Department of Economics A Test for Serial Dependence Using Neural Networks

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Abstract

Testing serial dependence is central to much of time series econometrics. A number of tests that have been developed and used to explore the dependence properties of various processes. This paper builds on recent work on nonparametric tests of independence. We consider a fact that characterises serially dependent processes using a generalisation of the autocorrelation function. Using this fact we build dependence tests that make use of neural network based approximations. We derive the theoretical properties of our tests and show that they have superior power properties. Our Monte Carlo evaluation supports the theoretical findings. An application to a large dataset of stock returns illustrates the usefulness of the proposed tests.

JEL Codes: C32, C33, G12 Keywords: Independence, Neural Networks, Strict Stationarity, Bootstrap, S&P500

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1 Introduction

Testing for and measuring serial dependence are of paramount importance to time series analysis (see e.g., Granger and Teräsvirta (1993), Robinson (1991), Tjostheim (1996)). A conventional measure of serial dependence is the autocorrelation function, which may overlook essential nonlinear features of time series that have zero autocorrelation. As Granger and Teräsvirta (1993) pointed out, there are few simple appropriate tools for analyzing nonlinear time series, although significant effort has been devoted to developing effective measures of and tests for serial dependence.

Accordingly there is a large literature on how to test independence among process and serial independence in particular, see, e.g., Hoeffding (1948), Baek and Brock (1992), Johnson and McClelland (1998), and Pinkse (1999). Tjostheim (1996) provides an excellent survey of the literature. Although there have been a number of distinct approaches to testing serial dependence there has been increasing interest recently on nonparametric entropy measures of and tests for serial dependence. This approach makes only minimal assumptions about the process under investigation avoiding, for example, normality and linearity assumptions that have been made in this context in existing literature. Prime examples of work that focuses on entropy measures are Robinson (1991), Hong and White (2005) and Fernandes and Neri (2007).

This paper provides new tests of serial dependence building on but moving away from this recent body of work. Our work is based on a fact that is essentially synonymous to the usual characterisation of serial dependence but has not been explored in the literature. In particular, letting y_t be a strictly stationary process, serial dependence is equivalent to $f_s(.,.) \neq g(.)g(.)$ for some s, where g(.) is the marginal density and $f_s(.,.)$ is the pairwise joint density for y_t and y_{t-s} , $s \in \{1, 2, ...\}$. Of course, zero autocorrelations are not evidence of lack of serial dependence. However, we base our analysis on the following fact. For any serially dependent process, there exist some functions $h_1(.)$ and $h_2(.)$ such that $Cov(h_1(y_t)h_2(y_{t-s})) \neq 0$. By generalising the definition of autocorrelation in this way, we can use a simple formulation for constructing tests for serial dependence. Of course, determining $h_1(.)$ and $h_2(.)$ is not trivial. Neural networks with their ability to approximate arbitrarily well unknown continuous functions are useful in this respect.

We provide algorithms for constructing appropriate estimates of $h_1(.)$ and $h_2(.)$ based on neural network approximations. We derive the relevant asymptotic theory for our test statistics under the null hypothesis that $f_s(.,.) = g(.)g(.)$ and also examine bootstrap approximations to the exact distribution of the test statistics. Further, we prove consistency of the test and perhaps surprisingly given the nonparametric nature of our analysis we show that the test statistic diverges at a parametric rate under any departure from the null hypothesis. This implies that our test is more powerful than other nonparametric tests which have slower rates of divergence.

The theoretical properties of our tests are reflected in their small sample properties. We compare our tests to a number of existing tests in the literature replicating exactly existing Monte Carlo studies. From this analysis we see that our tests dominate all other tests in all experiments considered that follow a number of dependence structures.

The paper is structured as follows: Section 2 provides the theoretical analysis underlying our tests and discusses their asymptotic properties. Section 3 provides the Monte Carlo analysis. Section 4 applies our tests to a large dataset of stock returns to examine the random walk hypothesis. Section 5 concludes. All proofs are relegated to an Appendix at the end of the paper.

2 Theoretical Considerations

Let y_t be a strictly stationary time series with marginal density g(.) and pairwise joint density $f_s(.,.)$ for y_t and y_{t-s} where $s \in \{1, 2, ...\}$. An important issue in nonlinear time series analysis is determining the presence of serial dependence in y_t . A standard set of tools for this aim examines the relationship between $f_s(.,.)$ and g(.) and in particular measures the deviation between $f_s(.,.)$ and g(.)g(.). Any evidence of deviation between these two quantities is taken to suggest evidence of serial dependence. Nonparametric estimates of the marginal and joint densities can be used to construct a variety of tests for serial dependence and have been the main tools used to investigate this issue.

This paper takes an alternative view in examining this issue, based on neural networks. We make the following assumptions:

Assumption 1 $f_s(.,.)$ and g(.) are continuous functions.

Assumption 2 y_t is strictly stationary.

Assumption 3 Let $f_s(.,.) \neq g(.)g(.)$. Then, y_t is a L_1 near epoque dependent (NED) process of size $-\phi$ for some $\phi > 0$, on some α -mixing process with mixing coefficients, α_{ℓ} , such that $\lim_{\ell \to \infty} \alpha_{\ell} = 0$.

Remark 1 The only noteworthy assumption is Assumption 3. We note that this assumption is extremely mild compared to assumptions made in related work such as, e.g., Assumption A.4 of Hong and White (2005).

We base our analysis on the following fact concerning dependent strictly stationary processes.

Lemma 1 Let $f_s(.,.) \neq g(.)g(.)$ on some interval with non-zero Borel measure. Let Assumptions 1-2 hold. Then, for every f_s , there exists at least one pair of continuous functions $(h_{1,f}(.), h_{2,f}(.))$ with $E(h_{i,f}(y_t)) = 0$, i = 1, 2 such that $E(h_{1,f}(y_t)h_{2,f}(y_{t-j})) \neq 0$.

Given the above Lemma, under dependence, there exist functions which when used to transform the data result in processes with non-zero correlation. Let the set of all pairs of functions referred to in Lemma 1 be given by \mathcal{H}_f . Given the bounded nature of correlation it is obvious that there exist a pair of functions in \mathcal{H}_f that maximises the absolute correlation $|Corr(h_{1,f}(y_t), h_{2,f}(y_{t-s}))|$ over \mathcal{H}_f . So the problem of testing for serial dependence is related to the problem of maximising $|Corr(h_{1,f}(y_t), h_{2,f}(y_{t-s}))|$ with respect to $(h_{1,f}(.), h_{2,f}(.))$.

This is of course a non-trivial optimisation. A number of alternative approaches for this problem can be envisaged. We focus on neural networks. We restate the problem as follows. Given the set of all pairs of continuous functions $(h_{1,f}(.), h_{2,f}(.))$ we need to determine a pair that maximises $|Corr(h_{1,f}(y_t), h_{2,f}(y_{t-s}))|$. We consider neural network approximations to continuous functions and in particular radial basis function (RBF) neural network (RBFNN) approximations for reasons that will be discussed below. The RBFNN series expansion takes the form

$$h_{i,f}(y) = \sum_{j=1}^{m} c_j \psi(y, t_{i,j}, \sigma_T)$$
(1)

where the RBF nodes, $\psi(y, t_{i,j}, \sigma_T)$, are radially symmetrical, integrable, bounded functions and $t_{i,j}$ are referred to as the centres of the RBFs. Examples include the Gaussian function of the form $\exp\left(-\left(\frac{||y-t_{i,j}||}{\sigma_T}\right)^2\right)$, or the multiquadratic function $\left(1+\left(\frac{||y-t_{i,j}||}{\sigma_T}\right)^2\right)^{-1}$, $\sigma_T > 0$, where ||.|| denotes Euclidean distance. Obviously, estimation of (1) is challenging since unlike standard series expansions, there are two problems that need attention: the first is that $\psi(y, t_j, \sigma_T)$ contain unknown parameters, in particular the centres, and the second is that the nodes are not ranked so that the choice of the nodes in the series expansion is not obvious.

We propose the following algorithm for using such approximations for our maximisation problem.

Algorithm 1 (RBF) Boosting algorithm

- 1. Let σ_T be some sequence such that $\sigma_T = o(1)$. We construct two initial sets of T s *RBF* nodes given by: $\Psi^{1,(1,\dots,T)} = \{\psi(y, y_{s+1}, \sigma_T), \psi(y, y_{s+2}, \sigma_T), \dots, \psi(y, y_T, \sigma_T)\}$ and $\Psi^{2,(1,\dots,T)} = \{\psi(y, y_1, \sigma_T), \psi(y, y_2, \sigma_T), \dots, \psi(y, y_{T-s}, \sigma_T)\}.$
- 2. These are ranked according to their ability to maximise $|Corr(\psi(y_t, y_{k_1}, \sigma_T), \psi(y_{t-s}, y_{k_2}, \sigma_T))|$ for $k_1 = s + 1, ..., T, k_2 = 1, ..., T - s$.
- 3. The pair of nodes that maximise the absolute correlation become the first pair node in the ranking of the nodes. Denote this pair of nodes by $(\psi(y_t, y_{S_{1,1}}, \sigma_T), \psi(y_{t-s}, y_{S_{2,1}}, \sigma_T))$. Let $\tilde{S}_{1,1} = \{S_{1,1}\}$ and $\tilde{S}_{2,1} = \{S_{2,1}\}$. Let $\Psi^{1,(1,\ldots,T)/\tilde{S}_{1,1}}$ and $\Psi^{2,(1,\ldots,T)/\tilde{S}_{2,1}}$ be the sets of nodes in $\Psi^{1,(1,\ldots,T)}$ and $\Psi^{2,(1,\ldots,T)}$ apart from the nodes indexed by the elements of $\tilde{S}_{1,1}$ and $\tilde{S}_{2,1}$.
- 4. Set i = 1 and $c_{1,1} = c_{2,1} = 1$.
- 5. The nodes in $\Psi^{1,(1,\ldots,T)/\tilde{S}_{1,i}}$ and $\Psi^{2,(1,\ldots,T)/\tilde{S}_{2,i}}$ are ranked according to their ability to increase

$$\left| Corr\left(\sum_{l=1}^{i} c_{1,l}\psi(y_{t}, y_{\mathcal{S}_{1,l}}, \sigma_{T}) + c_{1}^{k_{1}}\psi(y_{t}, y_{k_{1}}, \sigma_{T}), \sum_{l=1}^{i} c_{2,l}\psi(y_{t-s}, y_{\mathcal{S}_{2,l}}, \sigma_{T}) + c_{2}^{k_{2}}\psi(y_{t-s}, y_{k_{2}}, \sigma_{T})\right)\right|$$

where $k_1 \in \tilde{S}_{1,i}$ and $k_2 \in \tilde{S}_{2,i}$ and $c_1^{k_1}$ and $c_2^{k_2}$ are obtained numerically and constrained to be bounded.

