{2t} \end{array} \right] + K \left( \sum_{j=0}^{h-1} A^j \right) B_i
\]

which combined with (3.4), assuming \( \pi^* = 0 \), yields the rule for the interest rate, i.e.,

\[
i_t = \left[ K \left( \sum_{j=0}^{h-1} A^j \right) B \right]^{-1} KA^h \left[ \begin{array}{c} x_{1t} \\ x_{2t} \end{array} \right]
\]

assuming that \( \left[ K \left( \sum_{j=0}^{h-1} A^j \right) B \right]^{-1} \) exists. CII-targeting implies a systematic instrument rule and is not to be confused with the policy forecast assumption of a fixed interest rate in the targeting period. As time passes, the rationally expected development of the interest rate within the targeting period will be based upon the expected development of inflation beyond the initial targeting period. CII-targeting does not normally imply that inflation will return to the target level at the end of the targeting period since the end of the targeting period is changing through time. The structural forecast of inflation will hence move more gradually towards its target level than the length of the targeting horizon may indicate.

### 3.4. The Taylor rule

All the strategies presented so far have been of the 'use-all-information' type. Simple rules restrict the information set and respond just to only a few variables. One of the best known simple rule is that one of Taylor (1993), which can be stated as:

\[
i_t = a_\pi \pi^*_t + a_y y_t
\]

(3.5)

The reaction coefficients are \( a_\pi = 1.5 \) and \( a_y = .5 \) in Taylor’s original setup\(^{18}\). Several studies of US and German monetary policy making (Taylor (1993), Taylor (1998), Clarida and Gertler (1996) and Judd and Rudebusch (1998)) show that there is a close match between a Taylor type rule and the actual interest rate setting in the 1990’s. However, it seems like the Taylor rule does not sufficiently take into account that policy makers seem to smooth interest rate by letting past interest rates level have an effect on current interest rates. Judd and Rudebusch (1998) uses an error correction framework in estimating the US policy reaction function and finds evidence of (3.5) being a long run equilibrium relationship. Levin et al. (1998) investigates Taylor rules in the context of several models of the US economy. They find that by adding some response to lagged values of the interest rates in the Taylor rule, its performance improves and performs closer to the optimal rule. This dynamic form of the Taylor rule is explored by considering (3.5) as the equilibrium solution to the interest rate and that the interest rate over time equilibrium corrects to this value, i.e., \( \Delta i_t = (1-\rho_i)(a_\pi \pi^*_t + a_y y_t - i_{t-1}) \). The level of the interest rate would then be

\[
i_t = \rho_i i_{t-1} + (1-\rho_i)(a_\pi \pi^*_t + a_y y_t)
\]

(3.6)

\(^{18}\)This specification has been criticized by McCallum (1997b) as not being operative, since information is always revealed with a lag and policy makers cannot respond to measures unknown at the time of the interest rate decision. The implication of this could be that policy makers respond to the same variables with a one-quarter lag, i.e., \( i_t = a_\pi \pi^*_{t-1} + a_y y_{t-1} \). Simulation results (not reported here) show, however, that this specification does not result in any important differences from the current information specification and hence is not explored further in this paper.
which is termed a dynamic Taylor rule.

3.5. The Ball MCI-rule

The Taylor rule has been criticized by Ball (1998) as not being suitable for sufficiently open economies. He argues for the use of a monetary condition index as an intermediate target for monetary policy. Such a monetary condition index has been promoted by research in the Bank of Canada\(^{19}\). The index measures the stance of monetary policy by considering a weighted average of the real interest rate and the real exchange rate

\[
MCI_t = (1 - \omega) e_t + \omega r_t
\]  

Ball relates the MCI to a contingent intermediate target level of the form

\[
MCI_t = \beta_1 \pi_t^C + \beta_2 y_t + \beta_3 e_{t-1}
\]  

By using both (3.7) and (3.8), however, this may be expressed as a rule for the real interest rate, i.e.,

\[
r_t = \beta_1 \pi_t^C + \beta_2 y_t + \beta_3 e_{t-1} + \beta_4 e_t
\]  

where \(\beta_1 = \beta_1' / \omega, \beta_2 = \beta_2' / \omega, \beta_3 = \beta_3' / \omega\) and \(\beta_4 = -(1 - \omega) / \omega\).

As the Ball rule is stated in (3.9), there are several complications. The Ball rule requires more information than the Taylor rule since it responds to both the current and the one-period lagged real exchange rate deviation from the equilibrium real exchange rate. With optimal coefficients, this rule will surpass the Taylor rule just for this reason of extended information. Furthermore, as the Ball MCI rule is stated in terms of the real interest rate, it requires information about the expected development of inflation which, as indicated before, is a function of all the state variables in the model. In its original form, the Ball rule is thus not a simple rule. A third complication comes from the existence of a forward-looking variable in the information set; the current real exchange rate. Such forward-looking variables are influenced by expectations of future policy, which would partly depend on the forward-looking variables. This circularity may lead to multiple solution paths for these variables (Woodford (1994))\(^{20}\). This problem is, however, not restricted to the MCI rule, as both of the forecast based rules share this problem.

The Ball rule can seen as an extension to the Taylor rule that includes information about the real exchange rate - an extension that a priori seems reasonable since the real exchange rate play a significant role in the monetary policy transmission mechanism in a small, open economy. We explore the gains from extending the Taylor rule in this way by taking real exchange rate considerations into account, i.e.,

\[
i_t = (1 + \beta_1) \pi_t^C + \beta_2 y_t + \beta_3 e_{t-1} + \beta_4 e_t
\]  

where \(r_t\) is approximated by \(i_t - \pi_t^C\). Ball chooses the \(b\)-coefficients optimally in the context of a calibrated model and does not suggest general coefficients. We consider the benchmark values \(\beta_1 = \beta_2 = .5\) which are equal to that of the Taylor rule, and furthermore optimized values for \(\beta_3\) and \(\beta_4\).

\(^{19}\)See Freeman (1994).

