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The Elusive Persistence: Wage and Price Rigidities, the Phillips Curve, and Inflation Dynamics

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THE ELUSIVE PERSISTENCE:
Wage and price rigidities, the Phillips Curve, and inflation dynamics

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Abstract
We review the main New Keynesian inflation equations that have arisen as a result of aggregation from individual firms’ price rigidities. We find that, on the whole, they cannot account for inflation persistence, a key feature of the empirical dynamics of inflation, and with important policy implications. The only exception seems to be when price stickiness is combined with wage rigidity and staggering.

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1 Introduction
Though inflation has been drastically reduced in the last 15 years in industrial economies (a process known as the “great moderation”), it continues to be at the centre of macroeconomic debates. One of the primary reasons is its central place in macroeconomic policy-making, enshrined in inflation targeting and the mandates given to most (by now independent) Central Banks. There has therefore been a resurgence of interest in recent years for its theoretical and empirical properties.

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New Keynesian (NK) models build inflation from individual price and/or adjustment decisions that are costly and/or infrequent. While the earlier generation of NK models emphasised wage stickiness, newer generations have relied on price stickiness. Among these models, the best known is the Calvo (1983) model, which is based on infrequent and random price adjustment; due to its simplicity and tractability it has rapidly risen to canonical status in this strand of literature. Many other time-contingent models may be seen as extensions of it. McCallum (1997) calls it “the closest thing there is to a standard specification”. Clarida, Gali and Gertler (1999) place it at the centre of their influential review of monetary policy. In their recent review of empirical findings on inflation and their implications for macromodelling, Angeloni et al (2006) conclude that the Calvo equation is at least as consistent with empirical findings as its alternatives, and continues to play a fruitful role in the theoretical analysis of inflation. Its properties are mirrored in many of the more extended models.

However, the standard NK inflation framework, as exemplified in the Calvo model and its variants, involves well-known empirical problems:

- It fails to capture inflation persistence and inertia, which appears by now to be a stylised empirical fact (Fuhrer and Moore, 1995; Roberts, 1997; Gali and Gertler, 1999; Rudd and Whelan, 2006). Altissimo et al (2006) argue that this persistence weakens somewhat if structural breaks in inflation are taken into account in estimation, but even then, persistence does not disappear. By persistence, one means intrinsic dynamics, and not the extraneous ones that may be implicit on the RHS drivers of any inflation equation, such the output gap or marginal cost (a point also emphasised by Angeloni et al, 2006). Thus, persistence should most appropriately be assessed by the significance of the coefficient of lagged inflation in an inflation equation, and not by the autocorrelation of inflation which in general results from both intrinsic and extrinsic dynamics (Rudd and Whelan, 2006). The problem then is that empirical inflation equations invariably show a highly significant lagged inflation term, yet that term is entirely missing in most theoretically derived Calvo-style equations.

- Furthermore, its forward-looking component is often empirically rejected (Fuhrer, 1997; Rudd and Whelan, 2006). This however has been challenged in a series of papers by Gali and Gertler (1999, 2001, 2005), who argue that the empirical fit of the simple New Keynesian model is quite good and the forward-looking element in it crucial. However, the controversy regarding the significance of the forward-looking element remains. This is very important, because this element epitomises most succinctly the forward-looking nature of price setting, which is at the heart of the New Keynesian formulations.

- As pointed out by Mankiw (2001) and Mankiw and Reis (2002), it fails to give rise to
inverted U-shaped effects of monetary expansions, like those documented by Christiano, Eichenbaum and Evans (2005).

- Further, it implies expansionary effects of credibly announced disinflations (Ball, 1994) – as inflation is forward-looking, it reacts now to the announcement of less monetary growth in the future. As a result, the quantity m-p=y increases in the period between now and the actual implementation of the disinflationary stance.

- Interpreted as a NAIRU or “accelerationist framework” (the change in inflation to be related to a measure of output gap), then the output gap has the wrong sign (Gali and Gertler, 1999; Mankiw and Reis, 2002).

Among those properties (or shortcomings), one of the most fundamental may be argued to be the failure to capture inflation persistence. Persistence of inflation is very important for the conduct of monetary policy (Angeloni et al, 2006). A persistent inflation would not decrease right now (or not as much) by the anticipation of future disinflation and would not result in expansion now; furthermore, it has the potential to capture the rich dynamic adjustment of real output to monetary policy shocks (see Mankiw and Reis, 2002, Fig. IV).

This paper reviews the main NK inflation specifications, starting from the Calvo (1983) model and proceeding to variations and extensions, and examines their dynamic properties. The main emphasis is on the resulting inflation persistence and the degree to which inflation dynamics is forward or backward-looking. Secondarily, we examine whether the resulting specifications exhibit the NAIRU property, i.e. whether they result in a trade off between output and inflation or inflation change; and whether the sign of the correlation between output and inflation change conforms with data (i.e., positive), if the NAIRU property is exhibited.

Among the variants and extensions reviewed here are the following well known benchmarks:

- We begin with the Calvo (1983)-based time-contingent model of constant probability of price adjustment based on the fixed nominal price adjustment (or ”menu”) cost. A straightforward variant is derived in, among others, Tsoukis (2000). The Calvo (1983) equation is also shown to be compatible with a state-contingent (S,s) model in Gertler and Leahy (2005). Romer (1990) endogenises the probability of adjustment within the same framework.

- Mankiw and Reis (2001) and Devereux and Yetman (2002) ”sticky information” or ”predetermined pricing” model, which is based on the idea that it is full optimisation
over prices (and not price adjustment *per se*) that is infrequent, and that in between optimisations, prices are updated mechanistically by the rate of inflation (see below).

- Model of quadratic “real” costs of price adjustment (see Rotemberg, 1982), on the premises that some of the costs are on convincing markets and customers that price changes are necessary (see again below).


- The variant of the above strand that allows for measurement error in money and signal extraction of real money growth from error/noise (Collard and Dellas, 2006).

- Dotsey, King and Wolman (1999) and Dotsey and King (2006) provide a hybrid time- and state-contingent specification and provide a further step towards generalising the Calvo-derived inflation equation.

The variety of approaches in the literature is not surprising. It is motivated in part by our ignorance about the nature of price adjustment costs. A series of papers by a group of researchers (S. Dutta, D. Levy, M. Bergen, R. Venable and M. Zbaracki) investigate such costs for a variety of firms using data from the US. In a typical estimate, they find that “menu costs for the chain drugstore average $0.33 per price change which constitutes about 0.59 percent of revenues” (Dutta et al., 1999). These costs are comparable to findings from supermarkets (Levy et al., 1997). These costs are not trivial, particularly considering the very tight profit margins of such retailers. But in an interesting twist, investigating a large US manufacturing firm with a large network of salesmen, Zbaracki et al. (2004) find that the managerial costs (gathering information, holding meetings to finalise prices, informing the rest of the firm) are more than six times the physical (menu) costs of price changes, and that the customer costs (informing and convincing customers) are more than twenty times the physical costs. These numbers are substantially (and surprisingly) high, and suggest that lumpy menu costs, on which most of NK models have been built including the Calvo inflation model may not be the most important type of costs. Instead, one needs to seriously consider two other types of costs: informational and managerial costs, which imply that it is the optimisation that is costly, not the price change as such; and customer costs, which are aimed at informing and persuading customers that the price changes are warranted. Such
costs increase with the magnitude of price changes and may be approximated quadratically. The implications of these types of cost are considered below.

To facilitate comparison, this paper derives the various models as variants and extensions of the same basic framework; thus, one of its contributions is to unify a number of seemingly disparate models into a coherent whole. It can therefore be read as a review of the literature. Furthermore, spelling out explicitly the inflation implications of the various models of price adjustment is another contribution of this paper, as such implications have often not been derived outside the wider context (business cycle models, etc) in which price adjustment is embedded. Finally, this is not to forget the paper’s main aim, which is to examine and compare the dynamic properties of different inflation equations. This is done in an analytical manner, both in order to preserve intuition, but also because some of these models are observationally equivalent; so, this work logically precedes empirical estimation. Wherever possible, we derive closed-form solutions rather than structural equations; the coefficients in the latter may not be true indicators of persistence or forward-lookingness (Rudd and Whelan, 2006). To preamble, we find that only a limited range of models is able to deliver inflation persistence in line with what empirical work has uncovered. The variants that seem to have the biggest potential for doing so are the one that includes wage staggering and rigidity as well as price rigidity and the one incorporating inflation indexation of individual prices. The baseline Calvo model is introduced in Section 2. Extensions and variations for this baseline model are derived and reviewed in Section 3 which forms the main part of the paper. Finally, Section 3 concludes.

