The (Ir)relevance of the NRU for Policy Making:
The Case of Denmark*

Marika Karanassou
Queen Mary, University of London†
and IZA

Hector Sala
Universitat Autònoma de Barcelona‡
and IZA

Pablo F. Salvador
Universitat Autònoma de Barcelona§
and Universitat de Girona

20 November 2006

Abstract

We reconsider the central role of the natural rate of unemployment (NRU) in forming policy decisions. We show that the unemployment rate does not gravitate towards the NRU due to frictional growth, a phenomenon that encapsulates the interplay between lagged adjustment processes and growth in dynamic labour market systems. We choose Denmark as the focal point of our empirical analysis and find that the NRU explains only 33% of the unemployment variation, while frictional growth accounts for the remaining 67%. Therefore, our theoretical and empirical findings raise serious doubts as to whether the NRU should play a key instrumental role in policy making.

Key Words: Unemployment, Natural Rate of Unemployment, Labour Market Dynamics, Frictional Growth, Chain Reaction Theory.

JEL Classification Numbers: E22, E24, J21.

*Acknowledgments: Hector Sala is grateful to the Spanish Ministry of Education and Science for financial support through grant SEJ2006-14849/ECON.
†Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, UK; tel.: + (0)20 7882-5090; email: M.Karanassou@qmul.ac.uk; http://www.qmul.ac.uk/~ugte171/
‡Department d’Economia Aplicada, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain; tel: + 34 93 5812779; email: hector.sala@uab.es. Correspondence address.
§Departament d’Economia Aplicada, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain; + 34 93 5811680; email: pablo.salvador@uab.es.
1 Introduction

The natural rate of unemployment (NRU) plays a pivotal role in the decisions of policy makers. The influential contributions of Friedman and Phelps at the end of the 60s established that the Phillips curve is vertical in the long-run and marked the beginning of the "NRU era" in economic modelling. The term natural rate was coined by Friedman in 1968 and was described as a feature of the Walrasian market clearing general equilibrium.\(^1\)

On one hand, discussions about which labour market reforms are necessary draw heavily on the determinants of the NRU. On the other hand, the choice of contractionary or expansionary policy measures crucially depends on whether unemployment is below or above its natural rate. Proponents of the NRU paradigm assert that the natural rate is consistent with inflation stability and that unemployment gravitates towards it. This claim has major policy implications: when unemployment is perceived close to its natural rate any attempt to reduce it will only result in higher inflation.

Over the past 20 years the evolution of inflation and unemployment in most of the developed economies has put the NRU story under scrutiny. The relatively low and rather stable inflation rates imply that actual unemployment has been close to its natural rate. Therefore, given the rather high unemployment rates that persisted in the 80s and the 90s, the challenge for the NRU paradigm has been to identify the factors responsible for the rise in the natural rate. Blanchard (2006), in a journey through the decades, reviews the explanations offered to justify the NRU increases: high oil prices and slowdown in productivity in the 70s, persistence mechanisms in the 80s, labour market institutions in the 90s. Blanchard (2006) is a narrative of what we have learned and what we still do not know. He bravely points out that "One might have hoped that...we would now have an operational theory of unemployment. I do not think that we do." (p. 8).

This paper reassesses the role of the natural rate in policy making and argues that in the presence of frictional growth unemployment does not gravitate towards the NRU - instead, it can be described as chasing after a moving target. The phenomenon of frictional growth arises from the interplay between lagged adjustment processes and growth in multi-equation labour market models, and is thus a salient feature of the chain reaction theory (CRT) of unemployment models.

In Section 3 we develop a CRT model and show that the long-run unemployment rate is the sum of two components: the NRU and frictional growth. Therefore, the

\(^1\)Tobin (1998) argues that the NRU and NAIRU are not synonymous. In contrast, the view of Ball and Mankiw (2002) is that the two concepts are approximately synonyms. Karamassou, Sala, and Snower (2006) show that the NRU/NAIRU distinction becomes superfluous within their framework of "exogenous/endogenous" NRU models. It is important to note that our analysis does not hinge upon this issue which is beyond the scope of this paper.
predictions of the CRT models are in sharp contrast with those of dynamic single-equations where the NRU is the attractor of the unemployment rate. The reason for this substantial disparity is that single-equation models do not allow for frictional growth since all the labour market adjustments are supressed into the autoregressive coefficient(s) of the single unemployment rate equation, and the exogenous variables are stationary so that the right-hand side of this equation balances with the trendless unemployment rate.

Denmark is a particularly interesting case to study as it appears to refute the NRU predictions. It is one of the succesful economies in Europe having recovered, after experiencing serious unemployment problems, an unemployment rate close to full-employment levels that is half the European average. The Danish labour market is among the most flexible and dynamic ones accross Europe, resembling more the Anglo-Saxon model than the continental European labour markets. At the same time, like the rest of the Nordic economies, Denmark has a well-developed welfare state system with a very low degree of income inequality.

Our empirical model of the Danish labour market reveals that actual unemployment does not evolve around its natural rate - the NRU can only explain one third of the variation in unemployment, while frictional growth accounts for the remaining two thirds. In a nutshell, our analytic and empirical findings raise serious doubts about the importance of the NRU in policy modelling.

