VAR Modelling Approach and Cowles Commission Heritage

Duo Qin*
Economics Department
Queen Mary, University of London

March 2006

Abstract
This paper examines the rise of the VAR approach from a historical perspective. It shows that the VAR approach arises as a systematic solution to the issue of ‘model choice’ bypassed by Cowles Commission (CC) researchers, and that the approach essentially inherits and enhances the CC legacy rather than abandons or opposes it. It argues that the approach is not so atheoretical as widely believed and that it helps reform econometrics by shifting research focus from measurement of given theories to identification/verification of data-coherent theories, and hence from confirmatory analysis to a mixture of confirmatory and exploratory analysis.

JEL classification: B23, B40, C10, C30, C50

Keywords: VAR, macroeconometrics, methodology, rational expectations, structural model

* Thanks are due to C.L. Gilbert and S. Kochugovindan for their useful comments and to M. Kazaks, Y.G. Makedonis and R. Pierse for their help with background materials.
1. Introduction

It is a popular view that the VAR (Vector AutoRegressive) approach is an atheoretical methodology in opposition to the structural approach formalised by the Cowles Commission (CC). The VAR methodology was initiated by Sargent and Sims in the 1970s. Sims is recognised as its econometrics leader, e.g. see (Pagan, 1987). If one peruses Sims’ work on the VAR approach, one will not fail to notice striking features which stand at odds with the ‘atheoretical’ attribute, e.g. features such as Sims’ strong emphasis on identification and causality, two key concepts at the centre of the CC tradition, and Sims’ intimate interaction with pioneers of the Rational Expectation (RE) movement in macroeconomics.

This study examines the VAR approach from a historical perspective.¹ The focal point is the rise of the approach as evolved from the CC econometrics tradition. It covers a period up to the end of 1980s, in spite of the fact that the VAR approach was still developing during the 1990s.

The history epitomises the difficult journey where econometricians grope for a workable bridge between data, economics and statistical theory, while each of the three pillars is also evolving, and for a fine balance between applied need to have convenient and tractable models and conceptual desire to make the models theoretically consistent and interpretable. It shows that the VAR approach inherits and enhances, rather than abandons or opposes, the CC legacy to a great extent, that Sims has lived up to his own label of a ‘structural VAR’ modeler’ who gives ‘formal partial behavioural interpretations to statistically detailed models’ (1989), and that the VAR strategy essentially has offered, in response to the developments in macroeconomics, a systematic solution to the issue of

¹ For surveys of the VAR approach from a technical perspective, see e.g. (Cooley and LeRoy, 1985) and (Pagan, 1987); for its interaction with the RE movement, see (Sent, 1998).
‘model choice’, which was bypassed by the CC researchers. The ‘atheoretical’ and anti-CC attribute is thus an oversimplification of the VAR methodology.

The rest of the paper is organised as follows. Section 2 gives a brief description of how the so-called ‘reduced-form’ VAR came into being within the CC structural approach in the 1940s. Section 3 sketches alternative uses of VAR models from the late 1940s to the mid 1960s. Section 4 is focused on how VAR models became, in the RE movement, a prominent medium of testing theories and/or representing testable dynamic theories. Sections 4 and 5 depict the rise of the VAR approach and particularly the endeavour of VAR econometricians to preserve the CC heritage. The concluding section discusses methodological implications of the rise of the VAR approach.

2. Reduced-form VAR: the Cowles Commission Heritage

Standard regression-based econometrics was well established by the end of the 1960s. e.g. see (Gilbert and Qin, 2006). The foundational work was laid by researchers associated with the CC during the 1940s, see (Haavelmo, 1944), (Koopmans, 1950) and (Hood and Koopmans, 1953).

What the CC researchers accomplished for econometrics during the 1940s can be summarised from several perspectives. At a broad methodological level, their work becomes archetypical for researchers who attempt to bridge systematically empirical research and theory in a logically rigorous manner. Specifically, their research principle is to make all assumptions explicit to enable discovery of problems and revision of the assumptions in the light of these problems. Moreover, the assumptions are to be made as consistent as possible with knowledge of human behaviour and are classified into two types: the first are those which are statistically testable and the second are provisional working hypotheses, see (Marschak, 1946). At the disciplinary level of economics, they
delegate the job of formulating theoretic models to economists and, to econometricians, the task of specifying and estimating structural models based on already formulated theoretical models. The demarcation enables econometricians to leave aside the issue of ‘model choice’, see (Koopmans, 1950, pp 44-45), and focus their attention on how to measure the unknown coefficients of known ‘structural models’ in statistically best ways. This idea of founding econometrics upon theoretical models comes from Frisch, who is the first to advocate the ‘structural’ method, e.g. see (Frisch, 1937, 1938), in favour of the ‘historical’ method recommended by Tinbergen (1935), see (Qin, 1993; Chapter 2.2).

At the technical level of econometrics, the CC researchers build the econometric procedure within the framework of a simultaneous-equations model (SEM), which is regarded as the most general theoretical model form since it encompasses a dynamically extended Walrasian system:

\[ A_0 x_t = \sum_{i=1}^{p} A_i x_{t-i} + \varepsilon_t \]  

The procedure comprises primarily model specification via \( \varepsilon_t \) using Haavelmo’s probability approach, identification of the structural parameters of interest, which are contained mostly in the non-diagonal matrix, \( A_0 \), using rank and order conditions, and estimation of the structural parameters by full-information maximum likelihood (ML) or limited-information ML methods.