### 2 A baseline Inflation model: Calvo (1983)

Calvo (1983) presents a tractable and elegant specification of inflation explicitly derived from the microfoundations of infrequent individual price adjustment. It is notionally based on the existence of a fixed, lump-sum “menu cost” of price adjustment. As mentioned, this model and the resulting inflation equation remain at the forefront of modern macroeconomics; much subsequent work has built on it, while retaining its logic. We therefore take it as departure point, and subsequently extend it in various directions. The overarching question is what, if any, of those extensions can best account for inflation persistence.

We present a minor extension of the Calvo (1983) model (see Clarida, Gali and Gertler, 1999; Tsoukis, 2000) as our baseline model. Though the model is well known by now, we sketch it here as the same steps will be followed in all the extensions. It is assumed that there is a large number n of monopolistically competitive firms, which revise infrequently and randomly their prices because of menu costs. The time between consecutive adjustments is
random but exogenous: In each period, a proportion of firms \((1-\lambda)\), \(0< (1-\lambda) <1\), adjust their price. The probability of adjusting after \(s+1\) periods from the last adjustment will be given by the geometric distribution: \(f(s) = (1-\lambda)\lambda^s\), therefore the mean waiting time is \(\lambda/(1-\lambda)\). Because of the large number of firms, the same distribution determines the percentage of prices as of time \(t\) that were set at time \(t-s\). The price level at \(t\) will be approximated in a standard log-linear way as the sum of all individual prices holding at \(t\), \(p_t = \frac{1}{n} \sum_{i} p^i_t\), where \(p_t\) is the price level and \(p^i_t\) is the nominal price of firm \(i\), all in logs. Thus, using the distribution of \(s\), the price level will be:

\[
p_t = (1-\lambda) \frac{p^i_t}{1-\lambda L}
\]

where \(L\) is the lag operator.

In setting its nominal price at time \(t\), firm \(i\) maximises expected log profits; equivalently, given optimal prices (specified below), it minimises expected deviations of profits from maximal ones: those that will accrue with optimal prices at all times. These profit deviations are approximated by the squared actual-optimal log price deviations (see Walsh, 1998):

\[
\ell_t = E_{t-1} \sum_{s=0}^{\infty} (\lambda \theta)^s (p^i_t - \tilde{p}^i_{t+s})^2
\]

\(\ell_t\) is total lifetime costs from price adjustment at the point of setting a new price. \(\tilde{p}^i_{t+s}\) is the future log optimal price at \(t+s\). The discount factor \(0< \theta \leq 1\) is augmented by the probability \((\lambda)\) that the newly set price (at the beginning of period \(t\)) will last an additional period. The timing convention is that prices are set at the beginning of the period; \(E_{t-1}\) is expectations at the end of period \(t-1\). The FOC is:

\[
E_{t} \sum_{s=0}^{\infty} (\lambda \theta)^s (p^i_t - \tilde{p}^i_{t+s}) = 0
\]

This may be written more compactly as,

\[
p^i_t = (1-\lambda \theta) \frac{\tilde{p}^i_t}{1-\lambda \theta L^{-1}},
\]

where \(L^{-1}\) is the expected lead operator (e.g. \(L^{-1}x_t \equiv E_t x_{t+1}\)).

To proceed, we specify expected optimal prices. Various avenues can be followed here. The vintage imperfect competition literature (Ball, Mankiw and Romer, 1988; Blanchard and Kiyotaki, 1987; Dixon and Rankin, 1994;) specifies optimal prices as a positive function of output gap. More microfounded models (reviewed below) relate optimal prices to marginal cost. Gali and Gertler (1999) approximate the latter with labour’s share in GDP. As a start, we follow the first option and postulate prices as a function of the (observable) last period’s output gap:
\[ \hat{p}_t = p_t + \psi y_{t-1}, \quad \psi \geq 0 \]

\( y \) is the output gap (percentage excess real demand relative to the steady-state (growth) path). \( \psi \geq 0 \) is the responsiveness of optimal price to the output gap, closely linked to monopoly power and the notions of real price rigidity and strategic complementarity: \( \psi = 0 \) is associated with perfect competition, whereas the greater \( \psi \), the greater the monopolistic power of firms in a Dixit-Stiglitz framework and strategic complementarity in price setting, and the lower the degree of real price rigidity.

The key equations (1 – backward-looking price level), (3 – forward-looking newly set price) and (4 – optimal price) are now combined to yield:

\[
((1 - \lambda L)(1 - \lambda \theta L^{-1}) - (1 - \lambda)(1 - \lambda \theta)) p_t = (1 - \lambda)(1 - \lambda \theta)\psi y_{t-1}
\]

The LHS of (4) may be written as a polynomial of the form \( -L^{-1}(\alpha - \beta L + \lambda L^2) p_t \), with parameters \( \alpha \equiv \lambda \theta \) and \( \beta \equiv \lambda + \lambda \theta \). The roots are (where roots here are taken to be the inverse of the zeros of the lag polynomial):

\[
L_{1,2} = \frac{\beta \pm \sqrt{\beta^2 - 4\alpha\lambda}}{2\alpha} = \frac{\lambda + \lambda \theta \pm (\lambda - \lambda \theta)}{2\lambda \theta} = \begin{cases} \frac{1}{\theta} \\ 1 \end{cases}
\]

This polynomial is therefore written as: \( -L^{-1}(1 - L)(1 - \theta L^{-1})p_t \). As \( 1/\theta > 1 \), the first root is solved forward to yield a lead operator with coefficient \( \theta \); that is, the polynomial is rearranged as: \( \theta^{-1}(1 - L)(1 - \theta L^{-1})p_t \). Taking into account the timing convention and the appropriate inflation definition, this results in the inflation equation,

\[
\pi_{t-1} \equiv \Delta p_t = \frac{\omega y_{t-1}}{1 - \theta L^{-1}}, \quad \omega \equiv \theta(1 - \lambda)(1 - \lambda \theta)\psi > 0
\]

or, re-introducing expectations and shifting one period forward:

\[
\pi_t = \theta E_t \pi_{t+1} + \omega y_t
\]

Thus, the combination of backward-looking price level and forward-looking new price results in a unit root (hence, the price equation becomes an inflation equation) and an unstable root which is solved forward. The inflation equation of Calvo (1983) is derived without optimisation; alternatively, it represents the special case of no discounting (\( \theta = 1 \)) and unitary forward coefficient. We may now state:

**Observation 1:** In the baseline NK specification:

1. The inflation process is entirely forward-looking without any persistence (no lags);

2. With discounting (\( \theta < 1 \)), there is no NAIRU property. Steady-state inflation happens to be 0 here (as \( y = 0 \)), but with more microfoundations, steady-state inflation is positive
and influenced by monopoly power (see below sub-Section 2.6). With $\theta = 1$ as in Calvo (1983), we have the NAIRU property, but the correlation between $y_t$ and $\Delta \pi_t$ (change in inflation) has the wrong sign (see Gali and Gertler, 1999).

3 Extensions of the baseline Inflation model

3.1 Quadratic costs of adjustment

This specification augments the baseline model with the quadratic costs of price adjustment a la Rotemberg (1982, 1996) and Hairault and Portier (1993). Such a notion of price adjustment costs receives some support from the micro evidence in Zbaracki et al. (2004) who uncovered evidence of customer information costs: Firms try to avoid losing customers by appropriately informing them and trying to justify their decision of a price change. More generally, there may be costs associated with losing customers in "customer markets". Clearly, customers would rather see smaller rather than larger price adjustments, whenever those occur. The cost to the firm of upsetting customers by changing prices may be convex, i.e. rising faster than the magnitude of the price change itself; quadratic costs may be an approximation to such general convex costs arising from price adjustment. As is well known, quadratic costs result in frequent and small changes; hence, price adjustments may be somewhat more backward-looking, and as a result, inflation may exhibit more inertia. This sub-Section investigates whether that is the case by adding quadratic costs of price adjustment to the Calvo setup.

