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Abstract: Focusing on core-infrastructure capital *vis-à-vis* productive capital, we propose a macroeconomic method to determine both which type of capital shortage would be constraining potential output and what would be the optimal composition, or optimal ratio between these two types, of capital in any given period. This method is based on an adapted two-gap model, estimated via linear programming, and illustrated with the cases of Chile and Mexico over the 1950-2000 period. The results show that there appears to be an oscillating pattern over this period, with either type of capital shortage alternating each other. The results also show that, optimally, core infrastructure appears to support a variable level of productive investment over time. However, the shortage of productive capital would at least be as important as that of infrastructure capital, suggesting an optimal trade off between the two. That is, the social opportunity cost of investing in either type of capital would be determined by the gap between the optimal growth rates estimated from these two types of capital. For either type of capital, a macroeconomic shortage would mean that the economy as a whole is in a net state of shortage.

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Key Words: Capital Shortage, Potential Output, Two-Gap model, Linear Programming.

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Introduction

There appears to be enough evidence to suggest that infrastructures are an important contributor to growth and development. However, the issue as to whether an infrastructure capital shortage or a productive capital shortage constrains an economy's output is not so forthcoming. Neither is the issue as to what would be the optimal ratio between these two types of capital, or what would be the optimal share of infrastructure investment in gross fixed capital formation, to secure full utilisation in any given period. For such purpose, we propose a method that may help complementing existing approaches. We use a modification of a two-gap model, whose parameters are optimally estimated via linear programming, using data from Chile and Mexico between 1950 and 2000.

We first introduce the subject via both the importance of infrastructure and the standard assessment of shortages (section 1.1). We also characterise the problem of optimality when using standard production functions (section 1.2). Then we introduce the standard two-gap approach (section 2.1), explain the modifications required for our purpose and present the model structure (section 2.2). Next, we explain the estimation procedure and the aggregate variables used (section 3). Then the results from the application to Chile and Mexico are analysed for two outlooks: the two-gap approach over time (section 4.1) and against the infrastructure share in GFCF (section 4.2). And finally a short conclusion follows.

1. General Context

1.1 Infrastructure, Growth and Shortages

Since the early 1950s, several academic researchers have concluded that infrastructures, like transport and service networks, and education and health facilities, may play a crucial role in growth and development (e.g. Hirschman, 1958; Nurkse, 1954; Chenery, 1953). In addition, the geographic and demographic coverage of economic and social infrastructures, have always been considered as a fundamental basis of socio-economic development and as a precondition of economic growth. Indeed, these are normally presented as indicators of development by international agencies and development studies (e.g. ECLA; Todaro, 2003). Accordingly, most governments consider repairs and new investment in infrastructures as a strategic foundation for sustained growth and development (e.g. World Bank, 1997; 1994).

More specifically, infrastructures may represent a pre-condition for sustained growth, as a facilitating type of capital stock, which would allow the deployment and enhancement of directly productive physical capital and human capital. Infrastructures (or public capital) is then viewed as a complement to directly productive (or private capital), normally regarding the former as the necessary support of the latter, i.e. without appropriate infrastructures there would be little chance of stimulating, deploying and occupying efficiently the productive capacity of the economy. Thus, infrastructure investment would not only act as a direct complement, but also as an inducement, of directly productive investment. The latter issue came later to represent the “crowding-in”

hypothesis of economic literature, i.e. public investment crowds in, rather than out, private investment, *ceteris paribus* (Taylor, 1991; Bacha, 1990; Barro, 1989; Aschauer, 1988; Ortiz & Noriega, 1988; Blejer & Khan, 1984). In addition, when taking a multisectoral view, there could also be a productive trade off between sectors that compete in the use of the same fixed infrastructure stock, in the short and medium terms. For example, the manufacturing sector growth may be impaired if there were a sudden increase in infrastructure use by another sector, as this might create an overall shortage of infrastructures at the expense of manufacturing (e.g. Taylor, 1983).

In turn, authors like Aschauer (1989), Aaron (1990), Wickerman (1991), Munnell (1992), and many others, regard public infrastructures as an additional input in a production function, whose services would stimulate the productivity of both physical capital and human capital. In addition, empirical studies that have been carried for a number of countries, and especially for Japan by Ono (1987), Ohkawa & Kohama (1989) and Domoto (1992), appear to confirm both the importance of infrastructure capital in economic growth and the potential growth penalties from its shortage. The former author by comparing Japan with India allows for some useful generalisation about infrastructures and growth in the process of development. Gramlich (1994) in turn concentrates on the issue of infrastructure shortage and its evidence in the US, and surveys the literature on the issue. While criticising some of the available approaches, this study also lends support to the importance of infrastructure in economic growth and development. However, the issues as to whether an economy undergoes a shortage of infrastructure capital or a shortage of productive capital, and as to what would be an

optimal ratio between these two or an optimal share of infrastructure investment in gross fixed capital formation, are not so forthcoming.

Some authors have attempted to determine empirically whether there is an infrastructure insufficiency or excess, relative to productive capital, by using either non-economic techniques or theory-driven economic methods (see surveys by Nadiri, 1998 and Gramlich, 1994). Examples of the former are localised engineering measures of infrastructure requirement and local voting preferences as regards public works (Peterson, 1991; FHA, 1989). These are useful, but largely devoid of institutional or economic meaning and are far too confined to serve as a basis for a nationwide overview.

As regards economic methods, the two predominant ones are cost-benefit analysis (CBA) and econometric studies (Albala-Bertrand & Mamatzakis, 2004 and 2001; Nadiri & Mamuneas, 1998; Feltenstein & Ha, 1995; Gramlich, 1994; Munnell 1992; Aschauer, 1989). For a specific infrastructure project (e.g. a bridge), CBA may interpret the presence of an internal rate of return larger than the shadow interest rate as indirect evidence of an infrastructure shortage. But here the inter-geographic, inter-sectoral or general equilibrium repercussions from localised infrastructures are difficult to assess, which may make the exercise less consistent than required. Econometric studies, in turn, rely upon marginal productivities derived from well-behaved production or cost functions to assess the issue (Albala-Bertrand & Mamatzakis, 2004, 2001; Aschauer, 1993; Evans & Karras, 1993; Berndt & Hansson, 1992; Diewert, 1986), which might be problematic (Gramlich, 1994), as explained further below.

1.2. Optimality and Production Functions

Optimality. The main problem in assessing the existence of a macroeconomic shortage of capital is that this exercise may make little sense outside optimal or potential output. An economy could be operating sub-optimally due to reasons other than a shortage of physical capital, like foreign exchange constraints, skill insufficiency or political instability. In such a case, a potential shortage of infrastructure would appear to be less of a problem, as the economy would be unlikely using the available infrastructure stock. This is similar to the fiscal policy need to focus on the structural public deficit, rather than the actual deficit, the structural deficit being defined as that one which would prevail if the economy were at full employment, i.e. when the economy is operating at its stable potential. Therefore, to analyse the issue of capital shortage, the economy has to be assumed as operating without other constraints than the two types of capital themselves. That is, the issue would reduce to assess what would be the optimal output that could be achieved by using these two types of complementary capital in a potentially balanced way. And, if the right balance is not achieved, then what would be the resource waste, expressed in terms of foregone output.

This then requires the estimation of potential output, which always have serious drawbacks. There are two main approaches to such aim. One is via purely statistical de-trending techniques, which have little economic content, like univariate filters (e.g. Hodrick-Prescott's or wavelets) or the structural vector autoregression. An alternative approach is via aggregate production functions, like the Cobb-Douglas function, which

incorporate factor inputs into the analysis, making it more amenable for our capital analysis. These methods however produce significant variations in their results, which require much caution when used for fiscal or other policy purposes (Scciavillani & Swagel, 1999).