\(^{20}\)This paper investigates only the fundamental solutions to policy in accordance with the minimal state variables criterion, see McCallum (1999).
4. Policy evaluation

4.1. Policy objectives

We compare the effects of the different strategies by considering the unconditional standard deviations of some key macroeconomic measures. In order to rank the strategies, we need to decide on a loss function that reflects social welfare in a reasonable way. We explore two quadratic approximations for the underlying social loss function. The first loss function is the weighted sum of variability in three arguments: CPI inflation, aggregate output and change in the nominal interest rate, i.e.,

\[ L^S_{i}^{(A)} = E_t \sum_{s=0}^{\infty} \left[ (\pi_{t+s})^2 + (y_{t+s})^2 + \lambda^S_{\Delta i} (i_{t+s} - i_{t+s-1})^2 \right] \] (4.1)

where \( \lambda^S_{\Delta i} = .5 \) is our benchmark case. \( L^S_{i}^{(A)} \) is the standard loss function used in many monetary policy studies reflecting a concern for nominal as well as real and financial stability. As noted in the introduction, this loss function may not reflect the objectives of monetary policy if limiting adjustment costs is an important policy objective. In remedy of this, we propose a second loss function where introduces a concern for sectoral stability, i.e.,

\[ L^S_{i}^{(B)} = E_t \sum_{s=0}^{\infty} \left[ (\pi_{t+s})^2 + \eta (y^T_{t+s})^2 + (1 - \eta) (y^N_{t+s})^2 + \lambda^S_{\Delta i} (i_{t+s} - i_{t+s-1})^2 \right] \] (4.2)

which takes into account that adjustment costs are not aggregate phenomena. It assumes, however, that adjustment costs are of the same magnitude in both sectors so the weights in the loss function reflects the sector’s share of total output.

Throughout the paper we will report a measure for sectoral stability, the square root of a weighted average of the variance in each sector, i.e.,

\[ SOV = \sqrt{\eta \text{var}(y^T) + (1 - \eta) \text{var}(y^N)} \]

where the weights are the production share of the respective sector. This would be a good measure of adjustment costs if adjustment costs per unit of production are equal across the sectors and independent of whether resources are transferred between sectors or the pool of unemployed resources. This measure is also a vital part of loss function (4.2).

4.2. Discretionary optimizing targeting

Discretionary optimization is necessarily dependent on the weight attached to the different arguments in the CB’s loss function (3.1). It turns out that there are important differences between the outcomes depending on whether the central bank attaches a weight to interest smoothing or not. Even though the no-smoothing case may be unrealistic, it does provide a benchmark for comparison with the other cases. We therefore start by considering this case.

4.2.1. No instrument smoothing

Given a specific society loss function such as (4.1) or (4.2) and discretionary policy making, we know from among others Barro and Gordon (1983), Rogoff (1985), Svensson (1997) and Lockwood (1997) that there may
be reasons for the CB not to adopt them as its loss function. There might be a case for a weight conservative central banker or, equivalently, for a more strict inflation targeting framework. The inability to precommit may be an important problem in the presence of a model where forward-looking variables play an important role.

Table 4.1 shows the percentage standard deviations of key variables where the CB pursues a policy by minimizing the expected loss of (3.1) with $\lambda_{CB}^{\Delta i} = 0$. The right part of table C.1 displays the derived instrument rules for different values of the weight to output stabilization in the loss function.

<table>
<thead>
<tr>
<th>St. dev in %</th>
<th>$\lambda_{CB}^{y}$</th>
<th>$\pi_C$</th>
<th>$y$</th>
<th>$y^T$</th>
<th>$y^N$</th>
<th>SOV</th>
<th>$e$</th>
<th>$R$</th>
<th>$i$</th>
<th>$\Delta i$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strict targeting</td>
<td>0</td>
<td>0.676</td>
<td>1.374</td>
<td>2.654</td>
<td>1.969</td>
<td>2.161</td>
<td>4.059</td>
<td>.856</td>
<td>28.47</td>
<td>37.00</td>
<td>27.93</td>
</tr>
<tr>
<td>Flexible targ.</td>
<td>0.5</td>
<td>0.681</td>
<td>1.381</td>
<td>2.655</td>
<td>2.003</td>
<td>2.184</td>
<td>4.062</td>
<td>.849</td>
<td>28.56</td>
<td>37.06</td>
<td>27.84</td>
</tr>
<tr>
<td>Flexible targ.</td>
<td>1</td>
<td>0.694</td>
<td>1.387</td>
<td>2.663</td>
<td>2.036</td>
<td>2.209</td>
<td>4.072</td>
<td>.844</td>
<td>28.65</td>
<td>37.16</td>
<td>27.79</td>
</tr>
</tbody>
</table>

Table 4.1: Percentage standard deviations under discretionary optimizing targeting with no interest rate smoothing

When the interest rate is freely set in order to minimize the standard deviations of inflation and output, this implies a very active policy with a high degree of volatility in the change of the interest rate. Under the strict inflation targeting case, the single objective of the monetary policy maker is to minimize the variability of CPI inflation. The CB will, however, not be able to stabilize inflation completely. Other papers\(^{21}\) have argued that a strict inflation targeting framework implies using the direct exchange rate channel affecting the imported goods prices extensively to stabilize inflation and that this would lead to excessive real instability. The strong ability to target inflation completely is based upon the assumption that the nominal exchange rate feeds instantaneously into imported good prices and hence CPI inflation. Although there is a widespread view that the direct exchange rate plays an important part in affecting the inflation rate in the short to medium time, there is little empirical support for the view the exchange rate has an immediate effect on imported prices (see Naug and Nymoen (1995) and Dwyer et al. (1994)). Our assumption that the direct exchange rate channel starts affecting the imported prices gradually after a one period lag, significantly reduces the impact of this channel to a level we think is more appropriate. Our results indicate therefore that the undesirability of strict inflation targeting found in other open-economy models may be somewhat overstated. The case for a central bank that cares exclusively for inflation stability is then given some support in our model.

As the concern for output stability rises, inflation stability drops and, surprisingly, so does output stability, reinforcing the support for strict inflation targeting. It is difficult to see directly how the time inconsistency problem penalizes a flexible targeting central bank in our somewhat complex model. However, based on the results of Lockwood (1997) among others, one might guess that the existence of output persistence in our model is the key to understand our results. Simulations\(^{22}\) when the model exercises less non-traded output persistence confirms our guess. If output for some reason is below the natural rate, a concern about output stability creates an incentive to follow a more expansionary policy. However, as policy influences the economy

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\(^{21}\)See, e.g., Svensson (forthcoming), Batini and Haldane (1998) and Leitemo and Reisland (1999).

\(^{22}\)Available from the author upon request.
mainly through forward-looking components as the long real interest rate and the real exchange rate, the
monetary policy maker’s ability to stabilize is dependent on what degree he is able to influence expectations of
future policy. In the discretionary equilibrium, the policy maker reoptimizes in every period and cannot make
a binding commitment. The incentive to adjust the interest rate has an effect upon expectations of future
policy which adversely affect output stability. Persistence exasperates the problem, as it requires a stronger
policy reaction in order to have production return quickly to the natural level. The output-flexible inflation
targeter is penalized to an even greater extent for his incentive to lower the interest rate. The adverse effect
is, however, small.

This reasoning is reflected in the reaction function for the interest rate which is displayed in the left part
of table C.1 in the appendix. Higher flexibility leads to a somewhat stronger response to the output gaps in
each sector as reaction coefficients increase markedly. There are almost no significant relative changes to other
reaction coefficients, with the real exchange rate coefficients being an exception.

