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Revisiting Useful Approaches to Data-Rich Macroeconomic Forecasting

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Abstract

This paper revisits a number of data-rich prediction methods, like factor models, Bayesian ridge regression and forecast combinations, which are widely used in macroeconomic forecasting, and compares these with a lesser known alternative method: partial least squares regression. Under the latter, linear, orthogonal combinations of a large number of predictor variables are constructed such that these linear combinations maximize the covariance between the target variable and each of the common components constructed from the predictor variables. We provide a theorem that shows that when the data comply with a factor structure, principal components and partial least squares regressions provide asymptotically similar results. We also argue that forecast combinations can be interpreted as a restricted form of partial least squares regression. Monte Carlo experiments confirm our theoretical result that principal components and partial least squares regressions are asymptotically similar when the data has a factor structure. These experiments also indicate that when there is no factor structure in the data, partial least squares regression outperforms both principal components and Bayesian ridge regressions. Finally, we apply partial least squares, principal components and Bayesian ridge regressions on a large panel of monthly U.S. macroeconomic and financial data to forecast, for the United States, CPI inflation, core CPI inflation, industrial production, unemployment and the federal funds rate across different sub-periods. The results indicate that partial least squares regression usually has the best out-of-sample performance relative to the two other data-rich prediction methods.

Keywords: Macroeconomic forecasting, factor models, forecast combination, principal components, partial least squares, (Bayesian) ridge regression.
JEL classification: C22, C53, E37, E47.

1 Introduction

It has been a standard assumption in theoretical macroeconomic modeling that agents are processing all the available quantities of information when forming their expectations for the future.

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Also, policymakers traditionally have looked at a vast array of indicator series in the run-up to major policy decisions, or in the words of Lars Svensson (Svensson (2005)) about what central bankers do in practice: '(l)arge amounts of data about the state of the economy and the rest of the world ... are collected, processed, and analyzed before each major decision.' However, most traditional macroeconomic prediction approaches rarely consists of models that handle more than 10 variables, because it is either inefficient or downright impossible to incorporate a much larger number of variables in a single forecasting model and estimate it using standard econometric techniques. This failure of traditional macroeconomic forecasting methods prompted a new strand of research devoted to the theory and practice of alternative macroeconomic forecasting methods that utilize large data sets.

These alternative methods can be distinguished into two main categories. As, e.g., outlined in Hendry (1995), the methods of the first category involve inherently two steps: In the first step some form of variable selection is undertaken. The variables that are chosen are then used in a standard forecasting model. Recent developments in this line of research has focussed on automated model selection procedures in order to be better able to select the optimal predictors from large data sets; see Krolzig and Hendry (2001). An alternative group of forecasting methods consists of estimation strategies that allow estimation of a single equation model that utilizes *all* the information in a large data set and not just an 'optimal' subset of the available predictor series. This is a diverse group of forecasting methods ranging from factor-based methods to Bayesian regression and forecast combination. These two groups of methods inevitably overlap. However, we feel that the step of variable selection is, and involves methods that are, sufficiently distinct to merit separate mention and treatment. Instead, we focus in this paper on the latter group of data-rich forecasting methods.

Within the group of data-rich forecasting techniques, factor methods have gained a prominent place. These methods are related to the strict factor models used in finance, but, starting with Chamberlain and Rothschild (1983), they use weaker assumptions regarding the behavior of the idiosyncratic components, which allows the use of principal components in very large data sets to identify the common factors in such a data set. Stock and Watson (2002a) and Bai (2003) further formalized the underlying asymptotic theory. Stock and Watson (2002b) proved to be the starting point of a large empirical research output where, with mixed success, a limited number of principal components extracted from a large data set are used to forecast key macroeconomic variables. However, the use of principal components does not always guarantee that the information extracted from a large number of predictors is useful for forecasting. Boivin and Ng (2006) make it clear that if the forecasting power comes from a certain factor, this factor can be dominated by other factors in a large data set, as the principal components solely provide the best fit for the large data set and not for the target variable. This could explain why in some empirical applications principal components (PC) factor models are dominated by Bayesian regression and forecast combinations. Under Bayesian regression one essentially estimates a multivariate regression consisting of *all* predictor variables, but with the regression coefficients shrunken to a value close to zero. Starting with Bates and Granger (1969), forecast combination involves the use of subsets of predictor variables in distinct forecasting models and the production of multiple forecasts for the target variable, which are then averaged to produce a final forecast. The distinctive feature of these two approaches is that the information in a large data set is compressed such that this has explanatory power for the target variable. Note, however, that from an econometric perspective forecast combinations are *ad hoc* in nature, whereas it has been shown in De Mol *et al.* (2006) that Bayesian regression is theoretically related to PC-based factor models.

In this paper we revisit the use of principal components (PC), Bayesian regression and forecast combination for data-rich macroeconomic forecasting. In addition, we consider the use of the lesser known method of partial least squares (PLS), introduced by Herman Wold in the late 1970s, as a new data-rich approach that can be used with very large data sets for macroeconomic forecasting. PLS has similarities to PC analysis but its major advantage over PC is that it explicitly takes into account the target variable when constructing the factors which are used as summaries of the available large data set. One significant contribution of our paper is that we provide a theoretical result that relates PLS to PC for large data sets. Also, we argue that the range of forecast combination techniques can be seen as restricted versions of PLS. PLS, therefore, has explicit theoretical links with the currently used range of data-rich macroeconomic forecasting tools, i.e. PC, Bayesian regression and forecast combination. Next, we also consider in detail the properties of PLS, PC and Bayesian regression for forecasting using both Monte Carlo analysis and an empirical application to gauge the potential of each of these data-rich approaches.

In the remainder of this paper we have the following structure: Section 2 discusses the most frequently used data-rich methods for macroeconomic forecasting. This section also provides an overview of PLS and presents the theoretical result linking PLS to PC regression. Section 3 presents an extensive Monte Carlo study focusing on both in-sample and, more importantly, out-of-sample properties of PLS, PC and Bayesian regression. Section 4 presents an empirical application where PLS and the other data-rich forecasting methods are used on a large monthly US macroeconomic data set. Finally, Section 5 concludes.

2 Useful Data-rich Methods for Macroeconomic Forecasting

In this section we provide a selective review of existing data-rich methods for macroeconomic forecasting and suggest a possible alternative method.

2.1 Frequently Used Methods

A useful framework for studying existing methods is provided by the following forecasting equation

$$y_t = \alpha' x_t + \epsilon_t; \quad t = 1, \ldots, T, \tag{1}$$

where y_t is the target of the forecasting exercise, $x_t = (x_{1t} \cdots x_{Nt})'$ is a vector of dimension $N \times 1$ and thus $\alpha = (\alpha_1 \cdots \alpha_N)'$ is also $N \times 1$. It is assumed that the number of indicator variables N is too large for α to be determined by standard methods such as ordinary least squares (OLS).

Factor Methods

The most widely used class of data-rich forecasting methods are factor methods. Factor methods have been at the forefront of developments in forecasting with large data sets and in fact started this literature with the influential work of Stock and Watson (2002a). The defining characteristic of most factor methods is that relatively few summaries of the large data sets are used in forecasting equations which thereby becomes a standard forecasting equation as they only involve a few variables. The assumption is that the co-movements across the indicator variables can be captured by a $r \times 1$ vector of unobserved factors $F_t = (F_{1t} \cdots F_{rt})'$, i.e.

$$\tilde{x}_t = \Lambda' F_t + e_t \tag{2}$$

where $\tilde{x}_t$ may be equal to x_t or may involve other variables such as, e.g., lags and leads of x_t and Λ is a $r \times N$ matrix of parameters describing how the individual indicator variables relate to each of the r factors, which we denote with the terms 'loadings'. In (2) e_t represents a zero-mean $I(0)$ vector of errors that represent for each indicator variable the fraction of dynamics unexplained by F_t, the 'idiosyncratic components'. The number of factors is assumed to be small, meaning $r < \min(N,\ T)$. So, implicitly, in (1) $\alpha' = \tilde{\alpha}' \Xi \tilde{x}_t$, where $F_t = \Xi \tilde{x}_t$, which means that a small, r, number of linear combinations of $\tilde{x}_t$ represent the factors and act as the predictors for y_t. The main difference between different factor methods relate to how Ξ is obtained.

The use of principal components (PC) for the estimation of factor models is, by far, the most popular factor extraction method. It has been popularised by Stock and Watson (2002a,b), in the context of large data sets, although the idea had been well established in the traditional multivariate statistical literature. The method of principal components (PC) is simple. Estimates of Λ and the factors F_t are obtained by solving:

$$V(r) = \min_{\Lambda, F} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (\tilde{x}_{it} - \lambda_i' F_t)^2, \tag{3}$$

where λ_i is a $r \times 1$ vector of loadings that represent the N columns of $\Lambda = (\lambda_1 \cdots \lambda_N)$. One, non-unique, solution of (3) can be found by taking the eigenvectors corresponding to the r largest eigenvalues of the second moment matrix $X'X$, which then are assumed to represent the rows in Ξ, and the resulting estimate of Ξ provides the forecaster with an estimate of the r factors $\hat{F}_t = \hat{\Xi} \tilde{x}_t$. To identify the factors up to a rotation, the data are usually normalised to have zero mean and unit variance prior to the application of principal components; see Stock and Watson (2002a) and Bai (2003)

PC estimation of the factor structure is essentially a static exercise as no lags or leads of x_t are considered. On alternative is dynamic principal components, which, as a method of factor extraction, has been suggested in a series of papers by Forni, Hallin, Lippi and Reichlin (see, e.g., Forni *et al.* (2000, 2004) among others) is designed to address this issue. Dynamic principal components are extracted in similar fashion to static principal components but , instead of the second moment matrix, the spectral density matrices of the data at various frequencies are used. These are then used to construct estimates of the common component of the data set which is a function of the unobserved factors. This method used leads of the data and as a result its application to forecasting has been slow for obvious reasons. Recent work by the developers of the method has addressed this issue (see, e.g., Forni *et al.* (2005)). Another alternative way of factor estimation, assumes a parametric state space model for the data set, x_t. This follows earlier work by Stock and Watson (1989) who used state space models to extract factors via the Kalman filter and maximum likelihood estimation, for small data sets. Conventional wisdom suggested that such methods would be too computationally intensive for large data sets. Borrowing work from the engineering literature which again focused on small data sets, Kapetanios and Marcellino (2003) suggest using subspace algorithms to estimate factors from a state space model. This essentially uses OLS estimation to obtain estimates of the matrix coefficient in a multivariate regression of leads of x_t on lags of x_t. Then a reduced rank approximation to this estimated coefficient matrix provides estimates for the factors. In a Monte Carlo study Kapetanios and Marcellino (2003) found that subspace estimation compared favorably to static and dynamic principal components.

Bayesian Regression Estimation

Bayesian regression is a standard tool for providing inference for α in (1) and there exist a large variety of approaches for implementing Bayesian regression. We will provide a brief exposition of this method. A starting point is the specification of a prior distribution for α. Once this is in place standard Bayesian analysis proceeds by incorporating the likelihood from the observed data to obtain a posterior distribution for α which can then be used for a variety of inferential purposes, including, of course, forecasting.

A popular and simple implementation of Bayesian regression results in a shrinkage estimator for α in (1) given by

$$\hat{\alpha}_{BRR} = (X'^{-1}X'y \tag{4}$$

where $X = (x_1, ..., x_T)'$, $y = (y_1, .., y_T)'$ and v is a shrinkage scalar parameter. The shrinkage estimator (4) shrinks the OLS estimator, given by $(X'^{-1}X'y$ towards zero, thus enabling a reduction in the variance of the resulting estimator. This is a major feature of Bayesian regression that makes it useful in forecasting when large data sets are available. This particular implementation of Bayesian regression implies that elements of α are small but different from zero ensuring that all variables in x_t are used for forecasting. In this sense, Bayesian regression can be linked to other data-rich approaches. When a factor structure is assumed in the data, Bayesian regression through (4) will forecast y_t by projecting it on a weighted sum of all N principal components of X, with decaying weights, instead of projecting it on a limited number of r principal components with equal weights as in PC regression; see De Mol *et al.* (2006). Even if a factor structure does not hold for X, the implementation in (4) can be successful as it can be seen as a form of forecast combination. Other implementations use prior distributions for α that imply that only a few of the variables in x_t are actually used in forecasting, thereby closely relating Bayesian regression to variable selection methods.

Forecast Combination

A frequently used alternative forecasting tool when one is faced with multiple forecasts of the same variable is forecast combination. Forecast combinations have been used with great success across a wide array of applications, i.e. macroeconomic forecasting, empirical finance and several non-economic applications, prompting a rich line of research to uncover theoretical reasons for this success.[1] Despite that, the reasons for the empirical success of forecast combinations, in particular of simple forecast averages, are still poorly understood. Diversification across information sets when it is not feasible to pool the underlying individual information sets to construct a 'super' model that nests each of the underlying forecasting models has been mentioned as one reason. Also, a forecast combination can exhibit more adaptability than individual forecasts when structural changes occur if there is a heterogenous response to these changes. Related to

[1]Timmermann (2006) provides a comprehensive and up-to-date survey of the forecast combination literature.

that, in case of unknown misspecification bias combining individual forecasts can be seen as a way to make the forecast robust again this bias.

Although *ad hoc*, forecast combination provides an alternative data-rich forecasting method that is relatively simple to apply and which has been shown to be capable of rivalling other data-rich methods in terms of forecasting performance. For example, Faust and Wright (2007) show that forecast combination methods provide better out-of-sample performance than factor methods when applied to high-dimensional *real-time* panels of U.S. macroeconomic and financial data. Within our data-rich setting we combine models to forecast y with each of our N indicator variables x_i separately resulting in an aggregate of N forecasts for y:

$$\hat{y}_t = \sum_{i=1}^{N} \omega_i(\hat{\alpha}_i x_{it}) \tag{5}$$

where the $\hat{\alpha}_i$'s results from estimates of

$$y_t = \alpha_i x_{it} + \epsilon_{it}; \quad i = 1, \ldots, N$$

and ω_i is the weight of the i^{th} forecast model. Several approaches have been proposed to determine the weights of the individual models in the forecast combination, and it has been shown that this choice is a major factor in determining the success of a forecast combination. Granger and Ramanathan (1984) suggest regressing the individual forecasts in (5) on the target variable over a historical sample, where one can add the restriction that $\sum_{i=1}^{N} \omega_i = 1$, and the resulting parameter estimates can be used as weights in (5). More generally, by specifying a loss function one can derive weights ω_i that are optimal under that specific loss function, see e.g. Elliott and Timmermann (2004). So under mean squared forecast error (MSE) loss functions, the weights in (5) would be inversely related to the MSE's of the individual forecasts and also reflect the correlation across the individual forecast errors.

However, estimates of the weights in (5) using one of the earlier discussed methods, can often be substantially biased, which essentially reflects a 'generated regressor' problem as the input for the forecast combination are recursive generated forecasts from individual models. This explains why in general simple averages of individual forecasts, i.e. $\omega_1 = \cdots = \omega_N = \frac{1}{N}$ in (5), or combinations with weights determined by (Bayesian) shrinkage towards the simple average as in Wright (2003a,b) are shown to perform really well empirically. A successful variation on this has recently been introduced by Capistrán and Timmermann (2007): projection on equal-weighted mean (PEW). Under PEW one simply regresses in our context, with OLS, the target variable y_t on an intercept and a simple average of the individual forecasts in (5), resulting in the following forecast for y_t:

$$\hat{y}_t = \hat{\delta} + \hat{\beta}\bar{y}_t \tag{6}$$

with

$$\bar{y}_t = \frac{1}{N} \sum_{i=1}^{N} (\hat{\alpha}_i x_{it}).$$

The non-zero intercept and slope coefficients in (6) will correct for bias in the individual forecasts and allow for more flexibility than possible in the simple average case. Both the empirical and Monte Carlo applications in Capistrán and Timmermann (2007) indicate that PEW frequently outperforms other combination methods, including simple forecast averages and Bayesian shrinkage weights.

2.2 An Alternative: Partial Least Squares

Introduced by Herman Wold and co-workers between 1975 and 1982,[2] partial least squares (PLS) is a relatively new method for estimating regression equations, that has received much attention in a variety of disciplines and especially in chemometrics. The basic idea is similar to principal component analysis in that factors or components, which are linear combinations of the original regression variables, are used, instead of the original variables, as regressors. A major difference between PC and PLS is that, whereas in PC regressions the factors are constructed taking into account only the values of the x_t variables, in PLS, the relationship between y_t and x_t is considered as well in constructing the factors.

There are a variety of definitions for PLS and accompanying specific PLS algorithms that inevitably have much in common. We provide a brief description of these definitions and algorithms we feel are most appropriate for conveying the essence of PLS. A conceptually powerful way of defining PLS is to note that the PLS factors are those linear combinations of x_t, denoted by Λx_t, that give maximum covariance between y_t and Λx_t while being orthogonal to each other. Of course, in analogy to PC factors, an identification assumption is needed, to construct PLS factors, in the usual form of a normalization.

A simple algorithm to construct k_1 PLS factors is discussed among others, in detail, in Helland (1990). Assuming for simplicity that both y_t and x_t have been normalised to have zero mean, a simplified version of the algorithm is given below

Algorithm 1

1. *Set $u_t = y_t$ and $v_{i,t} = x_{i,t}$, $i = 1, ...N$. Set $j = 1$.*
2. *Determine the $N \times 1$ vector of indicator variable weights or loadings $w_j = (w_{1j} \cdots w_{Nj})'$ by computing individual covariances: $w_{ij} = Cov(u_t,\ v_{it})$, $i = 1, ..., N$. Construct the j-th PLS factor by taking the linear combination given by $w_j' v_t$ and denote this factor by $f_{j,t}$.*
3. *Regress u_t and $v_{i,t}$, $i = 1, ..., N$ on $f_{j,t}$. Denote the residuals of these regressions by $\tilde{u}_t$ and $\tilde{v}_{i,t}$ respectively.*
4. *If $j = k_1$ stop, else set $u_t = \tilde{u}_t$, $v_{i,t} = \tilde{v}_{i,t}$ $i = 1, .., N$ and $j = j + 1$ and go to step 2.*

This algorithm makes clear that PLS is computationally tractable for very large data sets. Once PLS factors are constructed y_t can be modeled or forecast by regressing y_t on $f_{j,t}$ $j = 1, ..., k_1$. Helland (1988, 1990) provide a general description of the partial least squares (PLS) regression problem. Helland (1988) shows that the PLS estimates of the regression coefficients, α, of the regression of y_t on x_t obtained implicitly via PLS Algorithm 1 and a regression of y_t on $f_{j,t}$ $j = 1, ..., m$, can be equivalently obtained by the following formula

$$\hat{\alpha}_{PLS} = V_{k_1}(V_{k_1}' X' X V_{k_1})^{-1} V_{k_1}' X' y \tag{7}$$

where $V_{k_1} = (X'y, X'XX'y, ..., (X'^{k_1-1}X'y)$, $X = (x_1, ..., x_T)'$ and $y = (y_1, ..., y_T)'$. A major question relates to the determination of the number of PLS factors, denoted by k_1 in Algorithm

[2] See, e.g., Wold (1982).

1. The PLS literature seems to have little to say on this question from a theoretical statistical point of view. A suggestion that seems to be partly adopted in the PLS literature is to set k_1 so that $V_{k_1}'X'XV_{k_1}$ is well-conditioned whereas $V_{k_1+1}'X'XV_{k_1+1}$ is not (see, e.g., Stoica and Söderström (1998)).

PLS regression does not seem to have been explicitly considered for large data sets (i.e. when N is assumed to tend to infinity). As a result there is a discrepancy between the large theoretical literature on PC regression for large data sets as developed in, e.g., Bai (2003) and theoretical results for PLS. Therefore, in what follows we provide an extension of a well known link between PLS and PC regression to the case where $N \to \infty$. We wish to show that PLS and PC regression analyses are equivalent as $N, T \to \infty$, under certain conditions. We use the framework of Stoica and Söderström (1998). Denote the PLS and PC regression estimates by $\hat{\alpha}_{PLS}$ and $\hat{\alpha}_{PC}$ respectively, based on (7) and

$$\hat{\alpha}_{PC} = \Lambda'(\Lambda X'X\Lambda'^{-1}\Lambda X'y \tag{8}$$

where Λ is the $r \times N$ matrix of linear combinations of the x_{it}'s that result from minimizing (3). Let the matrix norm we use be $||A|| = tr(A'^{1/2}$. Assume that

$$X'X = S\Psi S' + C_\delta \tag{9}$$

where $\Psi = diag(\psi_1, ..., \psi_r)$ for some $r < N$, $S'S = I$, C_δ denotes a term whose matrix norm is $O_p(\delta)$ and $\delta \to 0$. Then,

$$||\hat{\alpha}_{PLS} - \hat{\alpha}_{PC}|| = O_p(\delta). \tag{10}$$

An interesting aside that comes out of the above setup of Stoica and Söderström (1998) is that if the definition of k_1, suggested in the previous paragraph, is adopted then, for finite N at least, and assuming a reduced rank structure of the form (9) we have that $k_1 < r$.

We make the following assumptions

Assumption 1 *Let $\Sigma = \Sigma_N = [\sigma_{ij}]$ denote the $N \times N$ second moment matrix of X. Σ can be factorised as follows:*

$$\Sigma = \tilde{S}\tilde{\Psi}\tilde{S}' + R$$

where $\tilde{S}\tilde{S}' = I$, $\tilde{\Psi} = diag(\tilde{\psi}_{N1}, ..., \tilde{\psi}_{Nr})$, $r < N$ and $||R|| = o(N)$.