6. The pair of nodes that maximise the absolute correlation becomes the i + 1-th pair of nodes in the ranking of the nodes. Denote this pair of nodes by

 $(\psi(y_t, y_{\mathcal{S}_{1,i+1}}, \sigma_T), \psi(y_{t-s}, y_{\mathcal{S}_{2,i+1}}, \sigma_T))$ and their respective coefficients by $c_{1,i+1}$ and $c_{2,i+1}$. Let $\tilde{\mathcal{S}}_{1,i+1} = \tilde{\mathcal{S}}_{1,i} \cup \{\mathcal{S}_{1,i+1}\}$ and $\tilde{\mathcal{S}}_{2,i+1} = \tilde{\mathcal{S}}_{2,i} \cup \{\mathcal{S}_{2,i+1}\}$ where $\mathcal{S}_{1,i+1}$ and $\mathcal{S}_{2,i+1}$ denote the indices of the chosen nodes. Let $\Psi^{1,(1,\ldots,T)}/\tilde{\mathcal{S}}_{i+1}$ and $\Psi^{2,(1,\ldots,T)}/\tilde{\mathcal{S}}_{i+1}$ be the sets of nodes in $\Psi^{1,(1,\ldots,T)}$ and $\Psi^{2,(1,\ldots,T)}$ respectively apart from the nodes indexed by the elements of $\tilde{\mathcal{S}}_{1,i+1}$ and $\tilde{\mathcal{S}}_{2,i+1}$.

7. If i = m for some $m = m_T$ stop and select $\sum_{l=1}^{q_1} c_{1,l}\psi(y_t, y_{\mathcal{S}_{1,l}}, \sigma_T)$ and $\sum_{l=1}^{q_2} c_{1,l}\psi(y_t, y_{\mathcal{S}_{1,l}}, \sigma_T)$, as the function approximations for $h_{1,f}(.)$ and $h_{2,f}(.)$, where q_1 and q_2 are chosen so as to maximise the absolute correlation over 1, ..., m, else set i = i + 1 and go to Step 5.

A few remarks are in order.

Remark 2 This algorithm bears close resemblance to existing algorithms in the econometrics and statistics literature that are referred to as boosting algorithms (see, e.g., Kapetanios and Blake (2007) and Buhlmann (2006)). However, its use to approximate unknown functions so as to optimise a functional as we do in this paper is to the best of our knowledge novel.

Remark 3 The choice for m is not discussed in Algorithm 1. Theorem 1 suggests that, as long as m is finite or $m = o(T^{1/4})$, the test resulting from Algorithm 1 is well behaved, under the null. Theorem 3 states that the test is consistent and the test statistic diverges to infinity at a parametric rate of $T^{1/2}$, under the alternative hypothesis, $f_s(.,.) \neq g(.)g(.)$, for all finite m.

Remark 4 The sequence σ_T is left unspecified in Algorithm 1. The work of Park and Sandberg (1991) suggests that $\sigma_T = o(1)$. Given the lack of guidance on this choice from theory, it is reasonable to consider ad hoc data-based values following the practice established by Orr (1995) for RBF neural networks. Accordingly, in practice this tuning parameter is set such that $\sigma_T = \sigma$ where $\sigma = 2 \max_t |y_t - y_{t-1}|$.

Remark 5 The choice of the initial set of RBF nodes given by:

$$\Psi^{i,(1,...,T)} = \{\psi(y, y_1, \sigma_T), \psi(y, y_2, \sigma_T), \dots, \psi(y, y_T, \sigma_T)\}$$

may be straightforwardly generalised to $\Psi^{i,(1,\ldots,p_T)}$ where p_T is chosen to reflect a subset of the observations or possibly be of a larger order than T. The theory provides no constraints as long as $p_T \to \infty$ and the candidate centres of the functions in $\Psi^{i,(1,\ldots,p_T)}$ imply a partition of the support of y_t whose intervals tend uniformly to zero. The last restriction is automatically satisfied by using y_1, \ldots, y_T as candidate centres.

Next we provide a result that we will use to simplify our algorithm.

Lemma 2 Let $h_1(.)$ and $h_2(.)$ be continuous functions such that $E(h_1(y_t)) = E(h_1(y_t)) = 0$ and $E(h(y_t)h(y_{t-s})) \neq 0$. Then, for any positive RBF function $\psi(.,.,.)$, there exist some constants $c_{1,i}, c_{2,i}, t_{1,i}$ and $t_{2,i}, i = 1, ..., m$, such that the functions $\hat{h}_{1,m} = \sum_{l=1}^m c_{1,l}\psi(y_t, t_{1,l}, \sigma_m)$ and $\hat{h}_{2,m} = \sum_{l=1}^m c_{2,l}\psi(y_t, t_{2,l}, \sigma_m)$ approximate abritrarily well $h_1(.)$ and $h_2(.)$ respectively for any sequence $\sigma_m = o(1)$ as $m \to \infty$. Further, $\lim_{m\to\infty} \left| E(\hat{h}_1(y_t)\hat{h}_1(y_t)) \right| \neq 0$ implies that there exist constants $\tilde{t}_{1,i}$ and $\tilde{t}_{2,i}, i = 1, ..., m$, such that $\lim_{m\to\infty} \left| E(\tilde{h}_1(y_t)\hat{h}_1(y_t)) \right| \neq 0$ where $\tilde{h}_{1,m} = \sum_{l=1}^i \psi(y_t, \tilde{t}_{1,l}, \sigma_m)$ and $\tilde{h}_{2,m} = \sum_{l=1}^i \psi(y_t, \tilde{t}_{2,l}, \sigma_m)$. Following the above Lemma we can modify Algorithm 1 to restrict $c_{1,i} = c_{2,i} = 1$. We refer to this Algorithm as Algorithm 2.

Theorem 1 Let assumptions 1-2 hold. Let $\hat{\rho}$ denote the correlation obtained by applying Algorithm 1 to the observed data. Then, under the null hypothesis, and for $m = o(T^{1/4})$, $\sqrt{T}\hat{\rho}$ is asymptotically normally distributed with zero mean.

Of course, the above result applies straightforwardly to Algorithm 2 too, making it a reasonably computationally inexpensive algorithm as it does not involve any iterative estimation. The asymptotics of Theorem 1 are not that relevant for small samples both because a test based on these asymptotics is likely to overreject in small samples and also because the variance of the statistic is not easy to obtain. As a result we resort to the bootstrap. We apply the nonparametric bootstrap for i.i.d. sequences thereby imposing the null hypothesis on the bootstrap samples. The following Theorem gives a result on the theoretical properties of the bootstrap

Theorem 2 Let assumptions 1-2 hold. Let P_T denote the probability measure generating $y_1, ..., y_T$. Let $G_T(x)$ and $\hat{G}_T(x)$ denote the exact distribution of $\sqrt{T}\hat{\rho}$ and its bootstrap approximation respectively. Then, under the null hypothesis of serial independence

$$\lim_{T \to \infty} P_T \left[\sup_{x} \left| \hat{G}_T(x) - G_T(x) \right| > \varepsilon \right] = 0$$
(2)

for all $\varepsilon > 0$.

We next examine the power properties of Algorithm 2. We have the following Theorem.

Theorem 3 Let assumptions 1-3 hold. Under the alternative hypothesis of dependence given by $f_s(.,.) \neq g(.)g(.)$, and using Algorithm 2, $\sqrt{T}\hat{\rho} = O_p(T^{1/2})$, for all finite m.

Note the simplicity, generality and optimality of this result. The test is consistent against any hypothesis that does not satisfy $f_s(.,.) = g(.)g(.)$. In this sense it is nonparametric. Notice next the parametric rate of convergence implying that this test is more powerful that the nonparametric entropy based tests proposed in the literature of testing serial dependence. As a result we choose not to consider local power settings.

Remark 6 The test based on algorithm 2 is still relatively computationally intensive given that (i) a full search across all neural nodes takes place at each step of the Algorithm, (ii) the search is over a double grid since there are two unknown functions to be determined, and (iii) a bootstrap implementation is being considered. As a result we also consider a simplified Algorithm, referred to as Algorithm 3, in which we maximise $|Corr(h_f(y_t), h_f(y_{t-s}))|$, thereby searching for only one unknown function. The modifications needed to obtain this simplification from Algorithm 2 are obvious. Whereas, Algorithm 2 requires $O(T^2m)$ correlation evaluations for each bootstrap replication, Algorithm 3 only requires O(Tm) evaluations.

We refer to the bootstrap tests based on Algorithms 2 and 3, for lag s as $RBF_2(s)$ and $RBF_3(s)$.

Remark 7 Both RBF tests are informative in revealing information about the lag(s) at which there exists significant serial dependence. However, for testing the null hypothesis of serial independence, it is possible that two different lag orders may give conflicting conclusions. It is thus desirable to have a portmanteau test that uses multiple lags. For this purpose, we simply follow Hong and White (2005) and suggest using

$$RBF_i = \frac{1}{\sqrt{p}} \sum_{j=1}^p RBF_i(j), \quad j = 2, 3$$

Given the results of Theorems 1 and 2 on the asymptotic normality of the $RBF_i(s)$ tests and the validity of the bootstrap, we conclude that these results extend immediately to the RBF_i tests.

3 Monte Carlo Study

In this section we consider the finite sample performance of our new test. Comparability with results of Monte Carlo studies of other serial dependence tests is very important. Therefore, we follow exactly two recent Monte Carlo studies: Hong and White (2005) and Fernandes and Neri (2007). The exact replication of their Monte Carlo setup enables us to focus on our test and take the rejection probabilities under the alternative hypotheses from existing papers thus minimising computational cost.