The loss function now is:

$$\ell_t \equiv \phi (p^t_i - \bar{p}_{i-1} - p_t - p_{t-1})^2 + E_{t-1} \sum_{s=t}^{\infty} (\lambda \theta)^{s-t} (p^t_i - \tilde{p}^i_s)^2$$

(6)

$p^t_i$ is the newly set individual price as previously, and $\bar{p}_{i-1}$ is the price that existed last period which however was possibly carried over from previous periods (i.e., it was not a new quotation at that stage). Price adjustment costs are made up of the quadratic cost from being seen to raise prices faster than the general inflation, parameterised by $\phi > 0$, and the costs of being away from optimal price as long as the price is fixed at its current level. The baseline equation is the special case of no quadratic costs ($\phi = 0$) in this more general framework. The basic quadratic cost model emerges as the case of $\phi > 0$ and $\lambda = 0$ (Rotemberg, 1982).

Minimisation of (6) results in:

$$E_{t-1} \sum_{s=0}^{\infty} (\lambda \theta)^{s}(p^t_i - \tilde{p}^i_{t+s}) + \phi (p^t_i - \bar{p}_{i-1} - p_t + p_{t-1}) = 0$$
which may be written more compactly as:

\[
p_i^t \left( \frac{1}{1 - \lambda \theta} + \phi \right) = \phi \bar{p}_{i-1}^t + \frac{\bar{p}_{i-1}^t}{1 - \lambda \theta L^{-1}} + \phi (p_t - p_{t-1})
\]  

(7) replaces (3) as the optimal new price. It should be reminded that \( \bar{p}_{i-1}^t \) is the price that existed last period, not necessarily a newly set price at that stage.

To relate it to the price level at \( t-1 \), recall that "i" includes all firms whose prices were about to be reviewed the following period; it therefore encompasses a fraction \( (1-\lambda) \) of the prices prevailing at \( t-1 \). But these prices had been set in previous periods by the same frequency process as the general price level, therefore by the constancy of the adjustment probability across time and firms, they equalled the general price level, \( E_i \bar{p}_{i-1}^t = p_{t-1} \). To see this more formally, note that given the mass of unity of firms and the frequencies postulated above, we can convert this into previous new quotations by the formula:

\[
E_i \bar{p}_{i-1}^t = (1 - \lambda) \frac{p_{t-1}}{1 - \lambda L}
\]

Combining with the definition of price level (1), we then have \( E_i \bar{p}_{i-1}^t = p_{t-1} \), as stated.

Considering that firm is is "the typical" firm, we may more simply write \( \bar{p}_{i-1}^t = p_{t-1} \). Optimal prices continue to be given by (4). We therefore get from (7):

\[
\left( \frac{1 - \lambda L}{1 - \lambda} \right) p_t \left( \frac{1}{1 - \lambda \theta} + \phi \right) = \frac{p_t + \psi y_{t-1}}{1 - \lambda \theta L^{-1}} + \phi p_t
\]

Tedious algebra allows us to factorise this as:

\[
((1 - \lambda L)(1 - \lambda \theta L^{-1}) - (1 - \lambda)(1 - \lambda \theta)) p_t + \lambda \phi (1 - \lambda \theta)(1 - \lambda \theta L^{-1}) \pi_{t-1} = \frac{(1 - \lambda)(1 - \lambda \theta)(1 - \lambda \theta L^{-1})}{1 + \phi(1 - \lambda \theta)}
\]

(8) replaces (4) of the baseline formulation. Now, as before, the polynomial in \( p \) on the LHS of (8) can be factored as, \( \theta^{-1}(1 - \theta L^{-1}) \pi_{t-1} \), resulting in the inflation equation (at \( t \)):

\[
\pi_t \left( (1 - \theta L^{-1}) + \phi \lambda \theta (1 - \lambda \theta)(1 - \lambda \theta L^{-1}) \right) = \frac{\theta(1 - \lambda)(1 - \lambda \theta) \psi y_t}{1 + \phi(1 - \lambda \theta)}
\]

So, there is a richer structure on the LHS, but persistence does not arise as only leads of inflation emerge. And again, the model does not exhibit the NAIRU property.

**Observation 2:** Under fixed probability of adjustment and quadratic costs:

1. The lag/lead structure of the inflation equation is richer than under the benchmark NK model.
2. But it is entirely forward looking again, showing no persistence.
3. In the steady state, the level of inflation is associated with the output gap, so no NAIRU property emerges.
3.2 Optimal frequency of price adjustment

Within a time-contingent framework, Romer (1990) and Devereux and Yetman (2002) have optimised (minimised 7) over the probability of (non)adjustment $\lambda$. The loss function now is the recursive formula:

$$\ell_t = m + E_t \sum_{s=0}^{\infty} (\lambda \theta)^s \left\{ \left( p_i^t - \tilde{p}_{t+s}^i \right)^2 + (1 - \lambda) \lambda^{-1} \ell_{t+s+1} \right\}$$  (9)

$m > 0$ is the fixed menu cost (though a “nominal” cost of price adjustment, it is in fact assumed constant in real terms). The above stipulates that price adjustment costs are made up of the menu cost, the cost of being away from optimal price as long as the price is fixed at its current level, and the new cycle of costs when the price is set anew. The last term is weighted by the probability of a new cycle starting in exactly $s+1$ periods, as well as by the discount factor. The trade-off is between the menu cost, pulling towards the direction of more infrequent changes, and the larger deviations from optimal (that get bigger the longer the price stays fixed), pulling towards the direction of more frequent changes.

Romer (1990) and Devereux and Yetman (2002) proceed to derive optimal new prices and inflation equations, both of which are the same as above. As usual, inflation is driven by the exogenous output gap (or whatever other driving process one has in mind, like the money supply). Then, firms minimise their ex ante loss function (9). Because of the squared price deviations, both future trend inflation and its variability matter. As a result, the frequency of price adjustment increases in trend inflation and its variability. The overall structure of the inflation equation in terms of leads and lags is however not altered, so this feature is unlikely to alter its dynamic properties from the perspective of this paper.

3.3 Sticky information and predetermined prices

Here, a price sequence (as opposed to a single fixed price) is determined at the beginning of the planning period, with prices being allowed to evolve between periods in a predetermined manner. This framework has been proposed by Mankiw and Reis (2002), Devereux and Yetman (2003) and Yetman (2003). This idea is based on evidence by Zbaracki et al. (2004) who found that managerial and information-gathering costs are more important than “menu” costs: Thus, it seems that it is the re-optimisation that is more costly, and not the price change as such. On this basis, Mankiw-Reis (2002) and Devereux-Yetman (2003) essentially argue that optimisation is more infrequent than price change: At each (re-)optimisation, a whole sequence of future prices is determined, based on current expectations. In between re-optimisations, prices do change, but according to the sequence or plan already in place.
Existing plans are revised infrequently and probabilistically, with a constant probability of revision each period (much as in Calvo, 1983).

In this framework, the price level is given by the same weighted sum of individual prices as with the Calvo scheme:

$$p_t = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i p_i^t$$

The weights now reflect the probability \( (1-\lambda) \) of revising a given price plan (and not simply a given price); thus, the existing plans now were stipulated at different times in the past according to the above scheme. The prices holding now were stipulated \( i \)-periods ago on the basis of the then information:

$$p_i^t = E_{t-i} \tilde{p}^i = E_{t-i}(p_t + \psi y_{t-1})$$

Straightforward substitution yields:

$$p_t = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i E_{t-i}(p_t + \psi y_{t-1})$$

Mankiw and Reis (2001) further re-write this in terms of inflation:

$$\pi_{t-1} = \frac{\psi(1 - \lambda)y_{t-1}}{\lambda} + (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i E_{t-1-i}(\pi_{t-1} + \psi \Delta y_{t-1})$$

Mankiw and Reis (2002) argue that this structure possesses some desirable properties in terms of inflation persistence, output response to disinflations, and inflation-output correlation. But it must be pointed out that this framework also suffers from drawbacks. Firstly, it does not possess any parsimonious closed-form solution, and therefore does not lend itself to estimation. (Note that this equation cannot be cast in the Blanchard and Khan, 1980, canonical form. Mankiw and Reis, 2002, provide a solution by the method of undetermined coefficients, but it is not parsimonious.) Secondly, there is little intrinsic persistence in this model beyond the extrinsic one already embodied in the output process \( y_t \). To illustrate, note that under the assumption that \( y_t \) is white-noise, the price equation (10) is satisfied with an IMA(1,1) model for the price level,

$$p_t = p_{t-1} + a(y_t - \xi y_{t-1}), \quad a \equiv 1 - \lambda, \quad \xi \equiv \lambda/(2 - \lambda),$$

which exhibits no inflation persistence.

