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**Working Paper**

## Private health care as a supplement to a public health system with waiting time for treatment

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# MEMORANDUM

No 41/2000

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*By  
Michael Hoel and Erik Magnus Sæther*

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November 22, 2000

## **Private health care as a supplement to a public health system with waiting time for treatment**

By Michael Hoel\* and Erik Magnus Sæther\*\*

### **Abstract**

We consider an economy where most of the health care is publicly provided, and where there is waiting time for several types of treatments. Private health care without waiting time is an option for the patients in the public health queue. We show the effects of a tax (positive or negative) on private health care, and derive the socially optimal tax/subsidy. Finally, we discuss how the size of the tax might affect the political support for a high quality public health system.

*Key words:*

Private health care, public health care, health queues

*JEL classification numbers:* I111, I118

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## 1. Introduction

In several countries with dominantly public health care, there are often queues for some types of treatments. Patients who enter into such a queue sometimes have the option of using a private alternative to the public health care, thus avoiding the queue (see e.g. Cullis and Jones (1985), Iversen (1997)). However, by doing this they often incur larger costs, as they have to pay for the private treatment (directly or through a private supplementary insurance they previously have purchased), while the treatment in the public system would have been free or almost free.

An important issue in a system with predominantly public health care is how the government should treat alternative private treatment. It has been argued that a private alternative may undermine the public system (we return to this issue in Section 6), so that the government ought to discourage any private alternative. The most drastic form of “discouragement” would be to forbid various types of private treatment. A less drastic form of discouragement would be to impose a tax on private treatment. One could however also argue that those who choose the private alternative should be subsidized by the public health insurance. One argument for such subsidization is that in a public system, everyone has paid his or her mandatory insurance premium. Therefore, everyone should be entitled to compensation if they become ill. In particular, a person choosing the private system should be entitled to whatever it would have cost to treat this person in the public system.

The argument above for subsidizing private health care was based on fairness. However, even disregarding the issue of fairness, one could make an argument for such subsidization. By subsidizing the private alternative, the cost of this alternative will be lowered. Therefore more people will choose this alternative. If the subsidy is sufficiently below the cost of treatment in the public sector, there may be a net cost saving for the public sector. This cost saving could be used to expand the treatment capacity in the public health care system, and thus reduce queues for those who don't choose the private alternative. In other words, even if we give no weight to the interests of those who choose to use the private alternative, it might be sensible to partially subsidize treatment in the private sector. This reason for subsidizing private treatment is briefly discussed in Cullis and Jones (1985).

The present paper presents a very simple model where the arguments above for subsidizing private treatment are incorporated. The case in which we are only concerned with those who choose to stay in the public system comes out as a special case of the model. Moreover, in the model it is endogenously determined whether one ought to tax or subsidize private treatment.

Section 2 presents the basic model, and in Sections 3 and 4 we discuss some of the reasons why there may be queues for some types of treatment in the public health system. In Section 5 we show which parameters determine whether the optimal tax of private treatment is positive or negative. Finally, in Section 6 it is discussed how the introduction of a tax or subsidy might affect the political support for a high quality public health system.

## **2. The cost of waiting and the demand for private treatment**

Consider the simple case in which an exogenously given (and non-stochastic) number of cases requiring medical treatment of a particular type occur each year. Denote this number of cases by  $x$ . Moreover, assume that in the public sector there is a waiting time  $T$  before treatment is performed. Once treatment is given, it is free. The unit cost of treatment is assumed to be constant, denoted by  $q$ , in the public health system.

The private sector gives the same type of treatment, but without any waiting time, at a positive price  $p$ . Obviously, if there were no costs associated with waiting for treatment, everyone would prefer public to private treatment, since the former is free and the latter is not. There are, however, costs associated with waiting for treatment. One such cost could be that the medical condition deteriorates during the waiting time. The cost of this deterioration would either be a more severe treatment once the patient gets it, and/or a worse condition after treatment than the condition would have been after immediate treatment.<sup>1</sup>

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<sup>1</sup> In a study of patients admitted to hospital for elective orthopedic surgery in Norway, Rossvoll et.al. (1993) found that the probability of returning to work after surgery is strongly influenced by the length of time on the waiting list. A high proportion of the patients with a chronic orthopedic disorder were incapacitated for work while waiting.

In most countries a more relevant type of waiting cost is that patients suffer a welfare loss during the waiting period. This welfare loss could be either outright pain or various types of discomfort.<sup>2</sup> For instance, a person waiting for a knee operation would have to abstain from physical activities he/she otherwise would have undertaken. Another example could be a couple that does not wish to have more children, so that one of the persons wishes to be sterilized. During the waiting phase, the couple either must risk pregnancy or at least one of the persons would have to bear the inconveniences of preventive measures.<sup>3</sup> Additional health care cost may also be invoked in the form of care while waiting or the need for new tests and diagnosis.<sup>4</sup>

Whatever the background for the waiting costs, we shall assume that they are proportional to the waiting time.<sup>5</sup> The cost per unit of waiting time is assumed to vary among the population. We would expect this variation to be correlated to income variations, as a higher income typically will imply a higher willingness to pay to avoid waiting. However, waiting costs are also likely to vary among individuals for other reasons: An active skier or runner is likely to have considerably higher waiting costs for a knee operation than a person with a less active life style.

Denote the waiting cost per unit of waiting time for a particular person by  $\theta$ , so that the total waiting cost for this person is  $\theta T$ . The distribution of waiting costs across the population is given by the distribution function  $F(\theta)$ . The lowest and highest values of  $\theta$  are  $\alpha$  and  $\beta$ , respectively, so that  $F(\alpha)=0$  and  $F(\beta)=1$ .

From the assumptions above, it is straightforward to derive the demand for private treatment. A person will choose private treatment if and only if the waiting cost for public treatment ( $\theta T$ )

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<sup>2</sup> Hamilton et al. (1996) investigated the effect of waiting time for hip fracture surgery in Canada on post-surgery length of stay in hospital and inpatient mortality. They found no evidence of a detrimental impact caused by pre-surgery delay, but that surgery delay may lead to greater pre-surgery inpatient costs and more patient discomfort. Roy and Hunter (1996) studied 97 orthopaedic patients awaiting lower-limb surgery. 90 had pain, 44 significant night pains. Psychological and social problems were common. Only 11 were employed full-time. 68 required help with daily activities and 48 patients walked less than 120 metres in 12 minutes. The study also revealed that the planned procedure was no longer appropriate for 12 of the 97 patients.

<sup>3</sup> Using Norwegian data, Hørding et al. (1982) showed that the rate of abortions among women on waiting lists for sterilization was 3.4 times the rate in the normal population.

<sup>4</sup> Stern & Brown (1994) establish a significant relationship between failure to attend initial appointments and the length of time between referral and appointments in a child and family clinic.

exceeds the price of private treatment ( $p$ ). This gives the demand for private treatment, denoted by  $y$ , as

$$y(p, T) = x(1 - F(\frac{p}{T})) \quad (1)$$

For a sufficiently low price  $p$  everyone will choose private treatment, while if the price  $p$  is sufficiently high, no one will choose private treatment. Formally, it follows from (1) that

$$y(p, T) = x \quad \text{for} \quad p \leq \alpha T \quad (2)$$

$$y(p, T) = 0 \quad \text{for} \quad p \geq \beta T \quad (3)$$

The most interesting case is the when  $\alpha T < p < \beta T$ , implying  $0 < y < x$ . For this case the consumer surplus of those who choose private treatment is the total waiting time saved minus what they have to pay for the private treatment, i.e.

$$v(p, T) = x \int_{p/T}^{\beta} \theta T f(\theta) d\theta - py(p, T) \quad (4)$$

where  $f(\theta)$  is the density function for the distribution of  $\theta$  (i.e.  $f(\theta) \equiv F'(\theta)$ ). In Appendix A it is shown that this may be rewritten as

$$v(p, T) = x \int_p^{\beta T} (1 - F(\frac{i}{T})) di \quad (5)$$

Using (1), it is thus clear that the consumer surplus has the standard property that

$$v_p(p, T) = -y(p, T) \quad (6)$$

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<sup>5</sup> Notice that this assumption implies that the analysis of waiting lists by e.g. Lindsay and Figenbaum (1984) does not apply to the present case, as a crucial assumption in their analysis is that there is a positive fixed cost of joining the waiting list.



### 3. Why is there waiting time for treatment in the public system?

Waiting time in the public health system is often explained by some referral to limited public resources<sup>6</sup>. It is however not quite clear why a system with a queue should cost less than a system without. One obvious explanation is that demand for most types of health services fluctuates over time. If one were to dimension the capacity of the health system such that there never was any waiting time, there would be periods of idle capacity. This would be more costly than a system in which there always was full capacity utilization, and with a waiting time during periods of high demand<sup>7</sup>. However, if this were the only reason for having a waiting time, one would expect the waiting time to fluctuate between something close to zero and to, say, a month or two. The waiting times observed for many types of treatments are however considerably larger. More importantly, although they fluctuate, they are always bounded well away from zero. For instance, there were almost 2 million patients waiting for outpatient services, and more than 1 million patients registered for ordinary (inpatient) or day case admissions in the National Health Service in England in September 1999. Of the latter, 49% had been waiting for 3 months or more, and 26% had been waiting for more than 6 months. Similarly, in Norway the average waiting time for non-prioritised patients varied from about 3 months (outpatients) to about 4 months (day case and inpatients).<sup>8</sup> Clearly, cost savings due to better capacity utilization cannot explain waiting times of this length. Actual waiting times are thus often considerably longer than they need to be in order to achieve high/full capacity utilization.

