

# Department of Economics

## Changes in Predictive Ability with Mixed Frequency Data

Ana Beatriz Galvão

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Ana Beatriz Galvão\*

Department of Economics

Queen Mary, University of London

a.ferreira@qmul.ac.uk

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## Abstract

This paper proposes a new regression model—a smooth transition mixed data sampling (STMIDAS) approach—that captures recurrent changes in the ability of a high frequency variable in predicting a low frequency variable. The STMIDAS regression is employed for testing changes in the ability of financial variables in forecasting US output growth. The estimation of the optimal weights for aggregating weekly data inside the quarter improves the measurement of the predictive ability of the yield curve slope for output growth. Allowing for changes in the impact of the short-rate and the stock returns in future growth is decisive for finding in-sample and out-of-sample evidence of their predictive ability at horizons longer than one year.

Key words: smooth transition, MIDAS, predictive ability, asset prices, output growth

JEL codes: C22, C53, E44

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# 1 Introduction

Asset prices incorporate expectations on future economic activity because they are set based on expectations on future dividends and interest rates. This forward-looking characteristic suggests that bond and stock returns should be useful predictors of output growth (Harvey, 1988; Stock and Watson, 2003). Indeed, one of the most popular leading indicators of the US growth is the spread between long-term and short-term interest rates (Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002). In contrast, Stock and Watson (2003) conclude that stock returns have only marginal content for predicting output growth, although the results of Estrella and Mishkin (1998) suggest some power in predicting recessions at short horizons. Short-term interest rates are not as popular indicators as the spread, but recently Ang, Piazzesi and Wei (2006) argue that it is a better leading indicator than the spread from 1990 onwards.

In general, asset prices have predictive ability for economic activity, but the conclusion of the survey by Stock and Watson (2003) is that they are not always reliable. Estrella, Rodrigues and Schich (2003) report evidence of instability in the ability of the spread in predicting output growth, but no instability when the spread is used for predicting recessions. The measurement of predictive ability when there is a break is the object of study of Clark and McCracken (2005b). They show that breaks help understanding why some researchers find in-sample evidence of predictive ability, but no predictive content in the out-of-sample period. Out-of-sample tests of predictive ability have low power if the break towards no predictive ability occurs in the out-of-sample period.

This paper contributes to improve the measurement of the ability of asset returns in forecasting output growth. My new regression model is able to capture two important features of the ability of asset returns for predicting output growth: the predictive ability may be changing recurrently over time, and the information on asset returns may be available at higher frequencies than output growth.

Modelling recurrent changes over time is an alternative to modelling breaks. Although switching-regimes models may also capture breaks (Carrasco, 2002), there are economic reasons for adopting models with recurrent regimes. Changes in the predictive power of asset returns for output growth may be related to business cycle regimes. An inverted yield curve anticipates recessions, but an upward curve does not say much about booms or average growth. Bull and bear markets normally describe different regimes in the stock market. There is a popular saying that “the stock market correctly forecast nine of the last four recessions” (Harvey, 1988, p.39). However, this only makes sense if bear markets always lead to recessions. Recently, Sims and Zha (2006) have identified recurrent monetary policy regimes. They argue that monetary policy changes are better described

by processes with recurrent regimes than by process with break changes. Monetary policy changes may be also a candidate explanation for changes in the content of asset returns in predicting growth.

One usually aggregates financial variables before using them as predictors for quarterly output growth. A popular procedure is to take quarterly averages of monthly data. This imposes a restriction on how the information of the high frequency regressor is weighted inside the quarter for predicting economic activity. Averaging may not be the aggregation method that maximises the power of a high frequency variable in predicting a low frequency one.

The new regression model combines a non-linear time series regression model - smooth transition regressions (Teräsvirta, 1998) - with a MIXed Data Sampling approach - MIDAS (Ghysels, Santa-Clara and Valkanov, 2004). The mixed sampling approach allows for the direct use of high frequency data, while smooth transition allows changes in predictive ability over time. I show how to test for changes in predictive ability with mixed frequency data and how to estimate smooth transition MIDAS (STMIDAS) regressions.

The regression model can be extended by including an autoregressive term and multiple predictors. More flexible specifications allow us to answer questions on the predictive content of a variable in addition to an autoregressive term and/or another predictor. One can test for no predictive content of a predictor for a dependent variable with STMIDAS regressions by employing a bootstrap procedure to compute p-values, so that the computation of p-values takes into account the estimation of aggregation weights.

I use the STMIDAS regression to measure the ability of asset returns in predicting output growth. This approach builds on predictive regressions. On the one hand, some authors (Valkanov, 2003; Ang and Bekaert, 2007) have criticized long-run regressions to measure the predictive power when applied to highly persistent regressors that are correlated with autoregressive disturbances. On the other hand, Inoue and Kilian (2004) argue that in-sample tests of predictability may have more power than out-of-sample tests even under dynamic misspecification. In this paper, I use both in-sample and out-of-sample evaluation. The in-sample evaluation of predictive ability is based on Wald statistics, while the out-of-sample evaluation is based on tests of equal forecast accuracy and forecast encompassing with an autoregressive model as benchmark. Because there is an AR term in the STMIDAS regressions, the model is nested to the benchmark. As a consequence, the usual test statistics have a non-standard distribution. Thus the evaluation requires a bootstrap procedure to be able to test the predictive content of financial variables under different specifications as recommended by Clark and McCracken (2005a).

My empirical results obtained with STMIDAS improve our knowledge about the content in asset prices about future economic activity. In contrast to simple regression models, I find evidence that short-rates and stock returns have ability in predicting output growth at forecast horizons such as 8 and 12 quarters, both in-sample and out-of-sample. Averaging over quarters does harm the measurement of the predictive ability of the spread. There is more agreement between the in-sample and out-of-sample evidences of predictive ability when using STMIDAS regressions. The estimated chronology of changes of predictive ability sheds a light on the instability of financial variables as leading indicators and explains why results may change with the definition of the out-of-sample period. Finally, a novel empirical result is that stock returns have information in addition to the slope of the yield curve for predicting the growth of next 2-3 years. Only STMIDAS regressions are able to identify the information contained in high stock returns that is useful to predict growth.

The remaining of this paper is organized as follows. Section 2 presents the smooth transition MIDAS together with test procedures to identify changes in the predictive ability of financial variables for US GDP. A Monte Carlo exercise illustrates the properties of the estimator and testing procedures in small samples. In-sample evaluation of the predictive content of financial variables for output growth is presented in section 3. Section 4 presents the results of the out-of-sample evaluation using real-time data of output growth. Section 5 summarizes the main contributions and indicates some points for future research.

## 2 Smooth Transition MIDAS

### 2.1 MIDAS approach

Financial variables are available at high frequencies such as daily, weekly and monthly while the most popular measure of economic activity—output growth—is available quarterly. As a consequence, one aggregates financial variables in time before measuring their ability to predict the low frequency dependent variable. Using the regressor and the dependent variable with the same frequency, one can employ predictive regressions to measure the predictive power of the regressor at  $h$ -steps ahead (Estrella and Hardouvelis, 1991; Ang et al., 2006). An alternative exploited in this paper is the direct use of the high frequency predictor, letting aggregation weights to be estimated so that they maximise the ability the predictive content of the regressor.

The MIXed Data Sampling approach (MIDAS), proposed by Ghysels et al. (2004), permits the regression of low frequency data on high frequency data. Therefore, the

information on the weekly financial variables can be directly employed for checking the ability of the variables in predicting quarterly output growth.

A linear MIDAS regression to measure the predictive content of  $x$  for  $y_t$  at  $h$ -steps ahead is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} w(L^{1/m}) x_t^{(m)} + \varepsilon_{t+h} \quad (1)$$

where  $w(L^{1/m}) = \sum_{j=0}^{K-1} w(j) L^{j/m}$  is a polynomial of length  $K - 1$  in the lag operator  $L^{1/m}$

such that  $L^{j/m} x_t = x_{t-j/m}$ . The weights are identified if  $\sum_{j=0}^{K-1} w(j) = 1$ . When  $x_t^{(m)}$  is sampled weekly, for example, and only the information inside the quarter  $t$  is considered ( $m = 13 = K$ ), one has the following equation:

$$y_{t+h} = \beta_{0,h}^{(13)} + \beta_{1,h}^{(13)} \left[ w(0)x_t^{(13)} + w(1)x_{t-1/13}^{(13)} + \dots + w(12)x_{t-12/13}^{(13)} \right] + \varepsilon_{t+h}.$$

An advantage of MIDAS is that aggregation over time may smooth out information from the high frequency predictor that might otherwise help to predict  $y$ .

A problem is that the number of parameters in  $w(L^{1/m})$  increases with the frequency of predictor. A solution is the use of an exponential function for obtaining the weights. As a consequence, one has to estimate only the two parameters of the exponential function:<sup>2</sup>

$$w(j; \kappa) = \frac{\exp(\kappa_1 j + \kappa_2 j^2)}{\sum_{k=0}^{K-1} \exp(\kappa_1 k + \kappa_2 k^2)}.$$

The estimation of the function  $w(j; \kappa)$  together with the coefficients of the regression (1) implies that the best combination of information inside the quarter is employed for predicting  $y_{t+h}$ . Because of the smoothness of the weight function, the MIDAS regression can be estimated by nonlinear least squares.

While taking the average over the quarter equally weighs current information on  $x_t^{(m)}$ , similar weighting scheme only occurs with MIDAS approach when  $\kappa_1 = \kappa_2 = 0$ , so that  $w(0) = \dots = w(K - 1) = 1/K$ . In these circumstances, the MIDAS regression (1) nests:

$$y_{t+h} = \beta_{0,h} + \beta_{1,h} x_t + \varepsilon_{t+h}. \quad (2)$$

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<sup>1</sup>Of course not all quarters have 13 full weeks, consequently,  $m = 13$  is an approximation. In the empirical part, the observation of a specific week is the one from Friday (or the previous day that the market was open). As a result, I always use the last 13 observations from the date of the end of quarter as information at  $t$ .

<sup>2</sup>Ghysels, Santa-Clara and Valkanov (2006) use a beta function to compute the weights. For the shorter macroeconomic data, the exponential function is computationally easier to estimate. I use Gauss CML routines to estimate MIDAS regressions. More details on the estimation procedure are in Appendix B.

This type of regression was employed to measure the ability of the spread for predicting output growth (Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002; Ang et al., 2006), of dividend/price ratios for excess returns (see Cochrane (2005), ch. 20 for a survey), and of economic fundamentals for exchange rates (Kilian and Taylor, 2003). If  $\beta_{1,h} = 0$ ,  $x_t$  has no predictive content for  $y_{t+h}$ . This can be verified with a t-statistic after the estimation of the parameters. Although the coefficients can be consistently estimated by ordinary least squares, the t-statistic has to be computed using an estimator for  $var(\hat{\beta}_{1,h})$  robust to heteroscedasticity and autocorrelation. A consistent and popular choice is the Newey and West (1987) estimator. An alternative measure of the predictive power of  $x_t$  at each forecasting horizon is the  $R^2$  computed after estimating the forecasting regression (2).

In the context of MIDAS regressions, the null hypothesis of no predictive ability of  $x_t^{(m)}$  for  $y_{t+h}$  cannot be tested using the standard distribution of the t-ratio, because the parameters  $\kappa$  are unidentified under the null. Appendix A describes a procedure for computing p-values for the t-ratio by bootstrap. The contributions of the MIDAS regressions for the measurement of the ability of  $x_t^{(m)}$  in predicting  $y_{t+h}$  can be evaluated in comparison to regressions (eq. 2) and autoregressive models. The comparison of in-sample fit may use information criteria (Inoue and Kilian, 2006), while the out-of-sample performance may be compared in terms of means of squared forecast error.

## 2.2 Smooth Transition MIDAS

Switching regimes are a popular way of modelling nonlinear dynamics in regressions by using piecewise linear regimes linked by a transition function (Tong, 1990). When the transition between regimes is smooth and it depends on the size of an observed transition variable, switching-regime models are called smooth transition regressions (surveyed by Van Dijk, Teräsvirta and Franses (2002)). This type of non-linear approach permits the modelling of changes in the predictive content of a high frequency variable for a low frequency one in a simple way. The switches between regimes depend on the sign and the size of the weighted high frequency predictor.

Before writing the model with changes in predictive ability, I simplify the notation by writing the weighted sum of  $x_t^{(m)}$  as

$$x_{t(\kappa,m)}^{(m)} = \sum_{j=0}^{m-1} w(j, \kappa) L^{j/m} x_t^{(m)}, \quad (3)$$

and imposing the restriction that  $K = m$ , so that only the current information on  $x$  is employed for forecasting  $y$ . The smooth transition MIDAS (STMIDAS) regression is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,m)}^{(m)} \left[ 1 - G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c) \right] + \beta_{2,h}^{(m)} x_{t(\kappa,m)}^{(m)} \left[ G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c) \right] + \varepsilon_{t+h}, \quad (4)$$

where

$$G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c) = \frac{1}{1 + \exp(-(\gamma/\hat{\sigma}_x)(x_{t(\kappa,m)}^{(m)} - c))}.$$

The transition function  $G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c)$  is a logistic function that depends on the weighted sum of the explanatory variable in the current quarter. The function  $G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c)$  has values between 0 and 1. When the smoothing parameter  $\gamma$  is large, the function has all values equal to either 0 or 1. In the latter case, the function is similar to an indicator function that is zero when  $x_{t(\kappa,K)}^{(m)} \leq c$  and equal to 1 when  $x_{t(\kappa,K)}^{(m)} > c$ . Thus the impact of  $x_{t(\kappa,m)}^{(m)}$  in predicting  $y_{t+h}$  is  $\beta_{1,h}^{(m)}$  when the weighted sum of  $x_t^{(m)}$  is small and  $\beta_{2,h}^{(m)}$  when the weighted sum  $x_{t(\kappa,m)}^{(m)}$  is large. When  $\gamma$  is small but is not equal to zero, the impact of  $x_{t(\kappa,m)}^{(m)}$  in predicting  $y_{t+h}$  is a time-varying weighted sum of  $\beta_{1,h}^{(m)}$  and  $\beta_{2,h}^{(m)}$  depending on the value of  $G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c)$ .

