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U.S. natural rate dynamics reconsidered

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U.S. natural rate dynamics reconsidered

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U.S. natural rate dynamics reconsidered.*

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Abstract

Several features of the U.S. natural rate of unemployment are reconsidered through specification and testing of econometric models. Traditionally, the choice has been between a wage Phillips curve model, PCM, or an equilibrium correction wage curve model, WECM. The models proposed in this paper feature extended equilibrium correction which reduces the consequences for natural rate dynamics of choosing between wage models. In order for the difference between PCM and WECM to become important, the extended equilibrium correction mechanism must be ‘switched off’ by restrictions. These restrictions are rejected when tested. The analysis supports the original view that natural rates depend on the macroeconomic system, rather than just the wage Phillips curve. The analysis indicates a reduction of the natural rate in the course of the 1990s, due to low worker bargaining power and other structural changes. The estimated reduction is approximately 0.5 – 0.8 percentage points, which is less than existing results based on Phillips curve estimation.

Keywords: US unemployment, natural rate, NAIRU, equilibrium correction, Phillips curve.


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1 Introduction

There is little doubt that the natural rate counts as one of the most successful concepts in the history of macroeconomics. Governments and international organizations customarily refer to natural rate (or NAIRU) calculations in their discussions of employment and inflation prospects, and the existence of a natural rate consistent with a vertical long-run Phillips curve is crucial to the framework of modern monetary policy. In the US in particular, the empirical wage Phillips curve is the operational method for estimation of the natural rate, see Fuhrer (1995), Gordon (1997) and Blanchard and Katz (1999). Thus, the empirical wage Phillips curve is also the basis of the consensus view that the US natural rate of unemployment fell during the last decade of the previous century, see e.g., Blanchard (2005, pp 177-178). In this paper we analyse the methodological basis of the US natural rate from different angles.

In section 2 we present two models which are often contrasted in the way economists think about the natural rate: the standard North-American model of the natural rate with a wage Phillips curve, PCM, and a model with wage equilibrium correction, WECM. We then show that whether the PCM and the WECM really are the polar cases in terms of natural rate dynamics that for example Blanchard and Katz (1999) make them out to be, depends on the specification of other parts of the model. Following Bårdsen and Nymoen (2003), equilibrium correction elsewhere in the system, for example in price setting, implies that the dynamic properties of the PCM and WECM are qualitatively similar, in particular for the rate of unemployment. We dub this system property extended equilibrium correction, since both the PCM and (of course) the WECM are equilibrium correction systems themselves, albeit restricted. In section 3 of the paper, these points are illustrated empirically by dynamic simulation of different econometric models of the US rate of unemployment and its determinants.

The results support the view that extended equilibrium correction is a feature to be reckoned with. We demonstrate that the impact of choosing a Phillips curve equation or an equilibrium correction equation for wages on the natural rate dynamics may be overstated in the standard analysis. Only if the extended equilibrium correction mechanisms are omitted, which statistical tests indicate they should not, does the sharp distinction between the PCM and the WECM come back into full play.

In terms of further economic interpretation, our findings are consistent with a theoretical framework which allows a larger role for aggregate demand than in the standard model of the natural rate. Hence, the results are consistent with the view

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1 In the current literature, the term “Non-Accelerating Inflation Rate of Unemployment”, or NAIRU, is used as a synonym to the “natural rate of unemployment”. Historically, the need for a new term may have arisen because the macroeconomics rhetoric of the natural rate suggested inevitability, which is something of a strait jacket since the long run rate of unemployment is almost certainly conditioned by socio-economic factors, policy and institutions, see e.g., Layard et al. (1991, Chapter 1.3), which is also understood in this paper.

2 See Elmeskov and MacFarlan (1993), Scarpetta (1996) and OECD (1997, Chapter 1) for examples.

3 See the discussion in King (1998) for a central banker’s views.
that persistent demand shocks may affect the rate of unemployment beyond the period of the business cycle, see for example Ball (1997). We do not however subscribe to the view that there is less ‘hysteresis’ in USA than in continental Europe. In fact, ‘hysteresis’, either in its linear form (a unit-root in the rate of unemployment), or in its more genuine non-linear form (multiple equilibria), is not a conceptually meaningful entity within our modelling framework. Instead we interpret the comparative stability of the US natural rate as the joint outcome of demand effects and the flexibility of the US labour market. That said, our empirical model includes proxies for institutional developments and regime shifts, i.e., changes which also standard theory predicts should have an impact on the natural rate. In section 5 we discuss the stability of the natural rate in the period from 1990 to 2004. As noted above, the received view is that the natural rate was significantly reduced in the period. Our results confirm that a reduction may has taken place, but the estimated reduction is smaller than in existing studies. According to the model, unusual low worker bargaining power is one of the explanations for the lower natural rate. Section 6 concludes.

2 The natural rate as a system property

The main variables in this study are the logs of the following variables: wages per hour, denoted $w_t$, a price level variable, $p_t$, labour productivity, $z_t$, and a rate of unemployment, $u_t$. We base our modelling on the following two assumptions about the temporal properties:

A1. Non-stationarity: $w_t$ has a stochastic trend, while $\Delta w_t = w_t - w_{t-1}$ has no trend. Hence $w_t \sim I(1)$, reading integrated of degree 1. Likewise $p_t \sim I(1)$ and $z_t \sim I(1)$ as well.

A2. Cointegration: $w_t - p_t - t z_t - \mu_w \sim I(0)$, with $0 \leq t \leq 1$, and $u_t - \mu_u \sim I(0)$, possibly after removal of shifts in the respective means $\mu_w$ and $\mu_u$.

It is by now commonplace to regard nominal variables like $w_t$ as non-stationary. However, there is still the issue of how to model non-stationarity. The first assumption, A1, is essentially an assumption of local (or variable) trends in wages, prices and productivity variables. Hence, expected growth rate of e.g. productivity is a constant parameter, while the actual growth rate is stochastic. The alternative assumption would be a global or deterministic trend, which is less appealing on the grounds of realism. A variable trend assumption is tantamount to assuming that the variables become stationary after differentiation, and A1 states that the analysis is based on the premise that it is sufficient to difference $w_t$, $p_t$ and $z_t$ once to obtain stationarity.

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4 Røed (1997) instructively draws the distinction between genuine hysteresis as a non-linear and multiple equilibrium phenomenon, and the linear property of a unit root. Moreover, Cross (1995) has show that ‘hysteresis’ is not actually hysteresis (in its true meaning, as a non-linear phenomenon), and that proper hysteresis creates a time path for unemployment which is inconsistent with the natural rate hypothesis.

5 Many shocks cancel in such a vast economy, and disinflation periods have been short, perhaps due to good policy.
Economic theory, at least the kind of theory which is typically used to model US data on wages, inflation and unemployment, implies that there are relationships among the non-stationary integrated data. Hence, given the first assumption of integratedness, economic theory can be used to rationalize hypothesized cointegration relationships. In A2 above, there are two cointegration relationships. The first asserts the stationarity of the productivity corrected real wage. The second proposition in A2, $u_t - \mu_u \sim I(0)$, says that the rate of unemployment is stationary with a constant mean. However, in our interpretation, the mean can be conditional on regime shifts which can be represented by either deterministic variables or by strongly exogenous stochastic forcing variables.

Note that A2 is consistent with finding a ‘wage-curve’ in regressions between the real wage, the rate of unemployment (and productivity), see Blanchflower and Oswald (1994), but also with the converse finding, namely that the slope coefficient of the rate of unemployment is insignificant in such regressions.