To facilitate identification and estimation, a VAR model is derived as the ‘reduced-form’ of (2.1):

\[ x_t = \sum_{i=1}^{p} A_0^{-1} A_i x_{t-i} + A_0^{-1} \varepsilon_t = \sum_{i=1}^{p} \Pi_i x_{t-i} + u_t \]

---

2 On this period of the history, see e.g. (Christ, 1952, 1994), (Epstein, 1987), and (Qin, 1993).
When there are exogenous variables in the system, i.e. when the variable set, \( x \), is decomposed into \( x = (y, z) \) with \( z \) not to be modelled, the above two equations become:

\[
(2.1)' \quad A_0 y_t = \sum_{i=1}^{p} A_i y_{t-i} + \sum_{j=0}^{p} B_j z_{t-j} + \epsilon_t
\]

\[
(2.2)' \quad y_t = \sum_{i=1}^{p} \Pi_{1,i} y_{t-i} + \sum_{j=0}^{p} \Pi_{2,j} z_{t-j} + u_t
\]

Equation (2.2) is referred to as a ‘closed’ VAR as opposed to an ‘open’ VAR of (2.2)’. The reduced form is viewed as non-structural, because its coefficients, \( \Pi_i \), being functions of structural parameters, are not intuitively interpretable in terms of economics, and is regarded as a useful medium only for identification and estimation purposes, e.g. see (Mann and Wald, 1943), (Koopmans, 1950). This is how the ‘atheoretical’ label becomes attached to the VAR model.

3. Unrestricted VAR: from Simultaneity to Dynamics

At the time when the CC researchers formalised the econometric procedure, available macroeconomic theories were mostly static. The CC researchers thus devoted most of their attention to technical problems arising from simultaneity, i.e. problems concerning the unique estimatability of the contemporaneous structural matrix, \( A_0 \). The dynamic part of (2.1) or (2.1)’ was rarely addressed.\(^3\) In actual execution, the structural model was mostly assumed static with a static reduced form:

\[
(3.1) \quad A_0 y_t = B_0 z_t + \xi_t
\]

\(^3\) The CC researchers were aware of the problem of lacking rigorously formulated dynamic macroeconomic theories, e.g. see (Koopmans, 1957). In fact, a substantial part of the CC group, including Koopmans himself, moved into theoretical model building in the 1950s and led the way to the rise of dynamic equilibrium models and various growth theories. This line of research, however, contains a shift of agenda: the key task of characterising economic dynamics was recast as that of establishing conditions of stability or equilibrium of dynamic systems, see (Weintraub, 1991).
This is especially common in econometrics textbooks of the time, see e.g. (Johnston, 1963) and (Goldberg, 1964). As the VAR of (3.2) undercuts the potential of the reduced-form VAR of (2.2) to characterise the dynamics of a set of variables, residual autocorrelation has been frequently observed from applied modelling experiments. It is thus unsurprising to find the use of VAR as a primary model, instead of a reduced form, in the research of economic time-series problems.

An early pioneer to use VARs as a primary model to study economic time-series properties was G. Orcutt. Orcutt (1948) examined the sample series of each variable used in the Tinbergen’s model (1939) and identified a common AR(2) dynamic structure with a unit root in most of the series. Subsequently, this line of research became more focused on single-equation models, and particularly associated with the collective research on time-series economics at Cambridge University and then London School of Economics (LSE), e.g. see (Gilbert, 1989).

A more prominent use of the VAR model as a representation of dynamic systems is found in the works of H. Wold, e.g. see (Wold, 1954, 1960, 1964), (Wold and Juréen, 1953) and (Strotz and Wold, 1960). Starting from statistical research in stochastic processes, Wold was greatly inspired by the pioneering work on dynamic modelling by Wicksell, Moore and especially Tinbergen, see (Wold, 1954), and also (Hendry and Morgan, 1994). Wold was convinced that empirical causality was best mapped into a dynamic model describing a ‘chain of causation’. This model was effectively a VAR when there were more than one endogenous variable.  

\[ y_t = \Pi_{2,0} z_t + \nu_t \]

(3.2) This is shown clearly from the example given in (Wold, 1954) of a two-equation dynamic model of market demand, \( d \), supply, \( s \), and price, \( p \):  

\[
\begin{align*}
    s_t &= S(s_{t-1}, \ p_{t-1}, \ d_{t-1}, \ d_{t-2}, \ ...)+\ u_t, \\
    p_t &= P(s_{t-1}, \ p_{t-1}, \ d_{t-1}, \ ...)+\ v_t
\end{align*}
\]
method of sequence analysis in model construction, Wold (1960) formulated a causal chain model, in which contemporaneous causality among the endogenous variables was arranged in the manner of $y_i \leftarrow y_j$ for $j > i$, and cross-equation residuals were assumed uncorrelated with each other.\(^5\) Wold’s causal chain model can be seen as a special form of equation (2.1)’, with $A_0$ being an upper triangle matrix and the covariance matrix of the error terms being diagonal. Performing ‘a chain of substitutions’ on the model, a VAR model would result just as (2.2)’ from (2.1)’. But in contrast to an SEM, a causal chain model is always identifiable, and it can be estimated equation by equation using OLS in a ‘recursive’ manner.\(^6\)