4.2.2. Interest rate smoothing

In the above case, the need for interest rate changes are crucial in reaching the policy goals. Such a volatile
interest rate setting is seldom observed in modern industrialized countries which seem to regard interest
smoothing as an important part of the policy process. Table 4.2 shows the standard deviations and social
loss when the policy maker smooths interest rate movements and the implicit interest rate rule is displayed
in the right part of table C.1 in the appendix. CPI inflation variability increases markedly compared to the
no-smoothing case in the above section and so does aggregate output stability. When smoothing is an essential
part of policy maker’s and society’s loss function, the conclusion for the optimal degree of concern for output
stability in policy is changed. There is now a stronger case for introducing explicit output stabilization through
flexible targeting. If the market knows that it can expect less volatility in interest rate, excessive interest rate
changes in order to steer output towards its equilibrium level are punished and the strategy is more successful
in order to affect forward-looking variables in a favourable way. This effect of interest rate smoothing is recently
been investigated by Woodford (1999). This forward-looking effect also enhances CPI inflation variability.

<table>
<thead>
<tr>
<th>St. dev. in %</th>
<th>$\lambda_y^{CB}$</th>
<th>$\pi^C$</th>
<th>$y$</th>
<th>$y^T$</th>
<th>$y^N$</th>
<th>SOV</th>
<th>$\Delta t$</th>
<th>e</th>
<th>R</th>
<th>$r$</th>
<th>$L^S(A)$</th>
<th>$L^S(B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strict targeting</td>
<td>0</td>
<td>4.24</td>
<td>2.07</td>
<td>3.05</td>
<td>2.02</td>
<td>2.30</td>
<td>6.30</td>
<td>8.20</td>
<td>0.29</td>
<td>8.77</td>
<td>42.14</td>
<td>43.14</td>
</tr>
<tr>
<td>Flexible targ.</td>
<td>5</td>
<td>4.07</td>
<td>2.00</td>
<td>3.18</td>
<td>1.86</td>
<td>2.26</td>
<td>6.04</td>
<td>8.06</td>
<td>0.30</td>
<td>8.66</td>
<td>38.84</td>
<td>39.95</td>
</tr>
<tr>
<td>Flexible targ.</td>
<td>1</td>
<td>3.92</td>
<td>1.95</td>
<td>3.28</td>
<td>1.76</td>
<td>2.24</td>
<td>5.83</td>
<td>7.93</td>
<td>0.31</td>
<td>8.57</td>
<td>36.18</td>
<td>37.40</td>
</tr>
</tbody>
</table>

Table 4.2: Social loss and percentage standard deviations under discretionary optimizing targeting with interest
rate smoothing

We also note that it is the non-traded sector which gains from increased output stability. Although real
exchange rate volatility decreases with flexibility, traded output volatility increases - reflecting that there are
no monotonic link between these variables. First note that traded output is a predetermined variable and the
real exchange rate is forward-looking. From the inspection of the implied interest rate rules in the right part
of table C.1, policy is less aggressive but more persistent under flexible targeting. The expected future interest
rate differential response is thus less strong but more persistent. The exchange rate variability is reduced by the smoothed response but exasperated by the persistence. In our case, the first argument is stronger leading to lower exchange rate variability. The traded sector, however, does not benefit from the smoothed response to the same extent as production is predetermined one period in advance. The expected persistency of the exchange rate, however, creates more volatility in the sector.

By inspecting the impulse responses displayed in figures C.1 and C.2 we observe that shocks tend to have been stabilized after about 16-18 quarters. The traded sector is relatively strongly affected by shocks to the non-traded sector and wages - both resulting in an appreciated real exchange rate. The non-traded sector is less affected by any shock and is almost unaffected by shocks to traded output.

4.3. Forecast based rules

4.3.1. Inflation Forecast Responding Rule

We now turn to evaluate the inflation forecast responding rule (IFR-rule) in (3.3) where there is interest rate feedback from the rule consistent inflation forecast at a given horizon. Benchmark values of the coefficients in this rule are comparable to those used in BH; \( \phi = .5, \chi = 2 \) and \( h = 8 \). Table C.2 in the appendix displays the outcome of stochastic simulation of this rule for different length of the forecasting horizon.

The most striking result is that performance is quite independent of the length of the forecast horizon. This is in sharp contrast to the results of Batini and Haldane (1998) where the optimal forecast horizon of about 3-6 quarters performs clearly better than longer forecast horizons. The optimal forecast horizon in our model is 14 quarters which performance only marginally differs from our benchmark case. From the inspection of the impulse response functions in figure (C.4), we see that policy is able to stabilize the economy quickly and normally within 3-4 years. We would therefore expect \( \pi_{t+h_{l}}^{C} \) only to be small and insignificant number for a rather long horizon and so play only a small part in the determination of the interest rate. This explanation is reinforced by the fact that our model is to a large degree dependent on forward-looking variables, i.e., the (expected) future traded output, the long real interest rate in addition to domestic inflation and the real exchange rate. This feature act as a correcting mechanism as potential expected future interest rate reaction to the economy has a stabilizing effect through the contemporary forward-looking variables\(^{23}\). This mechanism reduces the need for future correcting policy. Such results hinge on the ability of the policy maker to commit to the rule as one would believe that the feature of this correcting mechanism would change if the policy makers were to reoptimize in every period.

Another way of explaining the unimportance of the length of the horizon becomes apparent when considering optimized parameters values of \( \chi \) in table C.3. We note that the performance of the rule only to a very limited degree is dependent on the forecast as the optimal value in general is far away from the benchmark value, being even negative for four and eight quarters forecast horizons. This suggests that the properties of this rule lies first and foremost in the will to keep real interest rates moderately stable - responding only to the past interest rate. Setting \( \chi = 0 \) produces a policy that only marginally differ from our benchmark case.

\(^{23}\)This expectation channel of monetary policy hinges critically on the assumption of rational, forward-looking expectations formation in our model.
The main reason for anyway having a forward-looking view in setting interest rates by considering a not too short forecast horizon lies in the interest rate smoothing objective. When \( h = \{0,1\} \) and policy responds to either the current inflation rate or the next period inflation forecast, the interest setting is excessively active as the forecast is having an important influence. Policy manages somewhat contrary to common belief to produce a relatively high degree of both nominal and real stability. This can partly be explained by the high degree of forward-lookingness in the model which act to stabilize the model on the grounds of future policy. Another reason may be that the model displays a high degree of real inertia - downsizing the impact effects of shocks. A longer horizon stabilizes the interest rate setting at the expense of more real and nominal instability.