The estimates of  $\kappa$  for regression (1) may be different from regression (4) because the weights are chosen to maximize the predictive power of  $x_t^{(m)}$  assuming that its predictive ability changes over time. Note that the weights are kept constant over time, but the impact of the weighted predictor is allowed to vary over time with the smooth transition. A discussion of the application of nonlinear least squares to estimate STMIDAS regressions is on Appendix B.

I would like to emphasize some important features of the STMIDAS regressions. In contrast to previous applications of non-linear time series models (see Anderson and Vahid (2001) for an application similar to this paper), I am not required to choose the delay, which is the lag of the transition variable, before estimating the smooth transition regression. For the purpose of modelling shifts in the predictive content of  $x_t^{(m)}$  for  $y$  at a given horizon, the weighted high frequency predictor  $x_{t(\kappa,m)}^{(m)}$  is the only reasonable candidate for transition variable.

Another feature is that STMIDAS regressions are designed for direct forecasting. Previous applications of non-linear time series models for verifying changes in the dynamic relationship between output growth and the spread (Galbraith and Tkacz, 2000; Anderson and Vahid, 2001; Galvão, 2006) have specified models only for one-step-ahead forecasts. Iterated forecasts for longer horizons are then obtained by bootstrap. Massimiliano, Stock and Watson (2006) show that only when longer lags are allowed, iteration generates more accurate forecasts than direct forecasting. In my application for measuring predictive ability, only the current quarter information about  $x_t^{(m)}$  is employed for forecasting. In this absence of longer lags, the direct computation of forecasts generates predictions that



are probably more accurate than iterating, specially for predicting at long horizons such as 2 and 3 years. In addition, one does not need a simulation procedure for computing the forecasts for  $h > 1$ . A simulation procedure is required when computing iterated forecasts because the conditional mean needs to be approximated numerically.

Another alternative for modelling switching regimes is to make the regimes dependent on a latent variable, which is controlled by a Markov process (Hamilton, 1989). A problem with this alternative is that a consistent estimation of the parameters requires maximum likelihood methods, which assumes normality of the residuals (see Kim and Nelson (1999) for a survey). This assumption needs a well-specified model. Because it is unlikely that a Markov-switching version of (2) will have well-behaved errors, the use of nonlinear least squares together with robust estimation of the variance is more adequate for testing changes in the ability of  $x_t^{(m)}$  in predicting  $y_{t+h}$ .

### 2.2.1 Testing for Changing Predictive Ability

I rewrite regression (4) as:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,K)}^{(m)} + \delta_h^{(m)} x_{t(\kappa,K)}^{(m)} \left[ G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c) \right] + \varepsilon_{t+h}, \quad (5)$$

where  $\delta_h^{(m)} = \beta_{2,h}^{(m)} - \beta_{1,h}^{(m)}$ . Thus a test for changes in the predictive ability of  $x_{t(\kappa,K)}^{(m)}$  on  $y_{t+h}$  has the null hypothesis  $\delta_h^{(m)} = 0$ . For testing this hypothesis, traditional methods cannot be used because the parameters  $\gamma$  and  $c$  are not identified under the null (Granger and Teräsvirta, 1993). This implies that the test for  $\delta_h^{(m)} = 0$  using STMIDAS estimates is severely oversized. Fortunately, the testing procedure proposed by Luukkonen, Saikkonen and Terasvirta (1988) can be applied. The testing procedure makes use of a Taylor approximation of the logistic function. Using a first-order approximation, the auxiliary regression for testing constant predictive ability (linearity) using equation (4) as the alternative hypothesis is:

$$y_{t+h} = \pi_{0,h}^{(m)} + \pi_{1,h}^{(m)} x_{t(\hat{\kappa},m)}^{(m)} + \pi_{2,h}^{(m)} \left( x_{t(\hat{\kappa},m)}^{(m)} \right)^2 + \varepsilon_{t+h}. \quad (6)$$

The null hypothesis is  $\pi_{2,h}^{(m)} = 0$ , assuming that  $\hat{\kappa}$  has been estimated under the null. This variable addition test has also power for detecting threshold linearity (Strikholm and Teräsvirta, 2005).

A problem of applying this approach for forecasting regressions is that the properties of the test are derived assuming that  $\varepsilon_{t+h}$  is *iid*. It is only reasonable to assume that this is the case when  $h = 1$ . For horizons longer than one quarter, I use estimates of  $var(\hat{\pi}_{2,h}^{(m)})$  robust to autocorrelation (and heteroscedasticity for  $h = 1, \dots, H$ ). This is also the usual approach when testing for no predictive content of  $x_t$  in  $y_{t+h}$  using in-sample estimates.

This testing approach may be oversized and underpowered when the sample is small. In the context of testing for no predictive ability, the results of Ang and Bekaert (2007) indicate that the Newey and West (1987) estimator is oversized in small samples (100) for large  $h$  (20). They suggest to employ the Hodrick (1992) estimator for computing the variance matrix because the implied t-test has the correct size. However, the t-statistic using the Hodrick (1992) estimator has very low power when  $h$  is large. I investigate the size and power properties of the use of Newey and West (1987) estimator to compute  $var(\hat{\beta}_{2,h}^{(m)})$  for testing changing predictive ability in section 2.3.2.

An approach similar to this one can be also applied to test changes in the predictive ability of a predictor that is sampled at the same frequency as the dependent variable. Assuming that  $m = 1$  in equation (4), one has the smooth transition specification of (2), that is,

$$y_{t+h} = \beta_{0,h} + \beta_{1,h}x_t [1 - G_t(x_t; \gamma, c)] + \beta_{2,h}x_t [G_t(x_t; \gamma, c)] + \varepsilon_{t+h}. \quad (7)$$

The test of the null of no changes in the predictive ability of  $x_t$  for  $y_{t+h}$  has this latter regression as alternative hypothesis. The auxiliary regression for testing no changes in predictive content is:

$$y_{t+h} = \pi_{0,h} + \pi_{1,h}x_t + \pi_{2,h}(x_t)^2 + \varepsilon_{t+h}. \quad (8)$$

I expect that the test for changes in predictive ability using regression (6) is more powerful than using regression (8). Ghysels et al. (2004) have shown that, based on a Hannan feasible estimator for regressions of same dependent variable but with predictors of low and high frequencies, the estimator for the impact is asymptotically more efficient with MIDAS regressions (1) than with the traditional regressions (2). The intuition of the validity of this result for testing changes in predictive ability is that because  $var(\hat{\pi}_{2,h}^{(m)})$  is inversely related to the variation of  $x_t^{(m)}$ , the disaggregation of the information from the regressor implies that the variation of  $x_t^{(m)}$  is computed using  $mT$  observations instead of  $T$ . Because it is expected that  $var(\hat{\pi}_{2,h}) \geq var(\hat{\pi}_{2,h}^{(m)})$ , tests for measuring the predictive ability with the MIDAS approach might be more powerful.

### 2.2.2 Inclusion of an autoregressive term

Specifications (2), (1) and (4) can be extended for allowing for autoregressive behaviour. If there is some weak memory in  $y_t$ , it is likely that the results of in-sample tests of no predictive ability of  $x_t^{(m)}$  for  $y_{t+h}$  do no change with the inclusion of an autoregressive term. However, when forecasting  $y_{t+h}$  out-of-sample, the autoregressive term may improve forecasts. Yet, the results by Ang et al. (2006) suggest that an autoregressive

term improves forecasts in short horizons ( $h = 1$ ), while it does not change the measurement of the predictive ability of the yield curve. Therefore, I also consider a STMIDAS specification with autoregressive term:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,m)}^{(m)} \left[ 1 - G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c) \right] + \beta_{2,h}^{(m)} x_{t(\kappa,m)}^{(m)} \left[ G_t(x_{t(\kappa,m)}^{(m)}; \gamma, c) \right] + \rho_h y_t + \varepsilon_{t+h}.$$

Clements and Galvão (2006) discuss the problem of including an autoregressive term in MIDAS modelling. Because of the polynomial in  $L^{1/m}$ , the lag structure with the inclusion of a lag dependent variable generates a “seasonal” behaviour on the effect of  $x_t^{(m)}$  for  $y_{t+h}$  with stronger peaks at the end of each quarter. The solution proposed was to use a common factor structure. However, when measuring changing predictive ability,  $K$  is equal to  $m$ , so that the lag structure of  $x_t^{(m)}$  does not go beyond a quarter. As a consequence, there is no problem in the inclusion of  $y_t$  on the left-hand side.

### 2.2.3 Combining High Frequency Predictors

The STMIDAS regressions (4) can be extended to incorporate two predictors  $x_{1,t}^{(m)}$  and  $x_{2,t}^{(m)}$ :

$$\begin{aligned} y_{t+h} = & \beta_{0,h}^{(m)} + \beta_{11,h}^{(m)} x_{1,t(\kappa_1,m)}^{(m)} \left[ 1 - G_{1,t}(x_{1,t(\kappa_1,m)}^{(m)}; \gamma_1, c_1) \right] \\ & + \beta_{12,h}^{(m)} x_{1,t(\kappa_1,m)}^{(m)} \left[ G_{1,t}(x_{1,t(\kappa_1,m)}^{(m)}; \gamma_1, c_1) \right] \\ & + \beta_{21,h}^{(m)} x_{2,t(\kappa_2,m)}^{(m)} \left[ 1 - G_{2,t}(x_{2,t(\kappa_2,m)}^{(m)}; \gamma_2, c_2) \right] \\ & + \beta_{22,h}^{(m)} x_{2,t(\kappa_2,m)}^{(m)} \left[ G_{2,t}(x_{2,t(\kappa_2,m)}^{(m)}; \gamma_2, c_2) \right] + \varepsilon_{t+h}. \end{aligned} \quad (9)$$

Each predictor is able to have a different type of switching behaviour over time, because two transition functions are estimated. This specification is useful for testing whether variable  $x_{2t}$  has predictive power for  $y_{t+h}$  in addition to  $x_{1t}$ . Appendix A describes a bootstrap procedure for computing p-values of a Wald test for testing additional predictive ability.

The test for changes in the predictive ability of both  $x_{1t}$  and  $x_{2t}$  employs an auxiliary regression based on a first-order Taylor approximation (Luukkonen et al., 1988):

$$\begin{aligned} y_{t+h} = & \pi_{0,h}^{(m)} + \pi_{1,h}^{(m)} x_{1,t(\hat{\kappa},m)}^{(m)} + \pi_{2,h}^{(m)} \left( x_{1,t(\hat{\kappa},m)}^{(m)} \right)^2 + \\ & \pi_{3,h}^{(m)} x_{2,t(\hat{\kappa},m)}^{(m)} + \pi_{4,h}^{(m)} \left( x_{2,t(\hat{\kappa},m)}^{(m)} \right)^2 + \varepsilon_{t+h}. \end{aligned} \quad (10)$$

The restrictions in this auxiliary regression for testing the null hypothesis of no changes in the impact of the predictors  $x_{1t}$  and  $x_{2t}$  for  $y_{t+h}$  are:

$$\pi_{2,h}^{(m)} = \pi_{4,h}^{(m)} = 0.$$

The Wald statistic is calculated using an estimator for the variance that is robust to autocorrelation and heteroscedasticity. This approach also allows for testing whether there is a change in predictability of  $x_{2t}$  for  $y_{t+h}$  without imposing any restriction on the measurement of predictive content of  $x_{1t}$ . Using the auxiliary regression (eq. 10), the null of the test is  $\pi_{4,h}^{(m)} = 0$ .

Specifications similar to (9) and (10) can be also written for aggregated data (set  $m = 1$ ). A competitor with constant parameters (R) is:

$$y_{t+h} = \beta_{0,h} + \beta_{11,h}x_{1,t} + \beta_{21,h}x_{2,t} + \varepsilon_{t+h}, \quad (11)$$

where  $\beta_{11,h}$  measures the impact of variable  $x_1$  in predicting  $y$  at horizon  $h$ .

### 2.3 Monte Carlo Evaluation

The objective of this Monte Carlo evaluation is to analyse the properties of nonlinear least squares (NLS) in the estimation of MIDAS regressions. Moreover, I check whether there is any gain from using high frequency data for testing changes in the predictive ability of the predictor for the dependent variable. I use data generating processes that are similar to the empirical relations between spread and output growth.

The process for  $x_t^{(m)}$  is an AR(1) with a large autoregressive coefficient (0.98) and a small drift (0.05). The value of  $m$  is set to 13 (weekly data), so at least  $mT$  observations of  $x_t^{(m)}$  are generated assuming that the disturbances are  $N(0, 1)$ . The persistence of the process of  $x_t^{(m)}$  decreases when aggregating the simulated values by averaging over blocks of  $m$  observations.

#### 2.3.1 Evaluation of NLS for estimating the parameters of STMIDAS regression

I specify parameters for the data generating processes (DGPs) with changing parameters such that in the first regime  $x_t^{(m)}$  has a stronger impact in predicting  $y_{t+h}$  than in the second regime, but  $x_t^{(m)}$  has some predictive content in both regimes. The values of the  $\beta_s$  are the same using both aggregated and disaggregated data, that is,  $\beta_{0,h} = \beta_{0,h}^{(m)} = 0.4$  and  $\beta_{1,h} = \beta_{1,h}^{(m)} = 0.50$ . The difference between the second and the first regime of the impact of the predictor for the dependent variable is  $\delta_h = \delta_h^{(m)} = -0.30$ . The threshold  $c$  is set to 2.3, which is near the unconditional mean of the  $x_t^{(m)}$  process.<sup>3</sup> The coefficients of the exponential function that defines the weights of each lag of the high frequency predictor are set such that the information of lag  $t - 5/13$  has higher weight.

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<sup>3</sup>The parameter  $\hat{\sigma}_x$  normally included in the logistic function to make  $\gamma$  scale free is assumed to be equal the square-root of unconditional variance of  $x_t^{(m)}$ , that is,  $\hat{\sigma}_x = 5$ .

Data are generated from the DGPs for forecasting horizons  $h = 1$  and 4. For  $h = 1$ , an autoregressive term is added ( $\rho = 0.20$ ); for  $h = 4$ , the autoregressive term is not included, but an MA(3) process is assumed for  $\varepsilon_{t+4}$ . These specifications are supported empirically. Because the MA terms are not estimated, the results for  $h = 4$  indicate the effect of this type of misspecification in the estimation. Finally, the disturbances of the  $y_{t+h}$  equation are standard normal.