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<sup>6</sup> The existence of waiting lists for medical care in Canada has been used as an argument against the single-payer option for health care reform in the United States. In a comparative study of the access to care, Mackillop *et al.* (1995) identify how long cancer patients wait for radiotherapy in Canada and the USA. They conclude that patients almost everywhere in Canada wait longer for radiotherapy than they do almost anywhere in the United States. In a related study, Coyte *et al.* (1994) compare waiting times for orthopaedic consultations and knee-replacement surgery. The median waiting time for an initial orthopaedic consultation was two weeks in the United States and four weeks in Canada (Ontario). The median waiting time for knee replacement after the operation had been planned was three weeks in the United States and eight weeks in Canada.

<sup>7</sup> Mobley and Magnussen (1998) present the need for excess capacity to ensure availability in private sector as an explanation of why they found no support to the hypothesis that private American hospitals in a competitive environment are more efficient than Norwegian public hospitals.

<sup>8</sup> See appendix D for a further discussion of waiting lists in England and Norway.

One way costs could be held down through queues is to let the treatment per unit of time be lower than the flow of new cases per unit of time. If all new cases were added to the queue, this would imply steadily increasing waiting times. If queues are caused by a lower flow of treatment than the flow of new cases per unit of time, the queue itself must have an effect on how the flow of cases translates into a flow demand for treatment in the public sector. One possibility is that the queue causes some people to exit from the queue before being treated.

The most drastic form of exit would be that patients die while waiting for treatment. Even though there are surveys confirming such deaths<sup>9</sup>, the longest queues typically are for medical cases that are not life threatening. A more positive possibility is that the illness heals on its own while waiting for treatment. To the extent that this occurs, the patient's cost of the queue is the postponement in recovery. A related possibility is that after experiencing a particular health defect for some time, a patient finds it less unpleasant than they initially find it. If medical treatment (e.g. an operation) has some risk of actually making the condition worse, this may imply that after a period of waiting the patient prefers to exit from the queue and accept the health defect.

Several of the possibilities above probably are relevant explanations of how the existence of a queue might reduce the flow demand for health treatment in the public sector. We shall ignore all of these possibilities in the present paper, and instead focus on what probably is a more important effect of waiting time in the public sector. As mentioned in the Introduction, we assume that there is a private alternative to public treatment for those who are willing to pay. The longer the waiting time, the more people choose the private alternative. The waiting time is thus an equilibrating mechanism making the demand for public treatment equal the supply, which is politically determined. In this Section we discuss health queues within a framework of standard welfare theory. In particular, we wish to see what type of considerations might make waiting time for treatment in the public sector part of a welfare maximizing policy.

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<sup>9</sup> Plump et al. (1999) examined the circumstances of death regarding patients who died in 1994 and 1995 while on waiting list for cardiac surgery in the Netherlands. They found that waiting lists for cardiac surgery engender high risks for the patients involved and approximately 100 deaths per year in this patient group was waiting list related. At least half of the deaths occur within the first six weeks.

Let the price of private treatment in the absence of a tax or subsidy be equal to  $mq$ . We assume that the parameter  $m \geq 1$ , although the sign of  $m-1$  is not obvious. There are at least two reasons why we may expect to find  $m > 1$ . One reason is that the private sector is assumed to have no waiting time, which implies that it must have a lower capacity utilization, since the need for treatment in reality will fluctuate over time. The second reason for  $m > 1$  is that in a health system where the private sector is only a supplement, there is reason to believe that competition will be less than perfect, thus making the equilibrium price exceed the unit cost. On the other hand, the private sector could be more efficient than the public sector. If this were true and the efficiency difference was sufficiently large, this could outweigh the two factors mentioned above, so that the net result was  $m < 1$ . The reason why we nevertheless assume that  $m \geq 1$  is that if  $m < 1$ , the public sector could purchase health services from the private sector instead of producing them. By doing this, the unit cost of publicly provided health services would be brought down to the price of privately produced services, thus making  $m=1$ .<sup>10</sup>

Assume that the public sector taxes or subsidizes treatment in the private sector at a rate  $t$  (i.e.  $t > 0$  is a tax and  $t < 0$  is a subsidy), so that the net price paid by users of the private system is  $p = mq + t$ . The total costs for the public sector related to the medical care under consideration consists of treatment costs plus the costs of subsidizing the private sector, or minus the revenue from taxing the private sector. Denoting the total costs by  $C$  we thus have

$$C = q(x - y(p, T)) - ty(p, T) \quad (7)$$

Nothing is lost by normalizing units so  $x=1$ . With this normalization we may rewrite (7) as

$$C = q - (q + t)y(p, T) \quad (8)$$

Total social costs of the health care under consideration are given by the sum of these public expenditures and the private health costs. These latter costs consist of waiting costs for those who choose to be treated in the public sector plus payment for treatment for those who choose to be

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<sup>10</sup> In spite of this argument, data from Norway indicate that for some types of treatment the price charged by private hospitals is considerably lower than the costs in public hospitals. See Appendix E for a further discussion of the costs

treated in the private sector. Assuming that the public expenditures are financed through distorting taxes, the public expenditures should be given a weight  $\lambda > 1$  reflecting these tax distortions (this weight is often referred to as the shadow cost of public funds). Total social costs of the health care under consideration are thus given by

$$W = \lambda C + py(p, T) + \int_x^{p/T} \theta T f(\theta) d\theta \quad (9)$$

In Appendix A it is shown that this may be rewritten as

$$W = \lambda C - v(p, T) + \int_x^{p/\alpha} \theta T f(\theta) d\theta \quad (10)$$

where  $v(p, T)$  is the consumers' surplus defined by (4).

If the government's objective is to minimize the social cost function given by (9) (or (10)), it is not optimal to have any waiting time. To see this, insert  $p = mq + t$  and (8) into (9) and denote the integral by  $I(T)$ :

$$W = \lambda[q - (q + t)y] + (qm + t)y + I(T) = (1 - y)\lambda q + ymq - (\lambda - 1)ty + I(T) \quad (11)$$

The term  $I(T)$  is non-negative, and equal to zero if either  $T = 0$  or  $T \geq p/\alpha$ . If  $\lambda q < mq$  it is optimal to have  $T = 0$ , implying  $y = 0$ . In this case public provision of health services costs less than private provision, even after the costs of distortionary taxation to cover the public treatment are accounted for. Given this, social costs are lowest when everyone uses the public treatment. Moreover, given this and our assumption that unit costs of public treatment are not affected by the waiting time, having a positive waiting time simply imposes waiting costs (making  $I(T)$  in (11) positive) without giving any benefits.

If  $\lambda q > mq$  the optimal  $T$  is equal to or larger than  $p/\alpha$ , implying  $y=1$ . With such a long waiting time everyone chooses private treatment, where there by assumption is no waiting time. In other words,  $T \geq p/\alpha$  is equivalent to letting treatment of the health care under consideration be fully privatized. The reason why this is optimal if  $\lambda q > mq$  is simply that private treatment is less costly than public treatment in this case.<sup>11</sup>

Given the simple objective of minimizing social costs defined by (9), it is not possible to justify waiting time for publicly provided health services. However, the objective function (9) misses an important point: All people are given the same weight in the social welfare function underlying the social cost function (9). However, if this were the case, there was no need for distortionary taxes. All public revenue could be raised by a fixed tax per person, which is a non-distortionary tax.<sup>12</sup> The reason why this type of non-distortionary tax is not used, is that there is a social concern for equity. An equal tax for all would not be considered satisfactory given the concern for equity. But a concern for equity must mean that different persons are given different weights in the social welfare function. The social cost function (9) should therefore be modified so that different persons must be given different weights. Let weights be normalized so that the weight given to those with the lowest weight is 1. It then must be true that the parameter  $\lambda$  must be larger than one and smaller than the weights given to those with the highest weights. If this were not true, social welfare could be increased by changing everyone's tax with a fixed amount (i.e. a non-distortionary tax change) and compensating the change in revenue by changing the distortionary components of the tax system. If e.g.  $\lambda$  exceeds the weights everyone has in the social welfare function, a tax reform of this type (with an increase in the non-distortionary component of the tax system) will raise social welfare.

If an optimally designed tax system includes distortionary taxes, we have implicitly given different welfare weights to different individuals. Moreover, the parameter  $\lambda$  will in this case lie

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<sup>11</sup> It also follows from (11) that  $W$  in this case is lower the higher the tax rate (since  $\lambda > 1$ ). The reason we get this result is that the demand for private treatment in this simple model remains unchanged as  $t$  increases, as long as  $T \geq p/\alpha$ . In reality, a large increase in the price of taxed private treatment would lead to a reduction in demand. One form of such a demand reduction would be substitution towards untaxed private treatment, for instance treatment abroad.