For our benchmark case, this rule generates a relative high degree of aggregate output stability relatively to the other rules which is not attained by excessive sectoral instability. Inflation variability is also relatively low compared to the other rules. However, this type of rule requires large movements in the interest rate for any horizons. Social loss is therefore relative high for both alternative loss functions. The impulse responses to different shocks are show in figure (C.4) in the appendix. Compared to the discretionary optimizing flexible targeting framework, policy is in generally more aggressive but still fairly similar - supporting the view that such forecast based rules replicate closely the discretionary optimizing policies, as noted by Batini and Haldane (1998).

4.3.2. Constant-interest-rate inflation forecast targeting

CII-targeting exhibits some of the same characteristics as IFR-rules in that the specific length of the targeting horizon is rather unimportant for social loss as long as it is not too short. The results with having a targeting horizon of 5-6 quarters differ only marginally from those with a horizon of 16 quarters. Table C.4 in the appendix displays the results of CII-targeting in some detail. As the length of the forecast horizon increases, policy becomes increasingly active and approaches no response to any state variable. For targeting horizons beyond 6 quarters, the inherent stability of the model stabilizes the policy consistent forecast of inflation at the appropriate forecast horizon towards its target and the implied interest rate under CII-targeting approaches its equilibrium level of zero - there is no need for an interest rate response at all. Hence, the interest smoothing objective in the loss function is the main reason for considering longer forecast horizons. This confirms the results in Leitemo (1999) where CII-targeting is explored in a small closed-economy model. A targeting horizon of only 3 quarter achieves superior nominal and real stability but needs a extremely active policy setting and only a small preference for interest smoothing makes this policy unattractive.

Targeting horizons with a reasonably degree of interest rate smoothing imply rather strong variability in both inflation and output in our model. The policy inactiveness results in these variables being persistently away from their respective equilibria. The traded sector output displays the highest degree of variability of all of the standard rules considered in this paper. Non-traded sector output is moderately stable and hence our measure of aggregate output stability underestimates the degree of sectoral variability, which is unreasonably(?) high.
4.4. Simple rules

4.4.1. The Taylor rule

The results from simulation is laid out in table C.5 in the appendix. The benchmark Taylor rule gives good stability of aggregate output and stabilizes CPI inflation at the same level as the forecast responding rules.

The non-traded sector shows superior stability relatively to traded sector stability. Traded sector volatility is high compared to most of the standard rules considered in this paper. This volatility is closely related to the traded sector being exposed to the exchange rate channel. Both inflation and output are fairly persistent in our model. As the interest rate responds intuitively to disequilibrium in both these variables, it still takes some time before output and inflation has returned to their respective equilibria. If output is below its natural output, say, it would call for a large negative interest rate differentials for a relatively long period. As the exchange rate responds to expected future interest differentials, the exchange rate depreciates strongly leading to a strong expansion in the traded sector. However, traded sector is relatively small and the expansion only partially offsets the disequilibrium in the labour market. The stability in the non-traded sector is therefore partially attained by traded sector variability which leads to a relatively high measure of sectoral variability. However, despite high traded sector variability, aggregate output volatility is low and interest rate movements are smoothed which give low social loss according to both (4.1) and (4.2). The impulse response functions displayed in (C.5) show that they are quite similar to those produced by the discretionary and forecast based rules. However, the reaction to a wage shock is somewhat smoother. As the rule includes the slow moving four-quarter inflation rate, a rising CPI inflation rate produces a high interest rate level for some time and thus an appreciated real exchange rate. Non-traded stability in the face of wage shocks is therefore attained at the cost of a more volatile traded sector output.

The optimized Taylor rule with respect to loss function (4.1) gives rather unintuitive results with a small coefficient for the inflation term and a negative coefficient for the output term. The gains from optimizing comes entirely from larger interest smoothing as both output and inflation variability are increased. By setting $\lambda_{S_i}^S = 0.01$ and hence reduce the importance of the interest rate smoothing objective, the optimized values seem to suggest a strong and positive response to the inflation term and a positive value for the output term.24 This suggest that the smoothing objective can indeed explain some of the oddities in the results from optimizing, but not all of them. It also seems to suggest that the transmission mechanisms in an open economy are complex and the aggregate demand reasoning behind the Taylor rule tells us only a part of the optimal monetary policy story. Although the standard rule works reasonably well, traded sector variability is relatively high.

The dynamic Taylor rule of (3.6) with $\rho_i = .5$ achieves smoother interest rate behaviour as intended at the cost of somewhat higher nominal and traded sector variability. More persistent interest rate behaviour increases real exchange rate variability and hence causes this result. Still its performance according to both our loss functions is superior to any of the other rules explored in this paper.

\[24\alpha^*_\pi = 5.35 \quad \text{and} \quad \alpha^*_y = .36 \quad \text{when} \quad \lambda_{S_i}^S = .01.\]
4.4.2. The Ball rule

The standard Ball rule does not involve any important changes compared to the Taylor rule in our model as shown in table C.6 in the appendix. No measures of output stability changes considerably and CPI inflation stability is also more or less unchanged. Interest becomes more smoothed and this persistence increases traded sector variability. Similar conclusions are reached in the optimizing cases, where coefficient values becomes rather strange as also noted for the Taylor rule. Optimization in general makes the traded sector more unstable as interest rate persistence increases.

This result tells us that there may not be any large advantages in reacting to disequilibrium in the real exchange rate even for small countries. Some modifications are, however, in order. If the foreign interest rate is more persistent than we have assumed in this paper, there seems to be a stronger role for real exchange rate information. If the foreign real interest rate (or risk premium) is more persistently away from its equilibrium, it may pay to react to the exchange rate as this would reduce the expected interest rate differential and such reduce the volatility of the real exchange rate. We do not, however, find evidence of a strongly persistent $r^*_t$ in our model.

4.5. Policy comparisons

Our search for a suitable inflation targeting strategy in an open economy setting seems to suggest that some form of interest rate policy rule dominates a discretionary strategy of inflation targeting. The discretionary strategies is hampered both by high CPI inflation volatility and relatively high aggregate output volatility. In figure 4.1 we plot how the regimes compare when only CPI inflation variability and aggregate output volatility
matter. The discretionary regimes are clearly outperformed. This conclusion is only moderately affected if we consider sectoral variability instead of aggregate output. The optimized rules now perform worse compare both to the standard interest rate rules and the discretionary optimizing regimes. However, both the standard Taylor, Ball and IFR rules do well.

However, our loss functions also include interest smoothing as an important argument. When taking this into account, the two forecast based strategies examined in this paper perform very differently. The inflation forecast responding rule performs badly due to the excessive policy activeness. CII-targeting, on the other hand, does very well as it requires only small policy responses at a targeting horizon of eight quarters.