The remaining of the paper is structured as follows. In Section 2 we first discuss the standard methodologies to estimate the NRU, and illustrate the conventional wisdom with a simple graph. We then provide a formal definition of the NRU. In Section 3 we use an analytic labour market model to explain the implications of the chain reaction theory (CRT) of unemployment for the NRU. In Section 4 we present the multi-equation dynamic model estimated for the Danish economy. In Section 5 we compute the NRU and discuss its relevance for policy making. Section 6 concludes.

2 The Natural Rate of Unemployment

2.1 The Conventional Wisdom

The standard unemployment rate models seek to explain movements in unemployment by distinguishing two components: (i) the so called "business cycle," i.e. the high-frequency unemployment movements which are induced by the effects of temporary shocks disrupting equilibrium, and (ii) the so called "trend" or NRU, i.e. the low-
frequency movements of unemployment which arise from changes in the permanent components of its determinants.

This compartmentalisation implies that the unemployment rate evolves around the NRU from which it only temporarily deviates. In other words, the natural rate serves as an attractor for the moving unemployment rate. The structuralists and institutionalists are two prominent and influential groups within this tradition. Both groups estimate single-equation unemployment rate models to identify the driving forces of the natural rate.\(^3\)

The structuralist perspective involves dynamic unemployment rate equations and asserts that the trajectory of unemployment is mainly determined by the structure of the economy, rather than by labour market lags (i.e. employment, real wage, and labour force adjustments).

This view was put forward by Phelps (1994) where the set of NRU determinants included (i) country-specific variables, such as real capital stock (normalised so that its trend is removed), real public debt, real government spending, tax rates, other institutional variables (replacement rate, duration of unemployment benefits), price markups induced by exchange rates, and some demographic variable (e.g. the proportion of population between 20 and 24 years old), and (ii) world variables, such as the real interest rate and the real price of oil.

Subsequent works of the structuralist proponents - see, among others, Phelps and Zoega (1998, 2001) and Fitoussi et al. (2000) - also included the slowdown of productivity (witnessed since the mid 70s), the share of social expenditures in GDP, the educational composition of the labour force, and asset valuation in the determination of unemployment.

The idea that labour market institutions are the main driving force of unemployment has significantly influenced academics and policy makers since the OECD Jobs Study was published in 1994. In general, the institutionalists argue that wage-push factors (such as unemployment benefits, firing restrictions, minimum wages, union power, and the tax wedge), and active labor market policies are responsible for the rise in unemployment. It is worth noting how far apart the institutionalist story stands from the Keynesian viewpoint that capital accumulation, demand factors and unemployment persistence are the driving forces of unemployment (see Stockhammer, 2004).


\(^3\)When the unemployment rate equations include the change in inflation on their right-hand side, they can be described as augmented Phillips curve models where the time-varying NRU changes are attributed to fundamentals.
the 1961-1995 period and find that shifts in labour market institutions explain around 55% of the rise in European unemployment (excluding Greece, Luxembourg and Eastern Europe).

According to Blanchard (2006, p. 31) "Changes in institutions did not appear able, however, to explain the evolution of unemployment rates over time." This of course may be due to the inability of quantitative indices to describe effectively the multiple dimensions of labour market institutions. The lack of annual time-series data on institutional variables and the observation that institutions do not vary much through time, also led researchers to adopt 5-year averages in their estimations (see, for example, Blanchard and Wolfers, 2000).

However, we should note that cross-country regressions and 5-year data averages in panel estimation completely disregard the role played by labour market dynamics in the evolution of the unemployment rate. The dismissal of dynamics in the analysis of the unemployment problem is justified by the macroeconomic consensus that the long-run equilibrium of the unemployment rate (NRU) and the short-run variations of actual unemployment around it are independent of one another.

Statistical filtering of the unemployment rate series is a popular technique to extract its "trend" component. In 1980 Hodrick and Prescott proposed their detrending method, commonly known as HP filtering (see Hodrick and Prescott, 1997). This is essentially a time-varying linear trend that changes smoothly over time. Although the univariate filters like the HP and band-pass (see Baxter and King, 1999) are used to decompose a series into its permanent and temporary components, they are unable to provide any insight on the driving forces of the "trend" component of the variable. This led to the development of multivariate HP filters, known as HPMV (see, for example, Chagny and Lemoine, 2004). Furthermore, the Kalman filter is another statistical technique that has been extensively used in Phillips curve models to estimate the time-varying non accelerating inflation rate of unemployment (TV-NAIRU).4

We illustrate the conventional wisdom with a simple example. Figure 1 plots the actual and natural rates of unemployment for Denmark over the 1973-2005 period. The NRU is computed by applying the HP filter to the actual unemployment rate series.5 The plot below aims at mimicking figure 18.2 in Phelps (1994) and figure 1 in Phelps and Zoega (1996) for the world economy, figure 4 in Holden and Nymoen (2002) for the Nordic countries, figure 2 in Batini and Greenslade (2006) for the UK, and figure 2 in Blanchard (2006) for the EU15. These figures were obtained by using the conventional

4Although the NRU and NAIRU are not synonymous, Karanassou, Sala, and Snower (2006) explain that the two concepts can be seen as the two sides of the same coin - the coin of the classical dichotomy. They also provide an overview of the various Phillips curve models and a discussion of their limitations.

5Filtering the actual series is equivalent to filtering the fitted values when the estimated model fits the data well.
approaches described above and yield a similar picture: the NRU closely tracks the actual unemployment rate.\textsuperscript{6} The NRU in figure 1 reproduces this feature.