Assumption 2 *For all $i, j = 1, ..., N$*

$$\sum_{t=1}^{T}(x_{i,t}x_{j,t} - \sigma_{i,j}) = O_p(T^{1/2})$$

Remark 1 *These assumptions deserve some comment. Assumption 1 states that the variables in X are asymptotically with respect to N collinear. It is instructive to compare this assumption with the standard factor assumption. In one sense this assumption is stronger than the standard factor assumption because in standard factor models the sum of all bounded eigenvalues of the covariance matrix is $O(N)$ whereas under assumption 1 it is $o(N)$. On the other hand this assumption is weaker. Under a standard factor assumption there can be only a finite number of unbounded eigenvalues, and hence unbounded singular values, for the covariance matrix of the data. In our case we can have an infinity of unbounded eigenvalues as long as the sum* *of all but the first r eigenvalues is $o(N)$. In particular the remainder term, R, can, in fact, be parametrised as a neglected 'weak' factor model whose eigenvalue characterisation allows for unbounded eigenvalues which, however, have to grow at a rate slower than N. Assumption 2 is a mild, high level, assumption. It is sufficient to have a central limit theorem for $(x_{i,t}x_{j,t} - \sigma_{i,j})$ for this assumption to hold.*

We have the following Theorem

Theorem 1 *Under Assumptions 1-2, and as $N, T \to \infty$ sequentially,*

$$||\hat{\alpha}_{PLS} - \hat{\alpha}_{PC}|| = o_p\left((NT)^{-1/2}\right) \tag{11}$$

Proof: In order the prove this result in our case we simply need to show that as $N, T \to \infty$, assumption (9) holds in probability asymptotically. We first note the following obvious fact: If

$$||\hat{\alpha}^n_{PLS} - \hat{\alpha}^n_{PC}|| = o_p(a_{N,T}), \tag{12}$$

for some sequence $a_{N,T}$, then

$$||\hat{\alpha}_{PLS} - \hat{\alpha}_{PC}|| = o_p(a_{N,T}/b_{N,T}), \tag{13}$$

where $\hat{\alpha}^n_{PLS}$ and $\hat{\alpha}^n_{PC}$ are the the PLS and PC regression estimates obtained when

$$X^n = \frac{1}{b_{N,T}} X$$

are used as regressors. Given this fact, we focus on $\hat{\alpha}^n_{PLS}$ and $\hat{\alpha}^n_{PC}$ where we set $b_{N,T} = (NT)^{1/2}$. First, we have that

$$X^{n\prime} X^n = \frac{1}{N}\Sigma + \frac{1}{N}\left(\frac{X^{n\prime}X^n}{T} - \Sigma\right)$$

But, by the factor assumption,

$$\frac{1}{N}\Sigma = \frac{1}{N}\tilde{S}\tilde{\Psi}\tilde{S}' + o(1)$$

where

$$\tilde{\psi}_{Ni} = O(N), \quad i = 1, \ldots, r.$$

The proof is complete if we show that

$$\left\|\frac{1}{N}\left(\frac{X^{n\prime}X^n}{T} - \Sigma\right)\right\| = o_p(1),$$

since, then, by (9) and (10), it follows that

$$||\hat{\alpha}^n_{PLS} - \hat{\alpha}^n_{PC}|| = o_p(1),$$

and then, by (12) and (13), (11) follows. But,

$$\left\| \frac{1}{N} \left(\frac{X^{n\prime} X^n}{T} - \Sigma \right) \right\| =$$

$$\left(\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\frac{1}{T} \sum_{t=1}^{T} (x_{i,t}x_{j,t} - \sigma_{i,j}) \right)^2 \right)^{1/2} = O_p(T^{-1/2}),$$

where the second equality follows by Assumption 2, proving the result. Q.E.D.

Garthwaite (1994) provides a rationale to cast (*ad hoc*) forecast combinations in terms of the above described PLS framework. Essentially what Garthwaite (1994) shows is that a general PLS algorithm like Algorithm 1 can be cast in terms of sequences of univariate regressions, i.e.

Algorithm 2

1. *Set* $u_t = y_t$ *and* $v_{i,t} = x_{i,t}$, $i = 1, ...N$. *Set* $j = 1$.
2. *Regress* u_t *on* $v_{i,t}$, $i = 1, ..., N$ *and denote the OLS estimate of the coefficient of each regression by* β_i. *Construct the* j*-th PLS factor by taking the weighted average of* $\beta_i v_{it}$: $f_{j,t} = \tilde{w}_j' v_t$ *with* $\tilde{w}_j = ((\beta_1 w_{1j}) \cdots (\beta_N w_{Nj}))'$ *where* $(w_{1j} \cdots w_{Nj})$ *are given.*
3. *Regress* u_t *and* $v_{i,t}$, $i = 1, ..., N$ *on* $f_{j,t}$. *Denote the residuals of these regressions by* $\tilde{u}_t$ *and* $\tilde{v}_{i,t}$ *respectively.*
4. *If* $j = k_1$ *stop, else set* $u_t = \tilde{u}_t$, $v_{i,t} = \tilde{v}_{i,t}$ $i = 1, .., N$ *and* $j = j + 1$ *and go to step 2.*

Hence, when in this algorithm one sets $(w_{1j} \cdots w_{Nj}) = (Var(v_{1t}) \cdots Var(v_{Nt}))$ Algorithm 1 follows, but if one assumes, as suggested by Garthwaite (1994), $(w_{1j} \cdots w_{Nj}) = (\frac{1}{N} \cdots \frac{1}{N})$ than in the *one-factor* case the Capistrán and Timmermann (2007) PEW forecast combination (6) follows. In general, forecast combinations can be interpreted through Algorithm 2 as restricted approximations to *one-factor* PLS regression, with alternative specifications for $(w_{1j} \cdots w_{Nj})$ and often with *zero* intercept and slope coefficients in the final forecast regression. This explains why forecast combinations often do very well compared to PC regressions within a factor environment, and why we would expect that PLS still would do well in a data-rich forecast environment where the factor assumption does not hold. Note, though, that PLS is much more general and it allows for several factors to be included in the forecast regression.

So given the aforementioned link between forecast combinations and PLS regression, Theorem 1 can then used to show that under certain conditions, i.e. the assumptions underlying Theorem 1 as well as an one-factor PLS regression specification, forecast combinations and PC regression are asymptotically identical. Similarly, as discussed in Section 2.1, De Mol *et al.* (2006) prove the existence of asymptotic equivalence between PC regression and Bayesian regression when the underlying data comply with a factor structure.[3] Thus, given that structure,

[3] It is worth noting that the assumption made by De Mol *et al.* (2006) on the idiosyncratic part of their entertained factor model bears similarities to our assumtion 1. In particular, just like the implication of our assumption 1, discussed in remark 1, the idiosyncratic component of their factor model can accommodate a residual 'weak' factor model in the sense that the eigenvalues implied by that factor model can be unbounded but have to grow at a rate slower than N.

Bayesian regression should, via Theorem 1, be asymptotically equivalent to PLS regression and, under the one-factor assumption, forecast combinations. Therefore, the introduction of the PLS regression framework provide a means to asymptotically tie together different existing data-rich forecasting methods and provides a theoretical rationale for the common empirical finding that different data-rich approaches have similar performance.

3 Monte Carlo Analysis

In this Section we present a Monte Carlo study of the finite sample properties of PLS regression for large data sets. We compare the performance of PLS to that of PCA regression and Bayesian regression. Details on these alternative methods are given in Section 4.

3.1 Monte Carlo Set-up

We wish to explore the finite sample relative performance of the various methods both for the case when the data have a factor structure and for the case where there is no factor structure. We consider the following setup:

$$y_t = \alpha' x_t + \epsilon_t, \quad t = 1, ..., T \tag{14}$$

with

$$x_t = \Lambda' f_t + u_t \tag{15}$$

where $x_t = (x_{1,t}, ..., x_{N,t})'$, $u_t = (u_{1,t}, ..., u_{N,t})'$, $\alpha = (\alpha_1, ..., \alpha_N)'$, $\Lambda = (\lambda_1, ..., \lambda_N)$ and $\lambda_i = (\lambda_{i,1}, ..., \lambda_{i,r})'$. We set $\alpha_i \sim NIID(0, 1)$. For the case where there is a factor structure we set $\lambda_{i,j} \sim NIID(0, 1)$. If there is no factor structure then in (15) $\lambda_{i,j} = 0$ for all i, j. $u_{i,t} \sim NIID(0, 1)$.[4] An important parameter for the Monte Carlo study is the R^2 of the regression equation (14). We control this by controlling the variance of ϵ_t. Therefore, we have that $\epsilon_t = \sqrt{cN}\varepsilon_t$ where $\varepsilon_t \sim NIID(0, 1)$. Setting $c = 1, 4, 9$ gives an R^2 of $0.5, 0.2$ and 0.1 respectively for the case of no factors. For the case of factors the R^2 is slightly higher but this is a minor deviation since $r << N$. We consider these values for R^2 to provide reasonable representations of empirically relevant situations. We set $r = 1, 4, 6$ and assume that the assumed number of PCA factors, k_2, is equal to the true number of factors, r, when carrying out PCA. For the case of PLS we know that the number of PLS factors, k_1, is smaller than the number of PCA factors (see, e.g., Stoica and Söderström (1998)). Therefore, we set k_1 to 1,2 and 3 to correspond to the number of PCA factors used. When the data do not have a factor structure we still keep the same number of assumed factors both for PCA and PLS. For the Bayesian regression we set the shrinkage parameter to qN where $q = 0.01, 1, 2, 5, 10$. Finally, we set $N, T = 20, 30, 50, 100, 200, 400$. We carry out 1000 Monte Carlo replications.

We evaluate the competing methods using one in-sample and one out-of-sample criterion. The in-sample criterion is the average R^2 over the Monte Carlo replications. The out-of-sample

[4] This setting implies that for the case of no factors the x_t variables are not correlated. This maybe considered restrictive. However, we have carried out an alternative Monte Carlo experiment where we allowed cross-sectional correlation in u_t by setting the 5 diagonals below and above the main diagonal of the covariance matrix of u_t to positive numbers, while ensuring positive-definiteness and symmetry for the covariance matrix. The results, which are not reported, are very similar to those reported in the next subsection for the case of no factors and are available upon request.

criterion is the relative forecast MSE compared to PCA regression. To construct this we estimate all models over T observations and we use the implied regression weights to do one-step ahead forecasting and get forecast errors over a forecast evaluation sample of 100 observations.

3.2 Monte Carlo Results

Results are reported in Tables 1-12. Tables 1-6 report results for the case where the data do not have a factor structure whereas Tables 7-12 report results for the case where the data have a factor structure. Starting with the former case we see that the average R^2 of PLS exceeds that of PCA for all cases considered which is expected given the absence of a factor structure. The average R^2 of Bayesian regression depends crucially on the shrinkage parameter.

The in-sample measure is not very informative for forecasting and therefore we focus on the relative forecast MSE presented in Tables 4-6. PLS outperforms massively PCA for all true R^2 considered. The degree of superiority for PLS is marginally more apparent for higher true R^2. It is also more apparent when the assumed number of PLS and PCA factors is higher. Moving to a comparison between PLS and Bayesian regression we see that PLS usually outperforms Bayesian regression. The only value of the shrinkage parameter for which this is not the case is 0.01, in which case Bayesian regression has extremely variable performance depending on the values of N and T. Given that we report relative forecast MSE the performance of Bayesian regression also depends on the assumed number of PCA factors. For this case we note that although, for high values of N and T, Bayesian regression can outperform PLS for this value of the shrinkage parameter, the reverse occurs on a massive scale for low values of N and T.

Moving on to the case where the data do have a factor structure we see that the performance of PLS and PCA is much more similar. This reflects the theoretical result we have proved in the previous section. The only other noteworthy feature for this case, given already available Monte Carlo work on the relative performance of PCA and Bayesian regression, is the confirmation of the superiority of Bayesian regression in a large number of cases.

To conclude we see that our Monte Carlo study suggests a great advantage for PLS compared to all other methods we considered in terms of forecasting performance, in the case where the data do not have a standard parsimonious representation such as a factor structure.

4 Empirical Applications

4.1 Implementation of the Data-Rich Methods and the Forecast Comparison

We follow standard practice in the macroeconomic forecasting literature and use as benchmark for our data-rich based forecasts an autoregressive (AR) model

$$\Delta y_{t+h,t} = \alpha^h + \sum_{i=1}^{p} \rho_i \Delta y_{t-i+1,t-i} + \epsilon_{t+h,t}, \quad t = 1, \ldots, T \tag{16}$$

with $\Delta y_{t+h,t} = y_{t+h} - y_t$ for $h > 0$ and $\Delta y_{t-i+1,t-i} = y_{t-i+1} - y_{t-i}$ for $i = 1, \ldots, p$. The number of lagged first differences p in (16) is determined by sequentially applying Schwarz (1978)'s Bayesian Information Criterion (BIC) starting with a maximum lag order of $p = p_{max}$ down to $p = 1$. Next, we use as benchmark the unconditional mean,

$$\Delta y_{t+h,t} = \alpha^h + \epsilon_{t+h,t}, \tag{17}$$

which implies a random walk (RW) forecast for the level of the forecast variable y_t. Our assessment of the forecasting performance of the data-rich methods relative to pure AR-based and random walk-based forecasts is based on the square root of the mean of the squared forecast errors (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{T - t_0 - h} \sum_{s=t_0}^{T-h} e^2_{s+h,s}}, \tag{18}$$

where $e_{s+h,s}$ is the forecast error of the model-generated prediction of a forecast variable relative to the *observed* $\Delta y_{t+h,t}$. In Section 4.3 we will report ratios of the RMSE of the respective data-rich forecasting approaches relative to the RMSE based on either (16) or (17). Superior out-of-sample performance of a data-rich method relative to these benchmarks is, obviously, indicated by a RMSE ratio smaller than one.

Our data-rich forecasts of h period-ahead changes in y_t are generated using a model that adds the information extracted from the N indicator variables in the $N \times 1$ vector $X_t = (x_{1,t} \cdots x_{N,t})$ to the benchmark models (16) and (17), i.e. respectively

$$\Delta y_{t+h,t} = \alpha^h + \beta^{h\prime} \mathcal{F}(X_t) + \sum_{i=1}^{p} \rho_i \Delta y_{t-i+1,t-i} + \epsilon_{t+h,t} \tag{19}$$

and

$$\Delta y_{t+h,t} = \alpha^h + \beta^{h\prime} \mathcal{F}(X_t) + \epsilon_{t+h,t}. \tag{20}$$

where β^h is $r \times 1$. In (19) and (20) $\mathcal{F}(X_t)$ represents a $r \times 1$ function of X_t that compresses the information in the N indicator variables, i.e. through principal components (PC), partial least squares (PLS) or by estimating the β^h's through Bayesian ridge regression (BRR, where $r = N$). We operationalize the construction of $\mathcal{F}(X_t)$ on our data sets as follows:

Principal Components

Following Stock and Watson (2002b) we take our $T \times N$ matrix of N indicator variables $X = (X_1' \cdots X_T')'$ and normalize this such that the variables are in zero-mean and unity variance space, which results in the $T \times N$ matrix $\tilde{X}$. Having done that, we compute the r eigenvectors of the $N \times N$ matrix $\tilde{X}'\tilde{X}$ that correspond to the first r largest eigenvalues of that matrix, which we assemble in the $N \times r$ matrix Λ^r. As discussed in Section 2.1, these eigenvectors are often used to approximate the common factors F that determine the series in X, i.e. $F = \frac{1}{\sqrt{N}} \tilde{X} \Lambda^r$. $\mathcal{F}(X_t)$ in (19) and (20) can therefore be approximated as:

$$\begin{pmatrix} \mathcal{F}'(X_1) \\ \vdots \\ \mathcal{F}'(X_T) \end{pmatrix} \equiv \begin{pmatrix} F_1' \\ \vdots \\ F_T' \end{pmatrix} = F = \frac{1}{\sqrt{N}} \tilde{X} \Lambda^r. \tag{21}$$

Our forecasting models will be updated based on an expanding window of historical data, which in case of the principal components-based models evolves as follows:

1. First forecast for all h is generated on t_0.
2. Extract r principal components F_t from the N indicator variables N over the sample $t = 1, \ldots, t_0 - h$.

3. Estimate either (19) or (20) with $\digamma(X_t) = F_t$ over the sample $t = 1, \ldots, t_0 - h$ for each h.

4. Extract r principal components F_t from the N indicator variables N over the sample $t = 1, \ldots, t_0$.

5. Generate for h the forecast $\Delta\hat{y}_{t+h,t}$ using the parameter estimates from step 3 and F_t from step 4.

6. Repeat for $t_0 + 1, \ldots, T - h$ for each h.

Bayesian Ridge Regression

When Bayesian ridge regression is used to compress the forecast information in the N indicator variables $\digamma(X_t)$ in (19) and (20) simply equals X_t, whereas β^h is estimated with a restricted estimator, i.e. in case of (20) we have

$$\Delta y_{t+h,t} = \alpha^h + \beta^{h'}_{BRR}\tilde{X}_t + \epsilon_{t+h,t}, \tag{22}$$

where

$$\hat{\beta}^h_{BRR} = \left(\tilde{X}'\tilde{X} + \nu I_N\right)\tilde{X}'\Delta\dot{Y}_h; \quad \Delta\dot{Y}_h = \left(I_T - \iota(\iota'^{-1}\iota')\right)\begin{pmatrix} \Delta y_{h+1,1} \\ \vdots \\ \Delta y_{T,T-h} \end{pmatrix}$$

with ν is the (scalar) ridge or shrinkage parameter, ι is a $T \times 1$ vector of ones and I_q is a q-dimensional identity matrix. As in the case of principal components-based regressions we use in (22) a normalized version of the $T \times N$ matrix of indicator variables $X = (X_1' \cdots X_T')'$, indicated with $\tilde{X}$, and we also demean $\Delta y_{t+h,t}$ first before we estimate β^h_{BRR}. By doing this we follow De Mol *et al.* (2006), as the ridge regression (22) can be interpreted as a Bayesian regression with a Gaussian prior. In case of (19) we have

$$\Delta y_{t+h,t} = \alpha^h + \beta^{h'}_{BRR}\tilde{X}_t + \sum_{i=1}^{p}\rho_i\Delta y_{t-i+1,t-i} + \epsilon_{t+h,t}, \tag{23}$$

with

$$\hat{\beta}^h_{BRR} = \left(\ddot{\tilde{X}}'\ddot{\tilde{X}} + \nu I_N\right)\ddot{\tilde{X}}'\Delta\ddot{Y}_h;$$
$$\Delta\ddot{Y}_h = \left(I_T - W(W'^{-1}W')\right)\Delta\dot{Y}_h,\ \ \ddot{\tilde{X}} = \left(I_T - W(W'^{-1}W')\right)\tilde{X}$$

where the projection is on the $T \times p$ matrix of lagged predictor variables $W = (W_1 \cdots W_T)'$, $W_t = (\Delta y_{t,t-1} \cdots \Delta y_{t-p+1,t-p})$. The projection of the demeaned predictor variable and the N normalized indicator variables is done to guarantee that the indicator variables are orthogonal to the autoregressive terms. For both (22) and (23) the values of $\hat{\alpha}^h, \hat{\rho}_1, \ldots, \hat{\rho}_p$ compatible with $\hat{\beta}^h_{BRR}$ can be retrieved simply by regressing $(\Delta y_{t+h,t} - \hat{\beta}^{h'}_{BRR}\tilde{x}_t)$ on an intercept term and, if applicable, $\Delta y_{t,t-1}, \ldots, \Delta y_{t-p+1,t-p}$.