<sup>12</sup> This is at least true if we ignore costs related to administration and enforcement of the tax.

somewhere between the lowest and highest of these different welfare weights. Given this extension of the simple objective function used above, it may be optimal to have positive waiting time. An example of such a case is given in the next Section.

#### 4 An example where it is optimal to have positive waiting time

Assume that a share  $\sigma$  of the population is “low income” with waiting costs  $\theta=\alpha$  and a share  $1-\sigma$  is “high income” with waiting costs  $\theta=\beta$ . These two groups are given weights  $\omega$  and  $1$ , respectively, in the social welfare function. Let the tax system be optimally designed. This means that social welfare cannot be increased by increasing or reducing a tax component which is equal for all (and thus non-distortionary) and adjusting the distortionary part of the tax system so that total revenue is unchanged. An optimally designed tax system of this type implies that

$$\lambda = \sigma\omega + 1 - \sigma \quad (12)$$

We assume that  $m=1$ , which may be interpreted as the private health sector being competitive and equally efficient as the public sector.

With these assumptions, the social cost function (9) takes the following form<sup>13</sup>:

$$W = \begin{cases} \lambda q + \sigma\omega\alpha T + (1-\sigma)\beta T & \text{for } 0 \leq T < \frac{q+t}{\beta} \\ \lambda[q - (q+t)(1-\sigma)] + (1-\sigma)(q+t) + \sigma\omega\alpha T & \text{for } \frac{q+t}{\beta} \leq T < \frac{q+t}{\alpha} \\ -\lambda t + (\sigma\omega + 1 - \sigma)(q+t) & \text{for } T \geq \frac{q+t}{\alpha} \end{cases} \quad (13)$$

It is clear from (12) and (13) that a public system with  $T=0$  and a fully privatized system (i.e.  $T$  so high that everyone chooses private treatment) give the same social cost in this case. Moreover, provided at least one of the groups chooses public treatment, it follows from (13) that  $W$  must be

minimized for either  $T=0$  or  $T=(q+t)/\beta$ . To see which of these two values of  $T$  gives the lowest value of  $W$ , we rewrite  $W$  for the case  $(q+t)/\beta \leq T < (q+t)/\alpha$  as (after inserting  $T=(q+t)/\beta$ )

$$W = \lambda q - \left[ (1-\sigma)\lambda - (1-\sigma) - \sigma\omega \frac{\alpha}{\beta} \right] (q+t) \quad (14)$$

We see from (12) and (14) that provided  $\alpha/\beta$  is sufficiently low, the term in brackets is positive. For  $q+t > 0$  the value of  $W$  is therefore in this case lower than  $\lambda q$ , which is the value of  $W$  when  $T=0$ .<sup>14</sup> In other words, if the difference in waiting costs between the two groups of the population is sufficiently large, it is optimal to have a waiting time that is just high enough to induce the high-income group to choose private treatment<sup>15</sup>, thus benefiting the persons with low waiting costs through the implied reduction of public health expenditures.

## 5 The optimal tax or subsidy

In the example in Section 4, it was never optimal to subsidize private treatment. This result is not generally true. In this Section we regard  $T$  as given and show that it may be optimal to subsidize private treatment. One interpretation of the given  $T$  is that it is the optimal waiting time derived from minimizing social costs of the type (9), except that different individuals are given different weight. Alternatively, we could simply take  $T$  as exogenous, and ask whether one should tax or subsidize private treatment, given the exogenous waiting time.

Since  $T$  is given, we may omit the last term in the expression (10) for the social cost. This cost may thus be rewritten as

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<sup>13</sup> For mathematical convenience, it is assumed that if a person is equally well off with private as with public treatment, he/she chooses private treatment.

<sup>14</sup> Notice also that  $W$  is lower the higher a tax  $t$  is (and the higher is  $T=(q+t)/\beta$ ). The reason we get this result is that the demand for private treatment in this simple model remains unchanged as  $t$  increases, as long as  $(q+t)/\beta$  remains constant, see footnote 11.

<sup>15</sup> In the formal analysis this is  $T=q/\beta$ , in practice it could be  $T$  “just above”  $q/\beta$ ; see also footnote 13.

$$V = C - \frac{1}{\lambda}v(mq + t) \quad (15)$$

The parameter  $\lambda$  may be given the same interpretation as before, i.e. as the shadow price of public funds. Alternatively, we could simply interpret  $1/\lambda$  directly as a parameter reflecting how much weight is given to the persons choosing private treatment relative to persons choosing public treatment. The extreme case of  $1/\lambda=0$  corresponds to giving no weight to those who choose private treatment. The opposite extreme,  $1/\lambda=1$ , is the case in which the private income of all citizens is given the same weight as the income of the public sector and the public health expenditures are financed through non-distortionary taxes.

It is useful to introduce the parameter  $\mu$  defined by

$$\mu = \frac{\lambda - 1}{\lambda} \quad (16)$$

which must lie between zero and one.  $\mu=0$  corresponds to the case of no distortionary taxes and equal weight to those choosing private treatment as to those choosing public treatment. The opposite extreme,  $\mu=1$ , corresponds to the case in which those choosing private treatment get no weight in the optimization problem

The objective function is to minimize  $V$  given by (15). Inserting (8) and  $p=mq-s$  into (15) gives

$$V(t) = q - (q+t)y(mq+t, T) - (1-\mu)v(mq+t) \quad (17)$$

This function is discussed in detail in Appendix B. There we make the assumption that  $0 < y(mq, T) < 1$ , i.e. that if the tax rate is zero, some but not all persons will choose private treatment. Given this assumption, we show that the properties of  $V(t)$  imply the following:

(a) It is never optimal to set the tax rate so high that no one chooses private treatment.



- (b) The optimal tax may be positive or negative (i.e. in the latter case it is optimal to subsidize private treatment).
- (c) If  $\mu=0$  it is optimal to subsidize private health care.
- (d) If it is optimal to subsidize private treatment( i.e. if the optimal tax is negative), the subsidy may be so large that everyone chooses private treatment.
- (e) If  $\mu=0$  and  $m=1$  the optimal subsidy is so large that everyone chooses private treatment.

If the function  $V(t)$  was convex, it would be straightforward to give necessary and sufficient conditions for the optimal tax to be positive or negative, and for an optimal subsidy to be so large that everyone chooses private treatment. However, the function  $V(t)$  is generally not convex, as  $y$  generally is not concave in  $p=mq+t$ . This follows from (1), which implies that for  $y$  to be concave the distribution function  $F$  would have to be convex. Convexity of a distribution function for all arguments giving  $0 < F < 1$  is not a particularly realistic assumption.

Differentiating (17) with respect to  $t$  gives

$$V'(t) = -\mu y(mq + t, T) - (q + t)y_p(mq + t, T) \quad (18)$$

It follows from (18) that

$$V'(t) > 0 \text{ if and only if } (q + t)(-y_p(mq + t, T)) > \mu y(mq + t, T) \quad (19)$$

Denote the demand elasticity (measured positively) for private treatment by  $\varepsilon(t)$ , i.e.

$$\varepsilon(t) = (-y_p) \frac{mq + t}{y} \quad (20)$$

Using (20), (19) may be rewritten as

$$V'(t) > 0 \text{ if and only if } \frac{mq + mt}{mq + t} \varepsilon(t) > \mu m \quad (21)$$

Denote the optimal tax by  $t^*$ . A sufficient condition for  $t^* < 0$  is that  $V'(t) > 0$  for all  $t \geq 0$ . Since  $m \geq 1$ , it therefore follows from (21) that

$$\text{A subsidy is optimal if } \varepsilon(t) > \mu m \text{ for all } t \geq 0 \quad (22)$$

Not surprisingly, we see that it is more likely to be optimal to subsidize private treatment the more weight we give to the persons choosing this option, i.e. the lower is  $\mu$ . If e.g.  $\mu = 0$  it is optimal to subsidize private treatment no matter how small the price elasticity for this treatment is (provided it is not zero). If on the other hand  $\mu = 1$ , and e.g.  $m = 1$ , the price elasticity must exceed 1 for it to be optimal to subsidize private treatment.

From (18) we see that  $V'(t) < 0$  if  $q + t \leq 0$  and  $\mu > 0$ . If  $\mu > 0$  it therefore cannot be optimal to have  $q + t \leq 0$ , i.e.

$$-t^* < q \text{ if } \mu > 0 \quad (23)$$

so that if a subsidy is optimal, it must be lower than  $q$  (i.e.  $q + t^* > 0$ ).

If it is optimal to have a subsidy, we cannot rule out the case in which the optimal subsidy is so large that everyone chooses private treatment. A sufficient condition to rule out this somewhat implausible case is that  $V'(t^+) < 0$  evaluated at the highest tax rate (i.e. lowest subsidy) giving  $y = 1$ . This tax rate is given by  $p = mq + t = \alpha T$ , i.e.  $t = \alpha T - mq$ . A sufficient condition for  $y(mq + t^*, T) < 1$  is therefore that  $V'((\alpha T - mq)^+) < 0$ . In Appendix B it is shown that this sufficiency condition may be written as

$$y(mq + t^*) < 1 \quad \text{if} \quad [\alpha T - (m-1)q](-y_p((\alpha T)^+, T)) < \mu \quad (24)$$

This sufficiency condition will certainly hold if either  $m$  is “large” or if  $\alpha=0$  (and  $m \geq 1$ ).