Table 1 presents the average biases of the NLS estimates of STMIDAS regressions computed with different samples sizes under no autocorrelation ( $h = 1$ ) and an MA in the errors ( $h = 4$ ). The description of the NLS procedure is in Appendix B. Even in samples as small as 100, the estimates of the  $\beta_h$ s and  $\delta$  are not significantly biased. The biases in the estimation of  $\kappa$  and  $\gamma$  only disappear for large  $T$  (1000), and they shrink slower for  $h = 4$ . These biases do not affect the measurement of the impact of  $x_t^{(m)}$  for  $y_{t+h}$ . However, it is likely that the estimates of  $\kappa$  and  $\gamma$  will be highly imprecise in small samples.

Figures 1 and 2 can help us to understand why large biases in  $\kappa$  and  $\gamma$  do not imply large biases in  $\beta_{1,h}$  and  $\delta_h$ . Figure 1 shows that differences in the values of  $\hat{\kappa}_1$  and  $\hat{\kappa}_2$  do not imply that the functions  $w(m, \hat{\kappa})$  are dramatically different if the proportion  $\hat{\kappa}_1/\hat{\kappa}_2$  is kept constant. The values of  $\kappa_1$  and  $\kappa_2$  used in the Figure 1 are based on the average estimates computed in the Monte Carlo for  $T = 100, 200, 500$ . Figure 2 shows that differences in  $\gamma$ , for a given value of threshold  $c$ , imply changes in the number of observations with values between 0 and 1. For small samples (120 in the Figure 2), it is unlikely that this makes a large difference in the estimates of  $\beta_{1,h}$  and  $\delta_h$ .

### 2.3.2 Evaluation of the Test for Changing Predictive Ability

I simulate data from linear and smooth transition specifications using aggregated and disaggregated data. The non-linear DGPs were described in the previous subsection. The linear DGPs have  $\beta_{0,h} = \beta_{0,h}^{(m)} = 0.4$ , and  $\beta_{1,h} = \beta_{1,h}^{(m)} = 0.25$ . The value of  $\beta_{1,h}$  is such that it is not equal to the average of  $\beta_{1,h}$  and  $\beta_{2,h}$  in the smooth transition specifications, which is 0.20.

Table 2 presents the rejection rates of the test for changes in predictive ability with MIDAS (6) and traditional regressions (8) under the alternative. The rejection rates are computed for data simulated from DGPs with constant and switching-regime parameters. In addition to the horizons  $h = 1, 4$ , I also check the properties of the test for  $h = 8$ , which assumes an MA(7) in the disturbances of the DGPs. The tests are computed using a Wald statistic for  $\pi_{2,h} = 0$  in the auxiliary regressions. The Newey-West estimator is employed for computing the variance-covariance matrix. Therefore, this Monte Carlo

evaluation investigates whether the proposed test is able to choose correctly between constancy and changes in predictive ability of  $x_t$  for  $y_{t+h}$  depending on the size of  $x_t$ , under the presence of autocorrelation in the residuals of the auxiliary regression.

The autocorrelation on the residuals strongly affects the properties of the test when the sample is small (100 observations). When the sample is large, there is some evidence that the test is over-sized. This is also common in tests for no predictive content of  $x_t$  for  $y_{t+h}$  (Ang and Bekaert, 2007). There is a clear trade-off between size and power when using a consistent estimator robust to heteroscedasticity and autocorrelation in the context of forecasting regressions with small samples. However, the results indicate that the test can be still used for detecting changes in predictive ability if it is done carefully in small samples.

The direct use of the high frequency observations of the predictor improves slightly the power of the test. The small gain in power is found for all  $h$ . Because the gain is small, one may argue that there is no advantage in using MIDAS regressions instead of traditional regressions for testing changes in the ability of a high frequency predictor in forecasting a low frequency dependent variable. However, if the weight function  $w(m, \hat{\kappa})$  that maximises the predictive content of a given predictor for  $y_{t+h}$  is significantly different from equal weighting, the use of MIDAS regressions may be still advantageous for detecting changing predictive ability. In the next section, the empirical exercise will confirm that this is the case in some situations.

### 3 Measurement of In-Sample Predictive Ability

In this section, STMIDAS regressions are employed for measuring the ability of financial variables in predicting output growth. Following the literature that starts with Estrella and Hardouvelis (1991), the dependent variable is  $y_{t+h} = (400/h)[z_{t+h} - z_t]$ , where  $z_t$  is the log of real GDP in dollars. The predictors are two factors of the yield curve and stock prices. The slope of the yield curve is measured using the spread:  $x_t = r_t^{(20)} - r_t^{(1)}$ , where  $r_t^{(20)}$  is the interest rate of a bond with maturity 20 quarters. The results do not change if the long-rate is the 10-year interest rate. The spread of the 10-year interest rate has been considered by Estrella and Hardouvelis (1991), while the one of the 5-year rate is employed by Ang et al. (2006). The stock returns are computed using the annual difference of the price index, that is,  $sr_t = 100(\ln(p_t) - \ln(p_{t-4}))$  with quarterly data.

The quarterly data on real GDP growth are from the 2005:Q3 vintage and I use data since 1970:Q1.<sup>4</sup> The regressors are sampled weekly ( $m = 13$ ). The interest rate data

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<sup>4</sup>The data is from the real-time dataset of the Philadelphia Fed:

are obtained from the FRED database in weekly frequencies.<sup>5</sup> The stock prices are the SP500 index obtained daily from Bloomberg. Weekly data is obtained by using the value of the last day of the week, while monthly data is obtained by averaging weekly data. Note that the stock returns are computed as  $sr_t^{(13)} = 100(\ln(p_t^{(13)}) - \ln(p_{t-52}^{(13)}))$  when data is weekly. The quarterly aggregated data are computed by averaging monthly data of a given quarter. The estimation uses values of  $h$  up to 12. As a consequence, the number in-sample observations is  $T = 123$  (almost 31 years of data) to be able to keep constant the number of observations for each forecast horizon.

Figure 3 presents quarterly data. The growth rate is computed as  $(100) [\log(z_t) - \log(z_{t-4})]$ . Negative spread and high interest rates lead negative growth, but the association with the short-rate is more evident only for the last two recessions (1991 and 2001). Large positive stock returns lead strong output growth. There are more periods of negative stock returns than recessions, consequently, stock returns may generate false alarms. From the financial variables analysed, the most popular indicator is the spread (Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002), the short-rate has been suggested recently (Ang et al., 2006) and stock returns have not been popular but the results by Estrella and Mishkin (1998) indicate that they are good for forecasting short horizons.

Although section 2.2.2 describes how to add an autoregressive term to MIDAS and STMIDAS regressions, this section will only present results with regressions without the autoregressive term. The qualitative results do not change with the inclusion of the autoregressive term.

### 3.1 The Disaggregation effect

Table 3 presents the results of the estimation of predictive regressions (R, eq. 2) and MIDAS regressions (eq. 1) with  $m = 13$  using the three predictors described for predicting output growth. Inoue and Kilian (2006) argue that the Schwarz information criterion is a powerful way of discriminating between forecasting models. Hence, Table 3 also presents the value of the Schwarz information criterion. P-values for the test of no predictive content of the predictors for the dependent variable are also shown in table 3. They are computed with t-distribution for regressions and bootstrapping for MIDAS regressions (see Appendix A). In both cases, the Newey-West estimator with truncation lag  $(h - 1)$  is employed to compute the standard errors.

The best predictor is the slope. Short-term interest rate and stock returns have pre-

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<http://www.phil.frb.org/econ/forecast/realindex.html>.

<sup>5</sup>The address is <http://research.stlouisfed.org/fred2/>.

dictive power only up to  $h = 4$ . Therefore, these in-sample results based on specifications with constant parameters do not support the finding of Ang et al. (2006) that the short-rate has a larger predictive power than the spread when the horizons are equal to 8 and 12. In addition, they confirm the results of Estrella and Mishkin (1998) that the stock returns only have predictive power for economic activity at short horizons.

The use of the exponential function to optimally weight weekly information inside the quarter increases the variation of output growth that is explained by the slope at  $h = 1$ : the  $R^2$  raises from 0.11 to 0.22. Moreover, a comparison of the information criteria (SIC) suggests the choice the MIDAS regression. There is no evidence of gains of using short-rate and stock returns sampled more frequently. Figure 4 presents the estimated weight functions  $w(m, \hat{\kappa})$  for  $h = 1$ . The weights are generally significant different from weighting equally each lag. The larger weights are normally given to the observations earlier in the quarter, suggesting that just released information on the financial variables does not have important contribution for forecasting next quarter output growth. The weight function of the short-term rate and the slope are similar, but the weights only imply significant gains in measuring the predictive content of the slope.

### 3.2 The Changing effect

Table 4 presents the estimates of the smooth transition (7) and the smooth transition MIDAS regression (4) for using the three financial variables for predicting output growth. The p-values of the tests for changes in predictive ability are also indicated. The Schwarz information criteria allow the comparison with the values of Table 3, so they help the inference on the existence of changes in predictive ability. Table 4 also shows Wald statistics and their p-values for testing the null that  $x_t^{(m)}$  has no ability in predicting  $y_{t+h}$ . The p-values for the Wald statistic computed with the STMIDAS regressions are obtained with the bootstrap procedure described in Appendix A.

The slope and the stock returns present changes in their ability in forecasting output at, respectively, horizons 1 and 4, and horizons 8 and 12. The information criterion chooses the STMIDAS regression (comparing also with estimates in Table 3) for capturing the predictive content of the slope at  $h = 1$  and of the stock returns at  $h = 12$ . The identified changes in the ability of the slope for forecasting output growth is not crucial for detecting its predictive power. However, when measuring the ability of stock returns for forecasting output growth at  $h = 8$ , the null of no predictive content is only rejected when changes in predictive ability are allowed for.

Another result from Table 4 is that the test of no predictive ability of  $x_t^{(m)}$  for  $y_{t+h}$  using STMIDAS regressions is more conservative than using smooth transition regressions



(STR). An explanation is that the use of standard distributions for the Wald statistic implies oversized tests in the case of STR. Because it is not clear this is the reason of this result, I will compare the out-of-sample performance of these models with an AR(1) benchmark in section 4.

### 3.3 Combining Indicators

The results in the previous subsections indicate that the spread is powerful predictor of future output growth while the short-term interest rate and stock returns have predictive power only at short horizons. Because STMIDAS regressions allow us to combine information of predictors with different timing of changes on their ability in predicting a variable, I combine the information on spread with each additional predictor — short-rate and the stock returns—at a time. The results in Ang et al. (2006) indicate that the combination of short-rate and spread generates good forecasts, while the results of Estrella and Mishkin (1998) indicate that the stock returns have additional predictive ability to the spread at short horizons.

Table 5 presents test statistics to verify whether the short-rate and the stock returns have additional predictive content when included in a regression that has already the spread as predictor. In addition, p-values of the test for no changes in the predictive power of the additional regressor are presented, assuming that the spread has a changing effect for output growth. There is now evidence of changes in the predictive ability of the short-rate and stock returns for output growth at all horizons. The shifts in the predictive ability over time are decisive for identifying the additional predictive content of the short-rate and the stock returns for output growth at long horizons. Therefore, the use of STMIDAS regressions improves the measurement of predictive content of these variables. However, the effect of switching-regimes on the measurement of the predictive content is larger than the effect of the direct use of high frequency data.

The information criteria suggest that it is better to combine the spread with the stock returns than combine it with the short-rate. Thus while it is true that the short-rate has additional information to the spread for forecasting output growth (as indicated by Ang et al. (2006)), the information in the stock returns is even more original. A deeper evaluation of the estimates is presented in the next section to understand the economic meaning of these results.

I checked whether the combination of information of the spread with the short-rate in the STMIDAS regression could be substituted by the use of the long-term interest rate as predictor, but the results (not shown) indicate a large increase of the information criterion.

### 3.4 Economic regimes and changes in the predictive ability over time

Figure 5 presents the estimates of the coefficient of each predictor over time using the STMIDAS regressions for different forecast horizons. The plots present  $\hat{\beta}_{1,h}^{(13)} \left[ 1 - G_t(x_{t(13,\hat{\kappa})}^{(13)}; \hat{\gamma}, \hat{c}) \right] + \hat{\beta}_{2,h}^{(13)} \left[ G_t(x_{t(13,\hat{\kappa})}^{(13)}; \hat{\gamma}, \hat{c}) \right]$  on the left axis and the aggregated predictor  $x_{t(13\hat{\kappa}),h}^{(13)}$  on the right axis for a set of forecast horizons. In general there are no differences in the estimates of the thresholds across horizons, but the smoothness of the transition function and the amplitude of shifts across regimes may change with  $h$ . The amplitude of changes increases with the horizon for the stock returns, but decreases with the horizon for the slope. This is in agreement with the tests for changes in ability in forecasting output growth for these predictors.

Specifically, the chronology of regime changes of using the slope as predictor indicates periods of low and high predictive ability. The regime of lower predictive ability has a higher frequency after 1980, so this is in agreement with the literature that finds instability in the predictive ability of the spread with data up to 1998 (Estrella et al., 2003). However, periods of low predictive ability can be identified as the high-growth recovery periods that occur after recessions. This association with business cycle regimes suggests that recurrent changes in predictive ability over time are more adequate to model the predictive ability of the spread than are structural breaks. The plot also indicates that the aggregation weights of the slope are changing across horizons. The estimates for the weights for forecasting one-year ahead are different than for forecasting 2-3 years. These differences in aggregation weights may improve the use of the slope as leading indicator for specific horizons.