We now give details on the Monte Carlo experiments of Hong and White (2005) and Fernandes and Neri (2007). Starting with Hong and White (2005), the experiments considered are given by

• Exp. 1A (IID, Size):

$$y_t = \epsilon_t, \ \epsilon_t \sim N(0, 1)$$

• Exp. 2A (AR1, Power):

$$y_t = 0.3y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 3A (ARCH1, Power):

$$y_t = h_t^{1/2} \epsilon_t, \quad h_t = 1 + 0.8y_{t-1}^2, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 4A (TARCH, Power):

$$y_t = h_t^{1/2} \epsilon_t, \quad h_t = 0.25 + 0.6h_{t-1} + 0.5y_{t-1}^2 I_{\{\epsilon_t < 0\}} + 0.2y_{t-1}^2 I_{\{\epsilon_t \ge 0\}}, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 5A (BILIN1, Power):

$$y_t = 0.8y_{t-1}\epsilon_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 6A (NMA1, Power):

$$y_t = 0.8\epsilon_{t-1}^2 + \epsilon_t, \quad \epsilon_t \sim N(0,1)$$

• Exp. 7A (TAR1, Power):

$$y_t = -0.5y_{t-1}I_{\{y_{t-1} < 1\}} + 0.4y_{t-1}I_{\{y_{t-1} \ge 1\}} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 8A (NAR1, Power):

$$y_t = 0.8\sqrt{|y_{t-1}|} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 9A (SIGN1, Power):

$$y_t = -I_{\{y_{t-1} < 0\}} + I_{\{y_{t-1} \ge 0\}} + 0.43\epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

Throughout, we consider sample sizes of 100 observations thereby focusing on the smaller sample size considered in Hong and White (2005). Results on rejection probabilities, for the tests based on Algorithms 2 and 3, are reported in Table 1. We set s = 1. We also report, in Table 1, the rejection probabilities for these experiments reported for $\mathcal{T}_n(1)$ in Hong and White (2005).

Moving on to the Monte Carlo study of Fernandes and Neri (2007), the experiments of that Monte Carlo study are given by

• Exp. 1B (AR2, Power):

$$y_t = 0.1 + 0.2y_{t-1} + \epsilon_t, \ \epsilon_t \sim N(0, 1)$$

• Exp. 2B (ARCH2, Power):

$$y_t = h_t^{1/2} \epsilon_t, \quad h_t = 0.1 + 0.2y_{t-1}^2, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 3B (NMA2, Power):

$$y_t = 0.2\epsilon_{t-1}\epsilon_{t-2} + \epsilon_t, \quad \epsilon_t \sim N(0,1)$$

• Exp. 4B (TAR2, Power):

$$y_t = (0.1 - 0.2y_{t-1})I_{\{y_{t-1} < 1\}} + (0.1 + 0.5y_{t-1})I_{\{y_{t-1} \ge 1\}} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 5B (BILIN2, Power):

$$y_t = 0.1 + 0.2y_{t-1}\epsilon_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 6B (NAR2, Power):

$$y_t = 0.1 + 0.2\sqrt{|y_{t-1}|} + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

• Exp. 7B (SIGN2, Power):

$$y_t = 0.1 + 0.2(-I_{\{y_{t-1} < 0\}} + I_{\{y_{t-1} \ge 0\}}) + \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

Throughout this set of experiments, we consider sample sizes of 500 observations thereby, again, focusing on the smaller sample size considered in Hong and White (2005). Results on rejection probabilities, for the tests based on Algorithms 2 and 3, are reported in Table 2. We focus solely on algorithm 3 for two reasons. Firstly, our results presented below show that Algorithm 2 is more powerful than Algorithm 3 for the Monte Carlo experiments of Hong and White (2005). Since we see that for the Monte Carlo experiments of Fernandes and Neri (2007) the test based on Algorithm 3 dominates all other tests, we feel it is redundant to incur further computational cost. This computational cost is the second reason. The test based on Algorithm 2 which searches for two rather than just one function is 25 times more expensive computationally compared to the test based on Algorithm 3, for sample sizes of 500 observations. We also report, in Table 2, the rejection probabilities for these experiments reported for the entropy based test of Hong and White (2005) in Hong and White (2005) for the values 1/2 and 4 for the tuning parameter q of their test. For all Monte Carlo experiments, the nominal significance level is 5%. We carry out 1000 Monte Carlo replications and 99 bootstrap replications throughout this section. We also set $m = [T^{0.249}]$, where [.]

denotes integer part. Throughout the section we set s = 1.

Results make interesting reading. $RBF_2(1)$ and $RBF_3(1)$ are extremely well behaved under the null hypothesis. For the power experiments of the Monte Carlo study of Hong and White (2005), both $RBF_2(1)$ and $RBF_3(1)$ are more powerful than all tests considered in that paper for all experiments. $RBF_2(1)$ is substantially more powerful than $RBF_3(1)$ for a number of experiments, but as we discussed earlier this comes at a quite considerable extra computational cost. In all other cases, $RBF_2(1)$ and $RBF_3(1)$ have essentially similar power properties. Notable examples of this are the linear AR model and the ARCH model, where $RBF_2(1)$ is marginally less powerful than $RBF_3(1)$. This is of course reasonable given that the same function (identity function and square function for the two models respectively) maximises $|Corr(h_{1,f}(y_t), h_{2,f}(y_{t-s}))|$ with respect to $(h_{1,f}(.), h_{2,f}(.))$ for each different model. Overall, $RBF_2(1)$ is either substantially more powerful than or as powerful as $RBF_3(1)$, but at an extra computational cost.

Moving on to the Monte Carlo of Fernandes and Neri (2007) we see again that the $RBF_3(1)$ is more powerful than the entropy based test of that paper for all experiments and all values of the tuning parameter of that test, q. It is also substantially more powerful than almost all other tests and almost all other experiments examined in the Monte Carlo study of Fernandes and Neri (2007). The only exception is the BDS test of Brock, Dechert, Scheinkman, and LeBaron (1996) and experiments ARCH2 and NMA2 where the BDS test marginally outperforms $RBF_3(1)$. However, the difference between these two tests for these two cases are negligible whereas for the rest of the cases the difference in performance between these two tests is very substantial in favour of $RBF_3(1)$.

4 Empirical Application to Stock Returns

In this section, we provide an empirical application that illustrates the potential of the new test to detect the presence of serial dependence. As it is sometimes difficult to draw meaningful conclusions from the empirical analysis of a single series for the performance of a new statistical test, we consider a large dataset such as the S&P 500. It has long been hypothesized that stock prices follow a (geometric) random walk possibly with a drift. We are interested in testing this hypothesis and in identifying important lags.

Data, obtained from Datastream, are weekly returns and span the period 01/01/1993-20/01/2004 comprising 575 weekly observations. We choose to consider only companies for

which data are available throughout the period leading us to have 412 series on which to use our test. We normalise the returns series to have mean equal to zero and variance equal to one prior to applying our test. Following the Mote Carlo evidence of the previous section, we use the $RBF_3(i)$ for the empirical analysis. We carry out 149 bootstrap replications. We set $m = [T^{0.249}]$. We consider a variety of lags to fully capture any serial dependence in the processes. In particular we look at 1, 2, 3, 4, 12 and 52 lags, which given the weekly frequency of the data, are expected to capture monthly, quarterly and yearly seasonalities. We report probability values for all these tests in Tables 3-5.

Looking at these Tables we note that there is extensive rejection of the null hypothesis for all lags considered. The extent of the rejection is much larger than than implied by pure chance. This implies a rejection of the random walk hypothesis. In particular, we have rejection of serial independence for 58.7% of the series considered at one lag, and 34.2%, 31.7%, 25.7%, 27.6% and 21.6% for 2, 3, 4, 12 and 52 lags respectively. Clearly, there is a decline in the extent of the evidence for serial dependence as the lag order increases, but this decline is quite slow.

5 Conclusions

Testing for serial dependence is important for time series analysis. Recent work on testing serial dependence has focused on nonparametric entropy measures. We take a different approach to that work by looking at a generalisation of the autocorrelation function. Using neural network approximations we are able to construct new tests of serial dependence. We derive the theoretical properties of our tests and show that they have superior power properties. Our Monte Carlo evaluation supports the theoretical findings. An application to a large dataset of stock returns illustrates the usefulness of the proposed tests.

A number of extensions are possible using our approach. Our tests of serial dependence should be in principle extensible, straightforwardly, to testing for dependence between different, possibly serially dependent, processes. Technical issues remain before this extension is feasible. In particular, we have relied on the independence of the process being tested under the null hypothesis. This independence simplifies greatly the analysis but cannot be relied on in the general case since dependent processes need to be analysable under the null hypothesis.

Another extension is more general. We have provided a strategy for estimating unknown functions that optimise functionals without relying on computationally intensive iterative optimisation techniques that lack robustness. This strategy has not been shown to provide consistent estimates of the unknown functions since we did not need such a result to prove the properties of our test under the alternative hypothesis. However, such a result would be of interest in itself and would generalise results in the statistics literature on boosting and related methods (see, e.g., Kapetanios and Blake (2007), Buhlmann (2006) and Temlyakov (2000)).

Finally, we have provided a possible basis for a generalised autocorrelation type measure of dependence based on our Lemma 1 that may be of independent interest for nonlinear time series analysis.

6 Appendix

6.1 Proof of Lemma 1

Let $y = y_t$ and $x = y_{t-j}$. We have that for any pair of functions $(h_{1,f}(.), h_{2,f}(.))$,

$$\int \int h_{1,f}(y)h_{2,f}(x)f_s(x,y)dxdy =$$

$$\int \int h_{1,f}(y)h_{2,f}(x)g(x)g(y)c(G(y),G(x))dxdy = 0$$
(3)

where c(.,.) is the copula density and G(.) the distribution function corresponding to $f_s(.,.)$ and g(.) respectively. We note that the copula density is given by

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$$

where C(u, v) is the copula function satisfying

$$f_s(.,.) = C(g(.), g(.)).$$

In the case of independence c(u, v) = 1. Then, (3) can be rewritten as

$$\int h_{1,f}(x) \left[\int h_{2,f}(y)g(y)c(G(y),G(x))dy \right] g(x)dx = 0$$

But, by the assumed dependence of y_t and y_{t-j} , and the fact that c(G(y), G(x)) is a continuous density, it follows that c(G(y), G(x)) is a non-constant function in both its arguments. Then, it follows that $\int h_{2,f}(y)g(y)c(G(y), G(x))dy$ cannot be zero for all possible $h_{2,f}(.)$. As a result there exists some $h_{2,f}(.)$ for which

$$\int h_{2,f}(y)g(y)c(G(y),G(x))dy = q(x)$$

such that q(x) is nonzero for some x and also non-constant. Then, similarly, given the nonconstancy of q(x), there exists $h_{1,f}(.)$ such that $\int h_{1,f}(x)q(x)g(x)dx$ is non-zero completing the proof of the Lemma.