Social loss is consistently higher under (4.2) than (4.1). Aggregate stability is achieved in all of the strategies at some expense of sectoral stability. This is in particular true for the optimized rules where sectoral instability is exploited to the limit in order to reduce loss. Furthermore, even though losses are smaller under the Ball MCI rule, the Taylor rule performs almost as good. Hence the complications introduced by treating the exchange rate as part of the policy instruments may outweigh the benefits from this type of rule. The Ball rule is hence not further examined in this paper. Optimized Taylor and Ball rules perform better mainly due to a larger degree of interest rate smoothing. The dynamic Taylor rule addresses the problem of having a rather active monetary policy effectively, and ranks as the best of the rules considered.

4.6. Structural considerations

In this section we study some aspects of the economic structure that may influence and be important to the outcome of inflation targeting. The two standard strategies that provide a reasonably degree of nominal and real stability, the Taylor rule and the IFR-rule are contrasted to the flexible discretionary optimizing targeting case.

The first aspect we consider is efficiency in the use of information in the wage setting process. Following BH, efficiency in this way is defined as the degree to which agents look forward. An efficient information wage (bargaining) process is characterized by an extended ability to take account of its effect upon future macroeconomic performance.

The second aspect considered is the effect of having the wage formation process depending on profitability in the traded sector. Finally, we study the outcome of inflation targeting when the agents in the traded sector discount the future to a smaller degree and hence become more forward-looking.

4.6.1. The degree of forward lookingness in the wage process

We have assumed that the real contract wage is set in such a way that the situation in the current period matters more than next and future periods by assigning a rather small value of $\phi = .2$. This asymmetry creates higher and more realistic inflation persistence than in a setup with more symmetrical treatment of the periods (Fuhrer (1997)). In this section we ask to what degree this imperfection in the labour market has an impact on macroeconomic performance under inflation targeting. We do this by studying the standard deviation of some of the key variables by varying this coefficient in the interval $\phi \in [.05,.5]$. This is depicted in figure 4.2.

The results seem to point consistently towards enhanced stability irrespective of the type of strategy.
In particular, traded sector output variability drops significantly as the wage setting process becomes more forward-looking. When wages reflect future development as well as current conditions, this has in particular a good influence on the traded sector which is most sensitive to changes in the real wages. Stabilizing the wage process is therefore of great importance if traded sector stability matters. Non-traded sector output stability is also increased but to a much lower degree. There is almost no change in CPI inflation variability. This suggests, not surprisingly, that the wage formation system may have an important influence on the choice of strategies and even targeting regimes.

4.6.2. The effects of wage response to competitiveness in the traded sector.

The wage process has been modelled in such a way that traded sector profitability, represented by the real traded goods price, has an important effect upon wages. If the labour unions organizing workers in the traded sector have a particular large influence on wage formation, we would expect that they would be concerned about the level of current and future cost competitiveness. This would have an important effect on current and expected future employment in this sector. Labour market imperfections, such as immobility of workers between the sectors could further strengthen this effect, since the alternative to work in the traded sector would then to a larger extent be unemployment.

Under the Taylor rule, the most notable effect of letting profitability play a more important role in the wage process, is the drop in traded output variability. When the conditions in the traded sector plays a
more important role in the wage setting process, monetary policy becomes more effective as the exchange rate channels play a more important role in the transmission mechanism. As traded output variability drops, both aggregate and, more importantly, sectoral output variability are reduced. On the other hand, CPI inflation becomes more volatile with increased wage response to profitability. This result is driven largely by the fact that CPI inflation is now more exposed to the indirect exchange rate channel. If the nominal exchange rate depreciates due to a foreign interest rate shock, traded sector profitability would temporary increase and hence affect wages and domestic inflation. Keep in mind that foreign interest rate shocks (or shocks to the risk premium) plays a major part in this model according to the assumption regarding the distributions of shocks. It is also interesting to see that the Taylor rule causes extensive variability in the traded sector if the wage setting process is independent of profitability in this sector. This would be true in the standard setup of the Fuhrer and Moore model.

Performance is in general less affected under the inflation forecast rule (IFR) and discretionary flexible optimization, but there are still signs of less variability in the traded sector when the profitability response increases. Aggregate stability and sectoral stability is consequently only affected to some degree. CPI inflation variability increases even more strongly than under a Taylor rule and policy becomes increasingly active.
4.6.3. Does a forward looking traded sector increase stability?

We have assumed that the traded goods producers discount future profit to a high degree. The discount rate, \( \delta \), is set to .5 in our benchmark case meaning a fairly short planning horizon. This could reflect low adjustment costs in production. If capital investment is irreversible or there are considerable start-up or shut-down costs in the production of traded goods, we would expect the (potential) producer to have a longer planning horizon and be more concerned with the expected future developments of prices and costs.

In this section we explore the performance of the three inflation targeting strategies when traded sector producers behave in a more (or less) forward-looking manner by studying the impact of different discount rates. We carry this out under the condition of a constant short-run elasticity of traded output to a permanent change in the real product price, i.e.,

\[
\beta(1 + \kappa + \kappa^2 + \ldots) = \frac{\beta}{1 - \kappa} = \beta_f
\]

where \( \beta_f = .8 \). A one percent perceived permanent increase in the real product price, reduces production with .8 percent. \( \beta \) then becomes a function of \( \kappa \), \( \beta(\kappa) = \beta_f(1 - \kappa) \). This assumption is made in order to distinguish changes in planning horizon from fundamental changes in the traded sector sensitivity to permanent real product price changes.

Figure 4.4: Policy performance at different degrees of traded sector forwardlookingness

Figure 4.4 plots the percentage standard deviations for key variables at different combinations of \( (\kappa, \beta(\kappa)) \) where \( \kappa \) is measured on the horizontal axis. Irrespective of the strategies used at inflation targeting,
traded sector stability is enhanced by a more forward looking traded sector. This has natural implications for aggregate output and total sectoral stability, in particular, which both are stabilized. There are only moderate changes to the rest of the variables considered.

If the performance of inflation targeting with respect to traded sector output stability is considered important, then the analysis seems to suggest that the choice of adopting inflation targeting may be heavily influenced by those factors that influence the optimal degree of forward-lookingness. Economies where the traded sectors is relatively more capital intensive or start- and shut-down costs may be important, should therefore be more confident about inflation targeting than other economies.

5. Conclusion

This paper has analyzed a number of different strategies that may be consistent with inflation targeting. Particular attention is given to how they affect the traded and non-traded sectors of the open economy and hence how firms in these sectors have to cope with adjustment due to macroeconomic reasons. It is natural to distinguish between the non-traded and the traded sectors of the economy because these sectors differ in how exposed they are to different channels of monetary policy.