The estimates of the impact of stock returns and short-rates on future output growth cross zero in some periods of time. This means that the sign of the impact of these predictors may change over time. In the case of the short-rate, there is a clear difference on the predictive content after 1991 because the regime with high/positive predictive power starts to be more frequent at long horizons ( $h = 8, 12$ ). Before 1991, the largest predictive power is at  $h = 4$  and the short-rate has a negative impact on future output growth. This shift in predictive ability may help to explain the novel result of Ang et al. (2006) on the predictive content of the short-rate using data up to 2001. I identify regimes that are related with inflationary cycles and monetary policy regimes. In the period of high inflation, short-term interest rates have a negative impact on next year output growth. In the period of low inflation (in general after 1991, but also the beginning of 1970's), short-term interest rates have a positive impact for growth in three years. These differences of sign and horizon imply that interest rates are strongly related with business cycles for the later part of the sample. Sims and Zha (2006) have also argued in favour

of a regime-switching model to characterize changes in the effect of monetary policy in the US. Their four-regime model has shifting dates similar to the ones in the second plot of Figure 5.

The impact of stock returns on future output growth is generally positive when the stock returns are large, and it is negative when they are small or negative. This implies that bear markets have a limited impact on future growth while bull markets have a stronger effect. The argument of Harvey (1988) on the use stock returns as leading indicators is based on the fact that a bear market should predict recessions. My results with no shifts in predictive ability suggest that the stock returns have no predictive content for growth at horizons longer than one year, but when changes in the impact are allowed for, I find that stock returns have strong power for forecasting growth when the stock market is booming. This is an interesting novel result because it recognises the instability of asset returns as leading indicator as argued by Stock and Watson (2003), but it shows that it has information for forecasting growth at long horizons, including the 90's economic boom.

## 4 Evaluation of Real-Time GDP growth forecasts

Results in the previous section indicate that the spread has predictive content for forecasting output growth at all horizons, and that the short-rates and the stock returns have some additional predictive content at long-horizons, when their impact in future output growth is allowed to change. The use of regressions for measuring the ability of a predictor for forecasting a variable at long horizons may be questioned because a persistence predictor may be spuriously correlated with the cumulated regressand (Valkanov, 2003). Another problem is that the tests for no predictive content of  $x_t$  on  $y_{t+h}$  may be oversized because of the properties of Newey-West estimator (Ang and Bekaert, 2007). Predictive regressions have some weakness as tools of the measurement of predictive ability. These criticisms are less of a problem with the MIDAS specification because the aggregation scheme may reduce the persistence of the predictor. Moreover, a bootstrap procedure is used to compute the p-values of the tests of no-predictability. The results of Inoue and Kilian (2004) and (2006) suggest that in-sample evidence of changes in predictive ability does not necessary imply gains in terms of out-of-sample forecasting. An explanation for that is the reduced power of out-of-sample tests of predictive ability due to the typically small out-of-sample periods. The sample size of my real-time forecasting exercise is of 13 years. Therefore, a warning on the results of the exercise that follows is that any break towards no predictive content in the end-of-sample will imply less power for the

out-of-sample evaluation in detecting any predictive content of a given regressor (Clark and McCracken, 2005b).

In this section, I evaluate the real-time forecasting performance of regressions and MIDAS models with and without changes in the parameters in similar fashion to the previous in-sample analysis. The benchmark in the comparison is an AR(1) model for output growth. The benchmark model is estimated for each  $h$  such that it can be used for direct forecasting. Because the autocorrelation in output growth may be important for getting significant reductions on forecasting errors at short horizons, I will also consider regression specifications with an autoregressive term. Data vintages of US real GDP growth from 1991:Q4 to 2004:Q3 (52 vintages) are applied to evaluate forecasts for horizons up to 4, and vintages up to 2002:Q4 (44 vintages) are used for forecasts with horizons 8 and 12. The forecast errors are computed using the 2005:Q3 vintage.

I employ two methods to compute forecasts. The first one is called “rolling”. This method keeps constant the window size ( $T = 90$ ), so that at each time a new vintage is used, it excludes the required observations from the beginning of the sample. The second method is called “recursive”: at each new vintage, a new observation is included, increasing the sample size. In this application, the sample size is  $T = 90$  using the first data-vintage but it is  $T = 142$  using the last vintage. “Rolling” is a forecasting method that is more robust to structural breaks when the regressors are exogenous. This is so because an increase in the number of observations before the break raises the bias in the estimation. However, when there is an autoregressive term, the shorter sample used with “rolling” methods will increase the parameter bias; thus, even with a break, “rolling” may not be the most adequate method (Pesaran and Timmermann, 2005). An advantage of “recursive” forecasting is the requirement of a large sample size to be able to find a strong evidence of changes in predictive ability such that it matters for out-of-sample forecasting. Previous out-of-sample measurements of the predictive ability of the yield curve for output growth are based on “rolling” forecasting (Ang et al., 2006) and also on “recursive” forecasting (Stock and Watson, 2003). In this paper, large differences in terms of root mean squared forecast errors between “rolling” and “recursive” methods are only detected at long horizons. This is explained by the use of direct forecasts since the estimates of the coefficients of the regressions are more sensitive to changes from inclusion of new information in longer than in shorter horizons. Therefore, I will only show the results of “rolling” forecasts at  $h = 8, 12$ .

#### 4.1 Measurement of comparative performance of forecasts

Forecasts of the regression models are compared with an AR(1) benchmark, implying that, under the null, the regressor has no predictive ability. The comparison of the accuracy of the benchmark forecaster (BF) with the performance from different regression specifications (RF) uses a quadratic loss function. It follows that the average differential of  $h$ -step-ahead forecasters is  $n^{-1} \sum_{t=1}^n (\hat{e}_{h,BF,t}^2 - \hat{e}_{h,RF,t}^2)$ , where  $\hat{e}$  is the forecast error. Using this differential, a t-statistic, computed with a HAC estimator of the variance, is applied for testing the null (Diebold and Mariano, 1995). In the case of non-nested models, such as the case that the RF forecaster has no autoregressive term, this statistic has normal distribution. Similar testing is employed for evaluating whether RF forecast encompasses BF, implying that the predictor has information in addition to the autoregressive term. The average differential of the forecasting encompassing test is  $n^{-1} \sum_{t=1}^n (\hat{e}_{h,BF,t}^2 - \hat{e}_{h,BF,t} \hat{e}_{h,RF,t})$  (Harvey, Leybourne and Newbold, 1998). The t-statistic with this differential is also normally distributed when the models are non-nested.

In the case of comparing regressions with the benchmark when they include an AR term, the competitor forecasters are nested under the null. Clark and McCracken (2005a) show how to compare direct forecasts of nested regressions with an F version of the usual accuracy and encompassing tests. The distributions of the statistics are data-dependent, and they show that a bootstrap procedure to compute p-values gives powerful tests with correct size. As a consequence, I use the following statistics to evaluate forecast accuracy and forecasting encompassing of regression specifications with AR term in comparison to an AR(1):

$$MSE-F_h = n \left( \frac{MSE_{h,BF} - MSE_{h,RF}}{MSE_{h,RF}} \right)$$

and

$$ENC-F_h = n \left( \frac{n^{-1} \sum_{t=1}^n (\hat{e}_{h,BF,t}^2 - \hat{e}_{h,BF,t} \hat{e}_{h,RF,t})}{MSE_{h,RF}} \right),$$

where  $MSE_{h,M}$  is the mean squared forecast error of the model M at  $h$ -steps ahead.

The p-values of these statistics are computed by bootstrap as described in Clark and McCracken (2005a). In the first step, the estimates of an AR(1) for  $y_t$  are used to simulate a sample of size  $T$  of  $y_{t+h}$  by bootstrapping the residuals of the AR(1) model. In the second step, the sample is divided to mimic in-sample and out-of-sample sizes employed in the computation of the statistic. Then the benchmark and the regression model are estimated either recursively or rolling over the artificial "out-of-sample period", forecasts

are computed, and the forecast accuracy and encompassing F statistics are calculated. In the third step, the empirical distributions of the statistics are used to compute the p-value of the tests.

## 4.2 MIDAS X R

Table 6 presents root mean squared forecast errors (RMSFE) of single regressions for predicting output growth with the indicated predictors as a ratio of AR(1), except for the "AR" column that has RMSFEs. The results are presented with and without autoregressive terms in the first and the second panels. The first value in brackets is the p-value for the test that the indicated regression model is more accurate than the AR(1). The second value in brackets is the p-value for the test that the indicated regression has additional information (encompass) for forecasting output growth with respect to the AR(1).

In contrast with the in-sample results obtained with the predictive regression (R), the regression with the slope is only more accurate than the AR(1) for forecasting output growth at  $h = 1$ . This disagreement between in-sample and out-of-sample results of the use of the slope as predictor is not detected when applying MIDAS regressions. When the latter regressions are employed for extracting the predictive content of the slope, there are significant gains of accuracy in comparison to the AR(1) for  $h = 4, 8$  and  $12$ . The out-of-sample results also confirm the in-sample gains from the estimation of aggregation weights for the spread when forecasting output growth. Similarly, the results when using the stock returns as predictor also confirm in-sample evidence that the estimation of aggregation weights reduces forecast accuracy.

A surprising result in comparison with the ones obtained in-sample is the evidence of predictive content of the short-rate for output growth of the next two and three years ( $h = 8, 12$ ). Looking at the second plot of Figure 5, the coincidence of the out-of-sample period (after 1991) with the period with larger frequency of the regime that the short-rate has predictive power for output growth at long horizons explains those results. This is an interesting evidence because it shows how the use of full-sample models that allow for changes in predictive ability may help us to understand disagreements between in-sample and out-of-sample results. Similar evidence is also obtained when using the stock returns for forecasting at  $h = 12$ .

## 4.3 STMIDAS X STR

Table 7 has the same structure as Table 6, but it presents the results of smooth transition regressions and STMIDAS regressions in comparison to the AR(1) benchmark. Compar-

ing the indications of the tests of equal accuracy with Table 6, the gains of allowing changes in predictive ability for forecasting output growth are small, but they are more important for the spread at  $h = 1$  (with MIDAS) and the stock returns at  $h = 1, 4$  (with R). Small gains of non-linearity in out-of-sample forecasting are broadly reported in the literature (Terasvirta, Medeiros and Van Dijk, 2005). Nonlinear models require a larger sample to be well estimated (explains why recursive is better than rolling) and some regimes may not occur in a short out-of-sample period

In contrast, if the only concern is the measurement of the predictive content of a candidate predictor for predicting a variable, the STMIDAS is recommended because it helps to better understand incompatibilities between in-sample and out-of-sample results.

#### 4.4 Additional predictive content

The RMSFE ratios presented in Table 8 allow us to check whether the evidence in Table 5 that the stock returns have strong additional predictive content with respect to the slope for forecasting output growth is repeated in real-time. The ratios in Table 8 are not computed using the AR(1) as benchmark. They use a regression with the described specification for the each column but only with the spread as predictor instead of the combination.

There are gains up to 35% in terms of RMSFE from the inclusion of stock returns for forecasting next quarter output growth. The gains of adding stock returns are larger than adding the short-rate. In agreement with the in-sample results, both predictors only have additional information for predicting growth at long horizons if the specification has shifting parameters. These results confirm the potential of using stock returns for predicting output growth of next 2-3 years. Recall that the third plot of Figure 5 indicates that only when the coefficient on the stock returns is allowed to shift, large returns imply strong growth in the future.

## 5 Concluding Remarks

The smooth transition MIXed Data Sampling (STMIDAS) regression improves the measurement of the predictive content of high frequency predictors on low frequency variables. I propose simple tests for detecting changes in ability of the high frequency predictor in predicting the low frequency variable. The estimation of the weights to aggregate the high frequency variable improves the extraction of the predictive content of the slope for output growth. The approach is also flexible enough to detect the predictive content of stock returns for forecasting output growth next two and three years. An important

advantage of the STMIDAS regressions is that their estimates help our understanding on the disagreements between in-sample and out-of-sample evidences of predictive ability.

The implication of this new modelling for the use of financial variables as leading indicators is that they have more predictive content than normally found. Even if there is instability on the predictive power (Stock and Watson, 2003), financial variables do have useful information for forecasting economic growth at long horizons.

The approach proposed in this paper could be also applied to measure predictive ability of fundamentals on exchange rates (Kilian and Taylor, 2003) and of dividends on stock returns (Ang and Bekaert, 2007). The advantage of allowing for changes in the predictive ability can help to identify periods in which the predictor has superior predictive content.



## A Testing Predictive Ability with MIDAS

The MIDAS regression to measure the predictive ability of a high frequency variable  $x$  for a low frequency variable  $y$  is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,m)}^{(m)} + \varepsilon_{t+h}, \quad (12)$$

with the notation described in equation (3). The null hypothesis of a test of no predictive content of  $x_{t(\kappa,m)}^{(m)}$  for  $y_{t+h}$  is  $\beta_{1,h}^{(m)} = 0$ . The problem is that under this null the parameters of the weight function,  $\kappa$ , are unidentified. The effect on the properties of the test is that it is severely oversized. Similar problem is found when testing for changes in predictive ability, that is, for non-linearity (Granger and Teräsvirta, 1993). Instead of using a LM test with an auxiliary regression obtained using the Taylor approximation of the non-linear regression model, I will use a bootstrap procedure. The bootstrap procedure is based on the procedure of Kilian (1999) applied for testing predictability in Kilian and Taylor (2003) and considered as a procedure robust to misspecification by Corradi and Swanson (2007).

The bootstrap procedure is employed to simulate data under the null in order to compute an empirical distribution for the test statistic. In the case of the MIDAS regression, I am interested in the data-dependent distribution of a t-statistic. The bootstrap procedure has also some similarities with the one employed for computing the empirical distribution of the Hansen sup-test for threshold non-linearity (Hansen, 2000).