When using production functions, potential output is defined as the level of output that is consistent with stable inflation and a “normal” capacity utilisation (EU, 2001; De Massi, 1997). Under this definition, output is assumed to fluctuate around potential, so potential output is synonymous with the trend growth of actual output, i.e. an econometrically fitted line across output fluctuations. This means that there should be a clear and strong positive correlation between actual output and inflation, so that points above the line should accelerate inflation, and below it decelerated it. But this might be a problem for developing countries like Chile and Mexico, as it can be shown that the correlation between output or its growth rate and inflation, with or without lags, is normally low and/or of the wrong sign. The latter means that high output is normally associated with low inflation, rather than with high inflation, as the above definition would suggest.

In such circumstances, to use an econometrically fitted production function would systematically underestimate the economy’s potential. To avoid this, potential output should represent a maximum rather than an average, which is why we use linear programming (LP) rather than econometrics for the purpose. In addition, we use a three-year moving average for all our variables so as to prevent any outlier from exerting undue influence on the maximum values. The LP is also estimated over 10 year-periods, with a

5-year overlapping, so that both any cycle would be incorporated and any odd sub-period would not influence the estimates unduly. As is shown later, our results compare well with the EU recommendation that any method to estimate potential output should first be based on some stable economic relationships; second, be simple, transparent and replicable; and lastly, not display pronounced pro-cyclical features (EU, 2001).

Production Functions. Authors who use standard production functions, mostly engage in theory-driven analysis, proceeding as follows (Nadiri, 1998). First, normally, a scenario of perfect competition is assumed to prevail and then a least-cost production function with well-known theoretical characteristics is selected. Second, this well-behaved production function, like a Cobb-Douglas, would be augmented with infrastructure capital, and then its theoretical characteristics would be exploited to analyse its econometrically estimated parameters (Gramlich, 1994).

Thus, a macroeconomic shortage of infrastructure capital might be assessed either by comparing the size of its estimated marginal productivity with that of the other participant factors or with some perfect-competition level of productivity equalisation or benchmark. The former may only hit the mark in extreme cases, while the latter might be too strong an assumption, especially in LDCs. In addition, a high positive productivity of infrastructure capital could also mean the existence of strong complementarity with productive capital or collinearity. A negative productivity might in turn be taken as a core-infrastructure surplus (Felstenstein & Ha, 1995), but that can also be the result of a sub-optimal estimation, as indicated above. Therefore, this way of establishing a cut-off

point to assess the existence of a shortage of infrastructures has plenty of difficulty, which is what we attempt to address here.

Lastly, by definition, a standard production function assumes full employment. Hence, to tackle the empirically observed idle capacity, the productive factor statistical series are often corrected so as to include only the actually employed factors, but such correction is not devoid of shortcomings. Our method, by addressing the issue of productive capacity in a different way, implicitly tackles this problem.

To conclude, our approach is somehow closer to the mainstream line that uses production functions to assess optimal productivities, as discussed above. We use a two-gap approach, which as customary uses a production function that depends only on capital, in a sense described later. We use as an illustration the cases of Chile and Mexico for the 1950-2000 period.

2. Two-Gap Analysis

2.1. Precedents

The two-gap model was originally devised as an *ex-ante* analytical framework to assess foreign exchange requirements for sustained growth (Chenery & Strout, 1966; McKinnon 1966; Chenery & Bruno, 1962). This represented an extension of the Harrod-Domar model to the open economy under the key assumption of complementarity, or at least weak substitutability, between foreign and domestic savings, in the short and medium

terms (Thirwall, 2003). Given a target GDP growth rate over a given period, it attempted to answer whether a country would undergo a potential shortage of foreign savings relative to domestic savings (“foreign exchange gap”), or a potential shortage of domestic savings relative to foreign savings (“saving gap”). The former case, with the help of estimated trends in domestic savings, trade and investment productivity, the complementary foreign aid/saving required to satisfy a given GDP growth rate could then be determined. If this aid (or foreign saving) was unattainable then the growth rate had to be revised down, at least in the short and medium term. Meanwhile, some policy package could be implemented to induce structural changes with a view to generate more foreign exchange inflows in the future.

At any time, a given amount of foreign savings may produce two different *ex-ante* valuations of potential output or growth: one coming from the saving-investment balance and the other from the trade balance. If the smaller of the two were the growth valuation from the trade balance, we would say that the economy is “foreign exchange constrained”, meaning that it experiences foreign exchange shortages. Conversely, we would say that the economy is “savings constrained” if domestic resources were not large enough to absorb or use productively the available foreign exchange. This type of mismatch is customarily called a “gap” rather than a “disequilibrium”, as the inconsistency would not be eradicated in the short and medium term by resorting to right prices or indeed to legislation. This means that the gap is associated to institutional structure, rather than to market disequilibria alone. The analytical framework of the two-gap model has however been rarely used outside the context of foreign exchange⁽¹⁾,

which is surprising, as it could address a number of other issues with advantage, as we intend to show.

2.2 *A Two-Gap Model to Assess a Capital Gap*

Focus. In our usage, there is the need for a few changes from the classical two-gap model. First, as we want to assess whether there has been a surplus or shortage of infrastructure in the recent past, the analysis has to be based on investment rather than saving, so we change the focus from the generation of savings to actual investment. On this count, the model becomes an *ex-post* analytical framework. But as we still want to assess the potential output or growth that would have been achieved from actual capital or investment, we can trace back an *ex-ante* element, which may have been affected by the *composition* of total capital or investment. Second, we shift the focus from foreign exchange to “core infrastructure” (hereafter, infrastructure capital or infrastructure investment). *Core infrastructure capital* is defined as all types of transport networks (including bridges, tunnels, terminals and the like), utility networks (including telecommunications, electricity, water, gas, oil, sewerage and the like), and power generating infrastructures (Diewert, 1986). Third, contrary to the standard two-gap analysis, our model focuses on the supply, rather than on the demand, side of the economy⁽²⁾.

Assumptions. Let us assume that both infrastructure capital and directly productive capital (hereafter, productive capital) are mutually complementary to each other for production,

i.e. one type of capital cannot be employed in the absence of the other type. The complementary, or at least weak substitutability, assumption is well supported in the literature, especially between “core infrastructure” capital and productive capital (Albala-Bertrand & Mamatzakis, 2001a; Mamatzakis, 1999; Feltenstein & Ha 1995; Domoto 1992; Munnell 1992)⁽³⁾. Let us also assume that any other requirement for production is fully available or passive, i.e. they accommodate to satisfy any targeted requirement. This allows focusing only on the two types of capital *vis-à-vis* potential output.

Aim. Given the assumptions above, and our discussion about potential output in the previous section, we seek to find the maximum output that can be derived from the actual availability of one type of capital, assuming that the other is fully available. That is, the aim would be to find both the maximum output from available infrastructure capital (K_I), assuming that productive capital (K_P) were not a constraint; and the maximum output from available productive capital (K_P), assuming that infrastructure capital (K_I) were not a constraint. This could also be transformed into output growth rates derived from investment ratios. The aim would then be to find both:

- (i) the maximum growth rate from the available infrastructure investment coefficient (I_I/Y), assuming that the productive investment coefficient (I_P/Y) were not a constraint, and
- (ii) the maximum growth rate from the available productive investment coefficient (I_P/Y), assuming that the infrastructure investment coefficient (I_I/Y) were not a constraint.

If the two types of capital were in equilibrium for production, i.e. matching their mutually required complements for the full employment of capital, then there would be *no gap* between the two. A gap arises only if their mutual economic requirements do not match, i.e. there is more of one type of capital than the other type can support or use. That is, there could be either more productive capital than what the available infrastructure capital can support, implying the existence of an *infrastructure shortage*. Or there could be potentially more infrastructure capital than what the available productive capital can use, implying an *infrastructure surplus* or a *productive capital shortage*. Therefore, production or growth cannot exceed the levels that the lower of the two types of capital can support or use. This means that for a gap to exist there should be a relative *excess* of one of the two types of *capital*, this excess remaining idle in production, as it cannot be supported or used by the other type of capital.