Several conclusions can be drawn from the analysis of our model. The most interesting finding is perhaps that the discretionary optimizing framework for inflation targeting does not seem to work well when realistic assumption about interest smoothing is imposed. This is true irrespective of whether the society loss function penalizes aggregate output fluctuation or total sectoral variability in addition to CPI inflation and unsmoothed interest rate paths. A reason for this may be that the discretionary strategy is heavily penalized if the state of the economy is determined in important ways by forward-looking variables. Persistency in output may enhance these problems.

The traded sector is generally more volatile than the non-traded sector. The real exchange rate plays an important part in the transmission on monetary policy onto the economy and the traded sector is affected accordingly. In our opinion, this would be a strong argument for studying monetary policy in the context of multi-sectoral models as one-sectoral models may seriously cover up important aspects of policy. We can, however, not conclude that extensive traded sector fluctuations are specific to inflation targeting. Leitemo and Røisland (1999) reports in a similar model that nominal exchange rate targeting may even imply a stronger volatility in the traded sector. We also show that traded sector variability depends on how forward-looking the agents of the economy are. Both a more forward-looking wage process and production decisions in the traded sector may contribute to stability. Stability will also be enhanced if wage formation responds more strongly to profitability in the traded sector.

Sectoral variability when defined as the weighted average of volatility in each sector is consistently above aggregate output volatility. Aggregate output fluctuation may therefore be a misleading measure of adjustment costs in the model as aggregate stability may be attained at the expense of sectoral instability. However, this effect may not be as strong as we initially expected as the ranking of the standard strategies in our model does not seem to be dependent on whether we use a loss function that penalizes sectoral variability (4.2) or aggregate output variability (4.1). However, when considering optimized Taylor or Ball rules, the conclusion is
reversed. Optimized rules exploits the possibility attaining aggregate output stability through higher sectoral
variability.

The standard Taylor rule performs reasonably well in comparison with discretionary optimizing targeting
and policy consistent inflation forecast responding rules in our model. The dynamic Taylor rule performs very
well resulting in a smoother interest rate path with only marginal influence on real and nominal stability.
The Ball critique of the Taylor rule in Ball (1998) receives only partial support in our model. There are
few advantages in adding past and current measures of real exchange rate in the reaction function and indeed
several disadvantages in a practical monetary policy setting. However, the optimized Taylor rule has unintuitive
coefficients that do not resemble the standard coefficients. This may reflect that the transmission channels
of monetary policy in an open economy are significantly more complicated than those in a relatively closed
economy.

The paper suggests that the main reason for having a rather lengthy forecast horizon in the forecast based
rules, lies in the interest smoothing objective. The paper does not give extensive support of the view that a
long forecast horizon may contribute to real and nominal stability, neither in the forecast responding strategy
nor in the constant-interest-rate inflation forecast targeting strategy.

The Bank of England publishes constant-interest-rate forecasts and explains its policy in relation to these
forecasts. This is of course no conclusive evidence of CII-targeting but may be a strong indicator. Its perfor-
mance in our model is among the best. However, this is mainly due to its ability to smooth the interest rate
movements. Its nominal and real stability properties are not so good. Indeed, the traded sector is severely ex-
posed to shocks and fluctuates extensively, as noted as a real world problem in our introduction. Our analysis
suggests that there may be several ways in conducting policy in order to achieve superior real and nominal
stability, but at the cost of a more active policy.
A. The time consistent policy optimization procedure

We here provide an explanation of the method of Backus and Drifill based upon Söderlind (1999). Start with the compact notation of our model in (2.16). Since we can write (2.19), we can also write \( x_{t+2|t} = H x_{t+1|t} \) where \( H \) still remains to be specified. By using this expression in (2.16), we get

\[
\begin{bmatrix}
I \\
H
\end{bmatrix}
\begin{bmatrix}
x_{1t+1|t} \\
x_{2t}
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_{1t} \\
x_{2t}
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} i_t
\]

which can be rewritten as

\[
\begin{bmatrix}
x_{1t+1|t} \\
x_{2t}
\end{bmatrix}
= \begin{bmatrix}
I & -A_{12} \\
H & -A_{22}
\end{bmatrix}^{-1}
\begin{bmatrix}
A_{11} & B_1 \\
A_{12} & B_2
\end{bmatrix} i_t
\]

and the forward looking variables can be extracted,

\[
x_{2t} = (A_{22} - HA_{12})^{-1}(HA_{11} - A_{21}) x_{1t} + (A_{22} - HA_{12})^{-1}(HB_1 - B_2) i_t
\]

(A.1)

where \( D \) and \( G \) are defined accordingly.

By substituting out for \( x_{2t} \) using (A.1) in (2.16), we can get an expression for the evolution of the backward looking variables,

\[
x_{1t+1} = (A_{11} + A_{12} D_t) x_{1t} + (B_1 + A_{12} G_t) i_t + u_{t+1}
\]

(A.2)

The central bank loss function (3.1) can be written in matrix notation,

\[
J_t = E_t \sum_{s=0}^{\infty} \begin{bmatrix}
x_{t+s} \\
\pi_{t+s}
\end{bmatrix}^\prime
\begin{bmatrix}
Q & U \\
U^\prime & R
\end{bmatrix}
\begin{bmatrix}
x_{t+s} \\
\pi_{t+s}
\end{bmatrix}
\]

where \( X = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}, Q = \begin{bmatrix} T_{\pi} \\ T_y \\ T_{\Delta} \end{bmatrix}, U = -\lambda_{t+1} T_{\pi}, \Delta \) and \( R = \lambda_{\Delta}, T_{\pi}, T_y \) and \( T_{\Delta} \) defines the relationship between the respective targeted variable and the vector of state variables, i.e. \( \pi_{t+1} = T_{\pi} X_t \).

This maximization problem is carried through stringent to our model as laid out in (2.16). The Bellman equation for this problem would be,

\[
J_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}^\prime
\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}
\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + 2 \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}^\prime
\begin{bmatrix} U_1 & U_2 \end{bmatrix}
\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \lambda_{t+1} R i_t + E_t \left[ x_{t+1} \right]^\prime V (x_{t+1} + v_{t+1})
\]

where \( V \) and \( v \) remain to be specified.

Substitution of \( x_{2t} \) from (A.1) and \( x_{1t+1} \) from (A.2) into the Bellman equation, will give us after some manipulation

\[
J_t = x_{1t}^\prime Q^* x_{1t} + 2 x_{1t+1}^\prime U^* i_t + i_t^\prime R^* i_t + E_t \left[ (A^* x_{1t} + B^* i_t + u_{t+1})^\prime V (A^* x_{1t} + B^* i_t + u_{t+1}) + v_{t+1} \right]
\]

(A.3)
where

\[ Q^* = Q_{11} + Q_{12}D + D'Q_{21} + D'Q_{22}D \]
\[ U^* = Q_{12}G + D'Q_{22}G + U_1 + D'U_2 \]
\[ R^* = R + G'Q_{22}G + G'U_2 + U_2'G \]

has the first order condition for the short interest rate,

\[
i_t = -(R^* + B^*V B^*)^{-1}(U^* + B^*V A^*)x_{1t} \tag{A.4}
\]

\[
= -Fx_{1t}
\]

which shows that policy is certainty equivalent, independent of the distribution of the error term in our model (see for example Currie and Levine (1993)).