![Figure 1. The NRU in Denmark according to the conventional view](image)

Observe that unemployment varies more between business cycles (identified by the peaks in 1978, 1983, 1993 and 2003) than within them. According to the mainstream view the changes between cycles are accounted by the "trend" component of unemployment, whereas the variations within cycles are attributed to the effects of temporary shocks. In other words, Figure 1 conforms with the conventional wisdom that unemployment evolves around its natural rate and thus the NRU can explain the large swings of the unemployment rate. As we show in Section 3, any single-equation unemployment rate model can produce a picture similar to that in Figure 1 since it has zero frictional growth (no interacting labour market lags and trendless exogenous variables).

### 2.2 Formal Definition

The natural rate of unemployment ($u^*$) is generally understood as the equilibrium value at which unemployment will stabilise in the long-run (see, for example, Ball and Mankiw, 2002). This definition is in line with the observation that the unemployment rate is trendless. When unemployment is modelled by a dynamic single equation, the natural rate is given by the steady-state unemployment rate.

\textsuperscript{6}In particular, Phelps (1994) and Phelps and Zoega (1996) apply the structuralist theory to compute the NRU, Holden and Nymoen (2002) estimate the NAWRU (non accelerating wage rate of unemployment), Batini and Greenslade (2006) use the Kalman filter to estimate the TV-NAIRU, and Blanchard (2006) constructs the NAIRU as $u^* = u + .5 (\Delta \pi)$, where $\Delta \pi$ is a 3-year moving average of the change in inflation.
For example, suppose that the unemployment rate is given by the following simple model:

\[ u_t = \alpha u_{t-1} + \gamma x_t + \varepsilon_t, \quad (1) \]

where \( x_t \) is an exogenous variable, \( \gamma \) is a constant, \( \varepsilon_t \) is a strict white noise error term (i.e. independently, identically distributed with zero mean and constant variance), and the autoregressive coefficient \( \alpha \) is less than one in absolute value.

Let us consider the following normalisation of the above equation:

\[ u_t = \frac{\gamma}{1 - \alpha} x_t - \frac{\alpha}{1 - \alpha} \Delta u_t + \frac{\varepsilon_t}{1 - \alpha}, \quad (2) \]

where \( \Delta \) denotes the difference operator.

We assume that in the long-run unemployment stabilises, so that \( \Delta u_t = 0 \) or \( u_t = u_{t-1} \), and all shocks are absorbed, so that \( \varepsilon_t = 0 \). In this case the NRU is given by

\[ u^n = \frac{\gamma}{1 - \alpha} x^{LR}, \quad (3) \]

where the superscript \( x^{LR} \) denotes the long-run value of the variable. It is commonly assumed that the exogenous variable stabilises in the long-run, and so the natural rate is simply the steady-state of the unemployment model.

In applied work, the unknown long-run value of the exogeneous variable, is replaced by its permanent component.\(^7\) We thus have the following definition.

\[ \text{→ Definition} \quad \text{The natural rate is the equilibrium unemployment rate at which there is no tendency for this rate to change at any time } t, \text{ given the permanent component values of the exogenous variables at that time.} \]

Note that the above definition applies to both single- and multi-equation models of the unemployment rate.

3 The Chain Reaction Theory of Unemployment

Like the structuralist and institutionalist theories, the chain reaction theory (CRT) aims at identifying the economic factors responsible for the evolution of the unemployment rate. But unlike the structuralist and institutionalist theories, the CRT is an interactive dynamics approach: it applies *dynamic multi-equation systems with spillover effects* to the labour market to explain the time path of unemployment. (The CRT was developed by Karanassou and Snower in 1993. See, among others, Karanassou and Snower, 1998.)

\(^7\) The permanent component of a series is usually obtained by filtering the series using the Hodrick-Prescott technique.
Since the unemployment rate is a nontrended variable, single-equation unemployment models have to use exogenous variables that do not display a trend. This is not the case when multi-equation labour market models are used - the only requirement is that each trended endogenous variable (e.g. employment, real wage, labour force) is balanced with the set of its explanatory variables.

In the context of multi-equation labour market models, changes in the unemployment rate are viewed as "chain reactions" of its responses to temporary and permanent labour market shocks. The unemployment responses work their way through a network of interacting lagged adjustment processes. These lagged adjustment processes are well documented in the literature and refer, among others, to: (i) employment adjustments arising from labour turnover costs (hiring, training and firing costs), (ii) wage/price staggering, (iii) insider membership effects, (iv) long-term unemployment effects, and (v) labour force adjustments. By identifying the various lagged adjustment processes, the CRT can explore their interactions and quantify the potential complementarities/substitutabilities among them.

In other words, the CRT postulates that the evolution of unemployment is driven by the interplay of lagged adjustment processes and the spillover effects within the labour market system. Spillover effects arise when shocks to a specific equation feed through the labour market system. The label "shocks" refers to changes in the exogenous variables.