Forecasts with (22) and (23) make use of model updates based on an expanding window of known historical data:

1. First forecast for all h is generated on t_0.

2. Estimate either (22) or (23) over the sample $t = 1, \ldots, t_0 - h$ for each h.

3. Generate for h the forecast $\Delta\hat{y}_{t+h,t}$ using the parameter estimates from step 2.

4. Repeat for $t_0 + 1, \ldots, T - h$ for each h.

Partial Least Squares

With partial least squares (PLS) regression, $\mathcal{F}(X_t)$ in (19) and (20) is constructed by computing r orthogonal combinations from the N indicator variables, where the weights of the individual indicator variables in the respective combinations are chosen such that the covariance with $\Delta y_{t+h,t}$ is maximized. The general PLS algorithm from Section 2.2 can be implemented for macroeconomic forecasting as follows:

Algorithm 3

1. *Denote, as before, the $T \times N$ matrix of indicator variables, each normalized to have a zero mean and unit variance, as $\tilde{X}$ and demean the predictor variable, i.e.*

$$\Delta\dot{Y}_h = \left(I_T - \iota(\iota'^{-1}\iota')\right) \begin{pmatrix} \Delta y_{h+1,1} \\ \vdots \\ \Delta y_{T,T-h} \end{pmatrix}.$$

2. *The r PLS factors $F_{1,t}^{PLS}, \ldots, F_{r,t}^{PLS}$ and their loadings $w_1, \ldots, w_r$ are iteratively build up through projections on lower order PLS factors followed by computing the covariances between the resulting residuals of the columns from $\tilde{X}$ and those of $\Delta\dot{Y}_h$:*

$$F_l^{PLS} = \tilde{X}_{l|l-1} w_l; \quad w_l = \frac{1}{T-1} \tilde{X}'_{l|l-1} \Delta\dot{Y}_{h,l|l-1} \quad \textit{for} \quad l = 1, \ldots, r \tag{24}$$

where for $l = 1$ (the first PLS factor)

$$\tilde{X}_{1|0} = \tilde{X}, \quad \Delta\dot{Y}_{h,1|0} = \Delta\dot{Y}_h,$$

and for $l > 1$

$$\tilde{X}_{l|l-1} = \left(I_T - F_{l-1}^{PLS}(F_{l-1}^{PLS'} F_{l-1}^{PLS})^{-1} F_{l-1}^{PLS'}\right) \tilde{X}_{l-1|l-2} \quad \textit{and}$$
$$\Delta\dot{Y}_{h,l|l-1} = \left(I_T - F_{l-1}^{PLS}(F_{l-1}^{PLS'} F_{l-1}^{PLS})^{-1} F_{l-1}^{PLS'}\right) \Delta\dot{Y}_{h,l-1|l-2}.$$

3. *Finally, we simply plug in the r PLS factors $F_t^{PLS} = (F_{1,t}^{PLS} \cdots F_{r,t}^{PLS})'$ from (24) in the predictive regression (20) which we estimate in the standard way:*

$$\Delta y_{t+h,t} = \alpha^h + \beta^{h'} F_t^{PLS} + \epsilon_{t+h,t}. \tag{25}$$

When lagged predictor variables are included in the predictive regression, as in (19), it slightly complicates the extraction of the PLS factors. As in the BRR case one needs to control for the effect of $\Delta y_{t,t-1}, \ldots, \Delta y_{t-p+1,t-p}$ on the covariances between $\Delta y_{t+h,t}$ and $x_{1,t}, \ldots, x_{N,t}$, but unlike BRR one cannot do that by simply projecting these on $\Delta y_{t,t-1}, \ldots, \Delta y_{t-p+1,t-p}$ as in the PLS case the relationship between $\Delta y_{t+h,t}$ and $x_{1,t}, \ldots, x_{N,t}$ is non-linear in nature. However, $\Delta y_{t+h,t}$ is linear in the r PLS factors and we can use that feature to adapt the above algorithm for the lagged predictor case:

Algorithm 4

1. *Estimate with OLS:*
$$\Delta\ddot{Y}_h = \ddot{F}^{PLS}\beta^h + \ddot{\varepsilon}, \tag{26}$$
where
$$\Delta\ddot{Y}_h = \left(I_T - \check{W}(\check{W}'\check{W})^{-1}\check{W}'\right)\Delta Y_h,\ \ddot{F}^{PLS} = \left(I_T - \check{W}(\check{W}'\check{W})^{-1}\check{W}'\right)F^{PLS}$$
and $\check{W} = (W \quad \iota)$ *with* W *the same as in (23) and* F^{PLS} *identical to (25) (so assuming no lagged predictors).*

2. *Estimate with OLS:*
$$\left(\Delta Y_h - F^{PLS}\hat{\beta}^h\right) = \check{W}\rho + \xi. \tag{27}$$

3. *Replace in (24)* $\Delta\dot{Y}_h$ *with* $\left(\Delta Y_h - \check{W}\hat{\rho}\right)$ *and compute the* r *PLS factors and corresponding loadings as in (24); call these* F^{PLS^*} *and* $w^* = (w_1^* \cdots w_r^*)$.

4. *Go back to step 1 using* F^{PLS^*} *from the previous step and repeat until the relative improvement in the sum of squared residuals* $\left(\Delta Y_h - F^{PLS^*}\hat{\beta}^h - \check{W}\hat{\rho}\right)'\left(\Delta Y_h - F^{PLS^*}\hat{\beta}^h - \check{W}\hat{\rho}\right)$ *is smaller than a chosen threshold* ς *(typically* ς *equals a number like* 10^{-3} *or* 10^{-4}*).*

5. *After convergence in step 4, use the resulting optimal PLS factors* F^{PLS^*} *in (19), which can be estimated in the standard way:*
$$\Delta y_{t+h,t} = \alpha^h + \beta^{h\prime}F_t^{PLS^*} + \sum_{i=1}^{p}\rho_i\Delta y_{t-i+1,t-i} + \epsilon_{t+h,t} \tag{28}$$

Finally, forecasts from (25) and (28) are generated as follows, again using an expanding window of historical data:

1. First forecast for all h is generated on t_0.

2. Extract r PLS factors F_t^{PLS} from the N indicator variables over the sample $t = 1, \ldots, t_0 - h$ for each h based on either Algorithm 3 or 4.

3. Estimate either (25) or (28) over the sample $t = 1, \ldots, t_0 - h$ for each h.

4. Extract r PLS factors F_t^{PLS} from the N indicator variables over the sample $t = 1, \ldots, t_0$ for each h using the corresponding loadings w_r from step 2 based on either Algorithm 3 or 4.

5. Generate for h the forecast $\Delta\hat{y}_{t+h,t}$ using the parameter estimates from step 3 and F_t^{PLS} from step 4.

6. Repeat for $t_0 + 1, \ldots, T - h$ for each h.

4.2 The Data Set and Variable Construction

Stock and Watson (2007) reorganize the large panel of macroeconomic, financial and survey-based indicator variables for the United States from Stock and Watson (2002b) and update the span of the data to the end of 2006. Both our forecast variables and our panel of indicator variables are extracted from Stock and Watson (2007)[5] and we focus on the 109 monthly series from this U.S. data set, which before transformation span a sample starting in January 1959 and ending in December 2006.

The panel of indicator variables consist of 104 series spanning real variables (sectoral industrial production, employment, subcomponents of unemployment and hours worked), nominal variables (subcomponents of consumer price index, producer price indexes, deflators, wages, money and credit aggregates), asset prices (interest rates, stock prices and exchange rates) and surveys; the 5 predictor variables (discussed below) are not included, but their subcomponents are included. The indicator variables are transformed such that they are $I(0)$, which in general means that the real variables are expressed in log first differences and we use simply first differences of series expressed in rates, such as interest rates and unemployment series; see Appendix A for more details. With respect to prices, wages, money and credit series we transform these into first differences of annual growth rates in order to guarantee that the dynamic properties of these transformed series are comparable to those of the rest of the indicator variable panel, as for example motivated in D'Agostino and Giannone (2006, Appendix B).[6] Hence, after transforming the indicator variables we end up with an effective span of the data that starts in February 1960 (i.e. 1960.2) and ends in December 2006 (i.e. 2006.12).

The aforementioned panel of indicator variables will be used to forecast appropriate transformations of U.S. CPI inflation, U.S. core CPI inflation (this is CPI inflation minus food and energy prices), U.S. industrial production, the U.S. unemployment rate and the U.S. federal funds rate. These five forecast variables are not part of the panel of predictors and are chosen to cover the full range of nominal and real developments in the U.S. economy. Table 13 provides an overview of the appropriate transformation of each forecast variable, which guarantees stationarity, for each of the respective versions of our predictive regressions (19) and (20). As described in the previous subsection, these forecasting models are updated based on an expanding window of data and all forecasts are direct forecasts for 4 horizons (in months): $h = 1$, $h = 3$, $h = 12$ and $h = 24$, which are horizons commonly analyzed in the literature. The forecast evaluation spans three samples: January 1972 - December 2006, January 1972 - December 1984 and January

[5]We are very grateful to Mark Watson who provided us with the underlying raw data from Stock and Watson (2007).

[6]This particular transformation acknowledges that series like log price levels and log money aggregate levels behave as if they are $I(2)$, possibly because of mean growth shifts due to policy regime shifts, financial liberalizations and other phenomena.

1985 - December 2006. The latter two sub-samples split the first sample in two around the start of the 'Great Moderation', e.g. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Sensier and van Dijk (2004) and Groen and Mumtaz (2007) all find evidence for a downward, exogenous, shift in the volatility of a large number of U.S. macroeconomic time series around 1985. This sample split is of particular importance for forecasting U.S. economic time series, as it has been shown that it is difficult for a lot of data-rich, approaches (including *Greenbook* projections from the Federal Reserve Board) to beat simple, non-structural benchmarks like RW and AR models after the occurrence of the 'Great Moderation'.[7]

4.3 Forecasting Results

As discussed in Section 4.1, we will assess the forecasting performance of our three data-rich forecast methods with two simple benchmark forecasts: those based on an autoregressive (AR) specification and those based on the unconditional mean or random walk (RW) model (respectively (16) and (17)). Before we discuss these forecast comparisons in more detail, it is of interest to assess for each forecast variable and each evaluation sample which benchmark would be more difficult to beat. Table 14 provides this assessment based on the data described in Section 4.2. Generally, the results in this table suggest that for industrial production the AR model is the most appropriate benchmark, for the federal funds rate it is the RW, and for the other forecast variables there is a more mixed picture with the AR model outperforming the RW model slightly at shorter horizons and *vice versa* at longer horizons.

The first set of evaluation results can be found in Table 15 and relate to forecasting changes in annual CPI inflation. Over the full 1972-2006 evaluation sample, the PLS model, in particular the one-factor specification, is the best performing model, although the principal components-based approach is a good second best. When we focus on the two sub-samples, the PLS approach is less dominant and either has to compete with the principal components approach (the 1985-2006 sample) or the Bayesian ridge regression (the 1972-1984 period); note, though, that when it loses out to either of these two the PLS approach still provides a close second best. Of course, the overall forecasting performance of the data-rich approaches is quite poor over the post-Great Moderation period.

For changes in annual core CPI inflation in Table 16, the one-factor PLS approach dominates in both the overall 1972-2006 evaluation sample and the 1972-1984 period. Even more striking than in the CPI inflation case is the overall poor performance of the data-rich models for 1985-2006, which possibly reflects a less volatile monetary policy regime that credibly attempts to stabilize inflation in the longer term; see e.g. Clarida *et al.* (2000).

The Great Moderation has less of a negative effect on the predictive performance of our data-rich methods for our real forecast variables, i.e. industrial production growth and unemployment rate changes; see the results for 1985-2006 in Tables 17-18. In case of industrial production growth (Table 17) the PLS model is the dominant approach relative to the AR benchmark for 1972-2006 and 1985-2006, and the AR benchmark outperforms in Table 14 the RW benchmark for industrial production growth throughout all evaluation samples. For 1972-1984 PLS has to compete with the other two approaches, although PLS is the only method for this evaluation sample that consistently outperforms the benchmarks at all horizons. The domination of the

[7]See, for example, D'Agostino *et al.* (2006) who compare PC-based, VAR-based and *Greenbook* forecasts for U.S. inflation and economic growth with simple benchmarks for both pre- and post-Great Moderation samples.

PLS approach *vis-à-vis* principal components and BRR becomes more striking for unemployment (Table 18): only for the two-year horizon in the 1972-1984 period is BRR able to perform better. Across all evaluation samples and horizons, PLS outperforms both benchmarks, which cannot be said for the other data-rich methods.

Finally, we turn to the results for the federal funds rate in Table 19. As the federal funds rate is in the end determined by the Federal Reserve Board, which sets the target for the federal funds rate by taking into account both nominal and real developments, one would expect that the data-rich methods would now have a performance somewhere in between that for the nominal and real forecasting variables. This seems certainly to be the case for principal components and BRR, although BRR performs poorly for the post-Great Moderation 1985-2006 period. PLS, however, performs well throughout the different evaluation samples and structurally outperforms the RW benchmark, which according to the results in Table 14 dominates the AR benchmarks for the federal funds rate.

The empirical forecast evaluation leads to a number of general observations. First, the PLS-based forecast models are, generally speaking, amongst the best performing models. Even in the few cases when they are outperformed by either PC- or BRR-based approaches, they are close competitors. Note also that in Tables 15-19 the performance of methods that use principal components and PLS factors are pretty close, with PLS usually having the edge over principal components-based regressions. These findings are almost identical to our Monte Carlo evaluation in Section 3.2 when we assume that the data complies with a factor structure. However, in that case the Monte Carlo simulations also indicated that BRR models outperform the rest, whereas in the empirical results of this subsection the BRR models hardly ever dominate in terms of forecasting performance.

5 Conclusions

In this paper we have revisited a number of approaches for prediction with many predictors that are widely used in macroeconomic forecasting and compare these with a less widely known alternative approach: partial least squares (PLS) regression. Under PLS regression, one constructs a number of linear combinations of the predictor variables such that the covariances between the target variable and each of these linear combinations are maximized.

We provide theoretical arguments for asymptotic similarity between principal components (PC) regression and PLS regression when the underlying data has a factor structure. More specifically, we prove that when the second moment matrix of the predictor variables is reduced rank in nature, differences between PC and PLS regressions disappear rapidly at a rate exceeding $(NT)^{1/2}$, where N (T) is the cross-section (time series) dimension of the data. We also argue that forecast combinations can be considered as a specific form of PLS regression. Hence, whether or not a large panel of predictors has a factor structure, we would expect PLS regression, like Bayesian ridge regression, to do well in macroeconomic forecasting.

An extensive Monte Carlo analysis, which compares PC regression and Bayesian ridge regression (for several ridge parameter values) with PLS regression, yields a number of interesting insights. Firstly, when we assume that the predictors relate to the target variable through a factor structure, PLS regression is shown to have an in-sample and out-of-sample performances that are at least comparable to, and often better than PC regression. PLS regression also compares well to Bayesian ridge regression under this data specification, although Bayesian ridge regression generally performs the best across the three approaches. When the relation between the predictors and the target variable does not comply with a factor structure, PLS regression clearly has the edge in terms of in-sample fit and out-of-sample forecasting performance.

Finally, we apply PC, PLS and Bayesian ridge regression on a panel of 104 U.S. monthly macroeconomic and financial variables to forecast, for the United States, CPI inflation, core CPI inflation, industrial production, unemployment and the federal funds rate, where these forecasts are evaluated across several sub-samples. PLS regression turns out to be generally the best performing prediction method, and even in the few cases when it is outperformed by PC regression or Bayesian ridge regression PLS regression still is a close competitor.

Table 1: No Factors, Average R^2 for $(k_1, k_2) = (1, 1)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.12	0.10	0.09	0.08	0.07	0.06	0.08	0.08	0.07	0.06	0.06	0.06	0.07	0.07	0.06	0.06	0.05	0.05
	30	0.10	0.09	0.07	0.06	0.05	0.04	0.07	0.06	0.05	0.05	0.04	0.04	0.05	0.05	0.04	0.04	0.04	0.04
	50	0.08	0.06	0.05	0.04	0.04	0.03	0.05	0.04	0.04	0.03	0.03	0.02	0.04	0.03	0.03	0.03	0.02	0.02
	100	0.06	0.05	0.04	0.03	0.02	0.02	0.04	0.03	0.03	0.02	0.01	0.01	0.03	0.02	0.02	0.02	0.01	0.01
	200	0.05	0.04	0.03	0.02	0.01	0.01	0.03	0.02	0.02	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01
	400	0.05	0.04	0.03	0.01	0.01	0.01	0.03	0.02	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.00
PLS	20	0.71	0.75	0.81	0.88	0.93	0.96	0.62	0.69	0.77	0.87	0.93	0.96	0.58	0.66	0.76	0.86	0.92	0.96
	30	0.68	0.71	0.76	0.83	0.90	0.94	0.55	0.61	0.70	0.81	0.89	0.94	0.49	0.58	0.68	0.80	0.88	0.94
	50	0.65	0.67	0.70	0.77	0.85	0.91	0.47	0.52	0.61	0.72	0.83	0.90	0.39	0.46	0.56	0.70	0.82	0.90
	100	0.64	0.64	0.66	0.70	0.77	0.84	0.40	0.43	0.50	0.60	0.72	0.82	0.29	0.35	0.42	0.56	0.70	0.81
	200	0.64	0.64	0.64	0.66	0.70	0.76	0.37	0.38	0.42	0.49	0.60	0.71	0.24	0.27	0.32	0.42	0.55	0.69
	400	0.65	0.64	0.64	0.64	0.65	0.70	0.35	0.36	0.38	0.42	0.49	0.59	0.21	0.23	0.25	0.32	0.42	0.55
BR(.01)	20	0.98	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00
	30	0.88	0.98	1.00	1.00	1.00	1.00	0.77	0.96	1.00	1.00	1.00	1.00	0.72	0.95	1.00	1.00	1.00	1.00
	50	0.79	0.86	0.98	1.00	1.00	1.00	0.60	0.72	0.96	1.00	1.00	1.00	0.51	0.67	0.95	1.00	1.00	1.00
	100	0.72	0.76	0.83	0.98	1.00	1.00	0.46	0.52	0.67	0.96	1.00	1.00	0.34	0.43	0.59	0.96	1.00	1.00
	200	0.69	0.71	0.75	0.83	0.98	1.00	0.39	0.43	0.50	0.66	0.97	1.00	0.26	0.31	0.39	0.59	0.96	1.00
	400	0.67	0.68	0.70	0.75	0.83	0.98	0.36	0.38	0.41	0.50	0.67	0.97	0.22	0.24	0.28	0.38	0.59	0.96
BR(1)	20	0.72	0.73	0.73	0.74	0.75	0.75	0.63	0.67	0.70	0.73	0.74	0.74	0.59	0.64	0.69	0.72	0.74	0.74
	30	0.72	0.73	0.73	0.74	0.75	0.75	0.59	0.64	0.68	0.72	0.73	0.74	0.53	0.60	0.66	0.70	0.73	0.74
	50	0.71	0.72	0.73	0.74	0.74	0.75	0.53	0.58	0.64	0.69	0.72	0.74	0.44	0.52	0.60	0.67	0.71	0.73
	100	0.70	0.71	0.73	0.73	0.74	0.74	0.44	0.49	0.56	0.64	0.70	0.72	0.32	0.39	0.48	0.60	0.67	0.71
	200	0.68	0.69	0.71	0.73	0.74	0.74	0.39	0.42	0.47	0.56	0.64	0.70	0.26	0.30	0.37	0.49	0.60	0.68
	400	0.67	0.68	0.69	0.71	0.73	0.74	0.36	0.38	0.41	0.47	0.56	0.64	0.22	0.24	0.28	0.36	0.49	0.60
BR(2)	20	0.59	0.58	0.57	0.56	0.56	0.56	0.50	0.52	0.53	0.54	0.55	0.55	0.46	0.49	0.52	0.53	0.55	0.55
	30	0.61	0.59	0.58	0.57	0.56	0.56	0.48	0.50	0.52	0.54	0.55	0.55	0.43	0.47	0.50	0.52	0.54	0.55
	50	0.63	0.61	0.60	0.58	0.57	0.56	0.46	0.48	0.51	0.53	0.54	0.55	0.38	0.42	0.47	0.51	0.53	0.54
	100	0.66	0.65	0.63	0.60	0.58	0.57	0.42	0.44	0.48	0.51	0.53	0.54	0.30	0.35	0.41	0.47	0.51	0.53
	200	0.67	0.67	0.66	0.63	0.60	0.58	0.38	0.40	0.43	0.48	0.51	0.53	0.26	0.29	0.33	0.41	0.47	0.51
	400	0.67	0.67	0.67	0.66	0.63	0.60	0.36	0.37	0.40	0.44	0.48	0.51	0.22	0.24	0.27	0.33	0.41	0.47
BR(5)	20	0.38	0.36	0.34	0.32	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.29	0.29	0.30	0.30	0.30	0.30
	30	0.42	0.38	0.35	0.33	0.32	0.31	0.32	0.32	0.31	0.31	0.31	0.31	0.28	0.29	0.30	0.30	0.30	0.30
	50	0.46	0.42	0.39	0.35	0.33	0.32	0.33	0.32	0.32	0.31	0.31	0.31	0.27	0.28	0.29	0.30	0.30	0.30
	100	0.55	0.50	0.45	0.39	0.35	0.33	0.34	0.34	0.33	0.32	0.31	0.31	0.25	0.27	0.28	0.29	0.30	0.30
	200	0.61	0.58	0.53	0.45	0.39	0.35	0.35	0.35	0.34	0.33	0.32	0.31	0.23	0.25	0.26	0.28	0.29	0.30
	400	0.64	0.63	0.60	0.53	0.45	0.39	0.35	0.35	0.35	0.35	0.33	0.32	0.21	0.22	0.24	0.26	0.28	0.29
BR(10)	20	0.24	0.22	0.20	0.19	0.18	0.18	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.17
	30	0.27	0.24	0.21	0.20	0.18	0.18	0.20	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.17
	50	0.32	0.28	0.24	0.21	0.19	0.18	0.23	0.21	0.20	0.19	0.18	0.18	0.19	0.18	0.18	0.18	0.18	0.17
	100	0.42	0.36	0.30	0.24	0.21	0.19	0.26	0.24	0.22	0.20	0.19	0.18	0.19	0.19	0.18	0.18	0.18	0.18
	200	0.52	0.46	0.39	0.30	0.24	0.21	0.30	0.28	0.25	0.22	0.20	0.19	0.20	0.20	0.19	0.18	0.18	0.18
	400	0.60	0.56	0.49	0.39	0.30	0.25	0.32	0.31	0.29	0.25	0.22	0.20	0.20	0.20	0.19	0.19	0.18	0.18

Notes: The entries are average R^2 across 1,000 Monte Carlo replications where a target variable y_t is fitted by N indicator variables x_{it}. The target and indicator variables are generated through DGPs (14) and (15) with $\lambda_{ij} = 0$ for all i, j; we impose different levels of fit in the DGPs. In case of PC k_2 factors are extracted, in case of PLS k_1 factors are extracted, and in case of Bayesian regression (BR) a shrinkage parameter $q \times N$ is used with $q = 0.1, 1, 2, 5, 10$.