The Model Structure. Given that actual growth is the outcome of both types of complementary investment (and all other productive factors), from actual aggregate data we cannot directly find either the *potential growth rate*, g_p , from the productive investment ratio, or the *potential growth rate*, g_f , from the infrastructure investment ratio. Therefore, we have first to find the *two optimal valuations* for the output or the growth rate that is associated with each type of capital or investment. We can then derive the *optimally required ratio* between the two investment types for unconstrained growth, as is shown later.

Let us first approach this issue by starting with a generic production function augmented with the services of infrastructure capital, as:

$$Y = AF(L, K_p, K_f) \tag{1}$$

Where Y is output, A is total factor productivity, L is labour, K_p is productive capital and K_f is infrastructure capital. This is supposed to be a “well-behaved” production function, i.e. one that exhibits diminishing returns to factor intensity. This type of function has been used in the literature to assess the contribution of infrastructure capital to output. But, in our context, we would be unable to find the optimal output that could be potentially achieved from each of the two complementary factors, K_p and K_f , as this is excluded in the definition of well-behaved production functions. They assume substitutability among the included factors so that they are always fully used. Thus, a gap could not possibly arise, as this requires complementarity between factors and therefore the possibility of idle capacity when the optimal proportion between factors is not observed. Therefore, we resort to a fixed-coefficient production function, like a Leontief function, which would be able to handle this issue to advantage.

With a view to concentrate only on the problem at hand, we include as productive factors only the two complementary types of capital, and then follow a Harrod-Domar growth model (Jones, 1975) or an AK endogenous growth model (Aghion & Howitt, 1998; Solow, 1994). Theoretically, this assumes that capital is either an all-dominant factor or a good proxy for all factors, including total factor productivity. Empirically, this simply

means that there is both a good correlation and a relatively stable relationship between output and capital over time. This is normally the case from actual data, but it is also stronger when either capital is corrected for idleness or actual output is replaced with its potential (Albala-Bertrand, 2003; Thirlwall, 2003), as in this paper. Following Williamson (1983), we generically describe our working function as:

$$Y = \text{Min} \left[\frac{K_p}{k_p}, \frac{K_f}{k_f} \right] \text{ (Production Function)} \quad (2)$$

Where K_i (i: p or f) are two types of complementary capital, k_i represents the optimal capital-output ratio, p: productive capital and f: infrastructure capital. Notice that this formulation allows either type of capital to remain idle, as either capital can only operate productively in complement with the other, i.e. once one capital type is fully used, any remnant of the other capital cannot be used, and therefore remains idle. In optimal equilibrium, without idle capacity, the following should hold:

$$Y = \frac{K_p}{k_p} = \frac{K_f}{k_f} \quad \text{(In equilibrium)} \quad (3)$$

Notice that (3) implies that $k_p/k_f = K_p/K_f$, i.e. in optimal equilibrium the relation between capital-output ratios (or average “productivities”) is the same as that between capital stocks. This means that if we can find empirically the optimal equilibrium value of the capita-output ratios, then we should also be able to find the optimal relation between capital stocks, and vice versa. Total capital is given by:

$$K = K_p + K_f = k_p Y + k_f Y = (k_p + k_f) Y = k Y \quad (4)$$

Assuming that the average capital-output ratio is the same as the incremental capital-output ratio, which is especially true when defined in optimal terms as here (Albala-Bertrand, 2003), then total investment would be given by:

$$I = I_p + I_f = \Delta K_p + \Delta K_f = \Delta K = (k_p + k_f) \Delta Y = k \Delta Y \quad (5)$$

Where Δ denotes variation and I denotes investment. From here, $k = \Delta K / \Delta Y = I / \Delta Y$ represents the optimal incremental capital-output ratio. We require also knowing the ratio $\varepsilon = I_p / I_f$, which represents an optimal structural relationship. Thus, given that $I = I_p + I_f$, then $I = (1 + \varepsilon) I_f$. Dividing equation (5) by Y and rearranging:

$$g^*_k = \frac{1}{k} v_k \quad (\text{optimal growth from total investment}) \quad (6)$$

Where $g^*_k = \Delta Y / Y$ is the optimal growth rate that comes from *total* investment (using a subindex “k”), and $v_k = I / Y$ or $v_k = (I_p / Y) + (I_f / Y) = v_p + v_f$ are the investment coefficients for productive and infrastructure capital formation, respectively. Therefore, replacing $I = (1 + \varepsilon) I_f$ in (5) leads to $g = [(1 + \varepsilon) / k Y] I_f$, or

$$g^*_f = \frac{1 + \varepsilon}{k} v_f \quad (\text{optimal growth from infrastructure investment}) \quad (7)$$

Where $g^*_f = \Delta Y/Y$ is the optimal growth rate that comes from *infrastructure* investment (using a subindex “F”), and $v_f = I_f/Y$ is the infrastructure investment coefficient. Once (6) and (7) have been estimated, a quantitative gap G can be built as:

$$G = g^*_k - g^*_f = \frac{1}{k} v_k - \frac{1+\varepsilon}{k} v_f = \frac{1}{k} [v_k - (1+\varepsilon)v_f] = \frac{1}{k} (v_p - \varepsilon v_f) \quad (8)$$

Where the gap G will be positive, negative or null, if g^*_k is larger, smaller or equal to g^*_f , respectively. The gap is then the result of the difference between the two growth rate valuations: one coming from total investment and another from infrastructure investment. Given that total investment includes infrastructure investment, the gap arises every time the *actual composition* of total investment is inconsistent with its *optimal composition*. In other words, the gap arises when the growth potential from infrastructure investment does not match that of productive investment. Only the smaller of the two growth rates would be attainable.

To further clarify, the gap G is equal to 0, when v_p is equal to εv_f . If so, we are consistently back to the investment ratio $\varepsilon = v_p/v_f = I_p/I_f$. This is the optimal relationship between the two investment types, which the economy should keep to secure a balanced growth, without any idle capacity. In turn, every time G is different from 0, the value of G represents potential growth rate lost or unattained, given a binding investment of either type. For example, if $G > 0$ then the foregone growth rate would be due to a binding

shortage of infrastructure investment, while if $G < 0$ the binding shortage would be from productive investment.

This gap can be calculated both against years and against the infrastructure investment ratio v_f . The former would account for both the shortage of either type of capital and its size for *every year* over the period considered. The latter would in turn account for both the shortage of either type of capital and the derived gap size for the *whole* period in consideration when taken on average. From the former, we would be able to learn in which years there was a likely potential shortage of either type of capital. From the latter, we would be able to determine (i) what would have been the optimal relation between the two types of investment, ε ; and (ii) for which infrastructure investment ratio there would have been an *overall* shortage of either type of capital in any period taken as a whole. The latter can also be represented in the customary two-gap graph, as will be shown later.

3. Estimation Procedure

Our next step is to estimate the optimal productivity parameters of equations (6) and (7). Let $\alpha = 1/k$ and $\beta = (1 + \varepsilon)/k$ be the optimal productivity parameters for total capital and infrastructure capital, respectively. That is, in equilibrium $\varepsilon = (\beta/\alpha) - 1$. Therefore, once the optimal α and β have been estimated, we can find the optimal value of the ratio ε , or what is equivalent the optimal composition of investment for full utilisation of capital.