However at the time, Wold’s causal chain model was largely overshadowed by the CC SEM during the dissemination of econometrics textbooks which were set within the CC paradigm. In applied macroeconometric modelling, the CC approach was also adhered to, at least overtly, by most of the modellers in governments and research institutions.\(^7\) This was notwithstanding the efforts made by Barger and Klein (1954) to build a quarterly model of the US economy by Wold’s causal chain method, as well as the practical revival of OLS as worries over ‘simultaneity bias’ were considerably eased by the ‘least squares’ reappraisal around the turn of 1960, e.g. see (Christ, 1960), (Waugh, 1961) and also (Qin, 1993; Chapter 6.3). Most of the macroeconometric models built at the time contained very limited dynamics. Whatever dynamics a model had was frequently an \textit{ad hoc} extension of

\[ u \text{ and } v \text{ are disturbances and could be inter-correlated.} \]

\(^5\) Wold (1964) observed that the assumption was generally satisfied in carefully built empirical models.

\(^6\) Notice that Wold did not use the taxonomy of ‘structural model’ versus ‘reduced form’ for VAR until the 1960s, and that he regarded a VAR could be the primary model as well, only whose form ‘coincides with the reduced form’, see (Wold, 1964; Introduction).

\(^7\) There was a growing enterprise of large-scale macroeconometric model building during the 1960s, see (Klein, 1971) and (Bodkin \textit{et al}, 1991) for more detailed historical accounts.
an *a priori* postulated static SEM of (3.1). Moreover, the model tended to contain numerous exogenous variables such that the SEM became in effect overidentified.

Such practice met forceful criticism from T.-C. Liu. Liu started his empirical modelling career having been trained in the CC structural paradigm. Exposed to complications in empirical modelling, Liu (1955) became wary of the practice of imposing overidentifying restrictions on an SEM for the mere purpose of securing the estimability of all the structural coefficients. He found the practice utterly arbitrary as evaluated against the economic reality where most of the structural relationships were unlikely to be identifiable. Acknowledging the reality, Liu (1960) maintained that the best practical strategy was to base a forecasting model on a virtually unrestricted reduced-form VAR, because such a VAR would best capture data characteristics. As the model could be easily estimated by OLS, Liu referred to such an unrestricted VAR as ‘the least squares reduced form’. Moreover, he (1960) advocated the use of higher frequency (i.e. quarterly instead of annual) data in model building so that ‘simultaneity would be less important than recursiveness’ (i.e. sequential causality). Here, Liu (1960) viewed the quarterly causal chain model experimented by Barger and Klein (1954) as ‘a fundamental reversal of the position underlying the simultaneous equation approach’, and suggested that the approach be further extended to substitute the heavily used residual autoregression scheme in that model by the unrestricted least squares reduced form.

Similarly to Wold, Liu’s voice was virtually lost during the consolidation and dissemination of the textbook econometrics. Nevertheless, the issues that Wold and Liu raised manifest the unsettling tension between dynamics and simultaneity in econometric modelling. The more attentive applied modellers are to time-series problems, the less substantial simultaneity becomes. The more the data is allowed to speak, the less textbook
preaching on SEMs is heeded. During this attention shift, the primary capacity of VAR becomes increasingly perceptible as a convenient means of characterising dynamics.

4. Observational equivalent VAR: Pursuit for testable dynamic theories

The early 1970s witnessed the start of a reformation of macroeconomics into dynamically testable structural models. The reformation is now widely known as the rational expectations (RE) movement. A major drive of the movement was to upgrade macroeconomics such that it would systematically match up with the CC structural econometrics. In the eyes of the key RE proponents, macroeconomics was weak both technically and methodologically, and the weakness lay mainly in the lack of dynamic and micro foundation in the Keynesian methodology. This was empirically revealed in poor forecasts of existing macroeconometric models, especially forecasts which could not even compete with those by means of simple time-series models, e.g. see (Lucas and Sargent, 1978) and (Sargent, 1980). The weakness was thought to be fundamentally rectifiable by the RE hypothesis (REH) originally formalised by Muth (1961), as the hypothesis was believed to offer a sound microeconomic behavioural base for generating dynamically more interesting and potentially empirically testable macro theories. Retrospectively, the movement can be regarded as the first systematic attempt in macroeconomics to execute Haavelmo’s (1944) conceptual argument for formulating theoretical models as stochastic and testable hypotheses.

