

Department of Economics

Using Extraneous Information and GMM to Estimate Threshold Parameters in TAR Models

George Kapetanios

Working Paper No. 494

July 2003

ISSN 1473-0278



Queen Mary
University of London

Using extraneous information and GMM to estimate threshold parameters in TAR models

George Kapetanios*
Queen Mary, University of London

April 2003

Abstract

A prominent class of nonlinear time series models are threshold autoregressive models. Recently work by Kapetanios (2000) has shown in a Monte Carlo setting that the superconsistency property of the threshold parameter estimates does not translate to superior performance in small samples. Another issue concerning inference for the threshold parameters relates to estimation of their standard errors. As the asymptotic distribution of the threshold parameters is neither normal nor nuisance parameter free, an outstanding issue is how to obtain standard errors and confidence intervals for them. This paper aims to address these issues. In particular, we suggest that using extraneous information on the location of the threshold parameters may lead to better estimates. The extraneous information comes in the form of moment conditions that relate residuals of standard threshold models to shocks driving other variables. Additionally the paper considers the problem of estimating standard errors and confidence intervals for threshold parameter estimates. We suggest use of the bootstrap for this problem.

Keywords: Threshold Models, GMM, Bootstrap

JEL Codes: C13, C22

*Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS. email: G.Kapetanios@qmul.ac.uk

1 Introduction

The investigation of nonlinearity in macroeconomics has been receiving increasing attention in the literature recently. As a result interest in the theoretical properties of nonlinear time series models has been increasing. A prominent class of nonlinear time series models are threshold autoregressive models. The main distinguishing characteristic of such models is the abrupt change in the dynamics of the process as it moves through regions of the state space. The parameters regulating these changes are the threshold parameters. The asymptotic properties of the threshold parameters estimated through least squares methods are non-standard. In particular, Chan (1993) has shown that these estimates are superconsistent and have non-standard asymptotic distributions. However, recently work by Kapetanios (2000) has shown in a Monte Carlo setting that the superconsistency property of the threshold parameter estimates does not translate to superior performance in small samples. On the contrary, it seems that due to peculiarities in the shape of the conditional least squares function the threshold parameter estimates suffer from biases and large variances for a variety of threshold models.

Another issue concerning inference for the threshold parameters relates to estimation of their standard errors. As the asymptotic distribution of the threshold parameters is neither normal nor nuisance parameter free, an outstanding issue is how to obtain standard errors and confidence intervals for them. One possibility for solving this problem is provided by Hansen (2001) as discussed in the text.

This paper aims to address these issues. In particular, we suggest that using extraneous information on the location of the threshold parameters may lead to better estimates. The extraneous information comes in the form of moment conditions that relate residuals of standard threshold models to shocks driving other variables. Perhaps surprisingly, we find that rather than including such variables in the model, considering them only for the estimation of the threshold parameters may lead to estimates with superior properties. The method we suggest may be interpreted as a particular instrumental variable estimator for the threshold parameters. We prove that this estimator is consistent. We also show that in a majority of the cases we consider in an extensive Monte Carlo the new estimation method works better than the standard one or the one where the extra variables are added to the model equation. Our results suggest that using distinct information sets for estimating coefficients and threshold parameters may improve the performance of the threshold parameter estimator.

Additionally the paper considers the problem of estimating standard errors and confidence intervals for threshold parameter estimates. We suggest use of the bootstrap for this problem. However, it is well known that discontinuities may lead to a breakdown of standard bootstrap techniques. An alternative bootstrap method based on subsampling is considered and compared to the standard bootstrap through Monte Carlo simulations. We find no evidence that the subsample bootstrap performs better.

The paper is structured as follows: Section 2 provides a description of the TAR model. Section 3 describes the new estimation method we suggest. Section 4 discusses the estimation of the standard errors and by implication, confidence intervals. Section 5 presents the setup and results of the Monte Carlo simulations. Finally, Section 6 concludes. The appendix contains the proof of the consistency of the new estimator.

2 Preliminaries

We focus on the following $TAR(m, p)$ threshold autoregressive model:

$$y_t = \sum_{i=1}^p \sum_{j=1}^m \alpha_{i,j} I_{\{r_{j-1} \leq x_t < r_j\}} y_{t-i} + \epsilon_t \quad (1)$$

where $r_0 = -\infty$ and $r_m = \infty$. The model has m regimes. In each of these regimes the autoregressive coefficients may take different values. The transition between regimes depends on x_t . In what follows we will assume that $x_t = y_{t-d}$, i.e. the transition depends on a lag of the process itself, controlled by the delay parameter d . These models are referred to as self-exciting threshold autoregressive (SETAR) models.

This model is estimated by least squares. More specifically, a grid of possible values for the vector of the threshold parameters is constructed, usually as a function of the observed values taken by the process. For example, selected quantiles of the process may be used. For each point in this grid, variables of the form $I_{\{r_{j-1} \leq y_{t-d} < r_j\}} y_{t-i}$ are constructed. These are then used to estimate the coefficients of the model for that particular grid point. The conditional sum of squares is calculated for each grid point and the final estimate for the threshold parameters is designated to be that point for which the conditional sum of squares is minimised. The respective coefficient estimates are then adopted as the final coefficient estimates.

The asymptotic properties of these estimates are as follows: The threshold parameters are T^{-1} -consistent (superconsistent) and, when normalised around the true parameters, have a nonstandard asymptotic distribution which is a functional of a compound Poisson process and involves nuisance parameters. The coefficient estimates have the usual $T^{1/2}$ rate of convergence and are asymptotically normal. In particular, their asymptotic distribution does not depend on whether the threshold parameters are known or estimated. More details may be found in Chan (1993). Because of the nuisance parameters involved in the asymptotic distribution of the threshold parameter estimates an alternative asymptotic theory has been developed. In particular, under the assumption that

$$\alpha_{i,j_1} - \alpha_{i,j_2} = cT^{-a}, \quad 0 < a < 1/2, \quad c \neq 0 \quad i = 1, \dots, p \quad (2)$$

Hansen (2001) has shown that the, appropriately normalised, threshold parameter estimate has a nonstandard asymptotic distribution which is free of nuisance parameters. We note that these results have been rigorously proven only for the case $m = 2$.

Abstracting from the asymptotic analysis, Monte Carlo evidence (see Kapetanios (2000)) suggests that even though the threshold parameters are superconsistent, their finite sample properties include large biases. So alternative estimation methods may be useful.

3 A GMM estimation method for the threshold parameter estimates

3.1 The new method

The starting point of our analysis is the realisation that moment conditions involving the threshold parameters may be a useful generalisation of the conditional least squares as a loss function for estimating the thresholds. We note the well known fact that the least squares estimator for the coefficients of a regression involving the vector of regressors \mathbf{x}_t is equivalent to the GMM estimator with moment conditions $E(\mathbf{x}_t u_t) = 0$ where u_t is the regression error term. In the case of the threshold autoregressive model the regressors are functions of the threshold parameters r_i . However, as the sample mean of $x_t \hat{u}_t$, where \hat{u}_t is the regression residual, is identically zero for any value of r_i , and therefore constant as a function of r_i , minimisation of the conditional least squares is used instead to give the threshold estimates in the standard

estimation method. Nevertheless, alternative moment conditions will have non zero sample means and these can be used as a basis for estimating the thresholds. We will concentrate on moment conditions of the form

$$E(z_{j,t}(r)u_t) = E(z_t I_{\{r_{j-1} \leq y_{t-d} < r_j\}} u_t) = 0 \quad (3)$$

where $r = (r_1, \dots, r_{m-1})'$.

We need to comment on the motivation of these moment conditions. What they are basically saying is that there exist variables which are correlated with the error term driving the process in some regimes but not others. A large number of economically meaningful examples can be easily cited. For example in the context of target zones for exchange rates variables such as, e.g., central bank reserves may be uncorrelated with the shocks of the exchange rate process in periods where the exchange rate is within its target zone but may start to be correlated with the shocks when the exchange rate approaches the bounds of the target zone. In business cycle modelling labour market tightness indicators may be correlated with shocks driving GDP in periods of strong expansion or recession but not in other times.

As we will see in the theoretical analysis of our method it is crucial that the moment conditions identify all the regimes. As a minimum there must be as many moment conditions as regimes. Otherwise, the estimator is not consistent. Of course, other forms of moment conditions are possible but for simplicity we will concentrate on these. It is easy to see that the setup we suggest is observationally equivalent to one where the variables z_t are included in the threshold model. One may therefore suggest that the model is augmented with these variables and reestimated using the standard method. This argument is valid but can be countered in principle even before the performance of the two alternatives is examined in a Monte Carlo study. When the z_t variables are included in the model a number of extra parameters (the coefficients of these variables) need to be estimated. These estimates may induce further variability and possibly small sample biases in the threshold parameter estimates through their effect on the residuals. Of course, excluding them from the model, will lead to biases for the coefficient estimates, as long as these variables are correlated with lags of y_t . We assume that the correlation between z_t and u_t , at particular points in time, arises out of the correlation between u_t and the shocks driving z_t .

This assumption is not far fetched and is defensible economically. In most economic examples, including those mentioned above, it is reasonable

to assume that only the serially uncorrelated shocks driving the z_t and the y_t processes are contemporaneously correlated. In any case if the correlation arises out of some other route the standard estimation method will suffer from biases as well. Under this assumption, the covariance matrix of the coefficient estimates from the model augmented with z_t will be block diagonal. But this still implies increased variability for the residuals and possibly inferior properties for the resulting threshold parameter estimates.

3.2 Theoretical Properties

Define the loss function concentrated with respect to the autoregressive coefficients as¹

$$L(\mathbf{y}; r) = \bar{\mathbf{m}}(\mathbf{y}; r)' \bar{\mathbf{m}}(\mathbf{y}; r) \quad (4)$$

where

$$\bar{\mathbf{m}}(\mathbf{y}; r)_j = 1/T \sum_{i \in \Gamma_j(r)} z_{j,t} \hat{u}_t(r) \quad (5)$$

$\Gamma_j(r)$ denotes the set of observations for which the j -th moment condition is specified to hold and $\hat{u}_t = y_t - x_t' \hat{\beta}(r)$. We assume that the moment conditions hold i.e. let $E_{j,t}(r)$ denote the event that at time t the j -th moment condition holds, then we assume that

$$E(z_{j,t} u_t | E_{j,t}(r^0)) = 0 \quad (6)$$

We also assume that for all j

$$E(z_{j,t} u_t | \overline{E_{j,t}(r^0)}) \neq 0 \quad (7)$$

This condition simply says that the instruments contain information about the location of the threshold parameters. We show the following Theorem in the Appendix

Theorem 1 *The estimator of the threshold parameter \hat{r} defined by the minimisation of the objective function $L(\mathbf{y}; r)$ is consistent*

¹In the Monte Carlo section we will consider a loss function of the form $\bar{\mathbf{m}}(\mathbf{y}; r)' \mathbf{W} \bar{\mathbf{m}}(\mathbf{y}; r)$ where the weighting matrix \mathbf{W} will be optimally chosen using standard GMM theory. But, for the time being we discuss the theoretical properties of the new method using this simpler objective function.

4 Bootstrapping the distribution of the threshold parameter

Given the results of Chan (1993), on the asymptotic distribution of the normalised threshold parameter estimate indicating that this distribution depends on nuisance parameters and is nonstandard it is clear that a simple inferential procedure for the threshold parameter estimate is not available unless one is willing to make further assumptions about the model such as those suggested by Hansen (2001).

An alternative approach for constructing standard errors and confidence intervals may be based on the bootstrap. The bootstrap relies on parametrically or non-parametrically resampling from the available data. The standard bootstrap procedure is in general able to provide an estimate (of the parameters) of the exact distribution of an estimator. Under mild assumptions this estimator is consistent. Further, under the assumption of asymptotic pivotalness (independence of the asymptotic distribution from nuisance parameters) the bootstrap estimator may converge more quickly to the true distribution compared to the asymptotic approximation. However, in the case of the standard estimator of the threshold parameter, the asymptotic distribution is not asymptotically pivotal. Further, it is not even clear whether the standard bootstrap estimator is consistent in this case, as Coakley and Fuertes (2002) claim. To explain in detail why consistency is in doubt, we denote the distribution of the threshold parameter estimate, \hat{r}_T , obtained by minimising the conditional sum of squares, by $\mathcal{L}_T(\hat{r}, F_T)$ where F_T denotes the joint distribution function of the sample y_1, \dots, y_T . We denote a generic parameter of $\mathcal{L}_T(\hat{r}, F_T)$ by $\theta_T(\hat{r}, F_T)$. For example, θ_T could be the variance of \hat{r}_T or the 95% quantile of its distribution. The crucial regularity condition for consistency of the standard bootstrap approach is the continuity of the mapping $F_T \rightarrow \theta_T$. To appreciate the difficulty of showing this for the threshold models we note that the asymptotic distribution of $T(\hat{r}_T - r^0)$ is given by that of the lower bound, M_- , of a random interval where a functional of two independent compound Poisson processes is minimised almost surely. This continuity condition is not necessary for consistency of the bootstrap but most theorems available on this subject assume it.

Although, we cannot prove it without actually deriving the distribution of M_- , which is a non trivial task, we conjecture that the mapping $F_\infty \rightarrow \theta_\infty$ is indeed continuous, for usually encountered statistics θ . We support our conjecture with the following points. The distribution of M_- depends on

r^0 via the following routes: (i) the rate of the two independent Compound Poisson processes and (ii) the definition of the independent increments of the compound Poisson processes. The parameters of the distribution of the compound Poisson process are continuous with respect to its rate. Secondly, we conjecture that the moments of the independent increments of the compound Poisson processes are continuous in r^0 . A simple example suggests why that might be the case. Consider the moments of the simple TAR process

$$y_t = ay_{t-1}I_{\{y_{t-1} > r^0\}} + \epsilon_t \quad (8)$$

started at $t = 0$, at $t = 1$. Then $y_1 = a\epsilon_0 I_{\{\epsilon_0 > r^0\}} + \epsilon_1$ and

$$E(y_1^m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (a\epsilon_0 I_{\{\epsilon_0 > r^0\}} + \epsilon_1)^m f(\epsilon_0) f(\epsilon_1) d\epsilon_1 d\epsilon_0 \quad (9)$$

Clearly, changes in r^0 affect only the indicator function and thereby only change the bounds of the integral with respect to ϵ_0 . This implies that the moments are continuous w.r.t. r^0 . Similar arguments can be used to give the result that the distribution of y_1 is continuous w.r.t r^0 . To see note that we need to derive $Prob(a\epsilon_0 I_{\{\epsilon_0 > r^0\}} + \epsilon_1 < x)$ which is given by

$$\begin{aligned} & \int_{-\infty}^{\infty} \int_{-\infty}^{x - a\epsilon_0 I_{\{\epsilon_0 > r^0\}}} f(\epsilon_1) f(\epsilon_0) d\epsilon_1 d\epsilon_0 = \\ & \int_{-\infty}^{r^0} \int_{-\infty}^x f(\epsilon_1) f(\epsilon_0) d\epsilon_1 d\epsilon_0 + \int_{r^0}^{\infty} \int_{-\infty}^{x - a\epsilon_0} f(\epsilon_1) f(\epsilon_0) d\epsilon_1 d\epsilon_0 \end{aligned} \quad (10)$$

The independent increments of the compound Poisson process are functions of the observations and the threshold parameters but similar arguments as those given above could be used to show the continuity of moments of these functions w.r.t. r^0 .

An alternative bootstrap approach, that is guaranteed to be consistent under the most mild of regularity conditions, is based on subsampling. More specifically, whereas for the standard bootstrap samples of size T , i.e. equal to the size of the original sample, are resampled, subsampling bootstrap methods suggest resampling samples of size T_m where $T_m \ll T$ and in particular, $T_m \rightarrow \infty$ and $T_m/T \rightarrow 0$ as $T \rightarrow \infty$. These methods are guaranteed to be consistent if the distribution of θ_T converges to a non-degenerate limit. No continuity conditions are needed. We will use the non-replacement algorithm for dependent data suggested by Politis and Romano (1994). The general non-replacement algorithm is also discussed in Bickel, Gotze, and

van Zwet (1997). In the case of dependent data a strong mixing condition is also needed. This is satisfied for TAR models as discussed in Hansen (2001) using results of Chan (1993). The algorithm works as follows: A choice of subsample size T_m is made. Then all possible blocks of consecutive data $\{y_i, y_{i+1}, \dots, y_{i+T_m-1}\}$, $i = 1, \dots, T - T_m + 1$ constitute the set of the bootstrap samples. As we see no subsample is sampled twice. The choice of the subsample size T_m reflects issues relating to the choice of the fixed block size in the standard bootstrap fixed block resampling for dependent data.

Note that in the above discussion we implicitly concentrate on the non-parametric bootstrap. This may at first seem at odds with the fact that we assume a parametric model of the TAR form. Nevertheless, the crucial innovation of the paper is the consideration of extraneous information through extra variables. At this stage we do not assume a parametric model for these variables. In fact, as we mention in the previous section there is a possibility that including those variables in the regression model rather than using them through GMM maybe counter-productive as far as estimating the threshold parameters. It is clear that we should make the same assumptions when constructing the bootstrap samples as when estimating the model. Further, as the z variables do not enter the regression model we cannot use a nonparametric bootstrap for them and a parametric one for y_t . The only alternative left is the nonparametric bootstrap both for z_t and for y_t . Nevertheless, if one is willing to make further assumptions about z_t in the form of a parametric model then that model should be used for the construction of the bootstrap samples. However, if such assumptions are made then it is clear that they should be used in the estimation of the threshold parameters and thus inevitably lead to a multivariate model. It is not clear whether such a model will improve the properties of the threshold parameter estimates.

5 Monte Carlo Study

5.1 Monte Carlo Setup

The new method we suggest is mainly concerned with improving the properties of the small sample threshold parameter estimates. A Monte Carlo evaluation of the new method compared with the existing method is therefore of paramount importance. We will consider three different methods of estimating the threshold parameters. The first, denoted as method A, is the new GMM based method. We minimise a weighted sum of squares of

the moment conditions using the standard GMM optimal weighting matrix which is the inverse of the second moment of the moment conditions. The second method, denoted as method B, estimates the following model

$$y_t = \sum_{i=1}^p \sum_{j=1}^m \alpha_{i,j} I_{\{r_{j-1} \leq y_{t-d} < r_j\}} y_{t-i} + \sum_{j=1}^m \sum_{s=1}^{m_z} \beta_{j,s} z_{s,t} I_{\{r_{j-1} \leq y_{t-d} < r_j\}} + \epsilon_t \quad (11)$$

Here, we incorporate the information that the z contain about the threshold parameters into the model for y_t . However, as mentioned before the data will then have to provide information about the parameters $\beta_{j,s}$ which, in the current context, are nuisance parameters. Finally, the third method, denoted method C, is the standard conditional sum of squares estimation method for TAR models. The model we use for generating the data is of the form

$$y_t = \sum_{j=1}^m \alpha_j I_{\{r_{j-1} \leq y_{t-1} < r_j\}} y_{t-1} + \epsilon_t, \quad m = 2, 3 \quad (12)$$

The variables $z_{j,t}$ are generated by

$$z_{j,t} = \delta z_{j,t-1} + u_{j,t} \quad (13)$$

where $(\epsilon_t, u_{j,t})$ is an i.i.d. process with contemporaneous covariance matrix $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ if regime j holds at time t and the identity matrix otherwise. $\rho = \delta = 0.5$ throughout the analysis. The parameter ρ introduces the dependence between y_t and z_t .