6.2 Proof of Lemma 2

The first part of the Lemma follows immediately from Park and Sandberg (1991). We now prove the second part. We have

$$\left| E(\hat{h}_{1}(y_{t})\hat{h}_{1}(y_{t})) \right| = \left| \sum_{l=1}^{m} \sum_{k=1}^{m} c_{1,l}c_{2,k} E\left(\psi(y_{t}, t_{1,l}, \sigma_{m})\psi(y_{t}, t_{1,l}, \sigma_{m})\right) \right| \le \sum_{l=1}^{m} \sum_{k=1}^{m} |c_{1,l}c_{2,k}| \left| E\left(\psi(y_{t}, t_{1,l}, \sigma_{m})\psi(y_{t}, t_{1,l}, \sigma_{m})\right) \right| > 0$$

But, then it immediately follows that

$$\sum_{l=1}^{m} \sum_{k=1}^{m} |E(\psi(y_t, t_{1,l}, \sigma_m)\psi(y_t, t_{1,l}, \sigma_m))| > 0$$

and by positivity of ψ that

$$\left|\sum_{l=1}^{m}\sum_{k=1}^{m}E\left(\psi(y_{t}, t_{1,l}, \sigma_{m})\psi(y_{t}, t_{1,l}, \sigma_{m})\right)\right| > 0$$

6.3 Proof of Theorem 1

We focus on the computationally infeasible but conceptually simpler case where the absolute correlation

$$\left| Corr\left(\sum_{l=1}^{m} c_{1,l} \psi(y_t, t_{1,l}, \sigma_T), \sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma_T) \right) \right|$$
(4)

is jointly maximised numerically with respect to $c_{1,l}$, $c_{2,l}$, $t_{1,l}$ and $t_{2,l}$ for l = 1, ..., m. Throughout, it is assumed that $m = m_T$ but this dependence is supressed. The above maximand is clearly equal to or larger than the absolute correlation obtained via Algorithm 1. It is clear that $c_{1,l}$, $c_{2,l}$, $t_{1,l}$ and $t_{2,l}$ are chosen so as to maximise

$$\frac{1}{T} \sum_{t=1}^{T} \sigma_{12}^{-1} \left[\left(\sum_{l=1}^{m} c_{1,l} \psi(y_t, t_{1,l}, \sigma_T) - \mu_{1,T} \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma_T) - \mu_{2,T} \right) \right] = (5)$$

$$\frac{1}{T} \sum_{t=1}^{T} D_t$$

where

$$\sigma_{12} = \left[\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{m}c_{1,l}\psi(y_t, t_{1,l}, \sigma_T) - \mu_{1,T}\right]^2\right]^{1/2} \left[\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{m}c_{2,l}\psi(y_t, t_{2,l}, \sigma_T) - \mu_{2,T}\right]^2\right]^{1/2} = A_1^{1/2}A_2^{1/2}$$
(6)

By the proof of the law of large numbers given in Theorem 19.1 of Davidson (1994) it follows that $A_i \to \sigma_i^2$, i = 1, 2 where

$$\mu_{i,T} = E\left(\sum_{l=1}^{m} c_{i,l}\psi(y_t, t_{i,l}, \sigma_T)\right)$$
$$\sigma_{i,T}^2 = E\left(\left(\sum_{l=1}^{m} c_{i,l}\psi(y_t, t_{i,l}, \sigma_T) - \mu_{i,T}\right)^2\right)$$

 $\mu_{i,T} \to \mu_i, \ \sigma_{i,T}^2 \to \sigma_i^2$ and both μ_i and σ_i^2 can be either finite or infinite. More specifically for Theorem 19.1 of Davidson (1994) it is required that

$$Var\left(\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{m}c_{i,l}\psi(y_{t},t_{i,l},\sigma_{T})-\mu_{i,T}\right]^{2}\right)=o(1), \quad i=1,2$$

But, by independence across t,

$$\begin{split} Var\left(\frac{1}{T}\sum_{l=1}^{T}\left[\sum_{l=1}^{m}c_{i,l}\psi(y_{t},t_{i,l},\sigma_{T})-\mu_{i,T}\right]^{2}\right) &\leq \frac{1}{T}Var\left(\left[\sum_{l=1}^{m}c_{i,l}\psi(y_{t},t_{i,l},\sigma_{T})\right]^{2}\right) \leq \\ T^{-1}E\left(\left(\left[\sum_{l=1}^{m}c_{i,l}\psi(y_{t},t_{i,l},\sigma_{T})\right]^{2}\right)^{2}\right) = \\ \frac{1}{T}\sum_{l=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m}c_{i,l}c_{i,k}c_{i,r}c_{i,s}\psi(y_{t},t_{i,l},\sigma_{T})\psi(y_{t},t_{i,k},\sigma_{T})\psi(y_{t},t_{i,r},\sigma_{T})\psi(y_{t},t_{i,s},\sigma_{T}) \leq \\ \frac{1}{T}\sum_{l=1}^{m}\sum_{k=1}^{m}\sum_{r=1}^{m}\sum_{s=1}^{m}|c_{i,l}c_{i,k}c_{i,r}c_{i,s}| \left|\psi(y_{t},t_{i,l},\sigma_{T})\psi(y_{t},t_{i,k},\sigma_{T})\psi(y_{t},t_{i,r},\sigma_{T})\psi(y_{t},t_{i,s},\sigma_{T})\right| \leq \\ T^{-1}m^{4}C_{\psi} = o(1) \end{split}$$

since by boundedness of ψ , $0 < C_{\psi} < \infty$, and also $c_{i,j}$ are constrained to be bounded.

As a result of the independence of the process y_t , only a subset of at most 4m observations (equal to the number of parameter to be chosen) are relevant for the maximisation in (4). The rest of the observations are not relevant. Let us denote the set of the time indices of the observations that have been used to choose $c_{1,l}$, $c_{2,l}$, $t_{1,l}$ and $t_{2,l}$ by \mathcal{O}_m Then,

$$\frac{1}{T} \sum_{t=1}^{T} \sigma_{12}^{-1} \left[\left(\sum_{l=1}^{m} c_{1,l} \psi((y_t, t_{1,l}), \sigma_T) - \mu_{1,T} \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma_T) - \mu_{2,T} \right) \right] = \frac{1}{T} \sum_{t \in \mathcal{O}_m} \sigma_{12}^{-1} \left[\left(\sum_{l=1}^{m} c_{1,l} \psi((y_t, t_{1,l}), \sigma_T) - \mu_{1,T} \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma_T) - \mu_{2,T} \right) \right] + \frac{1}{T} \sum_{t \notin \mathcal{O}_m} \sigma_{12}^{-1} \left[\left(\sum_{l=1}^{m} c_{1,l} \psi((y_t, t_{1,l}), \sigma_T) - \mu_{1,T} \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma_T) - \mu_{2,T} \right) \right] = B_1 + B_2 = B_1 + \frac{1}{T} \sum_{t \notin \mathcal{O}_m} B_{2,t}$$

By the fact that $m = o(T^{1/4})$, $\sqrt{T}B_1 = o_p(1)$. Therefore, we focus on B_2 . But in this case, $E(B_{2,t}) = 0$. Further, the conditions of Theorem 23.18 of Davidson (1994) are easily seen to be satisfied since by an argument similar to that used above for the law of large numbers, $B_{2,t}$ are uniformly L_2 -bounded implying the result of the Theorem.

6.4 Proof of Theorem 2

In order to prove Theorem 2 we use Theorem 2.2 of Horowitz (2002) which is a restatement of a result in Mammen (1992). Given the normality result of Theorem 1 and the linearity of $\hat{\rho}$ as a function of transforms of the observations, Mammen's result immediately implies (2).

6.5 Proof of Theorem 3

Using (5) we get that

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} D_t = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} E(D_t) + \frac{1}{\sqrt{T}} \sum_{t=1}^{T} (D_t - E(D_t))$$

Then it follows that it is sufficient to show that

$$\lim_{m \to \infty} Corr\left(\sum_{l=1}^m \psi(y_t, y_{\mathcal{S}_{1,l}}, \sigma_T), \sum_{l=1}^m \psi(y_{t-s}, y_{\mathcal{S}_{2,l}}, \sigma_T)\right) \neq 0$$

By Lemma 1 and 2 and under the alternative hypothesis of temporal dependence, there exist $t_{1,l}$ and $t_{2,l}$ such that

$$\lim_{m \to \infty} Corr\left(\sum_{l=1}^m \psi(y_t, t_{1,l}, \sigma_T), \sum_{l=1}^m \psi(y_{t-s}, t_{2,l}, \sigma_T)\right) \neq 0$$

To prove this theorem we will use similar analysis to that used in Theorem 1 of Temlyakov (2000). Note that the framework considered in Temlyakov (2000) is substantially different to the one we consider here and therefore we provide a distinct analysis in what follows. Let \mathcal{F}_f be a normed space with elements given by

$$(h,g) = \left(\sum_{l=1}^{\infty} c_{1,l}\psi(y_1, t_{1,l}, \sigma), \sum_{l=1}^{\infty} c_{2,l}\psi(y_2, t_{2,l}, \sigma)\right)$$

for some constants $c_{1,l}, c_{2,l} \in \{0, 1\}, t_{1,l}, t_{2,l}, l = 1, ..., and norm given by$

$$\|(h,g)\| = \psi_3 \left| \int \int \left(\sum_{l=1}^{\infty} \psi(y_1, t_{1,l}, \sigma) - \psi_1 \right) \left(\sum_{l=1}^{\infty} \psi(y_2, t_{2,l}, \sigma) - \psi_2 \right) f_s(y_1, y_2) dy_1 dy_2 \right|$$

where

$$\psi_{i} = \int \int \left(\sum_{l=1}^{\infty} \psi(y_{i}, t_{1,l}, \sigma) \right) f_{s}(y_{1}, y_{2}) dy_{1} dy_{2}, \quad i = 1, 2$$

and

$$\psi_3 = \psi_{3,1}^{-1/2} \psi_{3,2}^{-1/2},$$

$$\psi_{3,i} = \int \int \left(\sum_{l=1}^{\infty} \psi(y_i, t_{1,l}, \sigma) - \psi_i\right)^2 f_s(y_1, y_2) dy_1 dy_2, \quad i = 1, 2$$

It easily follows that the above norm is bounded. Define a dictionary to be a set of functions of the form

$$\mathcal{G} = \{(\psi(y_1, t_1, \sigma), \psi(y_2, t_2, \sigma)); t_1, t_2 \in \mathbb{R}\}$$