Starting from a typical static structural model (3.1), a RE model normally assumes that agents’ expectations of some of the endogenous variables, \( y^e \), play a non-negligible role in explaining \( y_t \):

---

8 The RM movement is commonly viewed as initiated by Lucas (1972). There are numerous studies on the history and the methodology of the RE movement, e.g. see (Maddock, 1984), (Sent, 1998), (Sheffrin, 1983); see also (Young and Darity, 2001) and (Sent, 1997; 2002) for the early history of RE models.
\[ A_0 y_t = B_0 z_t + C y_{t+k}^e + \epsilon_t \]
\[ y_t = \Pi_{2,0} z_t + \Pi_{3} y_{t+k}^e + u_t \]

where \( C \) contains normally a sub block of zeros to reflect that only the expectations of a subset of the endogenous variables matter, and \( \Pi_3 = A_0^{-1} C \). When \( k=0 \), forward expectations are absent.\(^9\) Under the REH, the latent expectations, \( y_t^e \), is assumed to be based on all the available information \( \{ I \} \):

\[ y_t^e = E(y_t \mid [I_{t-1}]) = E(y_t \mid \{ y_{t-1} \}, \{ z_{t-1} \}) \]
\[ \Rightarrow y_t - y_t^e = \nu_t \Rightarrow E(\nu_t \mid [I_{t-1}]) = 0 \quad \text{Var}(\nu_t \mid [I_{t-1}]) = \text{Var}(\nu_t) \]

The above equation shows that the REH is equivalent to assuming that the error term, \( \nu_t \), follows an innovative process. Take conditional expectation of the reduced form of (4.1):

\[ E(y_t \mid [I_{t-1}]) = (1 - \Pi_3)^{-1} \Pi_{2,0} E(z_{t-j} \mid [I_{t-1}]) = (1 - \Pi_3)^{-1} \Pi_{2,0} z_t^e \]

This equation reveals the equivalence of assuming RE of endogenous variables to that of the exogenous variables, \( z_t^e \). The equivalence makes transpire the necessity of modelling the latter expectations explicitly, which is often referred to as ‘completing’ the model, e.g. see (Wallis 1980):

\[ z_t^e = E(z_t \mid [I_{t-1}]) = \sum_{i=1}^{p} D_{1,i} y_{t-i} + \sum_{i=1}^{p} D_{2,i} z_{t-i} \]

Its incorporation into (4.3) results in a closed VAR:

\[ \begin{pmatrix} y_t \\ z_t \end{pmatrix} = \sum \begin{pmatrix} (1 - \Pi_3)^{-1} \Pi_{2,0} D_{1,i} \\ (1 - \Pi_3)^{-1} \Pi_{2,0} D_{2,i} \end{pmatrix} \begin{pmatrix} y_{t-i} \\ z_{t-i} \end{pmatrix} + \begin{pmatrix} u_{y} \\ u_{z} \end{pmatrix} \]

Notice that the introduction of the REH amounts to extending a static model into a dynamic model with \textit{a priori} tightly postulated structural parameters. In other words, the

\(^9\) The introduction of forward expectations brings a new technical issue into econometrics: the need for terminal conditions to assist unique solutions, e.g. see (Pesaran, 1987; Chapter 5). However, this issue does
REH tightens up all ‘theory-free’ lagged parameters in the traditional CC structural model of (2.1), mostly by cross-equation restrictions via carefully formulated $C$ in (4.1). By so doing, macroeconomists feel that they are finally able to provide econometricians with a coherent story base for measuring a closed, fully parameterised dynamic system, e.g. see (Evans and Honkapohja, 2005).

In regard to econometrics, one of the most provocative arguments in the RE movement is Lucas (1976) critique. Essentially, Lucas demonstrates that, under the REH, shifts in a macro policy instrument would affect the assumption of constancy of certain structural parameters, an assumption upon which macroeconomic model-based policy simulations were normally carried out. In terms of the RE model specified above, such policy shifts amount to value changes in $D_2$ of (4.4), when $z$ represents the policy instrument.\footnote{Notice that Lucas’ original presentation (1976) did not use parameter shifts as such. He analysed the policy variable in terms of its permanent and transitory components. A shift in the permanent component is equivalent to a parameter shift in the autoregressive representation of the variable.}

Obviously, the changes would transmit into the parameters of the $y$ equation in (4.5). Disregarding this transmission could lead to misleading conditional predictions of $y$ upon different policy shifts. Lucas critique has invoked a great deal of rethinking about the practice of \textit{a priori} categorisation of endogenous versus exogenous variables, as well as the dynamic implication of such causal categorisation.\footnote{Controversies and studies along this line lead to the redefinition of ‘exogeneity’ by Engle \textit{et al} (1983) in the time-series context.}

REH based models forcefully demonstrate the necessity of considering a closed dynamic model. However, empirical verification of such models turns out to be a hugely challenging enterprise. Aside from various technical complications that the REH has introduced, e.g. see (Pesaran, 1987), the issue of identifiability re-emerges as a paramount

\footnotetext[10]{Not compound our methodological discussion here. For simplicity, we will only consider RE models with current expectations hereafter.}
problem. One of the most influential expositions of the problem was provided by Sargent (1976). Sargent demonstrated that the VAR model of (4.5) set the empirical limit for testing the REHs versus conventional theories in the sense that both types were ‘observational equivalence’. In other words, both RE models and conventional structural models shared the same reduced-form VARs and hence were empirically undistinguishable from each other. This is apparent if the reduced form (4.5) of a RE model is compared with (2.2), the reduced form of a conventional structural model.\footnote{The RE model that Sargent (1976) used to demonstrate ‘observation equivalence’ is an alternative to (4.3)-(4.5). Instead of using the REH definition of information set based conditional expectation, he utilised the equivalent definition of the innovation process in (4.3) to obtain a RE model expressed in terms of unanticipated shocks of exogenous variables to the endogenous variables: 

\[
E(y_{t} | I_{t-1}) = \sum_{i=1}^{p} \Gamma_{1,i} y_{t-i} + \sum_{i=6}^{p} \Gamma_{2,i} (z_{i,t-1} - z'_{i-1})
\]