Given the assumption that $u_t - \mu_u \sim I(0)$ after removal of structural breaks, there exists a time series model of $u_t$ which is asymptotically stable. The natural rate hypothesis on its part, says that there is only one unemployment rate which can be reconciled with nominal stability of the economy, and that the natural rate equilibrium is asymptotically stable. Hence $\mu_u$ can be interpreted as the mean of the rate of unemployment, in other words, the equilibrium value which the rate of unemployment returns to asymptotically after a shock. This leads to several important questions that can only be answered by modelling the rate of unemployment, and thereby its mean, as a system property. In particular we need to know the economic mechanisms which stabilize the actual unemployment rate around its mean, and what kind of shocks to the system are likely to change the mean.

We first present the two best known models of the natural rate, which we refer to as the Phillips curve model, PCM, and wage equilibrium correction, WECM. The PCM in section 2.1 augments the standard model used to determine the natural rate in US macroeconometric models with a separate equation for the rate of unemployment, so that the model is capable of representing the dynamics of $u_t$ when it departs from its natural rate. In the WECM, in section 2.2, we only change the specification of the wage equation, thus focusing on the importance of the specification of the wage equation, as a Phillips curve or as an equilibrium correction model, for the determination of equilibrium unemployment rate, see Blanchard and Katz (1997). However, in section 2.3, and with reference to Bårdsen and Nymoen (2003), we introduce the idea that other equilibrating behaviour may intervene, and lessen the impact of the specification of wage setting on unemployment dynamics. We dub the resulting framework the extended equilibrium correction model of the natural rate.

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6 Formally, the solution of the linear difference equation of $u_t$ is unique when it has no roots on the unit circle, and the mean of $u_t$ is thus also unique and time independent. Hence the model of (linear) hysteresis of Blanchard and Summers (1986) is inconsistent with our set of modelling assumptions.
2.1 Phillips curve dynamics

To simplify the exposition, we set $\iota = 1$ in $w_t - p_t - \iota z_t - \mu_w \sim I(0)$. Hence we first consider the case where the wage share is stationary. We specify two dynamic systems of equations, with different economic interpretation, which both are consistent with assumptions A1 and A2 above. The first model is the PCM:

1. \[ \Delta w_t = \beta_{w0} - \beta_{w1} u_t + \beta_{w2} \Delta z_t + \beta_{w3} \Delta p_t + \varepsilon_{w,t}, \]
2. \[ u_t = \beta_{u0} + \beta_{u1} u_{t-1} + \beta_{u2} (w - p - z)_{t-1} - \beta_{u3} x_{u,t} + \varepsilon_{u,t}, \]
3. \[ \Delta p_t = \zeta (\Delta w_t - \Delta z_t) + (1 - \zeta) \Delta p_{t-1} + \varepsilon_{p,t}, \]
4. \[ \Delta z_t = g_z + \varepsilon_{z,t}, \]
5. \[ \Delta p_{t-1} = g_{pi} + \varepsilon_{pi,t}. \]

Equation (1) is the wage Phillips curve which is typically found to represent the relationship between aggregate (annual) wage inflation, and unemployment in the United States, see for example Blanchard and Katz (1999). Of course, $\Delta p_t$ is often replaced by expected inflation, i.e., $\Delta p_t^e$ or $\Delta p_{t+1}^e$, which are in turn approximated by, or instrumented by $\Delta p_{t-1}$. The role of inflation in the wage setting process is an important issue in empirical modelling, but the simple ‘simultaneous equations specification’ in (1) is convenient for our purpose. $\Delta z_t$ represents a possible effect of labour productivity on wage growth. $\varepsilon_{w,t}$ is a disturbance term, which without loss of generality can be taken to be normally distributed with a constant standard deviation, and uncorrelated with the other disturbances in the system ($\varepsilon_{u,t}, \varepsilon_{p,t}, \varepsilon_{pi,t}$ and $\varepsilon_{z,t}$).

It remains one of the great appeals of the PCM that the natural rate, the value of $u_t$ in a hypothesized steady-state situation, can be estimated from a single equation.\(^\text{7}\) Hence, estimation of a structural Phillips curve like equation (1) is the dominant strategy for estimation of the natural rate, and Staiger et al. (1997) is an important contribution.\(^\text{8}\) However, the standard approach does not address another important question: whether the estimated natural rate corresponds to an asymptotically stable equilibrium of the rate of unemployment, see Bårdsen et al. (2005, Ch 4.2). In order to investigate the stability issue, the wage Phillips curve model needs to be supplemented with an equation for the rate of unemployment, which is equation (2) above. The “catch-all” variable $x_{u,t}$ represents (a vector) of other factors than wages which affect the rate of unemployment. It might contain conventional demand side variables (foreign demand, changes in the domestic savings rate, and policy instruments), but also shocks that affect the supply of labour at the going real wage (for example demographic changes).

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\(^\text{7}\)As pointed out by Bårdsen et al. (2003), the single equation tradition extends to the literature on the New Keynesian Phillips curve. This may have obscured the fact that the weak identification of the New Keynesian Phillips curve hinges on the properties of a completing system, see Mavroeidis (2005).

\(^\text{8}\)Blanchard and Katz (1997) review the standard model of the natural rate in the following way (p. 60): U.S. macroeconometric models...determine the natural rate through two equations, a “price equation” ...and a “wage equation”. The “wage equation” specified in Blanchard and Katz is identical to our equation (1), albeit without the productivity term, and the “price equation” is the same as (3) but without the productivity and import price terms.
The three last equations of the PCM are even more stylized than the two first—their role is simply to close the system. Equation (3) gives price inflation as determined by the growth rate of domestic unit labour costs and of import prices (in dollars). Equations (4) and (5) specify productivity and import prices as random walks with expected growth rates $g_z$ and $g_{pi}$. Imported price growth $\Delta pi_t$ is in terms of domestic currency and the formulation in (5) is consistent with assuming that there is no pricing-to-market.

The PCM system is consistent with our main assumptions, in particular with cointegration in the form of $w_t - p_t - z_t - \mu_w \sim I(0)$ and $u_t - \mu_u \sim I(0)$. The essential step is to recognise that the PCM, despite the absence of equilibrium correction in the wage equation, is an equilibrium correction system. To see this, use equation (3) to substitute out the $\Delta p_t$ term in (1), giving a ‘semi’ reduced form equation for wage growth:

$$\Delta w_t = b_{w0} - b_{w1}u_t + b_{w2}\Delta z_t + b_{w4}\Delta p_i_t + \varepsilon'_t,$$

where $b_{w1} = \beta_{w1}/(1 - \beta_{w3}\zeta)$, $b_{w2} = (\beta_{w2} - \beta_{w3}\zeta)/(1 - \beta_{w3}\zeta)$ and $b_{w4} = \beta_{w3}(1 - \zeta)/(1 - \beta_{w3}\zeta)$. Next, substitute $u_t$ in (6) by the right hand side of equation (2), and note that the equilibrium correction coefficient of the lagged wage level term in the $\Delta w_t$ equation becomes $-\beta_{w1}\beta_{u2}$. Hence, as long as the PCM system displays both an effect from unemployment on wage growth, $-\beta_{w1} < 0$, and an effect of the wage level on unemployment, $\beta_{u2} > 0$, the dynamics of wages and unemployment are of the equilibrium correction type. Since equilibrium correction implies cointegration, and since cointegration corresponds to dynamic stability, it follows that a sufficient condition for stability is that $-\beta_{w1} < 0$ and $\beta_{u2} > 0$ hold jointly.

Formal dynamic analysis of the system (1)-(5) confirms that, subject to $-\beta_{w1} < 0$ and $\beta_{u2} > 0$ the PCM system has two stable roots and three unit roots. The unit roots represent the $I(1)$-ness of the price level index $p_t$, productivity $z_t$, and the import price index, $pi_t$. Consistent with A2, the two equilibrium values, corresponding to the means of the wage-share and unemployment are given by

$$\mu_{u,PCM} = \frac{\beta_{w0}}{\beta_{w1}} + \frac{(\beta_{w2} - 1)}{\beta_{w1}}g_z + \frac{(\beta_{w3} - 1)}{\beta_{w1}}g_{pi},$$

$$\mu_{w,PCM} = \frac{\beta_{u0}}{\beta_{u2}} + \frac{1 - \beta_{u1}}{\beta_{u2}}\mu_u + \frac{\beta_{u3}}{\beta_{u2}}x_u,$$

where we have added the PCM acronym to the subscript of the two means. The case of the vertical long-run Phillips curve is represented by $\beta_{w3} = 1$, and implies that the PCM natural rate of $\mu_{u,PCM}$ is independent of inflation, i.e., the usual implication of dynamic homogeneity of the wage Phillips curve.