To calculate the optimal output from available investment, we use a modification of the linear programming method proposed by Berg (1984) and applied by Marfan & Artiagoitia (1989) to estimate potential output⁽⁴⁾. The linear programme seeks to minimise the total gap between potential and actual output, for each given period, provided that actual output is smaller than or equal to potential output in each particular year. To this effect, we define the output of each year as the output in the previous year, minus the capacity-induced output lost to depreciation, plus the addition to output this year coming from the “productivity”⁽⁵⁾ of gross investment of the previous year. This definition would then be equivalent to that of the “perpetual inventory method” to accumulate physical capital (see Albala-Bertrand, 2003). That is:

$$Y_t^* = (1 - \gamma)Y_{t-1}^* + \alpha I_{t-1} \quad (9)$$

Where γ is the depreciation rate, the asterisk “*” indicates the optimal value of the variable, “t” represents time and α represents the productivity of investment. Then the first-difference equation derived from (9) is:

$$Y_t^* = (1 - \gamma)^t Y_0^* + \alpha \sum_{i=1}^t I_{i-1} (1 - \gamma)^{(t-i)} \quad (10)$$

Thus, the base year, Y_0^* , and the productivity, α , are the two parameters to estimate. The linear programme then takes the following shape:

Minimise:

$$Z = \sum_{t=1}^n (Y_t^* - Y_t) \quad (11)$$

or

$$Z = Y_0^* \sum_{t=1}^n (1-\gamma)^t + \alpha \sum_{t=1}^n \sum_{i=1}^t I_{i-1} (1-\gamma)^{(t-i)} - \sum_{t=1}^n Y_t$$

Subject to:

$$Y_t^* \geq Y_t$$

$$Y_0^*, \alpha \geq 0$$

Where the Y_t^* sequence is calculated by the programme via equation (10). Linear programming, contrary to econometrics, does not smooth fitted functions into average deviations, but picks the best or optimal combination of values, while ignoring the others (Choudhury & Kirkpatrick, 1994; Chiang, 1984; Dervis, 1982). The main drawback of the method is that the results cannot be tested with the statistical sophistication of econometric results. But, as a second best, the model can be tested by means of sensitivity analysis, i.e. changing marginally some parameters so that the stability of the results can be assessed. We do this by changing the depreciation parameter, as this is the only exogenous component in the system⁽⁶⁾.

We apply this linear programming model to estimate both the optimal base-year output and the optimal productivity of both total investment and infrastructure investment. These

optimal results would then correspond to a measure of potential output, as indicated earlier. Under certain conditions the base-year output would also imply a benchmark capital stock (see Albala-Bertrand, 2003). Following our previous notation, for total investment, we denote the productivity parameter as α , while for infrastructure investment, we denote it as β . We use a data set from Chile and Mexico for the period 1950 to 2000.

For this method, the series should be long enough to go across a cycle, i.e. so as to include both peaks and troughs, but short enough to let the assumption of fixed parameters hold. We satisfy these constraints by estimating the parameters over a 10-year period, with 5 overlapping years. That is, we repeat the calculation for 10 sub-periods, 1951-60, 1956-65, 1961-70, and so on. This will also allow us to assess the structural changes in the optimal ratio ($\varepsilon = v_p/v_f = I_p/I_f$) across sub-periods.

Furthermore, the basic series are transformed into three-year moving averages, allocating each average to the middle year, which would also prevent a rogue year from exerting undue influence on our optimal results. As a measure of output and total investment we use gross domestic product (GDP) and gross fixed capital formation (GFCF), respectively. And as a measure of “core infrastructure” we use series, derived from GFCF, especially built up for the purpose⁽⁷⁾.

Finally, without correcting for idle capacity, the correlation between total (or infrastructure) capital and actual output, as well as the stability of capital-output ratios

appears to be fairly acceptable for the purpose. This is even more so when output is set at its potential, as is further clarified below (see also Albala-Bertrand, 2003). Using basic data produced by Hofman (2000) for Chile and Mexico, the actual average correlation, over the same 10-year periods that we use to calculate our optimal parameters, is around 98 percent. In turn, the actual capital-output ratio exhibits variations of around 5 percent from the mean. At the level, of each sub-period, only in the two periods of high economic instability in Chile (1971-80) and Mexico (1981-90) does the actual correlation goes down to 58 and 78 percent respectively.

Had the capital series been corrected for actual capital usage, both the correlation and the stability of capital-output ratios would have significantly improved in any sub-period. Alternatively, we can use the capital series without idleness corrections and estimate what would have been the maximum output achievable in each sub-period. This should smooth out any output instability or recession.

The resulting optimal parameters would then correspond to a measure of *potential output*, associated to the actual capital stock and investment effort, within the optimal productive conditions of each of our 10-year periods and 5-year overlapping sub-periods. As indicated in section 1.2 above, this is the approach that we have adopted in this paper.

4. The Results

4.1. Gap Over Time

We use the same depreciation rates as proposed by Hofman (2000) for Chile and Mexico.

The linear programme produces the following optimal coefficients:

Table 1: Optimal Productivity Parameters

Period	CHILE		MEXICO	
	α	β	α	β
1951-55	0.39	1.89	0.62	2.93
1956-60	0.44	1.83	0.64	2.75
1961-65	0.47	2.41	0.82	2.40
1966-70	0.44	1.81	0.66	2.05
1971-75	0.46	1.34	0.65	2.03
1976-80	0.47	2.72	0.67	1.28
1981-85	0.32	1.31	0.45	0.84
1986-90	0.68	3.52	0.63	1.79
1991-95	0.63	3.02	0.97	2.44
96-2000	0.48	2.38	0.55	1.73
Average	0.48	2.2	0.66	2.0

Where the parameters α belong to the calculation when using GFCF, and β to that when using infrastructure investment. It shows that for both Chile and Mexico the optimal productivity of capital α appears to have significantly declined over the period 1981-85, which corresponds to the beginning of the “lost decade” on account of the debt crisis. Chile however recovered by mid 1980s, sustaining a larger than average optimal productivity for total investment until 2000, while Mexico had a more erratic recovery. The productivity of infrastructure investment shows a similar but more volatile pattern for both countries.

Tables 1 and 2 below show the series that are generated with the above optimal parameters over the 51 years contained in our ten periods above. The first column is *actual GDP*, the second is the *optimal GDP from total capital* (GDP_k^*), the third is the *optimal GDP from infrastructure capital* (GDP_f^*), and the fourth column is the *optimal gap* calculated as the difference between the two previous columns (Gap^*). In turn, the fifth column is the *feasible GDP*, which corresponds to the lower value of column [2] and column [3], and the sixth column is *Maximum GDP*, which corresponds to the higher value of column [2] and column [3]. The seventh column is the *optimal growth shortfall* (OptShort) calculated as the difference between maximum GDP and feasible GDP, on feasible GDP base. In turn, the eighth column is the *actual shortfall* (ActShort) calculated as the difference between feasible GDP and actual GDP, on actual GDP base. And finally the ninth column is the *total shortfall* (TotShort) calculated as the difference between maximum GDP and actual GDP, on actual GDP base.

The optimal gap in column [4] represents the foregone output due to either a shortage of core infrastructure capital or a shortage of productive capital at optimal levels. That is, when the optimal gap is negative, then the binding shortage would be infrastructure capital, as there would then be a *surplus* of productive capital and when the optimal gap is positive, then the binding shortage would be from productive capital. This is equivalent to say that when the *optimal gap* is negative, there would be a *shortage in the share* of infrastructure capital in total capital (GFCF), and vice versa.

The seventh column expresses this idea in percentage point of GDP. For our sample period, on yearly average, the *optimal GDP growth* foregone by a mismatch between the two types of capital would in percentage points be 2.5 and 1.8 for Chile and Mexico, respectively. The total shortfall in the ninth column would be necessarily higher, but the addition would correspond to factors other than capital shortages, e.g. inflation, balance of payments problems, capital flights, political instability, and the like. Over the sample period, on yearly average, the *total shortfall* would in percentage points be 7.0 and 5.6 for Chile and Mexico, respectively.