This type of representation enables macro economists to attach structural interpretation to the error term of a VAR model, e.g. as structural shocks due to unexpected news or policy manipulations.}

From the standpoint of applied econometricians, ‘observational equivalence’ implies essentially that a closed VAR is the most general data-based representation of model-consistent expectations, with both the RE model and the non-RE dynamic SEM as its special cases.\footnote{From the viewpoint of some macro theorists, however, the lack of identification power of econometrics in front of many theoretically interesting but parametrically sophisticated RE models shows the limitation of statistical hypothesis testing methods. As a result, empirical macroeconomics branches into two directions, see e.g. (Summers, 1991), one still pursuing macroeconometrics whereas the other abandoning statistical}

5. Rise of the VAR approach: Pursuit for data-coherent theories

In fact, unrestricted VARs had already been utilised as the model base of deriving econometric tests for theory evaluation prior to the RE movement. The ground-breaking
work was made by Granger in the late 1960s, in the form of a test which is now widely referred to as Granger-causality test.

Granger became first involved in economic time-series research led by Morgenstern at Princeton University, see (Phillips, 1997). The research fostered his interest in searching for a method to assess causality between two variables empirically, i.e. based upon their dynamic bivariate relationship. The search led him to the idea of defining such causality by means of cross-spectrum information put forward by physicist, N. Wiener. Granger adopted the idea and proposed a causality test which was set within a closed VAR model with white-noise errors:

$$\begin{align*}
y_t &= \sum_{i=1}^{p} \Pi_{11,i} y_{t-i} + \sum_{i=1}^{p} \Pi_{12,i} z_{t-i} + u_{1,t} \\
z_t &= \sum_{i=1}^{p} \Pi_{21,i} y_{t-i} + \sum_{i=1}^{p} \Pi_{22,i} z_{t-i} + u_{2,t}
\end{align*}$$

The test decides that $y$ is not causing $z$ when $\Pi_{21,i} = 0$, i.e. when the past $y$ exerts no influence or predicting power on $z_t$, see (Granger, 1969). Note that the notion of causality in the test departs from the static notion or the notion of simultaneity upon which mainstream economics definitions of causality have normally been specified.

The significance of Granger’s (1969) paper was recognised almost immediately by Sims for its intimate link with Wold’s causal chain model and also with the ‘strictly exogenous’ assumption. Sims (1972) applied Granger causality test to the money-income dynamic relationship in order to show how ‘unidirectional causality’ between money and income could be concluded from time-series information to infer about a priori theory. The application elicited a great deal of interest ‘because it came out in the peak of the monetarists-Keynesian controversy’ (Hansen, 2004). The test was also adopted by Sargent methods to develop computable dynamic general equilibrium models, e.g. see (Kydland and Prescott, 1991; 1996).
and Wallace (1973) to assess the validity of their extension of Cagan’s model of hyperinflation by the RE hypothesis. Subsequently, Granger causality test became a core tool among RE economists, see (Sent, 1998; Chapter 3).

The reformatory RE movement sent stimulus to many econometricians to reflect about methodological issues concerning macro-econometric practice. Lucas (1976) critique and Sargent’s observational equivalence were particularly provoking. ‘Test, test, test’ became the golden rules of macro-econometric research (Hendry, 1980). This research trend virtually abandoned the assumption that applied econometricians should have true models ‘about which nothing was unknown except parameter values’ (Hansen, 2004, p276). A prominent school of methodology to emerge from the trend was the VAR approach.

The rise of the VAR approach intertwined intimately with the RE movement. The methodological blueprint of the approach was set jointly by Sargent and Sims (1977). The title of their paper, ‘Business cycle modelling without pretending to have too much a priori economic theory’, declares clearly their choice of a data-driven stance. Interestingly, Sargent and Sims justified their choice by referring much of the existing practice in macroeconometric modelling as ‘measurement without theory’, a famous criticism with which Koopmans (1947) had charged the NBER (National Bureau of Economic Research) thirty years before for their data-driven modelling approach, see (Qin, 1993; Chapter 6.4). Acknowledging that there was no adequate a priori economic theory, Sargent and Sims proposed an alternative route, following Liu (1960), of starting econometric modelling from an ‘unrestricted’ reduced-form VAR which would capture adequately the important

---

14 Note that the test can be readily used to Equation (4.4) of the RE of the variable $z$.

15 Much of the crossbreed occurred at the University of Minnesota, where Sargent, Sims and Wallace were employed on the Economics faculty during the 1970s, e.g. see (Hansen 2004) and (Evans and Honkapohja 2005). Apart from university duty, they were also involved part-time at the Federal Reserve Bank of Minneapolis. Most of the influential papers on RE models and the VAR approach appeared first as the FRBM Working Papers, see the Bank website archive.
statistical regularities in data. Under this alternative, a key issue was to search for ways of simplifying the unrestricted VAR. Sargent and Sims resorted to \textit{a priori} postulated economic theories as the main source of the simplification, as the theories could be used as cross-equation linkages/restrictions. For example, they suggested the use of theoretical models which characterised economic dynamics by a few indices.\footnote{Interestingly, they viewed this type of models as \textquote{observable index models} and demonstrated that a typical Keynesian macro model could be deduced into a two-index (one nominal and one real) model and that a simple RE model could be seen as one-index (an unanticipated shock) model. The latter amounts to transforming the RE model (4.6) into a moving averages of the error term.}