We consider two different TAR models. The first is a two regime TAR model and the second is a symmetric three regime model where the lower and the upper regime have the same autoregressive coefficient and $r_1 = -r_2$. For the purposes of choosing the moment conditions for the upper and lower regime we assume, and construct the samples so, that the same moment conditions hold in these regimes. As a result both models are characterised by two autoregressive parameters and one threshold parameter. Previous work on the properties of the standard threshold parameter estimator by Kapetanios (2000) has suggested that two crucial dimensions with respect to which the performance of the estimator varies are (i) the autoregressive structure of the TAR(1,2) model and (ii) the value of the threshold parameter in the TAR(1,2) model. Due to the complexity associated with increasing m or p it is difficult to disentangle the effects of different parameter variations in such more complex models. The combinations of autoregressive parameters we consider, are given in Table 1. We also consider

Table 1: True autoregressive coefficients for Monte Carlo experiments

α_1	α_2
-0.6	0.6
-0.5	-0.2
0.5	0.2
-0.8	-0.5
-0.3	-0.1
0.8	0.6
0.3	0.1

the following values for the threshold parameter in the two regime model: $r = \{-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2\}$ and in the three regime symmetric model: $r_2 = \{0.2, 0.5, 1, 1.5, 2\}$. We present the mean, variance and proportion of observations belonging to the lower regime (the lower and upper regime in the case of the three regime model) in Tables 2 and 3. These numbers have been obtained by simulating a single sample of 5000 observations and calculating the relevant statistics. Note that in all cases the TAR models are stationary. All errors are pseudo-normal random variables. We concentrate on samples of sizes $T = 50, 100, 150$. Initial conditions are set to zero and 10 initial observations are dropped from each sample to minimise the effect of this choice of initial conditions.

We construct the grid of candidate threshold parameters as follows: For the two regime model we select $[T/5]$ equally spaced quantiles between the 10% and 90% quantile of the data. For the symmetric regime we construct the grid for r_2 by selecting $[T/5]$ equally spaced quantiles between the 55% and 90% quantile of the demeaned data and use $r_1 = -r_2$. These choices follows in general the literature. Further, as the grids are the same for every estimation method these choices should not, ex ante, bias any of our conclusions. Note that in some cases the minimum proportion of observations in some regime possible with the choice of grid we have made is higher than the true proportion as presented in Tables 2 and 3. For some sample sizes it is not possible to consider such grid points because we don't have enough observations. Nevertheless, examining the behaviour of the estimators for such parameter values is of interest given that many economic phenomena such as, e.g., business cycles where recessions (defined as a regime) are rare, resemble such threshold models.

For the standard bootstrap we use $R^b = 99$ replications and a block size

Table 2: Summary statistics of two regime TAR models

		r^0								
α_1	α_2	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
		Mean								
-0.6	0.6	0.188	0.318	0.482	0.580	0.610	0.545	0.430	0.255	0.155
-0.5	-0.2	0.017	0.038	0.071	0.094	0.096	0.084	0.068	0.040	-0.001
0.5	0.2	-0.045	-0.040	-0.158	-0.174	-0.184	-0.181	-0.170	-0.124	-0.064
-0.8	-0.5	0.023	0.032	0.086	0.081	0.105	0.077	0.088	0.049	0.039
-0.3	-0.1	0.015	0.030	0.040	0.031	0.079	0.070	0.037	0.024	0.009
0.8	0.6	-0.161	-0.204	-0.267	-0.400	-0.403	-0.311	-0.323	-0.295	-0.219
0.3	0.1	-0.033	-0.027	-0.051	-0.100	-0.129	-0.103	-0.054	-0.025	-0.050
		Variance								
-0.6	0.6	1.529	1.409	1.291	1.288	1.214	1.216	1.429	1.533	1.604
-0.5	-0.2	1.072	1.104	1.155	1.150	1.127	1.126	1.166	1.237	1.276
0.5	0.2	1.129	1.141	1.156	1.148	1.156	1.136	1.248	1.208	1.289
-0.8	-0.5	1.456	1.566	1.653	1.714	1.707	1.786	1.778	1.859	2.056
-0.3	-0.1	1.048	1.015	1.007	1.088	1.033	1.040	1.062	1.066	1.094
0.8	0.6	1.871	2.093	2.058	2.100	2.155	2.114	2.126	2.125	2.314
0.3	0.1	1.048	1.034	1.032	1.021	1.047	1.049	1.060	1.066	1.048
		Proportion in Lower Regime								
-0.6	0.6	0.033	0.057	0.093	0.173	0.293	0.486	0.682	0.842	0.926
-0.5	-0.2	0.027	0.067	0.166	0.296	0.473	0.652	0.802	0.905	0.960
0.5	0.2	0.032	0.084	0.212	0.376	0.575	0.732	0.853	0.927	0.971
-0.8	-0.5	0.044	0.104	0.199	0.336	0.484	0.626	0.760	0.851	0.908
-0.3	-0.1	0.024	0.063	0.149	0.302	0.463	0.667	0.824	0.921	0.973
0.8	0.6	0.088	0.175	0.290	0.464	0.601	0.704	0.812	0.892	0.932
0.3	0.1	0.027	0.072	0.170	0.342	0.555	0.720	0.841	0.933	0.978

Table 3: Summary statistics of symmetric three regime TAR models

		r_2^0				
α_1	α_2	2	1.5	1	0.5	0.2
		Mean				
-0.6	0.6	0.017	0.002	0.023	0.012	0.005
-0.5	-0.2	-0.004	0.023	-0.012	0.014	-0.002
0.5	0.2	-0.024	0.035	-0.001	0.012	-0.029
-0.8	-0.5	0.004	0.004	0.004	0.010	-0.001
-0.3	-0.1	0.018	-0.024	-0.008	0.024	0.011
0.8	0.6	0.033	-0.024	-0.025	0.061	0.080
0.3	0.1	-0.014	-0.021	-0.039	-0.011	-0.033
		Variance				
-0.6	0.6	1.594	1.487	1.519	1.494	1.595
-0.5	-0.2	1.132	1.185	1.261	1.297	1.319
0.5	0.2	1.135	1.205	1.227	1.304	1.300
-0.8	-0.5	1.908	2.336	2.755	2.737	2.883
-0.3	-0.1	1.050	1.034	1.096	1.105	1.102
0.8	0.6	2.084	2.370	2.641	2.795	2.733
0.3	0.1	1.006	1.031	1.069	1.108	1.109
		Proportion in Upper and Lower Regimes				
-0.6	0.6	0.108	0.220	0.415	0.688	0.877
-0.5	-0.2	0.062	0.166	0.368	0.654	0.858
0.5	0.2	0.064	0.167	0.363	0.676	0.856
-0.8	-0.5	0.130	0.306	0.544	0.754	0.908
-0.3	-0.1	0.051	0.138	0.336	0.633	0.845
0.8	0.6	0.160	0.318	0.516	0.763	0.896
0.3	0.1	0.045	0.141	0.339	0.632	0.849

of 10. Although the number of replications may appear low, it is comparable with the fixed number of replications of the subsample bootstrap for our sample sizes. For the subsample bootstrap the number of replications is fixed by the block size, b and is given by $T - b + 1$. We choose $b = \lceil T^{4/5} \rceil$. This choice seems to be a good compromise between number of replications and subsample size. For every experiment, $R = 500$ replications are carried out. Given the computational cost of a Monte Carlo study of nonlinear model estimators and the properties of bootstrap variances and confidence intervals, this number seems reasonable.

5.2 Monte Carlo Results

We present results for the following properties of the estimator for each method: (i) the estimated MSE and variance of the threshold parameter estimate. This is calculated as

$$MSE(\hat{r}) = 1/R \sum_{i=1}^R (\hat{r}^i - r^0)^2 \quad (14)$$

(ii) the estimated MSE of the bootstrap estimates of the variance of the threshold parameter estimator and (iii) the estimated MSE of the bootstrap estimates of the symmetric 95% confidence interval for the threshold parameter estimator. For the last two cases, the MSE is calculated as

$$MSE(\hat{T}(\hat{r})) = 1/R \sum_{i=1}^R (\hat{T}^i(\hat{r}) - T(r^0))^2 \quad (15)$$

where T denotes the statistic of interest (variance or quantile), \hat{T} denotes its estimate and $T(r^0)$ denotes the true value of T obtained via simulation over the R Monte Carlo replications. Results are presented in Tables 4-12 for the two regime TAR model and in Tables 13-21 for the three regime symmetric TAR model.

Results make interesting reading. We start by examining the two regime TAR model. We clearly see that Method A dominates in terms of performance all other methods. Out of 189 experiments in total, it has the minimum MSE in 132 cases. It has the minimum absolute bias in 167 cases. Finally, it has the minimum variance in 126 cases. Method C is never the best for any performance measure. Comparing methods A and B we see that method B works better for threshold parameter which are close to zero. In other words it works best for models with regimes which occur with roughly

similar probabilities. More specifically it works best only for $r^0 = 0$ with similar performance for $r^0 = 0.5, -0.5$. For all other values of r^0 , method A is overwhelmingly preferred. Even in the case of $r^0 = 0$ methods A and B have similar and very low biases. What distinguishes them is the variance of the estimator.

Moving on to the three regime TAR model, we see that again similar patterns emerge. Method A overwhelmingly dominates the other methods. Again Method C is never best. Method A has the lowest MSE in 72 out of 105 cases, the lower absolute bias in 92 cases and the lower variance in 65 cases. Method B works relatively better for cases where the three regimes have roughly equal probabilities of occurring (i.e. $r_2^0 = 1$). Overall, we must conclude that our conjecture in the previous section was right for this Monte Carlo design. Method A does not require of the data to provide estimates of the parameters of z_t in the model for y_t . Providing estimates for such parameters is clearly detrimental for threshold parameter estimation.

Moving on to the investigation of the bootstrap estimates of variance and quantiles of the threshold parameter estimates, we see that the standard bootstrap clearly outperforms the subsample bootstrap. In most cases, both for variances and for confidence intervals, the standard bootstrap estimates have lower MSEs than the subsample bootstrap estimates. More specifically, for the two regime TAR model the standard (subsample) bootstrap has lower MSE for the variance estimates in 426 (141) cases. The relevant numbers for the confidence intervals are 1001 (133). For the three regime symmetric TAR model we have that the standard (subsample) bootstrap estimate for the variance has lower MSE in 244 (71) cases whereas for the confidence intervals the relevant numbers are 562 (68). It is clear that either the subsample bootstrap is consistent whereas the standard bootstrap is not but this asymptotic result is swamped by the small sample inefficiency of the subsample bootstrap or that both bootstraps are consistent but the subsample bootstrap is inefficient. In either case it seems reasonable to suggest that the standard bootstrap be used. However, note that all these results are for the nonparametric bootstrap for the reasons explained in the previous section. These results could possibly be reversed if the parametric bootstrap were considered.

6 Conclusion

In this paper we have considered the problem of estimating the threshold parameter of TAR models using extraneous information via a GMM method. Previous work has shown that the threshold parameter estimator although superconsistent has poor sample properties. We find that, as expected, the GMM estimator improves upon the standard estimator when valid extraneous information is used. More surprisingly we find that including that information in the regression model as extra regressors may be counter productive. The best way to incorporate such information about the threshold parameter is via a GMM type set of moment conditions whose quadratic form is to be minimised in a second stage following estimation of the coefficients of the TAR model.

A further contribution of the paper is the introduction of the bootstrap to carry out inference about the threshold parameter estimate. As the standard bootstrap may be inconsistent for TAR models due to the discontinuity of the estimation objective function we suggest as an alternative a subsample bootstrap which is consistent under a very mild regularity condition and does not require continuity conditions of any form. However this bootstrap is less efficient than the standard bootstrap if the standard regularity conditions hold. We see that the standard bootstrap performs much better than the subsample bootstrap in small samples. As a result we conclude that even if the standard bootstrap is inconsistent (which we conjecture not to be the case using a heuristic argument) one may be better off, in terms of estimator MSEs, using it in small samples anyway.

Further research could concentrate on the following: (i) investigate formally the theoretical properties of the bootstrap in TAR models. We have made some arguments which could provide an avenue for further research. (ii) investigate whether the good properties of the newly proposed method extend to more complex TAR model with longer lags and more regimes. We conjecture that the superior performance of the new method may be diluted when models with more than 3 regimes are considered because of the need for many extraneous variables for the method to be consistent. Nevertheless more research is needed.

Appendix: Proof of Theorem 1

We wish to show that asymptotically

$$L(\mathbf{y}; r^0) < L(\mathbf{y}; r), \quad \forall r \quad (16)$$

in probability. To do that we will show that

$$(i) \ L(\mathbf{y}; r^0) \xrightarrow{p} 0, \text{ and}$$

$$(ii) \ L(\mathbf{y}; r) - L(\mathbf{y}; r^0) \xrightarrow{p} c(r) > 0 \ \forall r.$$

To show (i) it suffices to show that $\bar{m}(\mathbf{y}; r^0) \xrightarrow{p} 0$. This is straightforward to show by the Law of Large Numbers for strong mixing processes such as TAR processes (see Davidson (1994)). We now wish to show (ii). Using (i) we only need to show that

$$\|\bar{m}(\mathbf{y}; r^0) - \bar{m}(\mathbf{y}; r)\| \xrightarrow{p} c(r) > 0 \quad (17)$$

But

$$\|\bar{m}(\mathbf{y}; r^0) - \bar{m}(\mathbf{y}; r)\| = \left| 1/T \sum_{j=1}^m \sum_{t \in \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) - 1/T \sum_{j=1}^m \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r) \right| = \quad (18)$$

$$\left| \sum_{j=1}^m 1/T \sum_{t \in \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r) \right| \quad (19)$$

We examine the typical term of the above sum. We have

$$1/T \sum_{t \in \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r) = \left(1/T \sum_{t \in \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r^0) \right) + \quad (20)$$

$$\left(1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r) \right)$$

Concentrating on the first term of the above sum we have that if $\Gamma_j(r) \subset \Gamma_j(r^0)$ then both terms in the absolute value will tend to zero in probability. Although this can hold for some j it cannot hold for all j . Therefore, there exists some j for which $\Gamma_j(r^0) \subset \Gamma_j(r)$. As a result

$$1/T \sum_{t \in \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r^0) = 1/T \sum_{t \in \Gamma_j(r) \setminus \Gamma_j(r^0)} z'_t \hat{u}_t(r^0) \quad (21)$$

But this tends to a non-zero number since $E(z_{j,t}u_t|\overline{E_{j,t}(r^0)}) \neq 0$. Moving to the second term of the sum in (20) we have that

$$\begin{aligned} 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r^0) - 1/T \sum_{t \in \Gamma_j(r)} z'_t \hat{u}_t(r) &= 1/T \sum_{t \in \Gamma_j(r)} z'_t (\hat{u}_t(r^0) - \hat{u}_t(r)) = (22) \\ 1/T \sum_{t \in \Gamma_j(r)} z'_t (u_t + (x_t \hat{\beta}(r^0) - \hat{\beta}(r))) &= 1/T \sum_{t \in \Gamma_j(r)} z'_t u_t + 1/T \sum_{t \in \Gamma_j(r)} z'_t x_t (\hat{\beta}(r^0) - \hat{\beta}(r)) \end{aligned}$$

But the first term of this will tend to a nonzero number as long $\Gamma_j(r^0) \subset \Gamma_j(r)$ which will hold for some j . The above is not enough for consistency of the estimator. We also need stochastic equicontinuity of $\bar{m}(\mathbf{y}; r)$. This is defined to be that

$$\sup_{r \in R} \sup_{r' \in S(r, \delta)} |\bar{m}(\mathbf{y}; r) - \bar{m}(\mathbf{y}; r')| = 0 \quad (23)$$

in probability, for some $\delta > 0$ where R is a compact subset of the relevant Cartesian product of \mathbb{R} and $S(r, \delta)$ is an open ball of radius δ centered on r . Assuming that all of z_t , y_t and u_t are strong mixing processes with appropriate strong mixing coefficients gives the required result. To see this note that we can prove that the coefficient estimates are stochastically equicontinuous functions of r following Lemma 1 of Hansen (1996) and Lemma A.3 of Hansen (2001). From that the stochastic equicontinuity of our objective function follows using the strong mixing assumption for z_t .

References

- BICKEL, P. J., F. GOTZE, AND W. R. VAN ZWET (1997): “Resampling Fewer than n observations: Gains, Losses and Remedies for Losses,” *Statistica Sinica*, 7(1), 1–31.
- CHAN, K. S. (1993): “Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model,” *The Annals of Statistics*, 21(1), 520–533.
- COAKLEY, J., AND A. FUERTES (2002): “Bootstrap LR Tests of Sign and Amplitude Asymmetry,” *Birkbeck College Discussion Paper 4/2000*.
- DAVIDSON, J. (1994): *Stochastic Limit Theory*. Oxford University Press.
- HANSEN, B. (1996): “Inference when a Nuisance Parameter is not Identified Under the Null Hypothesis,” *Econometrica*, 64(2), 413–431.
- (2001): “Sample Splitting and Threshold Estimation,” *Econometrica*, 68(3), 575–604.

- KAPETANIOS, G. (2000): “Small Sample Properties of the Conditional Least Squares Estimator in SETAR Models,” *Economics Letters*, 69(3), 267–276.
- POLITIS, D. N., AND J. P. ROMANO (1994): “Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions,” *Annals of Statistics*, 22(4), 2031–2050.

Notes for Tables:

Table 10: Two regime TAR model. Results for Method A: Estimated MSE of bootstrap confidence interval ($\overset{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

Table 11: Two regime TAR model. Results for Method B: Estimated MSE of bootstrap confidence interval ($\overset{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

Table 12: Two regime TAR model. Results for Method C: Estimated MSE of bootstrap confidence interval ($\overset{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

Table 4: Two regime TAR model. Results for Method A: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the Method performs better than the other methods

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	1.947* (0.505)	1.547 (0.664)	0.980* (0.590)	0.594* (0.488)	0.616 (0.584)	0.623 (0.622)	0.704 (0.679)	1.077 (0.756)	1.770 (0.718)
	$(-0.5, -0.2)$	1.516* (0.334)	0.788* (0.363)	0.497* (0.407)	0.564* (0.553)	0.590 (0.589)	0.548* (0.532)	0.557* (0.477)	0.843* (0.425)	1.655* (0.432)
	$(0.5, 0.2)$	1.301* (0.354)	0.748* (0.406)	0.496* (0.442)	0.631 (0.627)	0.567 (0.565)	0.539* (0.512)	0.527* (0.403)	0.992* (0.397)	1.624* (0.416)
	$(-0.8, -0.5)$	1.350* (0.449)	0.933* (0.602)	0.680* (0.618)	0.761* (0.744)	0.871 (0.865)	0.925 (0.919)	0.885* (0.833)	1.043* (0.708)	1.474* (0.698)
	$(-0.3, -0.1)$	1.467* (0.287)	0.709* (0.323)	0.554* (0.435)	0.421* (0.419)	0.524 (0.524)	0.480* (0.475)	0.424* (0.342)	0.808* (0.363)	1.788* (0.402)
	$(0.8, 0.6)$	1.606* (0.852)	0.952* (0.843)	1.045* (1.032)	1.033 (1.031)	0.945 (0.920)	0.853* (0.822)	1.104* (0.892)	1.202* (0.749)	1.998* (0.946)
	$(0.3, 0.1)$	1.349* (0.306)	0.626* (0.317)	0.443* (0.377)	0.494* (0.481)	0.519 (0.517)	0.497* (0.481)	0.518* (0.381)	0.855* (0.352)	1.572* (0.330)
	$(-0.6, 0.6)$	1.409* (0.319)	0.670* (0.234)	0.295* (0.180)	0.258* (0.237)	0.347 (0.346)	0.401 (0.400)	0.420 (0.420)	0.413 (0.348)	1.094 (0.425)
100	$(-0.5, -0.2)$	0.987* (0.142)	0.370* (0.149)	0.167* (0.157)	0.267* (0.267)	0.367 (0.367)	0.281 (0.280)	0.216* (0.211)	0.301* (0.155)	0.845* (0.157)
	$(0.5, 0.2)$	1.002* (0.201)	0.339* (0.189)	0.263* (0.252)	0.357 (0.353)	0.338 (0.338)	0.255* (0.254)	0.186* (0.160)	0.424* (0.169)	0.910* (0.154)
	$(-0.8, -0.5)$	0.688* (0.185)	0.349* (0.219)	0.351* (0.347)	0.452 (0.452)	0.549 (0.548)	0.507 (0.500)	0.423* (0.419)	0.415* (0.342)	0.615* (0.306)
	$(-0.3, -0.1)$	1.043* (0.155)	0.322* (0.119)	0.209* (0.188)	0.311* (0.311)	0.329 (0.328)	0.303 (0.300)	0.180* (0.167)	0.346* (0.136)	0.959* (0.135)
	$(0.8, 0.6)$	0.603* (0.334)	0.400* (0.377)	0.487* (0.487)	0.589 (0.588)	0.612 (0.612)	0.544* (0.538)	0.354* (0.335)	0.489* (0.330)	0.815* (0.342)
	$(0.3, 0.1)$	0.926* (0.131)	0.352* (0.158)	0.195* (0.184)	0.307 (0.307)	0.311 (0.310)	0.267* (0.267)	0.196* (0.161)	0.398* (0.152)	1.051* (0.150)
	$(-0.6, 0.6)$	1.029 (0.180)	0.487 (0.155)	0.208 (0.133)	0.148* (0.144)	0.220 (0.213)	0.301 (0.301)	0.263 (0.258)	0.236 (0.223)	0.657 (0.263)
	$(-0.5, -0.2)$	0.768* (0.071)	0.235* (0.085)	0.133* (0.127)	0.190 (0.188)	0.270 (0.270)	0.220 (0.215)	0.147* (0.146)	0.205* (0.103)	0.701* (0.132)
150	$(0.5, 0.2)$	0.725* (0.096)	0.213* (0.121)	0.174* (0.173)	0.270 (0.268)	0.278 (0.271)	0.180 (0.178)	0.118* (0.111)	0.241* (0.085)	0.694* (0.091)
	$(-0.8, -0.5)$	0.535* (0.101)	0.202* (0.134)	0.199* (0.198)	0.310 (0.309)	0.392 (0.388)	0.441 (0.432)	0.306* (0.303)	0.207* (0.189)	0.340* (0.167)
	$(-0.3, -0.1)$	0.769* (0.063)	0.232* (0.074)	0.110* (0.103)	0.174* (0.167)	0.229 (0.229)	0.200 (0.198)	0.113* (0.110)	0.197* (0.067)	0.769* (0.075)
	$(0.8, 0.6)$	0.395* (0.207)	0.306* (0.296)	0.434 (0.425)	0.471 (0.466)	0.396 (0.396)	0.353* (0.352)	0.271* (0.264)	0.327* (0.235)	0.461* (0.176)
	$(0.3, 0.1)$	0.780* (0.079)	0.205* (0.082)	0.116* (0.116)	0.201 (0.200)	0.248 (0.247)	0.169* (0.165)	0.101* (0.093)	0.233* (0.075)	0.812* (0.080)