Let us concentrate on the Optimal Gap (column [4]). For Chile, setting aside the four years with perfect balance, the annual average loss of 2.5 points can be decomposed into 1.0 percentage points for the 16 years with a shortage of infrastructure investment and 3.7 percentage points for the 31 years with a shortage of productive investment. The results indicate that there appears to have been small shortages of infrastructure capital for the periods 1954-61, 1970-73 and 1993, and only larger ones for 1994-95, when output was growing stronger, demanding and not fully getting faster infrastructure investment. The remaining years however show significant shortages of productive capital. In turn, for Mexico, setting aside the seven years with perfect balance, the 1.8 average annual loss can be decomposed into 2.1 percentage points for the 20 years with an infrastructure shortage, and also 2.1 percentage points for the 24 years with a shortage of productive investment. This shows that there would have been infrastructure shortages for the years 1959-66, 1968-72, 1978-79 and 1992-96. The remaining years show variable shortages of productive capital.

TABLE 2a: CHILE: Gap and Growth Shortfall over Time (1986 constant \$ million)

Year	[1] GDP Actual	[2] GDPk* Optimal	[3] GDPf* Optimal	[4] [3]-[2] Gap*	[5] mnGDP* Feasible	[6] mxGDP* Maximum	[7] OptShort 100[4]/[5]	[8] ActShort 100[5-1]/[1]	[9] TotShort 100[6-1]/[1]
1950	1282953	1323300	1330100	6800	1323300	1330100	0.5	3.1	3.7
1951	1316973	1357700	1361200	3500	1357700	1361200	0.3	3.1	3.4
1952	1384121	1394200	1394900	700	1394200	1394900	0.1	0.7	0.8
1953	1435206	1435200	1435200	0	1435200	1435200	0.0	0.0	0.0
1954	1460638	1475100	1472000	-3100	1472000	1475100	-0.2	0.8	1.0
1955	1464766	1518000	1505600	-12400	1505600	1518000	-0.8	2.8	3.6
1956	1518306	1561200	1541200	-20000	1541200	1561200	-1.3	1.5	2.8
1957	1587240	1611900	1590600	-21300	1590600	1611900	-1.3	0.2	1.6
1958	1650403	1663100	1650400	-12700	1650400	1663100	-0.8	0.0	0.8
1959	1698568	1710600	1707600	-3000	1707600	1710600	-0.2	0.5	0.7
1960	1760238	1760200	1766100	5900	1760200	1766100	0.3	0.0	0.3
1961	1854098	1865200	1854100	-11100	1854100	1865200	-0.6	0.0	0.6
1962	1952482	1952500	1991600	39100	1952500	1991600	2.0	0.0	2.0
1963	2037941	2052800	2174300	121500	2052800	2174300	5.9	0.7	6.7
1964	2099866	2160700	2375500	214800	2160700	2375500	9.9	2.9	13.1
1965	2199588	2264200	2545400	281200	2264200	2545400	12.4	2.9	15.7
1966	2309859	2356800	2432000	75200	2356800	2432000	3.2	2.0	5.3
1967	2443604	2443600	2512000	68400	2443600	2512000	2.8	0.0	2.8
1968	2529955	2536300	2583800	47500	2536300	2583800	1.9	0.3	2.1
1969	2608431	2636300	2651000	14700	2636300	2651000	0.6	1.1	1.6
1970	2737629	2746900	2737600	-9300	2737600	2746900	-0.3	0.0	0.3
1971	2823683	2846700	2823700	-23000	2823700	2846700	-0.8	0.0	0.8
1972	2838332	2936900	2920700	-16200	2920700	2936900	-0.6	2.9	3.5
1973	2781904	3000200	3022200	22000	3000200	3022200	0.7	7.8	8.6
1974	2615800	3051600	3115700	64100	3051600	3115700	2.1	16.7	19.1
1975	2528647	3092400	3175800	83400	3092400	3175800	2.7	22.3	25.6
1976	2500418	3128700	3292000	163300	3128700	3292000	5.2	25.1	31.7
1977	2662452	3140100	3291000	150900	3140100	3291000	4.8	17.9	23.6
1978	2867274	3162300	3292800	130500	3162300	3292800	4.1	10.3	14.8
1979	3083358	3213700	3295400	81700	3213700	3295400	2.5	4.2	6.9
1980	3304315	3304300	3304300	0	3304300	3304300	0.0	0.0	0.0
1981	3298996	3320800	3299000	-21800	3299000	3320800	-0.7	0.0	0.7
1982	3179158	3335200	3301500	-33700	3301500	3335200	-1.0	3.8	4.9
1983	3045661	3327900	3326200	-1700	3326200	3327900	-0.1	9.2	9.3
1984	3106238	3293000	3367500	74500	3293000	3367500	2.3	6.0	8.4
1985	3262864	3262900	3423900	161000	3262900	3423900	4.9	0.0	4.9
1986	3434762	3850800	4028800	178000	3850800	4028800	4.6	12.1	17.3
1987	3656819	3945800	4160600	214800	3945800	4160600	5.4	7.9	13.8
1988	3949596	4077200	4269900	192700	4077200	4269900	4.7	3.2	8.1
1989	4229111	4283000	4385700	102700	4283000	4385700	2.4	1.3	3.7
1990	4541281	4541300	4541300	0	4541300	4541300	0.0	0.0	0.0
1991	4920466	4953100	5048000	94900	4953100	5048000	1.9	0.7	2.6
1992	5364325	5364300	5442200	77900	5364300	5442200	1.5	0.0	1.5
1993	5799712	5851600	5799700	-51900	5799700	5851600	-0.9	0.0	0.9
1994	6254736	6430600	6279300	-151300	6279300	6430600	-2.4	0.4	2.8
1995	6751234	7123000	6853100	-269900	6853100	7123000	-3.9	1.5	5.5
1996	7321525	7321500	7321500	0	7321500	7321500	0.0	0.0	0.0
1997	7776195	7994200	8086100	91900	7994200	8086100	1.1	2.8	4.0
1998	8032863	8698800	9065500	366700	8698800	9065500	4.2	8.3	12.9
1999	8250438	9332200	10031000	698800	9332200	10031000	7.5	13.1	21.6
2000	8511205	9886000	10939000	1053000	9886000	10939000	10.7	16.2	28.5
Average	3373061	3546978	3629718	82739	3533990	3642706	2.5	4.2	7.0

[1] GDP: Gross Domestic Product (3-year moving average)

[2] GDPk*: Optimal GDP From GFCF

[3] GDPf*: Optimal GDP From Infrastructure Investment

[4] Optimal Gap: [3]-[2]

[5] mnGDP*: minimum optimal value between [2] and [3] or Feasible GDP

[6] mxGDP*: maximum optimal value between [2] and [3]

[7] OptShort: Optimal Shortage: 100[4]/[5]

SIGN: Negative = infrastructure shortage or productive investment surplus

Positive = infrastructure surplus or productive investment shortage

[8] ActShort: Actual Shortage: 100[5-1]/[1]

[9] TotShort: Total Shortage: 100[6-1]/[1]