If we have an interior solution, i.e.  $y(mq+t^*, T) < 1$ , the optimal tax must be given by  $V'(t^*)=0$ .

From (18) we see that this gives

$$t^* = \frac{\mu y(mq + t^*, T)}{-y_p(mq + t^*, T)} - q \quad (25)$$

Using (20), this may be rewritten as

$$(\varepsilon(t^*) - \mu)t^* = (\mu m - \varepsilon(t^*))q \quad (26)$$

If  $\varepsilon(t^*) \neq \mu$  (which must be the case if  $m > 1$ , cf. (26)), we may rewrite (26) as

$$\frac{t^*}{q} = -\frac{\varepsilon(t^*) - \mu m}{\varepsilon(t^*) - \mu} \quad (27)$$

Using (22), (23) (24) (26) and (27) we can summarize our results as follows:

Case A)  $\mu = 0$  and  $m = 1$ :  $-t^* = q - \alpha T$

Case B)  $\mu = 0$  and  $m > 1$ :  $-t^* = q$   
or  
 $-t^* = mq - \alpha T$

Case C)  $\mu > 0$  and  $m = 1$ :  $\varepsilon(t^*) = \mu$   
or  
 $-t^* = q - \alpha T$

Case D)  $\mu > 0$  and  $m > 1$ :  $-t^* = \frac{\varepsilon(t^*) - \mu m}{\varepsilon(t^*) - \mu} q < q$

Notice that in case A we always have the corner solution giving a subsidy that is so large that everyone chooses private treatment. This corner solution is possible also in the other three cases, but in these cases it is also possible that the optimal tax makes some but not all persons choose private treatment. In case B the optimal tax is for sure negative, i.e. we have a subsidy. In cases C and D the optimal tax may be either positive or negative.

## **6 The political support for a high quality public health system**

From Section 5 it is clear that there are many cases in which it is optimal for the government to reimburse people for part of their expenditures on private health treatment, even though the public health system provides the same type of treatment. Clearly, such a subsidy will increase the use of private treatment instead of public treatment. Private health care will thus play a more important role when it is subsidized than when it is not.

In several countries there is a considerable opposition to letting private supplementary health care play an important role. Norway can serve as an interesting example, where the private-for-profit health care providers face a prohibitive tax in the form of legal regulation prohibiting new inpatient facilities (some beds were accepted before the law came into practice in 1986).<sup>16</sup> One reason for the opposition to private health care is that the private and sector compete for the same resources (doctors, nurses etc), so that an increased size of the private sector will make it more difficult for the public sector to recruit the personnel it needs. This argument is most valid in the short run, when the supply of different types of health personnel is more or less given. The model used in the present paper cannot shed any light on this argument, as the model used is a long-run model where unit costs are assumed constant both in the private and public sector.

Another complicating factor is the fact that many public surgeons also engage in private practice. Iversen (1997) concludes that “when consultants ration waiting-lists admissions, the waiting time will increase due to the private sector if public sector consultants are permitted to work in the private sector in their spare time”. We will not go into supply side effects in this model, including the issue of supplier-induced demand, the effect that with increased availability of resources,

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<sup>16</sup> However, the policy in Norway is not very consequent: The local governments and the National Insurance scheme are the key purchasers of private (outpatient) services to reduce the public waiting lists. During the last years there have been several initiatives to purchase privately provided services, also for inpatients. The Norwegian National Insurance scheme finances private health care services for employed on sick leave, restricted to those with a prognosis for a rapid return to work. Some counties in addition offer the whole population a choice between a free private or public treatment. There are also municipalities that provide their community with a free private health insurance scheme.

consultants (with asymmetric information) will respond by stimulating demand (Cullis, Jones and Propper, 2000).

Another type of argument is that as the private sector becomes more dominant, fewer people will be concerned with the quality of the public sector (Besley and Gouveia, 1994). According to this type of reasoning, this will in turn reduce the political support for a high quality public sector, implying that the quality of the public sector will gradually decline.

In the simple model used in the present paper, the only quality dimension of the public health system is the length of the waiting time for treatment. To see how subsidization of private treatment may affect the political support for a good public health system, we therefore calculate in what direction different persons would like the waiting time to change. More precisely, we consider a given initial waiting time  $T$ , and calculate the change in welfare different persons get from a small change in  $T$  from its initial value. Some persons would prefer a small reduction in  $T$  to a small increase, others would prefer a small increase. One could argue that the political support for a high quality public sector according to the present model is higher the larger is the group who prefers a small reduction in  $T$  to a small increase.

Assume that that total expenditures of the public health system are shared equally between everyone. From the discussion in Section 3, it is clear that a small change in taxes of the type “equal absolute change for all” does not contradict an optimal design of the tax system, provided the initial tax system is optimally designed.

The total expected costs of the health system for a person of type  $\theta$  consists of two terms. The first term is this person’s contribution to the expenditures of the public system. With the assumptions used in this Section this term is equal to  $C/N$  where  $N$  is the size of the population. The second term is the expected costs of waiting for treatment should the person become ill. The probability of becoming ill is  $x/N$ , and if this event occurs the cost is the lowest of waiting costs ( $=\theta T$ ) and the cost of treatment in the private sector ( $=p(t)=mq+t$ ). Denoting total expected cost for a person of type  $\theta$  by  $B$  we thus have

$$B = \frac{C}{N} + \frac{x}{N} \min[\theta T, p(t)] \quad (28)$$

where  $p(t) \equiv mq + t$ . Since both  $N$  and  $x$  are given, costs per person and costs per medical case are strictly proportional. It is slightly more conveniently to work with the latter cost, which we denote  $b(\theta, T, t) = BN/x$ . Inserting from (8) we thus have (using our normalization  $x=1$ ):

$$b(\theta, T, t) = q - (q+t)y(p(t)) + \min[\theta T, p(t)] \quad (29)$$

We know that  $y=0$  for  $T$  sufficiently small and that  $y=x=1$  for  $T$  sufficiently large. The exact limits for  $T$  are given by (2) and (3), which inserted into (29) give

$$b(\theta, T, t) = q + \theta T \quad \text{for} \quad T \leq \frac{p(t)}{\beta} \quad (30)$$

$$b(\theta, T, t) = -t + p(t) \equiv mq \quad \text{for} \quad T \geq \frac{p(t)}{\alpha} \quad (31)$$

For  $p(t)/\beta < T < p(t)/\alpha$  it follows from (1) that

$$b(\theta, T, t) = -t + (q+t)F\left(\frac{p(t)}{T}\right) + \min[\theta T, p(t)] \quad (32)$$

Differentiating this expression with respect to  $T$  gives

$$b_T = -(q+t)F' \cdot \frac{p(t)}{T^2} + \theta \quad \text{for} \quad T < \frac{p(t)}{\theta} \quad (33)$$

$$b_T = -(q+t)F' \cdot \frac{p(t)}{T^2} \quad \text{for} \quad T > \frac{p(t)}{\theta} \quad (34)$$

All persons with a value of  $\theta$  implying  $b_T(\theta, T, t) > 0$  will prefer a reduction in  $T$  to an increase. We assume that the initial  $T$  is such that some but not all persons choose private treatment even in the absence of a subsidy, i.e. that  $mq/\beta < T < mq/\alpha$ . From (33) and (34) it then follows that

$$b_T > 0 \quad \text{for} \quad (q+t)F' \cdot \frac{p(t)}{T^2} < \theta < \frac{p(t)}{T} \quad (35)$$

For an arbitrarily given value of  $T$ , it is not obvious that there exist any values of  $\theta$  giving  $b_T > 0$ . However, assume this is the case. Denote the share of the population that has  $\theta$ -values in the range given by (35) by  $R$ . The size of this share is

$$R = F\left(\frac{mq+t}{T}\right) - F\left(\frac{(q+t)(mq+t)}{T^2} F'\left(\frac{mq+t}{T}\right)\right) \quad (36)$$

Let us simplify the discussion by assuming  $m=1$ . For this case we can rewrite (36) as

$$R(z) = F(z) - F(z^2 F'(z)) \quad (37)$$

where we have defined the variable

$$z \equiv \frac{q+t}{T} \quad (38)$$

The size of the variable  $z$  is thus determined by the exogenous value of  $q$  and the policy choices  $T$  and  $t$ .

It immediately follows from (37) that

$$R(z) > 0 \quad \text{iff} \quad zF'(z) < 1 \quad (39)$$



We have already assumed that the sizes of  $T$  and  $t$  are such that some, but not all, persons choose private treatment. This means that we are restricting ourselves to the  $z$ -values satisfying  $\alpha < z < \beta$ . It is not obvious that there are any  $z$ -values within this range satisfying the inequality in (39). If there are no such  $z$ -values, that means that whatever value  $T$  has, everyone will prefer a small increase in  $T$  to a small reduction.

