Department of Economics

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Working Paper No. 592 March 2007 ISSN 1473-0278

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February 2007

Abstract

We propose a way to test the New Keynesian Phillips Curve (NKPC) without estimating the structural parameters governing the curve, i.e. price stickiness and firms' backwardness. Using this strategy we can test the NKPC avoiding the identification problems related to the GMM approach. We find that it does not exist a combination of the structural parameters which is consistent with US data. This result does not necessarily imply that the idea of a forward looking price setting behaviour should be entirely disregarded, as the rejection might be due to the failure of the joint hypothesis of rational expectations. Thus further research should be aimed at providing alternative models for agents' expectations.

JEL Classification: C32, E31

Keywords: VARs, Inflation, Phillips Curve

1 Introduction

Recently several papers have provided tests of the New Keynesian Phillips Curve (NKPC). The bunch of empirical evidence pursues a single-equation approach, uses the ex-post realized data to proxy ex-ante expectations, and estimates the NKPC via Generalized Methods of Moments (GMM). Galí and Gertler (1999) and Galí et al. (2001) have provided estimates of the NKPC clearly supporting the theory. Rudd and Whelan (2005a,b, 2006) have showed that these tests have low power against non-nested alternatives and have derived alternative tests on the closed form forward solution of the NKPC which find very limited role for forward looking expectations.

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These contradicting results may be due to the fact that single equation GMM estimation of rational expectations models may suffer of several problems, including lack of identification of the parameters, misspecification due to omitted variables, correlation of the instruments with the error term, or weak instruments. Pesaran (1987) and Beyer et al. (2005) provide a complete discussion of these issues. As a possible alternative to GMM, Fuhrer and Rudebush (2002) and Lindé (2005) have proposed Maximum Likelihood (ML) estimation. However, as emphasized by Cochrane (2001) there are no general theorems or Montecarlo exercises suggesting which one of the two works better.

In this paper we tackle these problems by proposing a simple test of the NKPC which avoids the estimation of the structural parameters measuring price stickiness and firms backwardness. Actually we do not estimate the structural parameters at all, we rather pursue a "brute force" strategy. First, we estimate an unrestricted VAR in inflation and marginal costs. Then, we show that the NKPC implies a set of restrictions on the VAR coefficients. The restrictions imply that the unrestricted VAR coefficients must equal some convolutions of the structural parameters. Finally, we test the restrictions via a simple Wald test.

In principle the Wald test of the restrictions would require the knowledge of the true values of the structural parameters. However, as both these parameters are bounded between 0 and 1, it is possible to grid search over the space they span and see whether there is any combination of them which is consistent with the data. Rudd and Whelan (2006) use a similar strategy to compute theoretical inflation, but they do not test for the restrictions. Using this strategy we can test the NKPC avoiding the identification problems related to the GMM approach.

We apply this procedure to US data, and we find that according to the Wald test it does not exist a combination of the structural parameters consistent with the data. The documented rejection is so strong that it can be hardly explained by small sample biases. However, this result does not necessarily imply that the idea of a forward looking price setting behaviour should be entirely disregarded, as the rejection of the NKPC may be due to the failure of the joint hypothesis of rational expectations.

2 Methodology and Results

In this section we describe our framework and implement the proposed test. First we briefly describe the NKPC. Then we show that it imposes a set of restrictions on a VAR in inflation and marginal costs. Finally we perform the test using US data.

2.1 The New Keynesian Phillips Curve

We refer to the NKPC in the formulation of Galí and Gertler (1999).[1] The starting point is an environment of monopolistically competitive firms that face some type of constraints on price adjustment. The constraint is that the price adjustment rule is time dependent, so in any given period each firm is able to adjust its price with a fixed probability $1-\theta$. Therefore, the parameter θ lies between 0 and 1 and measures price stickiness. Then, a fraction $1-\omega$ of the firms are forward looking and set prices optimally as in Calvo (1983). The remaining firms are backward looking and use a simple rule of thumb based on the recent history of aggregate price behavior. Thus, ω measures firms' backwardness and it also lies between 0 and 1. The presence of backward looking firms implies the presence of lagged values of inflation in the curve, and for this reason this version of the curve is called "hybrid", to distinguish it from the purely forward looking version, which can be considered a special case obtained by setting $\omega = 0$. A third structural parameter β is discounting future utility of consumption in the Euler equation.