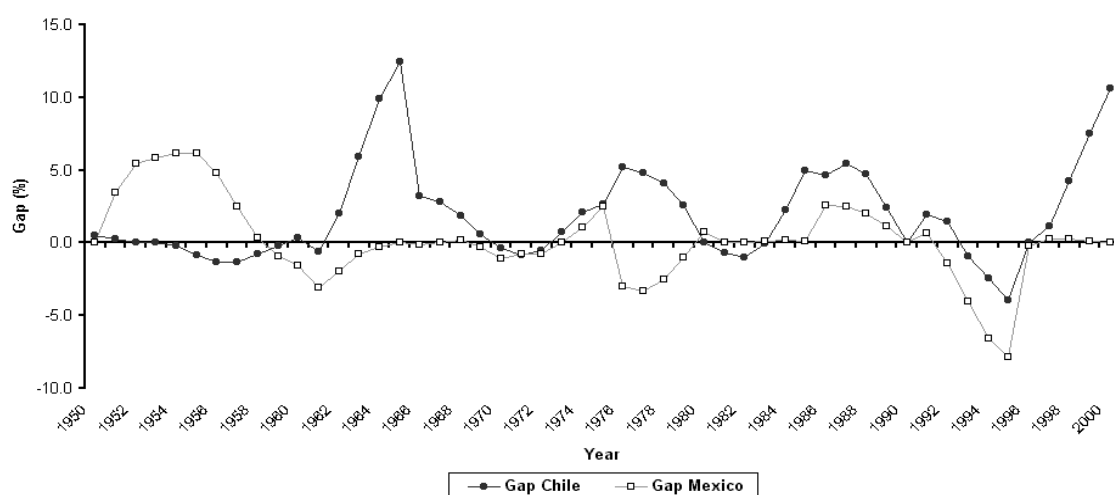
TABLE 2b: MEXICO: Gap and Growth Shortfall over Time (1970 constant \$ million)

Year	[1] GDP Actual	[2] GDPK* Optimal	[3] GDPF* Optimal	[4] [3]-[2] Gap*	[5] mnGDP* Feasible	[6] mxGDP* Maximum	[7] OptShort 100[4]/[5]	[8] ActShort 100[5-1]/[1]	[9] TotShort 100[6-1]/[1]
1950	129604	129600	129600	0	129600	129600	0.0	0.0	0.0
1951	132994	136940	141660	4720	136940	141660	3.4	3.0	6.5
1952	138121	144630	152500	7870	144630	152500	5.4	4.7	10.4
1953	144700	153030	162000	8970	153030	162000	5.9	5.8	12.0
1954	153865	161200	171150	9950	161200	171150	6.2	4.8	11.2
1955	166715	169650	180050	10400	169650	180050	6.1	1.8	8.0
1956	179406	179520	188220	8700	179520	188220	4.8	0.1	4.9
1957	191139	191140	195880	4740	191140	195880	2.5	0.0	2.5
1958	201078	203280	203930	650	203280	203930	0.3	1.1	1.4
1959	212146	215050	213160	-1890	213160	215050	-0.9	0.5	1.4
1960	223511	226960	223510	-3450	223510	226960	-1.5	0.0	1.5
1961	236542	247830	240450	-7380	240450	247830	-3.1	1.7	4.8
1962	250524	262800	257680	-5120	257680	262800	-2.0	2.9	4.9
1963	271224	278540	276490	-2050	276490	278540	-0.7	1.9	2.7
1964	294696	297190	296370	-820	296370	297190	-0.3	0.6	0.8
1965	318922	318920	318920	0	318920	318920	0.0	0.0	0.0
1966	339817	340430	340020	-410	340020	340430	-0.1	0.1	0.2
1967	364073	364070	364070	0	364070	364070	0.0	0.0	0.0
1968	389219	390760	391600	840	390760	391600	0.2	0.4	0.6
1969	416861	420280	419180	-1100	419180	420280	-0.3	0.6	0.8
1970	440862	452360	447500	-4860	447500	452360	-1.1	1.5	2.6
1971	469720	475490	471710	-3780	471710	475490	-0.8	0.4	1.2
1972	503066	506830	503070	-3760	503070	506830	-0.7	0.0	0.7
1973	541320	541320	541620	300	541320	541620	0.1	0.0	0.1
1974	577284	580930	587100	6170	580930	587100	1.1	0.6	1.7
1975	607792	625500	641110	15610	625500	641110	2.5	2.9	5.5
1976	634510	659520	640480	-19040	640480	659520	-3.0	0.9	3.9
1977	668512	699630	677330	-22300	677330	699630	-3.3	1.3	4.7
1978	715622	739480	721060	-18420	721060	739480	-2.6	0.8	3.3
1979	777000	784890	777000	-7890	777000	784890	-1.0	0.0	1.0
1980	842594	842590	848930	6340	842590	848930	0.8	0.0	0.8
1981	884818	884820	884820	0	884820	884820	0.0	0.0	0.0
1982	900082	927570	927860	290	927570	927860	0.0	3.1	3.1
1983	892470	959660	960790	1130	959660	960790	0.1	7.5	7.7
1984	873945	978590	980670	2080	978590	980670	0.2	12.0	12.2
1985	849630	991430	992730	1300	991430	992730	0.1	16.7	16.8
1986	829776	1068500	1096300	27800	1068500	1096300	2.6	28.8	32.1
1987	849997	1086400	1113200	26800	1086400	1113200	2.5	27.8	31.0
1988	919186	1102100	1124300	22200	1102100	1124300	2.0	19.9	22.3
1989	1026274	1119900	1133000	13100	1119900	1133000	1.2	9.1	10.4
1990	1143100	1143100	1143100	0	1143100	1143100	0.0	0.0	0.0
1991	1243549	1243500	1251900	8400	1243500	1251900	0.7	0.0	0.7
1992	1311034	1329400	1311000	-18400	1311000	1329400	-1.4	0.0	1.4
1993	1361065	1419100	1364400	-54700	1364400	1419100	-4.0	0.2	4.3
1994	1360560	1513200	1419500	-93700	1419500	1513200	-6.6	4.3	11.2
1995	1373708	1590200	1474600	-115600	1474600	1590200	-7.8	7.3	15.8
1996	1398411	1487800	1485200	-2600	1485200	1487800	-0.2	6.2	6.4
1997	1477119	1529500	1533500	4000	1529500	1533500	0.3	3.5	3.8
1998	1552154	1585600	1589500	3900	1585600	1589500	0.2	2.2	2.4
1999	1631138	1651100	1653200	2100	1651100	1653200	0.1	1.2	1.4
2000	1720554	1720600	1720600	0	1720600	1720600	0.0	0.0	0.0
Average	688863	726910	723206	-3704	719317	730800	1.8	3.7	5.6

- [1] GDP: Gross Domestic Product (3-year moving average)
- [2] GDPK*: Optimal GDP From GFCF
- [3] GDPF*: Optimal GDP From Infrastructure Investment
- [4] Optimal Gap: [3]-[2]
- [5] mnGDP*: minimum optimal value between [2] and [3] or Feasible GDP
- [6] mxGDP*: maximum optimal value between [2] and [3]
- [7] OptShort: Optimal Shortage: 100[4]/[5]
SIGN: Negative = infrastructure shortage or productive investment surplus
Positive = infrastructure surplus or productive investment shortage
- [8] ActShort: Actual Shortage: 100[5-1]/[1]
- [9] TotShort: Total Shortage: 100[6-1]/[1]

Hence, it seems that in Chile productive investment shortage was more detrimental to growth than infrastructure investment shortage, while for Mexico the two shortages appeared to have played a similar role in optimal growth losses. Put it another way, in Chile, the share of core infrastructure investment in total GFCF would have been mostly excessive or, which is equivalent, the share of productive investment in GFCF would have been mostly insufficient. In Mexico, the mismatching of both shares appears to have been detrimental to optimal growth rates. The graph below describes the story above:

GRAPH 1: Gap-Induced Foregone Growth Rate



The y-axis represents the optimal growth rate that would be foregone due to mismatches between investment types, while the x-axis represents years. The distance between zero and the curve represents the optimal growth rate lost in percentage points. Negative numbers represent the *positive* growth rate lost due to lack of infrastructure investment; while positive numbers represent positive growth rate lost due to lack of productive investment. That is, when the curve passes below zero, growth is constrained by a

shortage of infrastructures; while when it passes over zero, growth is constrained by a shortage of productive investment, i.e. there would be an unused surplus of infrastructure capital. The graph then shows the two gaps via an oscillating curve around equilibrium over time with variable peaks and troughs, as was explained above.