The thrust of the idea is that prices fail to catch up fully with the optimal ones simply because of the lack of up-to-date information based on infrequent optimisation, as opposed to a menu cost. As such, the model should probably be categorised as an instance of bounded rationality; Mankiw and Reis (2002) themselves do not seem averse to this idea.
(see their concluding paragraph). But the model surely exhibits less price stickiness than the standard menu-cost driven model, and therefore it would be counter-intuitive if it showed more inflation persistence.

But its main weakness may be its lack of realism: Most goods prices are spot prices, and sequences of future prices are not postulated, except in well known commodities markets and where order queues exist. Another difficulty is that prices are assumed predetermined at any given time, and do not adjust even if the economy-wide inflation has evolved since the price sequence was initially decided in a way that was not forecast then (due e.g. to a monetary policy or supply-side shock). Yet, it is logical and consistent with the evidence of Zbaracki et al. (2004) to assume that it is fairly straightforward for firms to adjust prices in line with average inflation. Is it then possible that a variant of this model whereby prices are simply updated by the rate of inflation, to account for realism, could give the elusive persistence? This is examined next.

3.4 A variant predetermined pricing scheme: Price indexation

Here, we let prices be decided by infrequent optimisation, but let them be updated by the rate of inflation in a rather mechanistic way (i.e., the originally set prices are fully indexed). In this way, the model would combine elements of the predetermined pricing and “sticky information” models with a greater degree of realism in that no future goods prices need to be stipulated. The model is consistent with the evidence of Zbaracki et al. (2004) mentioned above, and may be thought of as a simplified version of the predetermined pricing schemes envisaged by the ”sticky information” models: In between re-optimisations, the only updating is by the rate of inflation. Such a scheme is also embedded in some of the microfoundations-based variants of the Calvo (1983) scheme (e.g. Yun, 1996; Christiano et al., 2005), but with the exception of the latter, the implications for an inflation equation have not been spelt out. We follow more closely the exposition of the latter paper in order to facilitate comparison.

Here, the price setter’s loss function from the prices resulting from the current optimisation is (omitting the loss after future re-optimisations and expectations):

\[
\ell_t = (1 - \lambda) E_t \sum_{s=t}^{\infty} (\lambda \theta)^{s-t} \left\{ p_t + \sum_{v=0}^{s-1} \pi_{t-v} - p_t - \sum_{v=1}^{s} \pi_{t-v} - \psi y_{s-1} \right\}^2
\]

Now, \( \lambda \) is the probability that the current price scheme will be retained, as opposed to the probability of retaining a specific price in the standard Calvo model of sub-Section 2.1; thus, \( \lambda \) may be thought to be higher in this specification. The expression in the square brackets is simply \((p_s - \tilde{p}_s)^2\) - note the subscript \(s\) in both. As always, \( p_t \) is the price newly set, updated
mechanistically by the rate of inflation in subsequent periods to give . The new price is set with a view to nominal demand analysed as . The only difference to previous specifications is the timing conventions, which are as in Christiano et al. (2005 - henceforth CEE). Each new price, whether resulting from full optimisation or simply mechanically updated, is set at the beginning of the period in which it is revised. - Individual varieties i should be thought of as intermediate goods in this model. - Thus, for instance, if \( p_i^t \) is set at \( t \) from full optimisation, then the price next period will be \( p_i^t + \pi_t \) - purely mechanical updating by the rate of inflation in the relevant period. On the other hand, the price level (which may be thought of as the price of the final good, as in CEE) is set at the end of the period, hence next period it will be \( p_{t+1} = p_t + \pi_{t+1} \).

The sums inside the brackets can be unravelled as,

\[
\lambda \theta \pi_t + (\lambda \theta)^2 (\pi_t + \pi_{t+1}) + (\lambda \theta)^3 (\pi_t + \pi_{t+1} + \pi_{t+2}) + ... = \lambda \theta \pi_t (1 - \lambda \theta)^{-1} (1 - \lambda \theta L^{-1})^{-1}
\]

and, likewise, \( \lambda \theta \pi_{t+1} (1 - \lambda \theta)^{-1} (1 - \lambda \theta L^{-1})^{-1} \).

Minimisation of (11) with respect to \( p_i^t \) results in the following new optimised price:

\[
\frac{p_i^t - p_t}{1 - \lambda \theta} = \frac{\psi y_{t-1}}{1 - \lambda \theta L^{-1}} + \frac{\lambda \theta}{1 - \lambda \theta L^{-1}} (\pi_{t+1} - \pi_t) \tag{12}
\]

The price level is now:

\[
p_t = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i (p_{i-i}^t + \sum_{v=0}^{i-1} \pi_{t-i+v}) \tag{13}
\]

Here, the indicator “i” appears as both superscript and subscript, to emphasise that different firms set new price schemes at different times. Inflation indexation of the originally set prices in the manner described above is also accounted for. This then can be written more compactly with lag operator notation:

\[
p_t = (1 - \lambda) \frac{p_i^t}{1 - \lambda \theta L^{-1}} + \lambda \frac{\pi_{t-1}}{1 - \lambda \theta L^{-1}} \tag{14}
\]

This may trivially be re-written as:

\[
(1 - \lambda)(p_t - p_i^t) = -\lambda (\pi_t - \pi_{t-1}) \tag{15}
\]

Combining this information (new pricing scheme – 13, instantaneously optimal prices - 4, and the price level – 14”), results in the following inflation equation:

\[
\Delta \pi_t (1 - \theta L^{-1}) = \Lambda \psi y_{t-1} \tag{16}
\]

\[
\Lambda \equiv (1 - \lambda)/(\lambda(1 - \lambda \theta))
\]
where $\Delta$ is the difference operator. This formula corresponds exactly with CEE’s eq. (34). In more compact form, re-introducing expectations, this is expressed as:

$$\Delta \pi_t = \theta E_t \Delta \pi_{t+1} + \Lambda \psi y_{t-1}$$

(17)

It is the change in inflation that is related to future output gaps. Alternatively, we may solve this forward, to relate the change in inflation to future output gaps (Rudd and Whelan, 2006). In any case, the NAIRU or accelerationist property is now restored. Though the evidence may on balance be more against it than in favour (see the discussion in Graham and Snower, 2007), this property would appeal to many macroeconomists. Yet, there are two potential problems with an equation like (16). Firstly, the discount factor $\gamma$ may be indistinguishable from unity in finite samples. E.g., with a discount rate of 0.02 to 0.04 as is commonly assumed, $\gamma$ would be in the range of 0.96 to 0.98. Practically, inflation is predicted to be integrated of order two (assuming of course that the output gap $y$ is stationary); the data does not usually support even one unit root. Secondly, the second difference of inflation is predicted to be negatively related to the output gap. The modification explored below would go some way towards amending these defects.

To examine the persistence properties of (16), we may express the level of inflation as:

$$\pi_t = \theta_b \pi_{t-1} + \theta_f E_t \Delta \pi_{t+1} + \Lambda \psi y_{t-1}/(1 + \theta)$$

(18)

with the backward and forward coefficients $\theta_b \equiv 1/(1+\theta)$, $\theta_f \equiv 1-\theta_b$. Hence, persistence is now restored by the presence of the lagged inflation term in the inflation coefficient.