Table 5: Two regime TAR model. Results for Method B: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the method performs better than the other methods

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	2.582 (0.856)	1.521* (0.636)	0.998 (0.504)	0.812 (0.521)	0.465* (0.413)	0.254* (0.249)	0.288* (0.243)	0.522* (0.364)	1.179* (0.564)
	$(-0.5, -0.2)$	4.265 (0.777)	2.414 (0.828)	1.328 (0.707)	0.633 (0.526)	0.459* (0.457)	0.624 (0.564)	1.074 (0.676)	2.255 (0.907)	4.279 (0.978)
	$(0.5, 0.2)$	3.949 (0.840)	2.169 (0.823)	1.062 (0.642)	0.563* (0.511)	0.458* (0.450)	0.758 (0.577)	1.454 (0.803)	2.757 (0.950)	4.680 (0.993)
	$(-0.8, -0.5)$	4.217 (1.141)	2.741 (1.086)	1.413 (0.881)	0.941 (0.765)	0.612* (0.596)	0.801* (0.773)	1.342 (0.986)	2.204 (1.198)	4.031 (1.532)
	$(-0.3, -0.1)$	4.184 (0.757)	2.713 (0.827)	1.326 (0.711)	0.636 (0.519)	0.438* (0.437)	0.577 (0.492)	1.264 (0.722)	2.634 (0.847)	4.477 (0.789)
	$(0.8, 0.6)$	4.043 (1.333)	2.290 (1.173)	1.247 (0.994)	0.852* (0.836)	0.942* (0.898)	1.529 (1.180)	2.226 (1.397)	3.825 (1.666)	5.678 (1.849)
	$(0.3, 0.1)$	4.403 (0.789)	2.405 (0.783)	1.114 (0.644)	0.556 (0.499)	0.450* (0.447)	0.630 (0.513)	1.473 (0.747)	2.855 (0.871)	4.914 (0.830)
100	$(-0.6, 0.6)$	1.457 (0.485)	0.884 (0.393)	0.469 (0.291)	0.290 (0.220)	0.161* (0.153)	0.096* (0.093)	0.092* (0.087)	0.134* (0.111)	0.541* (0.245)
	$(-0.5, -0.2)$	4.080 (0.837)	2.115 (0.779)	0.822 (0.574)	0.320 (0.286)	0.188* (0.187)	0.260* (0.254)	0.600 (0.495)	1.490 (0.718)	3.897 (0.871)
	$(0.5, 0.2)$	4.013 (0.782)	1.759 (0.726)	0.516 (0.402)	0.260* (0.252)	0.217* (0.213)	0.438 (0.378)	0.954 (0.623)	2.480 (0.882)	4.390 (0.922)
	$(-0.8, -0.5)$	3.656 (1.065)	1.660 (0.910)	0.708 (0.541)	0.397* (0.373)	0.278* (0.275)	0.399* (0.385)	0.542 (0.487)	1.390 (0.942)	3.002 (1.363)
	$(-0.3, -0.1)$	4.431 (0.774)	2.199 (0.774)	0.631 (0.428)	0.367 (0.323)	0.211* (0.210)	0.290* (0.282)	0.713 (0.513)	2.172 (0.844)	4.217 (0.733)
	$(0.8, 0.6)$	3.131 (1.220)	1.522 (1.026)	0.635 (0.557)	0.406* (0.397)	0.440* (0.430)	0.829 (0.720)	1.459 (1.021)	2.580 (1.347)	4.962 (1.716)
	$(0.3, 0.1)$	4.125 (0.728)	1.893 (0.731)	0.749 (0.513)	0.247* (0.241)	0.201* (0.201)	0.328 (0.298)	0.839 (0.535)	2.458 (0.784)	4.303 (0.766)
150	$(-0.6, 0.6)$	0.987 (0.222)	0.463* (0.150)	0.166* (0.110)	0.180 (0.143)	0.079* (0.076)	0.041* (0.040)	0.020* (0.020)	0.039* (0.037)	0.350* (0.136)
	$(-0.5, -0.2)$	3.788 (0.743)	1.523 (0.639)	0.473 (0.367)	0.187* (0.180)	0.127* (0.127)	0.117* (0.117)	0.452 (0.384)	1.258 (0.697)	3.323 (0.837)
	$(0.5, 0.2)$	3.629 (0.746)	1.261 (0.666)	0.324 (0.271)	0.123* (0.122)	0.117* (0.116)	0.169* (0.162)	0.656 (0.457)	1.681 (0.700)	4.013 (0.919)
	$(-0.8, -0.5)$	3.124 (1.000)	1.203 (0.754)	0.384 (0.329)	0.194* (0.189)	0.174* (0.172)	0.181* (0.178)	0.307 (0.286)	0.812 (0.662)	1.906 (1.066)
	$(-0.3, -0.1)$	4.275 (0.721)	1.868 (0.736)	0.526 (0.377)	0.181 (0.172)	0.116* (0.115)	0.159* (0.156)	0.482 (0.392)	1.690 (0.717)	3.988 (0.763)
	$(0.8, 0.6)$	2.803 (1.263)	0.899 (0.730)	0.290* (0.276)	0.208* (0.207)	0.214* (0.206)	0.396 (0.370)	0.853 (0.709)	2.252 (1.319)	4.393 (1.837)
	$(0.3, 0.1)$	4.022 (0.726)	1.587 (0.680)	0.450 (0.368)	0.145* (0.141)	0.080* (0.080)	0.211 (0.201)	0.572 (0.449)	1.913 (0.734)	4.151 (0.752)

Table 6: Two regime TAR model. Results for Method C: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the method performs better than the other methods

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	2.736 (0.926)	1.647 (0.662)	1.234 (0.552)	1.021 (0.574)	0.719 (0.606)	0.492 (0.463)	0.493 (0.367)	0.722 (0.448)	1.388 (0.654)
	$(-0.5, -0.2)$	4.494 (0.746)	2.744 (0.778)	1.693 (0.730)	1.110 (0.752)	0.784 (0.778)	1.004 (0.793)	1.534 (0.709)	2.789 (0.822)	4.758 (0.876)
	$(0.5, 0.2)$	4.165 (0.763)	2.684 (0.783)	1.532 (0.813)	0.877 (0.766)	0.824 (0.789)	1.282 (0.779)	2.086 (0.857)	3.131 (0.884)	4.817 (0.934)
	$(-0.8, -0.5)$	4.213 (1.024)	2.977 (1.062)	1.987 (1.102)	1.357 (1.087)	1.103 (1.097)	1.326 (1.136)	2.007 (1.188)	3.162 (1.285)	4.526 (1.348)
	$(-0.3, -0.1)$	4.348 (0.693)	2.793 (0.726)	1.751 (0.688)	0.972 (0.667)	0.686 (0.685)	0.922 (0.634)	1.767 (0.726)	2.871 (0.800)	4.748 (0.796)
	$(0.8, 0.6)$	4.441 (1.368)	2.764 (1.282)	1.938 (1.406)	1.774 (1.715)	1.616 (1.493)	2.042 (1.335)	2.982 (1.428)	5.170 (1.845)	6.536 (1.730)
	$(0.3, 0.1)$	4.310 (0.703)	2.813 (0.748)	1.577 (0.733)	0.867 (0.730)	0.805 (0.774)	1.130 (0.746)	1.965 (0.793)	3.074 (0.738)	5.112 (0.778)
	$(-0.6, 0.6)$	1.570 (0.490)	0.927 (0.354)	0.661 (0.353)	0.548 (0.326)	0.540 (0.457)	0.280 (0.257)	0.256 (0.208)	0.248 (0.198)	0.687 (0.294)
100	$(-0.5, -0.2)$	4.341 (0.746)	2.703 (0.667)	1.517 (0.664)	0.956 (0.696)	0.665 (0.663)	0.803 (0.647)	1.275 (0.689)	2.320 (0.695)	3.867 (0.788)
	$(0.5, 0.2)$	4.313 (0.719)	2.495 (0.706)	1.382 (0.716)	0.849 (0.735)	0.724 (0.703)	1.196 (0.743)	1.661 (0.713)	2.953 (0.757)	4.374 (0.901)
	$(-0.8, -0.5)$	4.356 (0.940)	2.638 (0.936)	1.703 (0.939)	1.179 (0.930)	1.005 (1.004)	1.249 (1.026)	1.694 (1.073)	2.794 (1.134)	3.887 (1.261)
	$(-0.3, -0.1)$	4.374 (0.673)	2.898 (0.663)	1.645 (0.681)	0.943 (0.682)	0.592 (0.591)	0.815 (0.642)	1.509 (0.658)	2.866 (0.686)	4.436 (0.671)
	$(0.8, 0.6)$	4.301 (1.204)	2.570 (1.214)	1.835 (1.307)	1.305 (1.296)	1.487 (1.289)	2.291 (1.451)	2.841 (1.370)	4.009 (1.376)	5.667 (1.489)
	$(0.3, 0.1)$	4.604 (0.652)	2.625 (0.638)	1.480 (0.668)	0.861 (0.669)	0.664 (0.648)	0.978 (0.648)	1.627 (0.620)	2.895 (0.682)	4.619 (0.720)
	$(-0.6, 0.6)$	0.962* (0.167)	0.575 (0.183)	0.377 (0.218)	0.450 (0.286)	0.344 (0.306)	0.237 (0.207)	0.108 (0.097)	0.075 (0.070)	0.385 (0.139)
	$(-0.5, -0.2)$	4.255 (0.709)	2.514 (0.716)	1.443 (0.634)	0.983 (0.654)	0.705 (0.696)	0.857 (0.683)	1.271 (0.626)	2.284 (0.697)	3.780 (0.810)
150	$(0.5, 0.2)$	3.960 (0.691)	2.037 (0.664)	1.188 (0.641)	0.749 (0.668)	0.713 (0.657)	1.119 (0.684)	1.687 (0.704)	2.483 (0.751)	4.359 (0.824)
	$(-0.8, -0.5)$	3.868 (0.917)	2.262 (0.892)	1.734 (0.950)	1.145 (0.907)	0.993 (0.980)	1.098 (0.927)	1.323 (0.832)	2.283 (1.084)	3.259 (1.213)
	$(-0.3, -0.1)$	4.644 (0.694)	2.328 (0.621)	1.484 (0.605)	0.905 (0.647)	0.656 (0.650)	0.901 (0.649)	1.526 (0.632)	2.630 (0.646)	4.620 (0.708)
	$(0.8, 0.6)$	4.171 (1.214)	2.523 (1.226)	1.566 (1.190)	1.331 (1.327)	1.312 (1.197)	1.841 (1.180)	2.707 (1.266)	3.855 (1.404)	5.421 (1.685)
	$(0.3, 0.1)$	4.365 (0.659)	2.342 (0.661)	1.337 (0.605)	0.838 (0.667)	0.689 (0.665)	1.065 (0.635)	1.810 (0.731)	2.666 (0.626)	4.560 (0.681)

Table 7: Two regime TAR model. Results for Method A: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	0.181*	0.119*	0.086*	0.087*	0.069*	0.057*	0.094*	0.115*	0.149*
	$(-0.5, -0.2)$	0.067*	0.080	0.091	0.050*	0.044*	0.072*	0.085*	0.125	0.129
	$(0.5, 0.2)$	0.072	0.058*	0.063*	0.063*	0.050*	0.075*	0.078*	0.081*	0.100*
	$(-0.8, -0.5)$	0.272	0.225	0.356	0.271	0.209*	0.181*	0.293*	0.445	0.652
	$(-0.3, -0.1)$	0.077	0.077	0.050*	0.057*	0.037*	0.041*	0.086	0.080*	0.068*
	$(0.8, 0.6)$	0.290*	0.293*	0.360*	0.316*	0.262*	0.342*	0.293*	0.468*	0.386*
	$(0.3, 0.1)$	0.065*	0.060*	0.053*	0.037*	0.038*	0.048*	0.056*	0.064*	0.062*
	$(-0.6, 0.6)$	0.086*	0.081*	0.115*	0.076*	0.040*	0.045*	0.089*	0.151*	0.139*
100	$(-0.5, -0.2)$	0.043*	0.054*	0.098*	0.078*	0.044*	0.085*	0.084*	0.101*	0.113*
	$(0.5, 0.2)$	0.028*	0.042*	0.041*	0.037*	0.029*	0.045*	0.061*	0.044*	0.054*
	$(-0.8, -0.5)$	0.172*	0.249	0.289	0.265*	0.210*	0.242*	0.333	0.463	0.692
	$(-0.3, -0.1)$	0.033*	0.044*	0.051*	0.036*	0.037*	0.031*	0.062*	0.058*	0.052*
	$(0.8, 0.6)$	0.187*	0.254*	0.237*	0.206*	0.165*	0.204*	0.294*	0.271*	0.426*
	$(0.3, 0.1)$	0.030*	0.033*	0.041*	0.031*	0.033*	0.033*	0.041*	0.032*	0.031*
	$(-0.6, 0.6)$	0.041*	0.046*	0.055*	0.042*	0.022*	0.030*	0.068*	0.109*	0.082*
	$(-0.5, -0.2)$	0.024*	0.032*	0.029*	0.044*	0.035*	0.041*	0.037*	0.037*	0.038*
150	$(0.5, 0.2)$	0.011*	0.015*	0.023*	0.020*	0.016*	0.032*	0.023*	0.020*	0.017*
	$(-0.8, -0.5)$	0.094*	0.106*	0.174	0.254	0.176*	0.125*	0.186	0.267	0.357
	$(-0.3, -0.1)$	0.020*	0.021*	0.027*	0.029*	0.027*	0.021*	0.029*	0.025*	0.025*
	$(0.8, 0.6)$	0.099*	0.131*	0.151*	0.153*	0.182*	0.143*	0.147*	0.107*	0.131*
	$(0.3, 0.1)$	0.013*	0.016*	0.018*	0.023*	0.016*	0.021*	0.018*	0.013*	0.016*
50	$(-0.6, 0.6)$	0.206	0.208	0.140	0.116	0.152	0.159	0.170	0.229	0.212
	$(-0.5, -0.2)$	0.075	0.066*	0.086*	0.120	0.107	0.084	0.089	0.109*	0.106*
	$(0.5, 0.2)$	0.071*	0.078	0.088	0.151	0.119	0.100	0.093	0.110	0.136
	$(-0.8, -0.5)$	0.161*	0.205*	0.273*	0.257*	0.306	0.343	0.293	0.311*	0.359*
	$(-0.3, -0.1)$	0.077*	0.069*	0.078	0.069	0.095	0.074	0.062*	0.083	0.076
	$(0.8, 0.6)$	0.475	0.474	0.548	0.555	0.394	0.398	0.478	0.506	0.556
	$(0.3, 0.1)$	0.086	0.076	0.072	0.096	0.091	0.081	0.063	0.081	0.084
	$(-0.6, 0.6)$	0.231	0.246	0.278	0.201	0.109	0.108	0.188	0.310	0.230
100	$(-0.5, -0.2)$	0.088	0.100	0.141	0.121	0.086	0.121	0.128	0.166	0.145
	$(0.5, 0.2)$	0.092	0.132	0.096	0.098	0.073	0.110	0.125	0.136	0.164
	$(-0.8, -0.5)$	0.250	0.243*	0.286*	0.266	0.218	0.244	0.293*	0.387*	0.614*
	$(-0.3, -0.1)$	0.077	0.104	0.079	0.075	0.075	0.065	0.116	0.125	0.110
	$(0.8, 0.6)$	0.391	0.418	0.405	0.314	0.261	0.423	0.569	0.648	0.696
	$(0.3, 0.1)$	0.099	0.088	0.098	0.085	0.077	0.073	0.104	0.098	0.097
	$(-0.6, 0.6)$	0.193	0.200	0.186	0.121	0.094	0.108	0.177	0.256	0.240
	$(-0.5, -0.2)$	0.084	0.090	0.074	0.081	0.079	0.067	0.071	0.076	0.076
150	$(0.5, 0.2)$	0.067	0.069	0.082	0.053	0.058	0.082	0.076	0.101	0.102
	$(-0.8, -0.5)$	0.146	0.149	0.171*	0.220*	0.196	0.159	0.174*	0.236*	0.349*
	$(-0.3, -0.1)$	0.064	0.068	0.069	0.062	0.068	0.052	0.056	0.065	0.074
	$(0.8, 0.6)$	0.189	0.268	0.252	0.180	0.253	0.207	0.272	0.314	0.396
	$(0.3, 0.1)$	0.047	0.063	0.059	0.075	0.053	0.052	0.070	0.059	0.071

