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## Abstract

This paper investigates GLS detrending procedures for unit root tests against nonlinear stationary alternative hypotheses where deterministic components are assumed present in the series under investigation. It is found that the proposed procedures have considerable power gains in a majority of cases against both existing nonlinear unit root tests and standard unit root tests.

JEL Classification: C12, C22, F31.

Key Words: Detrending, Nonlinear Unit Root Tests, Nonlinearity, STAR Models, SETAR Models.

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# 1 Introduction

There is a growing dissatisfaction with the standard linear ARMA framework used to test for unit roots in time series, and increasingly alternative frameworks within which to test for unit roots are considered. For example, one alternative focuses on the use of panel data and its role in increasing the power of standard unit root tests. A good example is Abuaf and Jorion (1990), who use a panel data test to reject the joint hypothesis of unit roots in each of a group of real exchange rates against an alternative that they are all stationary. Another approach is to use an alternative form of stationarity to simple ARMA models. These include fractional integration and nonlinear transition dynamics (see e.g. Pesaran and Potter (1997)).

In this paper we extend recent work on testing for unit roots against particular nonlinear alternatives by Kapetanios, Snell, and Shin (2002) and Kapetanios and Shin (2002). We focus on a particular aspect of their analysis which is the detrending procedure they use. It is well known in the literature on linear unit root testing that inefficient detrending can reduce the power of the tests significantly and therefore render the tests less useful. We investigate the ability of efficient detrending procedures used in linear unit root tests to improve the power performance on nonlinear unit root tests.

The structure of the paper is as follows: Section 2 provides a description of the theoretical framework of nonlinear unit root testing. Section 3 discusses modifications to the existing procedures needed for the implementation of efficient detrending. Section 4 presents Monte Carlo evidence on the performance of the new procedures for a variety of experimental setups. Section 5 discusses an empirical application to real exchange rates. Finally,

Section 6 concludes.

## 2 Theoretical Framework

Recently, Kapetanios, Snell, and Shin (2002) and Kapetanios and Shin (2002) suggested two testing procedures to distinguish between nonstationary unit root processes and persistent but stationary nonlinear processes. The first considers the possibility that the alternative model follows a smooth transition autoregressive model and the second consider the alternative hypothesis that the nonlinear model belongs to the class of threshold models.

Both testing procedures have been found to be more powerful than standard unit root tests when the true data generation processes was a persistent but stationary nonlinear process. However, for the cases where demeaning or detrending of the data was carried out prior to the test, a significant loss of power was observed. As a result we investigate whether procedures which have been found useful in increasing the power of tests based on data whose deterministic components had been removed in the linear framework, may be useful in the nonlinear case.

Before proceeding we give a brief account of the alternative nonlinear models we consider.

### 2.1 STAR Models

We follow the nonlinear STAR framework considered by Kapetanios, Snell, and Shin (2002). More specifically, the model they consider is

$$\Delta y_t = \gamma y_{t-1} \{1 - \exp(-\theta y_{t-1}^2)\} + \epsilon_t, \quad (1)$$

where  $-2 < \gamma < 0$  and  $\epsilon_t$  is an iid error with zero mean and constant variance  $\sigma^2$ . To motivate the detrending procedure for the STAR model we consider the following modification of the above STAR model

$$y_t = \psi + \xi t + x_t, \quad t = 1, \dots, T, \quad (2)$$

$$\Delta x_t = \gamma x_{t-1} \{1 - \exp(-\theta x_{t-1}^2)\} + \epsilon_t, \quad (3)$$

We are now interested in testing the null hypothesis  $\theta = 0$  against the alternative hypothesis  $\theta > 0$ . Under the null,  $y_t$  follows a linear unit root process, whereas it is a nonlinear stationary STAR process under the alternative. For more details on the unit root testing procedure in the STAR framework see Kapetanios, Snell, and Shin (2002).

## 2.2 SETAR models

Here we follow the framework of Kapetanios and Shin (2002) More specifically, they consider the model,

$$\Delta y_t = \beta_1 y_{t-1} 1_{\{y_{t-1} \leq r_1\}} + \beta_2 y_{t-1} 1_{\{y_{t-1} > r_2\}} + \epsilon_t, \quad (4)$$

where  $-2 < \beta_1 < 0$ ,  $-2 < \beta_2 < 0$  and  $\epsilon_t$  is an iid error with zero mean and constant variance  $\sigma^2$  and  $r_1 < r_2$ . Again we modify their model to get the following model

$$y_t = \psi + \xi t + x_t, \quad t = 1, \dots, T, \quad (5)$$

$$\Delta x_t = \beta_1 x_{t-1} 1_{\{x_{t-1} \leq r_1\}} + \beta_2 x_{t-1} 1_{\{x_{t-1} > r_2\}} + \epsilon_t, \quad (6)$$

Here, the null hypothesis is of the form  $\beta_1 = \beta_2 = 0$  against the alternative hypothesis  $\beta_1 < 0$  or  $\beta_2 < 0$ . Under the null  $y_t$  follows a linear unit root process, whereas it is nonlinear stationary SETAR process under the alternative.