Defining π_t and s_t as inflation and marginal cost (in percent deviation from steady state) at time t, the NKPC states that:

$$\pi_t = \lambda s_t + \gamma_f E_t[\pi_{t+1}] + \gamma_b \pi_{t-1}, \tag{1}$$

where the parameters $\lambda, \gamma_f, \gamma_b$ are convolutions of the structural parameters β, θ, ω:

$$\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega\left[1-\theta(1-\beta)\right]},\ \gamma_f = \frac{\beta\theta}{\theta + \omega\left[1-\theta(1-\beta)\right]},\ \gamma_b = \frac{\omega}{\theta + \omega\left[1-\theta(1-\beta)\right]}. \tag{2}$$

Equation (1) is a second order difference equation. Provided that its characteristics roots δ_1 and δ_2 lie respectively inside and outside the unit circle, it has the following unique stable forward solution:

$$\pi_t = \delta_1 \pi_{t-1} + \psi \sum_{k=0}^{\infty} \delta_2^{-k} E_t s_{t+k}. \tag{3}$$

According to equation (3) actual inflation depends on past inflation and on expectations about future marginal costs. The parameters δ_1, δ_2, ψ are convolutions of $\lambda, \gamma_f, \gamma_b$, which

[1] A more general version of this model derived in Galí et al. (2001) includes the assumption of increasing marginal costs as in Sbordone (2002). Practically, this extension consists in multiplying the marginal costs by a factor reflecting the curvature of the production function and the elasticity of demand. We have carried the analysis for both these specifications, and results are virtually the same, thus in the paper we present results for the former specification only.

in turn (recall equation (2)) are convolutions of the structural parameters β, θ, ω:

$$\delta_1 = \frac{1}{\gamma_f}\left(-\frac{1}{2}\sqrt{1-4\gamma_b\gamma_f} + \frac{1}{2}\right), \ \delta_2 = \frac{1}{\gamma_f}\left(\frac{1}{2}\sqrt{1-4\gamma_b\gamma_f} + \frac{1}{2}\right), \ \psi = \frac{\lambda}{\delta_2\gamma_f}. \tag{4}$$

For a detailed discussion of the model see Galí and Gertler (1999).

2.2 Restrictions from the NKPC for a VAR

Equation (3) contains unknown expectational elements which can be proxied by using VAR projections.[2] Consider the following VAR in s_t and π_t:

$$\begin{bmatrix} s_t \\ s_{t-1} \\ ... \\ s_{t-p+1} \\ \pi_t \\ \pi_{t-1} \\ ... \\ \pi_{t-p+1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & ... & a_p & b_1 & b_2 & ... & b_p \\ 1 & 0 & ... & 0 & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 & 0 & ... & 0 \\ c_1 & c_2 & ... & c_p & d_1 & d_2 & ... & d_p \\ 0 & 0 & ... & 0 & 1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & 0 & 0 & ... & 0 \end{bmatrix} \begin{bmatrix} s_{t-1} \\ s_{t-2} \\ ... \\ s_{t-p} \\ \pi_{t-1} \\ \pi_{t-2} \\ ... \\ \pi_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ ... \\ 0 \\ u_{2t} \\ 0 \\ ... \\ 0 \end{bmatrix}, \tag{5}$$

compactly:

$$z_t = Az_{t-1} + u_t, \tag{6}$$

where $z_t = [s_t \ ... \ s_{t-p+1} \ \pi_t \ ... \ \pi_{t-p+1}]'$. Now define two $2p \times 1$ selector vectors g and h. The vector g contains all zeros except for its $p+1$ and $p+2$ elements which are respectively 1 and $-\delta_1$, thus $g'z_t = \pi_t - \delta_1\pi_{t-1}$. The vector h contains all zeros except for its first element, which is 1, thus $h'z_t = s_t$. Using this notation and substituting the expectational terms $E_t s_{t+k}$ with the VAR projections $h'A^k z_t$ equation (3) reads:

$$g'z_t = \psi\sum_{k=0}^{\infty}\delta_2^{-k}h'A^k z_t. \tag{7}$$

As $\delta_2^{-1} < 1$, the sum on the right-hand side of (7) converges and we have:

$$g'z_t = \psi h'(I - \delta_2^{-1}A)^{-1}z_t. \tag{8}$$

[2]The VAR projection method, as well as the Wald test of the restrictions implied by a present value model, were first proposed by Campbell and Shiller (1987) in the context of the expectations theory of interest rates.

Equation (8) is extensively used to compute "fundamental" inflation, i.e. the inflation consistent with the NKPC. Examples are Galí' and Gertler (1999), Sbordone (2002), Kurmann (2005), Rudd and Whelan (2006). All these papers use the unrestricted VAR estimates to project expectations but do not test the restrictions implied on the VAR by the NKPC. To obtain the restrictions, simply recognize that since (8) has to hold in general, it holds for any z_t:

$$g' = \psi h'(I - \delta_2^{-1} A)^{-1} \tag{9}$$

Postmultiplying both sides of (9) by $\delta_2(I - \delta_2^{-1} A)$ and using the fact that $\delta_1 + \delta_2 = 1/\gamma_f$ and $\delta_1 \delta_2 = \gamma_b/\gamma_f$ provides the following set of $2p$ restrictions:

$$c_1 = -\frac{\lambda}{\gamma_f};\ c_j = 0\ \forall\ j > 1;\ d_1 = \frac{1}{\gamma_f};\ d_2 = -\frac{\gamma_b}{\gamma_f};\ d_j = 0\ \forall\ j > 2. \tag{10}$$

2.3 Empirical evidence

Following Galí and Gertler (1999) we use quarterly data of the (log) labor income share (equivalently, real unit labor costs) in the non-farm business sector for s_t and the percent change in the GDP deflator for π_t. Data range from 1960:1 to 2006:4 and are plotted in Figure 1. The series are provided by the Bureau of Labor Statistics and the Bureau of Economic Analysis and are downloadable via Datastream.

First, we need to gauge the appropriateness of the VAR in (6) in describing the data. The Akaike Criterion selects 5 lags. Recursive residuals and parameter estimates are stable, and diagnostic tests provide evidence in favor of normality, no-autocorrelation, and homoscedasticity of the disturbances.[3] Choosing 5 lags implies setting the number of restrictions to 10, which is quite demanding. Therefore we have conducted the analysis also for more parsimonious specifications (with $2, 3, 4$ lags providing respectively $4, 6, 8$ restrictions to be tested). The evidence for all these cases is virtually the same.[4] This is due to the fact that the additional restrictions implied by richer dynamic specifications are mostly not rejected, while the rejection is systematically driven by the restrictions attached to first and second order lags, i.e. those related to the structural parameters.

We now turn to the test of the set of restrictions in (10). To perform such a test one should know the true values of the parameters λ, γ_f, γ_b, appearing on the right hand side of equation (10). If this would be the case, then one could simply check whether the

[3] All the test statistics are well below the critical values. In particular the LM test statistic (reported in Johansen, 1995 p.22) does not signal autocorrelation of any order up to order 8 (p-values 0.58, 0.82, 0.66, $0.74, 0.18$, $0.82, 0.92$, 0.94). The p-value of the White test for no heteroscedasticity is 0.12. The p-value of the normality test is 0.24.

[4] Results for the other specifications are available upon request.

unrestricted VAR coefficients are statistically different from the values consistent with the NKPC. However, the parameters λ, γ_f, γ_b are unknown. Still, we can exploit the fact that as shown in equation (2) they are functions of the structural parameters β, θ, ω.