Table 8: Two regime TAR model. Results for Method B: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	0.135*	0.161*	0.139	0.087*	0.125	0.208	0.281	0.300	0.397
	$(-0.5, -0.2)$	0.089*	0.098*	0.058*	0.077*	0.099*	0.062*	0.062*	0.097*	0.127*
	$(0.5, 0.2)$	0.145*	0.121*	0.054*	0.059*	0.100*	0.060*	0.089*	0.169*	0.185*
	$(-0.8, -0.5)$	0.210*	0.217*	0.411*	0.561	0.779	0.565	0.492*	0.447*	0.414*
	$(-0.3, -0.1)$	0.101*	0.122*	0.070*	0.042*	0.051*	0.043*	0.063*	0.112*	0.085*
	$(0.8, 0.6)$	0.447*	0.385*	0.447*	0.845	0.765	0.531*	0.423*	0.545*	0.667*
	$(0.3, 0.1)$	0.136*	0.115*	0.046*	0.039*	0.045*	0.037*	0.091*	0.172*	0.131*
	$(-0.6, 0.6)$	0.392	0.318	0.307	0.270	0.291	0.219	0.239	0.404	0.632
100	$(-0.5, -0.2)$	0.079*	0.052*	0.072*	0.221	0.260	0.260	0.176	0.115*	0.072*
	$(0.5, 0.2)$	0.060*	0.051*	0.166	0.232	0.240	0.200	0.095*	0.073*	0.087*
	$(-0.8, -0.5)$	0.171*	0.236*	0.770	0.988	1.209	1.181	1.471	0.930	0.598*
	$(-0.3, -0.1)$	0.065*	0.060*	0.110	0.115	0.165	0.137	0.075*	0.083*	0.052*
	$(0.8, 0.6)$	0.240*	0.547	1.381	1.952	2.135	1.560	1.132	0.755	0.491*
	$(0.3, 0.1)$	0.055*	0.053*	0.050*	0.145	0.159	0.162	0.071*	0.063*	0.058*
	$(-0.6, 0.6)$	0.608	0.376	0.362	0.234	0.218	0.119	0.107	0.176	0.325
	$(-0.5, -0.2)$	0.042*	0.051*	0.188	0.217	0.205	0.274	0.190	0.091*	0.067*
150	$(0.5, 0.2)$	0.052*	0.045*	0.197	0.235	0.233	0.334	0.187	0.098*	0.070*
	$(-0.8, -0.5)$	0.121*	0.319	0.882	0.938	0.934	1.084	1.525	1.191	1.048
	$(-0.3, -0.1)$	0.044*	0.046*	0.115	0.170	0.141	0.177	0.126	0.041*	0.048*
	$(0.8, 0.6)$	0.183*	0.743	1.639	1.779	1.971	2.207	1.875	0.630*	0.441*
	$(0.3, 0.1)$	0.056*	0.037*	0.102	0.150	0.179	0.145	0.102	0.044*	0.050*
50	$(-0.6, 0.6)$	0.444	0.207	0.118*	0.145	0.072*	0.027*	0.028*	0.053*	0.131*
	$(-0.5, -0.2)$	0.505	0.580	0.349	0.153	0.107	0.198	0.314	0.667	0.815
	$(0.5, 0.2)$	0.606	0.521	0.260	0.147	0.107	0.205	0.464	0.753	0.810
	$(-0.8, -0.5)$	1.118	0.939	0.510	0.349*	0.179*	0.358*	0.622	1.014	2.000
	$(-0.3, -0.1)$	0.501	0.561	0.402	0.165	0.100	0.144	0.406	0.591	0.493
	$(0.8, 0.6)$	1.494	0.941	0.600	0.394*	0.462*	0.963	1.502	2.156	2.881
	$(0.3, 0.1)$	0.528	0.485	0.290	0.144	0.116	0.154	0.420	0.659	0.561
	$(-0.6, 0.6)$	0.202*	0.154*	0.130*	0.081*	0.108*	0.072*	0.084*	0.151*	0.233*
100	$(-0.5, -0.2)$	0.334	0.244	0.092	0.097*	0.089*	0.074*	0.090*	0.152	0.279
	$(0.5, 0.2)$	0.220	0.197	0.076*	0.100*	0.096*	0.076*	0.110	0.296	0.333
	$(-0.8, -0.5)$	0.455	0.276	0.145*	0.218*	0.205*	0.167*	0.214*	0.254*	0.697
	$(-0.3, -0.1)$	0.258	0.245	0.069*	0.051*	0.073*	0.060*	0.081	0.315	0.190
	$(0.8, 0.6)$	0.633	0.355*	0.232*	0.269*	0.384*	0.344*	0.403*	0.652*	1.371
	$(0.3, 0.1)$	0.205	0.200	0.085	0.077*	0.066*	0.073*	0.079	0.248	0.208
	$(-0.6, 0.6)$	0.398*	0.237*	0.207*	0.124*	0.106*	0.054*	0.061*	0.117*	0.267*
	$(-0.5, -0.2)$	0.148	0.096	0.115*	0.117*	0.108*	0.153*	0.100*	0.116	0.167
150	$(0.5, 0.2)$	0.155	0.116	0.122*	0.140*	0.149*	0.213*	0.131*	0.149	0.236
	$(-0.8, -0.5)$	0.252	0.197*	0.324*	0.320*	0.264*	0.295*	0.443*	0.319*	0.304*
	$(-0.3, -0.1)$	0.163	0.143	0.082*	0.114*	0.087*	0.118*	0.102*	0.141	0.160
	$(0.8, 0.6)$	0.475	0.231*	0.429*	0.410*	0.430*	0.543*	0.497*	0.633	1.422
	$(0.3, 0.1)$	0.157	0.113	0.068*	0.105*	0.118*	0.108*	0.096*	0.149	0.157

Table 9: Two regime TAR model. Results for Method C: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r^0								
		-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	0.154*	0.227	0.156	0.106*	0.094*	0.146	0.231	0.260	0.313
	$(-0.5, -0.2)$	0.067*	0.073*	0.065*	0.065*	0.062*	0.071*	0.058*	0.088*	0.100*
	$(0.5, 0.2)$	0.079*	0.067*	0.083*	0.073*	0.069*	0.076*	0.097*	0.100*	0.104*
	$(-0.8, -0.5)$	0.202*	0.231*	0.301*	0.317*	0.314*	0.273*	0.305*	0.341*	0.485*
	$(-0.3, -0.1)$	0.054*	0.061*	0.055*	0.054*	0.051*	0.048*	0.057*	0.077*	0.085*
	$(0.8, 0.6)$	0.532*	0.584*	0.438*	0.627*	0.521*	0.540*	0.500*	0.676*	0.632*
	$(0.3, 0.1)$	0.060*	0.067*	0.059*	0.066*	0.081*	0.068*	0.088*	0.063*	0.065*
	$(-0.6, 0.6)$	0.326	0.370	0.219	0.205	0.098	0.191	0.172	0.307	0.440
100	$(-0.5, -0.2)$	0.058*	0.035*	0.036*	0.039*	0.041*	0.034*	0.046*	0.055*	0.054*
	$(0.5, 0.2)$	0.043*	0.044*	0.049*	0.044*	0.050*	0.050*	0.053*	0.055*	0.077*
	$(-0.8, -0.5)$	0.115*	0.140*	0.204*	0.239*	0.161*	0.179*	0.187*	0.241*	0.283*
	$(-0.3, -0.1)$	0.040*	0.036*	0.040*	0.041*	0.035*	0.036*	0.038*	0.040*	0.040*
	$(0.8, 0.6)$	0.219*	0.376*	0.333*	0.399*	0.425*	0.304*	0.375*	0.474*	0.405*
	$(0.3, 0.1)$	0.037*	0.036*	0.042*	0.033*	0.034*	0.036*	0.037*	0.038*	0.041*
	$(-0.6, 0.6)$	0.679	0.376	0.263	0.162	0.151	0.147	0.138	0.229	0.332
	$(-0.5, -0.2)$	0.045*	0.047*	0.039*	0.034*	0.039*	0.034*	0.040*	0.046*	0.050*
150	$(0.5, 0.2)$	0.039*	0.035*	0.042*	0.040*	0.042*	0.042*	0.039*	0.051*	0.049*
	$(-0.8, -0.5)$	0.075*	0.110*	0.115*	0.173*	0.126*	0.142*	0.334	0.158*	0.250*
	$(-0.3, -0.1)$	0.047*	0.029*	0.028*	0.031*	0.037*	0.029*	0.032*	0.033*	0.045*
	$(0.8, 0.6)$	0.169*	0.238*	0.310*	0.242*	0.396	0.376*	0.336*	0.283*	0.305*
	$(0.3, 0.1)$	0.041*	0.035*	0.032*	0.031*	0.033*	0.030*	0.047*	0.032*	0.029*
50	$(-0.6, 0.6)$	0.340	0.184*	0.122*	0.148	0.139	0.096*	0.077*	0.105*	0.164*
	$(-0.5, -0.2)$	0.271	0.289	0.231	0.231	0.271	0.292	0.202	0.305	0.327
	$(0.5, 0.2)$	0.266	0.247	0.293	0.244	0.262	0.243	0.344	0.369	0.384
	$(-0.8, -0.5)$	0.468	0.495	0.578	0.550	0.566	0.568	0.639	0.776	0.713
	$(-0.3, -0.1)$	0.220	0.229	0.187	0.193	0.193	0.158	0.235	0.313	0.297
	$(0.8, 0.6)$	1.043	0.771	0.931	1.607	1.029	0.784	0.905	1.887	1.613
	$(0.3, 0.1)$	0.219	0.256	0.227	0.235	0.280	0.242	0.298	0.225	0.270
	$(-0.6, 0.6)$	0.246*	0.222*	0.105*	0.098*	0.081*	0.121*	0.093*	0.185*	0.308*
100	$(-0.5, -0.2)$	0.191	0.130	0.109	0.142	0.125	0.102	0.129	0.118	0.159
	$(0.5, 0.2)$	0.140	0.137	0.152	0.139	0.139	0.164	0.130	0.160	0.253
	$(-0.8, -0.5)$	0.263	0.247	0.241	0.250	0.259	0.306	0.324	0.369	0.434
	$(-0.3, -0.1)$	0.141	0.127	0.128	0.143	0.111	0.112	0.112	0.141	0.125
	$(0.8, 0.6)$	0.470	0.528	0.557	0.586	0.573	0.755	0.593	0.769	0.681
	$(0.3, 0.1)$	0.122	0.119	0.145	0.104	0.110	0.114	0.105	0.127	0.148
	$(-0.6, 0.6)$	0.427*	0.229*	0.143*	0.109*	0.094*	0.086*	0.074*	0.160*	0.237*
	$(-0.5, -0.2)$	0.148	0.148	0.094	0.106	0.139	0.119	0.099	0.120	0.159
150	$(0.5, 0.2)$	0.102	0.102	0.104	0.113	0.102	0.120	0.121	0.148	0.169
	$(-0.8, -0.5)$	0.216	0.204	0.207	0.219	0.272	0.227	0.189*	0.307	0.377
	$(-0.3, -0.1)$	0.155	0.080	0.097	0.104	0.121	0.104	0.101	0.107	0.149
	$(0.8, 0.6)$	0.434	0.421	0.403	0.505	0.383*	0.396	0.535	0.621	1.089
	$(0.3, 0.1)$	0.118	0.109	0.091	0.126	0.110	0.102	0.145	0.091	0.120