3.1 A Simple CRT Model

We illustrate the workings of the CRT with the following model of labour supply, labour demand, and real wage equations:

\[ l_t = \alpha_2 l_{t-1} + \beta_2 z_t, \]
\[ n_t = \alpha_1 n_{t-1} + \beta_1 k_t - \gamma w_t, \]
\[ w_t = \beta_3 x_t - \delta u_t, \]

where \( l_t, n_t, \) and \( w_t \) denote the endogenous labour force, employment, and real wage, respectively; \( z_t \) is working age population, \( k_t \) is real capital stock, and \( x_t \) represents a wage push factor (e.g. benefits); the autoregressive parameters are \( 0 < \alpha_1, \alpha_2 < 1, \) and the \( \beta \)'s, \( \gamma, \) and \( \delta \) are positive constants. All variables are in logs and we ignore the error terms for ease of exposition. The unemployment rate (not in logs) is\(^8\)

\[ u_t = l_t - n_t. \]

\(^8\)Since labour force and employment are in logs, we can approximate the unemployment rate by their difference.
We should note that when either $\gamma$ or $\delta$ are zero in the toy model (4)-(6), labour market shocks do not spillover from labour supply to labour demand and vice versa. In other words, the influence of the exogenous variables ($k_t$ and $z_t$) on unemployment can be measured through individual analysis of the labour demand and supply equations. In particular, if unemployment does not influence wages ($\delta = 0$), then labour demand and supply shocks do not spillover to wages. As a result, capital stock changes do not affect labour force, and changes in working age population do not affect employment. If, on the other hand, $\gamma = 0$ shocks to wage setting do not affect employment and, consequently, do not spillover to unemployment. Thus the wage elasticity of demand provides the mechanism through which changes in the wage push factor $x_t$ feed through to unemployment. This can be seen clearly in the reduced form unemployment rate equation (13) derived below.

Let us rewrite the labour supply and demand equations (4)-(5) as

\begin{align}
(1 - \alpha_2 B) l_t &= \beta_2 z_t, \\
(1 - \alpha_1 B) n_t &= \beta_1 k_t - \gamma w_t,
\end{align}

where $B$ is the backshift operator. Substitution of (6) into (9) gives

\begin{equation}
(1 - \alpha_1 B) n_t = \beta_1 k_t - \gamma \beta_3 x_t + \gamma \delta u_t.
\end{equation}

Multiplying both sides of (8) and (10) by $(1 - \alpha_1 B)$ and $(1 - \alpha_2 B)$, respectively, gives

\begin{align}
(1 - \alpha_1 B) (1 - \alpha_2 B) l_t &= \beta_2 (1 - \alpha_1 B) z_t, \\
(1 - \alpha_1 B) (1 - \alpha_2 B) n_t &= \beta_1 (1 - \alpha_2 B) k_t - \gamma \beta_3 (1 - \alpha_2 B) x_t + \gamma \delta (1 - \alpha_2 B) u_t.
\end{align}

Finally, use the definition (7) and subtract (12) from (11) to obtain the reduced form unemployment rate equation:

\begin{equation}
(1 + \gamma \delta - \alpha_1 B) (1 - \alpha_2 B) u_t = \beta_2 (1 - \alpha_1 B) z_t - \beta_1 (1 - \alpha_2 B) k_t + \gamma \beta_3 (1 - \alpha_2 B) x_t.
\end{equation}

The term "reduced form" means that the parameters of the equation are not estimated directly - they are simply some nonlinear function of the parameters of the underlying labour market system.

\footnote{Note that (13) is dynamically stable since (i) products of polynomials in $B$ which satisfy the stability conditions are stable, and (ii) linear combinations of dynamically stable polynomials in $B$ are also stable.}
Alternatively, the reduced form unemployment rate equation (13) can be written as

\[ u_t = \phi_1 u_{t-1} - \phi_2 u_{t-2} - \theta_k k_t + \theta_x x_t + \alpha_2 \theta_k k_{t-1} - \alpha_1 \theta_z z_t - \alpha_2 \theta_x x_{t-1}, \quad (14) \]

where \( \phi_1 = \frac{a_1 + a_2(1+\delta)}{1+\gamma} \), \( \phi_2 = \frac{a_1 a_2}{1+\gamma} \), \( \theta_k = \frac{\beta_1}{1+\gamma} \), \( \theta_x = \frac{\beta_2}{1+\gamma} \), and \( \theta_z = \frac{\gamma \beta_3}{1+\gamma} \).

Parameterisations (13) and (14) of the reduced form unemployment rate equation show the following. First, the autoregressive parameters \( \phi_1 \) and \( \phi_2 \) embody the interactions of the employment and labour force adjustment processes (\( \alpha_1 \) and \( \alpha_2 \), respectively). Second, the short-run elasticities (\( \theta_k \), \( \theta_x \), and \( \theta_z \)) are a function of the feedback mechanisms that give rise to the spillover effects in the labour market system. Third, the interplay of the lagged adjustment processes and the spillover effects can be captured by the induced lag structure of the exogenous variables.

In applied work, as we discussed in Section 2, the NRU is defined as the equilibrium unemployment rate at which there is no tendency for this rate to change at any time \( t \), given the permanent component values of the exogenous variables at that time. In this sense, it represents the unemployment that would be achieved once all the lagged adjustment processes have been completed in response to the permanent components of the exogenous variables.

Therefore, the NRU is computed by setting the backshift operator \( B \) equal to unity in the unemployment rate equation (13):

\[ u^n_t = \frac{\beta_2 (1 - \alpha_1) \tilde{z}_t - \beta_1 (1 - \alpha_2) \tilde{k}_t + \gamma \beta_3 (1 - \alpha_2) \tilde{x}_t}{(1 + \gamma \delta - \alpha_1)(1 - \alpha_2)}, \quad (15) \]

where the \( \tilde{\cdot} \) above the variable denotes its permanent component. Naturally, the estimates of the NRU reflect the decision on which changes in the exogenous variables are permanent or temporary.