The parameter β is discounting future utility of consumption in the Euler equation, therefore we calibrate it to its steady state value, namely $1/(1+\bar{r})$ where $\bar{r}$ is the average one-period real interest rate. This value turns out to be 0.985.[5] Now consider the parameters ω and θ. Both these parameters lie between 0 and 1 by construction.[6] Then, by using a sufficiently thin grid over ω and θ it is possible to pin down a discrete collection of all the possible values of λ, γ_f, γ_b, and use them to perform a Wald test of the restrictions in (10). Rudd and Whelan (2006) use the same strategy to compute theoretical inflation, but they do not test for the restrictions.[7]

Results of the Wald tests are plotted in Figure 2. The x axis reports different values of the parameter θ, the y axis reports different values of the parameter ω. The z axis reports the (log) value of the Wald statistic. The black area on the floor of the picture is the 99% (log) critical value for the null of the validity of the NKPC restrictions. The surface of the Wald statistic lies well above the critical value for any admissible value of ω and θ. This means that regardless of the true value of the price stickiness and firms' backwardness in the economy, the unrestricted VAR estimates are statistically different from the values they should assume under the null of the validity of the NKPC. The rejection is so strong that it can be hardly explained by small sample biases.[8]

To shed light on the rejection of the joint restrictions, we have also looked at the individual t-ratios of the 10 restrictions at hand. The p-values associated to the t-ratios are displayed in Table 1. For the restrictions dependent on the structural parameters we report the *maximum* among all the p-values computed for any combination of ω and θ. The rejection of the joint restrictions is driven by the inconsistency with the data of the three restrictions related to the structural parameters. Indeed, the highest p-value reached by these restrictions is that attached to $d_2 = -\frac{\gamma_b}{\gamma_f}$ and it is only 0.048. All these results are very robust to the choice of the lag length of the VAR.

[5] Woodford (2001) suggests a value of $\beta = 0.99$. We have repeated the analysis with $\beta = 0.99$, 0.95, 0.90, and results are very robust to such modifications.

[6] Still, we have to exclude the values of ω and θ such that the NKPC is not well defined, i.e. it has not a stable forward solution. Stability requires $\delta_1 < 1$ and $\delta_2 > 1$. If $\beta < 1$ then $\delta_2 > 1$ for any value of ω and θ. Thus the NKPC is not defined whenever ω and θ are such that $\delta_1 \geq 1$. In practice, we exclude all the values of θ and ω such that $\delta_1 > 0.999$. We also exclude $\omega = \theta = 0$.

[7] Their framework is slightly different, as they study a model in which $\gamma_f + \gamma_b = 1$, and so they perform a grid search on the sole parameter γ_f. This case is obtained by setting $\beta = 1$ in our framework.

[8] Figure 2 is in logs, which reduces the visual impression of the distance between the statistic and the critical value. The minimum distance between the Wald statistic and the 99% percent critical value (23.21) is 44.

3 Concluding Remark

We provided a simple test of the NKPC which avoids the estimation of the structural parameters measuring the probability of not resetting prices and the portion of backward looking firms in the economy. According to a simple Wald test it does not exists a combination of price stickiness and firm backwardness which is consistent with the US data. It is important to say that this result does not necessarily imply that the idea of a forward looking price setting behaviour should be entirely disregarded, as the rejection might be due to the failure of the joint hypothesis of rational expectations.

Indeed, any test of the NKPC is a joint test of the model *and* of rational expectations. In particular, to build our test we have used VAR projections to proxy for agents' expectations. This approach is extensively used in the literature to compute "fundamental" inflation, i.e. the inflation consistent with the NKPC. By doing so we implicitly assume that agents form expectations in a model-consistent manner. However, even if agents are optimizing and forward looking they might form expectations in a different way. In this light our results do not necessarily exclude a role of future expected inflation in determining actual inflation, they rather suggest that further research should be aimed at providing alternative models for agents' expectations.

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Tables and Figures

Table 1: P-values of the individual NKPC restrictions.