Table 10: See notes in page 20

T	$(\alpha_1, \alpha_2)/r^0$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	$\begin{pmatrix} 0.222^* \\ (0.696) \end{pmatrix}$	$\begin{pmatrix} 0.211^* \\ (0.630^*) \end{pmatrix}$	$\begin{pmatrix} 0.152^* \\ (0.551^*) \end{pmatrix}$	$\begin{pmatrix} 0.195^* \\ (0.468^*) \end{pmatrix}$	$\begin{pmatrix} 0.173^* \\ (0.406^*) \end{pmatrix}$	$\begin{pmatrix} 0.226^* \\ (0.232^*) \end{pmatrix}$	$\begin{pmatrix} 0.240^* \\ (0.263^*) \end{pmatrix}$	$\begin{pmatrix} 0.319^* \\ (0.200^*) \end{pmatrix}$	$\begin{pmatrix} 0.291^* \\ (0.342^*) \end{pmatrix}$
	$(-0.5, -0.2)$	$\begin{pmatrix} 0.205^* \\ (0.367^*) \end{pmatrix}$	$\begin{pmatrix} 0.180^* \\ (0.391^*) \end{pmatrix}$	$\begin{pmatrix} 0.170^* \\ (0.339^*) \end{pmatrix}$	$\begin{pmatrix} 0.220^* \\ (0.200^*) \end{pmatrix}$	$\begin{pmatrix} 0.161^* \\ (0.144^*) \end{pmatrix}$	$\begin{pmatrix} 0.160^* \\ (0.110^*) \end{pmatrix}$	$\begin{pmatrix} 0.237^* \\ (0.106^*) \end{pmatrix}$	$\begin{pmatrix} 0.357^* \\ (0.108^*) \end{pmatrix}$	$\begin{pmatrix} 0.388^* \\ (0.133^*) \end{pmatrix}$
	$(0.5, 0.2)$	$\begin{pmatrix} 0.370^* \\ (0.347^*) \end{pmatrix}$	$\begin{pmatrix} 0.295^* \\ (0.367^*) \end{pmatrix}$	$\begin{pmatrix} 0.324^* \\ (0.321^*) \end{pmatrix}$	$\begin{pmatrix} 0.378^* \\ (0.221^*) \end{pmatrix}$	$\begin{pmatrix} 0.315^* \\ (0.182^*) \end{pmatrix}$	$\begin{pmatrix} 0.461^* \\ (0.192^*) \end{pmatrix}$	$\begin{pmatrix} 0.488^* \\ (0.131^*) \end{pmatrix}$	$\begin{pmatrix} 0.446 \\ (0.132^*) \end{pmatrix}$	$\begin{pmatrix} 0.492^* \\ (0.201^*) \end{pmatrix}$
	$(-0.8, -0.5)$	$\begin{pmatrix} 0.242^* \\ (1.269) \end{pmatrix}$	$\begin{pmatrix} 0.337^* \\ (0.530^*) \end{pmatrix}$	$\begin{pmatrix} 0.275^* \\ (0.639) \end{pmatrix}$	$\begin{pmatrix} 0.295^* \\ (0.305^*) \end{pmatrix}$	$\begin{pmatrix} 0.194^* \\ (0.262^*) \end{pmatrix}$	$\begin{pmatrix} 0.252^* \\ (0.204^*) \end{pmatrix}$	$\begin{pmatrix} 0.222^* \\ (0.264^*) \end{pmatrix}$	$\begin{pmatrix} 0.632 \\ (0.226^*) \end{pmatrix}$	$\begin{pmatrix} 0.959 \\ (0.299^*) \end{pmatrix}$
	$(-0.3, -0.1)$	$\begin{pmatrix} 0.140^* \\ (0.513) \end{pmatrix}$	$\begin{pmatrix} 0.223^* \\ (0.598) \end{pmatrix}$	$\begin{pmatrix} 0.143^* \\ (0.269^*) \end{pmatrix}$	$\begin{pmatrix} 0.156^* \\ (0.211^*) \end{pmatrix}$	$\begin{pmatrix} 0.136^* \\ (0.144^*) \end{pmatrix}$	$\begin{pmatrix} 0.198^* \\ (0.114^*) \end{pmatrix}$	$\begin{pmatrix} 0.267^* \\ (0.097^*) \end{pmatrix}$	$\begin{pmatrix} 0.284^* \\ (0.098^*) \end{pmatrix}$	$\begin{pmatrix} 0.297^* \\ (0.132^*) \end{pmatrix}$
	$(0.8, 0.6)$	$\begin{pmatrix} 1.414^* \\ (0.701^*) \end{pmatrix}$	$\begin{pmatrix} 1.514^* \\ (0.642^*) \end{pmatrix}$	$\begin{pmatrix} 1.446^* \\ (0.604^*) \end{pmatrix}$	$\begin{pmatrix} 0.839^* \\ (0.483^*) \end{pmatrix}$	$\begin{pmatrix} 1.032^* \\ (0.360^*) \end{pmatrix}$	$\begin{pmatrix} 0.783^* \\ (0.424^*) \end{pmatrix}$	$\begin{pmatrix} 1.178^* \\ (0.422^*) \end{pmatrix}$	$\begin{pmatrix} 1.230 \\ (0.517^*) \end{pmatrix}$	$\begin{pmatrix} 1.105^* \\ (0.807^*) \end{pmatrix}$
	$(0.3, 0.1)$	$\begin{pmatrix} 0.230^* \\ (0.533^*) \end{pmatrix}$	$\begin{pmatrix} 0.184^* \\ (0.372^*) \end{pmatrix}$	$\begin{pmatrix} 0.196^* \\ (0.331) \end{pmatrix}$	$\begin{pmatrix} 0.159^* \\ (0.237^*) \end{pmatrix}$	$\begin{pmatrix} 0.266^* \\ (0.157^*) \end{pmatrix}$	$\begin{pmatrix} 0.384^* \\ (0.134^*) \end{pmatrix}$	$\begin{pmatrix} 0.431^* \\ (0.104^*) \end{pmatrix}$	$\begin{pmatrix} 0.355^* \\ (0.126^*) \end{pmatrix}$	$\begin{pmatrix} 0.460 \\ (0.139^*) \end{pmatrix}$
	$(-0.6, 0.6)$	$\begin{pmatrix} 0.091^* \\ (0.599^*) \end{pmatrix}$	$\begin{pmatrix} 0.060^* \\ (0.610^*) \end{pmatrix}$	$\begin{pmatrix} 0.044^* \\ (0.873^*) \end{pmatrix}$	$\begin{pmatrix} 0.053^* \\ (0.525^*) \end{pmatrix}$	$\begin{pmatrix} 0.089^* \\ (0.340^*) \end{pmatrix}$	$\begin{pmatrix} 0.164^* \\ (0.222^*) \end{pmatrix}$	$\begin{pmatrix} 0.350^* \\ (0.108^*) \end{pmatrix}$	$\begin{pmatrix} 0.729^* \\ (0.114^*) \end{pmatrix}$	$\begin{pmatrix} 0.475^* \\ (0.127^*) \end{pmatrix}$
100	$(-0.5, -0.2)$	$\begin{pmatrix} 0.045^* \\ (0.499^*) \end{pmatrix}$	$\begin{pmatrix} 0.049^* \\ (0.568^*) \end{pmatrix}$	$\begin{pmatrix} 0.047^* \\ (0.982) \end{pmatrix}$	$\begin{pmatrix} 0.062^* \\ (0.479) \end{pmatrix}$	$\begin{pmatrix} 0.140^* \\ (0.130^*) \end{pmatrix}$	$\begin{pmatrix} 0.346^* \\ (0.069^*) \end{pmatrix}$	$\begin{pmatrix} 0.687 \\ (0.050^*) \end{pmatrix}$	$\begin{pmatrix} 0.919^* \\ (0.050^*) \end{pmatrix}$	$\begin{pmatrix} 1.146 \\ (0.058^*) \end{pmatrix}$
	$(0.5, 0.2)$	$\begin{pmatrix} 0.081^* \\ (0.407^*) \end{pmatrix}$	$\begin{pmatrix} 0.116^* \\ (0.466^*) \end{pmatrix}$	$\begin{pmatrix} 0.089^* \\ (0.342^*) \end{pmatrix}$	$\begin{pmatrix} 0.175^* \\ (0.248^*) \end{pmatrix}$	$\begin{pmatrix} 0.223^* \\ (0.126^*) \end{pmatrix}$	$\begin{pmatrix} 0.383^* \\ (0.062^*) \end{pmatrix}$	$\begin{pmatrix} 0.542^* \\ (0.062^*) \end{pmatrix}$	$\begin{pmatrix} 0.446^* \\ (0.064^*) \end{pmatrix}$	$\begin{pmatrix} 0.644^* \\ (0.080^*) \end{pmatrix}$
	$(-0.8, -0.5)$	$\begin{pmatrix} 0.144^* \\ (1.293) \end{pmatrix}$	$\begin{pmatrix} 0.073^* \\ (1.714) \end{pmatrix}$	$\begin{pmatrix} 0.096^* \\ (1.672) \end{pmatrix}$	$\begin{pmatrix} 0.095^* \\ (0.510) \end{pmatrix}$	$\begin{pmatrix} 0.152^* \\ (0.261^*) \end{pmatrix}$	$\begin{pmatrix} 0.575 \\ (0.119^*) \end{pmatrix}$	$\begin{pmatrix} 1.477 \\ (0.113^*) \end{pmatrix}$	$\begin{pmatrix} 3.033 \\ (0.089^*) \end{pmatrix}$	$\begin{pmatrix} 3.281 \\ (0.109^*) \end{pmatrix}$
	$(-0.3, -0.1)$	$\begin{pmatrix} 0.045^* \\ (0.412^*) \end{pmatrix}$	$\begin{pmatrix} 0.042^* \\ (0.745^*) \end{pmatrix}$	$\begin{pmatrix} 0.050^* \\ (0.495) \end{pmatrix}$	$\begin{pmatrix} 0.078^* \\ (0.290^*) \end{pmatrix}$	$\begin{pmatrix} 0.159^* \\ (0.156^*) \end{pmatrix}$	$\begin{pmatrix} 0.283^* \\ (0.095^*) \end{pmatrix}$	$\begin{pmatrix} 0.708^* \\ (0.046^*) \end{pmatrix}$	$\begin{pmatrix} 0.873^* \\ (0.039^*) \end{pmatrix}$	$\begin{pmatrix} 0.628^* \\ (0.048^*) \end{pmatrix}$
	$(0.8, 0.6)$	$\begin{pmatrix} 0.463^* \\ (0.871^*) \end{pmatrix}$	$\begin{pmatrix} 0.421^* \\ (0.800^*) \end{pmatrix}$	$\begin{pmatrix} 0.344^* \\ (0.558) \end{pmatrix}$	$\begin{pmatrix} 0.392^* \\ (0.301^*) \end{pmatrix}$	$\begin{pmatrix} 0.641^* \\ (0.199^*) \end{pmatrix}$	$\begin{pmatrix>0.819^* \\ (0.194^*) \end{pmatrix}$	$\begin{pmatrix>1.463^* \\ (0.176^*) \end{pmatrix}$	$\begin{pmatrix>1.475^* \\ (0.183^*) \end{pmatrix}$	$\begin{pmatrix>2.410 \\ (0.301^*) \end{pmatrix}$
	$(0.3, 0.1)$	$\begin{pmatrix} 0.063^* \\ (0.470^*) \end{pmatrix}$	$\begin{pmatrix} 0.067^* \\ (0.392^*) \end{pmatrix}$	$\begin{pmatrix} 0.066^* \\ (0.393^*) \end{pmatrix}$	$\begin{pmatrix} 0.086^* \\ (0.262^*) \end{pmatrix}$	$\begin{pmatrix} 0.199^* \\ (0.145^*) \end{pmatrix}$	$\begin{pmatrix} 0.328^* \\ (0.064^*) \end{pmatrix}$	$\begin{pmatrix} 0.436^* \\ (0.044^*) \end{pmatrix}$	$\begin{pmatrix} 0.366^* \\ (0.050^*) \end{pmatrix}$	$\begin{pmatrix} 0.404^* \\ (0.057^*) \end{pmatrix}$
	$(-0.6, 0.6)$	$\begin{pmatrix} 0.052^* \\ (0.441^*) \end{pmatrix}$	$\begin{pmatrix} 0.039^* \\ (0.449^*) \end{pmatrix}$	$\begin{pmatrix} 0.026^* \\ (0.507^*) \end{pmatrix}$	$\begin{pmatrix} 0.035^* \\ (0.513^*) \end{pmatrix}$	$\begin{pmatrix} 0.059^* \\ (0.318^*) \end{pmatrix}$	$\begin{pmatrix} 0.180^* \\ (0.204^*) \end{pmatrix}$	$\begin{pmatrix} 0.541^* \\ (0.079^*) \end{pmatrix}$	$\begin{pmatrix} 0.742^* \\ (0.072^*) \end{pmatrix}$	$\begin{pmatrix} 0.607^* \\ (0.118^*) \end{pmatrix}$
	$(-0.5, -0.2)$	$\begin{pmatrix} 0.029^* \\ (0.458^*) \end{pmatrix}$	$\begin{pmatrix} 0.026^* \\ (0.550^*) \end{pmatrix}$	$\begin{pmatrix} 0.036^* \\ (0.436^*) \end{pmatrix}$	$\begin{pmatrix} 0.048^* \\ (0.513) \end{pmatrix}$	$\begin{pmatrix} 0.148^* \\ (0.188^*) \end{pmatrix}$	$\begin{pmatrix} 0.371 \\ (0.062^*) \end{pmatrix}$	$\begin{pmatrix} 0.523^* \\ (0.046^*) \end{pmatrix}$	$\begin{pmatrix} 0.674^* \\ (0.028^*) \end{pmatrix}$	$\begin{pmatrix} 0.487^* \\ (0.037^*) \end{pmatrix}$
150	$(0.5, 0.2)$	$\begin{pmatrix} 0.055^* \\ (0.273^*) \end{pmatrix}$	$\begin{pmatrix} 0.068^* \\ (0.264^*) \end{pmatrix}$	$\begin{pmatrix} 0.075^* \\ (0.330^*) \end{pmatrix}$	$\begin{pmatrix} 0.170^* \\ (0.291^*) \end{pmatrix}$	$\begin{pmatrix} 0.269^* \\ (0.128^*) \end{pmatrix}$	$\begin{pmatrix} 0.411^* \\ (0.051^*) \end{pmatrix}$	$\begin{pmatrix} 0.368^* \\ (0.034^*) \end{pmatrix}$	$\begin{pmatrix} 0.339^* \\ (0.037^*) \end{pmatrix}$	$\begin{pmatrix} 0.348^* \\ (0.062^*) \end{pmatrix}$
	$(-0.8, -0.5)$	$\begin{pmatrix} 0.042^* \\ (1.184) \end{pmatrix}$	$\begin{pmatrix} 0.048^* \\ (1.440) \end{pmatrix}$	$\begin{pmatrix} 0.060^* \\ (1.585) \end{pmatrix}$	$\begin{pmatrix} 0.084^* \\ (1.234) \end{pmatrix}$	$\begin{pmatrix} 0.174^* \\ (0.272^*) \end{pmatrix}$	$\begin{pmatrix} 0.520 \\ (0.100^*) \end{pmatrix}$	$\begin{pmatrix} 1.285 \\ (0.076^*) \end{pmatrix}$	$\begin{pmatrix} 2.186 \\ (0.077^*) \end{pmatrix}$	$\begin{pmatrix} 2.769 \\ (0.067^*) \end{pmatrix}$
	$(-0.3, -0.1)$	$\begin{pmatrix} 0.029^* \\ (0.491^*) \end{pmatrix}$	$\begin{pmatrix} 0.026^* \\ (0.402^*) \end{pmatrix}$	$\begin{pmatrix} 0.029^* \\ (0.390^*) \end{pmatrix}$	$\begin{pmatrix} 0.044^* \\ (0.473^*) \end{pmatrix}$	$\begin{pmatrix} 0.135^* \\ (0.163^*) \end{pmatrix}$	$\begin{pmatrix} 0.358^* \\ (0.048^*) \end{pmatrix}$	$\begin{pmatrix} 0.452^* \\ (0.028^*) \end{pmatrix}$	$\begin{pmatrix} 0.504^* \\ (0.026^*) \end{pmatrix}$	$\begin{pmatrix} 0.457^* \\ (0.032^*) \end{pmatrix}$
	$(0.8, 0.6)$	$\begin{pmatrix} 0.293^* \\ (0.824^*) \end{pmatrix}$	$\begin{pmatrix} 0.289^* \\ (0.552^*) \end{pmatrix}$	$\begin{pmatrix} 0.363^* \\ (0.675) \end{pmatrix}$	$\begin{pmatrix} 0.350^* \\ (0.308^*) \end{pmatrix}$	$\begin{pmatrix} 0.662 \\ (0.168^*) \end{pmatrix}$	$\begin{pmatrix} 0.837 \\ (0.151^*) \end{pmatrix}$	$\begin{pmatrix} 1.087^* \\ (0.107^*) \end{pmatrix}$	$\begin{pmatrix} 0.736^* \\ (0.136^*) \end{pmatrix}$	$\begin{pmatrix} 1.268^* \\ (0.132^*) \end{pmatrix}$
	$(0.3, 0.1)$	$\begin{pmatrix} 0.043^* \\ (0.257^*) \end{pmatrix}$	$\begin{pmatrix} 0.040^* \\ (0.339^*) \end{pmatrix}$	$\begin{pmatrix} 0.037^* \\ (0.317^*) \end{pmatrix}$	$\begin{pmatrix} 0.072^* \\ (0.251^*) \end{pmatrix}$	$\begin{pmatrix} 0.224^* \\ (0.143^*) \end{pmatrix}$	$\begin{pmatrix} 0.329^* \\ (0.042^*) \end{pmatrix}$	$\begin{pmatrix} 0.314^* \\ (0.028^*) \end{pmatrix}$	$\begin{pmatrix} 0.339^* \\ (0.031^*) \end{pmatrix}$	$\begin{pmatrix} 0.333^* \\ (0.035^*) \end{pmatrix}$
50	$(-0.6, 0.6)$	$\begin{pmatrix} 1.084 \\ (0.695^*) \end{pmatrix}$	$\begin{pmatrix} 1.006 \\ (0.643) \end{pmatrix}$	$\begin{pmatrix} 0.958 \\ (0.662) \end{pmatrix}$	$\begin{pmatrix} 0.935 \\ (0.496) \end{pmatrix}$	$\begin{pmatrix} 0.835 \\ (0.547) \end{pmatrix}$	$\begin{pmatrix} 0.871 \\ (0.460) \end{pmatrix}$	$\begin{pmatrix} 0.816 \\ (0.606) \end{pmatrix}$	$\begin{pmatrix} 0.717 \\ (0.545) \end{pmatrix}$	$\begin{pmatrix} 0.701 \\ (0.816) \end{pmatrix}$
	$(-0.5, -0.2)$	$\begin{pmatrix>0.874 \\ (0.376) \end{pmatrix}$	$\begin{pmatrix>0.841 \\ (0.410) \end{pmatrix}$	$\begin{pmatrix>0.809 \\ (0.385) \end{pmatrix}$	$\begin{pmatrix>0.977 \\ (0.397) \end{pmatrix}$	$\begin{pmatrix>0.648 \\ (0.295) \end{pmatrix}$	$\begin{pmatrix>0.501 \\ (0.276) \end{pmatrix}$	$\begin{pmatrix>0.465 \\ (0.218) \end{pmatrix}$	$\begin{pmatrix>0.401 \\ (0.258) \end{pmatrix}$	$\begin{pmatrix>0.463 \\ (0.306) \end{pmatrix}$
	$(0.5, 0.2)$	$\begin{pmatrix>1.223 \\ (0.395) \end{pmatrix}$	$\begin{pmatrix>1.075 \\ (0.368) \end{pmatrix}$	$\begin{pmatrix>1.206 \\ (0.340) \end{pmatrix}$	$\begin{pmatrix>1.162 \\ (0.393) \end{pmatrix}$	$\begin{pmatrix>0.693 \\ (0.345) \end{pmatrix}$	$\begin{pmatrix>0.733 \\ (0.516) \end{pmatrix}$	$\begin{pmatrix>0.533 \\ (0.341) \end{pmatrix}$	$\begin{pmatrix>0.431^* \\ (0.240) \end{pmatrix}$	$\begin{pmatrix>0.581 \\ (0.451) \end{pmatrix}$
	$(-0.8, -0.5)$	$\begin{pmatrix>0.901 \\ (0.692^*) \end{pmatrix}$	$\begin{pmatrix>1.079 \\ (0.666) \end{pmatrix}$	$\begin{pmatrix>0.965 \\ (0.625^*) \end{pmatrix}$	$\begin{pmatrix>1.191 \\ (0.683) \end{pmatrix}$	$\begin{pmatrix>0.878 \\ (0.470) \end{pmatrix}$	$\begin{pmatrix>1.203 \\ (0.563) \end{pmatrix}$	$\begin{pmatrix>1.000 \\ (0.648) \end{pmatrix}$	$\begin{pmatrix>0.578^* \\ (0.508) \end{pmatrix}$	$\begin{pmatrix>0.647^* \\ (0.710) \end{pmatrix}$
	$(-0.3, -0.1)$	$\begin{pmatrix>0.671 \\ (0.505^*) \end{pmatrix}$	$\begin{pmatrix>0.909 \\ (0.524^*) \end{pmatrix}$	$\begin{pmatrix>0.762 \\ (0.396) \end{pmatrix}$	$\begin{pmatrix>0.691 \\ (0.348) \end{pmatrix}$	$\begin{pmatrix>0.545 \\ (0.335) \end{pmatrix}$	$\begin{pmatrix>0.510 \\ (0.283) \end{pmatrix}$	$\begin{pmatrix>0.432 \\ (0.181) \end{pmatrix}$	$\begin{pmatrix>0.522 \\ (0.197) \end{pmatrix}$	$\begin{pmatrix>0.528 \\ (0.353) \end{pmatrix}$
	$(0.8, 0.6)$	$\begin{pmatrix>3.019 \\ (0.827) \end{pmatrix}$	$\begin{pmatrix>3.232 \\ (0.662) \end{pmatrix}$	$\begin{pmatrix>3.139 \\ (0.811) \end{pmatrix}$	$\begin{pmatrix>2.022 \\ (0.852) \end{pmatrix}$	$\begin{pmatrix>2.123 \\ (0.706) \end{pmatrix}$	$\begin{pmatrix>1.166 \\ (0.936) \end{pmatrix}$	$\begin{pmatrix>1.864 \\ (0.920) \end{pmatrix}$	$\begin{pmatrix>1.016^* \\ (0.982) \end{pmatrix}$	$\begin{pmatrix>1.507 \\ (1.474) \end{pmatrix}$
	$(0.3, 0.1)$	$\begin{pmatrix>0.878 \\ (0.625) \end{pmatrix}$	$\begin{pmatrix>0.809 \\ (0.393) \end{pmatrix}$	$\begin{pmatrix>0.797 \\ (0.316^*) \end{pmatrix}$	$\begin{pmatrix>0.661 \\ (0.394) \end{pmatrix}$	$\begin{pmatrix>0.679 \\ (0.340) \end{pmatrix}$	$\begin{pmatrix>0.750 \\ (0.349) \end{pmatrix}$	$\begin{pmatrix>0.624 \\ (0.248) \end{pmatrix}$	$\begin{pmatrix>0.386 \\ (0.327) \end{pmatrix}$	$\begin{pmatrix>0.371^* \\ (0.331) \end{pmatrix}$
	$(-0.6, 0.6)$	$\begin{pmatrix>0.2>$								

Table 11: See notes in page 20

T	$(\alpha_1, \alpha_2)/r^0$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	(0.619*) (0.470*)	(0.273*) (0.540*)	(0.178*) (0.427*)	(0.103*) (0.301*)	(0.162*) (0.311*)	(0.113*) (0.682)	(0.319*) (0.505)	(0.610) (0.138*)	(0.710) (0.269*)
	$(-0.5, -0.2)$	(0.267*) (0.114*)	(0.211*) (0.102*)	(0.164*) (0.102*)	(0.154*) (0.118*)	(0.090*) (0.107*)	(0.139*) (0.122*)	(0.110*) (0.096*)	(0.112*) (0.137*)	(0.150*) (0.192*)
	$(0.5, 0.2)$	(0.595*) (0.113*)	(0.416*) (0.123*)	(0.236*) (0.162*)	(0.272*) (0.141*)	(0.306*) (0.134*)	(0.250*) (0.144*)	(0.258*) (0.160*)	(0.226*) (0.276*)	(0.409*) (0.270*)
	$(-0.8, -0.5)$	(0.245*) (0.199*)	(0.158*) (0.238*)	(0.154*) (0.251*)	(0.159*) (0.258*)	(0.273*) (0.251*)	(0.148*) (0.227*)	(0.155*) (0.198*)	(0.148*) (0.190*)	(0.158*) (0.237*)
	$(-0.3, -0.1)$	(0.338*) (0.113*)	(0.233*) (0.122*)	(0.181*) (0.092*)	(0.121*) (0.105*)	(0.107*) (0.102*)	(0.108*) (0.117*)	(0.166*) (0.097*)	(0.166*) (0.107*)	(0.153*) (0.130*)
	$(0.8, 0.6)$	(0.992*) (0.352*)	(0.706*) (0.311*)	(0.599*) (0.358*)	(0.680*) (0.357*)	(0.654*) (0.400*)	(0.707*) (0.403*)	(0.966*) (0.371*)	(0.858*) (0.467*)	(0.876*) (0.737*)
	$(0.3, 0.1)$	(0.403*) (0.120*)	(0.288*) (0.111*)	(0.236*) (0.137*)	(0.247*) (0.115*)	(0.230*) (0.110*)	(0.232*) (0.104*)	(0.249*) (0.137*)	(0.311*) (0.207*)	(0.337*) (0.169*)
	$(-0.6, 0.6)$	(0.123*) (0.858)	(0.073*) (0.773)	(0.038*) (1.308)	(0.049*) (1.404)	(0.104*) (1.209)	(0.383) (1.231)	(1.014) (0.378)	(3.034) (0.074)	(3.903) (0.140*)
100	$(-0.5, -0.2)$	(0.076*) (0.055*)	(0.056*) (0.063*)	(0.041*) (0.073*)	(0.120*) (0.119*)	(0.358) (0.424)	(0.238*) (0.097*)	(0.134*) (0.064*)	(0.086*) (0.081*)	(0.074*) (0.106*)
	$(0.5, 0.2)$	(0.120*) (0.120*)	(0.102*) (0.082*)	(0.102*) (0.140*)	(0.154*) (0.113*)	(0.222*) (0.153*)	(0.150*) (0.093*)	(0.139*) (0.070*)	(0.153*) (0.111*)	(0.135*) (0.114*)
	$(-0.8, -0.5)$	(0.137*) (0.092*)	(0.080*) (0.132*)	(0.182*) (0.406*)	(0.275*) (0.370*)	(1.057) (0.462)	(0.594) (0.177*)	(0.628*) (0.107*)	(0.278*) (0.091*)	(0.115*) (0.107*)
	$(-0.3, -0.1)$	(0.077*) (0.073*)	(0.061*) (0.080*)	(0.054*) (0.145*)	(0.055*) (0.071*)	(0.090*) (0.114*)	(0.160*) (0.058*)	(0.077*) (0.060*)	(0.075*) (0.085*)	(0.064*) (0.070*)
	$(0.8, 0.6)$	(0.384*) (0.207*)	(0.592*) (0.206*)	(1.194) (0.202*)	(1.281) (0.306*)	(1.546) (0.417*)	(0.669*) (0.281*)	(0.580*) (0.167*)	(0.647*) (0.218*)	(0.489*) (0.271*)
	$(0.3, 0.1)$	(0.112*) (0.065*)	(0.147*) (0.073*)	(0.073*) (0.078*)	(0.092*) (0.209*)	(0.223*) (0.090*)	(0.187*) (0.079*)	(0.095*) (0.054*)	(0.090*) (0.090*)	(0.118*) (0.104*)
	$(-0.6, 0.6)$	(0.057*) (3.481)	(0.057*) (3.519)	(0.017*) (3.341)	(0.055*) (1.705)	(0.139) (1.869)	(0.584) (1.051)	(2.160) (0.196)	(2.396) (0.042)	(3.392) (0.075*)
	$(-0.5, -0.2)$	(0.050*) (0.046*)	(0.041*) (0.057*)	(0.037*) (0.155*)	(0.082*) (0.145*)	(0.172*) (0.545)	(1.194) (0.080*)	(0.080*) (0.037*)	(0.051*) (0.045*)	(0.053*) (0.057*)
150	$(0.5, 0.2)$	(0.097*) (0.067*)	(0.086*) (0.078*)	(0.081*) (0.249*)	(0.414) (0.732)	(0.768) (0.246)	(1.250) (0.053*)	(0.176*) (0.040*)	(0.112*) (0.087*)	(0.086*) (0.090*)
	$(-0.8, -0.5)$	(0.049*) (0.064*)	(0.051*) (0.138*)	(0.179) (0.719)	(0.330) (1.452)	(1.129) (0.791)	(1.484) (0.433)	(1.715) (0.152)	(0.356*) (0.059*)	(0.225*) (0.102*)
	$(-0.3, -0.1)$	(0.046*) (0.046*)	(0.059*) (0.052*)	(0.030*) (0.193*)	(0.058*) (0.204*)	(0.263) (0.231)	(0.456) (0.097)	(0.076*) (0.030*)	(0.058*) (0.070*)	(0.043*) (0.067*)
	$(0.8, 0.6)$	(0.269*) (0.148*)	(0.460) (0.181*)	(1.027) (1.622)	(2.076) (1.133)	(2.451) (1.198)	(1.188) (0.425)	(0.779*) (0.103*)	(0.397*) (0.116*)	(0.403*) (0.298*)
	$(0.3, 0.1)$	(0.089*) (0.053*)	(0.067*) (0.059*)	(0.041*) (0.094*)	(0.100*) (0.343)	(0.820) (0.479)	(0.282) (0.050*)	(0.115*) (0.034*)	(0.067*) (0.088*)	(0.058*) (0.063*)
50	$(-0.6, 0.6)$	(2.237) (0.578)	(1.287) (0.581)	(1.079) (0.535)	(0.753) (0.608)	(0.986) (0.505)	(0.496) (0.251*)	(0.440) (0.164*)	(0.517*) (0.364)	(0.536*) (1.024)
	$(-0.5, -0.2)$	(1.620) (0.668)	(1.368) (0.736)	(1.243) (0.665)	(1.072) (0.658)	(1.037) (0.560)	(1.358) (0.660)	(1.067) (0.539)	(1.303) (0.904)	(1.495) (1.210)
	$(0.5, 0.2)$	(2.055) (0.563)	(1.676) (0.641)	(1.198) (0.827)	(1.401) (0.725)	(1.553) (0.593)	(1.388) (0.699)	(1.535) (0.814)	(1.321) (1.209)	(1.808) (1.169)
	$(-0.8, -0.5)$	(2.115) (1.038)	(1.447) (0.938)	(1.070) (1.186)	(1.266) (1.173)	(0.915) (0.945)	(1.823) (1.009)	(1.639) (1.027)	(1.807) (1.134)	(2.066) (1.594)
	$(-0.3, -0.1)$	(1.534) (0.787)	(1.430) (0.698)	(1.145) (0.645)	(0.883) (0.788)	(0.876) (0.660)	(0.943) (0.398)	(1.334) (0.585)	(1.206) (0.729)	(1.273) (0.829)
	$(0.8, 0.6)$	(3.054) (1.351)	(1.979) (1.373)	(2.234) (1.684)	(2.185) (1.285)	(2.024) (1.424)	(2.923) (1.662)	(4.017) (1.284)	(3.334) (1.750)	(3.620) (2.953)
	$(0.3, 0.1)$	(1.616) (0.722)	(1.343) (0.596)	(1.105) (0.811)	(1.119) (0.738)	(1.221) (0.583)	(1.245) (0.465)	(1.288) (0.669)	(1.476) (0.959)	(1.529) (0.783)
	$(-0.6, 0.6)$	(0.513) (0.645*)	(0.437) (0.597*)	(0.237) (0.545*)	(0.175) (0.441*)	(0.138) (0.408*)	(0.165*) (0.346*)	(0.284*) (0.107*)	(0.867*) (0.063*)	(1.003*) (0.284)
100	$(-0.5, -0.2)$	(0.563) (0.379)	(0.499) (0.314)	(0.316) (0.442)	(0.239) (0.371)	(0.204*) (0.202*)	(0.332) (0.168)	(0.542) (0.202)	(0.631) (0.334)	(0.622) (0.467)
	$(0.5, 0.2)$	(0.561) (0.440)	(0.530) (0.392)	(0.320) (0.375)	(0.333) (0.431)	(0.361) (0.189)	(0.809) (0.150)	(0.753) (0.204)	(0.914) (0.407)	(0.605) (0.469)
	$(-0.8, -0.5)$	(1.026) (0.539)	(0.504) (0.594)	(0.277) (0.461)	(0.374) (0.474)	(0.232*) (0.345*)	(0.461*) (0.430)	(0.719) (0.387)	(1.100) (0.330)	(1.559) (0.562)
	$(-0.3, -0.1)$	(0.532) (0.421)	(0.482) (0.426)	(0.171) (0.241)	(0.330) (0.415)	(0.352) (0.225)	(0.393) (0.216)	(0.476) (0.254)	(0.745) (0.369)	(0.495) (0.350)
	$(0.8, 0.6)$	(1.356) (0.692)	(0.713) (0.789)	(0.494*) (1.055)	(0.652*) (0.559)	(0.783*) (0.428)	(1.533) (0.378)	(1.763) (0.462)	(1.537) (0.698)	(2.239) (1.007)
	$(0.3, 0.1)$	(0.572) (0.338)	(0.603) (0.380)	(0.284) (0.360)	(0.266) (0.240)	(0.228) (0.269)	(0.395) (0.144)	(0.539) (0.158)	(0.658) (0.348)	(0.674) (0.416)
	$(-0.6, 0.6)$	(0.131) (1.493*)	(0.124) (1.350*)	(0.068) (1.366*)	(0.063) (0.620*)	(0.099*) (0.680*)	(0.236*) (0.287*)	(0.844*) (0.059*)	(1.084*) (0.026*)	(1.676*) (0.172)
	$(-0.5, -0.2)$	(0.250) (0.331)	(0.176) (0.366)	(0.108) (0.289)	(0.133) (0.372)	(0.203) (0.243*)	(0.493*) (0.132)	(0.480) (0.121)	(0.446) (0.218)	(0.328) (0.306)
150	$(0.5, 0.2)$	(0.291) (0.382)	(0.272) (0.351)	(0.134) (0.301)	(0.181*) (0.334*)	(0.334*) (0.159*)	(0.530*) (0.132)	(0.459) (0.148)	(0.503) (0.378)	(0.396) (0.399)
	$(-0.8, -0.5)$	(0.256) (0.552)	(0.150) (0.623)	(0.108*) (0.457*)	(0.177*) (0.522*)	(0.294*) (0.276*)	(0.413*) (0.210*)	(0.548*) (0.143*)	(0.612) (0.242)	(0.741) (0.461)
	$(-0.3, -0.1)$	(0.266) (0.386)	(0.206) (0.382)	(0.114) (0.263)	(0.114) (0.270)	(0.182*) (0.189*)	(0.304*) (0.094*)	(0.383) (0.132)	(0.274) (0.290)	(0.322) (0.298)
	$(0.8, 0.6)$	(0.568) (0.680)	(0.398*) (0.792)	(0.298*) (0.634*)	(0.507*) (0.432*)	(0.677*) (0.323*)	(0.803*) (0.198*)	(0.992) (0.407)	(1.467) (0.410)	(1.506) (1.024)
	$(0.3, 0.1)$	(0.321) (0.323)	(0.242) (0.340)	(0.158) (0.502)	(0.150) (0.235*)	(0.412*) (0.193*)	(0.282*) (0.099)	(0.366) (0.122)	(0.306) (0.370)	(0.237) (0.314)