Then, we can write

$$\mathcal{F}_{f} = \{ \oplus_{l=1}^{\infty} (\psi_{l,1}, \psi_{l,2}); (\psi_{l,1}, \psi_{l,2}) \in \mathcal{G} \}$$

where

By Lemmas 1 and 2 there exist element(s) in \mathcal{H}_f that have nonzero norm. Algorithm 2 can be schematised as follows: We choose an element $(\psi_{1,1}, \psi_{1,2})$ such that $\|(\psi_{1,1}, \psi_{1,2})\|$ is maximised. Then, we proceed by iteratively choosing $(\psi_{l,1}, \psi_{l,2})$ such that

$$\left\|\left(\oplus_{i=1}^{l-1}(\psi_{i,1},\psi_{i,2})\right)\oplus(\psi_{i,1},\psi_{i,2})\right\|$$

is maximised. The chosen element of \mathcal{F}_f is the one that maximises the norm over the set $\{\oplus_{l=1}^{1}(\psi_{l,1},\psi_{l,2}),...,\oplus_{l=1}^{m}(\psi_{l,1},\psi_{l,2})\}$. We first note that there can only exist element(s) in \mathcal{H}_f that have non zero norm if there exist elements in \mathcal{G} denoted by $(\psi_{1,1},\psi_{1,2})$ such that $\|(\psi_{1,1},\psi_{1,2})\| > 0$. The maximisations involved in Algorithm 2 imply that at least one such element will be picked as long sample moments converge to population moments, i.e., as

$$\frac{1}{T} \sum_{i=1}^{T} \left[\left(\sum_{l=1}^{m} c_{1,l} \psi(y_t, t_{1,l}, \sigma) \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma) \right) \right] \xrightarrow{p}$$

$$E \left(\left(\sum_{l=1}^{m} c_{1,l} \psi(y_t, t_{1,l}, \sigma) \right) \left(\sum_{l=1}^{m} c_{2,l} \psi(y_{t-s}, t_{2,l}, \sigma) \right) \right)$$

$$(7)$$

for finite *m*. Since $(\sum_{l=1}^{m} c_{1,l}\psi(y_1, t_{1,l}, \sigma))$ $(\sum_{l=1}^{m} c_{1,l}\psi(y_1, t_{2,l}, \sigma))$ is a bounded function, for finite *m*, (7) follows if we establish a law of large numbers for $\psi(y_t)$, where $\psi(.)$ is some bounded function. To establish this law of large numbers we use Theorem 19.11 of Davidson (1994). This requires three conditions: (i) $\psi(y_t)$ is a L_1 mixingale, (ii) $\psi(y_t)$ is uniformly integrable and (iii) the mixingale coefficients of $\psi(y_t)$ tend to zero. By Assumption 3, boundedness of $\psi(y_t)$ and Theorems 17.5 and 17.13 of Davidson (1994), we obtain conditions (i) and (iii). Condition (ii) follows immediately by boundedness of $\psi(y_t)$. From the above it follows immediately that the chosen element of \mathcal{F}_f has non zero norm thus proving the theorem.

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Table 1: Rejection Probabilities for Monte Carlo study of Hong and White (2005). $\mathcal{T}_n(1)$ is the entropy based test of Hong and White (2005). *best(Hong and White (2005))* refers to the best performing test in terms of rejection probabilities for a given experiment, of those tests considered and reported in Hong and White (2005).

Experiment	RBF3(1)	RBF2(1)	$\mathcal{T}_n(1)$	best(Hong and White (2005))
IID	0.050	0.047	-	-
AR1	0.699	0.604	0.140	0.140
ARCH1	0.887	0.875	0.376	0.612
TARCH	0.607	0.574	0.206	0.278
BILIN1	0.964	0.967	0.696	0.816
NMA1	0.411	0.607	0.340	0.348
TAR1	0.417	0.721	0.256	0.258
NAR1	0.495	0.477	0.170	0.170
SIGN1	0.625	0.616	0.608	0.608

Table 2: Rejection Probabilities for Monte Carlo study of Fernandes and Neri (2007). The second and third columns of the Table report rejection probabilities for the entropy based test of Fernandes and Neri (2007) and two values of q which is a tuning parameter for that test. *best(Fernandes and Neri (2007))* refers to the best performing test in terms of rejection probabilities for a given experiment, of those tests considered and reported in Fernandes and Neri (2007).

Experiment	RBF3(1)	q = 1/2	q = 4	best(Fernandes and Neri (2007))
AR2	0.978	0.209	0.447	0.447
ARCH2	0.864	0.210	0.156	0.903
NMA2	0.163	0.032	0.038	0.170
TAR2	0.454	0.166	0.323	0.323
BILIN2	0.976	0.387	0.508	0.788
NAR2	0.112	0.053	0.066	0.073
SIGN2	0.882	0.241	0.392	0.392

Table 3: Probability Values for S&P 500 Series and 1, 2, 3, 4, 12 and 52 lags (ABBOTT
LABS EASTMAN KODAK). (Significant p. values reported in bold typeface)