### 2.2 Wage equilibrium correction dynamics

As stated by Blanchard and Katz (1999), a well documented difference between Europe and the US is “the presence of an equilibrium correction term in the European but not in the US wage equation”\(^9\). Nevertheless, the dynamically stable

PCM system displays equilibrium correction. In the PCM, stability hinges on a single equilibrating mechanism, namely that the rate of unemployment is linked to the real wage, as in equation (1). Without this stabilizing mechanism, there can logically be no cointegration between wages, prices and productivity, and the rate of unemployment will not return to its (natural) long-run mean after a shock.

In the wage bargaining models another equilibrating mechanism is brought into the picture: wages are directly influenced by profits, in addition to the indirect channel through the unemployment rate. Accordingly, we replace the wage Phillips curve (1) by the equation

\[ \Delta w_t = \beta_{w0} - \beta_{w1}u_t + \beta_{w2}\Delta z_t + \beta_{w3}\Delta p_t - \theta_{w}(w - p - z)_{t-1} + \varepsilon_{w,t}, \]

where \( \theta_{w} \) is the equilibrium correction coefficient.\(^{10}\) Equilibrium correction models for wages and prices have a long history in econometrics. Sargan (1964, 1980) coined the term, and saw the formulation as an extension of the original Phillips curve. Later, it has been established that there is also a close correspondence between modelling wages in terms of cointegration and equilibrium correction, and a theoretical framework of the wage bargaining type, see Nymoen (1989, 1991) and , and Bårdsen et al. (2005, Ch. 4-6).

Wage equilibrium correction represents an adjustment mechanism which stabilizes wages at any given rate of unemployment. This feature is consistent with the main rationale of wage bargaining models, namely that firms and workers are engaged in a partly cooperative and partly conflictual sharing of the rents generated by the operation of the firms.

The two equilibrating mechanisms supporting the cointegration properties of \( w_t - p_t - z_t - \mu_w \sim I(0) \) and \( u_t - \mu_u \sim I(0) \) mean that the speed of adjustment will be faster in the case of the WECM than in the PCM case. Hence, if the PCM system is dynamically stable, then the WECM system is also stable, \textit{a fortiori}.

Another difference from the PCM is that the natural rate is a genuine system property in WECM—it can no longer be retrieved from the wage equation alone. Solving for the steady-state rate of unemployment gives

\[ \mu_{u,WECM} = \frac{\theta_{w}\{\beta_{w0} + \beta_{w3}x_u\} + \beta_{w2}\{\beta_{w0} + (\beta_{w2} - 1)g_z + (\beta_{w3} - 1)g_{pr}\}}{\theta_{w}(1 - \beta_{u1}) + \beta_{w1}\beta_{u2}}. \]

Note that a permanent change in the exogenous variable \( x_u \) (a shock which does not disappear) has an impact on the equilibrium rate of unemployment in this model, while it does not affect the PCM natural rate.

The role of \( x_u \) in the WECM equilibrium unemployment rate fits the idea that relatively permanent changes in unemployment might be due to structural breaks that occur intermittently, in line with our maintained view of the rate of unemployment as \( I(0) \) but subject to (infrequent) structural breaks. The PCM, while not inconsistent with this view, nevertheless would attribute the mean-shifting capability only to those structural changes which occur distinctly on the supply side.

\(^{10}\)Since it is unlikely to cause misunderstandings, and in order to keep notation at a reasonable level, we keep the same notation as in the PCM case, although, clearly this is a different model for wages.
(through shifts in the Phillips curve intercept $\beta_{u0}$). The difference between the PCM and WECM is thus one of degree, not of principle. The WECM might be said to allow the longer list of candidates for regime shifts—from different sectors of the macroeconomy. For example, if we associate the equations for wage, prices and productivity with the supply side of the macroeconomy and (2) with the demand side, then (10) allows permanent demand shocks to affect the equilibrium rate through $x_u$.

### 2.3 Extended equilibrium correction dynamics

Both the PCM and the WECM are equilibrium correction models of the natural rate. The difference is that the PCM implies a more restrictive stabilization process than the WECM. In the PCM, equilibrium correction takes place in the unemployment equation alone. The WECM has an additional stabilization mechanism in the wage equation itself. Moreover, the PCM is a special case of the WECM, since $\theta_w = 0$ removes equilibrium correction from the wage equation. Thus, $\mu_{u,WECM} = \mu_{u,PCM}$ subject to the restriction $\theta_w = 0$.

In the two models, the price setting equation has been kept deliberately simple, in so-called differenced form. As pointed out by Hendry et al. (1984), an equation in differenced form implies that the variable in question, the price level in our case, is always on its steady-state trajectory. This is unrealistic, goes against theory (e.g., Blanchard (1987)), and is an unnecessary constraint on an empirical model. Modern models of the wage-price inflation spiral instead model both wage and price inflation as influenced by past equilibria, see e.g., Nymoen (1991). We refer to such models as systems with extended equilibrium correction dynamics, i.e., relative to the PCM and WECM models above.

Bårdsen and Nymoen (2003) show that extended equilibrium correction makes the wage-price process become dynamically stable under quite general assumptions of parameter values, and also when the rate of unemployment is exogenous. and fixed11 As a corollary, we now state that when there is equilibrium correction in price setting, the restriction $\theta_w = 0$ no longer guarantees that the dynamic system has the properties of the PCM. Hence, even if there is a wage Phillips curve in the system, the expression for the natural rate of unemployment in (7) may not correspond to the true steady state implied by the extended equilibrium correction system. The reason is the presence of equilibrium correction in price setting, which adds extra stability to the system in a way that affects the mean rate of unemployment. In other words, with extended equilibrium correction, the natural rate is no longer determined by the parameters of the wage Phillips curve.

Extended equilibrium correction dynamics in wage- and price-setting models has a realistic ring to it, and the surveys of the literature in Kolsrud and Nymoen (1998) and Bårdsen and Nymoen (2003) indicate that there may have been an over-emphasis on the differences in wage dynamics. Specifically, even if there is a well defined empirical wage Phillips curve in the US, it does not imply that $\mu_{u,PCM}$ defined by that Phillips curve is a relevant parameter for the US natural rate of unemployment. This is because there may be equilibrium correction elsewhere in

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11See also the analysis of the conditional wage-price system in Bårdsen et al. (2005, Ch. 6.4).
the wage-price systems which dominate the wage Phillips curve, so that the implied mean of the rate of unemployment becomes a system property, more in line with \( \mu_{u,WECM} \) above.

In the next section, where we consider an empirical PCM for the US economy, we will see an instance of how price setting equilibrium correction influences the behaviour of the system—and the rate of unemployment in particular.

3 Inflation-unemployment dynamics in the US

In this section we specify econometric models with different equations for wage dynamics, and investigate their impact on natural rate dynamics. Based on the discussion above, we expect to find that the specification of the wage equation has a large impact on the natural rate and its dynamics if there is little equilibrium correction elsewhere in the completing macroeconomic model. Conversely, with extended equilibrium correction in the system as a whole, the consequences of choosing between a Phillips curve and a wage equilibrium correction model are less important.