Table 12: See notes in page 20

T	$(\alpha_1, \alpha_2)/r^0$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
50	$(-0.6, 0.6)$	(0.412*) (0.417*)	(0.174*) (0.552*)	(0.113*) (0.428*)	(0.112*) (0.263*)	(0.142*) (0.293*)	(0.130*) (0.356*)	(0.190*) (0.501)	(0.601) (0.145*)	(0.667) (0.241*)
	$(-0.5, -0.2)$	(0.113*) (0.134*)	(0.122*) (0.123*)	(0.151*) (0.100*)	(0.112*) (0.111*)	(0.102*) (0.096*)	(0.136*) (0.130*)	(0.097*) (0.101*)	(0.131*) (0.137*)	(0.117*) (0.122*)
	$(0.5, 0.2)$	(0.246*) (0.130*)	(0.257*) (0.160*)	(0.307*) (0.145*)	(0.209*) (0.172*)	(0.209*) (0.176*)	(0.299*) (0.157*)	(0.362*) (0.197*)	(0.272*) (0.202*)	(0.311*) (0.231*)
	$(-0.8, -0.5)$	(0.158*) (0.205*)	(0.186*) (0.196*)	(0.174*) (0.217*)	(0.150*) (0.214*)	(0.171*) (0.192*)	(0.154*) (0.201*)	(0.157*) (0.224*)	(0.168*) (0.177*)	(0.226*) (0.202*)
	$(-0.3, -0.1)$	(0.162*) (0.091*)	(0.107*) (0.109*)	(0.121*) (0.136*)	(0.118*) (0.118*)	(0.123*) (0.096*)	(0.152*) (0.100*)	(0.101*) (0.105*)	(0.168*) (0.109*)	(0.127*) (0.148*)
	$(0.8, 0.6)$	(1.157*) (0.374*)	(0.840*) (0.349*)	(0.692*) (0.392*)	(1.149*) (0.483*)	(0.859*) (0.479*)	(0.674*) (0.440*)	(0.847*) (0.417*)	(1.536*) (0.525*)	(0.916*) (0.502*)
	$(0.3, 0.1)$	(0.271*) (0.130*)	(0.178*) (0.120*)	(0.170*) (0.199*)	(0.172*) (0.139*)	(0.233*) (0.174*)	(0.216*) (0.177*)	(0.263*) (0.169*)	(0.156*) (0.141*)	(0.209*) (0.154*)
100	$(-0.6, 0.6)$	(0.140*) (0.927)	(0.091*) (1.443)	(0.044*) (1.264)	(0.053*) (0.654)	(0.059*) (0.250*)	(0.087*) (0.432)	(0.420) (0.422)	(2.020) (0.075)	(2.430) (0.142*)
	$(-0.5, -0.2)$	(0.069*) (0.071*)	(0.063*) (0.066*)	(0.044*) (0.061*)	(0.058*) (0.064*)	(0.061*) (0.067*)	(0.060*) (0.069*)	(0.079*) (0.087*)	(0.061*) (0.076*)	(0.074*) (0.076*)
	$(0.5, 0.2)$	(0.119*) (0.072*)	(0.103*) (0.090*)	(0.115*) (0.100*)	(0.118*) (0.083*)	(0.110*) (0.091*)	(0.122*) (0.080*)	(0.126*) (0.097*)	(0.154*) (0.097*)	(0.168*) (0.151*)
	$(-0.8, -0.5)$	(0.089*) (0.096*)	(0.079*) (0.096*)	(0.090*) (0.112*)	(0.083*) (0.106*)	(0.100*) (0.109*)	(0.100*) (0.102*)	(0.108*) (0.102*)	(0.123*) (0.093*)	(0.147*) (0.088*)
	$(-0.3, -0.1)$	(0.117*) (0.059*)	(0.054*) (0.091*)	(0.051*) (0.059*)	(0.073*) (0.055*)	(0.054*) (0.055*)	(0.062*) (0.069*)	(0.055*) (0.061*)	(0.061*) (0.067*)	(0.061*) (0.054*)
	$(0.8, 0.6)$	(0.424*) (0.184*)	(0.515*) (0.202*)	(0.476*) (0.213*)	(0.487*) (0.183*)	(0.467*) (0.211*)	(0.460*) (0.227*)	(0.464*) (0.197*)	(0.515*) (0.199*)	(0.488*) (0.291*)
	$(0.3, 0.1)$	(0.084*) (0.068*)	(0.077*) (0.063*)	(0.107*) (0.072*)	(0.088*) (0.076*)	(0.091*) (0.093*)	(0.097*) (0.074*)	(0.075*) (0.064*)	(0.094*) (0.081*)	(0.120*) (0.070*)
150	$(-0.6, 0.6)$	(0.061*) (4.550)	(0.074*) (3.485)	(0.021*) (1.856)	(0.033*) (0.625)	(0.054*) (0.563)	(0.056*) (0.795)	(0.546) (0.222)	(2.120) (0.029*)	(3.594) (0.084*)
	$(-0.5, -0.2)$	(0.049*) (0.053*)	(0.071*) (0.086*)	(0.039*) (0.050*)	(0.040*) (0.046*)	(0.047*) (0.064*)	(0.047*) (0.044*)	(0.051*) (0.042*)	(0.054*) (0.053*)	(0.045*) (0.073*)
	$(0.5, 0.2)$	(0.072*) (0.057*)	(0.084*) (0.053*)	(0.079*) (0.063*)	(0.075*) (0.055*)	(0.070*) (0.057*)	(0.074*) (0.051*)	(0.086*) (0.059*)	(0.085*) (0.093*)	(0.087*) (0.078*)
	$(-0.8, -0.5)$	(0.046*) (0.066*)	(0.045*) (0.087*)	(0.061*) (0.077*)	(0.092*) (0.071*)	(0.058*) (0.068*)	(0.057*) (0.081*)	(0.153*) (0.068*)	(0.097*) (0.067*)	(0.085*) (0.110*)
	$(-0.3, -0.1)$	(0.054*) (0.067*)	(0.056*) (0.041*)	(0.037*) (0.045*)	(0.046*) (0.041*)	(0.037*) (0.099*)	(0.043*) (0.044*)	(0.048*) (0.041*)	(0.057*) (0.057*)	(0.066*) (0.049*)
	$(0.8, 0.6)$	(0.324*) (0.121*)	(0.340*) (0.131*)	(0.384*) (0.119*)	(0.301*) (0.158*)	(0.335*) (0.141*)	(0.314*) (0.135*)	(0.282*) (0.142*)	(0.356*) (0.132*)	(0.475*) (0.218*)
	$(0.3, 0.1)$	(0.075*) (0.048*)	(0.072*) (0.055*)	(0.065*) (0.047*)	(0.053*) (0.052*)	(0.055*) (0.055*)	(0.080*) (0.046*)	(0.076*) (0.055*)	(0.060*) (0.059*)	(0.055*) (0.055*)
50	$(-0.6, 0.6)$	(1.340) (0.711)	(0.683) (0.644)	(0.565) (0.582)	(0.686) (0.546)	(0.747) (0.470)	(0.813) (0.465)	(0.543) (0.203*)	(0.564*) (0.220)	(0.561*) (0.513)
	$(-0.5, -0.2)$	(0.849) (0.485)	(0.864) (0.488)	(0.958) (0.347)	(0.784) (0.357)	(0.764) (0.404)	(1.007) (0.469)	(0.782) (0.425)	(0.963) (0.525)	(0.901) (0.476)
	$(0.5, 0.2)$	(1.059) (0.392)	(1.062) (0.478)	(1.282) (0.489)	(0.855) (0.425)	(0.841) (0.515)	(1.297) (0.408)	(1.648) (0.555)	(1.428) (0.583)	(1.342) (0.577)
	$(-0.8, -0.5)$	(1.107) (0.744)	(1.204) (0.461)	(1.254) (0.614)	(1.116) (0.749)	(1.308) (0.749)	(1.198) (0.749)	(1.237) (0.873)	(1.483) (0.588)	(1.205) (0.677)
	$(-0.3, -0.1)$	(0.858) (0.350)	(0.712) (0.343)	(0.766) (0.491)	(0.803) (0.366)	(0.742) (0.346)	(0.814) (0.359)	(0.737) (0.422)	(1.102) (0.413)	(0.871) (0.476)
	$(0.8, 0.6)$	(3.118) (0.720)	(2.541) (0.734)	(2.118) (0.944)	(3.509) (1.215)	(2.646) (1.150)	(1.988) (1.098)	(2.647) (0.925)	(4.635) (1.292)	(3.353) (1.149)
	$(0.3, 0.1)$	(1.079) (0.446)	(0.907) (0.386)	(0.809) (0.611)	(0.872) (0.423)	(1.011) (0.594)	(0.928) (0.549)	(1.157) (0.507)	(0.780) (0.402)	(1.069) (0.520)
100	$(-0.6, 0.6)$	(0.293) (0.672*)	(0.208) (0.569*)	(0.155) (0.440*)	(0.170) (0.326*)	(0.247) (0.314)	(0.203) (0.293*)	(0.280*) (0.102*)	(0.845*) (0.062*)	(1.000*) (0.274)
	$(-0.5, -0.2)$	(0.287) (0.486)	(0.321) (0.394)	(0.265) (0.334)	(0.289) (0.333)	(0.303) (0.410)	(0.298) (0.408)	(0.253) (0.421)	(0.316) (0.393)	(0.243) (0.430)
	$(0.5, 0.2)$	(0.435) (0.302)	(0.315) (0.431)	(0.342) (0.472)	(0.413) (0.351)	(0.388) (0.362)	(0.429) (0.417)	(0.467) (0.399)	(0.561) (0.438)	(0.402) (0.636)
	$(-0.8, -0.5)$	(0.405) (0.514)	(0.317) (0.538)	(0.353) (0.657)	(0.426) (0.494)	(0.392) (0.550)	(0.417) (0.632)	(0.562) (0.492)	(0.544) (0.657)	(0.599) (0.549)
	$(-0.3, -0.1)$	(0.422) (0.382)	(0.289) (0.446)	(0.251) (0.324)	(0.358) (0.386)	(0.279) (0.357)	(0.320) (0.359)	(0.231) (0.431)	(0.315) (0.349)	(0.270) (0.339)
	$(0.8, 0.6)$	(1.032) (0.824)	(0.922) (0.694)	(1.209) (0.991)	(1.369) (0.714)	(1.312) (0.763)	(1.351) (0.942)	(1.504) (0.899)	(1.662) (0.913)	(1.538) (1.144)
	$(0.3, 0.1)$	(0.292) (0.354)	(0.256) (0.352)	(0.423) (0.375)	(0.281) (0.343)	(0.306) (0.476)	(0.341) (0.386)	(0.253) (0.288)	(0.347) (0.373)	(0.442) (0.370)
150	$(-0.6, 0.6)$	(0.139) (2.046*)	(0.155) (1.379*)	(0.087) (0.675*)	(0.111) (0.324*)	(0.130) (0.239*)	(0.186) (0.301*)	(0.282*) (0.071*)	(0.963*) (0.030)	(1.825*) (0.212)
	$(-0.5, -0.2)$	(0.272) (0.364)	(0.315) (0.443)	(0.204) (0.324)	(0.245) (0.274)	(0.312) (0.381)	(0.328) (0.309)	(0.233) (0.242)	(0.282) (0.375)	(0.273) (0.429)
	$(0.5, 0.2)$	(0.240) (0.258)	(0.316) (0.324)	(0.288) (0.341)	(0.363) (0.241)	(0.304) (0.342)	(0.355) (0.285)	(0.446) (0.319)	(0.469) (0.369)	(0.356) (0.390)
	$(-0.8, -0.5)$	(0.277) (0.420)	(0.280) (0.450)	(0.273) (0.499)	(0.273) (0.597)	(0.447) (0.480)	(0.441) (0.561)	(0.302) (0.409)	(0.501) (0.376)	(0.663) (0.633)
	$(-0.3, -0.1)$	(0.301) (0.396)	(0.202) (0.269)	(0.207) (0.302)	(0.238) (0.250)	(0.184) (0.477)	(0.244) (0.291)	(0.255) (0.284)	(0.299) (0.359)	(0.348) (0.340)
	$(0.8, 0.6)$	(0.922) (0.623)	(0.779) (0.594)	(0.672) (0.611)	(1.097) (0.815)	(0.875) (0.717)	(0.700) (0.701)	(0.875) (0.717)	(1.352) (0.598)	(1.815) (0.923)
	$(0.3, 0.1)$	(0.292) (0.241)	(0.288) (0.308)	(0.263) (0.258)	(0.277) (0.304)	(0.216) (0.310)	(0.336) (0.249)	(0.351) (0.287)	(0.290) (0.326)	(0.235) (0.324)

Table 13: Three regime symmetric TAR model. Results for Method A: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the Method performs better than the other methods

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.907 (0.191)	0.336 (0.180)	0.202 (0.202)	0.186 (0.152)	0.174* (0.086)
	$(-0.5, -0.2)$	0.984* (0.111)	0.312* (0.116)	0.160 (0.146)	0.133* (0.116)	0.173* (0.091)
	$(0.5, 0.2)$	0.999* (0.117)	0.339* (0.129)	0.137* (0.134)	0.139* (0.124)	0.165* (0.086)
	$(-0.8, -0.5)$	0.764* (0.270)	0.333* (0.276)	0.325 (0.313)	0.358* (0.248)	0.343* (0.149)
	$(-0.3, -0.1)$	0.987* (0.095)	0.326* (0.095)	0.121* (0.107)	0.106* (0.096)	0.137* (0.077)
	$(0.8, 0.6)$	0.701* (0.257)	0.349* (0.278)	0.273* (0.273)	0.306* (0.235)	0.325* (0.142)
	$(0.3, 0.1)$	1.052* (0.098)	0.327* (0.100)	0.138* (0.122)	0.119* (0.106)	0.126* (0.068)
	$(-0.6, 0.6)$	0.558 (0.107)	0.183 (0.121)	0.152 (0.152)	0.088 (0.087)	0.048* (0.037)
100	$(-0.5, -0.2)$	0.706* (0.057)	0.147* (0.057)	0.104 (0.100)	0.092 (0.090)	0.051* (0.041)
	$(0.5, 0.2)$	0.694* (0.054)	0.166* (0.061)	0.100 (0.099)	0.092 (0.091)	0.054* (0.040)
	$(-0.8, -0.5)$	0.413* (0.107)	0.201 (0.188)	0.203 (0.203)	0.123* (0.120)	0.101* (0.068)
	$(-0.3, -0.1)$	0.806* (0.052)	0.189* (0.054)	0.085* (0.081)	0.080 (0.080)	0.049* (0.042)
	$(0.8, 0.6)$	0.329* (0.125)	0.211* (0.193)	0.239 (0.239)	0.166* (0.158)	0.111* (0.079)
	$(0.3, 0.1)$	0.772* (0.048)	0.184* (0.054)	0.088* (0.086)	0.073 (0.073)	0.046* (0.038)
	$(-0.6, 0.6)$	0.448 (0.070)	0.122 (0.084)	0.113 (0.113)	0.077 (0.076)	0.022* (0.021)
	$(-0.5, -0.2)$	0.580* (0.027)	0.116* (0.044)	0.074 (0.074)	0.066 (0.066)	0.022* (0.021)
150	$(0.5, 0.2)$	0.604* (0.032)	0.111* (0.042)	0.073 (0.073)	0.074 (0.074)	0.028* (0.026)
	$(-0.8, -0.5)$	0.283* (0.078)	0.107 (0.104)	0.178 (0.178)	0.090* (0.090)	0.046* (0.038)
	$(-0.3, -0.1)$	0.653* (0.028)	0.132* (0.032)	0.061 (0.061)	0.053 (0.053)	0.019* (0.019)
	$(0.8, 0.6)$	0.240* (0.093)	0.158* (0.155)	0.208 (0.208)	0.105* (0.105)	0.049* (0.039)
	$(0.3, 0.1)$	0.666* (0.027)	0.127* (0.029)	0.066* (0.066)	0.062 (0.062)	0.019* (0.019)
	$(-0.6, 0.6)$	0.448 (0.070)	0.122 (0.084)	0.113 (0.113)	0.077 (0.076)	0.022* (0.021)