Assume now that there exist values of  $z$  satisfying the inequality in (39) as well as the condition  $\alpha < z < \beta$ , i.e. giving a positive value of  $R$ . We want to see how the size of  $R$  is affected by the introduction of a subsidy, i.e. a reduction in  $t$  and thus in  $z$  (from (38)). Such a reduction in  $z$  will increase  $R$  if and only if  $R' < 0$ . From

(37) we obtain

$$R'(z) = F'(z) - F'(z^2 F'(z))(2zF'(z) + z^2 F''(z)) \quad (40)$$

Without any further assumption about the distribution function  $F$ , we know nothing about the sign or size of  $F''$ . In the general case it is therefore not possible to sign  $R'$ . Nevertheless, from (38) we have the following result: If an increase in  $T$  (which reduces  $z$ ) increases the share of the population who prefer a small reduction in  $T$  to a small increase, then a reduction in  $t$  (which also reduces  $z$ ) will also increase the share of the population who prefer a small reduction in  $T$  to a small increase.

We thus have the following rather weak conclusion: Assume that the private health sector is competitive and equally efficient as the public sector ( $m=1$ ) and that it is neither taxed nor subsidized. Consider an initial length of the waiting time for public treatment and subsidy for private treatment that gives some political support to a reduction in the waiting time ( $R > 0$ ). If one introduces a subsidy to private treatment, the political support for reducing the waiting time for public treatment may go up or down depending on what the initial waiting time is. If an increase in waiting time increases the political support for reducing the waiting time, then an introduction of a subsidy for private treatment will also increase the political support for reduced waiting time.

In Appendix C examples of specifications of the distribution function  $F$  are analyzed. From these examples we can draw the following conclusions:

- There exist distribution functions implying that  $R'(z) > 0$  whenever  $R(z) > 0$ . For these cases an increase in the subsidization of private health treatment will always reduce the political support for reducing the waiting time for public health treatment.
- There exist distribution functions implying that  $R'(z) < 0$  whenever  $R(z) > 0$ . For these cases an increase in the subsidization of private health treatment will always increase the political support for reducing the waiting time for public health treatment.
- There exist distribution functions implying that the sign of  $R'(z) < 0$  depends on the initial waiting time (even when one restricts oneself to waiting times implying  $R(z) > 0$ ). For these cases an increase in the subsidization of private health treatment will increase or reduce the political support for reducing the waiting time for public health treatment, depending on what the initial waiting time is.

In the discussion above, the initial value of  $T$  was arbitrarily given. One possible way to endogenize  $T$  would be to let the value of  $T$  be determined so that exactly half of the population preferred an increase in  $T$  to a reduction, the remaining half preferring a reduction. Formally, let  $T$  be determined by the a value  $z^*$  of  $z$  satisfying  $R(z^*) = 0.5$  and  $R'(z^*) < 0$ . For an exogenous value of  $t$ ,  $T$  thus follows from  $z^*$ , see (38). This value of  $T$  is locally stable: A small reduction in  $T$ , making  $z > z^*$ , will make  $R(z) < 0.5$ , so that a majority of the population would like  $T$  to be increased again. It is clear from the discussion above that it will not be possible to find such a  $z^*$  for an arbitrary distribution function  $F$ . However, it is shown in Appendix C that there exist distribution functions having such a  $z^*$ . For such a given  $z^*$ , any increase in the subsidy of the private sector will imply a reduction in  $T$  so that  $z$  is left unchanged equal to  $z^*$ . For such cases increased subsidization of private health care can therefore be said to increase the political support for high quality public health care.

The discussion in this Section could not give any decisive conclusion about how the introduction of a subsidy for private health care affects political support for high quality public health care.

However, it is certainly not obvious that the introduction of a subsidy will weaken such political support.

## Appendix A: Consumer surplus and social costs

Integrating by parts, we have

$$\int \theta T f(\theta) d\theta = \theta T F(\theta) - T \int F(\theta) d\theta \quad (\text{A1})$$

so that the first term in (4) may be written as (with the normalization  $x=1$ )

$$\begin{aligned} \int_{p/T}^{\beta} \theta T f(\theta) d\theta &= \beta T - \frac{p}{T} T F\left(\frac{p}{T}\right) - T \int_{p/T}^{\beta} F(\theta) d\theta \\ &= \beta T - p F\left(\frac{p}{T}\right) - T \int_p^{\beta T} \frac{1}{T} F\left(\frac{i}{T}\right) di \end{aligned} \quad (\text{A2})$$

Inserting this expression as well as (1) into and rearranging gives

$$v(p, T) = \beta T - p - \int_p^{\beta T} F\left(\frac{i}{T}\right) di = \int_p^{\beta T} \left(1 - F\left(\frac{i}{T}\right)\right) di \quad (\text{A3})$$

Which is identical to (5) when  $x=1$ .

The expression (9) may be rewritten as

$$W = \lambda C + p y(p, T) - v(p, T) + v(p, T) + \int_x^{p/T} \theta T f(\theta) d\theta \quad (\text{A4})$$

Inserting  $v$  from (4) gives

$$W = \lambda C - v(p, T) + \int_{p/T}^{\beta} \theta T f(\theta) d\theta + \int_x^{p/T} \theta T f(\theta) d\theta \quad (\text{A5})$$

which is equal to (10).

**Appendix B: a sufficient condition for the optimal tax implying positive treatment in both the public and private sector**

With the normalization  $x=1$  we may rewrite (1) and (2) as

$$\begin{aligned} y &= 1 \quad \text{for } p \leq T\alpha \\ y &= 0 \quad \text{for } p \geq T\beta \end{aligned} \tag{B1}$$

Using  $p=mq+t$ , this may be rewritten as

$$\begin{aligned} y &= 1 \quad \text{for } t \leq T\alpha - mq \\ y &= 0 \quad \text{for } t \geq T\beta - mq \end{aligned} \tag{B2}$$

Together with (17) this yields

$$V(t) = -t - (1 - \mu)v(mq + t, T) \quad \text{for } t \leq T\alpha - mq \tag{B3}$$

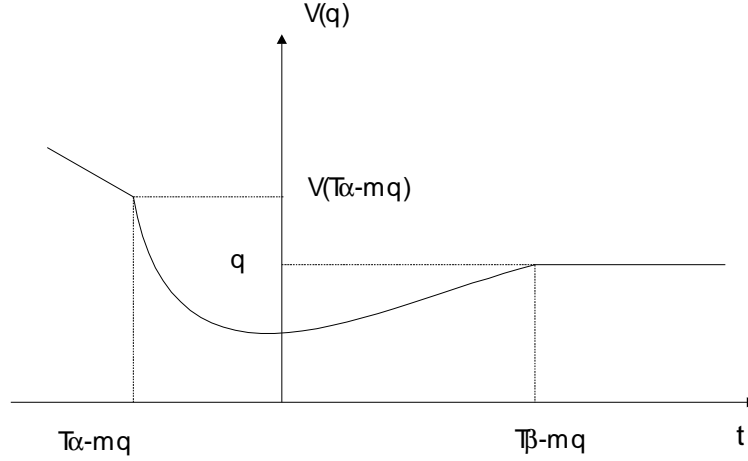
giving

$$V(T\alpha - mq) = mq - T\alpha - (1 - \mu)v(T\alpha, T) \tag{B4}$$

Similarly, we find

$$V(t) = q \quad \text{for } t \geq T\beta - mq \tag{B5}$$

A sketch of the curve for  $V(t)$  is drawn in Figure 1. It is assumed that without a subsidy or tax, some but not all persons will choose private treatment. This is equivalent to assuming that  $T\alpha - mq < 0 < T\beta - mq$ .



**Figure 1**

It is not obvious that the optimal  $t$ , denoted  $t^*$ , satisfies  $T\alpha - mq < t^* < T\beta - mq$ . However, this must be the case if  $V$  is declining immediately to the right of  $T\alpha - mq$  and rising immediately to the left of  $T\beta - mq$ , as in the figure.

To find the relevant one-sided derivatives we first differentiate (17):

$$V'(t) = -\mu y(mq + t, T) - (q + t) y_p(mq + t, T) \quad (\text{B6})$$

From (B6) it follows that

$$V'((T\beta - mq)^-) = -(q + T\beta - mq) y_p((T\beta)^-, T) \quad (\text{B7})$$

$$V'((T\alpha - mq)^+) = -\mu - (q + T\alpha - mq) y_p((T\alpha)^+, T) \quad (\text{B8})$$

Since  $q$  and  $T\beta - mq$  are positive and  $y_p$  is negative, it follows from (B7) that  $V(t)$  is rising with  $t$  immediately to the left of  $T\beta - mq$ . The optimal tax can therefore not be so high that it makes  $y=0$ , i.e.  $t^* < T\beta - mq$ .

A sufficient condition for the  $t^* > T\alpha - mq$  is that  $V(t)$  is declining immediately to the right of  $T\alpha - mq$ . This will be the case if  $V'((T\alpha - mq)^+) < 0$ .

It follows from (B8) that

$$V'((T\alpha - mq)^+) < 0 \quad \text{iff} \quad -y_p((T\alpha)^+, T)[\alpha T - (m-1)q] < \mu \quad (\text{B9})$$

**Appendix C: Examples of distribution functions giving different values of R' in Section 6**

Consider first the following specification of the distribution function:

$$F(\theta) = \frac{\theta - \alpha}{\beta - \alpha} \tag{C1}$$

with

$$F'(\theta) = \frac{1}{\beta - \alpha} \tag{C2}$$

i.e.  $F''=0$  for all  $\theta$ .