3.1 Data and empirical framework

Our operational measure of the rate of unemployment is the civilian unemployment rate, which is the commonly used unemployment variable in wage studies. The operational measure of the wage variable is the hourly manufacturing compensation rate, \( w_t \), and productivity is defined as value added per man hour, and is denoted \( z_t \), see the Appendix for details. In parallel to the theoretical model, we also include an import price index in the data set, it is denoted \( p_t \). The theoretical section abstracted from the difference between ‘consumer’ and ‘producer’ prices. In the empirical model we include both a consumer price index, \( p_t \), and a deflator of manufacturing value added, \( q_t \). This affects the econometric specification of the two hypotheses of wage setting.

The Phillips curve model (PCM). In this case, the hypothesis A2 carries over directly, but notably for the producer real wage. Hence \( w_t - q_t - z_t - \mu_w \sim I(0) \), which is the same as claiming that the wage share is stationary; and \( u_t - \mu_u \sim I(0) \). The implied equilibrium correction dynamics is that \( \Delta w_t \) adjusts with respect to \( u_{t-1} - \mu_u \), and that \( \Delta u_t \) adjusts with respect to \( w_{t-1} - q_{t-1} - z_{t-1} - \mu_w \).

The wage equilibrium model (WECM). Workers’ utility is linked to the consumer real wage, \( w_t - p_t \), while firms care about the producer real wage, \( w_t - q_t \). Theory nevertheless implies that, as long as unions have a strong bargaining power, the settled nominal wage will mainly reflect \( q_t \) and \( z_t \). Empirical tests on data from the Scandinavian small open economies lend support to this view. For the US case, where unions are different in character and bargaining is more fragmented, a better hypothesis may be that the productivity corrected consumer real wage is stationary, hence \( w_t - p_t - tz_t - \mu_w \sim I(0) \), with \( 0 \leq t < 1 \). The idea is that rather weak and uncoordinated unions manage to achieve a degree of compensation for increases in costs of living, but that workers only manage to extract a fraction of the productivity gains. Also in this case \( u_t - \mu_u \sim I(0) \), and in the equilibrium correction model, \( \Delta u_t \) adjusts with respect to \( w_{t-1} - q_{t-1} - z_{t-1} - \mu_w \), but wage setting is different from the PCM case and \( \Delta w_t \) is assumed to adjust with respect to \( w_{t-1} - p_{t-1} - tz_{t-1} - \mu_w \).
The empirical wage Phillips curve is well established on US data, so we start with this model. We first test the two PCM cointegration propositions, i.e., \( w_t - q_t - z_t - \mu_w \sim I(0) \) and \( u_t - \mu_u \sim I(0) \), and as a second step we specify a simultaneous equations model using the PCM equilibrium correcting mechanism as identifying restrictions. The methodology is discussed in detail in Bårdesen et al. (2005, Ch. 4-6).

3.2 A PCM model

Given the assumption of I(1)-ness of wages, prices and productivity, logical consistency of the Phillips curve model of the natural rate requires that wages, producer prices and productivity are cointegrated. In this subsection we first show that cointegration is supported by formal tests, and second estimate a dynamic model which is an extension of the standard model of section 2.1.

3.2.1 Cointegration analysis

To test the null hypotheses of no cointegration, i.e., \( w_t - q_t - z_t - \mu_w \sim I(1) \) and \( u_t - \mu_u \sim I(1) \) we estimate a 2nd order VAR for the three endogenous variables \( w_t \), \( q_t \) and \( u_t \). The test is conditional on productivity, \( z_t \).\(^{12}\) The sample period is 1962-2004. Formally, the Johansen (1995) approach to cointegration analysis suggests one or two cointegrating vectors. Our interpretation of this result is that \( w_t - q_t - z_t - \mu_w \sim I(0) \) holds strongly in the data, while \( u_t - \mu_u \sim I(0) \) is a weaker empirical cointegrating relationship. On this basis the cointegration rank is set to 2, and we proceed to test the 4 implied over-identifying restrictions. The likelihood ratio test statistic is \( \chi^2(4) = 5.75 \), with a p-value of 0.22 showing that the PCM restrictions on the cointegrating vectors are statistically acceptable.

These conclusions are supported by Dickey-Fuller tests. For the rate of unemployment in particular, a 2nd order Dickey-Fuller regression, augmented by three dummies for structural breaks (namely \( ken_t \), \( oil_{1,t} \) and \( pow_t \) which are explained below) is a statistically adequate model for inference, following the principles of Andreou and Spanos (2003). The Dickey-Fuller statistics of this model, calculated sequentially over the period from 1975-2004 are never lower (in absolute value) than 2.5 and the average of the sequence is much higher. The end of sample value is 4.7. Although the exact critical value is unknown in this case (because of the inclusion of dummies), these values of the Dickey-Fuller statistic are highly suggestive that a formal test would reject the unit-root.\(^{13}\) The estimated mean \( \mu_u \) is also stable, using the 1961-1975 sample we obtain \( \exp(\hat{\mu}_u) = 5.3\% \), and using the full sample 1962-2004 the estimated mean of the unemployment rate is \( \exp(\mu_{\hat{u}}) = 5.6\% \). Hence, both the VAR cointegrating analysis and the Dickey-Fuller tests corroborate the validity of the modelling assumptions of \( w_t - q_t - z_t - \mu_w \sim I(0) \) and \( u_t - \mu_u \sim I(0) \) stated in section 2.

\(^{12}\)The weak exogeneity of \( z_t \) may not hold in the data (an example of extended equilibrium correction), and in that case we loose statistical efficiency but this must be balanced against the gain in degrees of freedom.

\(^{13}\)Already, Perron (1989) showed that the inclusion or omission of dummy variables is important for the outcome of unit root tests.
3.2.2 Econometric PCM model

The next stage is to specify and estimate a PCM, with the lagged wage share and the lagged unemployment rate included as equilibrium correction terms. At this point in the analysis, we first estimate an unrestricted equilibrium correction model, which we dub \textit{p-ecm} and second we attempt to encompass the unrestricted system by an econometric model which has a wage Phillips curve as its core. This empirical model is an extension of the standard model in equation (1)-(5) above. The notable extensions of the theoretical model are:

1. **Two price indices**: Since there are two domestic price indices, the econometric PCM contains equations for both $\Delta q_t$ (producer prices) and $\Delta p_t$ (consumption price index).

2. **Extended equilibrium correction**: The cointegration analysis of the system made up of $w_t$, $q_t$ and $u_t$ shows evidence of equilibrium correction of $q_t$ with respect to the wage-share, so we expect to find such a relationship also in the simultaneous equations model. Moreover, since $p_t$ was not included in the VAR, equilibrium correction behaviour in $\Delta p_t$ may yet be revealed at this stage. Hence, the relationship in differences (3) in the standard PCM, is replaced by two equilibrium correction equations for $\Delta q_t$ and $\Delta p_t$. Finally, equilibrium correction may affect $\Delta z_t$ as well, in which case equation (4) in the standard model is replaced by an equilibrium correction equation.

3. **Structural breaks**: Variables representing shocks and intermittent structural breaks are included. First, we include a dummy ($ken$) which captures the Kennedy-Johnson administration policy to reduce unemployment. Second, there are two oil-price dummies, for 1974 and 1980 ($oil_{1,t}$ and $oil_{2,t}$) and in addition the annual rate of change of the oil price itself ($\Delta poil_t$) is an explanatory variable in the model. Third, a dummy representing periods of unusually high/low productivity growth, the 1990s in particular ($prod_t$). Fourth, a dummy ($pow_t$) representing the hypothesis that worker’s ability to take benefit of the industrial prosperity was significantly lower in the 1990s than in earlier US booms. This hypothesis has been influential, also in the policy process. For example, Pollin (2002) argues that in the mid 1990s, the leadership of the Federal Reserve was convinced that the decline in bargaining power was a prime cause of the surprisingly low inflationary pressure.\footnote{See for example (Greenspan): \url{www.bog.frb.us.boarddocs/hh/1997/July/Testimony.htm}, and (Yellen, then member of the Fed Board of Governors): \url{http://www.federalreserve.gov/FRM/Transcripts/1996/19960294Meeting.pdf}, p21.}

The \textit{p-ecm} column of table 1 shows the estimated standard errors (denoted $\hat{\sigma}_{\Delta w}$, $\hat{\sigma}_{\Delta u}$, $\hat{\sigma}_{\Delta p}$, $\hat{\sigma}_{\Delta q}$, $\hat{\sigma}_{\Delta z}$) of the 5 endogenous variables in the unrestricted equilibrium correction model. Below the estimated residual standard errors of each variable, the table shows two diagnostic tests based on the residual vector, for 1st order residual autocorrelation ($F_{AR(1-1)}$), and departure from normality ($\chi^2_{\text{normality}}$). The tests are vector versions of the well known single equation diagnostics, see Doornik and Hendry (2001a). The respective p-values are in brackets, and clearly we can proceed...
Table 1: Diagnostics for VAR and identified econometric models.