Under the null, the data generating process for  $y_t$  is:

$$y_t = (z_t - z_{t-1}) = \mu + \rho(z_{t-1} - z_{t-2}) + \epsilon_t, \quad (13)$$

where  $z_t$  is the log of real GDP in dollars. Data on  $z_t$  are used to estimate the conditional mean and obtain the residuals  $\hat{\epsilon}_t$ , which are then used to bootstrap  $B$  samples of size  $T + h$  of  $z_t$  in order to compute a sample of  $T$  observations of  $y_{t+h}$  (recall  $y_{t+h} = (400/h) [z_{t+h} - z_t]$ ). It is also required to simulate data of the predictor. Kilian and Taylor (2003) use a non-linear specification for  $x_t$ , and include past values of  $y$  as regressors. Because of the difference of frequencies between  $x$  and  $y$ , and the findings of non-linearities in financial time series, I use a threshold autoregressive model to generate  $x_t^{(m)}$ . Models with switching regimes have been employed previously for modelling interest rates (Ang and Bekaert, 2002). The two-regime threshold autoregressive model

is estimated for the weekly data using  $p = 2m$ , that is,

$$x_t^{(m)} = \left( \phi_0^{(1)} + \sum_{i=1}^p \phi_i^{(1)} x_{t-i}^{(m)} + \eta_t^{(1)} \right) I(x_{t-d} \leq c) + \left( \phi_0^{(2)} + \sum_{i=1}^p \phi_i^{(2)} x_{t-i}^{(m)} + \eta_t^{(2)} \right) I(x_{t-d} > c), \quad (14)$$

where  $c$  is threshold and  $d$  is the delay. The parameters are estimated by conditional least squares (Tsay, 1989) using a grid search for the threshold and the delay. The grid for the threshold has 100 points with the limits given by  $c_L$  and  $c_U$  defined such that there are at least 15% of the observations in each regime. The limits for the delay are  $d_L = 1$  and  $d_U = p$ . Using the estimates to simulate high frequency data on  $x_t^{(m)}$ , I bootstrap from the residuals  $\hat{\eta}_t^{(1)}$  and  $\hat{\eta}_t^{(2)}$  separately. This takes into account heteroscedasticity in the residuals of the threshold model.

Using each of the simulated sequences  $\{y_{t+h,i}\}_{i=1}^{i=T}$  and  $\{x_i^{(m)}\}_{i=1}^{i=mT}$ , the model under the alternative hypothesis (eq. 12) is estimated and the t-statistic  $\frac{\hat{\beta}_{1,h}^{(m)}}{\sqrt{\text{var}(\hat{\beta}_{1,h}^{(m)})}}$  is computed. Note that the Newey-West estimator with truncation lag  $h - 1$  is employed to compute the variance. The  $B$  replications allow us to compute the p-value for the t-statistic.

Similar procedure is employed for testing no predictive content of  $x_{t(\kappa,m)}^{(m)}$  for  $y_{t+h}$  using the STMIDAS. The specification under the alternative is:

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,K)}^{(m)} \left[ 1 - G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c) \right] + \beta_{2,h}^{(m)} x_{t(\kappa,K)}^{(m)} \left[ G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c) \right] + \varepsilon_{t+h}.$$

A Wald statistic is employed for the null that  $\beta_{1,h}^{(m)} = \beta_{2,h}^{(m)} = 0$ . Using the same data generating process described before, a STMIDAS regression is estimated in each replication and a Wald statistic is computed. The empirical distribution of the Wald statistic is used to compute p-values.

The testing of no predictive ability with the STMIDAS specification could be questioned on the grounds that if there is evidence of changes in predictive ability, there is also evidence of predictive ability because the test of no predictive content only adds an additional restriction on STMIDAS regression in comparison to the test for no changes in predictive ability. However, the test for changes in predictive ability presented in section 2.2.1 is based on an auxiliary regression. Although the results of the Monte Carlo evaluation in section 2.3.2 indicate that the testing procedure has power for smooth shifts in the predictive parameters, it does not require the computation of the model under the alternative. The proposed test for no predictive content requires the computation of the STMIDAS regression, while it assumes that a regime-switching specification is adequate to capture the predictive ability of  $x_{t(\kappa,m)}^{(m)}$  for  $y_{t+h}$ .

In the case of testing whether  $x_{2t}^{(m)}$  has predictive ability for  $y_{t+h}$  in a regression that has  $x_{1t}^{(m)}$ , I use also the two-regime TAR for  $x_{1t}$  under the null. However, the data generating process for  $y_{t+h}$  is a MIDAS with only  $x_{1t}^{(m)}$  as regressor. The specification under the alternative is the combining STMIDAS described in eq(9), as a consequence the restrictions under the null are  $\beta_{21,h}^{(m)} = \beta_{22,h}^{(m)} = 0$ . I assume that  $x_{2t}^{(m)}$  also follows a two-regime TAR, so values of the second high frequency regressor are also simulated using estimates of equation (14). One more time, the bootstrap procedure allows the computation of the distribution of the Wald statistic. Similar procedure is employed for testing whether  $x_{2t}^{(m)}$  has additional predictive ability with a combining MIDAS under the alternative, that is,

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{1,t(\kappa_r,m)}^{(m)} + \beta_{2,h}^{(m)} x_{2,t(\kappa,K)}^{(m)} + \varepsilon_{t+h}.$$

## B Estimation of STMIDAS

The parameters of the STMIDAS are collected in the vector  $\theta_h = [\beta_{0,h}^{(m)}, \beta_{1,h}^{(m)}, \beta_{2,h}^{(m)}, \rho_h, \kappa_1, \kappa_2, \gamma, c]'$ . So that the nonlinear regression is written as:

$$y_{t+h} = m(x_t^{(m)}, \theta_h) + \varepsilon_{t+h}.$$

The parameters of this regression can be consistently estimated by minimizing the sum of squared residuals:

$$Q_T(\theta_h) = T^{-1} \sum_{t=1}^T (y_{t+h} - m(x_t^{(m)}, \theta_h))^2,$$

because the function  $m(x_t^{(m)}, \theta_h)$  satisfies the identification and regularity conditions described in Hayashi (2000), ch. 7, proposition 7.4. Under additional conditions regarding the differentiability of  $m(x_t^{(m)}, \theta_h)$  and the behaviour of the Hessian  $h(\hat{\theta})$ , the NLS estimator  $\hat{\theta}_h$  is asymptotically normal, so that  $\sqrt{n}(\hat{\theta}_h - \theta_h) \rightarrow^d N(0, h(\theta_h)^{-1} \Sigma h(\theta_h)^{-1})$ .

The computation of the estimates can be simplified by concentrating the sum of squared residuals function with respect to  $\kappa, \gamma, c$ , so that the parameters in the vector  $\beta_h = [\beta_{0,h}^{(m)}, \beta_{1,h}^{(m)}, \beta_{2,h}^{(m)}, \rho_h]'$  can be computed with the least squares formula:

$$\hat{\beta}_h = \left( \sum_{t=1}^T x_{t(\hat{\kappa}, \hat{\gamma}, \hat{c})}^{(m)} x_{t(\hat{\kappa}, \hat{\gamma}, \hat{c})}^{(m)'} \right)^{-1} \sum_{t=1}^T x_{t(\hat{\kappa}, \hat{\gamma}, \hat{c})}^{(m)} y_t.$$

In practice, STMIDAS regressions use the estimates of MIDAS regressions as initial values for  $\kappa$ . Initial values for  $\kappa$  in MIDAS regression are obtained by a search over a grid of

values for  $\kappa_1, \kappa_2$  such that they imply different shapes for the weight function  $w(j, \kappa)$ . The initial values for  $\gamma$  and  $c$  in the STMIDAS regressions are also computed in a grid search. The optimisation procedure (with BFGS) imposes constraints in  $\gamma$  such that it is not too large or negative and in  $c$  such that it is not smaller (larger) than the 5% (95%) quantile of the empirical distribution of the weight high frequency predictor  $x_{t(\kappa)}^{(m)}$ .

The variance-covariance matrix of the estimates  $var(\hat{\theta}_h)$  is computed using the derivatives of the nonlinear function with respect to the parameters:

$$\frac{\delta m(x_t^{(m)}, \theta_h)}{\delta \theta_h'} = x_t^{(m)}(\theta_h).$$

This is so because under asymptotic linearity (conditions given in section 8.9.3 of Mittelhammer, Judge and Miller (2000)), I can compute the variance-covariance matrix as:

$$\widehat{var}(\hat{\theta}_h) = \left[ \sum_{t=1}^T x_t^{(m)}(\hat{\theta}_h) x_t^{(m)}(\hat{\theta}_h)' \right]^{-1} \Sigma \left[ \sum_{t=1}^T x_t^{(m)}(\hat{\theta}_h) x_t^{(m)}(\hat{\theta}_h)' \right]^{-1}.$$

An important comment on this formula is that  $\widehat{var}(\hat{\theta}_h)$  is computed using the variation of  $x_t$  across  $Tm$ . When computing the same formula using  $x_t$  aggregated in time instead of  $x_t^{(m)}$ , the variation across  $T$  observations is used.

An estimator for  $\Sigma$  that is consistent under autocorrelation and heteroscedasticity is the one by Newey and West (1987). The formula for the specific case of STMIDAS is written as:

$$\begin{aligned} \hat{\Sigma} &= \hat{\Gamma}(0) + \sum_{j=1}^p \left( 1 - \frac{j}{p+1} \right) (\hat{\Gamma}(j) + \hat{\Gamma}'(j)) \\ \hat{\Gamma}(j) &= \frac{1}{T} \sum_{t=j+1}^T \hat{\varepsilon}_{t+h} \hat{\varepsilon}_{t+h-j} \left( x_t^{(m)}(\hat{\theta}) x_t^{(m)}(\hat{\theta})' \right). \end{aligned} \quad (15)$$

Nonlinear least squares is also employed to estimate the MIDAS regression (equation 1) and the smooth transition regression (equation 7), while OLS is applied to the regression (2). In both cases, the estimator (15) is used to make the variance of the estimator robust to autocorrelation and heteroscedasticity.

## References

Anderson, H. M. and Vahid, F. (2001). Predicting the probability of a recession with nonlinear autoregressive leading indicator models, *Macroeconomic Dynamics* **5**: 482–505.

- Ang, A. and Bekaert, G. (2002). Regime switches in interest rates, *Journal of Business and Economic Statistics* **20**: 164–182.
- Ang, A. and Bekaert, G. (2007). Stock return predictability: Is it there?, *Review of Financial Studies*, *forthcoming* .
- Ang, A., Piazzesi, M. and Wei, M. (2006). What does the yield curve tell us about GDP growth?, *Journal of Econometrics* **131**: 359–403.
- Carrasco, M. (2002). Misspecified structural change, threshold and markov switching models, *Journal of Econometrics* **109**: 239–73.
- Clark, T. E. and McCracken, M. W. (2005a). Evaluating direct multistep forecasts, *Econometric Reviews* **24**: 369–404.
- Clark, T. E. and McCracken, M. W. (2005b). The power of tests of predictive ability in the presence of structural breaks, *Journal of Econometrics* **124**: 1–31.
- Clements, M. P. and Galvão, A. B. (2006). Macroeconomic forecasting with mixed frequency data: Forecasting US output growth, *University of Warwick, Working Paper Series* **773**.
- Cochrane, J. H. (2005). *Asset Pricing*, Princeton University Press, Princeton.
- Corradi, V. and Swanson, N. R. (2007). Nonparametric bootstrap procedures for predictive inference based on recursive estimation schemes, *International Economic Review* **forthcoming**.
- Diebold, F. X. and Mariano, R. S. (1995). Comparing predictive accuracy, *Journal of Business and Economic Statistics* **13**: 253–65.
- Estrella, A. and Hardouvelis, G. A. (1991). The term structure as a predictor of real economic activity, *Journal of Finance* **46**: 555–76.
- Estrella, A. and Mishkin, F. S. (1998). Predicting US recessions: Financial variables as leading indicators, *Review of Economics and Statistics* **80**: 45–61.
- Estrella, A., Rodrigues, A. P. and Schich, S. (2003). How stable is the predictive power of the yield curve? Evidence from Germany and the United States, *Review of Economics and Statistics* **85**: 629–44.
- Galbraith, J. W. and Tkacz, G. (2000). Testing for asymmetry in the link between the yield spread and output in the G-7 countries, *Journal of International Money and Finance* **19**: 657–672.
- Galvão, A. B. C. (2006). Structural break threshold VARs for predicting the probability of US recessions using the spread, *Journal of Applied Econometrics* **21**: 463–87.
- Ghysels, E., Santa-Clara, P. and Valkanov, R. (2004). The MIDAS touch: Mixed data sampling regressions, *University of North Carolina, mimeo* . ([www.unc.edu/eghysels/](http://www.unc.edu/eghysels/)).

- Ghysels, E., Santa-Clara, P. and Valkanov, R. (2006). Predicting volatility: How to get most out of returns data sampled at different frequencies, *Journal of Econometrics* **131**: 59–95.
- Granger, C. W. J. and Teräsvirta, T. (1993). *Modelling Nonlinear Economic Relationships*, Oxford University Press, Oxford.
- Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* **57**: 357–84.
- Hamilton, J. D. and Kim, D. H. (2002). A re-examination of the predictability of economic activity using the yield spread, *Journal of Money, Credit and Banking* **34**: 340–60.
- Hansen, B. E. (2000). Testing for linearity, in D. A. R. George, L. Oxley and S. Potter (eds), *Surveys in Economic Dynamics*, Blackwell, Oxford, pp. 47–72.
- Harvey, C. P. (1988). Forecasts of economic growth from the bond and stock markets, *Financial Analysts Journal* **45**: 38–45.
- Harvey, D. I., Leybourne, S. J. and Newbold, P. (1998). Tests for forecast encompassing, *Journal of Business and Economic Statistics* **16**: 254–59.
- Hayashi, F. (2000). *Econometrics*, Princeton University Press, Princeton.
- Hodrick, R. J. (1992). Dividend yields and expected stock returns: Alternative procedures for inference and measurement, *Review of Financial Studies* **5**: 829–853.
- Inoue, A. and Kilian, L. (2004). In-sample or out-of-sample tests of predictability: Which one should we use?, *Econometric Reviews* **23**: 371–402.
- Inoue, A. and Kilian, L. (2006). On the selection of forecasting models, *Journal of Econometrics* **130**: 273–306.
- Kilian, L. (1999). Exchange rate and monetary fundamentals: What do we learn from long-horizon regressions?, *Journal of Applied Econometrics* **14**: 491–510.
- Kilian, L. and Taylor, M. P. (2003). Why is so difficult to beat the random walk forecast of exchange rates?, *Journal of International Economics* **60**: 85–107.
- Kim, C.-J. and Nelson, C. R. (1999). *State-Space Model with Regime Switching*, MIT Press, Cambridge.
- Luukkonen, R., Saikkonen, P. and Terasvirta, T. (1988). Testing linearity against smooth transition autoregressive models, *Biometrika* **75**: 491–99.
- Massimiliano, M., Stock, J. and Watson, M. (2006). A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series, *Journal of Econometrics* **135**: 499–526.
- Mittelhammer, R. C., Judge, G. G. and Miller, D. J. (2000). *Econometric Foundations*, Cambridge University Press, Cambridge.
- Newey, W. and West, K. D. (1987). A simple positive semi-definite, heteroskedasticity

- and autocorrelation consistent covariance matrix, *Econometrica* **55**: 703–08.
- Pesaran, M. H. and Timmermann, A. (2005). Small sample properties of forecasts from autoregressive models under structural breaks, *Journal of Econometrics* **129**: 183–217.
- Sims, C. A. and Zha, T. (2006). Were there regime switches in U.S. monetary policy?, *American Economic Review*, *forthcoming* .
- Stock, J. H. and Watson, M. W. (2003). Forecasting output and inflation: The role of asset prices, *Journal of Economic Literature* **41**.
- Strikholm, B. and Teräsvirta, T. (2005). Determining the number of regimes in a threshold autoregressive model using smooth transitions, *SSE/EFI Working Paper Series in Economics and Finance* **n. 578**.
- Teräsvirta, T. (1998). Modeling economic relationships with smooth transition regressions, in A. Ullah and E. David (eds), *Handbook of Applied Economic Statistics*, Marcel Dekker, New York, pp. 507–52.
- Teräsvirta, T., Medeiros, M. and Van Dijk, D. (2005). Linear models, smooth transition autoregressions, and neural networks for forecasting macroeconomic time series: A re-examination, *International Journal of Forecasting* pp. 755–774.
- Tong, H. (1990). *Non-Linear Time Series: A Dynamical System Approach*, Oxford University Press, Oxford.
- Tsay, R. S. (1989). Testing and modeling threshold autoregressive processes, *Journal of the American Statistical Association* **84**: 231–40.
- Valkanov, R. (2003). Long-horizon regressions: Theoretical results and applications, *Journal of Financial Economics* **68**: 202–232.
- Van Dijk, D., Teräsvirta, T. and Franses, P. H. (2002). Smooth transition autoregressive models - A survey of recent developments, *Econometric Reviews* **21**: 1–47.