Inserting (c1) and (c2) into (35) gives

$$R(z) = \frac{z - \alpha}{\beta - \alpha} - \frac{\frac{z^2}{\beta - \alpha} - \alpha}{\beta - \alpha} \tag{C3}$$

which may be rewritten as

$$R(z) = \frac{(\beta - \alpha)z - z^2}{(\beta - \alpha)^2} \tag{C4}$$

From this equation we see the condition (39) in the present example becomes

$$R(z) > 0 \quad \text{iff} \quad z < \beta - \alpha \tag{C5}$$

Notice that the inequality above can only be consistent with the condition  $\alpha < z < \beta$  if  $\beta > 2\alpha$ . If  $\beta \leq 2\alpha$  no one will prefer a reduction in T to an increase, no matter what the initial size of T is (as long as T is so large that some persons choose private treatment).



Differentiation of (C4) gives

$$R'(z) = \frac{\beta - \alpha - 2z}{(\beta - \alpha)^2} \quad (C6)$$

which implies that

$$R'(z) < 0 \quad \text{for} \quad z > \frac{\beta - \alpha}{2} \quad (C7)$$

Together with (C4) we therefore have the following condition on the initial values of T and t for there to be some support for reducing T, and for this support to be increasing as the tax of the private sector is reduced:

$$R(z) > 0 \quad \text{and} \quad R'(z) < 0 \quad \text{for} \quad \max\left[\alpha, \frac{\beta - \alpha}{2}\right] < z < \beta - \alpha \quad (C8)$$

It is clear from (C8) that  $R' < 0$  whenever  $R > 0$  if  $\beta < 3\alpha$ . In the opposite case, the sign of  $R'$  will depend on  $z$ , i.e. on T.

To see that there exist distribution functions implying that  $R(z)$  is strictly increasing in  $z$  whenever for all  $z$ -values making  $R(z)$  positive, consider the example

$$F(\theta) = \theta^{1/3} \quad \text{where} \quad 0 \leq \theta \leq 1 \quad (C9)$$

It follows that

$$F'(\theta) = \frac{1}{3}\theta^{-2/3} \quad (C10)$$

so that

$$R(z) = z^{1/3} - \left(\frac{1}{3}z^{4/3}\right)^{1/3} \quad (C11)$$

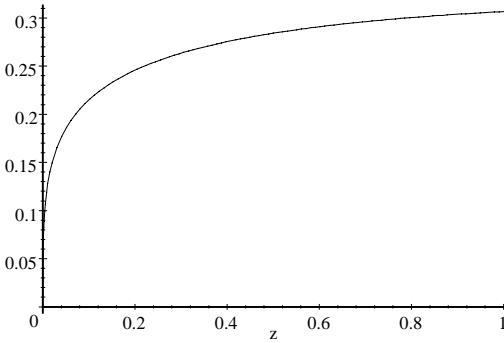
and

$$zF'(z) = \frac{1}{3}z^{\frac{1}{3}} < 1 \quad \forall 0 < z \leq 1,$$

It is thus clear that in this example  $R(z)$  is positive for all  $0 < z \leq 1$ . Differentiating (C11) yields

$$R'(z) = \frac{1}{3}z^{\frac{1}{3}} - \frac{1}{3}z^{\frac{1}{3}} \cdot \frac{4}{9}z^{-\frac{5}{9}} > 0 \quad \forall 0 < z \leq 1. \quad (\text{C12})$$

So we have an example that whenever  $R(z) > 0$ ,  $R'(z) > 0$ . The diagram of  $R(z)$  is shown in figure 2.



**Figure 2**

Finally, we want to give an example of a distribution function that has the property that there exists a value  $z^*$  giving  $r(z^*)=0$  and  $R'(z^*) < 0$ . Consider the distribution function

$$F(\theta) = \begin{cases} 1 - 1/\theta & 1 \leq \theta \leq 5 \\ \theta/25 + 0.6 & 5 \leq \theta \leq 10 \end{cases} \quad (\text{C13})$$

which is shown in figure 3. Then we have

$$F'(\theta) = \begin{cases} -1/\theta^2 & 1 \leq \theta \leq 5 \\ 1/25 & 5 \leq \theta \leq 10 \end{cases} \quad (\text{C14})$$

Implying that

$$R(z) = \begin{cases} 1 - 1/z & 1 \leq \theta \leq 5 \\ z/25 + 25/z^2 - 0.4 & 5 \leq \theta \leq 10 \end{cases} \quad (\text{C15})$$

It is straightforward to verify that  $\max(R(z))=0,8$  at  $z=5$ , and that  $R$  is strictly increasing for  $z < 5$  and strictly declining for  $z > 5$ , as illustrated in figure 4. Furthermore, there is a point  $z^*$  ( $z^* \approx 6.2$ ) where  $R(z^*)=0.5$  and  $R'(z^*) < 0$ .

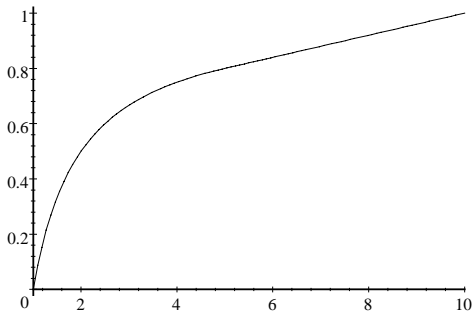


Figure 3

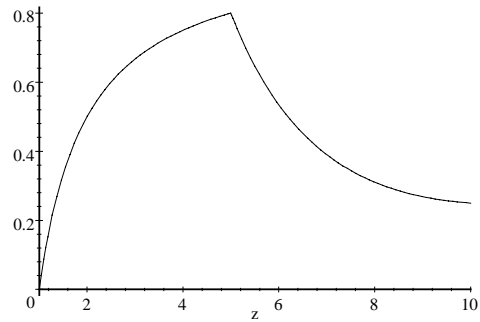


Figure 4

## **Appendix D: Examples of waiting lists, England and Norway.**

A normal allocation to a waiting list in a national health service will be as follows. After referral from the primary care physician, the patient will see a medical specialist working either in a public facility or in private practice. For non-urgent conditions the patient will be listed for inpatient or outpatient operation in public facilities or by private practitioners contracting with the health authorities. The waiting time begins from the date the clinician decided to admit the patient. The reliability of waiting lists has been criticised and they are sometimes referred to as the best misleading source of data on access to care, inaccurately registered and poorly monitored. Still prioritising and waiting lists are the accepted mechanism for allocation of public health care services.

### *Waiting lists in England*

1,084,157 patients were registered waiting for ordinary (inpatient) or day case admissions in the National Health Service (NHS) in England by the end of September 1999. 49% of the patients had been waiting for 3 months or more, 26% for 6 months or more and 5 % for over 12 months. For the 1,907,904 patients with referrals for outpatient services we can see a similar picture as for daycare and inpatient services, as presented in Table 1.

**Table 1, Waiting times, England, Qtr 2: to 30 Sep 1999**

Selected categories**	Outpatients*		Daycare			Ordinary inpatients		
	% of patients seen within		% of patients seen within			% of patients seen within		
	<3 months	< 6 months	<3 months	< 6 months	<12 months	<3 months	< 6 months	<12 months
All specialties	76	94	57	80	97	43	67	93
Trauma and orthopaedics	59	87	49	74	96	34	60	91
Ophthalmology	63	90	44	71	96	38	65	94
Rheumatology	64	97	86	95	99	69	84	97
Ear, Nose and Throat	66	92	58	81	97	41	67	94
Plastic surgery	69	88	56	77	95	38	62	91
Dermatology	71	93	77	91	99	69	86	99
Urology	73	94	67	86	98	56	76	95
Cardiology	75	97	59	82	98	59	82	97
Gastroenterology	75	95	82	95	100	68	87	97
Oral surgery	77	93	59	82	98	49	73	95
General medicine	80	97	77	93	100	74	89	99
Gynaecology	85	98	68	87	98	52	75	96
General surgery	86	97	60	81	97	48	71	94
Paediatrics	90	99	62	82	97	62	76	93
Mental illness	92	99	.	.	.	80	91	99
Cardiothoracic surgery	.	.	83	94	100	35	60	93
Paediatric surgery	.	.	56	79	98	50	73	95

Source: NHS Performance National Guide, UK. NHS Trust based.

\*Note. The outpatient data contains some estimated figures due to incomplete returns from Trust(s).

\*\*Note. See <http://www.doh.gov.uk/waitingtimes/booklist.htm> for information of all categories and number of patients in each group.

In interpreting the figures it should be noted that about half of the patients treated in NHS hospitals are emergency cases and therefore don't come from the waiting lists. One of the categories with a poor record is "Trauma and orthopaedics" where 66% of the inpatients are still waiting for admittance after 3 months of queuing. For those in need of daycare 51% are in the same situation, for outpatient services the percentage is 41%. Orthopaedics is an area with a large private supply of health care services, especially for the less complicated cases like arthroscopic knee surgery.