4.2. Gap against the Infrastructure Coefficient

To establish what would be the optimal infrastructure coefficient, v_f , and the optimal capital ratio ϵ for each of the sub-periods on average, we calculate the optimal growth rate equations at mutual full employment point. For this, noting that $v_k = v_p + v_f$, we first express equation (6) in terms of v_f , as $g^*_k = (1/k)v_p + (1/k)v_f$, we then use equation (7), already expressed in v_f terms, so that we can now have a solution system for growth against the infrastructure investment ratio. However, with a view to allow for depreciation, we transform equation (6) and equation (7) as:

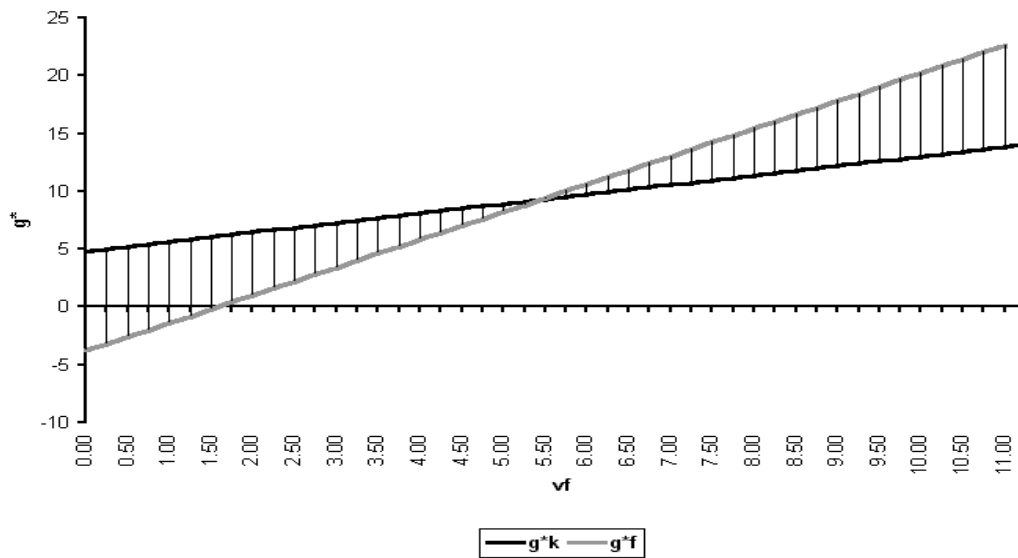
$$g^*_k = -\gamma_1 + \alpha v_k = (-\gamma_1 + \alpha v_p) + \alpha v_f \quad (12)$$

$$g^*_f = -\gamma_2 + \beta v_f \quad (13)$$

Where γ_1 and γ_2 are the respective depreciation rates, and α and β are the respective optimal coefficients for each growth rate valuation g^* . The former would give us the optimal growth rate derived from total GFCF (g^*_k), while the latter would for that derived from infrastructure investment (g^*_f). We aim to find the infrastructure

investment ratio that maximises optimal feasible growth. For this purpose, we calculate the optimal ratio v^*_f that would correspond to the average productive ratio v_p , for each sub-period. Given that at this point the two optimal growth rates must be equal, then we will have two equations and two unknowns, g^* and v^*_f , allowing the system to solve with a unique solution. The optimal capital ratio ε would then correspond to the ratio average-productive-coefficient to optimal-infrastructure-coefficient. The solution can also be presented in a more customary two-gap graph, as below:

GRAPH 2: Two-Gap Representation for each Sub-Period



The y-axis represents the growth rate, while the x-axis represents the infrastructure investment coefficient. The point where the two curves cross each other will determine the *optimal growth rate*, which also corresponds to the *optimal investment coefficient*. Any point *on* or *under* a single curve would correspond to a feasible growth rate. The former would be an optimal growth rate, while the latter would be a sub-optimal one for

constraints due to lack of complementary investment or other problems. Points above a single curve are not feasible, as they cannot be achieved in the context of the model. Therefore, for the overall consistency of the two curves acting simultaneously, only the points on or under the lower curve, i.e. that one that passes closer to the x-axis, at any level of infrastructure coefficient would be feasible. In the graph, this would correspond to the unfilled area under the curves. The lower curve between zero and the optimal infrastructure coefficient (i.e. the crossing point) corresponds to the optimal growth rate from the infrastructure coefficient, g^*_f . And the lower curve between the optimal infrastructure coefficient and infinity corresponds to the optimal growth rate from the total GFCF coefficient, g^*_k .

So if the achieved v_f is to the left of its optimal, then growth will be constrained by lack of infrastructure investment, imposing a foregone growth rate equal to the difference between the two curve at the point in question. Conversely, if the achieved v_f is to the right of its optimal, then there would be shortage of productive investment, as there would be an infrastructure surplus that remains idle. Thus, the shortages can be expressed in foregone growth rate in percentage points. In the graph, that would correspond to the length of the connecting lines between the two curves. Following conventional two-gap analysis, and allowing for our re-adaptation, the former shortage can be called the “infrastructure gap”, while the latter, the “productive gap”.

Every time there is a mismatch between the optimal and the actual infrastructure investment coefficient there would be a difference between the growth rates coming from

either type of investment. This difference, or gap between the two rates, represents a level of foregone growth rate to society. That is, on the left-hand side of equilibrium, the social opportunity cost of increasing the productive investment coefficient v_p would be wasteful to society, as it would only increase the gap without improving the feasible-optimal growth rate. However, an increase of v_f on the left-hand side of equilibrium would generate a higher feasible-optimal growth rate than the one on the right-hand-side section, as the g^*f curve is steeper than the g^*k curve. Thus, on the right hand side of equilibrium, only an increase of v_p (i.e. included the curve intercept) would be useful, while one of v_f would be unproductive and therefore wasteful to society. Notice that policies that act on the productivity parameters, α and β , can also affect mostly the curve slopes in economically convenient ways, just as policies that act on depreciation alone can affect the intercepts.

Therefore, other things being equal, when experiencing an infrastructure shortage, it would as expected pay to society to increase the investment coefficient on infrastructure at the expense of productive investment. While when experiencing an infrastructure surplus it would be more economically efficient to reinforce productive investment at the expense of infrastructure investment. The latter would make the g^*k curve jump up, both improving the feasible growth rate faster and shifting equilibrium to the right, i.e. reducing wasteful infrastructure investment.

The algebraic solutions from our data are presented in the table below. That is, given the average productive investment ratios, v_p , the equilibrium optimal growth rates, g^* , could only be reached if the optimal infrastructure investment ratios, v^*_f , were actually achieved, other things being equal. The optimal ratio $\epsilon^{(8)}$ gives a summary measure of medium-term structural change (i.e. around 5 year periods).