<table>
<thead>
<tr>
<th></th>
<th>p-ecm</th>
<th>gen-ecm</th>
<th></th>
<th>Table 2</th>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Identified PCM</td>
<td>Identified WECM</td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta w}$</td>
<td>0.78%</td>
<td>0.79%</td>
<td>0.59%</td>
<td>0.56%</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta u}$</td>
<td>8.64%</td>
<td>8.80%</td>
<td>8.14%</td>
<td>8.12%</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta p}$</td>
<td>0.67%</td>
<td>0.61%</td>
<td>0.62%</td>
<td>0.61%</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta z}$</td>
<td>1.17%</td>
<td>1.2%</td>
<td>1.15%</td>
<td>1.13%</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta q}$</td>
<td>1.70%</td>
<td>1.68%</td>
<td>2.28%</td>
<td>2.29%</td>
<td></td>
</tr>
<tr>
<td>$F_{AR(1-1)}$</td>
<td>0.72[0.81]</td>
<td>0.87[0.64]</td>
<td>0.87[0.65]</td>
<td>1.16[0.30]</td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{normality}$</td>
<td>14.57[0.15]</td>
<td>20.62[0.02]</td>
<td>10.846[0.37]</td>
<td>22.5[0.01]</td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{enc,p-ecm}$</td>
<td>50.316[0.38]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{enc,p-ecm}$</td>
<td>50.316[0.38]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{enc,p-ecm}$</td>
<td>82.11[0.006]</td>
<td>60.76[0.22]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{enc,p-ecm}$</td>
<td>53 restrictions</td>
<td>53 restrictions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The numbers in [ ] are p-values. The sample is 1962-2004.

on the basis that the disturbances are normally distributed, and test relevant restrictions on the p-ecm, with the aim of obtaining an identified simultaneous equations PCM model.

The column labelled Table 2 shows diagnostics for the identified PCM, consisting of the estimated equations shown in table 2 below. The model, estimated by FIML, corresponds to a set of restrictions on the p-ecm. Without any restrictions the model structure is unidentified, but the model in table 2 is over-identified, and we are particularly interested in whether this model is a valid parsimonious representation of the p-ecm—i.e., whether it is an encompassing model. A natural test statistic is the likelihood ratio test of the over-identifying restrictions, see Hendry et al. (1988). This test statistic is denoted $\chi^2_{enc,p-ecm}$ in table 1, and it shows that the 48 restrictions separating the unrestricted system from the identified PCM are jointly statistically acceptable, with a rather high p-value (columns 3 and 5 of table 1 are relevant for the discussion of the bargaining model below).

Turning to the individual equations, the first equation is the wage Phillips curve. Augmentation is in terms of consumer price growth, $\Delta p_t$, producer price growth, $\Delta q_t$, and productivity in the form of the two year growth rate, $\Delta z_t$, consistent with the idea that it is the persistent productivity changes that lead to increased wage growth.\(^{15}\) Note that the vertical long run Phillips curve restriction is imposed, i.e., the coefficients of $\Delta p_t$ and $\Delta q_t$ sum to unity. Both the change in unemployment and the lagged rate (literally $u_{t-2}$ but that does not affect the interpretation) are significant at conventional levels of significance.

In addition to the conventional augmentation, the Phillips curve specification contains three variables representing the effects on wages of stochastic shocks and

\(^{15}\) Staiger et al. (2002) report that without inclusion of productivity growth, their estimated real wage Phillips curve equations have unstable parameters. Our model is consistent with this observation.
The sample is 1962 to 2004. Estimation is by FIML.
Standard errors are in parentheses below the parameter estimates.
See table 1 for residual standard errors, diagnostic tests, and encompassing tests.

Table 2: The econometric PCM.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficients</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta w_t = )</td>
<td>0.0229 + 0.8855 ( \Delta p_t ) + 0.1145 ( \Delta q_t )</td>
<td>(0.0083) (0.0891) (-)</td>
</tr>
<tr>
<td>              + 0.3238 ( \Delta^2 z_t ) - 0.02818 ( \Delta u_t ) - 0.01731 ( u_{t-2} )</td>
<td>(0.0408) (0.00859) (0.00409)</td>
<td></td>
</tr>
<tr>
<td>              - 0.01188 ( \Delta^2 poil_t ) + 0.0237 ( oil_{2,t} ) + 0.0129 ( pow_t )</td>
<td>(0.00236) (0.00689) (0.00286)</td>
<td></td>
</tr>
<tr>
<td>( \Delta u_t = )</td>
<td>9.17 + 1.572 ( w - q - z )(<em>{t-1} ) - 0.3565 ( u</em>{t-1} ) + 3.331 ( \Delta p_{t-1} )</td>
<td>(2.25) (0.407) (0.666)</td>
</tr>
<tr>
<td>              + 0.2097 ( p - pi )(_{t-1} ) - 0.1764 ( ken_t )</td>
<td>(0.12) (0.0426)</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_t = )</td>
<td>0.02507 + 0.6097 ( \Delta p_{t-1} ) + 0.152 ( \Delta p_t )</td>
<td>(0.0108) (0.0636) (0.0205)</td>
</tr>
<tr>
<td>              + 0.01938 ( \Delta poil_t ) - 0.02385 ( p - pi )(<em>{t-1} ) - 0.01907 ( \Delta u</em>{t-1} )</td>
<td>(0.00523) (0.0102) (0.00777)</td>
<td></td>
</tr>
<tr>
<td>              - 0.009358 ( u_{t-1} ) - 0.02462 ( oil_{1,t} )</td>
<td>(0.00572) (0.00755)</td>
<td></td>
</tr>
<tr>
<td>( \Delta z_t = )</td>
<td>0.6106 + 0.1113 ( w - q - z )(<em>{t-1} ) + 0.04003 ( \Delta u</em>{t-1} ) + 0.02039 ( u_{t-2} )</td>
<td>(0.283) (0.0515) (0.00921) (0.00745)</td>
</tr>
<tr>
<td>              + 0.04525 ( p - pi )(_{t-1} ) + 0.0123 ( prod_t )</td>
<td>(0.0102) (0.00293)</td>
<td></td>
</tr>
<tr>
<td>( \Delta q_t = )</td>
<td>1.089 + 0.9744 ( \Delta (w - z) ) + 0.1975 ( w - q - z )(_{t-1} )</td>
<td>(0.54) (0.0403) (0.0982)</td>
</tr>
<tr>
<td>              + 0.02562 ( \Delta pi_t ) - 0.06179 ( oil_{2,t} )</td>
<td>(-) (0.0169)</td>
<td></td>
</tr>
</tbody>
</table>

Notes

- Structural breaks: The stochastic shock is the double difference in oil prices, showing that oil price shocks (small and large) depress annual wage shock. The OPEC-II oil price hike was of course special, and the positive and significant coefficient of \( oil_{2,t} \) shows that there was a one-off compensation for that event. Finally, the estimation results confirm the hypothesis of low worker ability to push for higher wages in the 1990s: \( pow_t \) is a significant explanatory variable, and it is zero or negative for most of the last decade of the previous century (see data appendix).