### 3.2 Long-Run Unemployment, NRU, and Frictional Growth

A salient feature of the CRT is that unemployment may substantially deviate from what is commonly perceived as its natural rate, even in the long-run. This was first pointed out by Karanassou and Snower (1997) and lies in sharp contrast with the conventional wisdom that the NRU is the attractor of the unemployment rate.

To elaborate this issue we use the labour market system (4)-(7) and make the plausible assumption that capital stock (\( k_t \)), the wage-push factor (\( x_t \)), and working age population (\( z_t \)) are growing variables with growth rates that stabilise in the long-run. (Note that the growth rates of log variables are proxied by their first differences, \( \Delta (\cdot) \), and recall that the superscript \( LR \) denotes the long-run value of the variable.)
Equation (7) implies that unemployment stabilises in the long-run, $\Delta u^{LR} = 0$, when

$$\Delta l^{LR} = \Delta n^{LR} = \lambda.$$  

(16)

In other words, the restriction that the growth rate of employment is equal to the growth rate of labour force, say $\lambda$, ensures unemployment stability in the long-run.$^{10}$

Let us substitute the wage (6) into labour supply (4) and labour demand (5), and rewrite the resulting equations as

$$l_t = \frac{\beta_2}{1 - \alpha_2} z_t - \frac{\alpha_2}{1 - \alpha_2} \Delta l_t,$$

(17)

$$n_t = \frac{\beta_1}{1 - \alpha_1} k_t - \frac{\gamma \beta_3}{1 - \alpha_1} x_t + \frac{\gamma \delta}{1 - \alpha_1} u_t - \frac{\alpha_1}{1 - \alpha_1} \Delta n_t$$

(18)

Substitution of the above equations into (7) and some algebraic manipulation yields the following expression for the unemployment rate:

$$u_t = \zeta \left( \frac{\beta_2}{1 - \alpha_2} z_t - \frac{\beta_1}{1 - \alpha_1} k_t + \frac{\gamma \beta_3}{1 - \alpha_1} x_t \right) + \zeta \left( \frac{\alpha_1}{1 - \alpha_1} \Delta n_t - \frac{\alpha_2}{1 - \alpha_2} \Delta l_t \right),$$

(19)

where $\zeta = \frac{1 - \alpha_1}{1 - \alpha_1 + \gamma \delta}$.

The long-run unemployment rate is obtained by imposing restriction (16) on parameterisation (19) of the reduced form unemployment rate equation:

$$u^{LR} = \zeta \left[ \frac{\beta_2}{1 - \alpha_2} z^{LR} - \frac{\beta_1}{1 - \alpha_1} k^{LR} + \frac{\gamma \beta_3}{1 - \alpha_1} x^{LR} \right] + \frac{(\alpha_1 - \alpha_2) \lambda}{(1 - \alpha_1)(1 - \alpha_2)}.$$  

(20)

Observe that the first term of (20) gives the NRU, whereas the second term of (20) captures frictional growth, i.e.,

long-run unemployment rate = NRU + frictional growth,

where frictional growth arises from the interplay between the lagged adjustment processes and the growing exogenous variables.

The long-run value ($u^{LR}$) towards which the unemployment rate converges reduces to the NRU only when frictional growth is zero. This occurs when (i) the exogenous

$^{10}$The above restriction can also be expressed in terms of the long-run growth rates of the exogenous variables:

$$\frac{\beta_1}{1 - \alpha_1} \Delta k^{LR} - \frac{\gamma \beta_3}{1 - \alpha_1} \Delta x^{LR} = \frac{\beta_2}{1 - \alpha_2} \Delta z^{LR} = \lambda.$$
variables have zero growth rates in the long-run (so that \( \lambda = 0 \)), or (ii) the labour
demand and supply equations have identical dynamic structures (so that \( \alpha_1 = \alpha_2 \)).

Therefore, frictional growth implies that under quite plausible conditions (e.g. differ-
ent labour demand and supply dynamics, and growing exogenous variables) the natural
rate is not an attractor of the moving unemployment. In these circumstances the ever
eclusive NRU is irrelevant for policy making.\(^\text{11}\)

4 A Dynamic Structural Model for Denmark

4.1 Data and estimation methodology

Our dataset is annual and covers the period 1973-2005. The OECD Economic Outlook
is our main source. Table 1 presents the group of variables used in the estimated
model.\(^\text{12}\)

<table>
<thead>
<tr>
<th>Table 1: Definitions of variables.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
</tr>
<tr>
<td>( n_t )</td>
</tr>
<tr>
<td>( l_t )</td>
</tr>
<tr>
<td>( w_t )</td>
</tr>
<tr>
<td>( u_t )</td>
</tr>
<tr>
<td>( k_t )</td>
</tr>
<tr>
<td>( r_t )</td>
</tr>
<tr>
<td>( g_t )</td>
</tr>
<tr>
<td>( z_t )</td>
</tr>
</tbody>
</table>

Source: OECD, Economic Outlook.

The estimation methodology is the autoregressive distributed lag (ARDL) approach
(also known as bounds testing approach). The ARDL was proposed by Pesaran (1997),
Pesaran and Shin (1999), and Pesaran, Shin and Smith (2001) as an alternative pro-
cedure to the standard cointegration analysis. The advantage of the ARDL is that
does not rely on whether the explanatory variables are integrated of order zero or one.
The voluminous literature on all the different types of unit root tests proposed since
the influential paper by Dickey and Fuller in Econometrica 1981, is a clear manifesta-
tion of the problems involved in correctly identifying the order of integration of a time
series. The ARDL approach avoids these pre-testing problems, while it gives consist-
tent estimates both in the short- and long-run. Thus, the ARDL, provides us with an
econometric tool to conduct our empirical analysis rigorously.