Table 2: No Factors, Average R^2 for $(k_1, k_2) = (2, 4)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.41	0.36	0.32	0.28	0.26	0.24	0.30	0.28	0.26	0.24	0.22	0.22	0.26	0.25	0.24	0.22	0.21	0.22
	30	0.33	0.29	0.25	0.22	0.19	0.17	0.24	0.21	0.19	0.17	0.17	0.15	0.18	0.18	0.16	0.15	0.15	0.15
	50	0.28	0.24	0.19	0.16	0.13	0.11	0.17	0.16	0.14	0.11	0.10	0.10	0.13	0.12	0.11	0.10	0.10	0.09
	100	0.23	0.18	0.14	0.11	0.08	0.07	0.14	0.11	0.09	0.07	0.06	0.05	0.10	0.08	0.07	0.06	0.05	0.05
	200	0.19	0.16	0.11	0.08	0.06	0.04	0.11	0.09	0.07	0.05	0.04	0.03	0.07	0.06	0.05	0.04	0.03	0.03
	400	0.17	0.13	0.09	0.06	0.04	0.03	0.09	0.07	0.05	0.03	0.03	0.02	0.06	0.04	0.03	0.02	0.02	0.02
PLS	20	0.85	0.90	0.94	0.98	0.99	1.00	0.77	0.86	0.93	0.98	0.99	1.00	0.74	0.84	0.92	0.98	0.99	1.00
	30	0.80	0.85	0.90	0.96	0.99	1.00	0.67	0.76	0.87	0.95	0.99	1.00	0.61	0.73	0.85	0.95	0.98	1.00
	50	0.75	0.79	0.84	0.92	0.97	0.99	0.55	0.64	0.75	0.89	0.96	0.99	0.47	0.58	0.72	0.88	0.96	0.99
	100	0.71	0.73	0.77	0.84	0.91	0.97	0.45	0.50	0.60	0.75	0.89	0.96	0.34	0.41	0.53	0.71	0.87	0.96
	200	0.68	0.70	0.72	0.77	0.84	0.91	0.39	0.43	0.48	0.60	0.75	0.88	0.26	0.30	0.37	0.52	0.71	0.87
	400	0.67	0.68	0.70	0.72	0.77	0.84	0.36	0.38	0.41	0.48	0.60	0.75	0.22	0.24	0.28	0.37	0.52	0.71
BR(.01)	20	0.98	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00
	30	0.88	0.98	1.00	1.00	1.00	1.00	0.77	0.96	1.00	1.00	1.00	1.00	0.72	0.96	1.00	1.00	1.00	1.00
	50	0.78	0.86	0.98	1.00	1.00	1.00	0.59	0.73	0.96	1.00	1.00	1.00	0.50	0.67	0.95	1.00	1.00	1.00
	100	0.72	0.76	0.83	0.98	1.00	1.00	0.46	0.53	0.66	0.97	1.00	1.00	0.35	0.42	0.59	0.96	1.00	1.00
	200	0.69	0.71	0.75	0.83	0.98	1.00	0.39	0.43	0.50	0.67	0.97	1.00	0.26	0.30	0.39	0.59	0.96	1.00
	400	0.67	0.68	0.70	0.75	0.83	0.98	0.36	0.38	0.41	0.50	0.67	0.97	0.22	0.24	0.28	0.39	0.59	0.96
BR(1)	20	0.72	0.73	0.74	0.74	0.75	0.75	0.63	0.67	0.70	0.73	0.74	0.74	0.59	0.64	0.69	0.72	0.73	0.74
	30	0.71	0.73	0.73	0.74	0.75	0.75	0.59	0.63	0.68	0.72	0.73	0.74	0.53	0.60	0.65	0.70	0.73	0.74
	50	0.71	0.72	0.73	0.74	0.74	0.75	0.52	0.58	0.64	0.69	0.72	0.74	0.44	0.52	0.60	0.67	0.71	0.73
	100	0.70	0.71	0.72	0.73	0.74	0.74	0.44	0.49	0.56	0.64	0.69	0.72	0.33	0.39	0.49	0.60	0.68	0.71
	200	0.68	0.70	0.71	0.73	0.74	0.74	0.39	0.43	0.47	0.56	0.64	0.70	0.26	0.30	0.37	0.49	0.60	0.68
	400	0.67	0.68	0.69	0.71	0.73	0.74	0.36	0.38	0.41	0.47	0.56	0.64	0.22	0.24	0.28	0.37	0.49	0.60
BR(2)	20	0.59	0.58	0.57	0.56	0.56	0.56	0.50	0.52	0.53	0.54	0.55	0.55	0.46	0.49	0.52	0.54	0.54	0.55
	30	0.60	0.59	0.58	0.57	0.56	0.56	0.49	0.50	0.52	0.54	0.55	0.55	0.43	0.47	0.50	0.53	0.54	0.55
	50	0.63	0.61	0.60	0.58	0.57	0.56	0.45	0.48	0.50	0.53	0.54	0.55	0.38	0.42	0.47	0.51	0.53	0.54
	100	0.66	0.65	0.63	0.60	0.58	0.57	0.41	0.44	0.47	0.51	0.53	0.54	0.31	0.35	0.41	0.47	0.51	0.53
	200	0.67	0.67	0.66	0.63	0.60	0.58	0.38	0.41	0.43	0.48	0.51	0.53	0.25	0.28	0.34	0.41	0.47	0.51
	400	0.67	0.67	0.67	0.66	0.63	0.60	0.36	0.37	0.39	0.43	0.48	0.51	0.22	0.24	0.27	0.33	0.41	0.47
BR(5)	20	0.38	0.36	0.34	0.32	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.28	0.29	0.30	0.30	0.30	0.31
	30	0.41	0.38	0.35	0.33	0.32	0.31	0.32	0.31	0.31	0.31	0.31	0.31	0.28	0.29	0.29	0.30	0.30	0.30
	50	0.46	0.42	0.38	0.35	0.33	0.32	0.33	0.32	0.32	0.31	0.31	0.31	0.27	0.28	0.29	0.30	0.30	0.30
	100	0.54	0.50	0.45	0.39	0.35	0.33	0.34	0.34	0.33	0.32	0.31	0.31	0.26	0.26	0.28	0.29	0.30	0.30
	200	0.61	0.58	0.53	0.45	0.39	0.35	0.35	0.35	0.34	0.33	0.32	0.31	0.23	0.24	0.26	0.28	0.29	0.30
	400	0.64	0.63	0.60	0.53	0.45	0.39	0.35	0.35	0.35	0.34	0.33	0.32	0.21	0.22	0.24	0.26	0.28	0.29
BR(10)	20	0.24	0.22	0.20	0.19	0.18	0.18	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.17
	30	0.27	0.24	0.21	0.19	0.18	0.18	0.21	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17
	50	0.32	0.28	0.24	0.21	0.19	0.18	0.22	0.21	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17
	100	0.41	0.36	0.30	0.24	0.21	0.19	0.26	0.24	0.22	0.20	0.19	0.18	0.19	0.19	0.18	0.18	0.18	0.17
	200	0.52	0.46	0.39	0.30	0.24	0.21	0.30	0.28	0.25	0.22	0.20	0.19	0.19	0.19	0.19	0.18	0.18	0.18
	400	0.60	0.56	0.49	0.39	0.30	0.24	0.32	0.31	0.29	0.25	0.22	0.20	0.19	0.19	0.20	0.19	0.18	0.18

Notes: See the notes for Table 1.

Table 3: No Factors, Average R^2 for $(k_1, k_2) = (3, 6)$

		R^2=.5						R^2=.2						— R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.54	0.50	0.43	0.40	0.37	0.35	0.41	0.41	0.38	0.35	0.34	0.33	0.36	0.35	0.33	0.33	0.32	0.31
	30	0.46	0.40	0.37	0.32	0.27	0.25	0.33	0.30	0.28	0.25	0.23	0.23	0.27	0.26	0.24	0.23	0.22	0.21
	50	0.38	0.33	0.27	0.22	0.19	0.17	0.26	0.23	0.20	0.17	0.15	0.15	0.19	0.17	0.16	0.15	0.14	0.13
	100	0.32	0.26	0.20	0.15	0.12	0.10	0.19	0.16	0.13	0.11	0.09	0.08	0.13	0.12	0.10	0.09	0.08	0.07
	200	0.28	0.22	0.16	0.11	0.08	0.06	0.16	0.13	0.10	0.07	0.05	0.05	0.10	0.08	0.07	0.05	0.04	0.04
	400	0.25	0.19	0.13	0.09	0.06	0.04	0.13	0.10	0.08	0.05	0.04	0.03	0.08	0.06	0.05	0.04	0.03	0.02
PLS	20	0.90	0.95	0.98	1.00	1.00	1.00	0.84	0.92	0.97	0.99	1.00	1.00	0.81	0.91	0.97	0.99	1.00	1.00
	30	0.84	0.90	0.95	0.99	1.00	1.00	0.72	0.83	0.93	0.99	1.00	1.00	0.66	0.80	0.93	0.99	1.00	1.00
	50	0.77	0.83	0.89	0.96	0.99	1.00	0.59	0.69	0.83	0.95	0.99	1.00	0.49	0.63	0.80	0.95	0.99	1.00
	100	0.72	0.75	0.80	0.89	0.96	0.99	0.45	0.52	0.64	0.82	0.95	0.99	0.35	0.42	0.56	0.79	0.94	0.99
	200	0.69	0.71	0.74	0.80	0.89	0.96	0.39	0.43	0.49	0.63	0.82	0.95	0.26	0.30	0.38	0.56	0.79	0.94
	400	0.67	0.68	0.70	0.74	0.81	0.89	0.36	0.38	0.41	0.49	0.64	0.82	0.22	0.24	0.28	0.38	0.56	0.79
BR(.01)	20	0.98	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	1.00
	30	0.88	0.98	1.00	1.00	1.00	1.00	0.77	0.96	1.00	1.00	1.00	1.00	0.72	0.95	1.00	1.00	1.00	1.00
	50	0.78	0.86	0.98	1.00	1.00	1.00	0.59	0.73	0.96	1.00	1.00	1.00	0.50	0.67	0.96	1.00	1.00	1.00
	100	0.72	0.75	0.83	0.98	1.00	1.00	0.46	0.53	0.66	0.96	1.00	1.00	0.35	0.42	0.59	0.96	1.00	1.00
	200	0.69	0.71	0.75	0.83	0.98	1.00	0.39	0.43	0.49	0.66	0.97	1.00	0.26	0.30	0.39	0.59	0.96	1.00
	400	0.67	0.68	0.70	0.75	0.83	0.98	0.36	0.38	0.41	0.50	0.67	0.97	0.22	0.24	0.28	0.38	0.59	0.96
BR(1)	20	0.72	0.73	0.73	0.74	0.75	0.75	0.63	0.67	0.70	0.73	0.74	0.74	0.59	0.64	0.69	0.72	0.73	0.74
	30	0.72	0.72	0.74	0.74	0.75	0.75	0.59	0.64	0.68	0.72	0.73	0.74	0.53	0.60	0.66	0.70	0.73	0.74
	50	0.71	0.72	0.73	0.74	0.74	0.75	0.53	0.58	0.64	0.69	0.72	0.74	0.44	0.52	0.60	0.67	0.71	0.73
	100	0.70	0.71	0.72	0.73	0.74	0.75	0.44	0.49	0.56	0.64	0.70	0.72	0.33	0.39	0.49	0.60	0.67	0.71
	200	0.68	0.70	0.71	0.73	0.73	0.74	0.39	0.42	0.47	0.56	0.64	0.70	0.26	0.30	0.36	0.49	0.60	0.68
	400	0.67	0.68	0.69	0.71	0.73	0.74	0.36	0.38	0.41	0.47	0.56	0.65	0.22	0.24	0.28	0.36	0.49	0.60
BR(2)	20	0.58	0.58	0.56	0.56	0.56	0.56	0.50	0.52	0.53	0.54	0.55	0.55	0.46	0.49	0.52	0.53	0.54	0.55
	30	0.60	0.59	0.58	0.57	0.56	0.56	0.49	0.51	0.53	0.54	0.54	0.55	0.43	0.47	0.50	0.53	0.54	0.55
	50	0.63	0.62	0.59	0.58	0.57	0.56	0.46	0.48	0.51	0.53	0.54	0.55	0.38	0.42	0.47	0.51	0.53	0.54
	100	0.66	0.65	0.63	0.60	0.58	0.57	0.41	0.44	0.48	0.51	0.53	0.54	0.31	0.35	0.41	0.47	0.51	0.53
	200	0.67	0.67	0.66	0.63	0.60	0.58	0.38	0.40	0.43	0.48	0.51	0.53	0.26	0.29	0.33	0.41	0.47	0.51
	400	0.67	0.67	0.67	0.66	0.63	0.60	0.36	0.37	0.39	0.43	0.48	0.51	0.22	0.24	0.27	0.33	0.41	0.47
BR(5)	20	0.38	0.36	0.33	0.32	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.28	0.29	0.30	0.30	0.30	0.30
	30	0.41	0.38	0.36	0.33	0.32	0.31	0.32	0.32	0.32	0.31	0.31	0.31	0.28	0.29	0.29	0.30	0.30	0.30
	50	0.46	0.43	0.38	0.35	0.33	0.32	0.33	0.33	0.32	0.31	0.31	0.31	0.27	0.28	0.29	0.30	0.30	0.30
	100	0.54	0.50	0.44	0.39	0.35	0.33	0.34	0.34	0.33	0.32	0.32	0.31	0.25	0.27	0.28	0.29	0.30	0.30
	200	0.61	0.58	0.53	0.45	0.39	0.35	0.35	0.35	0.34	0.33	0.32	0.31	0.23	0.25	0.26	0.28	0.29	0.30
	400	0.65	0.63	0.60	0.53	0.45	0.39	0.34	0.35	0.35	0.34	0.33	0.32	0.21	0.22	0.24	0.26	0.28	0.29
BR(10)	20	0.24	0.22	0.20	0.19	0.18	0.18	0.20	0.19	0.18	0.18	0.18	0.17	0.18	0.17	0.17	0.17	0.17	0.17
	30	0.27	0.24	0.22	0.20	0.19	0.18	0.21	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.17
	50	0.32	0.28	0.24	0.21	0.19	0.18	0.23	0.21	0.20	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.17
	100	0.41	0.36	0.30	0.24	0.21	0.19	0.26	0.24	0.22	0.20	0.19	0.18	0.19	0.19	0.18	0.18	0.18	0.17
	200	0.52	0.46	0.39	0.30	0.24	0.21	0.30	0.28	0.25	0.22	0.20	0.19	0.20	0.19	0.19	0.18	0.18	0.18
	400	0.60	0.56	0.49	0.39	0.30	0.25	0.32	0.31	0.29	0.25	0.22	0.20	0.19	0.20	0.19	0.19	0.18	0.18

Notes: See the notes for Table 1.

Table 4: No Factors, Relative MSE compared to PC for $(k_1, k_2) = (1, 1)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
	20	0.78	0.84	0.89	0.95	0.97	0.99	1.01	1.03	1.04	1.04	1.02	1.01	1.12	1.12	1.11	1.08	1.05	1.03
	30	0.69	0.76	0.83	0.91	0.96	0.97	0.96	0.99	1.02	1.03	1.03	1.02	1.09	1.11	1.12	1.10	1.06	1.04
PLS	50	0.61	0.67	0.75	0.85	0.92	0.96	0.90	0.94	0.99	1.03	1.04	1.03	1.04	1.06	1.09	1.11	1.09	1.06
	100	0.44	0.48	0.49	0.53	0.55	0.52	0.71	0.75	0.73	0.67	0.66	0.66	0.84	0.80	0.84	0.79	0.68	0.67
	200	0.38	0.37	0.37	0.36	0.31	0.24	0.65	0.63	0.59	0.52	0.41	0.30	0.77	0.74	0.69	0.59	0.46	0.32
	400	0.37	0.37	0.37	0.37	0.35	0.31	0.67	0.66	0.63	0.59	0.51	0.41	0.80	0.78	0.75	0.69	0.59	0.45
	20	2.07	1.24	0.96	0.96	0.97	0.99	3.81	1.98	1.31	1.10	1.04	1.02	4.67	2.35	1.44	1.17	1.07	1.03
	30	1.03	1.96	1.09	0.95	0.97	0.98	1.97	3.78	1.72	1.17	1.06	1.03	2.42	4.40	2.00	1.29	1.11	1.05
BR(.01)	50	0.61	0.86	1.97	0.98	0.95	0.97	1.15	1.65	3.73	1.47	1.14	1.05	1.39	2.00	4.52	1.68	1.22	1.09
	100	0.38	0.40	0.44	0.96	0.51	0.44	0.71	0.77	0.89	1.96	0.75	0.59	0.84	0.82	1.06	2.36	0.82	0.63
	200	0.33	0.30	0.26	0.18	0.04	0.01	0.62	0.59	0.52	0.35	0.07	0.01	0.75	0.71	0.62	0.42	0.09	0.02
	400	0.34	0.33	0.30	0.26	0.17	0.02	0.65	0.63	0.59	0.51	0.34	0.05	0.79	0.77	0.73	0.62	0.42	0.06
	20	0.73	0.80	0.86	0.93	0.96	0.98	0.94	0.96	0.97	0.99	0.99	1.00	1.04	1.03	1.02	1.01	1.01	1.00
	30	0.63	0.71	0.80	0.89	0.95	0.97	0.91	0.94	0.96	0.97	0.99	0.99	1.04	1.05	1.03	1.02	1.01	1.00
BR(1)	50	0.54	0.60	0.70	0.83	0.91	0.95	0.87	0.91	0.94	0.97	0.98	0.99	1.02	1.03	1.03	1.03	1.02	1.01
	100	0.38	0.40	0.43	0.49	0.55	0.56	0.69	0.70	0.70	0.66	0.65	0.64	0.82	0.77	0.80	0.73	0.67	0.66
	200	0.33	0.32	0.30	0.28	0.27	0.27	0.63	0.60	0.54	0.45	0.37	0.31	0.75	0.72	0.64	0.52	0.41	0.33
	400	0.35	0.33	0.31	0.29	0.28	0.27	0.65	0.64	0.60	0.53	0.44	0.36	0.79	0.77	0.73	0.64	0.52	0.40
	20	0.78	0.84	0.89	0.94	0.97	0.99	0.92	0.94	0.96	0.98	0.99	0.99	0.98	0.99	0.99	1.00	1.00	1.00
	30	0.69	0.77	0.84	0.91	0.96	0.98	0.89	0.92	0.94	0.97	0.98	0.99	0.98	0.99	0.99	1.00	1.00	1.00
BR(2)	50	0.59	0.66	0.76	0.86	0.93	0.96	0.85	0.88	0.92	0.95	0.98	0.99	0.97	0.97	0.98	0.99	1.00	1.00
	100	0.41	0.45	0.51	0.59	0.64	0.67	0.69	0.71	0.72	0.71	0.72	0.72	0.82	0.78	0.81	0.77	0.74	0.74
	200	0.35	0.35	0.35	0.38	0.41	0.43	0.64	0.61	0.58	0.53	0.50	0.48	0.76	0.73	0.68	0.60	0.54	0.50
	400	0.35	0.34	0.33	0.34	0.37	0.40	0.66	0.64	0.61	0.57	0.53	0.49	0.79	0.77	0.74	0.67	0.60	0.53
	20	0.87	0.91	0.94	0.97	0.98	0.99	0.94	0.96	0.97	0.99	0.99	1.00	0.97	0.98	0.98	0.99	1.00	1.00
	30	0.80	0.86	0.91	0.95	0.98	0.99	0.92	0.94	0.96	0.98	0.99	0.99	0.96	0.97	0.98	0.99	0.99	1.00
BR(5)	50	0.71	0.78	0.85	0.92	0.96	0.98	0.87	0.91	0.93	0.96	0.98	0.99	0.95	0.95	0.97	0.98	0.99	1.00
	100	0.51	0.59	0.66	0.75	0.79	0.81	0.74	0.77	0.80	0.81	0.83	0.84	0.84	0.83	0.85	0.85	0.85	0.85
	200	0.41	0.44	0.49	0.56	0.62	0.66	0.67	0.67	0.67	0.68	0.69	0.69	0.78	0.77	0.75	0.73	0.72	0.71
	400	0.37	0.38	0.41	0.48	0.56	0.62	0.67	0.66	0.66	0.66	0.67	0.68	0.80	0.79	0.77	0.74	0.73	0.71
	20	0.93	0.95	0.97	0.98	0.99	1.00	0.96	0.97	0.98	0.99	0.99	1.00	0.97	0.98	0.99	0.99	1.00	1.00
	30	0.88	0.92	0.95	0.97	0.99	0.99	0.95	0.96	0.97	0.98	0.99	1.00	0.97	0.98	0.98	0.99	1.00	1.00
BR(10)	50	0.82	0.86	0.91	0.95	0.98	0.99	0.91	0.94	0.96	0.98	0.99	0.99	0.96	0.96	0.98	0.99	0.99	1.00
	100	0.64	0.72	0.78	0.85	0.87	0.90	0.81	0.84	0.87	0.89	0.90	0.91	0.88	0.88	0.90	0.91	0.91	0.92
	200	0.51	0.56	0.63	0.71	0.77	0.80	0.73	0.74	0.76	0.79	0.81	0.82	0.82	0.82	0.82	0.83	0.83	0.83
	400	0.42	0.46	0.52	0.62	0.70	0.76	0.69	0.70	0.72	0.75	0.78	0.80	0.82	0.81	0.81	0.82	0.82	0.82

Notes: The entries are average one-period ahead, out-of-sample MSE ratios relative to principal components one-period ahead forecasts across 1,000 Monte Carlo replications. For the rest see the notes for Table 1.