To check robustness against the timing conventions of this model, let us now follow the standard convention of letting the price level be simply the average of all product varieties (which are themselves final goods in this interpretation). Thus, the price level is also set at the beginning of the period, at the same time as individual prices. Hence, the new price formula simplifies into the following, which replaces (13):

$$p_{t}^{i} - p_{t} = \psi y_{t-1}/(1 - \lambda \theta L^{-1})$$

(19)

At the same time, the price level is given by (14) and (15) as above. Equating (13') and (14'') by parts, we readily obtain:

$$\Delta \pi_t (1 - \lambda \theta L^{-1}) = \Lambda \psi y_{t-1}$$

(20)

This is fundamentally the same as (15), retaining the NAIRU property and the persistence evident in (16). However, the forward component is somewhat weakened (but recall that $\lambda<1$ is higher here than in the standard Calvo model). Yet, there is a fairer chance that $\lambda \theta$
will be empirically distinguishable from unity, and that inflation will be I(1) (as opposed to I(2) predicted by the CEE formulation examined above). We therefore have:

**Observation 3:**

1. We must record an open verdict as to whether predetermined prices as suggested by the "sticky information" models can deliver inflation persistence.

2. In contrast, the price indexation models, whereby prices are only updated by the rate of inflation, can deliver inflation persistence.

3. In such models, the NAIRU property (see 16) and the associated linear homogeneity between leads and lags (see 16') also hold.

### 3.5 Staggered wage and price setting combined.

It is probably fair to say that integration of price and wage rigidity remains a relatively underdeveloped area of the literature, though with notable exceptions such as Andersen (1998) and Ascari (2000, 2003). On whether staggered wage setting can deliver inflation persistence the literature does not speak with one voice, as noted above. Andersen (1998) suggests that it can, while Ascari (2000) is sceptical. Fuhrer and Moore (1995) and Roberts (1997) suggest this possibility under the auxiliary assumption of wages exhibiting strategic complementarity or catching up with other wages. The purpose of this Section is to embed a staggered wage structure a la Calvo (1983) into the main pricing framework.

The starting point is that, from microfoundations (as e.g. in Blanchard and Kiyotaki, 1987), the optimal price noted above should also incorporate an element arising from marginal cost. Thus, (4) is replaced by:

\[ \tilde{p}_t = p_t + \psi_1 y_{t-1} + \psi_2 mc_{t-1} + \mu \]  

(21)

Here, optimal price in real terms rises with the marginal cost \(mc\), aggregate demand, and the markup \(\mu\). So, \(\psi_1\) replaces the earlier \(\psi\), while \(\psi_2\) measures the new effect introduced here. We shall ignore the productivity element of the marginal cost, and therefore will set the marginal cost equal to the real wage, \(mc = w - p\). We shall also ignore the markup. Following the same steps as above, the following inflation equation is derived:

\[ \pi_t = \theta E_t \pi_{t+1} + (1 - \lambda)(1 - \lambda \theta)\{\psi_1 y_t + \psi_2 (w_t - p_t)\} \]  

(22)

The question now is whether the real wage under staggered wage adjustment introduces any new dynamics in this, otherwise standard, forward-looking inflation equation.
The structure of staggered wage setting is formally almost identical to Calvo’s (1983) staggered price setting. There are a fraction of unions $\nu$ that carry forward their negotiated wage for the next period, while a fraction $1-\nu$ renegotiate this wage. [The difference with Calvo is that this structure is unlikely to be probabilistic; thus, $\nu$ should best be considered an aggregate frequency rather than a probability.] Unions may also be thought of as having a discount factor $R \leq 1$. Thus, the current log wage level is a standard average of wages set in the past:

$$w_t = (1 - \nu) \frac{w_t^{\text{new}}}{1 - \nu L}$$

Next, we specify new wages. In view of the frequency of revision, the average of the newly set wages is (again suppressing expectations):

$$w_t^{\text{new}} = (1 - \nu R) \frac{p_t + \alpha(w_t - p_t) + \omega y_t}{1 - \nu RL^{-1}}$$

A new element here, exemplified by $\alpha \geq 0$, is the strategic complementarity or “catching up” behaviour of the newly set wages with the economy-wide average wages. Recognition of such behaviour (or “catching up with the Joneses”) has increased in recent years and has been investigated in various contexts (see Tsoukis, 2007). This serves to check the suggestion that only such behaviour generates the kind of inflation dynamics of interest here (Fuhrer and Moore, 1995; Roberts, 1997). $\omega$ now is the effect of excess demand on the nominal wage.

Combining and subtracting the price level from sides, and factoring the lag polynomial associated with the price level, we have the following expression for the real wage:

$$w_t p_t = \frac{\pi_t (1 - RL^{-1})}{(1 - \nu L)(1 - \nu RL^{-1})} + (1 - \nu)(1 - \nu R) \frac{\alpha(w_t - p_t) + \omega y_t}{(1 - \nu L)(1 - \nu RL^{-1})}$$

We then combine (XX) and (YY) to get:

$$\pi_t = \theta E\pi_{t+1} + (1 - \lambda)(1 - \lambda \theta)\{\psi_1 y_t + \psi_2[\pi_t (1 - RL^{-1}) - \omega (1 - \nu)(1 - \nu R)y_t]B^{-1}(L)\}$$

$$B(L) \equiv (1 - \nu L)(1 - \nu RL^{-1}) - \alpha (1 - \nu)(1 - \nu R)$$

It is instructive at this stage to see what the polynomial $B(L)$ looks like. In the polar case of no strategic complementarity ($\alpha = 0$), we have the obvious roots $\nu$ and $1/(\nu R)$. A marginal increase of $\alpha$ from $\alpha = 0$ (evaluating $B(L)$ at $\alpha = 0$) can then be easily seen to decrease the unstable root ($1/\nu R$) and therefore increase the forward-looking component ($\nu R$); importantly for our purposes, it also increases the stable (backward-looking root) $\nu$.

In the other polar case of full complementarity ($\alpha = 1$), we have full persistence with a unit root and an unstable root ($1/R > 1$), to be solved forward ($RL^{-1}$), exactly as in the baseline model.
Furthermore, if we equate the firm’s and union’s discount factors for convenience ($\theta=R$) in the polynomials, we get:

$$\pi_t(1 - RL^{-1})\{B(L) - \psi_2(1 - \lambda)(1 - \lambda\theta)\} = (1 - \lambda)(1 - \lambda\theta)\{\psi_1 B(L) + \psi_2 \omega(1 - \nu)(1 - \nu R)\}y_t$$

We now get richer, and potentially backward-looking, inflation dynamics. To see more clearly what is implied, we can build up a “special-to-general” typology:

- No marginal cost considerations and no strategic complementarity by unions ($\psi_2 = \alpha=0$): The $B(L)$ polynomial cancels out and the standard forward-looking equation results.

- No marginal cost considerations ($\psi_2=0$), but strategic complementarity by unions ($\alpha >0$): The multiplicative $B(L)$ polynomial on both sides changes in the way described above but it still cancels out. Hence, contrary to the suggestions in the literature, this feature alone is unable to introduce backward-looking dynamics and persistence to the baseline inflation model.

- Introducing marginal cost considerations, but without strategic complementarity ($\psi_2 >0$, $\alpha =0$): While $\psi_2$ affects the polynomials on both sides, the dynamics essentially hinges on the polynomial on the LHS. Again, we may ask how the roots of $B(L)$ evaluated at $\psi_2=0$ (in this case: $\nu$ and $\nu R$) change by a marginal increase in $\psi_2$. The effects are the same as with an incremental rise in $\alpha$, described above, and result in the enhancement of both the forward- and backward-looking dynamics. At the same time, the opposite effectively happens on the RHS (due the opposite sign of $\psi_2$), resulting in a reduction in the backward-looking root in particular. Hence, marginal cost considerations ($\psi_2 >0$) under the maintained assumption of staggered wage setting ($\nu >0$) are capable of delivering backward-looking inflation dynamics and persistence.

- In the general case ($\psi_2, \alpha >0$), both elements combine to enhance the stable (backward) root.

We may conclude as follows:

**Observation 4:**

1. Marginal cost considerations in price setting together with a staggered wage setting structure similar to the Calvo scheme are both necessary and sufficient in generating backward-looking inflation dynamics.
2. Strategic complementarity and catching up behaviour by unions is neither sufficient nor necessary for such dynamics, but it enhances persistence if it is generated by the conditions in (1) in the first place. These findings are in line with Andersen’s (1998) that staggered wages are more crucial for persistence than staggered prices; but they differ from those of Fuhrer and Moore (1995) and Roberts (1997) on the importance of catching-up union behaviour.