- The second equation in table 1 shows the empirical counterpart of equation (2) in the theoretical model. Hence, we first look for the role played by the lagged wage share. Indeed \( \beta_{w_2}(w - q - z)_{t-1} \) appears strongly in the estimated equation with a coefficient of 1.57 for \( \beta_{w_2} \). Second, the effect of the lagged unemployment rate is important for the stability of the system, and the fact that the coefficient corresponding to \( \beta_{u_1} \) in equation (2) is found to be precisely estimated (with a value of −0.36) is thus corroborating the cointegration results in support of \( u_t - \mu_u \sim l(0) \) and \( w_t - q_t - z_t - \mu_w \sim l(0) \).
In addition to the crucial significance of \((w - q - z)_{t-1}\) in the unemployment equation, we identify a positive effect of the rate of lagged inflation, \(\Delta p_{t-1}\), which we interpret as an effect of economic policy: higher inflation typically leads to a tightening of monetary policy and the rate of unemployment of course represents an important transmission mechanism. It is perhaps more surprising that we also estimate a positive effect of the lagged real exchange rate, defined as \((p - p_t)\). Hence, despite the vastness of the domestic US economy, its rate of unemployment does not appear to be completely sheltered from sustained loss of international competitiveness.

In the theoretical PCM model, the third equation was a “consumer price equation” cast in differenced form, as explained above. In table 2 there is a separate equilibrium correction model for \(\Delta p_t\). It would represent a price Phillips curve if it was not from the inclusion of \(p_{t-1} - p_{t-1}\) albeit with a small coefficient (below, we will present robustness tests of its impact on the dynamic behaviour of the system). This is the first encounter of extended equilibrium correction dynamics.

There is evidence of other extensions of equilibrium correction in the two last equations of the model. Productivity \(\Delta z_t\) equilibrium corrects rather strongly to the lagged wage share (it was assumed exogenous in the theoretical PCM model (1)-(5)), and it also depends on the lagged change and rate of unemployment. The last equation in table 2 shows the estimated manufacturing producer price equation. As can be seen from the specification, \(\Delta q_t\) is simultaneously determined with \(\Delta w_t\) through the change in unit labour costs. There is a small effect of the change in import prices as well, but a quite large equilibrium correction coefficient with respect to the lagged wage share.

In sum, the empirical PCM contains the kind of extended equilibrium correction which, according to the analysis in section 2, may cause natural rate dynamics to behave significantly different from the conventional Phillips curve dynamics. In section 4 we illustrate the dynamics of the empirical PCM by dynamic simulation. Before that, we consider whether an alternative to the wage Phillips curve can be formulated on this data set.

### 3.3 The WECM

The previous sections showed that the two hypotheses of \(w_t - q_t - z_t - \mu_w \sim I(0)\) and \(u_t - \mu_u \sim I(0)\) are supported by cointegration analysis, and that a corresponding dynamic PCM with desirable statistical properties—for example encompassing the \(p\)-\textit{ecm}—can be specified. Moreover, adding \(w_{t-1} - q_{t-1} - z_{t-1} - \mu_w\) to the wage Phillips curve gives an insignificant coefficient (the ‘t-value’ is 0.01). This findings are quite different from results on for example Scandinavian data, where the lagged wage share is typically found to be a significant equilibrium correction term in manufacturing sector wage equations, see e.g., Nymoen and Rødseth (2003) and Forslund et al. (2005).\(^{16}\)

However, as explained in section 3.1, there are other possible equilibrium correction specifications to consider. For example with weak and poorly organized

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\(^{16}\)In Bårdsen et al. (1998), the equilibrium correction framework is used to explain aggregate wage and price setting in Norway and the United Kingdom, with comparable and similar results for the two economies.
Table 3: The econometric model with a wage equilibrium correction equation, WECM.

\[
\begin{align*}
\Delta w_t &= -0.1198 + 0.8053 \Delta p_t + 0.1947 \Delta q_t \\
&\quad + 0.2955 \Delta z_t - 0.0094 \Delta^2 p_{oil_t} \\
&\quad + 0.0267 oil_{2,t} + 0.006145 pow_t - 0.02229 \Delta^2 u_t \\
&\quad - 0.0712 (w - p - 0.2z + 0.1u)_{t-1} \\
\end{align*}
\]

\[
\begin{align*}
\Delta u_t &= 8.951 - 0.1721 \kappa n_t - 0.3456 u_{t-1} \\
&\quad + 3.369 \Delta p_{t-1} + 1.537 (w - q - z)_{t-1} + 0.2097 (p - p^i)_{t-1} \\
\Delta p_t &= 0.02443 + 0.6231 \Delta p_{t-1} + 0.1594 \Delta p^i_t \\
&\quad + 0.0177 \Delta p_{oil_t} - 0.02021 (p - p^i)_{t-1} - 0.02366 \Delta u_{t-1} \\
&\quad - 0.00918 u_{t-1} - 0.02721 oil_{1,t} \\
\Delta z_t &= 0.6404 + 0.1173 (w - q - z)_{t-1} + 0.04593 (p - p^i)_{t-1} \\
&\quad + 0.04517 \Delta u_{t-1} + 0.0223 u_{t-2} + 0.01221 prod_t \\
\Delta q_t &= 1.016 + 0.9664 \Delta (w - z) + 0.1843 (w - q - z)_{t-1} \\
&\quad + 0.03361 \Delta p^i_t - 0.06224 oil_{2,t} \\
\end{align*}
\]

Notes

See table 2

unions, efficiency considerations may come to dominate wage setting. In efficiency wage theory, a main implication is that wages are set by firms in the light of the reference wage (\(\bar{w}_t\)) and worker’s probability of getting a job elsewhere, see Forslund et al. (2005). The job probability can be approximated by the unemployment rate, while at the aggregate level \(\bar{w}_t\) is probably linked to the the general price level of the economy and to productivity, hence we have the following alternative to \(w_{t-1} - q_{t-1} - z_{t-1} - \mu_w\) as a predictor of \(\Delta w_t\):

\[
(11) \quad w_t - p_t - \iota z_t - \lambda u_t - \mu_{wp} \sim l(0), 0 \leq \iota < 1, \lambda \geq 0,
\]

which can be viewed as a linear combination of the two \(l(0)\) variables \(w_t - p_t - \iota z_t \sim l(0)\) and \(u_t \sim l(0)\).

Analysis of a VAR consisting of \(w_t, p_t\) and \(z_t\), keeping \(u_t\) as non-modelled, does not yield very clear results, but setting \(\iota = 0.2\) and \(\lambda = 0.1\) gives an equilibrium correction variable which at least has some stationary traits. Importantly, when the
equilibrium correction variable is added to the unrestricted equilibrium correction system, to form a generalized unrestricted reduced form, gen-ecm, the PCM of table 2 is no longer an encompassing model: the p-value of the encompassing test falls from 0.38 to 0.006, cf. the entry for $\chi^2_{\text{enc,gen-ecm}}$ in the bottom row of table 1. Hence, there is need for a different structural model to account for the force of $w_{t-1} - p_{t-1} - 0.2z_{t-1} - 0.1u_{t-1}$ in gen-ecm.

In terms of specification, the only difference between the econometric PCM and WECM is the wage equation. But in order to check how the point estimates of the coefficients of other equations in table 2 are affected, all 5 of the WECM equations are reported in table 3. The equilibrium correction wage equation includes $w_{t-1} - p_{t-1} - 0.2z_{t-1} - 0.1u_{t-1}$, and although the equilibrium correction coefficient is quite small numerically, its ‘t-value’ is still significant ($-5.9$).