Once a simplified VAR was obtained, model performance was assessed by means of impulse response analysis, on the basis of the mathematical equivalence between a VAR and a moving average (MA) of innovation shocks, e.g. (4.5) could be rewritten as:

\[
\begin{pmatrix}
y \\
z
\end{pmatrix}_t = \sum_{j=0}^{\infty} \begin{pmatrix}
\Gamma_{11} & \Gamma_{12} \\
\Gamma_{21} & \Gamma_{22}
\end{pmatrix} \begin{pmatrix}
u_y \\
u_z
\end{pmatrix}_{t-j}
\]

The MA form was considered attractive not only empirically but also theoretically, since the error term had acquired the interpretation of unanticipated structural shocks under RE modelling, see (Sargent, 1976) and also Footnote 12.

A fuller methodological exposition of the Sargent-Sims joint venture was in Sims’ (1980a) critique on the existing econometric practice, which is now widely considered as the manifesto of the VAR approach. Here, Sims started his arguments again by reviving Liu’s (1960) critique on the arbitrary imposition of identification restrictions. He deemed these identification restrictions \textquote{incredible} on the ground that they lacked sound theoretical underpinning and would therefore not serve to bridge data with theory. However, Sims went further than Liu. He used the recent RE movement to highlight the importance of having empirical models as an adequate base for theory evaluation, not just as a means of forecasting. Sims believed that empirical models should primarily capture, as best as
possible, the information of regular, dynamic interaction among macro time series. This prerequisite sustained his proposal to carry out macro-econometric modelling under ‘an alternative strategy’.

Macroeconometrics under this strategy consists of three steps. The first step is to start from an unconstrained closed VAR model to summarise the data. The second step is to simplify the VAR by ‘limiting the nature of cross-dependencies between variables’. Granger causality test can serve as a power means to assist the simplification. But more importantly, economic theories are introduced here as possibly feasible restrictions on cross-equation parameters. The Bayesian approach tried by Leamer (1973) is considered as a promising method to implement the imposition of simplifying restrictions, as the method can duly reflect the uncertainty of the restrictions in the form of Bayesian priors. The final step is to perform impulse response analysis of the simplified VAR based on its MA transformation (5.2). The analysis is to serve a dual purpose. It is a convenient way for modellers to check whether the resulting model possesses ‘reasonable economic interpretation’; it is also an intuitive way for model users to conduct policy simulations as well as conditional forecasting.

The new VAR strategy is designed essentially to combat the uncertainty in structural model formulation and specification, which is effectively the issue of ‘model choice’ left aside by the CC group. In Sims’ view, the VAR strategy can reduce the uncertainty by filtering only those data-coherent theories, hence ‘making economics credible’ (Sims, 1987). Note that Sims considers this filtering process as a crucial aspect of ‘identification’, an aspect beyond and enhancing the scope of the CC concept of identification embodied in the rank and order conditions. Interestingly, there is great resemblance between Sims’ idea of identification and the identification step of the Box and Jenkins (1970) time-series methodology. Nevertheless, the aim of the strategy remains unchanged from the CC course
– to provide, as much as possible, theoretically and statistically sound structural models which would be particularly useful for policy analyses.

6. Structural VARs: Restoring CC heritage

Sims’ alternative strategy elicited wide interest and his critique brought about heated methodological debates. The VAR strategy was branded ‘atheoretical macroeconometrics’, see (Cooley and LeRoy, 1985) and described as ‘dissented vigorously from the Cowles Commission tradition’ (Pagan, 1987). The strategy was criticised for reliance on non-structural VARs and on Granger causality test to verify exogeneity. Amid all these controversies, however, Sims’ methodological pursuit remained amazingly close to the CC spirit. This is manifest from both his intensive engagement in applied macroeconomic issues and his devotion to make macro-econometric models as theoretically sound as possible to ensure the usefulness of the models for policy analyses.

In respect to macroeconomics, Sims kept close track of topical issues, especially those discussed intensely by RE economists. For example, he made a careful comparison of the policy implications of monetarist versus non-monetarist theories against time-series data evidence (1980b); he wrote a lengthy and convincing exposition on why Lucas critique should not be taken as a taboo against using econometric models for policy analyses (1982); he attempted persistently to try and match empirical findings with dynamic equilibrium theories, see e.g. (Sims, 1983) and (Leeper and Sims, 1994). Note also that Sims’ applied research interest was shared by other adherents of the VAR approach, e.g. see (Blanchard and Watson, 1984), (Bernanke, 1986), (Litterman and Weiss, 1985).

In econometrics, Sims and his followers devoted their concerted efforts to defend the new VAR approach in line with the CC structural modelling spirit, e.g. see Sims ardent expositions (1986; 1987; 1989) on why VAR models could be well used for policy
analyses as well as forecasting. Their efforts led to a conscientious movement to transform the VAR approach into a structural VAR (SVAR) approach.