Table 5: No Factors, Relative MSE compared to PC for $(k_1, k_2) = (2, 4)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PLS	20	0.86	0.88	0.95	0.96	0.98	0.99	1.18	1.17	1.14	1.07	1.04	1.01	1.33	1.31	1.22	1.13	1.06	1.02
	30	0.74	0.79	0.87	0.94	0.96	0.98	1.12	1.15	1.17	1.11	1.06	1.02	1.29	1.32	1.30	1.18	1.10	1.04
	50	0.60	0.66	0.75	0.87	0.94	0.97	1.02	1.07	1.15	1.16	1.09	1.06	1.16	1.25	1.33	1.29	1.16	1.07
	100	0.45	0.39	0.45	0.48	0.51	0.52	0.75	0.75	0.69	0.69	0.66	0.63	0.78	0.84	0.99	0.82	0.81	0.62
	200	0.40	0.35	0.31	0.26	0.18	0.10	0.68	0.63	0.55	0.43	0.27	0.13	0.79	0.74	0.66	0.50	0.31	0.14
	400	0.40	0.37	0.33	0.29	0.24	0.17	0.71	0.67	0.62	0.54	0.41	0.26	0.83	0.79	0.75	0.65	0.49	0.30
BR(.01)	20	2.21	1.28	1.00	0.97	0.98	0.99	3.77	1.99	1.29	1.10	1.04	1.01	4.58	2.27	1.43	1.15	1.07	1.03
	30	1.15	2.06	1.11	0.96	0.97	0.98	1.97	3.79	1.76	1.18	1.06	1.03	2.42	4.47	2.03	1.28	1.11	1.05
	50	0.65	0.92	2.00	1.02	0.96	0.97	1.18	1.69	3.80	1.47	1.14	1.05	1.38	2.02	4.58	1.70	1.22	1.09
	100	0.40	0.43	0.48	1.07	0.53	0.50	0.79	0.73	0.81	1.80	0.76	0.57	0.89	0.90	1.13	2.57	0.83	0.62
	200	0.39	0.34	0.29	0.19	0.04	0.01	0.68	0.62	0.54	0.36	0.07	0.01	0.80	0.74	0.64	0.44	0.09	0.02
	400	0.40	0.36	0.33	0.27	0.18	0.02	0.71	0.67	0.62	0.52	0.35	0.05	0.83	0.80	0.74	0.63	0.42	0.06
BR(1)	20	0.78	0.84	0.89	0.94	0.97	0.98	0.95	0.96	0.97	0.98	0.99	0.99	1.01	1.00	1.01	1.00	1.00	1.00
	30	0.70	0.75	0.82	0.90	0.95	0.97	0.93	0.94	0.96	0.98	0.98	0.99	1.02	1.03	1.02	1.01	1.00	1.00
	50	0.58	0.64	0.73	0.84	0.91	0.95	0.89	0.91	0.94	0.97	0.98	0.99	1.02	1.04	1.04	1.03	1.01	1.00
	100	0.42	0.43	0.46	0.52	0.56	0.60	0.77	0.70	0.68	0.67	0.66	0.64	0.86	0.83	0.82	0.73	0.70	0.66
	200	0.40	0.36	0.33	0.30	0.28	0.27	0.69	0.63	0.57	0.46	0.38	0.32	0.80	0.75	0.67	0.54	0.42	0.34
	400	0.40	0.37	0.34	0.31	0.28	0.27	0.71	0.67	0.62	0.55	0.45	0.37	0.83	0.80	0.75	0.65	0.53	0.41
BR(2)	20	0.84	0.88	0.92	0.95	0.98	0.99	0.93	0.94	0.96	0.98	0.99	0.99	0.96	0.96	0.98	0.99	0.99	0.99
	30	0.76	0.81	0.86	0.93	0.96	0.98	0.91	0.92	0.95	0.97	0.98	0.99	0.97	0.98	0.98	0.99	0.99	0.99
	50	0.64	0.70	0.78	0.87	0.93	0.96	0.87	0.89	0.92	0.95	0.98	0.99	0.97	0.98	0.99	0.99	0.99	0.99
	100	0.46	0.49	0.55	0.62	0.66	0.70	0.78	0.71	0.71	0.73	0.73	0.73	0.85	0.83	0.82	0.77	0.76	0.75
	200	0.41	0.39	0.38	0.40	0.43	0.44	0.70	0.65	0.61	0.55	0.51	0.49	0.80	0.76	0.70	0.62	0.55	0.51
	400	0.40	0.38	0.36	0.36	0.38	0.41	0.71	0.68	0.64	0.59	0.54	0.50	0.83	0.80	0.76	0.68	0.60	0.54
BR(5)	20	0.94	0.95	0.97	0.98	0.99	0.99	0.95	0.96	0.97	0.98	0.99	0.99	0.94	0.95	0.97	0.98	0.99	0.99
	30	0.88	0.90	0.93	0.96	0.98	0.99	0.93	0.94	0.96	0.98	0.98	0.99	0.95	0.96	0.97	0.98	0.99	0.99
	50	0.78	0.83	0.88	0.93	0.96	0.98	0.90	0.91	0.94	0.97	0.98	0.99	0.95	0.96	0.97	0.98	0.99	0.99
	100	0.60	0.63	0.72	0.77	0.82	0.84	0.83	0.78	0.81	0.83	0.85	0.86	0.87	0.87	0.87	0.86	0.87	0.86
	200	0.48	0.50	0.53	0.60	0.65	0.68	0.73	0.71	0.70	0.70	0.71	0.71	0.83	0.80	0.77	0.75	0.73	0.72
	400	0.43	0.42	0.44	0.50	0.57	0.63	0.72	0.70	0.68	0.68	0.69	0.69	0.84	0.81	0.79	0.76	0.74	0.72
BR(10)	20	1.00	1.00	0.99	0.99	1.00	1.00	0.97	0.97	0.98	0.99	0.99	1.00	0.95	0.96	0.97	0.98	0.99	0.99
	30	0.97	0.97	0.97	0.99	0.99	1.00	0.96	0.97	0.97	0.98	0.99	0.99	0.96	0.96	0.98	0.98	0.99	0.99
	50	0.89	0.92	0.94	0.97	0.98	0.99	0.94	0.94	0.96	0.98	0.99	0.99	0.96	0.97	0.97	0.98	0.99	0.99
	100	0.75	0.77	0.85	0.88	0.92	0.92	0.90	0.85	0.88	0.90	0.92	0.93	0.90	0.92	0.92	0.92	0.94	0.93
	200	0.60	0.63	0.69	0.76	0.80	0.82	0.79	0.79	0.80	0.82	0.83	0.84	0.87	0.85	0.85	0.85	0.85	0.85
	400	0.49	0.51	0.56	0.65	0.73	0.78	0.75	0.74	0.75	0.78	0.80	0.82	0.85	0.84	0.83	0.83	0.83	0.83

Notes: See the notes for Table 4.

Table 6: No Factors, Relative MSE compared to PC for $(k_1, k_2) = (3, 6)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PLS	20	0.96	0.98	0.98	0.98	0.99	0.99	1.39	1.35	1.22	1.08	1.04	1.02	1.55	1.48	1.30	1.14	1.06	1.03
	30	0.83	0.86	0.93	0.96	0.98	0.98	1.29	1.36	1.31	1.14	1.06	1.04	1.46	1.59	1.48	1.24	1.09	1.04
	50	0.65	0.72	0.82	0.94	0.95	0.97	1.10	1.23	1.34	1.26	1.12	1.06	1.26	1.44	1.55	1.41	1.20	1.08
	100	0.43	0.41	0.45	0.51	0.49	0.51	0.87	0.82	0.86	0.90	0.61	0.57	0.85	0.87	0.93	0.86	0.81	0.58
	200	0.43	0.38	0.31	0.22	0.12	0.05	0.72	0.65	0.56	0.40	0.20	0.07	0.83	0.76	0.66	0.48	0.23	0.07
	400	0.44	0.39	0.34	0.28	0.21	0.12	0.74	0.69	0.64	0.53	0.38	0.19	0.85	0.81	0.76	0.64	0.46	0.22
BR(.01)	20	2.31	1.33	1.02	0.98	0.99	0.99	3.72	1.96	1.31	1.08	1.04	1.02	4.32	2.29	1.43	1.14	1.06	1.03
	30	1.19	2.14	1.13	0.98	0.97	0.98	2.04	3.75	1.73	1.17	1.07	1.03	2.31	4.50	2.00	1.27	1.10	1.04
	50	0.70	0.95	2.06	1.03	0.96	0.97	1.21	1.72	3.72	1.46	1.13	1.05	1.36	2.04	4.42	1.68	1.20	1.08
	100	0.49	0.43	0.49	1.03	0.51	0.50	0.77	0.77	0.86	1.84	0.79	0.63	0.92	0.92	1.03	2.45	0.92	0.63
	200	0.43	0.37	0.31	0.20	0.04	0.01	0.72	0.66	0.56	0.37	0.07	0.01	0.82	0.76	0.66	0.45	0.09	0.02
	400	0.44	0.39	0.34	0.27	0.18	0.02	0.74	0.69	0.64	0.53	0.35	0.05	0.85	0.81	0.76	0.64	0.43	0.06
BR(1)	20	0.82	0.86	0.91	0.95	0.97	0.99	0.94	0.95	0.97	0.98	0.99	0.99	0.98	0.99	0.99	0.99	0.99	1.00
	30	0.73	0.78	0.84	0.92	0.95	0.98	0.93	0.95	0.96	0.97	0.99	0.99	1.01	1.01	1.01	1.00	1.00	1.00
	50	0.62	0.67	0.75	0.85	0.92	0.96	0.92	0.93	0.95	0.97	0.98	0.99	1.01	1.03	1.03	1.02	1.01	1.00
	100	0.51	0.45	0.48	0.53	0.58	0.61	0.75	0.73	0.71	0.64	0.66	0.67	0.88	0.85	0.79	0.74	0.73	0.67
	200	0.44	0.39	0.35	0.31	0.29	0.28	0.73	0.67	0.59	0.48	0.38	0.32	0.82	0.77	0.68	0.55	0.42	0.35
	400	0.44	0.40	0.35	0.31	0.29	0.28	0.74	0.70	0.64	0.56	0.46	0.37	0.85	0.81	0.76	0.66	0.53	0.41
BR(2)	20	0.88	0.91	0.93	0.96	0.98	0.99	0.92	0.93	0.96	0.97	0.99	0.99	0.93	0.95	0.96	0.98	0.99	0.99
	30	0.80	0.84	0.88	0.94	0.97	0.98	0.91	0.93	0.95	0.96	0.98	0.99	0.95	0.96	0.97	0.98	0.99	0.99
	50	0.68	0.73	0.80	0.89	0.94	0.97	0.90	0.90	0.92	0.95	0.97	0.99	0.96	0.97	0.98	0.99	0.99	0.99
	100	0.55	0.52	0.58	0.63	0.69	0.72	0.76	0.75	0.75	0.72	0.74	0.75	0.87	0.85	0.81	0.78	0.79	0.75
	200	0.46	0.43	0.41	0.42	0.44	0.45	0.74	0.69	0.63	0.57	0.52	0.49	0.83	0.78	0.72	0.63	0.56	0.52
	400	0.45	0.41	0.38	0.37	0.39	0.42	0.74	0.70	0.66	0.60	0.54	0.50	0.85	0.81	0.77	0.69	0.61	0.54
BR(5)	20	0.98	0.98	0.98	0.99	0.99	1.00	0.94	0.95	0.96	0.98	0.99	0.99	0.92	0.93	0.95	0.97	0.98	0.99
	30	0.93	0.94	0.95	0.98	0.99	0.99	0.93	0.94	0.96	0.97	0.99	0.99	0.93	0.94	0.96	0.97	0.98	0.99
	50	0.82	0.86	0.90	0.94	0.97	0.98	0.92	0.92	0.94	0.96	0.98	0.99	0.94	0.95	0.96	0.98	0.99	0.99
	100	0.69	0.68	0.76	0.79	0.83	0.86	0.82	0.82	0.86	0.85	0.86	0.86	0.90	0.89	0.88	0.86	0.89	0.87
	200	0.54	0.54	0.57	0.62	0.67	0.70	0.77	0.75	0.73	0.72	0.72	0.72	0.86	0.82	0.79	0.76	0.74	0.73
	400	0.48	0.46	0.46	0.51	0.59	0.64	0.76	0.73	0.70	0.69	0.69	0.70	0.86	0.83	0.80	0.77	0.74	0.73
BR(10)	20	1.05	1.03	1.01	1.00	1.00	1.00	0.96	0.97	0.97	0.98	0.99	0.99	0.92	0.94	0.96	0.97	0.98	0.99
	30	1.02	1.00	1.00	1.00	1.00	1.00	0.96	0.97	0.97	0.98	0.99	0.99	0.94	0.95	0.96	0.98	0.99	0.99
	50	0.94	0.95	0.96	0.98	0.99	0.99	0.95	0.96	0.96	0.98	0.99	0.99	0.95	0.96	0.97	0.98	0.99	0.99
	100	0.85	0.83	0.90	0.89	0.92	0.95	0.90	0.90	0.95	0.94	0.94	0.92	0.93	0.93	0.94	0.91	0.95	0.93
	200	0.67	0.69	0.73	0.78	0.82	0.84	0.84	0.83	0.83	0.84	0.85	0.85	0.89	0.88	0.87	0.86	0.86	0.86
	400	0.54	0.55	0.58	0.67	0.74	0.79	0.79	0.77	0.77	0.79	0.81	0.82	0.88	0.86	0.85	0.84	0.84	0.84

Notes: See the notes for Table 4.

Table 7: Factors, Average R^2 for $(k_1, k_2) = (1, 1)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.31	0.31	0.31	0.32	0.31	0.30	0.21	0.23	0.22	0.22	0.21	0.21	0.17	0.16	0.17	0.16	0.15	0.15
	30	0.32	0.30	0.30	0.29	0.30	0.30	0.22	0.21	0.22	0.21	0.22	0.21	0.15	0.16	0.15	0.14	0.15	0.15
	50	0.30	0.29	0.29	0.29	0.30	0.30	0.21	0.21	0.20	0.19	0.21	0.20	0.14	0.13	0.14	0.14	0.14	0.14
	100	0.30	0.31	0.28	0.29	0.28	0.29	0.20	0.20	0.20	0.20	0.20	0.21	0.13	0.13	0.13	0.12	0.13	0.14
	200	0.30	0.30	0.29	0.28	0.28	0.29	0.19	0.20	0.20	0.20	0.20	0.19	0.14	0.13	0.13	0.13	0.13	0.13
	400	0.30	0.28	0.29	0.28	0.28	0.29	0.19	0.21	0.20	0.19	0.19	0.19	0.13	0.13	0.12	0.13	0.13	0.12
PLS	20	0.54	0.52	0.50	0.51	0.50	0.49	0.43	0.44	0.43	0.44	0.44	0.44	0.39	0.39	0.40	0.41	0.42	0.42
	30	0.50	0.47	0.45	0.44	0.43	0.44	0.38	0.37	0.37	0.36	0.37	0.37	0.31	0.32	0.32	0.33	0.33	0.33
	50	0.46	0.42	0.40	0.39	0.39	0.38	0.33	0.32	0.31	0.30	0.31	0.30	0.25	0.25	0.25	0.25	0.24	0.25
	100	0.43	0.40	0.36	0.35	0.33	0.34	0.29	0.27	0.27	0.26	0.25	0.26	0.21	0.20	0.19	0.18	0.20	0.19
	200	0.41	0.39	0.35	0.32	0.31	0.31	0.27	0.26	0.24	0.24	0.23	0.22	0.19	0.18	0.17	0.16	0.16	0.16
	400	0.41	0.37	0.35	0.31	0.31	0.30	0.26	0.25	0.23	0.22	0.21	0.21	0.18	0.17	0.15	0.15	0.15	0.14
BR(.01)	20	0.98	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	1.00
	30	0.91	0.99	1.00	1.00	1.00	1.00	0.82	0.97	1.00	1.00	1.00	1.00	0.76	0.96	1.00	1.00	1.00	1.00
	50	0.85	0.90	0.99	1.00	1.00	1.00	0.66	0.78	0.97	1.00	1.00	1.00	0.56	0.71	0.96	1.00	1.00	1.00
	100	0.80	0.83	0.87	0.99	1.00	1.00	0.56	0.61	0.73	0.97	1.00	1.00	0.42	0.49	0.64	0.96	1.00	1.00
	200	0.77	0.79	0.82	0.88	0.99	1.00	0.50	0.53	0.59	0.73	0.97	1.00	0.35	0.39	0.46	0.64	0.96	1.00
	400	0.76	0.77	0.79	0.82	0.88	0.99	0.47	0.50	0.53	0.59	0.73	0.97	0.32	0.33	0.37	0.46	0.64	0.96
BR(1)	20	0.80	0.81	0.81	0.82	0.82	0.82	0.70	0.74	0.76	0.79	0.79	0.80	0.66	0.70	0.74	0.76	0.77	0.78
	30	0.80	0.80	0.81	0.82	0.82	0.82	0.67	0.70	0.75	0.77	0.79	0.79	0.59	0.65	0.70	0.74	0.77	0.78
	50	0.79	0.80	0.81	0.81	0.82	0.82	0.61	0.66	0.71	0.75	0.78	0.79	0.51	0.57	0.65	0.72	0.75	0.77
	100	0.78	0.79	0.80	0.81	0.81	0.82	0.55	0.58	0.65	0.71	0.76	0.78	0.41	0.47	0.55	0.65	0.72	0.75
	200	0.77	0.78	0.79	0.80	0.81	0.81	0.50	0.53	0.57	0.65	0.71	0.75	0.35	0.38	0.44	0.55	0.65	0.72
	400	0.76	0.77	0.78	0.79	0.80	0.81	0.47	0.50	0.52	0.57	0.64	0.71	0.32	0.33	0.37	0.45	0.55	0.65
BR(2)	20	0.70	0.70	0.69	0.69	0.69	0.69	0.59	0.62	0.63	0.64	0.64	0.65	0.55	0.56	0.59	0.61	0.61	0.61
	30	0.72	0.71	0.70	0.69	0.69	0.69	0.59	0.60	0.62	0.63	0.64	0.64	0.51	0.54	0.57	0.59	0.61	0.62
	50	0.74	0.72	0.71	0.70	0.70	0.69	0.56	0.58	0.60	0.62	0.64	0.64	0.46	0.49	0.54	0.57	0.59	0.61
	100	0.75	0.75	0.73	0.71	0.70	0.69	0.53	0.55	0.58	0.60	0.62	0.64	0.39	0.43	0.48	0.53	0.57	0.60
	200	0.76	0.77	0.76	0.73	0.71	0.70	0.49	0.52	0.54	0.58	0.60	0.62	0.35	0.37	0.42	0.48	0.54	0.58
	400	0.76	0.76	0.77	0.76	0.74	0.72	0.47	0.50	0.51	0.54	0.58	0.61	0.32	0.33	0.36	0.42	0.48	0.53
BR(5)	20	0.55	0.54	0.53	0.52	0.51	0.51	0.44	0.45	0.45	0.45	0.44	0.45	0.39	0.39	0.40	0.40	0.40	0.40
	30	0.58	0.55	0.54	0.52	0.51	0.51	0.46	0.45	0.45	0.45	0.45	0.44	0.38	0.39	0.39	0.40	0.40	0.41
	50	0.62	0.59	0.56	0.53	0.53	0.52	0.46	0.46	0.45	0.44	0.45	0.44	0.37	0.37	0.38	0.39	0.40	0.40
	100	0.67	0.65	0.60	0.56	0.53	0.52	0.47	0.46	0.46	0.45	0.45	0.45	0.34	0.36	0.37	0.37	0.39	0.40
	200	0.72	0.70	0.66	0.60	0.56	0.54	0.46	0.47	0.47	0.46	0.45	0.44	0.33	0.34	0.35	0.37	0.38	0.39
	400	0.74	0.73	0.71	0.66	0.60	0.56	0.46	0.48	0.48	0.47	0.46	0.45	0.31	0.32	0.33	0.36	0.37	0.38
BR(10)	20	0.43	0.43	0.41	0.41	0.40	0.39	0.34	0.34	0.34	0.34	0.33	0.33	0.29	0.29	0.29	0.29	0.28	0.28
	30	0.48	0.44	0.43	0.41	0.41	0.41	0.37	0.35	0.35	0.34	0.34	0.33	0.29	0.29	0.29	0.28	0.29	0.29
	50	0.51	0.48	0.45	0.43	0.43	0.42	0.37	0.36	0.35	0.34	0.35	0.33	0.29	0.28	0.28	0.29	0.28	0.28
	100	0.58	0.55	0.49	0.46	0.43	0.42	0.40	0.38	0.37	0.35	0.35	0.35	0.29	0.29	0.29	0.28	0.28	0.29
	200	0.65	0.62	0.56	0.49	0.46	0.44	0.42	0.42	0.39	0.37	0.35	0.34	0.30	0.29	0.29	0.28	0.28	0.29
	400	0.71	0.68	0.64	0.56	0.50	0.46	0.44	0.45	0.43	0.39	0.37	0.36	0.30	0.30	0.29	0.30	0.29	0.28

Notes: The entries are average R^2 across 1,000 Monte Carlo replications where a target variable y_t is fitted by N indicator variables x_{it}. The target and indicator variables are generated through DGPs (14) and (15) with $\lambda_{ij} \sim NIID(0,1)$ for all i,j assuming $r = k_2$ factors; we impose different levels of fit in the DGPs. In case of PC k_2 factors are extracted, in case of PLS k_1 factors are extracted, and in case of Bayesian regression (BR) a shrinkage parameter $q \times N$ is used with $q = 0.1, 1, 2, 5, 10$.