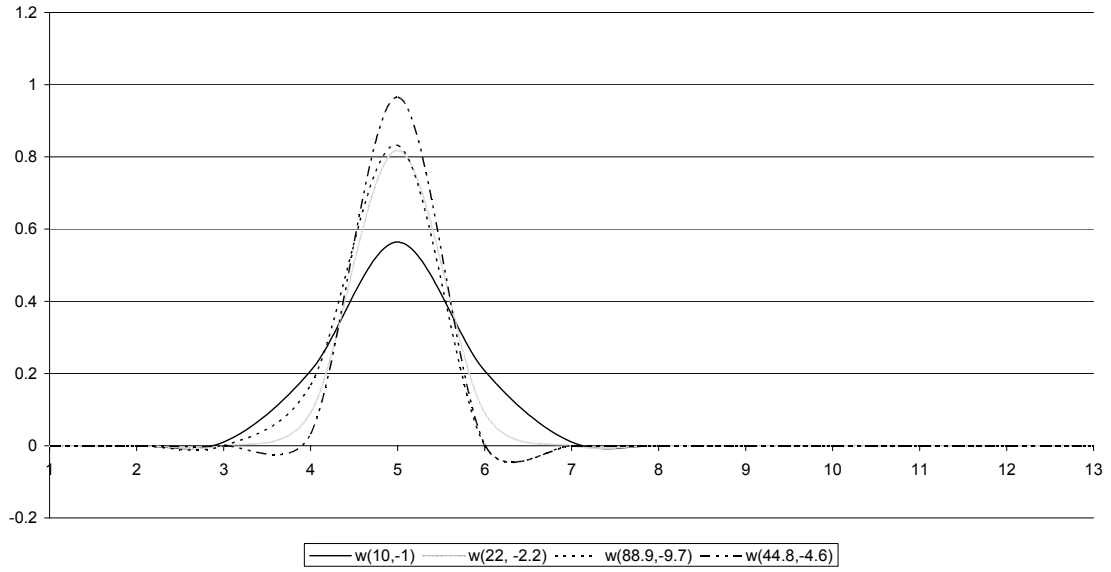


Figure 1: Weight Functions for different values of  $\kappa_1$  and  $\kappa_2$  and similar ( $\kappa_1/\kappa_2$ ).

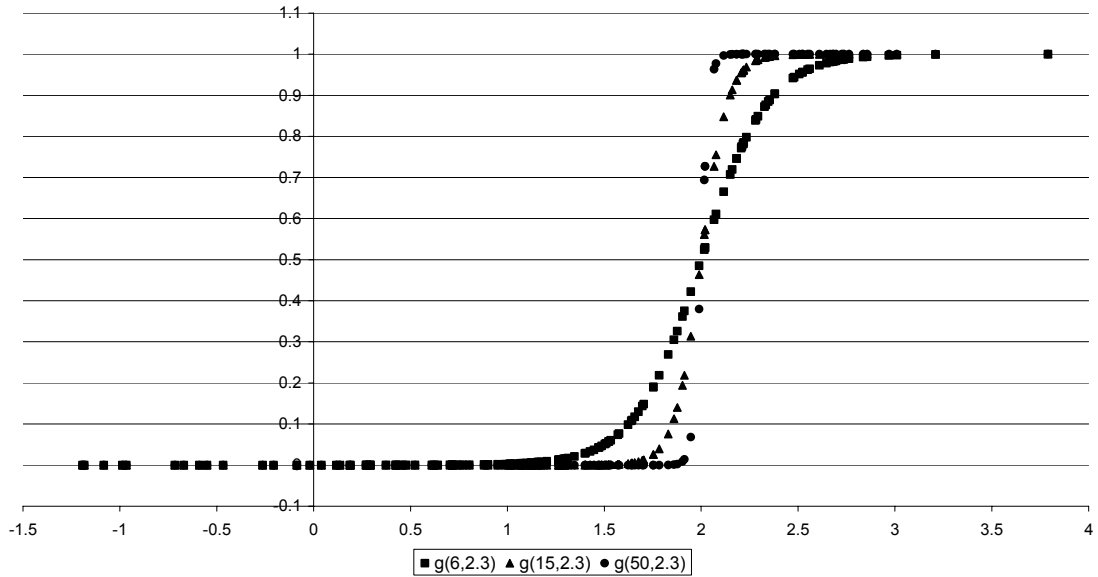


Figure 2: Exponential function for different values of  $\gamma$  (6, 15, 60) with  $T = 140$ .



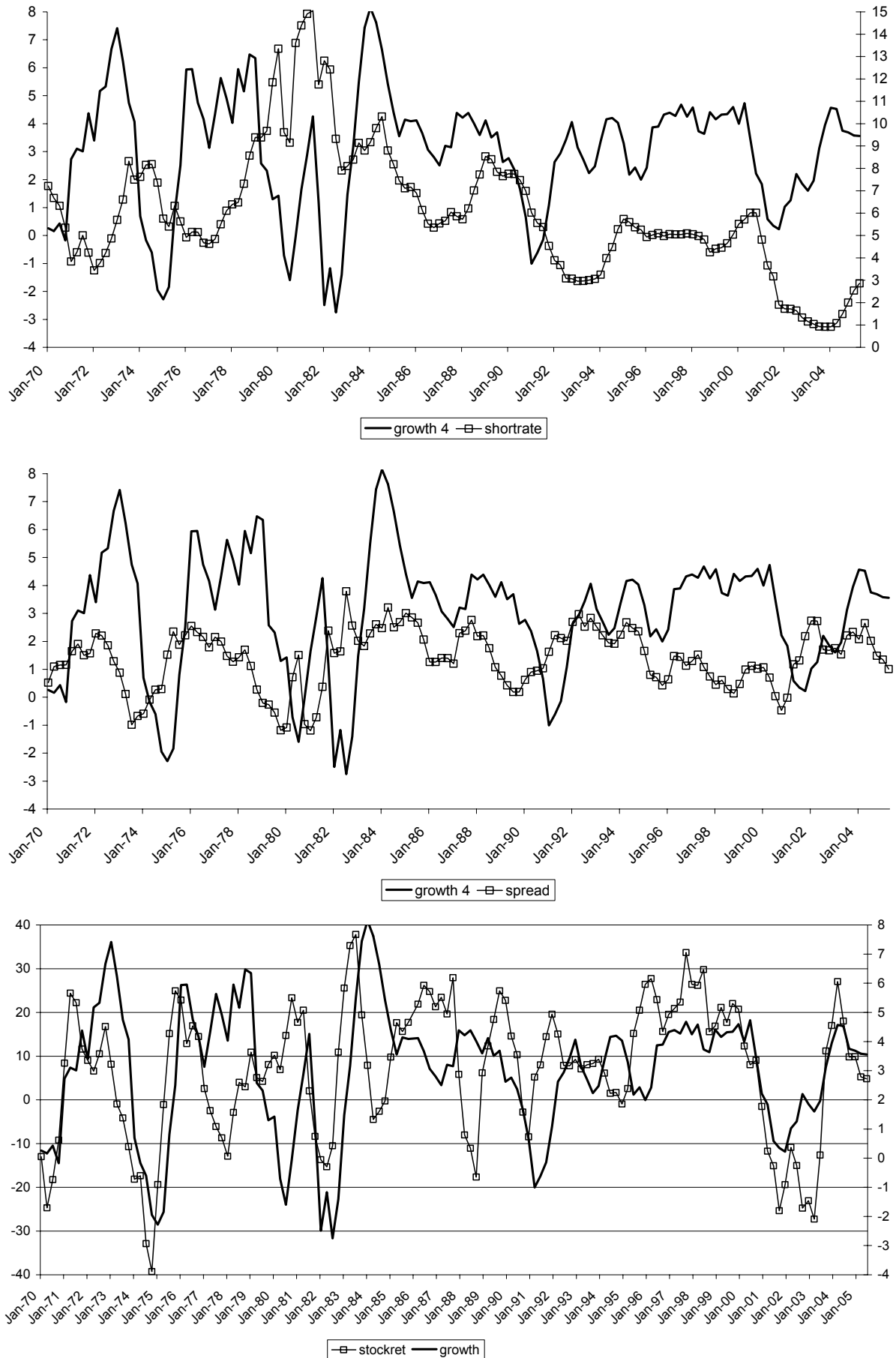


Figure 3: Output growth ( $100 * (\log(y_t) - \log(y_{t-4}))$ ) and quarterly predictors: short-rate ( $r_t^1$ ) in the first panel, spread ( $s_{20} = r_t^{20} - r_t^1$ ) in the second panel, and stock returns ( $100(\ln(p_t) - \ln(p_{t-4}))$ ) in the third panel.

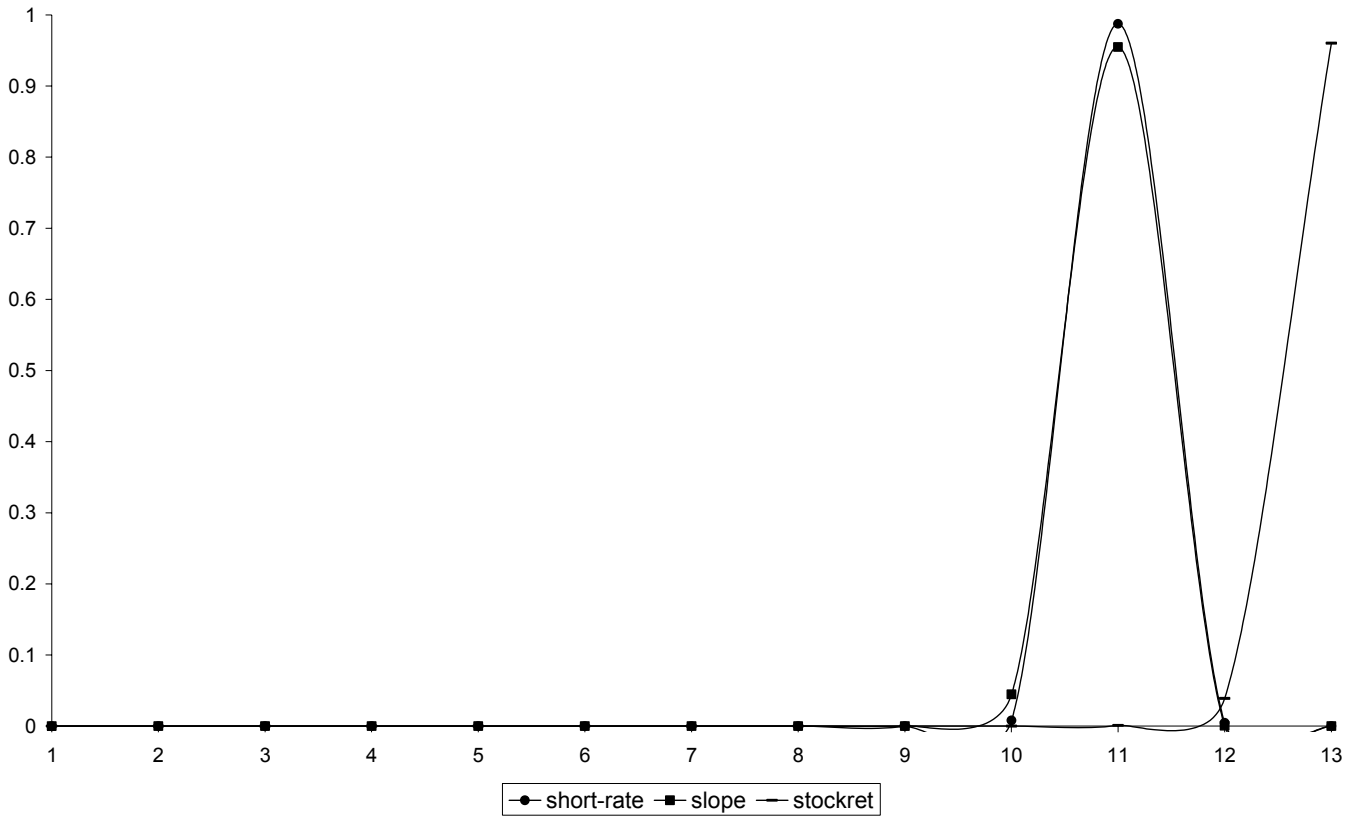


Figure 4: Estimated weight functions for MIDAS ( $m = 13$ ) for predicting next quarter output growth ( $h=1$ ) using the short-rate, the spread and stock returns.