$c_1 = -\frac{\lambda}{\gamma_f}$	$c_2 = 0$	$c_3 = 0$	$c_4 = 0$	$c_5 = 0$	$d_1 = \frac{1}{\gamma_f}$	$d_2 = -\frac{\gamma_b}{\gamma_f}$	$d_3 = 0$	$d_4 = 0$	$d_5 = 0$
0.001*	0.142	0.305	0.355	0.487	5.4e-11*	0.048*	0.089	0.002	0.007

The table displays the p-values associated with the individual NKPC restrictions. The values denoted by (*) depend on $\lambda, \gamma_f, \gamma_b$, which in turn depend on θ and ω. For these cases is reported the *maximum* among all the p-values computed for any value of θ and ω. Figures are rounded to third decimal.

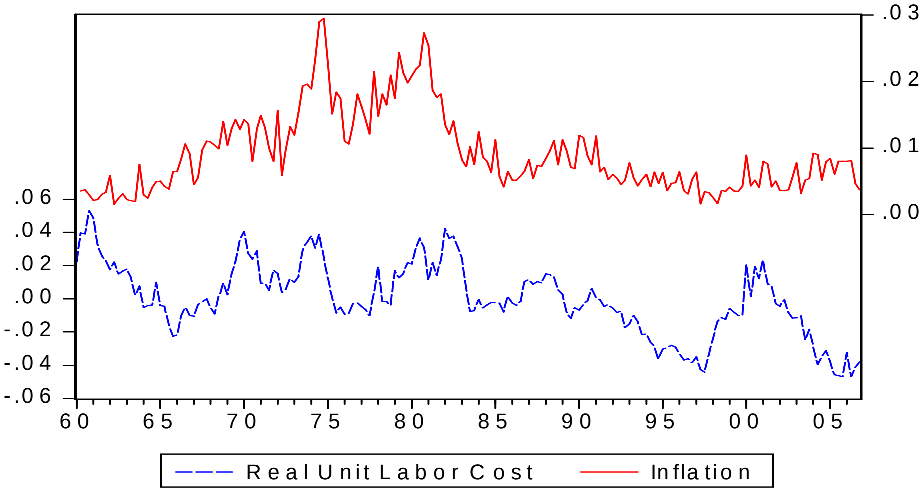

Figure 1: Data. The variable π_t (right axis) is quarterly GDP Inflation. The variable s_t (left axis) is (log) Real Unit Labor Cost in the nonfarm business sector. The series used to compute these variables are (Datastream codes in parenthesis): Real GDP (USGNPNFMD), Nominal GDP (USGNPNFMB), Real GDP non-farm business sector (USOEXP03D), Nominal GDP non-farm business sector (USOEXP03B), and Nominal Unit Labor Cost (USULCNBSE).

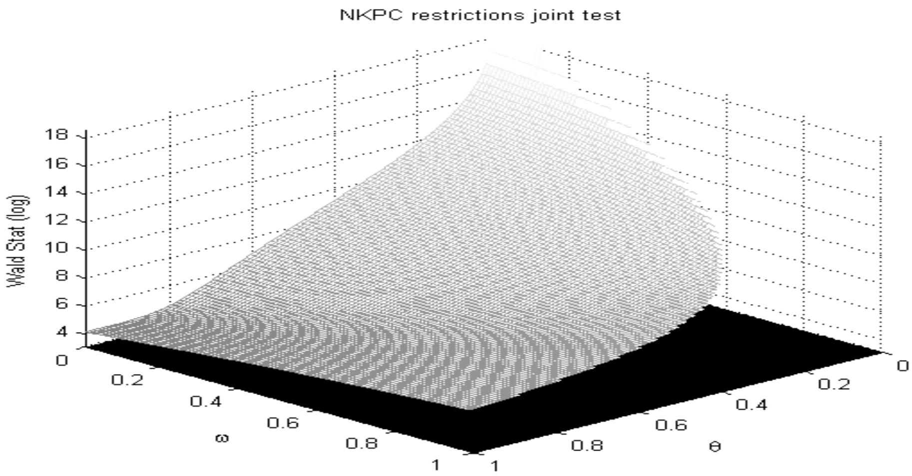

Figure 2: Logs of the Wald statistic and of the 99% critical value (black area) for the NKPC restrictions. Computed for any admissible value of the degree of price stickiness (θ) and of the portion of backward-looking firms in the economy (ω). The values of θ and ω such that the NKPC is not well defined have been excluded.

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