Table 8: Factors, Average R^2 for $(k_1, k_2) = (2, 4)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.72	0.73	0.72	0.70	0.70	0.70	0.61	0.60	0.60	0.60	0.59	0.58	0.48	0.50	0.50	0.48	0.50	0.48
	30	0.72	0.70	0.69	0.69	0.69	0.69	0.59	0.57	0.56	0.56	0.56	0.56	0.47	0.46	0.46	0.45	0.45	0.46
	50	0.70	0.68	0.69	0.68	0.69	0.68	0.56	0.55	0.56	0.54	0.54	0.55	0.43	0.43	0.43	0.42	0.41	0.42
	100	0.69	0.69	0.67	0.67	0.67	0.67	0.53	0.53	0.54	0.52	0.53	0.53	0.41	0.41	0.41	0.41	0.39	0.40
	200	0.68	0.67	0.66	0.66	0.65	0.66	0.55	0.54	0.51	0.52	0.53	0.51	0.40	0.39	0.39	0.39	0.39	0.39
	400	0.69	0.69	0.67	0.66	0.66	0.66	0.52	0.52	0.52	0.52	0.51	0.53	0.40	0.39	0.38	0.38	0.39	0.38
PLS	20	0.81	0.81	0.81	0.80	0.79	0.80	0.71	0.72	0.72	0.73	0.73	0.73	0.62	0.64	0.66	0.67	0.68	0.68
	30	0.79	0.78	0.77	0.76	0.76	0.76	0.67	0.66	0.66	0.66	0.67	0.66	0.57	0.57	0.59	0.59	0.60	0.60
	50	0.77	0.74	0.74	0.72	0.73	0.73	0.63	0.62	0.62	0.61	0.61	0.62	0.50	0.51	0.51	0.51	0.51	0.52
	100	0.75	0.73	0.71	0.70	0.70	0.69	0.58	0.58	0.58	0.56	0.57	0.57	0.46	0.46	0.46	0.45	0.45	0.46
	200	0.74	0.72	0.70	0.68	0.67	0.67	0.59	0.57	0.55	0.55	0.55	0.53	0.44	0.42	0.42	0.42	0.42	0.42
	400	0.74	0.73	0.70	0.68	0.67	0.67	0.56	0.55	0.54	0.54	0.52	0.54	0.42	0.42	0.40	0.39	0.40	0.39
BR(.01)	20	0.99	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00
	30	0.96	0.99	1.00	1.00	1.00	1.00	0.89	0.98	1.00	1.00	1.00	1.00	0.83	0.97	1.00	1.00	1.00	1.00
	50	0.92	0.95	0.99	1.00	1.00	1.00	0.79	0.86	0.98	1.00	1.00	1.00	0.68	0.79	0.97	1.00	1.00	1.00
	100	0.90	0.91	0.94	0.99	1.00	1.00	0.72	0.76	0.84	0.98	1.00	1.00	0.59	0.64	0.74	0.97	1.00	1.00
	200	0.89	0.90	0.91	0.94	0.99	1.00	0.70	0.72	0.74	0.83	0.98	1.00	0.54	0.56	0.61	0.75	0.97	1.00
	400	0.88	0.89	0.90	0.91	0.94	0.99	0.67	0.69	0.71	0.75	0.83	0.98	0.51	0.52	0.55	0.61	0.75	0.97
BR(1)	20	0.91	0.92	0.92	0.92	0.92	0.92	0.83	0.85	0.87	0.88	0.89	0.89	0.76	0.80	0.84	0.85	0.86	0.87
	30	0.91	0.91	0.91	0.92	0.92	0.92	0.81	0.83	0.85	0.87	0.88	0.88	0.72	0.76	0.81	0.83	0.85	0.86
	50	0.90	0.90	0.91	0.91	0.92	0.92	0.76	0.79	0.83	0.86	0.87	0.88	0.64	0.70	0.76	0.81	0.83	0.84
	100	0.89	0.90	0.90	0.91	0.91	0.92	0.71	0.74	0.79	0.82	0.85	0.87	0.58	0.62	0.68	0.76	0.80	0.83
	200	0.89	0.89	0.90	0.90	0.91	0.91	0.70	0.72	0.73	0.78	0.83	0.85	0.54	0.55	0.60	0.68	0.75	0.80
	400	0.88	0.89	0.89	0.90	0.91	0.91	0.67	0.69	0.70	0.74	0.78	0.83	0.51	0.52	0.55	0.60	0.68	0.75
BR(2)	20	0.86	0.87	0.87	0.86	0.86	0.86	0.78	0.78	0.80	0.81	0.81	0.81	0.69	0.72	0.74	0.75	0.76	0.76
	30	0.87	0.86	0.86	0.86	0.86	0.86	0.76	0.77	0.78	0.79	0.80	0.80	0.67	0.69	0.72	0.73	0.74	0.75
	50	0.87	0.87	0.87	0.86	0.86	0.86	0.73	0.75	0.77	0.78	0.79	0.79	0.61	0.65	0.68	0.71	0.72	0.74
	100	0.88	0.88	0.87	0.86	0.86	0.86	0.70	0.72	0.75	0.76	0.78	0.78	0.57	0.59	0.64	0.68	0.70	0.72
	200	0.88	0.88	0.88	0.87	0.86	0.86	0.70	0.71	0.72	0.74	0.77	0.77	0.53	0.55	0.58	0.64	0.67	0.70
	400	0.88	0.89	0.89	0.88	0.87	0.86	0.67	0.68	0.70	0.73	0.74	0.77	0.50	0.52	0.54	0.58	0.64	0.67
BR(5)	20	0.78	0.78	0.78	0.76	0.76	0.76	0.68	0.68	0.69	0.69	0.68	0.68	0.58	0.60	0.62	0.61	0.62	0.62
	30	0.80	0.79	0.78	0.78	0.77	0.77	0.68	0.68	0.68	0.68	0.68	0.68	0.58	0.59	0.60	0.60	0.60	0.61
	50	0.82	0.80	0.79	0.78	0.78	0.78	0.68	0.67	0.68	0.68	0.67	0.68	0.55	0.57	0.58	0.58	0.58	0.59
	100	0.84	0.83	0.81	0.79	0.78	0.78	0.67	0.68	0.68	0.67	0.67	0.67	0.54	0.55	0.56	0.57	0.57	0.58
	200	0.86	0.85	0.83	0.81	0.79	0.78	0.68	0.68	0.67	0.67	0.68	0.66	0.52	0.52	0.54	0.56	0.56	0.57
	400	0.88	0.88	0.86	0.84	0.81	0.79	0.67	0.67	0.68	0.68	0.67	0.68	0.50	0.51	0.52	0.53	0.56	0.56
BR(10)	20	0.69	0.70	0.69	0.68	0.68	0.68	0.60	0.59	0.60	0.60	0.59	0.59	0.50	0.51	0.52	0.51	0.53	0.52
	30	0.74	0.72	0.71	0.71	0.70	0.70	0.62	0.61	0.60	0.60	0.60	0.60	0.51	0.51	0.52	0.51	0.51	0.52
	50	0.76	0.74	0.74	0.72	0.72	0.72	0.62	0.61	0.62	0.61	0.61	0.61	0.50	0.51	0.51	0.51	0.49	0.51
	100	0.80	0.78	0.76	0.74	0.73	0.73	0.63	0.63	0.63	0.61	0.61	0.61	0.50	0.50	0.51	0.51	0.50	0.50
	200	0.83	0.82	0.79	0.76	0.74	0.73	0.66	0.65	0.63	0.62	0.62	0.60	0.50	0.49	0.50	0.50	0.50	0.49
	400	0.86	0.85	0.83	0.79	0.76	0.74	0.65	0.66	0.65	0.64	0.62	0.62	0.49	0.50	0.50	0.49	0.50	0.49

Notes: See the notes for Table 7.

Table 9: Factors, Average R^2 for $(k_1, k_2) = (3, 6)$

		R^2=.5						R^2=.2						R^2=.1					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PC	20	0.85	0.83	0.83	0.81	0.81	0.82	0.74	0.73	0.72	0.72	0.72	0.72	0.64	0.63	0.63	0.63	0.62	0.61
	30	0.82	0.81	0.80	0.80	0.80	0.79	0.70	0.70	0.69	0.68	0.68	0.68	0.59	0.59	0.58	0.57	0.58	0.58
	50	0.81	0.80	0.79	0.79	0.78	0.78	0.67	0.67	0.67	0.67	0.66	0.66	0.55	0.55	0.53	0.53	0.54	0.54
	100	0.80	0.79	0.77	0.77	0.77	0.77	0.66	0.64	0.64	0.65	0.64	0.63	0.52	0.52	0.52	0.51	0.51	0.52
	200	0.80	0.79	0.77	0.77	0.76	0.76	0.66	0.65	0.63	0.63	0.63	0.63	0.51	0.49	0.50	0.50	0.50	0.49
	400	0.80	0.78	0.77	0.77	0.76	0.76	0.65	0.65	0.63	0.62	0.63	0.62	0.51	0.50	0.50	0.50	0.49	0.49
PLS	20	0.91	0.90	0.91	0.90	0.90	0.91	0.83	0.84	0.85	0.86	0.86	0.87	0.76	0.78	0.81	0.82	0.83	0.83
	30	0.88	0.88	0.87	0.87	0.87	0.87	0.78	0.79	0.80	0.80	0.81	0.82	0.69	0.71	0.73	0.75	0.77	0.77
	50	0.87	0.86	0.85	0.85	0.84	0.84	0.74	0.75	0.75	0.76	0.76	0.76	0.63	0.64	0.65	0.67	0.69	0.70
	100	0.85	0.84	0.82	0.82	0.81	0.81	0.71	0.70	0.70	0.71	0.71	0.72	0.57	0.58	0.59	0.60	0.62	0.64
	200	0.85	0.84	0.81	0.80	0.80	0.80	0.70	0.69	0.67	0.67	0.68	0.69	0.55	0.54	0.54	0.55	0.57	0.59
	400	0.85	0.83	0.81	0.80	0.78	0.79	0.69	0.68	0.66	0.65	0.66	0.66	0.54	0.53	0.53	0.53	0.53	0.55
BR(.01)	20	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00
	30	0.97	0.99	1.00	1.00	1.00	1.00	0.90	0.99	1.00	1.00	1.00	1.00	0.85	0.98	1.00	1.00	1.00	1.00
	50	0.95	0.96	1.00	1.00	1.00	1.00	0.83	0.89	0.99	1.00	1.00	1.00	0.74	0.83	0.98	1.00	1.00	1.00
	100	0.93	0.94	0.95	1.00	1.00	1.00	0.78	0.81	0.86	0.99	1.00	1.00	0.64	0.70	0.78	0.98	1.00	1.00
	200	0.92	0.93	0.94	0.96	1.00	1.00	0.76	0.78	0.80	0.87	0.99	1.00	0.61	0.62	0.68	0.79	0.98	1.00
	400	0.92	0.92	0.93	0.94	0.96	1.00	0.75	0.76	0.77	0.80	0.87	0.99	0.59	0.60	0.63	0.69	0.79	0.98
BR(1)	20	0.94	0.94	0.95	0.95	0.95	0.95	0.88	0.90	0.91	0.92	0.92	0.92	0.82	0.85	0.87	0.89	0.89	0.90
	30	0.93	0.94	0.94	0.94	0.95	0.95	0.84	0.87	0.89	0.90	0.91	0.92	0.77	0.81	0.85	0.87	0.88	0.89
	50	0.93	0.93	0.94	0.94	0.94	0.94	0.81	0.84	0.87	0.89	0.90	0.91	0.71	0.76	0.80	0.84	0.86	0.87
	100	0.92	0.93	0.93	0.94	0.94	0.94	0.78	0.80	0.83	0.87	0.89	0.90	0.64	0.68	0.74	0.80	0.84	0.86
	200	0.92	0.93	0.93	0.93	0.94	0.94	0.76	0.77	0.79	0.83	0.86	0.89	0.61	0.62	0.67	0.73	0.80	0.83
	400	0.92	0.92	0.92	0.93	0.93	0.94	0.75	0.76	0.77	0.80	0.83	0.86	0.59	0.60	0.63	0.68	0.73	0.80
BR(2)	20	0.91	0.91	0.91	0.91	0.91	0.91	0.83	0.85	0.85	0.86	0.86	0.86	0.77	0.78	0.80	0.81	0.81	0.82
	30	0.91	0.91	0.90	0.90	0.91	0.90	0.81	0.83	0.84	0.84	0.85	0.85	0.73	0.75	0.77	0.79	0.80	0.81
	50	0.92	0.91	0.91	0.91	0.90	0.90	0.79	0.81	0.83	0.84	0.84	0.84	0.68	0.72	0.74	0.76	0.78	0.79
	100	0.92	0.92	0.91	0.90	0.90	0.90	0.77	0.78	0.80	0.82	0.83	0.83	0.63	0.66	0.70	0.73	0.76	0.77
	200	0.92	0.92	0.92	0.91	0.90	0.90	0.76	0.77	0.78	0.80	0.81	0.83	0.60	0.62	0.65	0.70	0.73	0.75
	400	0.92	0.92	0.92	0.92	0.91	0.90	0.75	0.76	0.76	0.78	0.80	0.81	0.59	0.60	0.62	0.66	0.69	0.73
BR(5)	20	0.85	0.84	0.84	0.83	0.83	0.84	0.76	0.77	0.76	0.77	0.77	0.77	0.68	0.69	0.70	0.70	0.70	0.70
	30	0.86	0.85	0.84	0.84	0.84	0.84	0.75	0.76	0.76	0.76	0.76	0.76	0.66	0.67	0.68	0.68	0.69	0.69
	50	0.88	0.86	0.86	0.85	0.84	0.84	0.75	0.76	0.76	0.76	0.76	0.75	0.64	0.65	0.65	0.66	0.67	0.67
	100	0.89	0.88	0.86	0.85	0.85	0.84	0.75	0.75	0.75	0.75	0.75	0.74	0.61	0.62	0.64	0.65	0.65	0.66
	200	0.90	0.90	0.89	0.87	0.85	0.84	0.75	0.75	0.75	0.75	0.74	0.75	0.60	0.60	0.62	0.63	0.64	0.64
	400	0.91	0.91	0.90	0.89	0.87	0.85	0.75	0.75	0.75	0.75	0.75	0.74	0.59	0.59	0.61	0.63	0.63	0.64
BR(10)	20	0.78	0.76	0.76	0.75	0.75	0.76	0.69	0.69	0.68	0.69	0.69	0.68	0.61	0.60	0.61	0.61	0.61	0.60
	30	0.80	0.80	0.78	0.78	0.78	0.78	0.69	0.70	0.70	0.69	0.69	0.69	0.60	0.60	0.61	0.60	0.62	0.61
	50	0.83	0.82	0.81	0.81	0.80	0.79	0.70	0.71	0.71	0.71	0.70	0.70	0.59	0.60	0.59	0.59	0.60	0.60
	100	0.86	0.85	0.83	0.82	0.81	0.81	0.72	0.71	0.71	0.71	0.70	0.69	0.58	0.59	0.59	0.59	0.59	0.59
	200	0.89	0.88	0.85	0.83	0.82	0.81	0.73	0.73	0.71	0.71	0.70	0.70	0.58	0.57	0.58	0.59	0.58	0.58
	400	0.90	0.89	0.88	0.86	0.83	0.82	0.74	0.74	0.73	0.71	0.71	0.70	0.58	0.58	0.59	0.59	0.58	0.58

Notes: See the notes for Table 7.

Table 10: Factors, Relative MSE compared to PC for $(k_1, k_2) = (1, 1)$

		R^2=.5						R^2=.33						R^2=.2					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PLS	20	0.92	0.96	1.00	1.02	1.04	1.05	1.01	1.02	1.05	1.06	1.07	1.07	1.06	1.07	1.08	1.09	1.09	1.10
	30	0.90	0.94	0.98	1.01	1.03	1.03	0.99	1.01	1.02	1.03	1.05	1.05	1.03	1.04	1.05	1.06	1.06	1.07
	50	0.88	0.93	0.96	0.99	1.01	1.02	0.96	0.98	1.00	1.02	1.03	1.03	1.01	1.02	1.03	1.03	1.04	1.04
	100	0.86	0.87	0.91	0.95	0.96	0.96	0.91	0.91	0.98	0.96	0.96	0.97	0.97	0.96	0.99	0.98	0.99	0.98
	200	0.83	0.87	0.91	0.94	0.96	0.97	0.91	0.92	0.94	0.95	0.96	0.96	0.94	0.95	0.96	0.97	0.96	0.97
	400	0.84	0.88	0.92	0.96	0.97	0.98	0.92	0.95	0.96	0.97	0.98	0.98	0.95	0.96	0.97	0.98	0.98	0.98
BR(.01)	20	2.00	1.16	0.96	0.96	0.97	0.99	3.71	1.95	1.30	1.10	1.04	1.02	4.55	2.35	1.46	1.17	1.06	1.03
	30	1.02	1.94	1.07	0.94	0.96	0.98	1.92	3.82	1.72	1.18	1.07	1.03	2.34	4.55	1.99	1.28	1.10	1.05
	50	0.59	0.85	1.93	1.00	0.94	0.97	1.12	1.62	3.83	1.46	1.13	1.05	1.35	1.99	4.52	1.68	1.21	1.09
	100	0.37	0.37	0.42	0.95	0.47	0.45	0.69	0.72	0.90	1.90	0.69	0.54	0.91	0.86	1.07	2.14	0.91	0.68
	200	0.32	0.29	0.26	0.18	0.04	0.01	0.62	0.58	0.51	0.35	0.07	0.01	0.75	0.71	0.62	0.43	0.09	0.02
	400	0.34	0.32	0.30	0.26	0.17	0.02	0.65	0.63	0.59	0.51	0.34	0.05	0.79	0.77	0.72	0.62	0.42	0.06
BR(1)	20	0.71	0.78	0.86	0.93	0.96	0.98	0.94	0.95	0.97	0.98	0.99	0.99	1.04	1.03	1.02	1.01	1.00	1.00
	30	0.62	0.70	0.80	0.89	0.94	0.97	0.90	0.94	0.96	0.98	0.99	0.99	1.04	1.04	1.03	1.02	1.00	1.00
	50	0.53	0.59	0.70	0.83	0.90	0.95	0.86	0.90	0.93	0.96	0.98	0.99	1.01	1.04	1.04	1.03	1.01	1.01
	100	0.38	0.38	0.41	0.48	0.52	0.57	0.67	0.67	0.69	0.63	0.63	0.61	0.86	0.80	0.81	0.71	0.72	0.68
	200	0.33	0.31	0.29	0.28	0.27	0.26	0.62	0.59	0.54	0.45	0.36	0.31	0.75	0.72	0.64	0.53	0.40	0.33
	400	0.34	0.33	0.31	0.29	0.27	0.27	0.65	0.63	0.60	0.53	0.44	0.36	0.79	0.77	0.72	0.64	0.52	0.40
BR(2)	20	0.76	0.82	0.89	0.94	0.97	0.98	0.92	0.93	0.95	0.98	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99
	30	0.68	0.76	0.84	0.91	0.95	0.98	0.88	0.92	0.94	0.97	0.98	0.99	0.98	0.98	0.99	0.99	0.99	1.00
	50	0.58	0.65	0.75	0.86	0.92	0.96	0.85	0.87	0.91	0.95	0.97	0.99	0.96	0.98	0.99	0.99	1.00	1.00
	100	0.41	0.43	0.48	0.57	0.62	0.67	0.68	0.69	0.71	0.69	0.71	0.71	0.85	0.80	0.81	0.75	0.78	0.75
	200	0.34	0.33	0.35	0.37	0.40	0.43	0.63	0.61	0.57	0.53	0.50	0.47	0.76	0.73	0.67	0.60	0.53	0.50
	400	0.35	0.34	0.33	0.34	0.37	0.40	0.65	0.64	0.61	0.57	0.52	0.49	0.79	0.77	0.73	0.67	0.59	0.53
BR(5)	20	0.86	0.90	0.94	0.97	0.99	1.00	0.93	0.95	0.96	0.98	0.99	0.99	0.97	0.97	0.98	0.99	0.99	0.99
	30	0.79	0.85	0.90	0.95	0.98	0.99	0.91	0.93	0.96	0.98	0.99	0.99	0.96	0.97	0.98	0.99	0.99	0.99
	50	0.70	0.77	0.85	0.92	0.96	0.98	0.87	0.90	0.93	0.96	0.98	0.99	0.94	0.96	0.97	0.98	0.99	0.99
	100	0.51	0.56	0.63	0.73	0.77	0.81	0.73	0.75	0.78	0.80	0.83	0.83	0.86	0.84	0.85	0.84	0.86	0.85
	200	0.40	0.42	0.48	0.55	0.62	0.65	0.66	0.66	0.66	0.67	0.68	0.69	0.78	0.77	0.74	0.73	0.71	0.71
	400	0.37	0.38	0.40	0.47	0.55	0.61	0.67	0.66	0.65	0.66	0.67	0.68	0.80	0.78	0.76	0.74	0.72	0.71
BR(10)	20	0.94	0.96	0.98	1.01	1.01	1.02	0.96	0.98	0.98	0.99	1.00	1.00	0.97	0.98	0.98	0.99	0.99	0.99
	30	0.88	0.92	0.96	0.99	1.00	1.01	0.94	0.96	0.98	0.99	0.99	1.00	0.97	0.97	0.98	0.99	0.99	0.99
	50	0.80	0.86	0.91	0.96	0.98	1.00	0.91	0.93	0.96	0.98	0.99	1.00	0.95	0.97	0.98	0.99	0.99	1.00
	100	0.64	0.69	0.76	0.83	0.87	0.90	0.79	0.82	0.85	0.88	0.90	0.90	0.89	0.88	0.90	0.90	0.92	0.91
	200	0.49	0.54	0.62	0.70	0.76	0.79	0.71	0.73	0.75	0.78	0.80	0.82	0.81	0.82	0.81	0.82	0.82	0.83
	400	0.42	0.45	0.51	0.61	0.70	0.76	0.69	0.70	0.72	0.75	0.78	0.80	0.81	0.81	0.81	0.81	0.82	0.82

Notes: The entries are average one-period ahead, out-of-sample MSE ratios relative to principal components one-period ahead forecasts across 1,000 Monte Carlo replications. For the rest see the notes for Table 7.