It now remains to find an expression for \( H \) which is achieved by inserting (A.4) into (A.1) and we arrive at

\[
x_{2t} = (D - GF)x_{1t}
\]
\[
= Hx_{1t}
\]

where \( H = (D - GF) \).

By inserting the optimal instrument rule into our Bellman equation, the value function can be written as

\[
J_t = x_{1t}' \left[ Q^* - U^*F - F'U^* + F'R^*F + (A^* - B^*F)'V(A^* - B^*F) \right] x_{1t} + \\
E_t u_{t+1}'V u_{t+1} + Ev_{t+1}
\]

\[
= x_{1t}'V x_{1t} + E_t u_{t+1}'V u_{t+1} + Ev_{t+1}
\]

where \( V = Q^* - U^*F - F'U^* + F'R^*F + (A^* - B^*F)'V(A^* - B^*F) \).

By iterating on the equation for \( D, G, A^*, B^*, Q^*, U^*, R^*, F, C \) and \( V \), and given that convergence is achieved, we have found the discretionary optimizing solution.

B. Tables and figures
### C. Tables and figures

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<td>.119</td>
<td>.113</td>
</tr>
<tr>
<td>$y^T_t$</td>
<td>.100</td>
<td>.098</td>
<td>.096</td>
</tr>
<tr>
<td>$y^T_{t-1}$</td>
<td>.009</td>
<td>.008</td>
<td>.008</td>
</tr>
<tr>
<td>$y^y_t$</td>
<td>.035</td>
<td>.033</td>
<td>.032</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>.195</td>
<td>.184</td>
<td>.176</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>.565</td>
<td>.535</td>
<td>.510</td>
</tr>
<tr>
<td>$\pi^{im}_{t-1}$</td>
<td>-.035</td>
<td>-.033</td>
<td>-.032</td>
</tr>
<tr>
<td>$\pi^{im}_t$</td>
<td>.487</td>
<td>.460</td>
<td>.438</td>
</tr>
<tr>
<td>$\pi^{im}_{t-1}$</td>
<td>.141</td>
<td>.134</td>
<td>.127</td>
</tr>
<tr>
<td>$u_t$</td>
<td>2.267</td>
<td>2.143</td>
<td>2.042</td>
</tr>
<tr>
<td>$u_{t-1}$</td>
<td>.883</td>
<td>.836</td>
<td>.796</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>.193</td>
<td>.185</td>
<td>.179</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>-.005</td>
<td>-.005</td>
<td>-.005</td>
</tr>
<tr>
<td>$c_{t-1}$</td>
<td>-.149</td>
<td>-.138</td>
<td>-.128</td>
</tr>
<tr>
<td>$i_{t-1}$</td>
<td>.307</td>
<td>.319</td>
<td>.329</td>
</tr>
</tbody>
</table>

Table C.1: Discretionary derived interest reaction function with (right) and without interest smoothing
Inflation Forecast Responding Rule with benchmark coefficients: $\rho = 0.5$ and $\chi = 2$.

<table>
<thead>
<tr>
<th>Std dev. in %</th>
<th>$\pi^C$</th>
<th>$y$</th>
<th>$y^T$</th>
<th>$y^N$</th>
<th>$SOV$</th>
<th>$i$</th>
<th>$\Delta i$</th>
<th>$e$</th>
<th>$R$</th>
<th>$r$</th>
<th>$L^{S(A)}$</th>
<th>$L^{S(B)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 0$</td>
<td>1.958</td>
<td>1.486</td>
<td>2.330</td>
<td>1.468</td>
<td>1.725</td>
<td>12.045</td>
<td>9.972</td>
<td>5.147</td>
<td>.482</td>
<td>8.682</td>
<td>55.763</td>
<td>56.529</td>
</tr>
<tr>
<td>$h = 1$</td>
<td>2.037</td>
<td>1.542</td>
<td>2.260</td>
<td>1.601</td>
<td>1.789</td>
<td>12.858</td>
<td>11.054</td>
<td>4.837</td>
<td>.453</td>
<td>7.692</td>
<td>67.619</td>
<td>68.441</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>2.412</td>
<td>1.621</td>
<td>2.277</td>
<td>1.668</td>
<td>1.839</td>
<td>11.967</td>
<td>9.617</td>
<td>5.425</td>
<td>.392</td>
<td>6.147</td>
<td>54.685</td>
<td>55.441</td>
</tr>
<tr>
<td>$h = 4$</td>
<td>2.559</td>
<td>1.658</td>
<td>2.255</td>
<td>1.698</td>
<td>1.853</td>
<td>11.721</td>
<td>9.135</td>
<td>5.600</td>
<td>.370</td>
<td>5.862</td>
<td>51.022</td>
<td>51.706</td>
</tr>
<tr>
<td>$h = 8$</td>
<td>2.595</td>
<td>1.673</td>
<td>2.253</td>
<td>1.696</td>
<td>1.851</td>
<td>11.701</td>
<td>9.105</td>
<td>5.539</td>
<td>.363</td>
<td>5.729</td>
<td>50.985</td>
<td>51.611</td>
</tr>
<tr>
<td>$h = 12$</td>
<td>2.568</td>
<td>1.665</td>
<td>2.284</td>
<td>1.671</td>
<td>1.843</td>
<td>11.624</td>
<td>9.082</td>
<td>5.512</td>
<td>.368</td>
<td>5.787</td>
<td>50.610</td>
<td>51.236</td>
</tr>
<tr>
<td>$h = 16$</td>
<td>2.557</td>
<td>1.661</td>
<td>2.290</td>
<td>1.666</td>
<td>1.842</td>
<td>11.617</td>
<td>9.085</td>
<td>5.508</td>
<td>.369</td>
<td>5.814</td>
<td>50.593</td>
<td>51.226</td>
</tr>
<tr>
<td>$h = 20$</td>
<td>2.557</td>
<td>1.661</td>
<td>2.289</td>
<td>1.667</td>
<td>1.842</td>
<td>11.621</td>
<td>9.091</td>
<td>5.510</td>
<td>.369</td>
<td>5.815</td>
<td>50.618</td>
<td>51.253</td>
</tr>
</tbody>
</table>