3.6 (More) Microfoundations under time-contingent price adjustment

A newer generation of models (Yun, 1996; Ergec, Henderson and Levin, 2000; CEE, 2005; Dotsey and King, 2006) has embedded time-contingent price and wage stickiness in the context of micro-founded, intertemporally optimising models. Our intention is to examine whether such microfoundations are capable of delivering inflation inertia. The setup is monopolistic competition; product varieties are assumed to have pair-wise elasticities of substitution $\varepsilon > 1$. Demand for individual firm producing variety $i$ is given by:

$$Y^i = \left(\frac{P^i}{P}\right)^{-\varepsilon}Y$$

Time-subscripts are suppressed; the $i$-superscript indicates the individual firm, while no superscript indicates economy-wide averages. Aggregate output is a standard Dixit-Stiglitz aggregate of all individual varieties. The associated price level is given by:

$$P = \left(\sum_i (P^i)^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$$

For tractability, this may be log-linearised to yield the standard:

$$p = \sum_i p^i$$

The firm’s static problem may be written as

$$\text{Max } p^i (P^i/P)Y^i - mc^iY^i$$

Where $mc^i$ is the firm’s marginal cost. In view of individual product demand, the solution is the textbook:

$$P^i/P = (1 + \mu)mc^i$$

with the markup $\mu$ defined as $\mu \equiv \varepsilon/(\varepsilon - 1) - 1$. With a standard production function (ignoring capital and productivity) of $Y^i = (L^i)^{\alpha}$, $0 < \alpha < 1$, the marginal cost is $mc^i = (W/P)/(\alpha(L^i)^{\alpha-1})$, where $W/P$ is the real wage.

The complication that price stickiness of course introduces is that the price is there to stay for some time, hence the price maximises the present value of profits. In particular,
substituting for output demand and introducing time-subscripts, the firm sets the price such as to maximise profits over the horizon over which the price now set is relevant:

$$\max_{P_t} \sum_{s=0}^{\infty} \mathbb{E}_t \left( \frac{P_t^s}{P_{t+s}} \right)^{1-\varepsilon} Y_{t+s} - m c_{t+s} \left( \frac{P_t^s}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s}$$

Note that the individual price is only subscripted \(-t-\) as it stays fixed throughout the period under consideration. This equation, at the core of micro-founded sticky price models (e.g. Yun, 1996; Dotsey and King, 2006), replaces the quadratic profit loss minimisation of the baseline model. (Yun, 1996 and CEE also allow for the price now set to be indexed but this feature was examined above and is ignored here.) The notable other features here are the expectations and the stochastic discount factor $\vartheta_t + s \theta_s \lambda_s$. This takes into account the purpose of the firm, which is to maximise consumers’ (its shareholders’) utility. Hence, future profits are valued the current-value marginal utility of wealth (consumers’ current value Lagrange multiplier), $\vartheta_t + s \theta_s$, as well as the subjective discount factor. But future profits continue to be weighted by the (constant) probability $\lambda_s$ that the currently set price will be carried forward to the next period.

This maximisation yields:

$$P_t^s = (1 + \mu) \frac{\sum_{s=0}^{\infty} \theta^s \lambda^s \mathbb{E}_t \left( \vartheta_{t+s} P_{t+s} Y_{t+s} + m c_{t+s} \right)}{\sum_{s=0}^{\infty} \theta^s \lambda^s \mathbb{E}_t \left( \vartheta_{t+s} P_{t+s}^{s-1} Y_{t+s} \right)}$$

Note that this equation encompasses the static solution as the special case under no price stickiness ($\lambda = 0$).

To derive an empirically implementable specification, we let the marginal utility of wealth be inversely depend on aggregate output, $\vartheta_{t+s} = (Y_{t+s})^{-\zeta}$, $0 < \zeta < 1$. Furthermore, we define the following discount factor:

$$\Gamma_{t+s} \equiv \prod_{v=1}^{s} (1 + \pi_{t+v})^{\varepsilon-1}(1 + g_{t+v})^{1-\zeta} \approx \prod_{v=1}^{s} (1 + (\varepsilon - 1) \pi_{t+v} + (1 - \zeta) g_{t+v})$$

where $\pi_{t+v} \equiv P_{t+v} / P_{t+v-1} - 1$ is the inflation rate and $g_{t+v}$ is analogously the growth rate of aggregate output. Thus,

$$P_{t+s}^{s-1} Y_{t+s}^{1-\zeta} = \Gamma_{t+s} P_{t+s}^{s-1} Y_{t+s}^{1-\zeta}$$

and we may write the above as:

$$P_t^s = (1 + \mu) \frac{\sum_{s=0}^{\infty} \theta^s \lambda^s \mathbb{E}_t \left( \Gamma_{t+s} P_{t+s} c_{t+s} \right)}{\sum_{s=0}^{\infty} \theta^s \lambda^s \mathbb{E}_t \left( \Gamma_{t+s} \right)} = (1 + \mu) \frac{\sum_{s=0}^{\infty} \theta^s \lambda^s (E_t \left( \Gamma_{t+s} \right) (E_t P_{t+s} c_{t+s}) (1 + Cov1))}{\sum_{s=0}^{\infty} \theta^s \lambda^s \mathbb{E}_t \left( \Gamma_{t+s} \right)}$$

The second equality is derived from first principles, and spels out the Jensen inequality effect of the expectation of the product. The associated covariances (subsumed under the
Cov1 term) are assumed proportional to expected values (as the variables involved are not covariance-stationary, except the real marginal cost, which is assumed stationary). Analogously, we let the covariances involved in the discount factor between the growth and inflation products (under Cov2) depend on the order of magnitude of those products, so that we have:

\[ E_t \Gamma_{t+s}^\gamma(\gamma_t(1 + Cov2))^s, \quad \gamma_t \equiv E_t[1 + (\varepsilon - 1)\pi_{t+v} + (1 - \zeta)g_{t+v}] \]

Note that \( \gamma_t \) represents the expectation of the discount factor conditional on information at \( t \), hence it is given a subscript \( t \) as a reminder that it depends on time \( t \) but not further (i.e., not \( s \) or \( v \)). In this light, the above is written as,

\[
P_t^i = (1 + \mu)(1 + Cov1) \sum_{s=0}^{\infty} \frac{(\theta \lambda \gamma_t(1 + Cov2))^s(E_t P_{t+s}^1)(E_t mc_{t+s}^i)}{\sum_{s=0}^{\infty} (\theta \lambda \gamma_t(1 + Cov2))^s},
\]

where the composite discount factor involved here is assumed less than unity for the sums to converge. This equation has a similar format for optimal new prices with the baseline specification. There are also important differences: Apart from a numerically different factors and the dependence of the discount factor on information at \( t \), there is also the important dependence of the discount factor on inflation.

To make progress, let us further assume that the conditional expectations may be analysed as unconditional expectations plus deviations from them (in tildes); in the case of the non-stationary price level, the deviation is proportional to the trend. Thus,

\[ \gamma_t = \gamma + \tilde{\gamma}_t, \quad E_t P_{t+s} = EP_{t+s}(1 + \tilde{p}_{t+s}) = EP_t(1 + \pi)^s(1 + \tilde{p}_{t+s}), \quad mc_{t+s}^i = mc + m\tilde{c}_{t+s}^i \]

Linear trend prices are indicated by the unconditional expectations sign, \( EP_{t+s} \). Letting trend inflation be subsumed by \((\gamma + \pi)\), we can now linearise the numerator as follows:

\[
P_t^i / EP_t = M \sum_{s=0}^{\infty} (\lambda \Theta)^s(1 + E_t \tilde{p}_{t+s}^+ E_t mc_{t+s}^i) \sum_{s=0}^{\infty} (\lambda \Theta)^s
\]

\[ M \equiv (1 + \mu)(1 + Cov1)(1 + mc), \quad \Theta \equiv (\gamma + \pi)\theta(1 + Cov2) \]

In other words, to first approximation, the effect of variations of \( \gamma_t = \gamma + \tilde{\gamma}_t \) cancel out as they affect the numerator and denominator symmetrically. Finally, the newly set price \( P_t^i \) as a ratio of the linear trend price level may be written as a percentage deviation,

\[ 1 + \tilde{p}_t^i \equiv P_t^i / EP_t, \]

20
and the above is finally re-written in terms of lag operators:

\[ 1 + \tilde{p}_t = M (1 - \lambda \Theta) \frac{1 + \tilde{p}_t^c i_t}{(1 - \lambda \Theta L^{-1})} \]

The conditional operator has now been suppressed. This is in the same format, but not identical to the baseline specification of the newly set price. Apart from the difference in the discount factor, there is also the markup due to monopolistic competition (absent in the baseline specification), and the Jensen inequality effects (covariance terms) due to expectations; the latter two elements are implicit in M. To gain an idea of what this implies, in the trend steady state, when \( \tilde{p}_{t+s} = m \tilde{c}_{t+s} = 0 \), newly set individual prices will be higher than the trend average price level by \( \tilde{p}_t = M - 1 \), i.e. by the markup and the Jensen inequality effect (which is positive).