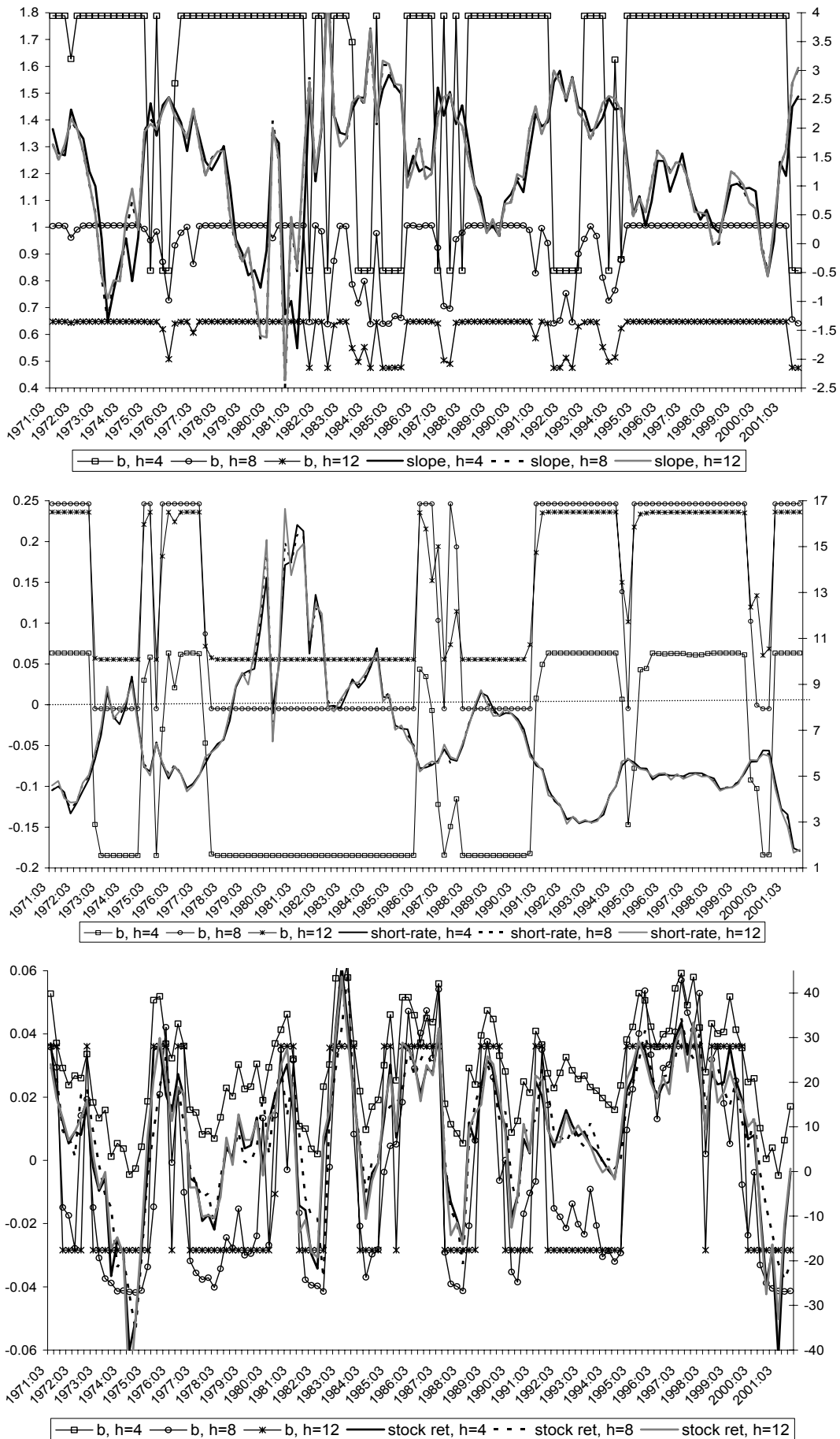


Figure 5: Estimates of ability in predicting US GDP growth using STMIDAS regressions ( $m=13$ ) for  $h=4, 8$  and  $12$ : slope in first panel, short-rate in the second panel and stock returns in the third panel. Weekly data aggregated by  $w(j, \hat{\kappa})$  estimated for each  $h$  is plotted in the secondary axis.

Table 1: Biases of NLS estimates of STMIDAS

H = 1							
T	$\beta_{0,h}$	$\beta_{1,h}$	$\delta_h$	$\gamma$	$\mathbf{c}$	$\kappa_1$	$\kappa_2$
100	-0.029	0.014	-0.073	202.220	-0.211	43.279	-2.216
200	-0.013	0.007	-0.005	159.790	-0.320	22.048	-1.114
500	-0.009	0.001	-0.001	50.125	-0.071	3.166	-0.159
1000	-0.007	0.000	0.001	8.068	-0.017	0.050	-0.002
h = 4							
T	$\beta_{0,h}$	$\beta_{1,h}$	$\delta_h$	$\gamma$	$\mathbf{c}$	$\kappa_1$	$\kappa_2$
100	-0.070	0.016	-0.077	204.298	0.286	67.390	-3.779
200	-0.051	0.019	-0.027	197.580	-0.153	41.664	-2.139
500	-0.026	0.017	-0.023	118.185	-0.429	12.594	-0.633
1000	-0.015	0.004	-0.007	43.823	-0.103	2.637	-0.135

$$y_{t+h} = \beta_{0,h}^{(m)} + \beta_{1,h}^{(m)} x_{t(\kappa,K)}^{(m)} + \delta_h^{(m)} x_{t(\kappa,K)}^{(m)} \left[ G_t(x_{t(\kappa,K)}^{(m)}; \gamma, c) \right] + \varepsilon_{t+h}$$

$$\beta_{0,h}^{(13)} = 0.4; \beta_{1,h}^{(13)} = 0.5; \beta_{2,h}^{(13)} = -0.3; \gamma = 6; c = 2.3; \kappa_1 = 10; \kappa_2 = -0.5;$$

The biases are computed using 1000 replications for different sample sizes T. The frequency of x is 13 times the frequency of y. When  $h = 1$ , there is an additional autoregressive term (=0.2). When  $h = 4$ , there is an MA(3) in the DGP disturbances.

Table 2: Rejection Rates of Test of Changing Predictive Ability

T	DGP with Constant Parameters		DGP with Changing Parameters	
	STR	STMIDAS	STR	STMIDAS
h = 1				
100	0.076	0.076	0.902	0.913
200	0.072	0.070	0.988	0.991
500	0.068	0.073	1	1
h = 4				
100	0.165	0.148	0.547	0.565
200	0.141	0.139	0.734	0.776
500	0.108	0.105	0.957	0.973
h = 8				
100	0.178	0.174	0.472	0.479
200	0.167	0.136	0.634	0.678
500	0.084	0.097	0.892	0.921

The rejection rates are computed using 1000 replications for different sample sizes T. The STMIDAS DGP is described in the notes of Table 1. The DGP for STR (smooth transition regression) is equal to the STMIDAS DGP with  $m=1$  and  $\kappa_1=\kappa_2=0$ . The linear DGPs have  $\beta_{0,h}^{(13)} = 0.4; \beta_{1,h}^{(13)} = 0.3$ . When  $h = 4(8)$ , there is an MA(3)(7) in the DGP disturbances. The indicated models are the ones under alternative hypothesis. The test is the one based on equation (5).

Table 3: In-sample results with predictive and MIDAS regressions

Predictor	Slope				Short-rate				Stock Returns			
Model	$\beta_1$	$\beta_1=0$	$R^2$	$SIC_2$	$\beta_1$	$\beta_1=0$	$R^2$	$SIC_2$	$\beta_1$	$\beta_1=0$	$R^2$	$SIC_2$
$h = 1$												
R	1.069 (.278)	[0.00]	0.11	2.421	-0.383 (.142)	[0.01]	0.09	2.446	0.093 (0.021)	[0.00]	0.16	2.370
MIDAS (m=13)	1.334 (.242)	[0.00]	0.22	2.364	-0.454 (.118)	[0.02]	0.14	2.471	0.072 (0.019)	[0.04]	0.11	2.497
$h = 4$												
R	1.211 (.222)	[0.00]	0.33	1.300	-0.344 (.111)	[0.00]	0.17	1.519	0.047 (0.018)	[0.01]	0.09	1.603
MIDAS (m=13)	1.185 (.217)	[0.02]	0.35	1.357	-0.314 (.140)	[0.12]	0.15	1.622	0.031 (0.018)	[0.22]	0.05	1.728
$h = 8$												
R	0.937 (.158)	[0.00]	0.38	0.584	-0.185 (.111)	[0.10]	0.09	0.960	0.018 (0.012)	[0.13]	0.03	1.030
MIDAS (m=13)	0.786 (.149)	[0.00]	0.31	0.761	-0.144 (.124)	[0.38]	0.06	1.075	0.015 (0.012)	[0.32]	0.02	1.113
$h = 12$												
R	0.622 (.127)	[0.00]	0.29	0.164	-0.063 (.096)	[0.51]	0.02	0.485	0.013 (0.009)	[0.15]	0.02	0.481
MIDAS (m=13)	0.544 (.114)	[0.00]	0.25	0.292	-0.041 (.104)	[0.68]	0.01	0.574	0.008 (0.009)	[0.44]	0.01	0.569

The sample size is kept constant across horizons with an effective sample from 1970:Q3 to 2002:Q1 ( $T = 123$ ). R is a single regression model (eq. 2) and MIDAS with  $m=13$  uses interest rates sampled weekly (eq. 1).  $\beta_1$  measures the impact of the predictor for  $yt+h$ . Robust standard errors are between parentheses. The value in brackets in the  $\beta_1=0$  column is the p-value of the test with the indicated null hypothesis. The p-value is computed using the t-distribution for regressions (R) and by the bootstrap procedure of Appendix A for MIDAS regressions.  $SIC_2$  is the Schwarz information criterion computed counting also the parameters of the MIDAS weight function.

Table 4: In-sample results with smooth transition regressions and smooth transition MIDAS regressions

Predictor	Slope						Short-rate						Stock returns						
Model	Lin Test	$\beta_1$	$\beta_2$	$\beta_1=0;$ $\beta_2=0;$	$R^2$	$SIC_2$	Lin Test	$\beta_1$	$\beta_2$	$\beta_1=0;$ $\beta_2=0;$	$R^2$	$SIC_2$	Lin Test	$\beta_1$	$\beta_2$	$\beta_1=0;$ $\beta_2=0;$	$R^2$	$SIC_2$	
h = 1																			
STR	[.049]	1.787 (0.546)	0.122 (0.479)	7.61 [.02]	0.16	2.479	[.325]	-0.926 (0.361)	-0.469 (0.155)	7.89 [.01]	0.15	2.456	[.950]	0.065 (0.054)	0.106 (0.031)	19.87 [.00]	0.16	2.483	
STMIDAS (m=13)	[.002]	2.504 (0.495)	0.870 (0.2342)	27.92 [0.03]	0.31	2.357	[.046]	-1.567 (0.399)	-0.571 (0.126)	21.26 [0.05]	0.18	2.538	[.529]	0.112 (0.030)	0.050 (0.033)	17.24 [.02]	0.12	2.604	
h = 4																			
STR	[.002]	1.999 (0.262)	0.868 (0.220)	62.85 [.00]	0.44	1.235	[.146]	-1.507 (0.749)	-0.426 (0.104)	16.45 [.00]	0.20	1.593	[.134]	-0.002 (0.034)	0.077 (0.027)	8.26 [.02]	0.12	1.594	
STMIDAS (m=13)	[.016]	1.788 (0.227)	0.838 (0.156)	61.96 [.02]	0.47	1.261	[.275]	0.063 (0.305)	-0.185 (0.161)	8.42 [0.12]	0.18	1.703	[.068]	-0.010 (0.071)	0.116 (0.351)	0.14 [.94]	0.07	1.819	
h = 8																			
STR	[.047]	1.287 (0.149)	0.742 (0.201)	82.29 [.00]	0.42	0.628	[.461]	0.134 (0.139)	-0.071 (0.106)	5.01 [0.08]	0.14	1.108	[.004]	-0.026 (0.016)	0.090 (0.050)	6.49 [.04]	0.10	1.063	
STMIDAS (m=13)	[.245]	1.006 (0.145)	0.638 (0.161)	52.36 [.02]	0.34	0.838	[.590]	0.246 (0.136)	-0.005 (0.099)	8.11 [0.16]	0.13	1.113	[.000]	-0.042 (0.019)	0.065 (0.018)	15.37 [.09]	0.12	1.129	
h = 12																			
STR	[.502]	0.796 (0.149)	0.585 (0.118)	42.32 [.00]	0.30	0.258	[.750]	0.188 (0.096)	0.026 (0.079)	2.27 [0.32]	0.08	0.543	[.001]	-0.033 (0.018)	0.041 (0.012)	11.58 [.00]	0.17	0.436	
STMIDAS (m=13)	[.701]	0.646 (0.152)	0.475 (0.111)	26.36 [.06]	0.26	0.394	[.605]	0.236 (0.119)	0.056 (0.065)	3.93 [0.29]	0.07	0.632	[.002]	-0.028 (0.014)	0.036 (0.012)	9.84 [0.19]	0.16	0.529	

The sample size is kept constant across horizons with an effective sample from 1970:Q3-2002:Q1 ( $T = 123$ ). STR is a smooth transition model using quarterly predictors (eq. 7). The smooth transition MIDAS (STMIDAS) with  $m=13$  uses the predictors sampled weekly (eq. 5). The column 'lin test' has the p-value of the test of no changes in the ability of the predictor in forecasting output growth at  $h$ -steps ahead, computed using auxiliary regressions (eq. 6 and 8).  $\beta_{1,h}$  measures the impact of the predictor for  $yt+h$  in the first regime.  $\beta_{2,h}$  is the equivalent measure of the second regime. Robust standard errors are between parentheses. The column " $\beta_1=0; \beta_2=0$ " has the Wald statistic for the indicated null hypothesis. P-values in brackets are computed using the chi-squared distribution for the STR and a bootstrapped distribution for the STMIDAS, as described in Appendix A.  $SIC_2$  is the Schwarz information criterion computed taking into account the parameters of transition function and of MIDAS weight function.