The coefficients of the other explanatory variables in the wage equation change very little compared to the PCM case, which indicates that the equilibrium correction term is relatively orthogonal with respect to the variables of the Phillips curve. The coefficient of the bargaining power dummy is an exception. The estimated coefficient halved, probably because the equilibrium correction term itself is related to bargaining power.$^{17}$ With the wage equation in place, the econometric WECM easily encompasses the gen-ecm, as shown by the value of $\chi^2_{\text{enc,gen-ecm}}$ in the right column of table 1. Direct inspection of the other structural equations confirms that their estimated coefficients change very little compared to the PCM case, hence any differences in the dynamic behaviour of the two models can be directly related to the different specifications of the wage equations.

4 Natural rate dynamics

The last section discussed two dynamic models of the US natural rate. The only difference between the two models is the specification of the wage equation. It is a wage Phillips curve in the PCM in table 2, and an equilibrium correction wage equation in the model in table 3. According to a view shared by most economists, this difference is essential and should have important implications for the dynamics of the equilibrium wage, see Blanchard and Katz (1999).

However, we have hypothesized that extended equilibrium correction may come to dominate unemployment dynamics even though the structural wage equation is a Phillips curve. In this section we illustrate the empirical relevance of these ideas using the estimation results of the previous section.

The solutions of the econometric models PCM and WECM are easily found by dynamic simulation. By choosing initial values from 1962 and 1963 the solution approaches the long run means in the 1990s, i.e., there is little trait of the initial conditions in the solution for the most recent part of the sample period.$^{18}$ Hence,

$^{17}$The other main difference between the two wage equations is the presence of $\Delta^2u_t$ in the WECM, but that difference is due to the specification of the error correction term, with $u_{t-1}$ rather than $u_{t-2}$ as table 2 would suggests.

$^{18}$This is confirmed by starting the simulation 10 years later, in 1974. For $\Delta u_t$ and $u_t$ in particular, the solution of the 1990’s is not much affected. There are some large roots in the solution though. In the WEM the non trivial roots are 0.98, 0.94, 0.8 and 0.7 (a complex pair) and in the PCM: 0.94, 0.79, 0.67 and 0.31 (a complex pair).
for the rate of unemployment in particular, the solution reflects the long run mean implied by the dynamic systems, including the effects of the stochastic shocks (oil price effects in wages for example, $\Delta^2 p_{oil_t}$), and deterministic structural breaks: the dummies representing low bargaining power ($pow_t$) and unusually high productivity growth ($prod_t$) in particular.

In figure 1 we first show the WECM solution of $\Delta w_t$ (panel a) and $u_t$ (panel b). The model’s solution is seen to fit the data very well. For the rate of wage inflation some of the good fit at the end of the period is of course due to inclusion of the two dummies: $pow_t$ because it is an explanatory variable in the wage equation itself, and $prod_t$ because that dummy helps keep the solution for $\Delta z_t$ on track, and productivity growth in turn affects wages in the model, as we have seen.

The solution of the rate of unemployment in panel b) is of course central to the theme of this paper. The solution follows the actual rate of unemployment down to 3.5%, corresponding to 1.25 on the log scale of panel b). This is due to the $ken_t$ dummy variable in the model. The solution then tracks the period from the mid 1970s to the late 1980s when the rate of unemployment, though fluctuating is above the level of the 1960s for most of the time. In the last 15 years of the period, the solution for the unemployment rate is relatively constant.

The constancy of the simulated unemployment rate for this period, and therefore “lack of fit”, is not a sign of model failure, but reflects that by starting the simulation in 1964 the simulated values for 1990-2004 are probably quite close to the model’s steady state. According to the model, the explanatory variables with most influence on the steady state are the bargaining and power productivity dummies, $pow_t$ and $prod_t$, though these two variables affect the solution for the rate of unemployment only indirectly, through their effect on the solution for the wage-share. Hence, according to the model, the main structural breaks during the last 15 years of the sample have an indirect and muted effect on the solution for the rate of unemployment.

Having looked at the solution of the WECM, we next turn to the differences between the solution of that model and of the PCM. As noted above, the consensus view is that the difference is likely to be large, because there are difference wage setting dynamics in the two models. However, panel c) and d) of figure 1, with scatter plots of the simulated values of $\Delta w_t$ and $u_t$ of the two models, tell a different story: the solutions of the PCM and the WECM are nearly identical. This is due to extended equilibrium correction, which dominates the effects of the different specifications of the wage equation.

In order to specify a PCM which behaves distinctively different from the WECM, and more in line with the textbook case of a vertical long run Phillips curve, the equilibrating mechanisms of the model must be restricted much more than in table 2. Hence we consider a restricted econometric PCM which corresponds to the theoretical PCM of section 2.1, where there are no extended equilibrium correction in the $\Delta p_t$ or $\Delta z_t$ equations. Therefore, in the restricted PCM, the coefficients of $p_{t-1} - p_{t-1}$ in the $\Delta p_t$ and $\Delta z_t$ equations are set to zero, along with for example the coefficient of the lagged wage share in the $\Delta q_t$ equation. It should come as no surprise to find that these additional restrictions are statistically rejected. The value of the $\chi^2_{enc,p=ecm}$ statistic (cf. table 1) with 57 degrees of freedom is 128, which is significant at any significance level. Hence, unlike the econometric PCM and WECM,
which encompass their respective unrestricted equilibrium correction systems, the restricted PCM is firmly rejected by the statistical tests.

Figure 2 shows how the three different models respond to a shock to unemployment (of magnitude corresponding to a reduction from 5% to 4.5%) which is not reversed, i.e., a counterfactual experiment. The PCM of table 2 shows the most vigorous wage response, cf. panel a) of the figure, corresponding to a lowering of the annual rate from 5% to 4.4% in the third year after the shock for example. There is less marked difference between the responses of the WECM and the PCM in panel b), showing the inflation response, which is due to the direct effect of the rate of unemployment in the $\Delta p_t$ equation of both models.

The difference between the three models also becomes apparent in panels c) and d) of the figure. In the case of the WECM and PCM there is a sharp increase in the rate of unemployment, which transforms into a little more than 1 percentage point change in the natural rate (since the shock is not reversed). This kind of response cannot be reconciled with the standard Phillips curve model, which only allows shocks within the Phillips curve itself to affect the mean unemployment rate. Nevertheless, the responses in figure 2 happen for perfectly logical reasons since the econometric WECM and PCM are in fact quite similar due to extended equilibrium correction.

The graph for the restricted PCM in panel c) shows the conventional response.
Figure 2: Accumulated dynamic multipliers of the econometric models PCM, WECM, and the restricted PCM, to a 0.5 percentage point reduction in the unemployment percentage.

5 Stability of the natural rate

In an important paper, Frisch (1936) anticipated the day when it would become common among economists to define (and measure) ‘normal’ or natural values of economic variables by the values of the variables in a stationary state. As pointed out above, there has been little development in that direction in the estimation of the natural rate of unemployment. The dominant strategy has been to estimate the natural rate from partial models, as in the case of the wage Phillips curve. While econometrically sophisticated, as in Staiger et al. (1997), these studies do not address the important issue of dynamic stability of the rate of unemployment around the estimated natural rate. Moreover, completely ad hoc methods for measuring the
natural rate have also gained currency, see e.g., Holden and Nymoen (2002) for an appreciation of one of the OECD’s methods.

As pointed out by Fair (2005), part of the explanation for the slow progress in the direction that Frisch foresaw lies in the low confidence in the steady state properties of estimated macroeconometric models. In this paper we have illustrated at least one feature of econometric models of the natural rate of unemployment which is crucial for the evaluation of steady state properties, namely the specification of equilibrium correction mechanisms. We regard this mainly as a methodological contribution, and the detailed specification of the models can no doubt be contested. That said, we still believe that the main picture, and that extended equilibrium correction dominates the standard Phillips curve dynamics, will prove to be robust to any change in the details of the specification.