In fact, the second step of the VAR approach drew almost the full research attention from Sims and the VAR approach followers. From his earlier investigation in the money-income causality, Sims noticed that VAR-based inference on causality was very sensitive to the number of variables included. While it was theoretically ideal to include as many variables as possible in the starting VAR, such inclusion was seriously hampered in practice by the limited parameter space under finite data samples. Development of a general method, therefore, which would enable simplification of the unrestricted VAR without disturbing its data coherence and its logical consistency of the stochastic specification, took priority in the research agenda.

As mentioned in the previous section, Sims was attracted by the Bayesian approach for its explicit expression of uncertainty of a priori restrictions in terms of Bayesian priors. Moreover, much of the motives underlying Leamer’s (1978) specification search under the Bayesian approach was shared by the Sims and his collaborators. Initial experiment with a Bayesian VAR approach was explored by Litterman (1979) around the time Sims was preparing his critique. The approach was more fully explored by Doan et al (1984). In this joint work, a small number of Bayesian priors are imposed on each equation of the unrestricted VAR to help simplify mainly the lag lengths (both own lags and lags of other variables) and time variations of the parameters, and the equations are estimated one by one using the same set of priors to keep the symmetry of the VAR. A data-coherent and

17 See also Sargent’s (1981) sympathetic interpretation of Sims’ position.
18 In this case, the causality result changes with the money-income relationship when interest rate is added into the VAR, see e.g. (Sims 1980b; 1989) and (Hansen, 2004).
19 See (Qin, 1996) for a historical description of the rise of the Bayesian econometrics.
simplified VAR will result after numerous experiments through adjusting the priors. Although the simplification is not explicitly guided by economic theories, the resulting cross-equation relations are believed to embody theoretical interdependence, based on the argument that dynamic stochastic theories were special cases of a general VAR.

Once the simplified VAR model is interpreted as an empirical counterpart of a dynamic equilibrium model, it becomes justifiable to apply the model for economic analyses, including policy analyses. The main method used for such analyses is impulse response analysis, as shown earlier in (Sargent and Sims, 1977). However, the economic interpretability of each impulse shock entails cross-equation independence between equation shocks in the simplified VAR. When the VAR residuals are found to be cross-equation correlated, the proposed solution originally is to re-order the equations by Wold’s causal chain method and to use statistical methods to orthogonalise the residuals, i.e. to transform $u_t$ in (5.2) by a triangular matrix defined statistically, say by Cholesky decomposition method, e.g. see (Sims, 1980a):

$$x_t = \sum_{j=0}^{\infty} \Pi^j u_{t-j} = \sum_{j=0}^{\infty} \Pi^j \Psi^{-1} \Psi u_{t-j} = \sum_{j=0}^{\infty} (\Pi^j \Psi^{-1}) u_{t-j}$$

However, the orthogonalisation method and the imposition of Bayesian priors tried by Doan et al (1984) met with the most vehement attack for the very thin economic interpretability that the two methods were able to sustain. Efforts to slake the criticism led VAR researchers to a rapid retreat to Sargent and Sims’ (1977) position of seeking simplifying restrictions mainly from economic theories.

A structural approach for identifying the error terms, i.e. to make them independent across equations, was initiated by Blanchard and Watson (1984) and extended by Bernanke

---

20 Several interesting results emerged in the Doan et al (1984) experiment, including the assumption of a unit root in each variable, i.e. the prior on the own first lag taking the value of one, and most of the parameter estimates revealed little time-varying feature when time-varying priors and Kalman filter were used.
(1986). The approach essentially exploits the relationship of $A_0 u_t = \varepsilon_t$ in (2.2), or (4.1), i.e. to ensure that $A_0$ contains adequate restrictions derived from \textit{a priori} static theory to enable the orthogonal structural errors, $\varepsilon_t$, to be identified from the ‘reduced-form’ errors, $u_t$. As for cross-equation parameter restrictions, Blanchard and Quah (1989) proposed to utilise the money neutrality postulate as a restriction on the long-run parameter embedded in a VAR, as differentiated from demand shocks, which were assumed to only exert transitory, i.e. short-run, impact on output. Their proposal established a direct link between SVAR models and Engle and Granger (1987) cointegration theory. As a result, research thrived in SVARs with embedded cointegrating long-run conditions, e.g. see (King \textit{et al}, 1991).

All the endeavour of the SVAR research aims ultimately at maintaining and strengthening the economic validity of the simplified-VAR-based impulse analysis for policy purposes. In case when not all of the structural error terms could be orthogonalised after the imposition of all the available structural restrictions, SVAR advocates would take the explicit position to assume that the error correlation should totally reflect structural interdependence rather than the ‘passive responses’ between equation disturbances, see (Hansen, 2004).

7. Concluding Reflection

This historical investigation demonstrates that the VAR approach arises as a methodological revision and renovation of the CC structural modelling approach. Stimulated by the RE movement in macroeconomics, the VAR approach offers a systematic procedure to tackle the issue of ‘model choice’ bypassed by the CC researchers. The procedure embodies a synergy of various preceding methods explored by prominent econometricians and statisticians alike, such as Tinbergen’s ‘kitchen work’ (1937), Liu’s (1960) appreciation of a general VAR, Leamer’s sinning ‘in the basement’ (1978), as well as Granger’s time-series approach to causality (1969) and the concept of model
identification in Box and Jenkins’ time-series methodology (1970). However, the synergy has not abandoned the CC tradition of theory allegiance, policy-oriented research target and technical rigour.