### *Waiting lists in Norway*

In Norway over 77% of the referred patients are admitted within 3 months. Still, by the end of August 1999, 270 000, or 6% of the total Norwegian population was in a health queue for somatic or psychiatric health services<sup>17</sup>.

**Table 2. Patients waiting for treatment in Norway August 31, 1999.**

	Total on waiting list	Patients with guarantee	Guarantee violations
Somatic	263,955	24,160	5,382
Psychiatric	5,666	1,404	479
Total	269,621	25,564	5,861

Source: Norwegian Patient Register.

From 1987, patients waiting for treatment in Norwegian hospitals are given different degrees of priority, from zero to immediate. As from 1997, prioritised patients, not in need of emergency care, are presented a policy guarantee that they will be treated within three months. At the National level, 20% of all patients accepted for treatment are entitled to such a guarantee. There is a considerable variation in the frequency with which the patients are given this treatment guarantee between hospitals and regions.<sup>18</sup> The violation rate is correlated with the inclusion policy. According to Kristoffersen and Piene (1997) the reason for this discrepancy may be varying composition of the population, different extents of day surgery or different economic strategic thinking. Probably the main reason is that the criteria for giving a waiting list guarantee are not accepted as operational. This leads to different medical judgements when evaluating the applications for treatment at a hospital. In the group of patients with a treatment guarantee 90% are admitted within the guarantee period<sup>19</sup>.

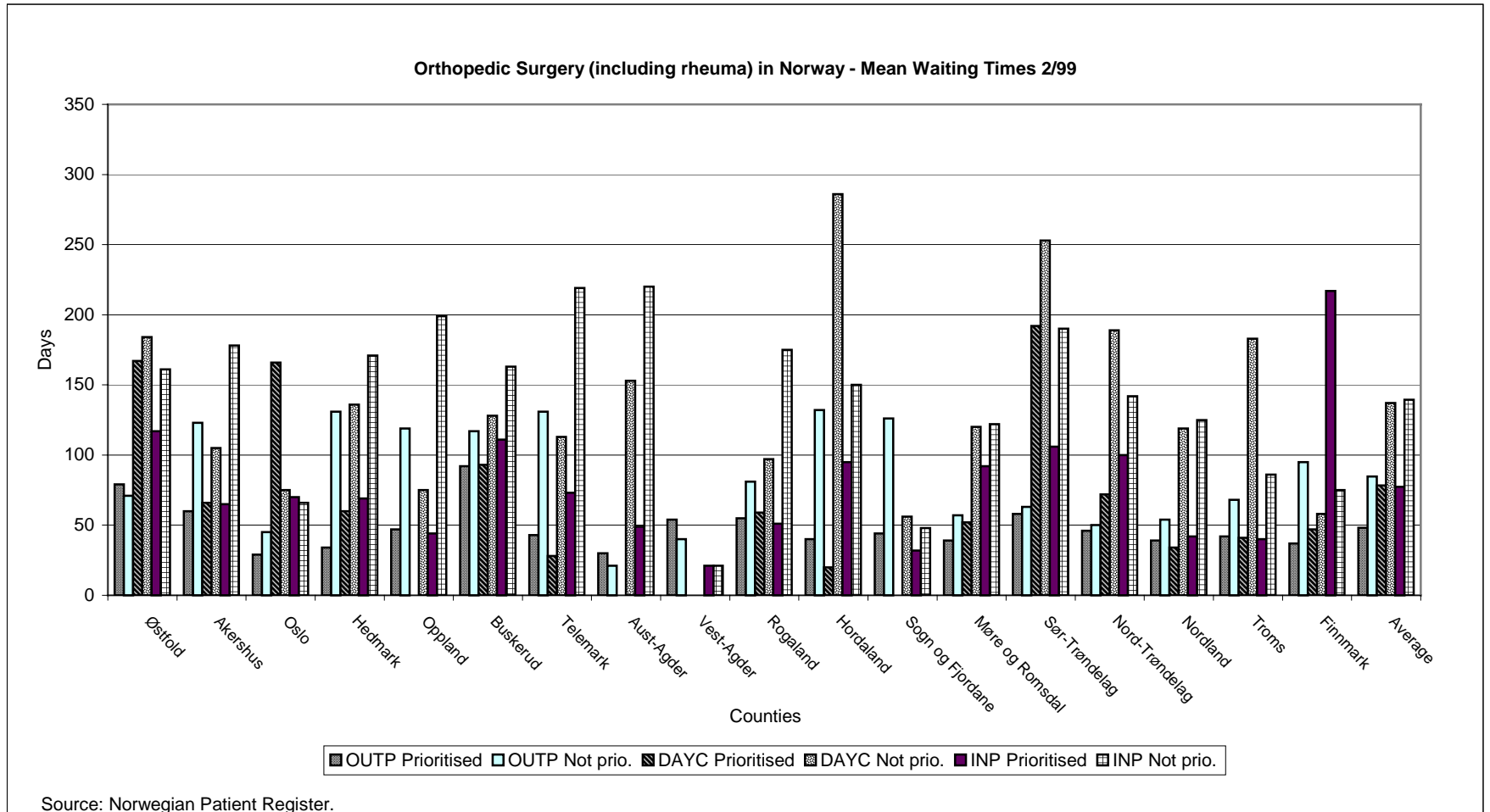
<sup>17</sup> 59 percent of all inpatient care was emergency care in 1998; in 1999 this had increased to 70% (Dagens medisin 2/17/2000).

<sup>18</sup> In the case of gynaecological patients the frequency varies between counties from 1% to 94%, for urological patients from 43% to 100%, for orthopaedic patients from 21% to 89% and for otorhinolaryngological patients from 21% to 89%. Kristoffersen & Piene (1997) (Based on the previous six months guarantee.)

<sup>19</sup> 22% of the violations of the guarantee are in the area of orthopaedic surgery, followed by 12% in urology. The distribution of the violations over care levels is 69% waiting for outpatient treatment, 6% for day care and 25% inpatients.

Figure 5 demonstrates the significant variation between the counties within the area of orthopaedic surgery. In all counties the waiting time is shorter for prioritised patients than the others, but patients willing to travel to other districts may be able to reduce their expected waiting time significantly, e.g. from a mean of 256 days of waiting for non-prioritised day care in Hordaland to a mean of 56 days in the neighbour district Sogn og Fjordane. Private services are primarily demanded by non-prioritised patients, and are of highest supply in the central regions like Oslo and Hordaland (Bergen).

Looking at the development of outpatient waiting times for orthopaedic surgery in Norway from 1996 to 1999 in Figure 6, we notice a 30% increase in the number of referred patients waiting for admittance registered in each four-month period. In spite of a growing number of patients, there is not a similar increase in waiting times. The fluctuations in the data in the beginning of 1996 and the period 2/97 are mainly due to incomplete data registration and change of data routines.