In none of the models considered above is the natural rate a completely constant and invariant entity. However, there is a difference between our framework and the time varying NAIRU model of Gordon (1997). In the time varying natural rate model, all shocks, small and large, influence the estimated natural rate. However, small and random shocks, by their very nature, either vanish or are counteracted by other shocks, usually after only a short while. Thus, a method which feeds such disturbances into the estimated natural rate may induce too much volatility in the estimated natural rate. It is a different matter with intermittent and large shocks, and with events which are usually recognized as important in contemporary economic analysis and debate. Our modelling framework allows such structural changes to affect the estimated natural rate.

As we have seen, both the estimated PCM and the WECM, but not the restricted PCM (which corresponds to the standard text book model), have steady states which are dynamically stable. On this basis, following Frisch’s suggestion, we may interpret the values obtained by dynamic simulation, as reasonably good approximations to the models’ implied natural rate, for example $\mu_{u,WECM}$ in the notation of section 2.

However, there is still the issue about the credibility of using simulated values to represent the natural rate. For one thing, even if one accepts our method in principle, there is a question whether the simulated values actually correspond to the steady state or whether they are influenced by the initial conditions, or by other factors which should not, by our definition, influence the natural rate. Above we showed that the initial conditions do not have much impact on the simulated values in figure 1, panel b). However, the simulated values of other variables of the model may nevertheless have an influence on the solution of the rate of unemployment in the 1989-2004 period.

In order to check the robustness of the model solution qua natural rate we have calculated a natural rate estimate based on the semi-reduced form for the rate of unemployment. The resulting estimated equation is similar to the augmented Dickey-Fuller regression of section 3.2.1, but with the lagged values of the wage share and the real exchange rate as additional variables. The natural rate estimate of this model is obtained by setting the explanatory variables equal to their average values. In this way, we ‘cut off’ the link between the reduced form of the rate of
unemployment and the rest of the model.\textsuperscript{19}

Figure 3 shows the actual rate of unemployment as the dotted line. The estimate from the semi-reduced form, denoted \textit{RF ‘natural rate’} in the graphs, is shown as a thick line, and we have added the associated 95\% confidence interval of the mean, denoted as \( \pm 2se \) in the graph. \textit{RF ‘natural rate’} has been estimated recursively, hence the starting point of the lines are based on the 29 observations from 1961 to 1989. The end-point of the curve shows the results for the full sample from 1961 to 2004. The impression is that this estimate is very stable over the 16 year period from 1989-2004. Figure 3 also shows the simulated values of WECM, i.e., the same solution as shown in figure 1 (panel b), but transformed to percentages. As argued above, due to asymptotic stability of the model, these simulated values approximate the implied natural rate of the full model, \( \hat{\mu}_{u,WECM} \).

As explained above, the estimated WECM includes two of the most cited candidates for a reduction of the ‘natural rate’ in the 1990s: the reduction of workers’ bargaining power and the unusually high productivity growth. The two factors have statistically and numerically significant effects on wage growth in our estimated model, and logically they are part of the explanation why the line for the dynamic solution of WECM lies below the line of \textit{RF ‘natural rate’} after 1993. The highest

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{The civilian unemployment rate together with two measures of the natural unemployment rate: the solution of the estimated WECM, see panel b) of figure 1, and the mean unemployment rate from a semi-reduced form from WECM, with \( \pm 2 \) standard errors, denoted \( \pm 2se \) in the graph.}
\end{figure}

\textsuperscript{19}This can be seen as a variant of the procedure proposed in Frisch (1936), as an alternative to the full steady state, for estimation of natural values of variables by modifying structural macroeconomic models, see Fair (2005).
\( \hat{\mu}_{u,\text{WECM}} \) is 5.9% (in 1991), and the lowest is 5.1% (in 1999). This is however a smaller estimated reduction in the natural rate than the best evidence based on estimation of Phillips curves, Staiger et al. (2002), which suggests a 1.5 percentage point reduction between 1992 and 2000. Moreover, while the graphs of Staiger et al. (2002) indicate a continued fall in the natural rate also after 2002, our estimate suggests that the natural rate might have been stabilized in the course of the period 2000-2004.

6 Conclusions

This paper has discussed methodological and substantive issues relating to the empirical assessment of the US natural rate, starting by noting that the methodology underlying the consensus view is based on a highly restrictive model of wages, prices and the rate of unemployment, dubbed PCM above. Another model, called WECM, with equilibrium correction in wage setting is an alternative hypothesis to the wage Phillips curve. The consensus view is that the PCM and WECM are polar cases, but we show that this is only true if one abstracts from equilibrium correction behaviour elsewhere in the system, in price setting in particular. We dub this system feature extended equilibrium correction. Thus, even if the wage Phillips curve is the preferred model of US wage dynamics, it does not follow that the natural rate can be estimated from the empirical wage Phillips curve alone. Due to extended equilibrium correction, the PCM dynamics might become almost indistinguishable from the dynamics of the WECM. Our econometric versions of PCM and WECM confirm this feature, and only in the case where we tailor the PCM dynamics by imposing restriction that are rejected by statistical tests, do the dynamics correspond to the standard analysis.

It is a widely held view that differences in wage dynamics between the USA and Europe go a long way towards explaining the different behaviour of the unemployment rates on the two sides of the Atlantic. Hence research has focused on the specification of wage equations. One belief shared by a majority of economists is that the relatively swift adjustment of the US unemployment rate, and its apparent insulation from shocks originating in demand sector of the economy, is due to supply side behaviour which is represented by a wage Phillips curve. While not wanting to downplay the importance of wage dynamics for the comparative macroeconomic performance of different economies, our results show that the wage Phillips curve and the error correcting wage equation may not be the polar cases that the standard framework will have us believe. In our analysis, equilibrium correction elsewhere in the system may be (almost) as important for natural rate dynamics as wage setting dynamics. Thus, the comparative stability of the US natural rate may be less of a Phillips curve property, and more of a system property than the conventional analysis suggests. Specifically, the natural rate may have stayed constant because demand shocks have had a tendency to disappear in this vast economy, sometimes with the aid of good economic policies. Unlike the incumbent model of the US natural rate, our empirical results do not rule out the possibility that a sufficiently large and persistent demand shock, may have long lasting effects on US unemployment.
References


### A Data definitions

*Economic time series*

A main data source has been the AMECO (Annual Macro-Economic) database of the European Commisions’s Directorate General for Economic and Financial Affairs (DG ECFIN). The other main sources are EcoWin and Economagic.

- $W_t$— Compensation of employees, manufacturing industry, nominal USD. AMECO code: HMCMW. The AMECO series was spliced by the Bureau of Labor Statistics Employment cost index, manufacturing, private industry. Source: Economagic.
- $P_t$— Consumer price index. 1995=100. AMECO code ZCPIN.
- $Q_t$— Price deflator of gross value added, manufacturing industry, 1995=100. AMECO code PVGM.
- $Z_t$— Labour productivity, output per hour of all persons, manufacturing. EcoWin code ew:usa09102
- $PI_t$— Price deflator on imports of goods and services. 1995=100. AMECO code PMGS.
$U_t$ – Unemployment rate, in percent. Civilian unemployment, Source: Economagic, St.Louis Fed.

$POIL_t$ – Price of West Texas Intermediate Crude, USD Per Barrel. Source: Economagic

As explained in the text, lower case letter refer to the logarithm of the original variables above, $u_t = \log(U_t)$ for example.

**Dummies.**

$pow_t$ – Bargaining power dummy, see section 3.2 for motivation. It is 1 in 1962 and 1964; $-0.5$ in 1995; $-1$ in 1996 and 1997; $-0.5$ in the years 1998-2001; and $-0.5$ in 2003. Otherwise zero.


$oil_{1t}$ – 1 in 1974, otherwise zero.

$oil_{2t}$ – 1 in 1980, otherwise zero.

$ken_t$ – 1 Kennedy-Johnson dummy, 1 in the period 1965-1969, zero elsewhere.