Table 14: Three regime symmetric TAR model. Results for Method B: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the Method performs better than the other methods

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.572* (0.170)	0.092* (0.061)	0.041* (0.040)	0.158* (0.122)	0.490 (0.200)
	$(-0.5, -0.2)$	1.786 (0.157)	0.507 (0.181)	0.121* (0.106)	0.186 (0.147)	0.402 (0.178)
	$(0.5, 0.2)$	1.712 (0.171)	0.610 (0.173)	0.161 (0.131)	0.169 (0.132)	0.411 (0.164)
	$(-0.8, -0.5)$	1.140 (0.335)	0.360 (0.251)	0.212* (0.211)	0.551 (0.350)	1.031 (0.376)
	$(-0.3, -0.1)$	1.742 (0.151)	0.577 (0.158)	0.133 (0.107)	0.111 (0.097)	0.300 (0.127)
	$(0.8, 0.6)$	1.213 (0.326)	0.489 (0.258)	0.287 (0.286)	0.526 (0.298)	0.928 (0.306)
	$(0.3, 0.1)$	1.782 (0.151)	0.667 (0.161)	0.161 (0.116)	0.130 (0.109)	0.319 (0.140)
	$(-0.6, 0.6)$	0.350* (0.086)	0.026* (0.020)	0.013* (0.013)	0.071* (0.066)	0.300 (0.194)
100	$(-0.5, -0.2)$	1.743 (0.194)	0.429 (0.186)	0.084* (0.077)	0.078* (0.071)	0.244 (0.150)
	$(0.5, 0.2)$	1.705 (0.214)	0.471 (0.193)	0.099* (0.086)	0.085* (0.081)	0.273 (0.157)
	$(-0.8, -0.5)$	0.925 (0.320)	0.197* (0.165)	0.110* (0.110)	0.272 (0.219)	0.810 (0.434)
	$(-0.3, -0.1)$	1.845 (0.172)	0.613 (0.197)	0.087 (0.077)	0.059* (0.059)	0.215 (0.141)
	$(0.8, 0.6)$	0.954 (0.328)	0.351 (0.239)	0.227* (0.227)	0.452 (0.298)	0.929 (0.405)
	$(0.3, 0.1)$	1.940 (0.190)	0.566 (0.189)	0.105 (0.092)	0.073* (0.069)	0.195 (0.125)
	$(-0.6, 0.6)$	0.260* (0.040)	0.011* (0.009)	0.006* (0.006)	0.035* (0.034)	0.188 (0.140)
	$(-0.5, -0.2)$	1.704 (0.220)	0.311 (0.165)	0.037* (0.037)	0.043* (0.042)	0.197 (0.144)
150	$(0.5, 0.2)$	1.684 (0.223)	0.299 (0.146)	0.063* (0.059)	0.055* (0.055)	0.221 (0.145)
	$(-0.8, -0.5)$	0.869 (0.353)	0.089* (0.083)	0.055* (0.055)	0.158 (0.143)	0.536 (0.359)
	$(-0.3, -0.1)$	2.008 (0.194)	0.476 (0.187)	0.050* (0.048)	0.038* (0.037)	0.124 (0.096)
	$(0.8, 0.6)$	0.751 (0.302)	0.199 (0.156)	0.164* (0.164)	0.302 (0.235)	0.769 (0.387)
	$(0.3, 0.1)$	1.914 (0.187)	0.487 (0.185)	0.069 (0.064)	0.046* (0.046)	0.134 (0.102)

Table 15: Three regime symmetric TAR model. Results for Method C: Estimated MSE of threshold parameter estimator (Estimated variance in subscript). Stars denote case where the Method performs better than the other methods

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.629 (0.194)	0.114 (0.077)	0.052 (0.051)	0.267 (0.194)	0.653 (0.213)
	$(-0.5, -0.2)$	1.792 (0.160)	0.655 (0.183)	0.196 (0.153)	0.276 (0.177)	0.483 (0.177)
	$(0.5, 0.2)$	1.782 (0.165)	0.710 (0.163)	0.243 (0.171)	0.237 (0.172)	0.518 (0.180)
	$(-0.8, -0.5)$	1.354 (0.338)	0.490 (0.303)	0.351 (0.350)	0.728 (0.382)	1.177 (0.380)
	$(-0.3, -0.1)$	1.861 (0.142)	0.736 (0.156)	0.208 (0.137)	0.187 (0.142)	0.420 (0.145)
	$(0.8, 0.6)$	1.427 (0.273)	0.619 (0.303)	0.347 (0.347)	0.593 (0.355)	0.935 (0.319)
	$(0.3, 0.1)$	1.858 (0.140)	0.753 (0.147)	0.233 (0.122)	0.186 (0.139)	0.386 (0.137)
	$(-0.6, 0.6)$	0.394 (0.112)	0.032 (0.024)	0.024 (0.024)	0.169 (0.147)	0.551 (0.241)
100	$(-0.5, -0.2)$	1.935 (0.185)	0.789 (0.214)	0.235 (0.170)	0.214 (0.175)	0.481 (0.207)
	$(0.5, 0.2)$	1.881 (0.188)	0.710 (0.202)	0.262 (0.177)	0.213 (0.186)	0.415 (0.185)
	$(-0.8, -0.5)$	1.345 (0.331)	0.491 (0.308)	0.360 (0.360)	0.656 (0.419)	1.108 (0.454)
	$(-0.3, -0.1)$	1.987 (0.147)	0.877 (0.172)	0.271 (0.148)	0.169 (0.155)	0.359 (0.168)
	$(0.8, 0.6)$	1.387 (0.314)	0.676 (0.311)	0.357 (0.353)	0.635 (0.398)	1.000 (0.402)
	$(0.3, 0.1)$	2.068 (0.161)	0.822 (0.176)	0.275 (0.170)	0.172 (0.154)	0.335 (0.153)
	$(-0.6, 0.6)$	0.297 (0.058)	0.013 (0.009)	0.009 (0.009)	0.105 (0.100)	0.467 (0.249)
	$(-0.5, -0.2)$	1.993 (0.202)	0.673 (0.225)	0.220 (0.164)	0.197 (0.188)	0.394 (0.186)
150	$(0.5, 0.2)$	1.910 (0.207)	0.678 (0.232)	0.260 (0.178)	0.217 (0.193)	0.400 (0.204)
	$(-0.8, -0.5)$	1.246 (0.402)	0.382 (0.271)	0.328 (0.320)	0.510 (0.403)	0.958 (0.445)
	$(-0.3, -0.1)$	2.117 (0.154)	0.877 (0.203)	0.300 (0.171)	0.179 (0.173)	0.325 (0.173)
	$(0.8, 0.6)$	1.438 (0.367)	0.630 (0.300)	0.357 (0.336)	0.617 (0.408)	0.975 (0.433)
	$(0.3, 0.1)$	2.085 (0.172)	0.915 (0.179)	0.276 (0.159)	0.170 (0.162)	0.302 (0.162)

Table 16: Three regime symmetric TAR model. Results for Method A: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.008*	0.007*	0.007*	0.007*	0.020
	$(-0.5, -0.2)$	0.004*	0.005*	0.003*	0.007*	0.008*
	$(0.5, 0.2)$	0.003*	0.003*	0.004*	0.004*	0.004*
	$(-0.8, -0.5)$	0.026*	0.034*	0.040*	0.057	0.086
	$(-0.3, -0.1)$	0.003*	0.004*	0.003*	0.003*	0.004*
	$(0.8, 0.6)$	0.023*	0.025*	0.022*	0.026*	0.033
	$(0.3, 0.1)$	0.003*	0.003*	0.002*	0.002*	0.003*
100	$(-0.6, 0.6)$	0.007*	0.008*	0.005*	0.008*	0.015
	$(-0.5, -0.2)$	0.005*	0.008*	0.005*	0.004*	0.007*
	$(0.5, 0.2)$	0.003*	0.004*	0.003*	0.004*	0.004*
	$(-0.8, -0.5)$	0.034*	0.033*	0.038*	0.063	0.087
	$(-0.3, -0.1)$	0.003*	0.004*	0.003*	0.002*	0.003*
	$(0.8, 0.6)$	0.026*	0.021*	0.015*	0.020*	0.022*
	$(0.3, 0.1)$	0.003*	0.003*	0.002*	0.002*	0.002*
150	$(-0.6, 0.6)$	0.005*	0.006*	0.005*	0.005*	0.006*
	$(-0.5, -0.2)$	0.004*	0.004*	0.004*	0.004*	0.003*
	$(0.5, 0.2)$	0.002*	0.002*	0.003*	0.002*	0.002*
	$(-0.8, -0.5)$	0.022*	0.034	0.028*	0.044	0.041
	$(-0.3, -0.1)$	0.002*	0.003*	0.002*	0.003*	0.002*
	$(0.8, 0.6)$	0.017*	0.018*	0.012*	0.020*	0.014*
	$(0.3, 0.1)$	0.002*	0.002*	0.001*	0.002*	0.001*
50	$(-0.6, 0.6)$	0.023	0.018	0.024	0.015	0.019*
	$(-0.5, -0.2)$	0.008	0.009	0.010	0.011	0.012
	$(0.5, 0.2)$	0.007	0.009	0.008	0.008	0.010
	$(-0.8, -0.5)$	0.058	0.081	0.095	0.053*	0.061*
	$(-0.3, -0.1)$	0.006	0.006	0.006	0.006	0.006
	$(0.8, 0.6)$	0.037	0.038	0.038	0.037	0.032*
	$(0.3, 0.1)$	0.006	0.006	0.007	0.005	0.006
100	$(-0.6, 0.6)$	0.015	0.012	0.013	0.011	0.015*
	$(-0.5, -0.2)$	0.008	0.010	0.008	0.009	0.010
	$(0.5, 0.2)$	0.008	0.008	0.006	0.007	0.011
	$(-0.8, -0.5)$	0.048	0.050	0.052	0.059*	0.049*
	$(-0.3, -0.1)$	0.007	0.006	0.005	0.004	0.005
	$(0.8, 0.6)$	0.039	0.032	0.027	0.028	0.037
	$(0.3, 0.1)$	0.009	0.006	0.004	0.005	0.006
150	$(-0.6, 0.6)$	0.013	0.012	0.007	0.010	0.011
	$(-0.5, -0.2)$	0.009	0.007	0.007	0.006	0.005
	$(0.5, 0.2)$	0.006	0.007	0.006	0.006	0.005
	$(-0.8, -0.5)$	0.026	0.029*	0.029	0.040*	0.028*
	$(-0.3, -0.1)$	0.006	0.007	0.005	0.005	0.004
	$(0.8, 0.6)$	0.033	0.026	0.018	0.030	0.027
	$(0.3, 0.1)$	0.006	0.006	0.003	0.003	0.004

Table 17: Three regime symmetric TAR model. Results for Method B: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.008*	0.029	0.047	0.028	0.015*
	$(-0.5, -0.2)$	0.004*	0.004*	0.011	0.010*	0.006*
	$(0.5, 0.2)$	0.004*	0.004*	0.006*	0.007*	0.005*
	$(-0.8, -0.5)$	0.036*	0.067	0.141	0.087*	0.059*
	$(-0.3, -0.1)$	0.003*	0.003*	0.004*	0.005	0.004*
	$(0.8, 0.6)$	0.028*	0.031	0.039*	0.042*	0.034*
	$(0.3, 0.1)$	0.003*	0.003*	0.003*	0.003*	0.003*
100	$(-0.6, 0.6)$	0.024	0.016	0.021	0.043	0.017
	$(-0.5, -0.2)$	0.004*	0.004*	0.015	0.023	0.012
	$(0.5, 0.2)$	0.006*	0.003*	0.013	0.017	0.006*
	$(-0.8, -0.5)$	0.022*	0.091	0.207	0.175	0.054*
	$(-0.3, -0.1)$	0.003*	0.004*	0.009	0.011	0.004*
	$(0.8, 0.6)$	0.024*	0.048	0.077	0.050	0.037*
	$(0.3, 0.1)$	0.005*	0.003*	0.005	0.008	0.003*
150	$(-0.6, 0.6)$	0.027	0.005*	0.007	0.042	0.031
	$(-0.5, -0.2)$	0.005*	0.005*	0.023	0.025	0.012
	$(0.5, 0.2)$	0.006*	0.007	0.015	0.018	0.007*
	$(-0.8, -0.5)$	0.013*	0.113	0.200	0.259	0.096
	$(-0.3, -0.1)$	0.004*	0.003*	0.015	0.010	0.009
	$(0.8, 0.6)$	0.019*	0.082	0.100	0.109	0.042*
	$(0.3, 0.1)$	0.004*	0.003*	0.010	0.009	0.005
50	$(-0.6, 0.6)$	0.015	0.011*	0.012*	0.012*	0.020
	$(-0.5, -0.2)$	0.012	0.015	0.006*	0.010	0.014
	$(0.5, 0.2)$	0.015	0.013	0.007	0.007	0.011
	$(-0.8, -0.5)$	0.077	0.056*	0.048*	0.093	0.094
	$(-0.3, -0.1)$	0.010	0.010	0.005	0.005*	0.007
	$(0.8, 0.6)$	0.063	0.030*	0.046	0.059	0.049
	$(0.3, 0.1)$	0.011	0.013	0.006	0.005	0.008
100	$(-0.6, 0.6)$	0.017*	0.009*	0.006*	0.012*	0.017*
	$(-0.5, -0.2)$	0.013	0.010	0.007*	0.009*	0.009*
	$(0.5, 0.2)$	0.017	0.012	0.007*	0.008*	0.008
	$(-0.8, -0.5)$	0.034	0.027*	0.044*	0.057*	0.108
	$(-0.3, -0.1)$	0.010	0.016	0.006*	0.006*	0.007
	$(0.8, 0.6)$	0.040	0.023*	0.032*	0.033*	0.073
	$(0.3, 0.1)$	0.015	0.013	0.004*	0.005*	0.005
150	$(-0.6, 0.6)$	0.027*	0.005	0.003*	0.010*	0.012*
	$(-0.5, -0.2)$	0.019	0.008	0.010*	0.008*	0.008*
	$(0.5, 0.2)$	0.020	0.005*	0.008*	0.009*	0.007
	$(-0.8, -0.5)$	0.044	0.049*	0.041*	0.051*	0.066*
	$(-0.3, -0.1)$	0.015	0.011	0.008*	0.005*	0.005*
	$(0.8, 0.6)$	0.025	0.026*	0.025*	0.027*	0.050
	$(0.3, 0.1)$	0.013	0.010	0.006*	0.005*	0.005*

Table 18: Three regime symmetric TAR model. Results for Method C: Estimated MSE of bootstrap variance of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	(α_1, α_2)	r_2^0				
		2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	0.006*	0.022	0.040	0.009*	0.011*
	$(-0.5, -0.2)$	0.004*	0.005*	0.005*	0.007*	0.006*
	$(0.5, 0.2)$	0.004*	0.004*	0.004*	0.005*	0.005*
	$(-0.8, -0.5)$	0.040*	0.050*	0.063*	0.062*	0.052*
	$(-0.3, -0.1)$	0.002*	0.003*	0.003*	0.003*	0.003*
	$(0.8, 0.6)$	0.032*	0.030*	0.040*	0.039*	0.036*
	$(0.3, 0.1)$	0.003*	0.003*	0.004*	0.003*	0.003*
	$(-0.6, 0.6)$	0.019*	0.018	0.024	0.017	0.007*
100	$(-0.5, -0.2)$	0.005*	0.005*	0.005*	0.005*	0.005*
	$(0.5, 0.2)$	0.004*	0.004*	0.003*	0.004*	0.004*
	$(-0.8, -0.5)$	0.023*	0.039*	0.046*	0.037*	0.037*
	$(-0.3, -0.1)$	0.002*	0.003*	0.003*	0.003*	0.003*
	$(0.8, 0.6)$	0.025*	0.033*	0.035*	0.029*	0.032*
	$(0.3, 0.1)$	0.002*	0.003*	0.003*	0.002*	0.002*
	$(-0.6, 0.6)$	0.027*	0.007*	0.012	0.030	0.005*
	$(-0.5, -0.2)$	0.004*	0.005*	0.005*	0.004*	0.004*
150	$(0.5, 0.2)$	0.005*	0.006*	0.003*	0.003*	0.003*
	$(-0.8, -0.5)$	0.024*	0.034	0.055	0.042*	0.035*
	$(-0.3, -0.1)$	0.002*	0.006*	0.002*	0.003*	0.002*
	$(0.8, 0.6)$	0.017*	0.030	0.040*	0.030*	0.028*
	$(0.3, 0.1)$	0.003*	0.003*	0.002*	0.002*	0.002*
50	$(-0.6, 0.6)$	0.019	0.011*	0.017*	0.019	0.019
	$(-0.5, -0.2)$	0.012	0.015	0.010	0.016	0.013
	$(0.5, 0.2)$	0.013	0.011	0.014	0.012	0.012
	$(-0.8, -0.5)$	0.066	0.079	0.092	0.105	0.099
	$(-0.3, -0.1)$	0.008	0.010	0.007	0.008	0.008
	$(0.8, 0.6)$	0.045	0.048	0.071	0.069	0.055
	$(0.3, 0.1)$	0.009	0.008	0.007	0.007	0.008
	$(-0.6, 0.6)$	0.021	0.014*	0.010*	0.012*	0.022
100	$(-0.5, -0.2)$	0.014	0.016	0.010	0.012	0.016
	$(0.5, 0.2)$	0.012	0.015	0.011	0.012	0.010
	$(-0.8, -0.5)$	0.049	0.057	0.071	0.081	0.100
	$(-0.3, -0.1)$	0.007	0.011	0.007	0.008	0.010
	$(0.8, 0.6)$	0.049	0.037	0.053	0.063	0.072
	$(0.3, 0.1)$	0.009	0.011	0.010	0.008	0.007
	$(-0.6, 0.6)$	0.032	0.008	0.006*	0.012*	0.023
	$(-0.5, -0.2)$	0.015	0.019	0.008	0.011	0.011
150	$(0.5, 0.2)$	0.016	0.020	0.010	0.011	0.014
	$(-0.8, -0.5)$	0.074	0.029*	0.047*	0.068	0.085
	$(-0.3, -0.1)$	0.007	0.019	0.009	0.010	0.010
	$(0.8, 0.6)$	0.057	0.029*	0.047	0.055	0.077
	$(0.3, 0.1)$	0.011	0.011	0.007	0.008	0.008