In view of the log-linear price level (**) and staggered price setting, the standard equation for the price level closes the model:

\[ (1 - \lambda L) p_t = (1 - \lambda) p_t^i \]

Subtracting (1-\( \lambda \)) the log trend price level, \( E \tilde{p}_t \), from both sides of (1), and considering (33), we get an equation in deviation terms \( (\tilde{p}_t \equiv p_t - E \tilde{p}_t) \):

\[ (1 - \lambda L) \tilde{p}_t \lambda \pi = (1 - \lambda) \tilde{p}_t^i \]

where \( \pi \) is trend inflation. The two equations (88) and (99) are combined in:

\[ \tilde{p}_t \frac{M - 1 - \lambda \pi / (1 - \lambda)}{M (1 - \lambda \Theta) (1 - \lambda)} \frac{\tilde{p}_t^c i_t}{(1 - \lambda L)(1 - \lambda \Theta L^{-1})} \]

A number of implications concerning trend inflation, not present in the baseline model, are obvious now. In the trend steady state whereby \( \tilde{p}_t = 0 \), we readily have:

\[ \pi = (M - 1)(1 - \lambda) / \lambda \]

Trend inflation increases in the degree of monopoly power and the markup; it also decreases with the degree of price stickiness, \( \lambda \). As pointed out by Kiley (2000) and Devereux and Yetman (2002), price adjustment may be more frequent under higher trend inflation, hence \( \lambda = \lambda(\pi) \), with \( \lambda'(\pi) < 0 \) (see Konieczny and Skrzypacz 2005). This may give multiple equilibria in the trend inflation process derived above. A further implication concerns the correlation between inflation and inflation uncertainty. It is a stylised empirical finding that this correlation is positive. This may be due to factors outside the model, e.g. monetary uncertainty rises with the rise of inflation. Another possibility is that the frequency of
price adjustment increases with trend inflation, as mentioned; as a result, monetary shocks are transmitted more rapidly, and inflation uncertainty rises (Devereux and Yetman, 2002). Here, an additional effect of this nature appears, namely the variance of inflation is among the covariance terms embedded in $M$. Thus, depending on the (unspecified here) functional form of $M(\text{Var}(\pi))$, we may get multiple equilibria of inflation and its associated uncertainty. Multiple equilibria are interesting at least on two grounds. First, this is effectively the suggestion of the literature on credibility of monetary policy, with the degree of credibility dictating which equilibrium is selected. Secondly, there is evidence (Altissimo et al, 2006) that the inflation process is characterised by structural breaks in its trend. It is an asset of this model that it can give rise to such possibilities, that are not present in the baseline model. The implications of this model of price adjustment for multiple inflation equilibria and the inflation-uncertainty correlation are to be further pursued in future work.

Finally, we come to the dynamic properties of the above model. It is readily obvious that if $M=1$, the lag structure of (34) would be identical to the baseline model of sub-section 2.1 with two roots, a unit backward one and a forward one equal to $\Theta$. Hence, the inflation equation (in deviations from trend form) would be entirely forward-looking with no persistence. A rise of $M$ from its baseline value of unity can then be seen by straightforward algebra to increase the unstable root $(1/\Theta)$ and hence to decrease the forward-looking element of the process. Importantly, it also decreases the unit root, implying that the above cannot be cast in terms of inflation. Hence, in the general case of $M>1$, there would be rich price level (in deviations form) dynamics, but no inflation persistence, as the above process cannot even deliver an inflation equation. But for small deviations of $M$ from unity, the dynamics may be indistinguishable from those of the baseline model.

That persistence drops under forward-looking monopolistically competitive pricing should not be a surprise. To begin with, this structure is in its basics analogous to the baseline model. Secondly, markup pricing implies in a sense even more forward-lookingness, in the sense that any shock to the markup elicits a multiple price response. Hence, prices and inflation adjust more robustly in anticipation to future developments, and are bound less by the past. We may summarise as follows:

**Observation 5:**

1. Microfoundations based on an intertemporally optimising, monopolistically competitive model with time-contingent pricing allow a potentially fruitful decomposition of inflation into trend and deviations components.

2. Furthermore, the two components may interact (deviations via its variances) in such a way as to deliver multiple equilibria.
3. Regarding the deviations component though, this class of models fails to deliver persistence in the inflation equation and may not even deliver an inflation equation at all.

Altissimo et al (2005) find that allowing for structural breaks in inflation, its persistence drops (but it is still there). This suggests an interaction between trend inflation and dynamics that is suggested as a possibility by this model. But the possibility that equation (34) may be just a rich price equation and not an inflation one should surely count as a weakness of this model from the perspective addressed in this paper. Such models then should not be relied on if inflation dynamics is the main point under investigation, but they may well be part of the story in other respects. Below, we investigate whether similar models but with state-contingent, instead of price-contingent, pricing may account better for inflation dynamics.

3.7 Time-contingent price adjustment and signal extraction

Collard and Dellas (2006) introduce signal extraction of the monetary impulses in the context of a microfounded model with Calvo pricing, similar to the one reviewed in the previous sub-Section. The starting point is twofold. Firstly, they criticise the inclusion of purely mechanical backward-looking price indexation when full price realignment is not happening in microfounded models like CEE (2005) and Smets and Wouters (2004). (The same criticism can also be applied to many others including the sticky information models of Mankiw and Reis, 2001; and Devereux and Yetman, 2002.) In particular, they take issue with the fact that such a non-rational mechanism is critical for reproducing the stylised observations of CEE.

Secondly, they provide evidence that data on money supply is characterised by considerable measurement error; further, that this measurement error has a significant impact on output and inflation. In the spirit of the well known Lucas island model, agents are assumed to observe their own variables, but to be unable to observe aggregate outcomes perfectly due to the measurement error in the money data. Thus, there is scope for signal extraction to filter out the noise due to measurement error in money. They show that such signal extraction can generate the Christiano et al. (2005) empirical observations without reliance on backward-looking inflation indexation of prices. But otherwise, their model and inflation predictions are similar to those reported in the previous sub-Section.
3.8 Variable probability of price adjustment and state-contingent (S,s) models

A different strand of models are those based on state-contingent (S,s) price adjustment rules (a recent example being Gertler and Leahy, 2005, building on Danziger, 1999). To gain tractability, Dotsey and King (2006), and Dotsey, King and Wolman (1999) present a hybrid time- and state-contingent model, of the type that receives empirical support by Konieczny and Skrzypacz (2005), whereby the probability of adjustment is allowed to vary with the state of the firm (demand, time during which the price has stayed unaltered, etc), and the firm takes that into account in its optimisation. This problem yields very similar equations determining the price level and the optimal newly set price as before, with the crucial difference that probability of adjustment \((1-\lambda)\) varies over time. Specifically, we have:

\[
P_t = (1 + \mu) \frac{\sum_{s=0}^{\infty} \theta^s \Lambda^{t,t+s} E_t (\delta_{t+s} P_{t+s} Y_{t+s} m_{t+s}^i)}{\sum_{s=0}^{\infty} \theta^s \Lambda^{t,t+s} E_t (\delta_{t+s} P_{t+s-1} Y_{t+s})}
\]

where \(\Lambda^{t,t+s}\) is the proportion of prices set at \(t\) and still in place at \(t+s\). It is given by the recursive formula \(\Lambda^{t,t+1} = \frac{1}{\Sigma \Lambda^{t,t+v}}\), \(\Lambda^{t,t+s} = \lambda^{t+s} \Lambda^{t,t+s-1}\), where \(\lambda^{t+s}\) is the probability that a price set at \(t\) and lasted till \(t+s-1\) will last one more period (to \(t+s\)). If \(\lambda\) is constant, this boils down to the \(\Lambda^{t,t+s} = (1-\lambda)\lambda^{s-1}\) of the baseline specification. Here, \(\lambda\) is allowed in general to depend both on \(t\) and the time during which this price has been in place, \(s\).