Table 11: Factors, Relative MSE compared to PC for $(k_1, k_2) = (2, 4)$

		R^2=.5						R^2=.33						R^2=.2					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PLS	20	0.90	0.97	1.00	1.02	1.03	1.06	0.99	1.01	1.06	1.08	1.07	1.05	1.04	1.09	1.08	1.07	1.13	1.10
	30	0.95	0.94	0.95	1.00	1.01	1.02	1.01	1.03	0.99	1.05	1.06	1.06	1.07	1.02	1.07	1.08	1.09	1.11
	50	0.92	0.89	0.96	0.99	1.04	1.06	0.97	0.98	1.00	1.04	1.02	1.02	0.99	1.03	1.04	1.03	1.04	1.07
	100	0.77	0.97	0.88	0.89	1.02	1.00	0.90	0.92	1.00	0.88	0.98	1.03	0.97	0.98	0.97	1.27	1.02	1.00
	200	0.82	0.84	0.87	0.92	0.93	0.95	0.94	0.94	0.92	0.96	0.98	0.93	0.96	0.94	0.95	0.97	0.96	0.94
	400	0.85	0.89	0.89	0.95	0.96	0.95	0.90	0.92	0.95	0.98	0.95	1.00	0.93	0.95	0.95	0.98	0.98	0.97
BR(.01)	20	1.94	1.17	0.96	0.95	0.98	0.99	3.55	1.86	1.26	1.08	1.04	1.02	4.41	2.16	1.37	1.14	1.06	1.03
	30	1.01	1.92	1.07	0.95	0.96	0.98	1.84	3.73	1.67	1.16	1.06	1.02	2.21	4.34	1.90	1.27	1.10	1.04
	50	0.61	0.82	1.90	0.98	0.95	0.96	1.11	1.58	3.61	1.43	1.13	1.05	1.32	1.91	4.42	1.61	1.21	1.08
	100	0.36	0.40	0.42	1.02	0.49	0.49	0.83	0.72	0.97	1.76	0.71	0.62	0.90	0.92	1.04	2.56	0.90	0.61
	200	0.36	0.32	0.27	0.18	0.04	0.01	0.67	0.61	0.53	0.36	0.08	0.02	0.78	0.73	0.64	0.43	0.09	0.02
	400	0.37	0.34	0.31	0.26	0.17	0.02	0.68	0.66	0.61	0.51	0.34	0.05	0.82	0.78	0.73	0.63	0.42	0.06
BR(1)	20	0.73	0.81	0.87	0.92	0.96	0.98	0.93	0.95	0.96	0.97	0.98	0.98	1.02	1.01	1.00	0.99	0.99	0.98
	30	0.65	0.72	0.81	0.90	0.94	0.97	0.91	0.94	0.96	0.97	0.98	0.99	1.03	1.03	1.02	1.01	1.00	0.99
	50	0.56	0.61	0.70	0.83	0.91	0.95	0.88	0.91	0.93	0.96	0.98	0.99	1.02	1.03	1.04	1.02	1.01	1.00
	100	0.37	0.41	0.42	0.50	0.55	0.58	0.79	0.68	0.74	0.65	0.65	0.65	0.87	0.84	0.80	0.77	0.71	0.66
	200	0.36	0.34	0.30	0.28	0.28	0.27	0.67	0.62	0.55	0.46	0.37	0.31	0.79	0.74	0.66	0.53	0.41	0.33
	400	0.37	0.35	0.32	0.30	0.28	0.27	0.68	0.66	0.61	0.54	0.45	0.36	0.82	0.78	0.74	0.65	0.52	0.40
BR(2)	20	0.78	0.85	0.90	0.94	0.97	0.99	0.91	0.92	0.95	0.96	0.97	0.97	0.96	0.96	0.96	0.97	0.97	0.97
	30	0.71	0.78	0.85	0.92	0.96	0.98	0.89	0.91	0.94	0.96	0.98	0.98	0.97	0.98	0.98	0.99	0.98	0.98
	50	0.61	0.68	0.76	0.86	0.93	0.96	0.86	0.89	0.91	0.95	0.97	0.99	0.97	0.97	0.99	0.99	0.99	0.99
	100	0.41	0.46	0.50	0.59	0.65	0.68	0.78	0.70	0.76	0.71	0.73	0.73	0.86	0.83	0.81	0.79	0.77	0.74
	200	0.37	0.36	0.35	0.38	0.41	0.43	0.68	0.63	0.59	0.54	0.50	0.48	0.79	0.75	0.69	0.60	0.54	0.50
	400	0.38	0.36	0.34	0.35	0.37	0.40	0.69	0.66	0.63	0.57	0.53	0.49	0.82	0.79	0.75	0.68	0.60	0.53
BR(5)	20	0.91	0.96	0.99	1.01	1.03	1.03	0.94	0.94	0.96	0.96	0.97	0.98	0.94	0.94	0.94	0.95	0.95	0.95
	30	0.85	0.89	0.93	0.98	1.00	1.01	0.92	0.94	0.96	0.97	0.98	0.99	0.95	0.96	0.96	0.97	0.97	0.97
	50	0.74	0.80	0.86	0.93	0.97	0.99	0.88	0.91	0.93	0.96	0.98	0.99	0.95	0.96	0.97	0.98	0.98	0.99
	100	0.52	0.59	0.66	0.75	0.79	0.82	0.81	0.77	0.82	0.81	0.83	0.84	0.87	0.86	0.86	0.86	0.85	0.85
	200	0.43	0.45	0.49	0.56	0.62	0.66	0.71	0.68	0.68	0.68	0.69	0.69	0.81	0.78	0.76	0.73	0.72	0.71
	400	0.40	0.39	0.42	0.48	0.56	0.61	0.70	0.68	0.67	0.66	0.67	0.68	0.83	0.80	0.78	0.75	0.73	0.71
BR(10)	20	1.07	1.11	1.12	1.13	1.14	1.14	0.99	0.98	1.01	1.00	1.01	1.01	0.95	0.96	0.95	0.95	0.96	0.96
	30	1.00	1.01	1.03	1.07	1.08	1.08	0.97	0.98	1.00	1.00	1.02	1.01	0.96	0.97	0.97	0.98	0.98	0.98
	50	0.87	0.91	0.96	1.00	1.02	1.03	0.93	0.95	0.97	0.99	1.00	1.00	0.96	0.97	0.98	0.98	0.99	0.99
	100	0.65	0.72	0.79	0.86	0.89	0.91	0.86	0.84	0.88	0.89	0.90	0.91	0.90	0.90	0.91	0.91	0.91	0.91
	200	0.52	0.57	0.63	0.71	0.77	0.80	0.75	0.75	0.77	0.79	0.81	0.82	0.84	0.83	0.83	0.82	0.83	0.83
	400	0.45	0.47	0.52	0.62	0.70	0.76	0.72	0.72	0.73	0.76	0.78	0.80	0.84	0.82	0.82	0.82	0.82	0.82

Notes: See the notes for Table 10.

Table 12: Factors, Relative MSE compared to PC for $(k_1, k_2) = (3, 6)$

		R^2=.5						R^2=.33						R^2=.2					
	T/N	20	30	50	100	200	400	20	30	50	100	200	400	20	30	50	100	200	400
PLS	20	0.93	0.93	1.00	1.00	1.08	1.06	1.06	1.08	1.09	1.13	1.14	1.15	1.13	1.11	1.13	1.17	1.17	1.18
	30	0.85	0.89	0.96	0.98	1.05	1.03	0.96	1.03	1.06	1.06	1.11	1.12	1.08	1.08	1.12	1.16	1.19	1.17
	50	0.84	0.89	0.91	1.04	1.02	1.07	0.95	0.97	1.02	1.09	1.08	1.10	1.01	1.03	1.07	1.11	1.15	1.13
	100	0.77	0.89	0.85	0.86	1.03	0.91	0.90	0.95	0.94	0.90	0.84	0.84	0.97	0.95	0.94	0.90	0.91	0.98
	200	0.73	0.81	0.82	0.86	0.88	0.86	0.89	0.88	0.87	0.89	0.88	0.84	0.93	0.91	0.90	0.90	0.86	0.82
	400	0.76	0.78	0.83	0.92	0.88	0.89	0.89	0.89	0.89	0.89	0.92	0.87	0.94	0.94	0.95	0.96	0.92	0.88
BR(.01)	20	2.00	1.16	0.97	0.96	0.97	0.98	3.63	1.86	1.25	1.07	1.03	1.02	4.08	2.10	1.36	1.13	1.05	1.03
	30	1.01	1.98	1.09	0.96	0.96	0.98	1.79	3.67	1.63	1.14	1.06	1.02	2.10	4.34	1.84	1.23	1.10	1.04
	50	0.60	0.84	1.97	0.98	0.95	0.97	1.10	1.55	3.65	1.43	1.12	1.05	1.27	1.89	4.36	1.60	1.19	1.08
	100	0.42	0.46	0.48	1.00	0.56	0.47	0.81	0.74	0.90	1.95	0.73	0.60	0.87	0.88	1.06	2.40	0.83	0.59
	200	0.40	0.34	0.28	0.18	0.04	0.01	0.70	0.63	0.54	0.36	0.07	0.02	0.80	0.74	0.65	0.44	0.09	0.02
	400	0.40	0.36	0.32	0.27	0.17	0.02	0.72	0.67	0.62	0.52	0.35	0.05	0.83	0.80	0.74	0.63	0.42	0.06
BR(1)	20	0.75	0.81	0.88	0.93	0.96	0.97	0.93	0.95	0.95	0.96	0.97	0.97	0.99	0.99	0.99	0.98	0.97	0.97
	30	0.67	0.74	0.82	0.91	0.95	0.97	0.93	0.94	0.96	0.97	0.98	0.98	1.02	1.02	1.01	1.00	1.00	0.99
	50	0.56	0.62	0.72	0.83	0.92	0.96	0.89	0.90	0.94	0.97	0.98	0.99	1.00	1.04	1.04	1.02	1.00	1.00
	100	0.44	0.46	0.45	0.49	0.57	0.59	0.80	0.71	0.71	0.66	0.65	0.67	0.85	0.82	0.83	0.77	0.69	0.66
	200	0.40	0.35	0.32	0.29	0.28	0.27	0.70	0.64	0.56	0.46	0.37	0.31	0.81	0.75	0.67	0.54	0.41	0.33
	400	0.41	0.36	0.33	0.30	0.28	0.27	0.72	0.67	0.62	0.55	0.45	0.36	0.83	0.80	0.75	0.65	0.52	0.40
BR(2)	20	0.81	0.86	0.91	0.95	0.98	0.99	0.91	0.92	0.93	0.95	0.96	0.96	0.93	0.94	0.95	0.94	0.94	0.95
	30	0.73	0.80	0.86	0.93	0.96	0.98	0.90	0.92	0.94	0.96	0.97	0.98	0.96	0.97	0.97	0.98	0.98	0.98
	50	0.62	0.68	0.78	0.87	0.94	0.97	0.87	0.88	0.92	0.95	0.97	0.99	0.96	0.98	0.98	0.98	0.99	0.99
	100	0.48	0.50	0.53	0.59	0.66	0.69	0.80	0.73	0.74	0.72	0.73	0.75	0.85	0.82	0.83	0.80	0.75	0.74
	200	0.42	0.38	0.37	0.38	0.41	0.43	0.70	0.65	0.60	0.54	0.50	0.48	0.81	0.76	0.70	0.61	0.54	0.50
	400	0.41	0.37	0.35	0.35	0.37	0.40	0.72	0.68	0.63	0.58	0.53	0.49	0.83	0.80	0.75	0.68	0.60	0.53
BR(5)	20	0.98	1.00	1.02	1.03	1.06	1.07	0.94	0.94	0.95	0.97	0.97	0.98	0.90	0.91	0.93	0.92	0.92	0.93
	30	0.89	0.93	0.97	1.00	1.02	1.03	0.93	0.94	0.96	0.97	0.98	0.98	0.94	0.94	0.95	0.96	0.96	0.96
	50	0.76	0.81	0.88	0.94	0.98	1.00	0.90	0.91	0.94	0.96	0.98	0.99	0.94	0.96	0.97	0.97	0.98	0.99
	100	0.59	0.63	0.68	0.75	0.80	0.83	0.83	0.79	0.82	0.82	0.84	0.86	0.87	0.86	0.87	0.87	0.85	0.85
	200	0.47	0.47	0.50	0.57	0.62	0.66	0.73	0.70	0.68	0.69	0.69	0.69	0.83	0.79	0.77	0.74	0.72	0.71
	400	0.43	0.41	0.42	0.48	0.56	0.62	0.73	0.70	0.68	0.67	0.68	0.68	0.84	0.81	0.78	0.75	0.73	0.71
BR(10)	20	1.19	1.20	1.21	1.19	1.23	1.22	1.01	1.01	1.01	1.03	1.04	1.04	0.91	0.92	0.94	0.93	0.93	0.94
	30	1.08	1.11	1.12	1.12	1.15	1.15	0.99	1.00	1.01	1.03	1.03	1.02	0.95	0.96	0.96	0.98	0.97	0.97
	50	0.92	0.95	1.00	1.04	1.06	1.07	0.95	0.96	0.98	1.00	1.00	1.02	0.96	0.97	0.98	0.98	0.99	0.99
	100	0.73	0.77	0.82	0.87	0.91	0.92	0.89	0.87	0.89	0.90	0.91	0.93	0.90	0.91	0.92	0.92	0.92	0.91
	200	0.56	0.59	0.64	0.72	0.77	0.80	0.78	0.77	0.77	0.80	0.81	0.82	0.86	0.84	0.83	0.83	0.83	0.83
	400	0.48	0.48	0.53	0.62	0.70	0.76	0.75	0.73	0.74	0.76	0.79	0.80	0.85	0.84	0.82	0.82	0.82	0.82

Notes: See the notes for Table 10.

Table 13: Transformation of the forecast variables

Y_t	$\Delta y_{t,t-1}$	$\Delta y_{t+h,t}$
CPI index	$\Delta \ln Y_{t,t-12} - \Delta \ln Y_{t-1,t-13}$	$\Delta \ln Y_{t+h,t+h-12} - \Delta \ln Y_{t,t-12}$
core CPI index	$\Delta \ln Y_{t,t-12} - \Delta \ln Y_{t-1,t-13}$	$\Delta \ln Y_{t+h,t+h-12} - \Delta \ln Y_{t,t-12}$
Industrial Production index	$\Delta \ln Y_{t,t-1}$	$\Delta \ln Y_{t+h,t}$
Unemployment rate	$\Delta Y_{t,t-1}$	$\Delta Y_{t+h,t}$
Federal Funds rate	$\Delta Y_{t,t-1}$	$\Delta Y_{t+h,t}$

Notes: The table illustrates the transformation of a forecast variable Y_t, indicated in the first column, for use in prediction regressions (19) and (20).

Table 14: Relative forecast performance of the benchmark models: AR *versus* RW

h	*CPI inflation*	*core CPI infl.*	*Ind. Prod.*	*Unemployment*	*Fed. Funds Rate*
			January 1972 - December 2006		
1	0.8343	0.9094	0.9185	0.9617	1.0074
3	0.9938	0.8960	0.8833	0.8992	1.0625
12	1.0302	1.0270	0.9039	0.9871	1.0794
24	1.1023	1.0349	0.8966	1.0379	1.0483
			January 1985 - December 2006		
1	0.8347	0.9014	0.9993	0.9946	0.9157
3	0.9551	0.9265	0.8653	0.9507	1.0099
12	1.0780	1.1252	0.7257	0.9592	1.0318
24	1.0517	1.0889	0.6602	1.0237	1.0526
			January 1972 - December 1984		
1	0.8339	0.9110	0.8747	0.9403	1.0142
3	0.8763	0.9975	0.8902	0.8800	1.0678
12	0.9935	1.0259	0.9950	0.9956	1.0931
24	1.0978	1.0302	1.1477	1.0399	1.0376

Notes: The table reports the ratio of the root mean squared prediction error (18) of the autoregressive model (16) *vis-à-vis* the random walk model (17) for each of our forecast variables (see Table 13)) at each horizon *h* (in months).

Table 15: Forecast evaluation for CPI inflation

	PC				PLS			BRR			
h	*1 PC*	*2 PC*	*4 PC*	*6 PC*	*1 PLS*	*2 PLS*	*3 PLS*	*N*	*5N*	*10N*	*20N*
					January 1972 - December 2006						
					Benchmark: RW						
1	0.9440	0.9291	0.9138	0.9293	**0.9114**	0.9208	0.9333	1.0684	1.0231	1.0000	0.9751
3	0.8950	0.9064	0.8951	0.8975	**0.8678**	0.9003	0.9213	1.0529	1.0211	1.0008	0.9777
12	0.8281	0.8411	0.8308	0.8559	**0.8101**	0.8233	0.8446	1.0090	0.9552	0.9268	0.8983
24	0.8974	0.9102	0.8659	0.8933	**0.8441**	0.8476	0.8636	0.9761	0.9301	0.9045	0.8792
					Benchmark: AR						
1	0.9650	0.9804	0.9795	0.9847	**0.9643**	1.0044	1.0565	1.2076	1.1337	1.1141	1.0849
3	0.9404	0.9582	0.9692	0.9754	**0.9229**	0.9769	1.0059	1.1377	1.0994	1.0782	1.0568
12	0.8166	0.8541	0.8201	0.8360	0.8032	**0.8031**	0.8251	0.9831	0.9341	0.9085	0.8827
24	0.8193	0.8223	**0.7694**	0.7942	0.7709	0.7701	0.7884	0.8673	0.8336	0.8150	0.7956
					January 1985 - December 2006						
					Benchmark: RW						
1	0.9747	0.9594	0.9395	0.9431	0.9510	**0.9372**	0.9508	1.0372	1.0200	1.0082	0.9923
3	**0.9837**	1.0149	1.0088	0.9972	0.9928	1.0230	1.0263	1.1211	1.1047	1.0945	1.0808
12	**1.0143**	1.0180	1.1165	1.1323	1.0158	1.0729	1.1061	1.2494	1.2235	1.2064	1.1830
24	1.0765	1.0205	1.0884	1.1553	**1.0127**	1.0546	1.1322	1.4302	1.3693	1.3165	1.2494
					Benchmark: AR						
1	0.9867	0.9777	0.9587	**0.9481**	0.9707	0.9487	1.0340	1.0782	1.0659	1.0555	1.0414
3	0.9828	0.9733	0.9602	**0.9580**	0.9696	0.9853	1.0681	1.1658	1.1603	1.1458	1.1220
12	0.9401	0.9165	0.9798	0.9918	**0.9024**	0.9433	0.9859	1.1498	1.1191	1.0973	1.0708
24	1.0045	0.9737	1.0086	1.1228	**0.9446**	1.0047	1.0787	1.3602	1.2993	1.2517	1.1893
					January 1972 - December 1984						
					Benchmark: RW						
1	0.9132	0.8985	0.8881	0.9156	**0.8710**	0.9045	0.9160	1.0979	1.0260	0.9920	0.9580
3	0.8282	0.8244	0.8082	0.8229	**0.7703**	0.8067	0.8432	1.0058	0.9616	0.9331	0.9020
12	0.7509	0.7679	**0.7098**	0.7417	0.7242	0.7221	0.7413	0.9267	0.8617	0.8278	0.7954
24	0.8456	0.8657	0.7984	0.8039	0.7791	0.7615	0.7570	0.7747	0.7436	0.7337	**0.7284**
					Benchmark: AR						
1	**0.9433**	0.9830	0.9995	1.0193	0.9579	1.0559	1.0780	1.3219	1.1963	1.1685	1.1258
3	0.9059	0.9461	0.9757	0.9883	**0.8848**	0.9705	0.9552	1.1162	1.0506	1.0236	1.0040
12	0.7443	0.8083	0.7374	0.7589	0.7504	**0.7336**	0.7472	0.9155	0.8564	0.8275	0.7999
24	0.7795	0.7826	0.7134	0.7148	0.7196	0.6922	0.6959	0.7052	0.6780	0.6687	**0.6631**

Notes: The table reports the ratio of the root mean squared prediction error (18) of either a version of (19) *vis-à-vis* autoregressive model (16) or a version of (20) *vis-à-vis* the random walk model (17) for CPI inflation (see Table 13)) at each horizon h (in months). Versions of (19) and (20) depend on the usage of principal components (PC), partial least squares (PLS) or Bayesian ridge regression (BRR) to compress the information in the panel of predictor variables; see Section 4.1. In each case we use several sub-variants, depending either on the number of principal components, PLS factors or shrinkage parameters (BRR), where the shrinkage parameters is assumed to be proportional to the number of predictors in the panel (N = 104). The best performing method relative to the benchmarks are highlighted in **bold**.