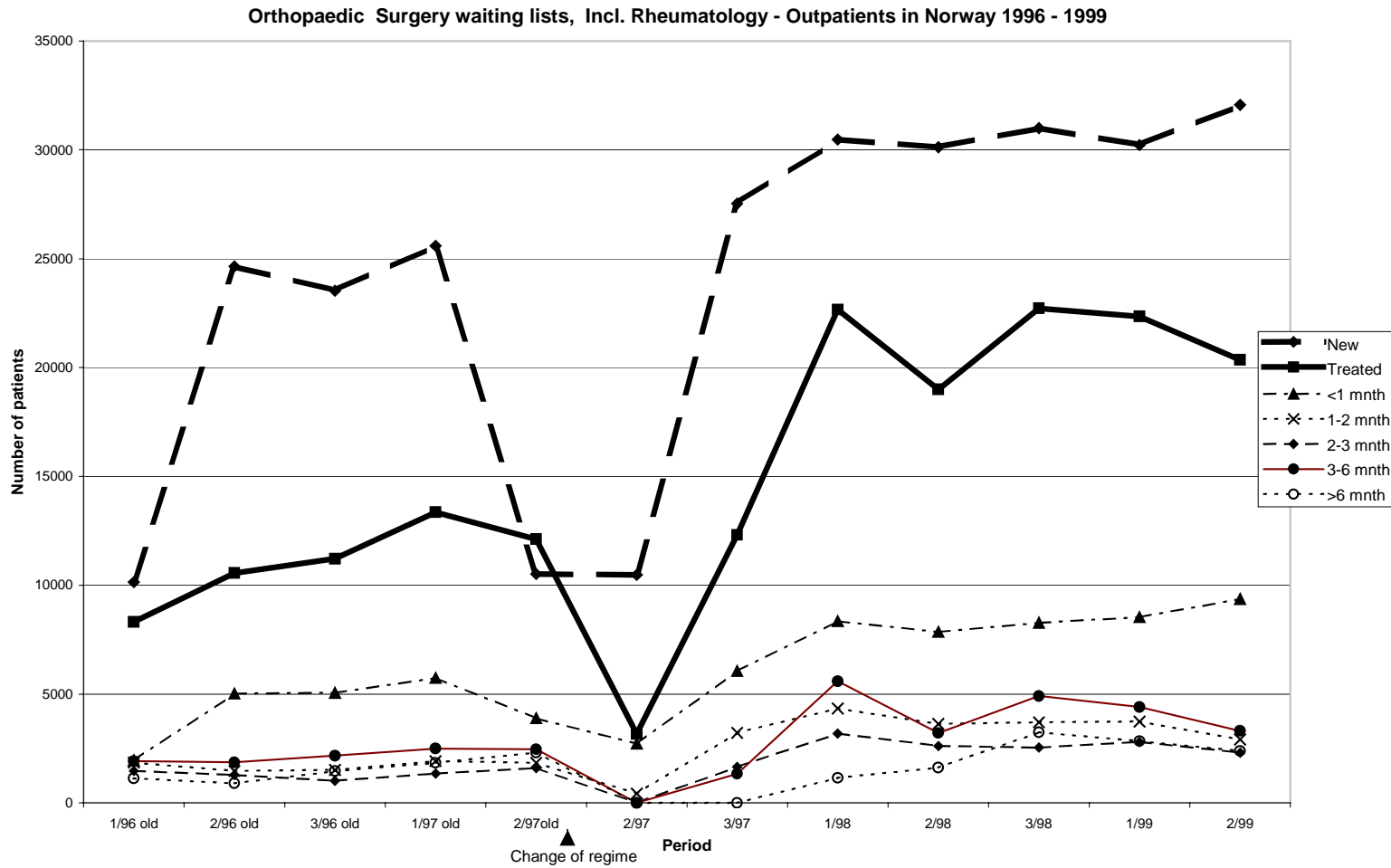


**Figure 5.**



Figure 6.

Source: Norwegian Patient Register.



## **Appendix E: The costs of private and public health services in Norway**

In Norway the health services are mainly financed and provided by the government. Inpatient services are provided free of charge, outpatient services are provided against a minimal fee, often in special wards at the hospitals or contracted out to private specialists. As presented in Appendix D there are waiting lists for almost all non-acute health services that are publicly provided. It is prohibited to supply privately financed inpatient services, with the exemption of some small hospitals accepted prior to 1986. However, there is ample supply of outpatient services for those who have the willingness to pay. Most private health services are paid out of pocket, as private health insurance schemes cover only a minimal share of the population.

In section 3 we assumed that  $m \geq 1$ , i.e. a higher cost in the privately financed care than the public alternative. Looking at some of the cases with longest waiting lists in Norway, we however find  $m < 1$  in almost all of them, based on a comparison of the adjusted average DRG costs in public facilities in Norway with the average prices in privately financed practises in Oslo (Table 3).

**Table 3. A cost comparison of health services provided by public hospitals and privately financed specialists in Norway.**

Category	Case	Waiting time in days DRG		Public Private		Price in private practise/		Public DRG cost code
		Mean		Median		cost	average price	
					NOK	NOK	m	
Gastroenterology	Inguinal Hernia	163	112	162	14 710	12 233	0.83	
	Cholecystectomy	141	91	494	51 486	25 500	0.50	
Rheumatology	Arthritis - knee replacement total	136	83	209	105 518	73 650	0.70	
	Arthritis - knee replacement			209	105 518	56 800	0.54	
	Arthritis - hip replacement total	223	173	209	105 518	70 665	0.67	
	Hybrid hip replacement			209	105 518	76 300	0.72	
	Hip revision			209	105 518	130 000	1.23	
Orthopaedics	Knee meniscal surgery	159	112	232	9 901	9 523	0.96	
	Knee ligament surgery	221	162	222	35 078	25 650	0.73	
General surgery	Disk surgery /prolaps disci	153	100	215	60 821	33 600	0.55	
	Varicose veins	263	208	119	11 598	10 250	0.88	
Urology	Sterilisation, male	209	153	351	7 921	3 700	0.47	
Gynaecology	Sterilisation, female	202	155	362	11 881	10 233	0.86	
	Uterine Prolapse	215	173	359	26 875	32 500	1.21	
	Uterus resection	116	75	358	58 558	32 500	0.56	
Ear, Nose and Throat	Tonsillectomy/adenoidectomy	196	137	59	12 164	10 333	0.85	
Ophthalmology	Cataract	239	201	39	9 052	12 133	1.34	

Sources: Norwegian Ministry of Social Welfare and Health and Norwegian Patient Register. Private prices are from a survey of services provided by physicians and hospitals not included in the public health plan in the Oslo area. (Average price from one to six private providers ).

E.g. a patient with an arthritis related hip problem must wait in average 223 days from he/she is referred by their physician to the surgery is provided. The patients not willing to wait can pay out of pocket or by private insurance and be operated immediately. This price varied in 1999 from NOK 70,000 to NOK 80,000 with the material and type of hip replacement. The estimated cost for this Diagnosis Related Group (DRG) in the public sector was NOK 105,500, making  $m \approx 0.75$ <sup>20</sup>. This DRG however also covered knee replacements, where NOK 73,650 was the private average price, and hip revision, priced to 130.000. Simple comparisons like this should of

<sup>20</sup> Informal estimations of the cost of a normal hip replacement without complications in one of the public hospital wards specializing in elective services is around NOK 60.000

course be used with care, but may indicate that it will be more efficient to purchase private services than provide them in public facilities. On the other hand, cataract surgery with a mean waiting time of 239 days, seem to have a higher price in the private market than the average public cost. One reason may be the existing high share of private contracted provision within ophthalmology. The public DRG cost codes are not adjusted for teaching responsibilities or case mix (increased  $m$ ) or the fact that the capital cost not is part of the DRG cost (reduced  $m$ ).

A comparison of the main cost drivers in private and public hospitals (Bjørnenak and Pettersen, 2000) may indicate that a small complementary private health service may adapt to lower costs than the public. Even though the public sector has a much higher total volume of services, the private sector is allowed to single out larger series of similar services and provide them in a streamlined organisation. The public sector must relate to a higher complexity both with regards to breadth of diagnosis and depth of severity. They must provide emergency care and function as a teaching hospital, whereas the private hospitals can focus on the provision of elective services. The private providers generally have a younger and leaner organisation with a higher efficiency or throughput of patients. Many of the private practitioners also have their main position in a public hospital and get their medical update and time for research there. Analysis of the cost drivers gives reason to expect variation of costs between public and private facilities, and within public facilities with different functioning. With an average hospital cost per corrected stay normalised to 1.00, the cost in Norway varies with the patient mix from a high of 1.16 in the largest teaching hospital with a very high breadth- and depth-complexity to a low of 0.71 in a small hospital focusing on elective services (Samdata, 1998).

Jørgenvåg *et.al.* (2000), compare private practitioners with a public contract and hospital outpatient clinics within the area of treatment costs, patient mix, size and scope of practice. They conclude that private practitioners have an average cost per consultation that is considerably lower than the public. Their analysis of practices in ophthalmology, ear, nose and throat, and internal medicine indicate, as shown in Table 4, a  $m$  of 0.7 – 0.8, or even as low as 0.6 to 0.7 if they estimate corrections for the differences in payment practices over years. The variations in case mix does not explain the cost variation, except for ophthalmology with more resource

demanding patients in public practice. Cost related to facilities and major medical equipment is not included in this analysis.

**Table 4. Cost estimates of publicly purchased health services, provided by public outpatient clinics and private practitioners.**

		Ear, Nose and Throat	Nose Ophthalmology	Dermatologi	Gynaecology	General surgery	Internal medicine
Norway (NOK)	Public outpatient clinic	562	588				747
	Private practitioner	442	482				507
	<i>M</i>	0.79	0.82				0.68
Sweden (SEK)	Public outpatient clinic	501	623	469	568	609	568
	Private practitioner	429	487	388	534	556	523
	<i>M</i>	0.86	0.78	0.83	0.94	0.91	0.92

Data from Jørgenvåg *et.al.* (2000) and (Svalander *et. al.*, 1997)

Aasand *et.al.* (2000) compares costs of (adeno)tonsillectomies in 120 children in Norway. Surgery included in the study was performed on an inpatient as well as an outpatient basis as in public as well as private clinics. The private clinic rented facilities from the public hospital in evenings and weekends. With comparable patient satisfaction the private initiative used 63% of the manpower both for physicians and registered nurses. The private average cost in 1996 was NOK 6130. Hagen and Hatling (1996) compare cost and efficiency in public and private psychiatric practices in Norway. The public outpatient clinics have a professional staff 10 times the size of personnel in private practices. The total number of consultations is however only three times the number of psychiatric patients consulted in private practice. The study indicates a cost per consultation in public practice that is on average 2.5 times the size of the private alternative ( $m \approx 0.4$ ). Differences in case mix and teaching responsibilities explain some of this variation.

A larger Swedish study by Svalander *et. al.*, (1997) compared the costs within six medical specialities between physicians in 40 private practices and 20 public hospital outpatient clinics (Table 4). They conclude that the cost per consultation is by far lower in private practice. With the exception of gynaecology, the private groups are also more cost efficient after compensating for patient mix. The parameter  $m$  varies around 0.8-0.9. Even though the physicians in private practice receive a higher income, there is less use of other personnel, the facilities are more cost efficient, and they pay less for supplies and over-head.

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