\(^{11}\) Elusive in the sense that while the NRU is a charming idea, it is most often hard to agree on its
value at any point in time. This issue has also been raised recently in The Economist, 30 September
2006, p. 108.

\(^{12}\) Our wider set of explanatory variables also included oil prices (source: IMF), financial wealth
(source: Bloomberg), several public sector variables (such as direct and indirect taxes, the fiscal wedge,
social security benefits and contributions), alternative measures of competitiveness, consumption, and
real money balances. However, we were unable to find any influence of these variables on the Danish
labour market.
In line with the CRT, we estimate a structural vector autoregressive distributed lag model to analyse the trajectory of the unemployment rate: \(^{13}\)

\[
\mathbf{A}_0 \mathbf{y}_t = \sum_{i=1}^{2} \mathbf{A}_i \mathbf{y}_{t-i} + \sum_{i=0}^{2} \mathbf{D}_i \mathbf{x}_{t-i} + \mathbf{e}_t, \tag{21}
\]

where \(\mathbf{y}_t\) is a \((3 \times 1)\) vector of endogenous variables, \(\mathbf{x}_t\) is a \((4 \times 1)\) vector of exogenous variables, the \(\mathbf{A}_i\)’s and \(\mathbf{D}_i\)’s are \((3 \times 3)\) and \((4 \times 4)\), respectively, coefficient matrices, and \(\mathbf{e}_t\) is a \((3 \times 1)\) vector of strict white noise error terms.

Our labour market system (21) comprises labour demand, wage setting, and labour supply equations. Each equation is estimated following the ARDL approach and passes the standard misspecification and structural stability tests. To account for potential endogeneity and cross equation correlation we estimate the labour market model with 3SLS.

### 4.2 Estimated equations

Using the estimated three-equations model (see Table 2 below) and the unemployment equation (7), we obtain the fitted values of unemployment. Figure 2 plots the actual and fitted values of the unemployment rate and shows that our estimation tracks the data reasonably well. We should emphasize that a good fit is much harder to obtain when dynamic multi-equation labour market models are being estimated instead of single unemployment rate equations. This is because of the numerous interactions of the endogenous variables that take place when we solve the model for the unemployment rate. Table 2 presents our estimated equations.\(^{14}\)

The labour demand equation is quite standard. Employment depends on capital stock, real wages, and public expenditures. Labour demand is more sensitive to changes in the real wage than to changes in capital stock (the long-run elasticities are -1 and 0.6, respectively). Phelps (1994, ch. 17) popularised the inclusion of public expenditures in single-equation unemployment rate models, and its strong influence on the Danish economy comes as no surprise. The public sector is responsible for the production of the vast majority of services, it accounts for almost a third of total employment, and public consumption represents around 40% of total public expenditure (see Madsen (1999)). A one percentage point increase in the ratio of public expenditures to GDP will boost employment by 1.2%, in the long-run.

\(^{13}\)The dynamic system (21) is stable if, for given values of the exogenous variables, all the roots of the determinantal equation

\[
|\mathbf{A}_0 - \mathbf{A}_1 B - \mathbf{A}_2 B^2| = 0
\]

lie outside the unit circle. Note that the estimated equations in Section 4.2 below satisfy this condition. \(^{14}\)The OLS results do not differ substantially from the 3SLS ones and are available upon request.
Furthermore, observe that the employment and wage equations display low persistence (the autoregressive coefficients are 0.18 and 0.32, respectively) indicating a quick speed of adjustment to economic disturbances. This reflects the high degree of flexibility which characterises the Danish labour market (the employment protection legislation is among the less strict in the OECD countries).

Wage setting is influenced by unemployment, capital deepening \((k_t - n_t)\), and the interest rate. As expected, unemployment exerts downward pressure on the real wage with a semi-elasticity of -0.60 in the short-run. In addition, if the unemployment rate goes up by 1 percentage point, wages fall by 0.9% in the long-run. The effect of capital deepening on wages is captured by a long-run coefficient of 0.46.\(^\text{15}\) The impact of the interest rate on wages is positive (0.56 in the long-run).\(^\text{16}\) However, since wages enter negatively in labour demand, the relation between the interest rate and unemployment has the expected negative sign.

It is important to remark that neither tax variables nor social security benefits were found to influence the wage equation. This may be due to the emphasis of the Danish system on active labour market policies (ALMPs) - Denmark is the country with the highest GDP percentage of ALMPs expenditures. When this is coupled with loose employment protection legislation, standard labour market institutions (i.e., taxes and

---

\(^{15}\) Capital deepening is regarded as a good proxy for labour productivity. The advantage of using capital deepening instead of productivity in our model is that we avoid dealing with an additional endogenous variable in our estimation.

\(^{16}\) We regard the positive association of the real wage with the interest rate as a result of the procyclicality of the two variables. In booming times, a tight labour market puts upward pressure on wages, and the monetary authorities raise interest rates to control for inflation.
benefits) become less relevant to wage setting.

Table 2: Denmark, 3SLS, 1973-2005.