Table 19: Three regime symmetric TAR model. Results for Method A: Estimated MSE of bootstrap confidence interval ($\frac{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	$(\alpha_1, \alpha_2)/r_2^0$	Standard bootstrap					Subsample bootstrap				
		2	1.5	1	0.5	0.2	2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	(0.026*) (0.200*)	(0.020*) (0.111*)	(0.005*) (0.124*)	(0.002*) (0.135*)	(0.002*) (0.291)	(0.094) (0.482)	(0.085) (0.261)	(0.060) (0.326)	(0.032) (0.191)	(0.022) (0.227*)
	$(-0.5, -0.2)$	(0.020*) (0.068*)	(0.021*) (0.094*)	(0.005*) (0.074*)	(0.002*) (0.088*)	(0.002*) (0.117*)	(0.118) (0.169)	(0.075) (0.246)	(0.074) (0.212)	(0.023) (0.161)	(0.026) (0.189)
	$(0.5, 0.2)$	(0.041*) (0.068*)	(0.025*) (0.071*)	(0.013*) (0.080*)	(0.002*) (0.090*)	(0.001*) (0.115*)	(0.113) (0.153)	(0.084) (0.197)	(0.043) (0.221)	(0.013) (0.149)	(0.012) (0.179)
	$(-0.8, -0.5)$	(0.017*) (0.626*)	(0.011*) (0.355*)	(0.007*) (0.320*)	(0.006*) (0.406*)	(0.007*) (0.722)	(0.130) (1.012)	(0.110) (0.680)	(0.112) (0.869)	(0.083) (0.518)	(0.062) (0.468*)
	$(-0.3, -0.1)$	(0.025*) (0.058*)	(0.023*) (0.050*)	(0.008*) (0.055*)	(0.001*) (0.058*)	(0.001*) (0.077*)	(0.080) (0.163)	(0.055) (0.126)	(0.064) (0.112)	(0.023) (0.111)	(0.009) (0.144)
	$(0.8, 0.6)$	(0.039*) (0.384*)	(0.022*) (0.320*)	(0.009*) (0.244*)	(0.004*) (0.274*)	(0.004*) (0.336)	(0.110) (0.892)	(0.093) (0.767)	(0.038) (0.571)	(0.027) (0.404)	(0.013) (0.308*)
	$(0.3, 0.1)$	(0.033*) (0.062*)	(0.035*) (0.057*)	(0.014*) (0.060*)	(0.002*) (0.063*)	(0.001*) (0.089*)	(0.095) (0.151)	(0.092) (0.133)	(0.054) (0.169)	(0.020) (0.150)	(0.007) (0.119)
100	$(-0.6, 0.6)$	(0.093*) (0.066*)	(0.051*) (0.048*)	(0.016*) (0.058*)	(0.001*) (0.107*)	(0.000*) (0.465)	(0.122) (0.186)	(0.072) (0.118)	(0.063) (0.169)	(0.014) (0.144)	(0.005) (0.269*)
	$(-0.5, -0.2)$	(0.098*) (0.059*)	(0.168) (0.041*)	(0.021*) (0.036*)	(0.002*) (0.079*)	(0.000*) (0.215)	(0.107) (0.148)	(0.138*) (0.116)	(0.047) (0.118)	(0.017) (0.117)	(0.005) (0.166*)
	$(0.5, 0.2)$	(0.075*) (0.040*)	(0.108*) (0.032*)	(0.035*) (0.043*)	(0.002*) (0.089*)	(0.000*) (0.141*)	(0.121) (0.090)	(0.146) (0.083)	(0.047) (0.083)	(0.011) (0.100)	(0.002) (0.178)
	$(-0.8, -0.5)$	(0.342) (0.163*)	(0.134) (0.201*)	(0.006*) (0.160*)	(0.001*) (0.670)	(0.001*) (1.069)	(0.211*) (0.310)	(0.122*) (0.412)	(0.089) (0.321)	(0.020) (0.382*)	(0.011) (0.405*)
	$(-0.3, -0.1)$	(0.077*) (0.026*)	(0.090*) (0.026*)	(0.027*) (0.029*)	(0.003*) (0.063*)	(0.000*) (0.114*)	(0.103) (0.063)	(0.099) (0.076)	(0.048) (0.057)	(0.014) (0.103)	(0.005) (0.117)
	$(0.8, 0.6)$	(0.330*) (0.181*)	(0.069*) (0.157*)	(0.021*) (0.162*)	(0.001*) (0.233*)	(0.002) (0.449)	(0.365) (0.328)	(0.081) (0.343)	(0.041) (0.283)	(0.008) (0.247)	(0.001*) (0.414*)
	$(0.3, 0.1)$	(0.088*) (0.028*)	(0.084*) (0.027*)	(0.042*) (0.029*)	(0.002*) (0.066*)	(0.000*) (0.108*)	(0.143) (0.078)	(0.120) (0.092)	(0.053) (0.085)	(0.009) (0.072)	(0.003) (0.141)
150	$(-0.6, 0.6)$	(0.154*) (0.040*)	(0.096*) (0.034*)	(0.041*) (0.050*)	(0.003*) (0.117*)	(0.000*) (0.344)	(0.242) (0.124)	(0.123) (0.089)	(0.058) (0.166)	(0.016) (0.131)	(0.002) (0.316*)
	$(-0.5, -0.2)$	(0.226*) (0.022*)	(0.124*) (0.021*)	(0.075*) (0.028*)	(0.003*) (0.106)	(0.000*) (0.196)	(0.316) (0.068)	(0.147) (0.070)	(0.077) (0.079)	(0.017) (0.101*)	(0.001) (0.183*)
	$(0.5, 0.2)$	(0.093*) (0.025*)	(0.100*) (0.028*)	(0.058*) (0.029*)	(0.004*) (0.112*)	(0.000*) (0.117*)	(0.217) (0.065)	(0.215) (0.086)	(0.070) (0.079)	(0.010) (0.112)	(0.001) (0.188)
	$(-0.8, -0.5)$	(0.586) (0.125*)	(0.355) (0.098*)	(0.014*) (0.119*)	(0.001*) (0.733)	(0.000*) (1.352)	(0.389*) (0.292)	(0.190*) (0.267)	(0.089) (0.265)	(0.017) (0.371*)	(0.008) (0.495*)
	$(-0.3, -0.1)$	(0.113*) (0.020*)	(0.118*) (0.016*)	(0.063*) (0.021*)	(0.004*) (0.081)	(0.000*) (0.128*)	(0.231) (0.070)	(0.172) (0.053)	(0.071) (0.077)	(0.017) (0.080*)	(0.002) (0.164)
	$(0.8, 0.6)$	(0.292*) (0.159*)	(0.161*) (0.113*)	(0.038*) (0.147*)	(0.001*) (0.340)	(0.000*) (0.389*)	(0.416) (0.358)	(0.172) (0.250)	(0.043) (0.272)	(0.005) (0.256*)	(0.001) (0.418)
	$(0.3, 0.1)$	(0.097*) (0.016*)	(0.128*) (0.016*)	(0.061*) (0.019*)	(0.007*) (0.078)	(0.000*) (0.083*)	(0.238) (0.051)	(0.284) (0.054)	(0.063) (0.065)	(0.020) (0.076*)	(0.001) (0.163)

Table 20: Three regime symmetric TAR model. Results for Method B: Estimated MSE of bootstrap confidence interval ($\frac{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	$(\alpha_1, \alpha_2)/r_2^0$	Standard bootstrap					Subsample bootstrap				
		2	1.5	1	0.5	0.2	2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	(0.011*) (0.143*)	0.208 (0.109)	0.139 (0.564)	0.003* (0.218*)	0.005* (0.171*)	0.321 (0.306)	0.116* (0.093*)	0.055* (0.124*)	0.038 (0.221)	0.087 (0.298)
	$(-0.5, -0.2)$	0.001* (0.079*)	0.002* (0.084*)	0.003* (0.128)	0.002* (0.097*)	0.003* (0.101*)	0.058 (0.227)	0.084 (0.261)	0.076 (0.118*)	0.046 (0.262)	0.061 (0.257)
	$(0.5, 0.2)$	0.001* (0.073*)	0.002* (0.070*)	0.002* (0.085*)	0.002* (0.097*)	0.002* (0.086*)	0.049 (0.228)	0.075 (0.200)	0.056 (0.158)	0.031 (0.166)	0.038 (0.211)
	$(-0.8, -0.5)$	0.008* (0.365*)	0.008* (0.484)	0.011* (0.737)	0.008* (0.452*)	0.012* (0.406*)	0.117 (0.779)	0.166 (0.464*)	0.215 (0.459*)	0.149 (0.754)	0.150 (0.805)
	$(-0.3, -0.1)$	0.001* (0.061*)	0.002* (0.060*)	0.002* (0.059*)	0.001* (0.072*)	0.002* (0.076*)	0.050 (0.193)	0.074 (0.212)	0.070 (0.134)	0.040 (0.151)	0.039 (0.144)
	$(0.8, 0.6)$	0.005* (0.419*)	0.005* (0.279*)	0.005* (0.292*)	0.007* (0.316*)	0.006* (0.287*)	0.057 (1.220)	0.085 (0.598)	0.056 (0.900)	0.050 (0.849)	0.047 (0.818)
	$(0.3, 0.1)$	0.001* (0.089*)	0.002* (0.051*)	0.002* (0.061*)	0.001* (0.048*)	0.001* (0.049*)	0.049 (0.301)	0.060 (0.199)	0.058 (0.135)	0.029 (0.154)	0.036 (0.175)
100	$(-0.6, 0.6)$	0.405 (0.063*)	1.012 (0.037)	0.515 (0.322)	0.004* (0.351)	0.001* (0.113*)	0.229* (0.142)	0.389* (0.030*)	0.172* (0.059*)	0.026 (0.147*)	0.037 (0.228)
	$(-0.5, -0.2)$	0.001* (0.051*)	0.002* (0.037*)	0.004* (0.072)	0.005* (0.109*)	0.001* (0.113*)	0.034 (0.168)	0.065 (0.118)	0.074 (0.067*)	0.027 (0.148)	0.032 (0.128)
	$(0.5, 0.2)$	0.001* (0.078*)	0.002* (0.042*)	0.014* (0.065*)	0.001* (0.170)	0.001* (0.073*)	0.025 (0.207)	0.055 (0.137)	0.024 (0.075)	0.017 (0.095*)	0.015 (0.128)
	$(-0.8, -0.5)$	0.002* (0.200*)	0.014* (0.389)	0.006* (0.732)	0.002* (0.480*)	0.003* (0.262*)	0.101 (0.338)	0.122 (0.184*)	0.125 (0.292*)	0.052 (0.514)	0.079 (0.733)
	$(-0.3, -0.1)$	0.001* (0.034*)	0.002* (0.033*)	0.005* (0.034*)	0.002* (0.117)	0.001* (0.032*)	0.037 (0.115)	0.059 (0.125)	0.053 (0.060)	0.032 (0.081*)	0.017 (0.180)
	$(0.8, 0.6)$	0.002* (0.200*)	0.004* (0.248*)	0.003* (0.317)	0.003* (0.217*)	0.003* (0.226*)	0.048 (0.434)	0.060 (0.321)	0.037 (0.280*)	0.027 (0.427)	0.026 (0.515)
	$(0.3, 0.1)$	0.001* (0.043*)	0.001* (0.035*)	0.002* (0.026*)	0.001* (0.048*)	0.000* (0.039*)	0.028 (0.165)	0.035 (0.131)	0.049 (0.079)	0.019 (0.100)	0.015 (0.124)
150	$(-0.6, 0.6)$	1.090 (0.030*)	0.465 (0.020)	0.406 (0.212)	0.010* (0.542)	0.001* (0.156*)	0.721* (0.077)	0.329* (0.018*)	0.159* (0.041*)	0.022 (0.126*)	0.017 (0.196)
	$(-0.5, -0.2)$	0.001* (0.038*)	0.003* (0.024*)	0.318 (0.049*)	0.012* (0.221)	0.000* (0.055*)	0.031 (0.183)	0.075 (0.112)	0.155* (0.065)	0.016 (0.105*)	0.015 (0.207)
	$(0.5, 0.2)$	0.001* (0.041*)	0.003* (0.031*)	0.027* (0.047*)	0.002* (0.246)	0.000* (0.035*)	0.030 (0.151)	0.043 (0.099)	0.030 (0.048)	0.014 (0.095*)	0.010 (0.184)
	$(-0.8, -0.5)$	0.003* (0.115*)	0.089* (0.212)	0.169 (1.099)	0.003* (0.841)	0.002* (0.222*)	0.084 (0.355)	0.119 (0.135*)	0.078* (0.202*)	0.047 (0.371*)	0.052 (0.611)
	$(-0.3, -0.1)$	0.001* (0.029*)	0.004* (0.022*)	0.031* (0.021*)	0.007* (0.166)	0.001* (0.050*)	0.026 (0.163)	0.063 (0.110)	0.046 (0.041)	0.019 (0.071*)	0.015 (0.143)
	$(0.8, 0.6)$	0.002* (0.126*)	0.028* (0.191)	0.010* (0.360)	0.002* (0.287*)	0.002* (0.144*)	0.045 (0.339)	0.034 (0.188*)	0.028 (0.200*)	0.016 (0.354)	0.022 (0.619)
	$(0.3, 0.1)$	0.000* (0.026*)	0.002* (0.029*)	0.034* (0.020*)	0.005* (0.084)	0.000* (0.030*)	0.021 (0.119)	0.033 (0.121)	0.038 (0.041)	0.011 (0.082*)	0.008 (0.165)

Table 21: Three regime symmetric TAR model. Results for Method C: Estimated MSE of bootstrap confidence interval ($\frac{2.5\%}{97.5\%}$) of the threshold parameter estimator. First block of data is for standard bootstrap. Second block is for subsample bootstrap. Stars denote case where the respective bootstrap method has lower MSE.

T	$(\alpha_1, \alpha_2)/r_2^0$	Standard bootstrap					Subsample bootstrap				
		2	1.5	1	0.5	0.2	2	1.5	1	0.5	0.2
50	$(-0.6, 0.6)$	(0.009*) (0.159*)	(0.093*) (0.081*)	(0.088) (0.430)	(0.002*) (0.110*)	(0.003*) (0.127*)	(0.303) (0.300)	(0.153) (0.095)	(0.051*) (0.139*)	(0.035) (0.376)	(0.063) (0.356)
	$(-0.5, -0.2)$	(0.001*) (0.070*)	(0.002*) (0.085*)	(0.002*) (0.085*)	(0.002*) (0.089*)	(0.002*) (0.084*)	(0.044) (0.198)	(0.051) (0.270)	(0.044) (0.194)	(0.043) (0.244)	(0.055) (0.236)
	$(0.5, 0.2)$	(0.001*) (0.077*)	(0.001*) (0.074*)	(0.001*) (0.088*)	(0.001*) (0.084*)	(0.002*) (0.093*)	(0.036) (0.241)	(0.029) (0.252)	(0.024) (0.344)	(0.021) (0.293)	(0.029) (0.252)
	$(-0.8, -0.5)$	(0.005*) (0.413*)	(0.007*) (0.379*)	(0.010*) (0.377*)	(0.010*) (0.383*)	(0.007*) (0.355*)	(0.060) (0.910)	(0.092) (0.674)	(0.134) (0.818)	(0.163) (0.738)	(0.100) (0.933)
	$(-0.3, -0.1)$	(0.001*) (0.055*)	(0.001*) (0.072*)	(0.001*) (0.057*)	(0.001*) (0.057*)	(0.002*) (0.056*)	(0.042) (0.165)	(0.033) (0.249)	(0.040) (0.187)	(0.029) (0.170)	(0.035) (0.190)
	$(0.8, 0.6)$	(0.004*) (0.307*)	(0.006*) (0.300*)	(0.006*) (0.337*)	(0.006*) (0.343*)	(0.004*) (0.303*)	(0.033) (0.732)	(0.043) (0.812)	(0.036) (1.107)	(0.050) (1.091)	(0.031) (0.983)
	$(0.3, 0.1)$	(0.001*) (0.060*)	(0.001*) (0.055*)	(0.001*) (0.100*)	(0.001*) (0.055*)	(0.001*) (0.054*)	(0.027) (0.203)	(0.036) (0.205)	(0.025) (0.119)	(0.028) (0.162)	(0.020) (0.152)
100	$(-0.6, 0.6)$	(0.096*) (0.062*)	1.030 (0.028*)	0.376 (0.181)	(0.002*) (0.080*)	(0.001*) (0.062*)	(0.213) (0.152)	0.463* (0.033)	0.143* (0.070*)	0.033 (0.257)	0.030 (0.243)
	$(-0.5, -0.2)$	(0.001*) (0.047*)	(0.001*) (0.040*)	(0.001*) (0.045*)	(0.001*) (0.044*)	(0.001*) (0.048*)	(0.015) (0.197)	(0.021) (0.153)	(0.033) (0.171)	(0.030) (0.195)	(0.030) (0.184)
	$(0.5, 0.2)$	(0.001*) (0.074*)	(0.001*) (0.040*)	(0.001*) (0.045*)	(0.001*) (0.048*)	(0.000*) (0.052*)	(0.016) (0.274)	(0.022) (0.180)	(0.017) (0.195)	(0.012) (0.225)	(0.011) (0.139)
	$(-0.8, -0.5)$	(0.001*) (0.197*)	(0.002*) (0.307*)	(0.002*) (0.202*)	(0.002*) (0.210*)	(0.002*) (0.204*)	(0.038) (0.516)	(0.035) (0.332)	(0.070) (0.764)	(0.090) (0.578)	(0.072) (0.679)
	$(-0.3, -0.1)$	(0.000*) (0.027*)	(0.001*) (0.031*)	(0.001*) (0.034*)	(0.001*) (0.035*)	(0.001*) (0.028*)	(0.017) (0.153)	(0.019) (0.153)	(0.019) (0.118)	(0.021) (0.150)	(0.021) (0.135)
	$(0.8, 0.6)$	(0.001*) (0.190*)	(0.002*) (0.265*)	(0.001*) (0.197*)	(0.001*) (0.185*)	(0.002*) (0.219*)	(0.018) (0.466)	(0.011) (0.366)	(0.017) (0.563)	(0.015) (0.719)	(0.012) (0.695)
	$(0.3, 0.1)$	(0.001*) (0.034*)	(0.001*) (0.048*)	(0.000*) (0.036*)	(0.000*) (0.035*)	(0.001*) (0.035*)	(0.019) (0.136)	(0.015) (0.183)	(0.015) (0.189)	(0.016) (0.119)	(0.014) (0.098)
150	$(-0.6, 0.6)$	0.703 (0.031*)	0.620 (0.013*)	0.571 (0.174)	(0.001*) (0.170)	(0.001*) (0.040*)	0.507* (0.083)	0.406* (0.024)	0.259* (0.039*)	0.022 (0.163*)	0.020 (0.269)
	$(-0.5, -0.2)$	(0.001*) (0.038*)	(0.001*) (0.028*)	(0.001*) (0.038*)	(0.001*) (0.031*)	(0.001*) (0.039*)	(0.014) (0.198)	(0.018) (0.184)	(0.027) (0.125)	(0.016) (0.198)	(0.014) (0.132)
	$(0.5, 0.2)$	(0.001*) (0.036*)	(0.001*) (0.028*)	(0.000*) (0.031*)	(0.000*) (0.034*)	(0.000*) (0.030*)	(0.011) (0.175)	(0.015) (0.135)	(0.016) (0.198)	(0.007) (0.136)	(0.013) (0.201)
	$(-0.8, -0.5)$	(0.001*) (0.133*)	(0.001*) (0.145*)	(0.002*) (0.134*)	(0.001*) (0.140*)	(0.001*) (0.155*)	(0.033) (0.523)	(0.064) (0.291)	(0.053) (0.620)	(0.038) (0.515)	(0.057) (0.947)
	$(-0.3, -0.1)$	(0.000*) (0.025*)	(0.000*) (0.031*)	(0.001*) (0.022*)	(0.000*) (0.019*)	(0.001*) (0.026*)	(0.012) (0.132)	(0.017) (0.207)	(0.018) (0.159)	(0.018) (0.148)	(0.015) (0.165)
	$(0.8, 0.6)$	(0.001*) (0.125*)	(0.001*) (0.180*)	(0.001*) (0.184*)	(0.001*) (0.152*)	(0.001*) (0.153*)	(0.009) (0.504)	(0.007) (0.307)	(0.016) (0.362)	(0.010) (0.636)	(0.011) (0.713)
	$(0.3, 0.1)$	(0.000*) (0.026*)	(0.000*) (0.028*)	(0.000*) (0.020*)	(0.000*) (0.025*)	(0.000*) (0.023*)	(0.012) (0.153)	(0.014) (0.156)	(0.009) (0.122)	(0.010) (0.104)	(0.009) (0.123)

**This working paper has been produced by
the Department of Economics at
Queen Mary, University of London**

**Copyright © 2003 George Kapetanios
All rights reserved.**

**Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096 or Fax: +44 (0)20 8983 3580
Email: j.conner@qmul.ac.uk
Website: www.econ.qmul.ac.uk/papers/wp.htm**