			P. V	alues						P. V.	alues		
Company Name/s	1	2	3	4	12	52		1	2	3	4	12	52
ABBOTT LABS	0.020	0.020	0.523	0.007	0.644	0.074	ADC TELECOM	0.000	0.060	0.007	0.322	0.765	0.738
ADOPE SVS	0.020	0.570	0.020	0.067	0.544	0.004	ADVD MICPO DEVC	0.678	0.000	0.001	0.205	0.161	0.628
ADOBE 515.	0.880	0.370	0.034	0.007	0.344	0.094	ADVD.MICIO DEVC.	0.078	0.215	0.000	0.295	0.101	0.038
AES	0.013	0.007	0.000	0.000	0.060	0.812	AFLAC	0.000	0.027	0.000	0.201	0.013	0.154
AIR PRDS.& CHEMS.	0.107	0.832	0.154	0.000	0.503	0.020	ALBERTO CULVER 'B'	0.007	0.101	0.168	0.315	0.181	0.416
ALBERTSONS	0.047	0.866	0.195	0.537	0.195	0.497	ALCOA	0.000	0.134	0.282	0.020	0.792	0.007
ALLEGHENY EN.	0.000	0.047	0.000	0.007	0.040	0.973	ALLEGHENY TECHS.	0.013	0.034	0.168	0.383	0.530	0.013
ALLERGAN	0.336	0.195	0.081	0.161	0.074	0.826	ALLIED WASTE INDS.	0.128	0.826	0.275	0.134	0.953	0.034
ALLTEL	0.013	0.121	0.007	0.591	0.040	0.007	ALTERA	0.060	0.322	0.174	0.074	0.188	0.148
ALTRIA CP	0.107	0.383	0.034	0.148	0.107	0.966	AMBAC FINANCIAL	0.000	0.617	0.054	0.087	0.047	0.174
AMEDADA HESS	0.107	0.303	0.604	0.140	0.107	0.300	AMED ELEC DWD	0.000	0.017	0.004	0.061	0.041	0.174
AMERADA HESS	0.919	0.309	0.004	0.313	0.202	0.349	AMER.ELEC.FWR.	0.000	0.013	0.007	0.208	0.000	0.248
AMERICAN EXPRESS	0.007	0.336	0.228	0.644	0.027	0.107	AMER.GREETINGS 'A'	0.315	0.490	0.946	0.577	0.067	0.383
AMERICAN INTL.GP.	0.000	0.074	0.067	0.322	0.067	0.000	AMER.POWER CONV.	0.302	0.946	0.456	0.356	0.295	0.336
AMGEN	0.034	0.168	0.000	0.497	0.000	0.000	AMSOUTH BANC.	0.013	0.564	0.966	0.973	0.007	0.154
ANADARKO PETROLEUM	0.034	0.638	0.000	0.000	0.013	0.772	ANALOG DEVICES	0.013	0.000	0.000	0.034	0.221	0.221
ANDREW	0.872	0.423	0.946	0.289	0.221	0.020	ANHEUSER - BUSCH COS.	0.000	0.007	0.040	0.577	0.000	0.040
AON	0.000	0.275	0.396	0.013	0.000	0.597	APACHE	0.054	0.148	0.322	0.067	0.201	0.336
APPLE COMPUTERS	0.060	0.201	0.114	0.340	0.040	0.148	APPLERA APPD BIOS	0.000	0,000	0.087	0.416	0.134	0.584
ADDUED MATS	0.000	0.201	0.114	0.191	0.040	0.140	ADCHED DANIS	0.000	0.101	0.007	0.410	0.134	0.199
AFFLIED MAIS.	0.000	0.370	0.330	0.181	0.295	0.208	ARCHER - DANLS.	0.087	0.121	0.047	0.785	0.525	0.188
ASHLAND	0.544	0.430	0.349	0.738	0.906	0.242	AT & T	0.758	0.007	0.060	0.772	0.866	0.047
AUTODESK	0.779	0.161	0.047	0.624	0.013	0.134	AUTOMATIC DATA PROC.	0.013	0.604	0.926	0.087	0.181	0.262
AUTONATION	0.040	0.638	0.792	0.879	0.054	0.275	AUTOZONE	0.698	0.074	0.960	0.557	0.148	0.121
AVERY DENNISON	0.168	0.121	0.295	0.007	0.221	0.409	AVON PRODUCTS	0.497	0.134	0.000	0.611	0.161	0.013
BAKER HUGHES	0.027	0.919	0.201	0.289	0.047	0.121	BALL	0.007	0.007	0.007	0.060	0.705	0.906
BANK OF AMERICA	0.000	0.074	0.436	0.195	0.034	0.034	BANK OF NEW YORK	0.087	0.007	0.248	0.188	0.362	0.121
BANK ONE	0.154	0.114	0.101	0.168	0.490	0.004	BARD C B	0.060	0.013	0.940	0.523	0.738	0.658
DANK ONE	0.134	0.114	0.101	0.108	0.490	0.020	DANTED INTE	0.000	0.013	0.940	0.525	0.738	0.005
BAUSCH & LOMB	0.436	0.752	0.879	0.544	0.611	0.752	BAATER INTL.	0.060	0.094	0.255	0.242	0.597	0.295
BB & T	0.000	0.040	0.013	0.000	0.040	0.604	BEAR STEARNS	0.007	0.000	0.040	0.772	0.027	0.691
BECTON DICKINSON & .CO.	0.000	0.477	0.060	0.611	0.020	0.423	BED BATH & .BEYOND	0.000	0.000	0.020	0.000	0.013	0.745
BELLSOUTH	0.000	0.040	0.195	0.161	0.007	0.329	BEMIS	0.034	0.208	0.517	0.745	0.732	0.953
BEST BUY CO.	0.470	0.832	0.040	0.000	0.463	0.953	BIG LOTS	0.027	0.027	0.349	0.685	0.362	0.691
BIOGEN IDEC	0.188	0.040	0.899	0.168	0.007	0.826	BIOMET	0.020	0.758	0.799	0.919	0.738	0.698
BLSVS	0.047	0.060	0.456	0.570	0.074	0.013	BLACK & DECKER	0.040	1 000	0.718	0.879	0.362	0.101
H & P PLOCK	0.400	0.141	0.105	0.199	0.601	0.507	PMC SOFTWARE	0.074	0.426	0.020	0.161	0.000	0.000
DODING	0.490	0.141	0.195	0.188	0.091	0.097	DOISE CASCADE	0.074	0.430	0.020	0.101	0.000	0.000
BOEING	0.074	0.121	0.188	0.295	0.497	0.013	BOISE CASCADE	0.309	0.228	0.564	0.027	0.282	0.638
BOSTON SCIENTIFIC	0.409	0.711	0.383	0.034	0.678	0.067	BRISTOL MYERS SQUIBB	0.013	0.000	0.013	0.262	0.121	0.148
BROWN - FORMAN 'B'	0.195	0.134	0.013	0.114	0.966	0.570	BRUNSWICK	0.027	0.879	0.530	0.732	0.570	0.054
BURL.NTHN.SANTA FE C	0.000	0.067	0.007	0.470	0.174	0.007	BURLINGTON RES.	0.034	0.148	0.000	0.221	0.000	0.765
CAMPBELL SOUP	0.013	0.544	0.094	0.067	0.503	0.188	CARDINAL HEALTH	0.591	0.215	0.087	0.617	0.114	0.060
CARNIVAL	0.034	0.161	0.074	0.007	0.168	0.349	CATERPILLAR	0.201	0.215	0.691	0.987	0.027	0.322
CENDANT	0.597	0.047	0.128	0.919	0.671	0.168	CENTERPOINT EN	0.000	0.000	0.081	0.094	0.007	0.013
CENTEY	0.685	0.685	0.728	0.206	0.725	0.040	CENTURVIEI	0.007	0.215	0.564	0.052	0.621	0.067
CUADIES COUVAD	0.000	0.000	0.730	0.000	0.720	0.040	CULADEED ONE FINI	0.001	0.215	0.004	0.305	0.001	0.007
CHARLES SCHWAB	0.054	0.034	0.732	0.926	0.678	0.047	CHARTER ONE FINL.	0.000	0.946	0.993	0.396	0.839	0.389
CHEVRONTEXACO	0.000	0.268	0.181	0.591	0.007	0.315	CHIRON CORP	0.168	0.000	0.013	0.000	0.007	0.503
CHUBB	0.000	0.067	0.000	0.624	0.121	0.664	CIGNA	0.812	0.141	0.054	0.698	0.409	0.336
CINCINNATI FIN.	0.000	0.000	0.000	0.007	0.228	0.497	CINTAS	0.087	0.087	0.047	0.020	0.013	0.698
CIRCUIT CITY STORES	0.396	0.470	0.456	0.678	0.020	0.302	CISCO SYSTEMS	0.047	0.007	0.000	0.007	0.530	0.013
CITIGROUP	0.020	0.195	0.195	0.168	0.000	0.403	CITIZENS COMMS.	0.000	0.074	0.369	0.597	0.396	0.617
CLEAR CHL.COMMS.	0.027	0.040	0.195	0.007	0.557	0.799	CLOBOX	0.007	0.027	0.416	0.013	0.040	0.349
CMS ENERGY	0.013	0.027	0.020	0.013	0.020	0.510	COCA COLA	0.000	0.550	0.000	0.591	0.242	0.034
COCA COLA ENTS	0.149	0.021	0.141	0.010	0.149	0.700	COLCATE DAIM	0.000	0.007	0.124	0.114	0.729	0.141
COMCASE 141	0.140	0.004	0.141	0.012	0.140	0.199	CONFRICA	0.000	0.001	0.141	0.114	0.155	0.141
COMCAST 'A'	0.027	0.436	0.336	0.336	0.987	0.188	COMERICA	0.067	0.309	0.141	0.638	0.215	0.121
COMPUTER ASSOCS.INTL.	0.141	0.060	0.275	0.054	0.336	0.510	COMPUTER SCIS.	0.242	0.819	0.121	0.705	0.027	0.134
COMPUWARE	0.933	0.128	0.174	0.094	0.034	0.564	COMVERSE TECH.	0.081	0.973	0.275	0.329	0.872	0.517
CONAGRA	0.054	0.000	0.490	0.107	0.611	0.879	CONCORD EFS	0.611	0.839	0.047	0.000	0.899	0.966
CONOCOPHILLIPS	0.047	0.228	0.081	0.268	0.570	0.128	CONS.EDISON	0.000	0.007	0.020	0.000	0.409	0.698
CONSTELLATION EN.	0.181	0.007	0.047	0.329	0.040	0.027	COOPER INDS.	0.040	0.872	0.980	0.007	0.752	0.329
COOPER TIRE BUB	0.470	0.074	0.523	0.013	0.772	0.047	ADOLPH COORS 'B'	0.362	0.403	0.161	0.188	0.866	0.698
COBNING	0.020	0.013	0.020	0.181	0,000	0.000	COUNTRYWIDE FINL	0.725	0.812	0.846	0.336	0.850	0.181
CDANE	0.624	0.010	0.060	0.101	0.500	0.050	CONTRACTOR INCL.	0.007	0.012	0.040	0.639	0.047	0.101
CUMMING	0.024	0.019	0.000	0.228	0.510	0.955		0.007	0.034	0.652	0.038	0.047	0.369
COMMINS	0.228	0.282	0.000	0.000	0.450	0.705		0.779	0.074	0.094	0.490	0.027	0.054
DANA	0.007	0.000	0.121	0.000	0.564	0.000	DANAHER	0.275	0.000	0.007	0.074	0.000	0.000
DEERE & CO.	0.087	0.034	0.463	0.000	0.993	0.013	DELL	0.430	0.705	0.000	0.007	0.973	0.993
DELTA AIR LINES	0.007	0.376	0.315	0.013	0.664	0.027	DELUXE	0.302	0.651	0.007	0.866	0.564	0.000
DILLARDS 'A'	0.094	0.933	0.577	0.799	0.691	0.557	DOLLAR GENERAL	0.510	0.691	0.913	0.141	0.054	0.309
DOMINION RES.	0.181	0.020	0.000	0.134	0.007	0.329	DONNELLEY R. R.	0.638	0.336	0.953	0.584	0.349	0.161
DOVER	0.752	0.000	0.315	0.443	0.081	0.154	DOW CHEMICALS	0.007	0.020	0.013	0.315	0.174	0.280
DOW JONES & CO	0.752	0.611	0.515	0.443	0.001	0.104	DTE ENERCY	0.007	0.020	0.124	0.010	0.114	0.209
DUW JOINES & .CO	0.013	0.611	0.530	0.772	0.114	0.819	DIE ENERGY	0.000	0.423	0.134	0.034	0.423	0.248
DU PONT ET DE NEMOURS	0.000	0.752	0.013	0.161	0.027	0.054	DUKE ENERGY	0.000	0.020	0.007	0.007	0.074	0.624
DYNEGY 'A'	0.000	0.087	0.027	0.020	0.289	0.067	EASTMAN KODAK	0.450	0.383	0.685	0.027	0.020	0.235

Table 4: Probability Values for S&P 500 Series and 1, 2, 3, 4, 12 and 52 lags (EATON - NORTH FORK BANCORP.). (Significant p. values reported in bold typeface)