<table>
<thead>
<tr>
<th>Dependent variable: $n_t$ coefficient</th>
<th>Dependent variable: $w_t$ coefficient</th>
<th>Dependent variable: $l_t$ coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ 11.6 [0.000]</td>
<td>$c$ 5.34 [0.000]</td>
<td>$c$ 1.24 [0.000]</td>
</tr>
<tr>
<td>$n_{t-1}$ 0.18 [0.000]</td>
<td>$w_{t-1}$ 0.32 [0.000]</td>
<td>$l_{t-1}$ 0.90 [0.000]</td>
</tr>
<tr>
<td>$\Delta n_{t-1}$ 0.61 [0.000]</td>
<td>$\Delta w_{t-1}$ 0.44 [0.001]</td>
<td>$\Delta l_{t-1}$ 0.76 [0.000]</td>
</tr>
<tr>
<td>$w_t$ -0.58 [0.000]</td>
<td>$u_t$ -0.60 [0.000]</td>
<td>$\Delta w_t$ -0.04 [0.032]</td>
</tr>
<tr>
<td>$w_{t-1}$ -0.30 [0.052]</td>
<td>$k_t - n_t$ 0.31 [0.000]</td>
<td>$\Delta u_{t-1}$ -0.04 [0.035]</td>
</tr>
<tr>
<td>$k_t$ 0.48 [0.000]</td>
<td>$r_t$ 0.38 [0.000]</td>
<td>$w_t$ 0.02 [0.004]</td>
</tr>
<tr>
<td>$\Delta k_t$ 1.78 [0.001]</td>
<td></td>
<td>$\Delta w_t$ -0.03 [0.035]</td>
</tr>
<tr>
<td>$\Delta k_{t-1}$ 1.14 [0.083]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_t$ 1.02 [0.001]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta g_t$ -0.89 [0.012]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta g_{t-1}$ 0.95 [0.003]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

s.e. 0.010 0.009 0.001

P-values in square brackets; $\Delta$ is the difference operator; s.e. is the standard error of the regression.

In contrast to labour demand and wage setting, inertia in labour supply decisions is large, with a persistence coefficient of 0.90. Labour supply is driven by the unemployment rate, real wage and participation rate.

In particular, it is the change rather than the level of unemployment that enters the labour force equation. This is commonly referred to as the discouraged workers’ effect, here with a long-run coefficient of -0.80. The wage incentive appears to activate labour supply with a long-run elasticity 0.20.

Finally, it is through the participation rate instead of the working-age population that we can capture demographic influences on the labour supply movements. Our understanding of this finding is that the participation rate also embodies the society’s attitude towards the labour market. In this sense, it is partly the social norms that induce participation rates to be among the highest in the Nordic countries and among the lowest in the Mediterranean ones.

5 The NRU in Denmark

Given the definition of the natural rate in section 2, we compute the NRU along the lines of equation (15). That is, we set the lagged (period $t-i$) values of the endogenous variables equal to their period $t$ values, and solve the labour market system by using only
the permanent components of the exogenous variables. Recall that our model consists of the estimated equations given in Table 2, and the unemployment rate equation (7).

5.1 Permanent and Temporary Components of the Exogenous Variables

We estimate the kernel density functions of the determinants of unemployment to disentangle their permanent and temporary components and identify the number and longevity of the regimes embedded in each variable. We should note that in this context, the term "permanent component" is not a universal concept - it only applies to our sample period.

A time series with different regimes is characterised by a multimodal density of its frequency distribution, the number of modes corresponding to the number of regimes. In particular, a unimodal kernel density indicates that a unique regime exists with mean equal to the value of the mode. On the other hand, a variable with two regimes displays a bimodal kernel density with a "valley point" dividing the observations in the sample. The data points are grouped in the two regimes depending on whether they lie to the left or to the right of the "valley point". The kernel density analysis of the two-regime case can easily be extended to account for three or more regimes.

Naturally, when the time series display one regime, this is taken to be permanent. For multimodal kernel densities we distinguish between permanent and temporary regimes and identify them as follows. The variable starts in one regime (say, A) in the beginning of the sample, and then moves to another regime (say, B) at some later point in time. If the variable reverses to regime A before the end of the sample, then regime B is temporary and regime A is permanent. On the other hand, if the variable stays in regime B by the end of the sample then both regimes are permanent ones.

The mean values of the identified permanent regimes give our estimates of the permanent components of the exogenous variables used in the computation of the NRU. It is important to note that the kernel density analysis can be carried out only when the time series is stationary. When the variable is growing (e.g. capital stock), the analysis is performed on its first difference from which we then recover the level of the variable.

The plots of the kernel density functions in the first column of Figure 3 reveal the number of regimes for each of the exogenous variables of the labour market model in Table 2. The plots in the second column of Figure 3 display the actual series (solid lines) and the mean values of their permanent regimes (dotted lines).

Bianchi and Zoega (1998) use Kernel density functions to examine the regime-mean shifts of unemployment in 15 OECD countries. Raurich, Sala and Sorolla (2006) apply the Kernel density analysis to compare the relationship of unemployment and capital accumulation in the EU and the US.
Figure 3. Actual and long-run values of the exogenous variables

a. Kernel density analysis of capital accumulation

b. Capital accumulation

Permanent regime, mean = 3.6%

Actual trajectory (solid line)
Permanent component (dotted line)

Actual trajectory (solid line)
Permanent component (dotted line)

Regime change at 30.5%

Permanent regime, mean = 27.6%

Temporary regime, mean = 30.5%

Regime change at 30.5%

c. Kernel density analysis of public expenditures
d. Public expenditures

Regime change at 30.0%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%

Regime change at 77.5%
According to Figures 3a-b, the growth rate of capital stock has been in a single regime throughout the sample with mean 3.6%. In contrast, the bimodal kernel densities of public expenditures, interest rates, and participation rates reveal the existence of two regimes (see Figures 3c, 3e, and 3g).