Table 5: In-sample results on Additional predictors in regressions with the slope.

Additional Predictor: Model	Short-rate			Stock Returns		
	Lin Test	$\beta_{21}=0;$ $\beta_{22}=0;$	SIC <sub>2</sub>	Lin Test	$\beta_{21}=0;$ $\beta_{22}=0;$	SIC <sub>2</sub>
h = 1						
R		-1.66 [0.10]	2.423		4.910 [.00]	2.267
MIDAS (m=13)		-1.756 [0.08]	2.442		3.431 [.03]	2.353
STR	[0.040]	7.64 [.02]	2.628	[0.211]	22.96 [.00]	2.429
STMIDAS (m=13)	[0.094]	3.73 [.74]	2.563	[0.031]	15.77 [.07]	2.463
h = 4						
R		-1.528 [.13]	1.280		2.889 [.00]	1.189
MIDAS (m=13)		-1.163 [.32]	1.437		1.417 [.20]	1.423
STR	[0.028]	4.34 [.11]	1.315	[0.000]	13.48 [.00]	1.184
STMIDAS (m=13)	[0.030]	6.91 [.30]	1.388	[0.000]	7.65 [.15]	1.305
h = 8						
R		-0.500 [0.62]	0.617		1.329 [0.19]	0.580
MIDAS (m=13)		-0.119 [.92]	0.878		0.957 [.24]	0.853
STR	[0.007]	11.37 [.01]	0.699	[0.000]	44.35 [.00]	0.534
STMIDAS (m=13)	[0.026]	15.18 [.09]	0.938	[0.000]	30.82 [.01]	0.883
h = 12						
R		0.579 [.44]	0.193		1.022 [.31]	0.170
MIDAS (m=13)		0.869 [.34]	0.382		.688 [.60]	0.393
STR	[0.035]	12.56 [.00]	0.291	[0.000]	18.93 [.00]	0.418
STMIDAS (m=13)	[0.005]	23.38 [.07]	0.443	[0.000]	24.47 [.02]	0.322

The sample size is kept constant across horizons with an effective sample from 1970:Q3 to 2002:Q1 ( $T = 123$ ). R is a linear regression model (eq. 11) with quarterly predictors. MIDAS with  $m=13$  uses predictors sampled weekly. STR is the smooth transition version of R. STMIDAS is the smooth transition version of MIDAS with two predictors (eq. 9). The entries labeled with “lin test” are p-values for tests of changing predictive ability based on auxiliary regression (eq. 10) for the additional predictor, assuming that there are changes in the coefficient of the slope. The column “ $\beta_{21}=0; \beta_{22}=0$ ” has the t(Wald) statistic for the null that the additional predictor has no predictive power. P-values in brackets are computed using the t (chi-squared) distribution for the R(STR) and bootstrapped distributions for the MIDAS and STMIDAS, as described in Appendix A. SIC<sub>2</sub> is the Schwarz information criterion computed taking into account the parameters of transition function and of MIDAS weight function.

Table 6: Real-Time Forecasting: Predictive regressions and Midas regressions against AR

		Slope				Short-rate				Stock returns				
	Roll	AR	R		MIDAS		R		MIDAS		R		MIDAS	
h = 1		1.121	1.251		1.430		1.302		1.227		0.865		1.319	
			[.97]	<b>[.02]</b>	[1.0]	<b>[.07]</b>	[.96]	<b>[.02]</b>	[.95]	<b>[.03]</b>	[.12]	<b>[.00]</b>	[1.0]	[.11]
h = 4		1.388	1.297		0.943		1.197		1.068		0.881		1.096	
			[.94]	[.36]	[.20]	<b>[.04]</b>	[.85]	<b>[.03]</b>	[1.0]	[.99]	<b>[.11]</b>	<b>[.01]</b>	[.85]	[.64]
h = 8		1.538	0.946		1.096		0.967		1.304		1.057		1.023	
			[.23]	<b>[.06]</b>	[.90]	[.74]	[.34]	[.18]	[.99]	[.98]	[.87]	[.66]	[.68]	[.44]
h = 8	✓	1.615	0.984		0.920		0.998		0.969		1.044		1.127	
			[.44]	[.16]	<b>[.00]</b>	<b>[.00]</b>	[.49]	[.13]	<b>[.06]</b>	<b>[.01]</b>	[.85]	[.67]	[.79]	[.51]
h= 12		1.447	1.072		1.268		1.409		1.442		1.038		0.990	
			[.81]	[.58]	[1.0]	[.98]	[.99]	[.99]	[.98]	[.97]	[.70]	[.46]	[.43]	[.23]
h= 12	✓	1.398	1.028		0.925		1.406		1.050		1.121		0.845	
			[.67]	[.41]	[.34]	[.17]	[.99]	[.99]	[.76]	[.57]	[.77]	[.43]	[.17]	<b>[.09]</b>
with AR term														
h = 1		1.121	0.948		1.001		0.975		1.035		0.835		1.100	
			<b>[.01]</b>	<b>[.01]</b>	[.26]	[.37]	<b>[.01]</b>	<b>[.00]</b>	[.89]	[.94]	[.12]	<b>[.00]</b>	[.98]	[.99]
h = 4		1.388	1.295		0.948		1.204		1.122		0.885		1.090	
			[.99]	[.14]	<b>[.05]</b>	<b>[.01]</b>	[.95]	<b>[.02]</b>	[.89]	[.94]	[.11]	<b>[.01]</b>	[.84]	[.64]
h = 8		1.538	1.013		0.930		0.955		1.239		1.033		1.046	
			[.53]	<b>[.05]</b>	<b>[.05]</b>	<b>[.06]</b>	<b>[.09]</b>	[.11]	[.92]	[.97]	[.87]	[.82]	[.73]	[.83]
h = 8	✓	1.615	1.046		1.046		0.981		0.993		1.077		1.167	
			[.70]	[.16]	[.67]	[.77]	[.22]	[.28]	[.28]	[.47]	[.81]	[.88]	[.91]	[.72]
h= 12		1.447	1.018		1.170		1.307		1.322		1.017		0.994	
			[.54]	[.66]	[.93]	[.97]	[.95]	[.99]	[.95]	[1.0]	[.70]	[.67]	[.36]	[.44]
h= 12	✓	1.398	0.993		0.881		1.306		1.023		1.074		0.939	
			[.28]	[.42]	<b>[.03]</b>	<b>[.04]</b>	[.95]	[.99]	[.34]	[.60]	[.78]	[.39]	<b>[.08]</b>	[.17]

The forecasts are computed using increasing sample sizes (recursive forecasts) except when indicated in the column labelled with “roll”, which indicates rolling windows of same size. Q3The entries for h=1 and h=4 are computed with data vintages from 1991:Q4 to 2004:Q3. The entries for h=8 and h=12 are computed only with vintages from 1991:Q4-2002:Q3. The entries for the AR(1) are RMSFEs. The entries for the regressions are ratios to the AR(1) RMSFE. R is a linear regression model with quarterly predictors (eq. 2). MIDAS with uses predictors sampled weekly (eq. 1). “with AR term” means that an autoregressive term was also include in the regression (section 2.2.1). The values in brackets are p-values for the test of equal forecast accuracy and forecasting encompassing with the AR(1) under the null. The p-values for the regressions with no AR term were computed using the t-distribution. The p-values for the regressions with AR term were computed by bootstrap for an F statistic (see section 4.1).



Table 7: Real-Time Forecasting: smooth transition regressions and smooth transition MIDAS against AR

		Slope		Short-rate		Stock returns			
	Roll	AR	STR	STMIDAS	STR	STMIDAS	STR	STMIDAS	
h = 1		1.121	1.333	1.332	1.325	1.242	0.957	1.239	
			[.99] [.03]	[.99] [.05]	[.96] [.00]	[.95] [.02]	[.36] [.00]	[.98] [.11]	
h = 4		1.388	1.153	0.933	1.559	1.188	0.847	1.121	
			[.78] [.16]	[.10] [.01]	[.96] [.04]	[.95] [.84]	[.02] [.01]	[.93] [.23]	
h = 8		1.538	1.097	1.144	0.953	1.701	1.119	1.132	
			[.75] [.33]	[.97] [.87]	[.39] [.14]	[.97] [.96]	[.82] [.55]	[.96] [.64]	
h = 8	✓	1.615	1.105	1.024	1.108	0.888	1.176	1.129	
			[.80] [.38]	[.80] [.38]	[.69] [.19]	[.12] [.02]	[.91] [.66]	[.84] [.43]	
h= 12		1.447	1.180	1.212	2.366	1.262	1.231	0.934	
			[.96] [.70]	[1.0] [.97]	[.95] [.94]	[.94] [.42]	[.95] [.72]	[.30] [.04]	
h= 12	✓	1.398	1.144	1.145	2.508	0.870	1.338	0.979	
			[.92] [.76]	[.56] [.25]	[.94] [.95]	[.20] [.03]	[.97] [.56]	[.43] [.20]	
With AR term									
h = 1		1.121	0.998	0.967	0.933	1.033	0.847	1.105	
			[.02] [.01]	[.03] [.17]	[.00] [.01]	[.42] [.33]	[.00] [.00]	[.95] [.91]	
h = 4		1.388	0.882	0.944	1.593	1.362	0.815	1.156	
			[.78] [.05]	[.03] [.04]	[.97] [.03]	[.94] [.94]	[.00] [.01]	[.82] [.29]	
h = 8		1.538	1.185	0.942	0.954	1.697	1.087	1.263	
			[.85] [.41]	[.07] [.11]	[.10] [.04]	[.98] [.99]	[.74] [.55]	[.96] [.96]	
h = 8	✓	1.615	1.171	1.012	0.956	0.961	1.172	1.156	
			[.80] [.54]	[.71] [.91]	[.13] [.11]	[.14] [.18]	[.87] [.77]	[.85] [.59]	
h= 12		1.447	1.116	1.134	2.447	1.050	1.196	0.859	
			[.78] [.78]	[.70] [.66]	[.99] [.93]	[.29] [.10]	[.89] [.80]	[.01] [.01]	
h= 12	✓	1.398	1.122	0.995	2.643	0.986	1.356	1.041	
			[.58] [.73]	[.14] [.25]	[.99] [.95]	[.13] [.16]	[.92] [.66]	[.43] [.57]	

The forecasts are computed using increasing sample sizes (recursive forecasts) except when indicated in the column labelled with “roll”, which indicates rolling windows of same size. Q3The entries for h=1 and h=4 are computed with data vintages from 1991:Q4 to 2004:Q3. The entries for h=8 and h=12 are computed only with vintages from 1991:Q4-2002:Q3. The entries for the AR(1) are RMSFEs. The entries for the regressions are ratios to the AR(1) RMSFE. STR is a smooth transition regression model with quarterly predictors (eq. 7). STMIDAS is a smooth transition model with predictors sampled weekly (eq. 4). “with AR term” means that an autoregressive term was also include in the regression (section 2.2.1). The values in brackets are p-values for the test of equal forecast accuracy and forecasting encompassing with the AR(1) under the null. The p-values for the regressions with no AR term were computed using the t-distribution. The p-values for the regressions with AR term were computed by bootstrap for an F statistic (see section 4.1).

Table 8: Real-Time Forecasting: Regressions combining the slope and an additional predictor against regressions with only the slope.

	Roll	Slope + Short-rate				Slope + Stock Returns			
		R	MIDAS	STR	STMIDAS	R	MIDAS	STR	STMIDAS
h = 1		1.052	<b>0.901</b>	1.125	1.108	<b>0.647</b>	<b>0.646</b>	<b>0.692</b>	<b>0.835</b>
h = 4		1.060	1.478	1.171	1.324	<b>0.893</b>	1.017	1.156	1.303
h = 8		1.099	1.065	1.007	<b>0.931</b>	1.129	1.096	<b>0.979</b>	0.999
h = 8	✓	1.193	1.618	<b>0.911</b>	<b>0.920</b>	1.105	1.594	--	--
h = 12		1.310	1.823	1.207	1.587	1.008	1.125	<b>0.968</b>	1.074
h = 12	✓	1.425	2.471	1.091	1.312	1.091	2.054	0.998	1.086
With AR									
h = 1		1.000	0.915	1.019	0.999	<b>0.780</b>	<b>0.722</b>	<b>0.810</b>	<b>0.843</b>
h = 4		0.938	1.346	1.097	1.300	<b>0.898</b>	1.006	1.173	1.303
h = 8		1.054	1.245	0.963	1.317	1.054	1.291	<b>0.894</b>	1.214
h = 8	✓	1.144	1.043	<b>0.831</b>	<b>0.806</b>	1.095	1.493	<b>0.791</b>	<b>0.852</b>
h = 12		1.282	1.889	1.127	1.664	1.062	1.219	1.005	1.148
h = 12	✓	1.405	3.054	1.066	1.222	1.070	2.044	<b>0.802</b>	<b>0.915</b>

The forecasts are computed using increasing sample sizes (recursive forecasts) except when indicated in the column labelled with “roll”, which indicates rolling windows of same size. The entries for h=1 and h=4 are computed with data vintages from 1991:Q4 to 2004:Q3. The entries for h=8 and h=12 are computed only with vintages from 1991:Q4-2002:Q3. R is a linear regression model with quarterly predictors. MIDAS with uses predictors sampled weekly. STR is a smooth transition regression model with quarterly predictors. STMIDAS is a smooth transition model with predictors sampled weekly. For combining specifications see section 2.2.3. “with AR term” means that an autoregressive term was also include in the regression. The entries are ratios of the RMSFE of the combining regression to a regression with only the slope. Ratios that imply a gain larger than 5% in favour of the combination are emboldened.

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**Department of Economics  
Queen Mary, University of London  
Mile End Road  
London E1 4NS  
Tel: +44 (0)20 7882 5096  
Fax: +44 (0)20 8983 3580  
Web: [www.econ.qmul.ac.uk/papers/wp.htm](http://www.econ.qmul.ac.uk/papers/wp.htm)**