Table 3: Optimal Solutions and Gaps per Sub-Period

Period	CHILE						MEXICO					
	g^*	v^*f	vf	vp	ϵ	Gap*	g^*	v^*f	vf	vp	ϵ	Gap*
	Optimal	Optimal	Actual	Average	vp/v^*f	%points	Optimal	Optimal	Actual	Average	vp/v^*f	%points
1951-55	2.9	3.8	3.7	16.4	4.3	-0.1	4.0	2.6	5.2	11.2	4.2	5.7
1956-60	4.1	4.6	4.1	16.7	3.6	-0.7	5.9	3.4	4.0	13.1	3.8	1.1
1961-65	3.8	3.3	6.4	15.9	4.8	6.2	9.3	5.4	4.6	12.1	2.2	-1.3
1966-70	3.4	3.9	5.2	15.2	3.8	1.7	7.4	5.5	5.4	13.7	2.5	-0.3
1971-75	2.4	4.7	5.6	11.7	2.5	0.8	7.0	5.5	5.8	14.2	2.6	0.4
1976-80	0.6	1.8	3.3	11.1	6.1	3.4	10.3	11.0	7.8	12.8	1.2	-1.9
1981-85	-0.3	3.2	4.3	13.7	4.2	1.1	3.3	8.0	8.2	11.4	1.4	0.1
1986-90	5.9	3.0	4.3	14.7	4.9	3.7	2.0	2.9	4.4	9.3	3.2	1.7
1991-95	10.2	4.9	4.6	20.9	4.3	-0.8	12.0	6.2	3.5	11.7	1.9	-4.0
96-2000	6.5	4.6	7.2	21.4	4.6	5.0	3.1	3.6	3.7	11.8	3.3	0.1
Average	4.0	3.8	4.9	15.8	4.3	2.5	6.4	5.4	5.3	12.1	2.6	1.8

Gap*: foregone growth rates at optimal level expressed in percentage points
 negative numbers correspond to infrastructure shortage
 positive numbers correspond to productive investment shortage

It can be seen that in Chile a unit of infrastructure capital is capable of optimally carrying or sustaining a larger amount of productive capital than in Mexico, i.e. Chile's average ϵ is much larger than that of Mexico, i.e. 4.3 against 2.6. This is also true for most sub-periods, especially from 1976-80 on, which appears to indicate that in general infrastructure capacity is more efficient in Chile than in Mexico. And contrary to Mexico, Chile in practice appears to have been just under or significantly over the required equilibrium infrastructure investment coefficient, for each sub-period. The gaps, however, expressed in average percentage point losses of growth rate per sub-period, were larger in Chile than Mexico, but the source quite different. For Chile, this was

mostly due to a lack of a productive investment that could have used up the running surplus of infrastructure capacity. In other words, the share of infrastructure investment in GFCF was systematically excessive. For Mexico, on the other hand, there was a significant lack of an infrastructure capacity that could have facilitated the deployment of the existing surplus of productive investment. So Mexico's share of infrastructure investment in GFCF was often insufficient.

Conclusion

Answering the question in the title, in the context of our proposed method, the wrong composition of capital or investment can indeed constrain potential output or growth. The cases of Chile and Mexico over the 1950-2000 period were used as an illustration of the approach. The results show that the shortage in the share of infrastructure capital appears to exhibit an oscillating pattern, with shortages following surpluses. In Chile, potential output appears to be due mostly to shortages of productive capital, rather than that of infrastructure capital. Mexico, however, shows some significant potential output losses due to shortage of infrastructure capital. But these potential losses might still be relatively small as compared to other reasons why the Chilean and Mexican economies did not achieve their feasible optimal growth rates.

The alternation of shortages above suggests that the imbalances between capital types tend to be corrected by the economic system itself over variable periods. This also means that the timing of the corrections could be improved. That is, within our framework, in

terms of growth rates, when there is a shortage of core infrastructures, the opportunity cost to society would be better served by devoting resources to infrastructure at the expense of productive investment. Likewise, when there is a core-infrastructure surplus, it would be less costly to society to devote resources to productive investment at the expense of infrastructure investment than to further increase the latter. Either policy would both increase optimal growth and reduce the wasteful gap.

A macroeconomic shortage of either infrastructure capital or productive capital implies that the economy as a whole would be in a net state of shortage, but does not necessarily imply that there would be a shortage everywhere. Our method may represent a useful contribution at macroeconomic level, so that once a shortage has been established, a more focused analysis can be used to discriminate between geographic locations, economic sectors, and infrastructure or productive components.

NOTES

(1) The easy credit to developing countries from 1973 until early 1980s virtually eradicated this model from the economic literature, but it came back with renewed strength after the 1982 Mexico moratorium that heralded the world debt crisis. Given that there was now a serious fiscal constraint, as governments were forced to serve the debt, the model was extended to deal with the so-called “fiscal gap”, i.e. a third valuation for *ex-ante* output or growth. Since then the model is resorted to from time to time in academic literature to analyse constrained growth (Taylor, 1991; Bacha, 1984 and 1990; Chisari & Fanelli, 1990; Eyzaguirre, 1989; Fanelli & Frankel 1989; and others). But the World Bank has always used it for its growth programming exercises, whether in its original form or a modified version of it, currently called the “Revised Minimum Standard Model” or RMSM (Agenor, 2000; Khan et al, 1990; Michalopoulos, 1987).

(2) It should be pointed out that the standard, *ex-ante*, demand-determined model can still be entertained for policy analysis. In this case, what is required is (i) to focus on the generation of saving for earmarked infrastructure purposes, which might not be free from strong assumptions, and (ii) to take the potential level of savings as the maximum achievable investment.

(3) Although the complementarity assumption may hold reasonably well for composite infrastructure investment/capital and composite productive investment/capital over reasonable time periods, it may not hold equally well for components of each capital. It is clear that, say, the share of telecommunications in infrastructure has dramatically

changed in the last fifteen years, which may mean a strong substitutability within this type of composite capital. Therefore, our method may not work, unless corrections are entertained, at this level.

(4) Berg (1984) used it to estimate potential or capacity output in various manufacturing sectors. The method may overestimate capacity in some heterogeneous sectors, but the results appear to be strongly correlated with actual plant information. In turn, Marfan and Artiagoitia (1989) used it for macroeconomic analysis with acceptable results.

(5) Notice that this average “productivity” of capital is measured by the ratio of total output (or addition to output) to total capital (or investment). That is, we attribute total output to total capital, but total output is the result of many other factors contributing to it. Therefore, the coefficient is not actually the productive capacity of capital alone, but an assumedly and/or reportedly stable ratio, which could be generally corrected by making explicit all factor contributions (see Thirwall 2003), although this is not required here. In addition, investment is assumed to become productive with one-year lag.

(6) To test the sensitivity of the linear programming model, we actually tried some combinations with two, four and five percent depreciation. The gap may increase in some calculations, but the overall pattern remains pretty stable.

(7) All our data come directly from Chilean and Mexican official institutions (e.g. Planning Ministry, Central Bank, Public Works Ministry and National Institute of

Statistics, INEGI, etc.) and also from a compilation by Moguillansky (1999) for Chile who use the same sources above. For Mexico, Ernesto Piedras kindly supplied the “core infrastructure” data, which was part of his PhD thesis on the subject at the LSE. All series have been deflated, via PPI and GDP deflator, for 1986 and 1970 for Chile and Mexico, respectively. For the stock of infrastructure capital, we use a definition that is confined to transport, sewage and utilities (e.g. water, electricity, and gas). That is, the so-called “core infrastructure” (Diewert 1986). The depreciation rates used were the ones suggested by Hofman (2000). Finally, strictly speaking, what we call private capital is actually non-infrastructure capital, as this incorporates both private capital and non-core public capital. This allows us to measure both the *share* of core-infrastructure investment in total GFCF and that of core-infrastructure capital in the total capital stock.

(8) The optimal ratio between investments, ϵ , can be calculated either by first finding the equilibrium v_f , given v_p , or by equalising the two optimal growth rate equations above (equation (12) and (12)') and then solving for ϵ . The calculation of $\epsilon = (I_p/I_f) = (v_p/v_f)$ then becomes: $\epsilon = [(\gamma_1 - \gamma_2)/\alpha v_f] + [(\beta/\alpha) - 1]$. Notice that if the difference between depreciation rates is small, the second term alone will approximate well the actual value of the optimal ratio ϵ .

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