However, as with the stochastic subjective discount factor earlier on, so this innovation will not have any effects (to a first-order approximation) on the new price set at \(t\), simply because this factor appears in both numerator and denominator. So, in a linearised form around the benchmark \(\Lambda^{t,t+s} = \lambda^s\), the new price (as a deviation from the trend price level) will again be given by (BB) above.

Where the varying probability of time adjustment may make a difference is in the definition of the price level. We now have,

\[
P_t = \left( \sum_s \Lambda^{t-s,t} (P_{t-s}^t)^{1-\epsilon} \right)^{1/(1-\epsilon)}
\]

where again all past new prices are weighted by the probability that they are still in place. Log-linearisation around \(\lambda^{t,v} = \lambda \forall t,v\), yields:

\[
(1 - \epsilon)p_t = \frac{(1 - \lambda)(1 - \epsilon)p_t^i}{1 - \lambda L} + \sum_s \Lambda^{t-s,t} \tilde{\lambda}^{t-s,t}
\]

where it should be reminded that \(\epsilon > 1\); or

\[
(1 - \epsilon)p_t = \frac{(1 - \lambda)(1 - \epsilon)p_t^i + \tilde{\lambda}^{t,t}}{1 - \lambda L}
\]
where $\tilde{\lambda}^{t,t+v} \equiv (\lambda^{t,t+v} - \lambda)/\lambda$ is the log deviation of the probability of non-adjustment in the relevant period. It may be thought of as a negative function of demand, marginal cost and $v$, the time during which this price has been in place. So, in a very simple manner, we could postulate:

\[ \tilde{\lambda}^{t,t+v} = -\xi_1\tilde{y}_{t+v} - \xi_2m\tilde{c}_{t+v} - \xi_3v \]

Inserting above, we get:

\[ (1 - \varepsilon)p_2 = \frac{(1 - \lambda)(1 - \varepsilon)p_t - \xi_1\tilde{y}_t - \xi_2m\tilde{c}_t}{1 - \lambda L} \frac{\xi_3}{(1 - \lambda)^2} \]

or

\[ (1 - \varepsilon)(1 - \lambda L)p_2 = (1 - \lambda)(1 - \varepsilon)p_t - \xi_1\tilde{y}_t - \xi_2m\tilde{c}_t - \xi_3/(1 - \lambda)^2 \]

Subtracting again $(1-\varepsilon)(1-\lambda)$ the log trend price level, $E_{\pi_t}$, from both sides as above, we get:

\[ (1 - \varepsilon)(1 - \lambda L)\tilde{p}_2 + (1 - \varepsilon)\lambda\pi = (1 - \lambda)(1 - \varepsilon)\tilde{p}_t - \xi_1\tilde{y}_t - \xi_2m\tilde{c}_t - \xi_3/(1 - \lambda)^2 \]

Finally, combining with the equation of the newly set price (ZZ) which is the same as in the time-contingent microfoundations, we obtain:

\[ \tilde{p}_t M - 1 - \lambda\pi/(1 - \lambda) + M(1 - \lambda\Theta)(1 - \lambda)\frac{\tilde{p}_t + m\tilde{c}_t}{(1 - \lambda L)(1 - \lambda\Theta L^{-1})} + \frac{\xi_1\tilde{y}_t + \xi_2m\tilde{c}_t + \xi_3/(1 - \lambda)^2}{(\varepsilon - 1)(1 - \lambda L)} \]

Taking all the lead and lag denominators to the LHS, we note that this class of models delivers the same inflation dynamics (in deviations) to those of the time-contingent microfounded models reviewed immediately above. Hence, the time-varying probabilities of adjustment of the hybrid time/state-contingent model do not add more inflation persistence to the model. Secondly, the extraneous dynamics is richer, with more forward-looking elements of the marginal cost and/or the output gap (this emerges if ones takes the polynomials of the denominator to the LHS). This is because in addition to the standard effect of higher costs and demand pushing optimal new prices up, we now have the novel effect that these factors also precipitate price adjustment, hence past prices and costs considerations matter less, and future ones more. Finally, the trend inflation rate is now higher, adjusted upwards by the $\xi_3$ term, again because erosion of the probability of price stickiness implies faster upward adjustment. We therefore have:

**Observation 6:**

The inflation dynamics arising from state-contingent pricing models are quite similar to those arising from time-contingent models.
4 Conclusions

In the era of inflation targeting, understanding the dynamics of inflation is as crucial as ever. Among the issues of interest is inflation persistence because of its important implications for the effectiveness (time, force, etc) of monetary policy. Empirical research is informed by a host of micro-founded models based on individual price stickiness feeding into price level and inflation dynamics. However, it is increasingly clear that the dynamic properties of such microfounded models are at odds with key empirical properties of inflation. In particular, theoretical models find it very difficult to rationalise the inflation persistence (i.e., the backward-looking element) in empirical inflation equations.

In parallel with this difficulty, there is rather little concrete information as to what exactly the price adjustment costs consist of. Recent work has cast doubt on the notion that price adjustment cost is made up of a (lump-sum) “menu cost” and has emphasised instead managerial and information gathering costs, and customer information costs. If so, the traditional notion of infrequent price adjustment is weakened in favour of, correspondingly, notions of bounded rationality whereby full optimisation occurs infrequently and rules of thumb are followed in the interim, and quadratic costs of price adjustment, which rise with the magnitude of the price change. Yet a different avenue is to intertwine the dynamics of inflation with those of wage adjustment, which would have important implications on price dynamics.

This paper has analysed the dynamic properties of inflation, particularly its persistence, in a range of microfounded models featuring a variety of costs of price adjustment and associated mechanics of price stickiness. In common with much of the literature, the organising framework is the tractable and elegant Calvo (1983) model, featuring a constant probability of price adjustment. With a minor variation, we have called this our baseline model. Then, we have extended it into various directions that encompass the various types of price adjustment costs that the micro studies have highlighted. We argued that analytical work is important because many of these models are observationally equivalent.

Our main finding is captured in the title: Few models are able to deliver inflation persistence that is typically found in the literature (Fuhrer and Moore, 1995; Roberts, 1995). Instead, they yield purely forward-looking dynamics for inflation, of the types that have been attacked by Fuhrer (1997), Rudd and Whelan (2006), and others. The only exception is if inflation dynamics incorporates staggered wage setting emphasised in the first-generation New Keynesian models such as Taylor (1979, 1980) and now nearly forgotten. Thus, the paper’s main message is a simple one. For realistic inflation dynamics, one needs to jointly model price and wage rigidities.
Inflation persistence may of course result from some form of bounded rationality, taking the form of either purely backward-looking expectations (Roberts, 1997), or inertia by price setters (Gali and Gertler, 1999), or “rational inattention” (Sims, 2003), or “sticky information” (Mankiw and Reis, 2002). It is somewhat paradoxical that, after a generation during which work along the New Keynesian lines has sought to uncover the precise causes of price inertia, we are now moving towards the direction of simply postulating it. New Keynesians have upheld the rationality postulate and have emphasised forward-looking price setting. For these tenets to be compatible with the inflation persistence observed in the data, the number of models available to the theorist is rather limited. One conclusion of this paper is that models that combine both price and wage stickiness and/or models of price indexation hold the biggest promise in resolving the apparent tension between theoretical rigour and sound empirical performance. Another conclusion is that the literature suggests a number of other, ”exotic”, possibilities, like multiple equilibria and structural breaks, the interplay between trend and dynamics of inflation, or between inflation level and its variability. We end with a plea for more theory-based models that analyse these important empirical questions.

References