Table 16: Forecast evaluation for core CPI inflation

	PC				PLS			BRR			
h	*1 PC*	*2 PC*	*4 PC*	*6 PC*	*1 PLS*	*2 PLS*	*3 PLS*	*N*	*5N*	*10N*	*20N*
					January 1972 - December 2006						
					Benchmark: RW						
1	0.9787	0.9384	0.9574	0.9504	**0.9109**	0.9631	0.9651	1.2467	1.1775	1.1361	1.0894
3	0.9503	0.9167	0.9318	0.9340	**0.8329**	0.8828	0.9020	1.1053	1.0420	1.0107	0.9782
12	0.8512	0.9750	0.9958	1.0141	**0.7928**	0.8657	0.8935	1.0904	1.0222	0.9865	0.9495
24	0.8363	0.8619	0.8282	0.8425	**0.8220**	0.8621	0.9063	1.0040	0.9598	0.9319	0.9029
					Benchmark: AR						
1	0.9893	**0.9677**	1.0111	1.0203	0.9803	1.0516	1.0560	1.2930	1.2203	1.1802	1.1217
3	0.9587	0.9275	0.9356	0.9189	**0.8697**	0.8808	0.9175	1.0596	1.0170	0.9950	0.9711
12	0.8346	0.9631	0.9748	0.9768	**0.7806**	0.8428	0.8694	1.0463	0.9945	0.9624	0.9265
24	0.8089	0.8381	0.8037	0.8043	**0.7957**	0.8362	0.8744	0.9749	0.9323	0.9047	0.8754
					January 1985 - December 2006						
					Benchmark: RW						
1	**0.9991**	1.0649	1.0592	1.0696	1.0525	1.0630	1.1549	1.5885	1.5161	1.4656	1.4001
3	**1.0270**	1.2869	1.3188	1.2906	1.1525	1.1733	1.1855	1.6692	1.5536	1.4830	1.4064
12	**1.1841**	1.3313	1.5884	1.5856	1.2658	1.3608	1.3023	1.6925	1.5659	1.5012	1.4295
24	1.1129	**1.1117**	1.5048	1.5467	1.1487	1.3837	1.4891	1.6831	1.6374	1.5862	1.5184
					Benchmark: AR						
1	**1.0076**	1.0308	1.0256	1.0262	1.0214	1.0519	1.1818	1.4999	1.4469	1.4064	1.3385
3	**1.0438**	1.1884	1.1978	1.2123	1.0940	1.1312	1.2094	1.6346	1.5353	1.4740	1.4052
12	**1.1097**	1.2269	1.4571	1.4239	1.1726	1.2693	1.2065	1.5380	1.4514	1.3918	1.3202
24	**1.0245**	1.0261	1.3693	1.4246	1.0599	1.2665	1.3638	1.5588	1.4927	1.4437	1.3818
					January 1972 - December 1984						
					Benchmark: RW						
1	0.9747	0.9118	0.9363	0.9254	**0.8807**	0.9425	0.9237	1.1688	1.0996	1.0603	1.0181
3	0.9426	0.8740	0.8874	0.8942	**0.7966**	0.8508	0.8710	1.0324	0.9766	0.9513	0.9253
12	0.8121	0.9357	0.9318	0.9580	**0.7385**	0.8170	0.8592	1.0237	0.9642	0.9344	0.9046
24	0.7933	0.8221	**0.7422**	0.7507	0.7692	0.7896	0.8291	0.8825	0.8489	0.8301	0.8117
					Benchmark: AR						
1	0.9857	**0.9553**	1.0083	1.0192	0.9723	1.0516	1.0303	1.2498	1.1722	1.1320	1.0754
3	0.9518	0.9017	0.9101	0.8897	**0.8482**	0.8559	0.8881	0.9905	0.9556	0.9391	0.9216
12	0.7983	0.9314	0.9195	0.9281	**0.7326**	0.7980	0.8398	0.9941	0.9475	0.9198	0.8897
24	0.7709	0.8039	0.7257	**0.7213**	0.7480	0.7703	0.8025	0.8765	0.8408	0.8193	0.7973

Notes: See the notes for Table 15.

Table 17: Forecast evaluation for industrial production

	PC				PLS			BRR			
h	*1 PC*	*2 PC*	*4 PC*	*6 PC*	*1 PLS*	*2 PLS*	*3 PLS*	*N*	*5N*	*10N*	*20N*
					January 1972 - December 2006						
					Benchmark: RW						
1	0.8775	0.8600	0.8557	0.8547	**0.8437**	0.8589	0.8615	1.1243	1.0127	0.9672	0.9276
3	0.8658	0.8216	0.8287	0.8412	0.7784	**0.7668**	0.7836	1.0538	0.9444	0.8981	0.8564
12	0.9182	1.0414	1.0621	1.0657	**0.7912**	0.8558	0.8630	1.0785	1.0022	0.9673	0.9324
24	0.9138	1.0734	1.0875	1.0929	0.9037	**0.8575**	0.8963	1.0131	0.9691	0.9480	0.9253
					Benchmark: AR						
1	0.9581	0.9388	0.9291	0.9303	**0.9239**	0.9246	0.9664	1.2110	1.1275	1.0774	1.0308
3	0.9839	0.9289	0.9310	0.9411	0.8870	**0.8752**	0.9036	1.2080	1.1060	1.0605	1.0149
12	1.0218	1.1922	1.2204	1.1878	**0.8768**	0.9598	0.9514	1.1584	1.1133	1.0876	1.0567
24	1.0156	1.1505	1.1803	1.1840	1.0030	**0.9657**	0.9994	1.1511	1.1196	1.0950	1.0653
					January 1985 - December 2006						
					Benchmark: RW						
1	0.9664	0.9758	0.9822	0.9751	**0.9610**	0.9876	0.9810	1.1179	1.0852	1.0652	1.0426
3	0.8450	0.8486	0.8562	0.8628	**0.8196**	0.8737	0.9021	1.1527	1.1024	1.0690	1.0296
12	**0.7217**	0.7791	0.7878	0.7980	0.7422	0.8619	0.9127	1.1417	1.1000	1.0701	1.0311
24	**0.6902**	0.7044	0.7216	0.7312	0.7699	0.7432	0.8405	1.1055	1.0625	1.0295	0.9849
					Benchmark: AR						
1	0.9743	0.9685	0.9874	0.9831	**0.9528**	0.9650	1.0141	1.1502	1.1331	1.1023	1.0706
3	0.9861	0.9784	0.9874	0.9880	**0.9443**	1.0287	1.0490	1.3272	1.2904	1.2598	1.2187
12	1.0242	1.0794	1.0815	1.0794	**0.9759**	1.2166	1.2446	1.6380	1.5960	1.5586	1.5046
24	1.0820	**1.0725**	1.0761	1.0867	1.1274	1.1642	1.2677	1.6543	1.6082	1.5657	1.5054
					January 1972 - December 1984						
					Benchmark: RW						
1	0.8289	0.7950	0.7839	0.7867	**0.7776**	0.7858	0.7941	1.1276	0.9739	0.9136	0.8636
3	0.8737	0.8086	0.8161	0.8307	0.7605	**0.7181**	0.7309	1.0115	0.8732	0.8199	0.7765
12	1.0180	1.1660	1.1946	1.1940	**0.8043**	0.8338	0.8139	1.0110	0.9087	0.8715	0.8420
24	1.1578	1.4260	1.4413	1.4439	1.0704	0.9961	0.9845	0.8976	**0.8549**	0.8559	0.8697
					Benchmark: AR						
1	0.9473	0.9186	**0.8884**	0.8937	0.9043	0.8969	0.9336	1.2495	1.1237	1.0607	1.0037
3	0.9828	0.9066	0.9070	0.9206	0.8634	**0.8081**	0.8420	1.1593	1.0276	0.9747	0.9265
12	1.0218	1.2216	1.2589	1.2169	0.8299	0.8448	**0.8187**	0.9288	0.8784	0.8583	0.8400
24	0.9913	1.1835	1.2218	1.2184	0.9508	0.8605	0.8604	0.8562	0.8312	0.8181	**0.8101**

Notes: See the notes for Table 15.

Table 18: Forecast evaluation for unemployment

	PC				PLS			BRR			
h	*1 PC*	*2 PC*	*4 PC*	*6 PC*	*1 PLS*	*2 PLS*	*3 PLS*	*N*	*5N*	*10N*	*20N*
					January 1972 - December 2006						
					Benchmark: RW						
1	0.9101	0.8927	0.8977	0.9073	**0.8856**	0.8879	0.8923	1.0738	1.0115	0.9809	0.9495
3	0.8669	0.8095	0.8119	0.8227	0.7964	**0.7705**	0.8018	0.9908	0.9154	0.8823	0.8500
12	0.9949	1.0483	1.0516	1.0424	**0.8338**	0.8434	0.8759	1.1440	1.0647	1.0254	0.9835
24	1.0522	1.1398	1.1686	1.1453	0.9195	**0.8471**	0.8950	1.0952	1.0231	0.9888	0.9539
					Benchmark: AR						
1	0.9380	0.9057	0.9169	0.9228	0.9061	**0.9009**	0.9182	1.1588	1.0848	1.0452	1.0055
3	0.9777	0.9031	0.9065	0.9352	0.8889	**0.8616**	0.8935	1.1236	1.0553	1.0204	0.9839
12	1.0099	1.0542	1.0494	1.0438	**0.8606**	0.8645	0.8873	1.1004	1.0408	1.0116	0.9798
24	1.0113	1.0761	1.1071	1.0861	0.8926	**0.8221**	0.8618	0.9718	0.9367	0.9170	0.8955
					January 1985 - December 2006						
					Benchmark: RW						
1	0.9825	0.9850	0.9848	0.9867	**0.9761**	0.9781	0.9875	1.1489	1.1203	1.1030	1.0813
3	0.9095	0.8959	0.8918	0.8897	**0.8741**	0.8926	0.9357	1.1207	1.0996	1.0786	1.0483
12	0.9422	0.9602	0.9393	0.9332	**0.8257**	0.9385	0.9403	1.3248	1.2620	1.2137	1.1511
24	1.0501	1.0090	1.0537	1.0204	1.0248	**0.9305**	1.0132	1.3772	1.3150	1.2652	1.1994
					Benchmark: AR						
1	0.9645	0.9584	0.9596	0.9592	0.9545	**0.9534**	0.9721	1.1642	1.1349	1.1147	1.0901
3	0.9626	0.9484	0.9477	0.9415	**0.9095**	0.9483	0.9839	1.2068	1.1785	1.1571	1.1279
12	0.9865	1.0078	0.9825	0.9543	**0.8646**	1.0096	0.9726	1.3550	1.3078	1.2679	1.2134
24	1.0320	0.9999	1.0305	0.9874	1.0153	**0.9224**	0.9844	1.2612	1.2246	1.1898	1.1414
					January 1972 - December 1984						
					Benchmark: RW						
1	0.8614	0.8294	0.8384	0.8536	**0.8237**	0.8261	0.8269	1.0238	0.9367	0.8957	0.8563
3	0.8509	0.7757	0.7806	0.7966	0.7665	**0.7211**	0.7480	0.9399	0.8395	0.8002	0.7665
12	1.0121	1.0740	1.0850	1.0765	0.8353	**0.8030**	0.8486	1.0655	0.9799	0.9459	0.9146
24	1.0430	1.2186	1.2438	1.2390	0.8714	0.7931	0.8218	0.9078	0.8196	0.7974	**0.7881**
					Benchmark: AR						
1	0.9188	0.8667	0.8856	0.8963	0.8705	**0.8621**	0.8782	1.1551	1.0481	0.9933	0.9414
3	0.9837	0.8833	0.8884	0.9321	0.8801	**0.8222**	0.8527	1.0857	0.9978	0.9560	0.9156
12	1.0181	1.0667	1.0686	1.0701	0.8586	**0.8086**	0.8550	1.0006	0.9356	0.9116	0.8904
24	1.0030	1.1536	1.1878	1.1822	0.8395	0.7597	0.7914	0.7837	0.7508	**0.7449**	0.7456

Notes: See the notes for Table 15.

Table 19: Forecast evaluation for the federal funds rate

	PC				PLS			BRR			
h	*1 PC*	*2 PC*	*4 PC*	*6 PC*	*1 PLS*	*2 PLS*	*3 PLS*	*N*	*5N*	*10N*	*20N*
					January 1972 - December 2006						
					Benchmark: RW						
1	0.9146	0.9175	0.9107	0.8897	0.8541	**0.8374**	0.8555	1.0380	0.9827	0.9537	0.9233
3	0.9228	0.9279	0.9560	0.9747	0.8920	**0.9048**	0.9196	1.1090	1.0512	1.0199	0.9873
12	0.9346	0.9490	0.9281	0.9345	**0.8939**	0.9424	0.9757	1.3180	1.2454	1.1993	1.1461
24	0.9719	1.0426	0.9947	1.0146	**0.9262**	1.0004	1.0154	1.2854	1.2179	1.1769	1.1309
					Benchmark: AR						
1	1.0347	1.0387	0.9664	0.9325	0.9713	**0.8400**	0.8593	1.0032	0.9682	0.9467	0.9222
3	0.9416	1.0253	1.0525	0.9795	0.9717	**0.9211**	0.9214	1.0222	0.9909	0.9783	0.9582
12	0.8516	0.8602	0.8474	0.8344	**0.8136**	0.8694	0.8944	1.1647	1.1172	1.0861	1.0478
24	0.9622	0.9739	0.9288	0.9425	**0.8724**	0.9513	0.9665	1.1305	1.0978	1.0727	1.0420
					January 1985 - December 2006						
					Benchmark: RW						
1	**0.8445**	0.8528	1.2544	1.2335	0.8909	1.1798	1.3448	1.6444	1.5520	1.5019	1.4433
3	0.7551	0.7880	1.0271	1.0339	**0.7546**	1.1476	1.2513	1.7976	1.6781	1.5973	1.4968
12	0.8224	0.8613	0.9935	0.9964	**0.8102**	1.0153	1.0762	1.3096	1.2410	1.2044	1.1675
24	0.9303	0.9447	0.9363	0.9516	**0.8836**	0.9970	1.0492	1.2708	1.1958	1.1535	1.1091
					Benchmark: AR						
1	**0.9997**	1.0731	1.3156	1.2996	1.0316	1.3079	1.4715	1.8148	1.7453	1.6981	1.6343
3	0.9492	0.9620	1.0767	1.0707	**0.9482**	1.1327	1.3825	1.6464	1.5490	1.4716	1.4299
12	0.7993	0.8188	0.9445	0.9398	**0.7939**	0.9752	1.0251	1.2399	1.2065	1.1837	1.1538
24	0.8724	0.8839	0.8847	0.8820	**0.8404**	0.9461	0.9928	1.1432	1.1030	1.0754	1.0445
					January 1972 - December 1984						
					Benchmark: RW						
1	0.9198	0.9223	0.8781	0.8569	0.8511	**0.8044**	0.8046	0.9746	0.9233	0.8966	0.8694
3	0.9372	0.9394	0.9492	0.9692	0.9040	**0.8773**	0.8819	1.0172	0.9688	0.9454	0.9234
12	0.9531	0.9590	**0.8920**	0.8985	0.9000	0.9031	0.9293	1.3084	1.2355	1.1862	1.1268
24	0.9605	1.0409	0.9758	0.9800	**0.8812**	0.9310	0.9179	1.1764	1.1122	1.0737	1.0314
					Benchmark: AR						
1	1.0369	1.0364	0.9397	0.9039	0.9673	**0.8009**	0.8045	0.9275	0.8959	0.8770	0.8567
3	0.9393	1.0296	1.0504	0.9710	0.9726	0.9002	**0.8704**	0.9463	0.9247	0.9217	0.9047
12	0.8505	0.8548	0.8044	**0.7886**	0.8003	0.8234	0.8411	1.1337	1.0803	1.0459	1.0037
24	0.9746	0.9740	0.9104	0.9153	**0.8326**	0.8910	0.8789	1.0143	0.9928	0.9740	0.9492

Notes: See the notes for Table 15.

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Table A.1: Transformation of the predictor variables

Transformation code	Transformation X_t of raw series Y_t
1	$X_t = Y_t$
2	$X_t = \Delta Y_{t,t-1}$
3	$X_t = \Delta Y_{t,t-12} - \Delta Y_{t-1,t-13}$
4	$X_t = \ln Y_t$
5	$X_t = \Delta \ln Y_{t,t-1}$
6	$X_t = \Delta \ln Y_{t,t-12} - \Delta \ln Y_{t-1,t-13}$

Appendices

A Data Set

The data set used for forecasting are the monthly series from the panel of U.S. indicator series as employed in Stock and Watson (2007), but excluding our five forecast variables: CPI inflation, core CPI inflation, (aggregate) industrial production, (aggregate) unemployment rate and the (effective) federal funds rate. In order to be sure that these predictor variables are $I(0)$, the underlying raw series need to be transformed such that this is the case; generally we employ the same transformation as Stock and Watson (2007), except for the bulk of the nominal series where we follow, e.g., D'Agostino and Giannone (2006) and use first differences of twelve-month transformations of the raw series. Table A.1 summarizes our potential transformations for the raw series.

Hence, we are using as predictor variables the following 104 series, which span the sample January 1959 - December 2006 before the appropriate transformations are applied, and we refer to Stock and Watson (2007) for more details regarding data construction and sources:

Series Y_t	**Transformation:** ***(See Table A.1)***
INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL	5
INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS	5
INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS	5
INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS	5
INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS	5
INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT	5
INDUSTRIAL PRODUCTION INDEX - MATERIALS	5
INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS	5
INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS	5
INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)	5
INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES	5
INDUSTRIAL PRODUCTION INDEX - FUELS	5
NAPM PRODUCTION INDEX (PERCENT)	1
CAPACITY UTILIZATION - MANUFACTURING (SIC)	1
AVG HRLY EARNINGS, PROD WRKRS, NONFARM - GOODS-PRODUCING	6
AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION	6
AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG	6
REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - GOODS-PRODUCING	5

REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION	5
REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG	5
EMPLOYEES, NONFARM - TOTAL PRIVATE	5
EMPLOYEES, NONFARM - GOODS-PRODUCING	5
EMPLOYEES, NONFARM - MINING EMPLOYEES, NONFARM - CONSTRUCTION	5
EMPLOYEES, NONFARM - MFG	5
EMPLOYEES, NONFARM - DURABLE GOODS	5
EMPLOYEES, NONFARM - NONDURABLE GOODS	5
EMPLOYEES, NONFARM - SERVICE-PROVIDING	5
EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES	5
EMPLOYEES, NONFARM - WHOLESALE TRADE	5
EMPLOYEES, NONFARM - RETAIL TRADE	5
EMPLOYEES, NONFARM - FINANCIAL ACTIVITIES	5
EMPLOYEES, NONFARM - GOVERNMENT	5
INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)	2
EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF	2
CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)	5
CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)	5
UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)	2
UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)	5
UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)	5
UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)	5
UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)	5
UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)	5
AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING	1
AVG WKLY OVERTIME HOURS, PROD WRKRS, NONFARM - MFG	2
HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)	4
HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,U)SA	4
HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.	4
HOUSING STARTS:MIDWEST(THOUS.U.)S.A.	4
HOUSING STARTS:SOUTH (THOUS.U.)S.A.	4
HOUSING STARTS:WEST (THOUS.U.)S.A.	4
INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)	2
INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)	2
INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)	2
INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)	2
INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)	2
BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)	2
BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)	2
INTEREST RATE SPREAD: 6-MO. TREASURY BILLS MINUS 3-MO. TREASURY BILLS	1
INTEREST RATE SPREAD: 1-YR. TREASURY BONDS MINUS 3-MO. TREASURY BILLS	1
INTEREST RATE SPREAD: 10-YR. TREASURY BONDS MINUS 3-MO. TREASURY BILLS	1
INTEREST RATE SPREAD: AAA CORPORATE MINUS 10-YR. TREASURY BONDS	1
INTEREST RATE SPREAD: BAA CORPORATE MINUS 10-YR. TREASURY BONDS	1
MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)	6
MZM (SA) FRB St. Louis	6
MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP)(BIL$,SA)	6
MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)	6
DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)	6
DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)	6
Commercial and Industrial Loans at All Commercial Banks (FRED) Billions $ (SA)	6
CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)	6
Personal Consumption Expenditures, Price Index (2000=100) , SAAR	6
Personal Consumption Expenditures - Durable Goods, Price Index (2000=100), SAAR	6
Personal Consumption Expenditures - Nondurable Goods, Price Index (2000=100), SAAR	6
Personal Consumption Expenditures - Services, Price Index (2000=100) , SAAR	6
PCE Price Index Less Food and Energy (SA) Fred	6
PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)	6
PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)	6
PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)	6
PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)	6
Real PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)	5
SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)	6
Real SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)	5
PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)	6

PPI Crude (Relative to Core PCE)	5
NAPM COMMODITY PRICES INDEX (PERCENT)	1
UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)	5
FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)	5
FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)	5
FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)	5
FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)	5
S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)	5
S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)	5
S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)	2
S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)	2
COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE	5
S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)	2
U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)	2
PURCHASING MANAGERS' INDEX (SA)	1
NAPM NEW ORDERS INDEX (PERCENT)	1
NAPM VENDOR DELIVERIES INDEX (PERCENT)	1
NAPM INVENTORIES INDEX (PERCENT)	1
NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)	5
NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)	5

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