Why has the VAR approach been widely regarded as ‘atheoretical macroeconometrics’ in rivalry with the CC structural modelling methodology? On the face of it, the ‘atheoretical’ attribute seems to be an over-generalisation of the unrestricted VAR used at the initial step to the entire approach. Historically, two key factors have probably contributed greatly to the over-generalisation. First, the initial step constitutes the most drastic proposition of the VAR approach, as it advocates a reversal of the structural→reduced-form sequence, a reversal with serious methodological implication of backtracking Frisch’s (1937) ‘structural’ method in favour of Tinbergen’s (1935) ‘historical’ method. Secondly, the polemics that VAR advocators have employed to criticise the macroeconometric practice at the time encouraged the public view that they demarcated their methodological position from the CC methodology. This is by no means to ascribe the ‘atheoretical’ misnomer to VAR advocators themselves. The historical era that they found themselves in is considerably different from the early 1940s, when the first generation of econometricians were compelled to formalise econometric practice in order to gain its recognition in the economics discipline, e.g. see (Gilbert and Qin, 2006). Textbook econometrics had already been established under the banner of the CC approach at the turn of 1970. Any attempts to challenge it would require substantive justification and rationalisation.

Interestingly, the inherent weakness of the CC structural methodology is highlighted by the RE movement, which has actually set out to emulate the CC econometrics. Growth of REH-based models has brought protean connotations to the terminology of ‘structural’ models. In comparison to REH-based models, those old and simple macroeconomic models
are considered by many as ‘non-structural’ for lack of explicit scheme of optimizing behaviour of representative agents, e.g. see (Sims, 1991). Once macroeconomics proliferates in the form of stochastic, dynamic equilibrium models, econometricians are faced not only with more complicated parameters to measure, but also with various testable hypotheses to verify and compare. The moving ‘structural’ post makes it no longer possible for econometricians to anchor their starting position on given ‘true’ models. The VAR alternative is avant-garde in this light. It marks an explicit shift of attention among macroeconometric modellers – from measurement of given theories towards identification and verification of data-permissible theories. Following suit is a shift of research methodology in macroeconometric modelling – from mainly confirmatory analysis towards a mixture of confirmatory and exploratory analyses.

It is only with respect to its exploratory component that the VAR approach may be seen as ‘atheoretical’. The methodology is otherwise starkly faithful to the CC tradition. As shown in the previous section, VAR modellers have devoted a great deal of their efforts to mend the link between structural and reduced-form models; they have placed the issue of structural identification on top of their research agenda; they have attached far greater importance to theory testing and policy analysis than to forecasting; moreover, they are convinced that the ‘ideal model’ is one which ‘contains a fully explicit formal behavioural interpretation of all parameters’, ‘connects to the data in detail’, ‘takes account of the range of uncertainty about the behavioral hypotheses invoked’ and ‘includes a believable probability model that can be used to evaluate the plausibility, given the data, of various behavioural interpretations’ (Sims, 1989).

In short, the history reveals that the rise of the VAR approach constitutes more of a reformation than a revolution of macroeconometrics. This is the most evident from the fact that both VAR and CC researchers share the same fundamental conviction that
macroeconometric modelling should always be formulated, specified and estimated in a system of interdependent equations. While the CC group emphasise economic interdependence at a static level and correspond that with a Walrasian equilibrium model, VAR researchers accentuate dynamic interdependence and correspond that with a dynamic general equilibrium model. Noticeably, the latter correspondence sustains crucially the validity of using the impulse response method for policy analyses, as the method entails an unequivocal ‘structural’ interpretation of the error terms. The interpretation assumes away the possible existence of any theoretical ignorance or omission, which should be contained inseparably in the residuals of any fitted structural models. However, the general invalidity of this assumption is apparently camouflaged by the system-of-equations stance. In constructing a system of adequately dynamic stochastic equations where all variables involved are endogenised, VAR modellers are prone to the belief that misspecification is absent through fully stochastic dynamic specification of a multiple-equation system guided by, and consistent with, theory, e.g. see Sargent’s (1981) interpretation of economic time series. The ceteris paribus clause well associated with any theories is forgotten. In comparison, modellers who follow the Bayesian approach or the LSE dynamic specification approach have taken a more agnostic position than the VAR modellers.


22 Interestingly, Sims (1991) criticized modellers following the LSE approach for not being really structural.
References


Hendry, D.F. and M.S. Morgan (1994) The ET interview: Professor H.O.A. Wold: 1908-


Klein, L.R. (1971) Forecasting and policy evaluation using large scale econometric models:
the state of the art. In M.D. Intriligator (ed.) *Frontiers of Quantitative Economics*.
Amsterdam: North-Holland, pp. 133-63.

**29**, 161-79.


McGraw-Hill.

Kydland, F. and E. Prescott (1991) The econometrics of the general equilibrium approach


Leeper, E.M. and C.A. Sims (1994) Towards a modern macroeconomic model usable for