Table C.2: Standard deviation under an inflation forecast responding rule

Optimized inflation forecast responding rules, $\rho = 0.5$

<table>
<thead>
<tr>
<th>Std dev. in %</th>
<th>$\chi^*$</th>
<th>$\pi^C$</th>
<th>$y$</th>
<th>$y^T$</th>
<th>$y^N$</th>
<th>$SOV$</th>
<th>$i$</th>
<th>$\Delta i$</th>
<th>$e$</th>
<th>$L^{S(A)}$</th>
<th>$L^{S(B)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 4$</td>
<td>-5.17</td>
<td>2.237</td>
<td>1.558</td>
<td>2.655</td>
<td>1.410</td>
<td>1.804</td>
<td>9.047</td>
<td>5.103</td>
<td>48.351</td>
<td>49.178</td>
<td></td>
</tr>
<tr>
<td>$h = 8$</td>
<td>-17.04</td>
<td>2.297</td>
<td>1.575</td>
<td>2.451</td>
<td>1.517</td>
<td>1.796</td>
<td>9.150</td>
<td>5.357</td>
<td>49.621</td>
<td>50.366</td>
<td></td>
</tr>
<tr>
<td>$h = 12$</td>
<td>7.80</td>
<td>2.602</td>
<td>1.676</td>
<td>2.271</td>
<td>1.681</td>
<td>1.846</td>
<td>9.056</td>
<td>5.518</td>
<td>50.587</td>
<td>51.186</td>
<td></td>
</tr>
<tr>
<td>$h = 16$</td>
<td>225.79</td>
<td>2.264</td>
<td>1.568</td>
<td>2.544</td>
<td>1.396</td>
<td>1.755</td>
<td>8.794</td>
<td>5.208</td>
<td>46.256</td>
<td>46.877</td>
<td></td>
</tr>
</tbody>
</table>

Table C.3: Optimal response to the inflation forecast and variability of key variables
Constant-interest-rate inflation forecast targeting; \( \pi_{t+h|t(t)} = 0 \).

<table>
<thead>
<tr>
<th>Std dev. in %</th>
<th>( \pi^C )</th>
<th>( y )</th>
<th>( y^T )</th>
<th>( y^N )</th>
<th>SOV</th>
<th>( i )</th>
<th>( \Delta i )</th>
<th>( e )</th>
<th>( R )</th>
<th>( r )</th>
<th>( L^{S(A)} )</th>
<th>( L^{S(B)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 2 )</td>
<td>unstable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h = 3 )</td>
<td>.96</td>
<td>1.39</td>
<td>2.26</td>
<td>1.79</td>
<td>1.92</td>
<td>21.11</td>
<td>24.00</td>
<td>3.91</td>
<td>.70</td>
<td>18.79</td>
<td>290.88</td>
<td>292.64</td>
</tr>
<tr>
<td>( h = 4 )</td>
<td>3.75</td>
<td>2.05</td>
<td>2.31</td>
<td>2.84</td>
<td>2.71</td>
<td>16.94</td>
<td>15.22</td>
<td>6.37</td>
<td>.24</td>
<td>4.62</td>
<td>134.13</td>
<td>137.31</td>
</tr>
<tr>
<td>( h = 5 )</td>
<td>3.89</td>
<td>2.07</td>
<td>5.57</td>
<td>1.31</td>
<td>3.01</td>
<td>1.28</td>
<td>1.35</td>
<td>7.39</td>
<td>.24</td>
<td>13.14</td>
<td>20.32</td>
<td>25.10</td>
</tr>
<tr>
<td>( h = 6 )</td>
<td>3.75</td>
<td>2.02</td>
<td>5.35</td>
<td>1.31</td>
<td>2.90</td>
<td>.14</td>
<td>.15</td>
<td>7.26</td>
<td>.25</td>
<td>11.83</td>
<td>18.13</td>
<td>22.48</td>
</tr>
<tr>
<td>( h = 7 )</td>
<td>3.75</td>
<td>2.02</td>
<td>5.33</td>
<td>1.31</td>
<td>2.90</td>
<td>.008</td>
<td>.01</td>
<td>7.26</td>
<td>.25</td>
<td>11.73</td>
<td>18.13</td>
<td>22.42</td>
</tr>
<tr>
<td>( h = 8 )</td>
<td>3.75</td>
<td>2.02</td>
<td>5.33</td>
<td>1.31</td>
<td>2.90</td>
<td>.002</td>
<td>.001</td>
<td>7.26</td>
<td>.25</td>
<td>11.73</td>
<td>18.13</td>
<td>22.43</td>
</tr>
<tr>
<td>( h = 16 )</td>
<td>3.75</td>
<td>2.02</td>
<td>5.33</td>
<td>1.31</td>
<td>2.90</td>
<td>.000</td>
<td>.000</td>
<td>7.26</td>
<td>.25</td>
<td>11.73</td>
<td>18.13</td>
<td>22.44</td>
</tr>
</tbody>
</table>

Table C.4: Standard deviation under constant-interest-rates inflation forecast targeting

<table>
<thead>
<tr>
<th>Taylor Rules</th>
<th>Standard deviations % (quarterly)</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y^T )</td>
<td>( y^N )</td>
</tr>
<tr>
<td>Dynamic rule</td>
<td>3.864</td>
<td>.966</td>
</tr>
<tr>
<td>Optimized rule</td>
<td>4.947</td>
<td>1.009</td>
</tr>
<tr>
<td>Optimized dynamic rule</td>
<td>5.038</td>
<td>.955</td>
</tr>
</tbody>
</table>

Table C.5: Results from simulation of Taylor rules

<table>
<thead>
<tr>
<th>Ball MCI rule</th>
<th>Standard deviations % (quarterly)</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y^T )</td>
<td>( y^N )</td>
</tr>
<tr>
<td>Benchmark rule: ( b_\pi = 1.5 ); ( b_y = .5 ); ( b_{\epsilon \pi} = .28 ); ( b_{\epsilon \epsilon} = .03 )</td>
<td>3.415</td>
<td>1.110</td>
</tr>
<tr>
<td>Optimized rule: ( b_\pi^* = .23 ); ( b_y^* = -1.78 ); ( b_{\epsilon \pi}^* = .19 ); ( b_{\epsilon \epsilon}^* = .09 )</td>
<td>4.989</td>
<td>1.103</td>
</tr>
</tbody>
</table>

Table C.6: Results from simulation of Ball rule
Figure C.1: Impulse responses strict targeting discretionary optimization with interest smoothing
Figure C.2: Impulse responses flexible ($C_B = 1$) targeting discretionary optimization with interest smoothing.
Figure C.3: Impulse responses policy consistent inflation forecasting rules, forecast horizon $= 14$ qrt.
Figure C.4: Impulse responses constant-interest-rate inflation forecast targeting, forecast horizon = 8 qrt.
Figure C.5: Impulse response standard current information Taylor rule.
Figure C.6: Impulse responses standard Ball MCI rule.
References