Public expenditures and interest rates are characterised by one permanent and one (higher) temporary regime, the duration of which is indicated by the shaded areas in Figures 3d and 3f. The temporary regime of public expenditures refers to the expansionary fiscal policy during the economic downturn at the end of the 70s and early 80s, and is well documented in the literature.\footnote{See Madsen (1999), Green-Pedersen and Lindbom (2005).}

The temporary, albeit prolonged, regime of high interest rates was induced by the contractionary monetary policy response of the central bank to (i) the high inflation rates brought by the oil price shocks of the 70s, and (ii) the rise in German interest rates by the Bundesbank to control inflation in the aftermath of the German unification. By the mid 90s Denmark, like the rest of Europe, softened its monetary policy and has since then witnessed interest rate levels similar to the ones before the oil price crises.

On the other hand, the two regimes of the participation rates are both permanent (see Figure 3h). The "low" regime with mean 76% lasts until 1982 when the participation rate enters the high regime with mean 81%.

5.2 The (ir)relevance of the NRU

As we already explained, to compute the NRU we substitute the exogenous variables by their permanent trajectories (identified in the previous section), set the lags of the endogenous variables equal to their contemporaneous values, and solve the resulting labour market model for the unemployment rate. Figure 4 plots the NRU in Denmark versus the actual unemployment rate series.

According to our analysis, the NRU in Denmark rose from values below 5% in the early 70s to a peak of 6.3% in 1987 and 1988. The subsequent period was characterised by a slow but steady decline of the NRU reaching 4.1% in 2005. In other words, the time path of the NRU has been rather flat since the 70s, never exceeding 6% or falling below 4%.

Notwithstanding the different approaches, our results are in accordance with Holden and Nymoen (2002) and Nymoen and Rødseth (2003) who show that institutional or wage-pressure factors explain only a small fraction of the variation of unemployment in the Nordic countries. Our results are also in line with Henry, Karanassou and Snower (2000) who find that the NRU in the UK was reasonably stable around 4% over the
1964-1997 period with a mild peak in the mid 80s.¹⁹

We should remark that the above Figure 4 and Figure 5 in HKS convey a very similar picture regarding the trajectory of the NRU in Denmark and the UK. This should come as no surprise since it is widely acknowledged that Denmark, unlike the other Nordic economies, shares some of the UK and US features. First, its economic downturns follow closely those experienced by the Anglo-Saxon economies: the slump in the aftermath of the first and second oil price shocks, the recession of the early 90s, and the slowdown of the early 00s. Second, Denmark, UK, and the US, are the economies with the lowest level of employment protection. In sharp contrast, Sweden, Japan and Freece are at the other end of the employment protection spectrum.

Plougmann and Madsen (2005) point out that, although the Scandinavian Model (high tax rates, a comprehensive social security system, a universal insurance benefit system and low degrees of wage and income inequality) has not changed substantially, the natural rates of unemployment in Denmark and Sweden may have even decreased over the past decades. This offers support to the chain reaction theory perspective versus the conventional belief that institutional variables (some of which are closely linked to the welfare state) are the main driving forces of the unemployment rate.

The viewpoint of the unemployment problem portrayed in Figure 4 is at odds with the conventional wisdom: the NRU does not account for the large increases in unemployment (3 percentage points in the early 80s, and 5 percentage points in the early 90s). In particular, we find that the NRU explains only 33% of the unemployment variation,²⁰ and so frictional growth accounts for the remaining 67%.

¹⁹The difference between our approach and the one in Henry, Karanassou, and Snower (2000) is that we use the kernel density function to extract the permanent components of the exogenous variables.
²⁰This is the $R^2$ obtained by regressing the fitted values of our estimated model on the NRU.
6 Conclusions

Should the NRU dictate the decisions of policy makers? The theoretical and empirical models in this paper lead to a negative answer.

We first analysed a chain reaction theory (CRT) model and showed that the unemployment rate does not gravitate towards the NRU. This is due to frictional growth, a phenomenon that encapsulates the interplay between lagged adjustment processes and growth in dynamic labour market systems.

We then chose Denmark as the focal point of our empirical analysis and found that the NRU is not the most important factor for explaining the movements of unemployment through time, since it can only explain one third of its variation. Our methodology differs from that of the conventional wisdom labour market models in two main respects: (i) we estimate a multi-equation (as opposed to a single-equation) dynamic labour market model that allows growing exogenous variables to interact with the persistence mechanism of the system, and (ii) we estimate the kernel density function of each exogenous variable to disentangle its trend and business-cycle components (as opposed to filtering the variables to extract their trend).

Our findings indicate that the preoccupation of macroeconomists with the NRU, derived from the estimation of single unemployment rate equations, serves as an end to itself and does not provide the means to understand what really matters for the evolution of unemployment.

How then, can one meaningfully address the unemployment problem? We argue that future work should estimate chain reaction theory models, and measure the unemployment contributions of the "usual suspects" (e.g. wage-push factors) along with those of growing exogenous variables (such as capital stock).

References