	P. Values							P. Values					
Company Name/s	1	2	3	4	12	52		1	2	3	4	12	52
EATON	0.899	0.013	0.866	0.174	0.497	0.376	ECOLAB	0.000	0.235	0.054	0.148	0.523	0.154
EDISON INTL.	0.000	0.013	0.027	0.275	0.047	0.617	EL PASO	0.007	0.047	0.020	0.013	0.000	0.107
ELECTRONIC ARTS	0.020	0.973	0.040	0.336	0.497	0.987	ELECTR. DATA SYSTEMS	0.013	0.081	0.550	0.557	0.040	0.839
EMC	0.020	0.020	0.000	0.671	0.671	0.107	EMERSON ELECTRIC	0.000	0.013	0.000	0.007	0.007	0.000
ENGELHARD	0.027	0.799	0.839	0.852	0.423	0.403	ENTERGY	0.040	0.034	0.671	0.034	0.054	0.792
EOG RES.	0.094	0.121	0.899	0.034	0.007	0.933	EQUIFAX	0.544	0.302	0.242	0.114	0.436	0.067
EXELON EXYON MODIL	0.060	0.195	0.879	0.597	0.463	0.067	EAPRESS SURIPIS 'A'	0.027	0.852	0.168	0.081	0.846	0.738
EANNIE MAE	0.000	0.000	0.000	0.470	0.651	1.000	FAMILY 5.51K5.	0.242	0.128	0.262	0.396	0.430	0.711
FAINIL MAE FEDERATED DEPT STRS	0.007	0.289	0.134	0.034	0.027	0.330	FREDDIE MAC FEDEX	0.013	0.485	0.505	0.174	0.067	0.349
FIETH THIRD BANCORP	0.141	0.000	0.195	0.397	0.309	0.013	FIRST DATA	0.071	0.349	0.034	0.228	0.235	0.893
FIRST TEN NAT	0.007	0.148	0.396	0.000	0.007	0.658	FIRSTENERGY	0.054	0.120	0.275	0.004	0.456	0.000
FISEBV	0.020	0.235	0.054	0.517	0.785	0.174	FLEETBOSTON FINL	0.000	0.081	0.329	0.141	0.376	0.121
FORD MOTOR	0.154	0.060	0.658	0.040	0.081	0.651	FOREST LABS.	0.094	0.302	0.510	0.450	0.926	0.060
FORTUNE BRANDS	0.054	0.107	0.523	0.940	0.577	0.658	FPL GROUP	0.000	0.013	0.047	0.309	0.027	0.799
FRANK.RES.	0.000	0.497	0.168	0.094	0.000	0.893	GANNETT	0.000	0.000	0.007	0.074	0.000	0.081
GAP	0.248	0.000	0.000	0.470	0.141	0.215	GEN.DYNAMICS	0.228	0.980	0.201	0.262	0.362	0.060
GENERAL ELECTRIC	0.000	0.409	0.181	0.215	0.322	0.691	GEN.MILLS	0.711	0.859	0.228	0.154	0.000	0.295
GENERAL MOTORS	0.946	0.322	0.141	0.114	0.268	0.738	GENUINE PARTS	0.000	0.792	0.685	0.201	0.081	0.322
GENZYME	0.174	0.966	0.631	0.711	0.020	0.919	GEORGIA PACIFIC	0.000	0.007	0.107	0.107	0.060	0.000
GILLETTE	0.007	0.638	0.034	0.040	0.819	0.443	GOLDEN WEST FINL.	0.007	0.020	0.356	0.477	0.081	0.819
GOODRICH	0.268	0.201	0.403	0.678	0.577	0.362	GOODYEAR TIRE	0.034	0.188	0.007	0.040	0.087	0.000
GRAINGER W W	0.013	0.027	0.000	0.040	0.067	0.007	GT.LAKES CHM.	0.268	0.570	0.067	0.940	0.081	0.262
HALLIBURTON	0.013	0.463	0.054	0.013	0.490	0.832	HARLEY - DAVIDSON	0.054	0.309	0.671	0.510	0.718	0.235
HARRAHS ENTM.	0.322	0.886	0.899	0.336	0.168	0.362	HASBRO	0.047	0.436	0.423	0.040	0.060	0.966
HCA	0.000	0.000	0.745	0.356	0.087	0.161	HEALTH MAN.AS.A	0.040	0.101	0.242	0.188	0.242	0.235
HEINZ HJ	0.007	0.530	0.195	0.013	0.779	0.617	HERCULES	0.007	0.081	0.624	0.893	0.973	0.007
HERSHEY FOODS	0.007	0.161	0.570	0.221	0.846	0.289	HEWLETT - PACKARD	0.007	0.000	0.128	0.436	0.557	0.584
HILTON HOTELS	0.973	0.201	0.886	0.101	0.215	0.060	HOME DEPOT	0.000	0.725	0.638	0.034	0.785	0.597
HONEYWELL INTL.	0.638	0.356	0.832	0.792	0.376	0.329	HUMANA	0.007	0.114	0.322	0.047	0.490	0.919
HUNTINGTON BCSH.	0.000	0.000	0.034	0.235	0.000	0.309	ILLINOIS TOOL WKS.	0.013	0.148	0.067	0.530	0.624	0.174
INGERSOLL - RAND	0.336	0.067	0.027	0.785	0.081	0.000	INTEL	0.685	0.114	0.007	0.772	0.168	0.013
INTL.BUS.MACH.	0.423	0.262	0.020	0.550	0.812	0.705	INTL.FLAV.& FRAG.	0.812	0.094	0.020	0.060	0.282	0.336
INTL.GAME TECH.	0.698	0.705	0.973	0.765	0.148	0.946	INTL.PAPER	0.423	0.181	0.248	0.054	0.463	0.000
INTERPUBLIC GP.	0.000	0.020	0.275	0.013	0.054	0.128	ITT INDUSTRIES	0.000	0.443	0.000	0.503	0.698	0.698
JP MORGAN CHASE & .CO.	0.000	0.195	0.060	0.181	0.128	0.443	JEFFERSON PILOT	0.007	0.013	0.007	0.114	0.020	0.268
JOHNSON & JOHNSON	0.000	0.121	0.953	0.013	0.691	0.953	JOHNSON CONTROLS	0.020	0.054	0.121	0.893	0.322	0.201
JONES APPAREL GROUP	0.443	0.013	0.886	0.195	0.617	0.067	KB HOME	0.000	0.000	0.027	0.054	0.503	0.872
KELLOGG	0.040	0.000	0.020	0.040	0.658	0.336	KERR - MCGEE	0.027	0.013	0.074	0.000	0.497	0.188
KEYCORP KIMPEDIN CLADK	0.000	0.034	0.020	0.040	0.000	0.020	KEISPAN	0.007	0.087	0.101	0.611	0.389	0.074
KIMBERLY - CLARK	0.000	0.557	0.718	0.007	0.792	0.161	KINDER MORGAN KANS	0.013	0.000	0.000	0.248	0.987	0.248
KLA TENCOR	0.007	0.114	0.094	0.007	0.154	0.859	KNIGHT - RIDDER	0.047	0.376	0.436	0.000	0.309	0.027
I ECCETTI: DI ATT	0.000	0.074	0.550	0.617	0.570	0.058	LUIVEU	0.047	0.651	0.087	0.080	0.000	0.148
LIMITED BRANDS	0.034	0.020	0.004	0.017	0.047	0.208	LILLI ELI LINCOLN NAT	0.880	0.051	0.557	0.081	0.940	0.091
LINEAD TECH	0.342	0.000	0.289	0.013	0.040	0.313	LIZ CLAIROPNE	0.000	0.000	0.007	0.000	0.208	0.000
LOEWS	0.000	0.320	0.107	0.054	0.557	0.012	LNA PACIFIC	0.303	0.140	0.121	0.973	0.872	0.920
LOWE'S COMPANIES	0.013	0.029	0.953	0.591	0.275	0.060	LSLLOGIC	0.074	0.376	0.121	0.235	0.953	0.678
MANOR CARE	0.302	0.289	0.745	0.523	0.215	0.208	MARATHON OIL	0.007	0.010	0.255	0.027	0.168	0.121
MARSH & MCLENNAN	0.000	0.013	0.027	0.054	0.000	0.919	MARSHALL & ILSLEY	0.000	0.000	0.000	0.000	0.000	0.456
MASCO	0.000	0.034	0.040	0.060	0.315	0.013	MATTEL	0.000	0.765	0.094	0.007	0.054	0.430
MAXIM INTEGRATED PRDS	0.034	0.074	0.309	0.275	0.577	0 248	MAY DEPT STORES	0 154	0.913	0.208	0 436	0.013	0.302
MAYTAG	0.107	0.094	0.544	0.799	0.658	0.785	MBIA	0.000	0.302	0.302	0.188	0.356	0.020
MBNA	0.007	0.154	0.477	0.772	0.188	0.866	MCCORMICK & .CO NV.	0.007	0.195	0.805	0.477	0.101	0.987
MCDONALDS	0.456	0.141	0.530	0.698	0.027	0.134	MCGRAW - HILL CO.	0.000	0.181	0.584	0.691	0.383	0.242
MEADWESTVACO	0.020	0.893	0.121	0.054	0.711	0.356	MEDIMMUNE	0.960	0.101	0.376	0.510	0.705	0.114
MEDTRONIC	0.000	0.839	0.047	0.275	0.081	0.027	MELLON FINL.	0.174	0.074	0.617	0.946	0.356	0.000
MERCK & .CO.	0.047	0.651	0.114	0.826	0.711	0.456	MEREDITH	0.007	0.114	0.000	0.275	0.497	0.738
MERRILL LYNCH & .CO.	0.013	0.000	0.611	0.208	0.034	0.040	MGIC INVT	0.007	0.040	0.013	0.463	0.181	0.047
MICRON TECH.	0.141	0.302	0.134	0.060	0.490	0.450	MICROSOFT	0.040	0.007	0.013	0.020	0.309	0.705
MILLIPORE	0.362	0.000	0.208	0.000	0.503	0.000	MOLEX	0.188	0.503	0.134	0.174	0.007	0.846
MOTOROLA	0.013	0.013	0.040	0.000	0.013	0.295	NABORS INDS.	0.034	0.054	0.074	0.859	0.000	0.000
NAT.CITY	0.000	0.000	0.000	0.000	0.000	0.221	NATIONAL SEMICON.	0.658	0.322	0.020	0.544	0.235	0.503
NAVISTAR INTL.	0.872	0.262	0.490	0.987	0.523	0.772	NEW YORK TIMES 'A'	0.000	0.275	0.403	0.128	0.020	0.436
NEWELL RUBBERMAID	0.047	0.134	0.114	0.242	0.993	0.000	NEWMONT MINING	0.000	0.664	0.926	0.000	0.946	0.691
NEXTEL COMMS.A	0.000	0.362	0.000	0.000	0.128	0.805	NICOR	0.597	0.148	0.792	0.564	0.282	0.745
NIKE 'B'	0.000	0.040	0.228	0.221	0.664	0.262	NISOURCE	0.000	0.007	0.000	0.000	0.000	0.013
NOBLE	0.000	0.195	0.624	0.611	0.872	0.000	NORDSTROM	0.087	0.664	0.007	0.940	0.141	0.087
NORFOLK SOUTHERN	0.034	0.013	0.000	0.054	0.027	0.000	NORTH FORK BANCORP.	0.000	0.027	0.054	0.617	0.013	0.973

			P. V	alues						P. V	alues		
Company Name/s	1	2	3	4	12	52		1	2	3	4	12	52
NTHN TRUST	0.000	0.624	0.134	0.007	0.134	0.047	NORTHROP CRUMMAN	0.128	0.181	0.501	0.201	0.826	0.557
NOVELI	0.000	0.024	0.134	0.1007	0.134	0.041	NOVELLUS SYSTEMS	0.120	0.101	0.031	0.201	0.047	0.557
NUCOD	0.732	0.304	0.000	0.128	0.134	0.074	NOVELLUS SISIEMS	0.101	0.000	0.020	0.000	0.047	0.584
NUCOR	0.195	0.899	0.228	0.866	0.282	0.000	OCCIDENTAL PIL.	0.993	0.846	0.980	0.302	0.020	0.034
OFFICE DEPOT	0.852	0.201	0.570	0.107	0.644	0.671	OMNICOM GP.	0.000	0.000	0.027	0.007	0.114	0.134
ORACLE	0.020	0.275	0.081	0.121	0.047	0.221	PACCAR	0.094	0.027	0.322	0.698	0.617	0.523
PALL	0.772	0.933	0.148	0.396	0.013	0.000	PARAMETRIC TECH.	0.007	0.356	0.329	0.617	0.362	0.114
PARKER - HANNIFIN	0.007	0.000	0.436	0.564	0.973	0.456	PAYCHEX	0.000	0.020	0.020	0.141	0.007	0.839
PENNEY JC	0.000	0.000	0.711	0.523	0.000	0.550	PEOPLES ENERGY	0.020	0.362	0.013	0.242	0.564	0.329
PEOPLESOET	0.188	0.013	0.282	0.128	0.383	0.141	PEPSICO	0.000	0.081	0.174	0.870	0.020	0.242
DEDVINELMED	0.100	0.015	0.202	0.120	0.305	0.141	DEIZED	0.000	0.001	0.174	0.671	0.020	0.242
FERRINELMER	0.034	0.390	0.121	0.034	0.705	0.839	FFIZER	0.000	0.255	0.034	0.071	0.980	0.800
PG & .E	0.000	0.020	0.000	0.040	0.013	0.396	PHELPS DODGE	0.000	0.054	0.060	0.067	0.369	0.007
PINNACLE WEST CAP.	0.060	0.027	0.020	0.315	0.020	0.564	PITNEY - BOWES	0.013	0.591	0.866	0.282	0.262	0.456
PLUM CREEK TIMBER	0.000	0.054	0.463	0.094	0.154	0.013	PMC - SIERRA	0.007	0.000	0.013	0.007	0.007	0.013
PNC FINL.SVS.GP.	0.040	0.396	0.940	0.752	0.255	0.114	PPG INDUSTRIES	0.007	0.221	0.154	0.826	0.772	0.188
PPL	0.094	0.020	0.000	0.362	0.537	0.000	PRAXAIR	0.087	0.564	0.128	0.349	0.872	0.020
PROCTER & GAMBLE	0.013	0.248	0.993	0.000	0.436	0.705	PROGRESS EN	0.067	0.235	0.040	0.020	0.054	0.154
PROGRESSIVE OHIO	0.013	0.087	0.107	0.322	0.000	0.020	PROVIDIAN FINI	0.000	0.000	0.013	0.007	0.060	0.040
DUD SED ENTED CD	0.015	0.007	0.107	0.024	0.000	0.020	DULTE HOMES	0.000	0.000	0.010	0.101	0.000	0.040
FUB.SER.ENTER.GF.	0.000	0.007	0.013	0.074	0.007	0.081	PULLE HOMES	0.034	0.038	0.403	0.101	0.840	0.470
QUALCOMM	0.141	0.638	0.919	0.000	0.047	1.000	RADIOSHACK	0.101	0.248	0.027	0.074	0.188	0.302
RAYTHEON 'B'	0.027	0.430	0.013	0.000	0.940	0.114	REEBOK INTL.	0.013	0.000	0.879	0.819	0.000	0.329
REGIONS FINL.	0.000	0.027	0.054	0.020	0.000	0.040	ROBERT HALF INTL.	0.356	0.752	0.812	0.094	0.013	0.007
ROCKWELL AUTOMATION	0.013	0.705	0.081	0.456	0.128	0.000	ROHM & HAAS	0.007	0.691	0.027	0.940	0.034	0.477
ROWAN COS.	0.497	0.544	0.081	0.295	0.067	0.007	BYDEB SYSTEM	0.000	0.899	0.248	0.490	0.611	0.362
SAFECO	0.000	0.000	0.000	0.000	0.034	0.564	SAFEWAY	0.309	0.557	0.248	0.336	0.034	0.463
CADALEE	0.000	0.000	0.000	0.000	0.729	0.169	SPC COMMUNICATIONS	0.005	0.007	0.240	0.101	0.477	0.149
SARA LEE	0.330	0.202	0.235	0.000	0.738	0.108	SBC COMMUNICATIONS	0.027	0.013	0.007	0.121	0.477	0.148
SCHERING - PLOUGH	0.034	0.074	0.087	0.060	0.221	0.101	SCHLUMBERGER	0.007	0.013	0.047	0.000	0.000	0.000
SCIENTIFIC ATLANTA	0.201	0.725	0.718	0.020	0.604	0.651	SEALED AIR	0.268	0.154	0.208	0.993	0.718	0.376
SEARS ROEBUCK & .CO.	0.020	0.060	0.000	0.799	0.054	0.926	SEMPRA EN.	0.007	0.034	0.054	0.020	0.007	0.477
SHERWIN - WILLIAMS	0.356	0.805	0.020	0.161	0.812	0.040	SIGMA ALDRICH	0.221	0.758	0.705	0.336	0.074	0.826
SLM	0.000	0.067	0.081	0.007	0.134	0.584	SNAP - ON	0.530	0.389	0.779	0.671	0.101	0.000
SOLECTBON	0.060	0.034	0.000	0.000	0.195	0.007	SOUTHERN	0.007	0.000	0.060	0.027	0.000	0.034
SOUTHTBUST	0 101	0.000	0.000	0.000	0.013	0.946	SOUTHWEST AIBLINES	0.215	0.906	0.638	0.966	0.101	0.946
SPRINT	0.101	0.000	0.007	0.000	0.121	0.072	ST HDE MED	0.210	0.060	0.047	0.694	0.101	0.671
CT DATE	0.000	0.289	0.007	0.820	0.121	0.975	ST.JODE MED.	0.000	0.000	0.041	0.024	0.040	0.071
ST.PAUL	0.000	0.007	0.013	0.530	0.000	0.007	STANLEY WORKS	0.195	0.034	0.081	0.805	0.067	0.215
STAPLES	0.000	0.235	0.020	0.148	0.423	0.658	STARBUCKS	0.060	0.094	0.268	0.040	0.020	0.416
STARWOOD HTLS.& .RESORTS	0.007	0.067	0.430	0.309	0.980	0.101	STATE STREET	0.087	0.772	0.557	0.195	0.832	0.114
STRYKER	0.148	0.208	0.477	0.423	0.013	0.094	SUN MICROSYSTEMS	0.711	0.000	0.027	0.839	0.671	0.007
SUNGARD DATA SYSTEMS	0.040	0.242	0.470	0.255	0.886	0.242	SUNOCO	0.040	0.792	0.242	0.101	0.671	0.295
SUNTRUST BANKS	0.040	0.000	0.081	0.060	0.000	0.705	SUPERVALU	0.517	0.020	0.423	0.174	0.040	0.107
SVMANTEC	0.000	0.315	0.168	0.360	0.087	0.121	SVMBOL TECHS	0.007	0.054	0.000	0.067	0.101	0.007
STMANTEO SVNOVUS FINI	0.000	0.010	0.105	0.305	0.907	0.121	STMBOL TEORS.	0.001	0.034	0.000	0.007	0.101	0.001
5 INOVUS FINL.	0.000	0.040	0.725	0.295	0.893	0.195	51500	0.000	0.344	0.107	0.899	0.919	0.899
T ROWE PRICE GP.	0.007	0.597	0.698	0.174	0.772	0.987	TARGET	0.087	0.000	0.094	0.550	0.020	0.644
TECO ENERGY	0.000	0.000	0.007	0.000	0.000	0.973	TEKTRONIX	0.000	0.195	0.121	0.054	0.107	0.846
TELLABS	0.007	0.000	0.114	0.067	0.497	0.020	TEMPLE INLAND	0.369	0.490	0.054	0.463	0.329	0.067
TENET HLTHCR.	0.000	0.893	0.933	0.409	0.409	0.584	TERADYNE	0.000	0.188	0.329	0.000	0.141	0.000
TEXAS INSTS.	0.020	0.013	0.000	0.000	0.148	0.349	TEXTRON	0.000	0.584	0.644	0.201	0.201	0.416
THEBMO ELECTBON	0.027	0.020	0.128	0.591	0.946	0.067	THOMAS & BETTS	0.188	0.188	0.960	0.664	0 195	0 107
TIFFANY & CO	0.007	0.027	0.047	0.188	0.376	0.946	TIME WARNER	0.953	0.349	0.228	0.544	0.114	0.946
TIX COS	0.215	0.000	0.041	0.007	0.810	0.260	TOPCHMARK	0.105	0.045	0.149	0.012	0.124	0.129
TJA COS.	0.313	0.000	0.000	0.007	0.812	0.309	TORCHMARK	0.195	0.007	0.148	0.013	0.134	0.128
TOYS R US HOLDINGS CO.	0.463	0.188	0.758	0.262	0.107	0.000	I RIBUNE	0.007	0.000	0.034	0.101	0.208	0.651
TXU	0.000	0.000	0.060	0.047	0.329	0.993	TYCO INTL.	0.013	0.966	0.134	0.174	0.148	0.483
US BANCORP	0.000	0.007	0.007	0.027	0.201	0.000	UNION PACIFIC	0.034	0.188	0.081	0.557	0.047	0.235
UNION PLANTERS	0.074	0.128	0.846	0.000	0.289	0.624	UNISYS	0.993	0.262	0.007	0.161	0.007	0.161
UNITEDHEALTH GP.	0.040	0.074	1.000	0.852	0.376	0.792	US.STEEL	0.638	0.275	0.685	0.705	0.772	0.074
UNITED TECHNOLOGIES	0.013	0.020	0.282	0.450	0.812	0.550	UNOCAL	0.020	0.141	0.389	0.181	0.107	0.027
UNUMPROVIDENT	0.010	0.416	0.101	0.376	0.081	0.000	UST	0.013	0.013	0.007	0.003	0.208	0.591
VE	0.477	0.410	0.101	0.570	0.124	0.105	VEDIZON COMMS	0.015	0.010	0.161	0.555	0.200	0.051
	0.141	0.013	0.094	0.377	0.134	0.195	VERIZON COMMS.	0.000	0.000	0.101	0.077	0.020	0.966
VIACOM 'B'	0.020	0.295	0.477	0.148	0.060	0.711	VULCAN MATERIALS	0.013	0.926	0.268	0.262	0.161	0.040
WACHOVIA	0.000	0.007	0.087	0.128	0.034	0.081	WALGREEN	0.000	0.020	0.208	0.651	0.087	0.362
WAL MART STORES	0.000	0.000	0.000	0.000	0.013	0.638	WALT DISNEY	0.128	0.060	0.497	0.309	0.530	0.812
WASHINGTON MUTUAL	0.007	0.020	0.027	0.034	0.329	0.047	WASTE MAN.	0.228	0.376	0.000	0.523	0.322	0.034
WELLS FARGO & .CO	0.000	0.195	0.342	0.000	0.000	0.725	WENDY'S INTL.	0.148	0.705	0.456	0.752	0.094	0.141
WEYERHAEUSER	0.376	0.423	0.242	0.430	0.819	0.430	WHIRLPOOL	0.114	0.866	0.134	0.872	0.034	0.946
WILLIAMS COS	0.010	0.124	0.000	0.19	0.019	0.400	WINN DIVIE STOC	0.020	0.000	0.501	0.260	0.770	0.240
WILLIAMS COS.	0.000	0.134	0.000	0.013	0.040	0.900	WINN - DIALE SIRS.	0.020	0.047	0.391	0.309	0.179	0.202
WORTHINGTON INDS.	0.107	0.396	0.678	0.953	0.711	0.550	WRIGLEY WILLIAM JR.	0.000	0.309	0.376	0.322	0.953	0.221
W Y E'TH	0.000	0.000	0.107	0.289	0.765	0.114	XCEL ENERGY	0.060	0.000	0.208	0.383	0.221	0.765
XEROX	0.007	0.007	0.000	0.000	0.000	0.000	XILINX	0.195	0.128	0.208	0.074	0.342	0.919
ZIONS BANCORP	0.081	0.020	0.201	0.705	0.054	0.094	3M	0 000	0.154	0.463	0.141	0.181	0.148

Table 5: Probability Values for S&P 500 Series and 1, 2, 3, 4, 12 and 52 lags (NTHN.TRUST - 3M). (Significant p. values reported in bold typeface)



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