## 3 Improved Detrending

### 3.1 The Linear Case

Elliott, Rothenberg, and Stock (1996) investigated the issue of efficient detrending for linear unit root tests following previous work by Dufour and King (1991), King (1988) and King (1980). Their work was motivated by asymptotic local power considerations. More specifically, they derived point optimal unit root tests for trended data for specific local alternative hypotheses. The model they consider is of the form

$$y_t = \beta z_t + x_t \quad (7)$$

$$x_t = \alpha x_{t-1} + \epsilon_t \quad (8)$$

where  $z_t$  is a deterministic component and  $\epsilon_t$  is an i.i.d. process with finite zero-frequency spectral density and variance  $\sigma^2$ . Since the focus is on local alternative hypotheses, the autoregressive parameter is re-expressed as  $\alpha = 1 - c/T$  anticipating the necessary rate of convergence for local power analysis. They test the null hypothesis,  $H_0 : \alpha = 1$ , against local alternative hypotheses of the form  $H_{\bar{c}} : \alpha = \bar{\alpha} \equiv 1 - \bar{c}/T$ , where  $\bar{c}$  is a constant and  $T$  is the number of observations. Note the distinction between  $c$  which is the local alternative parameter and  $\bar{c}$  which is the value under which tests are constructed. Point optimal likelihood ratio tests of the form

$$L_T^* = \min_{\beta} L(\bar{\alpha}, \beta) - \min_{\beta} L(1, \beta)$$

for the null hypothesis against the local alternative, may be constructed where  $L(\bar{\alpha}, \beta) = (y_{\bar{\alpha}} - z_{\bar{\alpha}}\beta)' \Sigma^{-1} (y_{\bar{\alpha}} - z_{\bar{\alpha}}\beta)$  is the loglikelihood of the local alternative hypothesis,  $y_{\bar{\alpha}} = (y_1, y_2 - \bar{\alpha}y_1, \dots, y_T - \bar{\alpha}y_{T-1})$ ,  $z_{\bar{\alpha}} = (z_1, z_2 - \bar{\alpha}z_1, \dots, z_T - \bar{\alpha}z_{T-1})$  and  $\Sigma$  is the covariance matrix of  $\epsilon_1, \dots, \epsilon_T$ . The term  $(y_{\bar{\alpha}} - z_{\bar{\alpha}}\beta)' \Sigma^{-1} (y_{\bar{\alpha}} - z_{\bar{\alpha}}\beta)$  may be viewed as a weighted sum of squared

residuals coming from a constrained GLS regression where the autoregressive coefficient  $\bar{\alpha}$  has been imposed.

Elliott, Rothenberg, and Stock (1996) suggest modifying the standard Dickey-Fuller test by applying it to residual data  $y_t^d$  whose deterministic component has been removed. More specifically,  $y_t^d = y_t - \hat{\beta}z_t$  where  $\hat{\beta}$  is obtained from a regression of  $y_{\bar{\alpha}}$  on  $z_{\bar{\alpha}}$ . No rigorous theoretical account of whether this test is close to the point optimal test is given. However, simulation analysis indicates that the test performs much better than standard unit root tests. Note that this test is designed with a particular local alternative in mind represented by the choice of  $\bar{\alpha}$  used for detrending. Elliott, Rothenberg, and Stock (1996) suggest that choosing  $\bar{\alpha}$  such that the asymptotic local power of the test, when faced with the local alternative for which it was designed, is equal to 0.5. This is the test they settle on for their finite sample analysis.

Clearly the motivation for the modified DF test lies in the fact that the detrending carried out takes specific account of the local alternative against which we wish the test to be powerful. We suggest that the same detrending approach is taken for the nonlinear unit root tests. In particular we suggest that the nonlinear tests be applied to the residual series  $y_t^d$  obtained following GLS detrending. Clearly this is not optimal since the nonlinear nature of the alternative hypothesis is not taken into account. Take for example the STAR model in (2)-(3). A local alternative version of it could be constructed by specifying  $\theta = \bar{c}/T$ . A first problem with this is that the  $T^{-1}$  rate involved in the construction of the local alternative is not necessarily appropriate since  $\theta$  enters in the exponential rather than linearly (see also Park and Phillips (2001)). Further, it is not clear how to devise a generalised quasi-differencing

scheme, that does not involve knowledge of  $x_t$  which would enable the construction of series  $\tilde{y}_{\bar{\theta}}$  and  $\tilde{z}_{\bar{\theta}}$  comparable to  $y_{\bar{\alpha}}$  and  $z_{\bar{\alpha}}$  for the linear case such that  $\tilde{y}_{\bar{\theta}} - \beta\tilde{z}_{\bar{\theta}} = \epsilon_t$ . Another problem is that any detrending taking account of the nonlinear structure of the alternative would also involve taking a stance on the values of the nuisance (in this case) parameters such as  $\gamma$  for STAR models or  $r_1$  and  $r_2$  in the case of the SETAR alternative. As a result, the linear local alternative detrending may provide a useful approximation to the nonlinear local alternative.

### 3.2 Implementation and Asymptotic Theory

In this subsection we give more details on the construction of the tests and discuss their asymptotic properties under the null hypothesis. We concentrate on two cases of deterministic components in the rest of the paper following the literature on both linear and nonlinear unit root tests. These are the cases  $z_t = 1$  and  $z_t = (1, t)'$ , i.e. demeaning and detrending the series. GLS detrending of the original series, as suggested by Elliott, Rothenberg, and Stock (1996), using a given  $\bar{c}$  results in the residual series  $y_t^\mu$  and  $y_t^\tau$  where the superscripts  $\mu$  and  $\tau$  denote demeaning and detrending respectively. Then, Elliott, Rothenberg, and Stock (1996) show that under the local alternative hypothesis,  $T^{-1/2}y_{[rT]}^\mu \Rightarrow \sigma W_c(r)$ ,  $r \in [0, 1]$  and  $T^{-1/2}y_{[rT]}^\tau \Rightarrow \sigma V_c(r, \bar{c})$ , where  $W_c(r) = \int e^{c(r-s)} dW(s)$ ,  $V_c(r, \bar{c}) = W_c(r) - r [\lambda W_c(1) + 3(1 - \lambda) \int s W_c(s) ds]$ ,  $\lambda = (1 - \bar{c}) / (1 - \bar{c} + \bar{c}^2 / 3)$  and  $W(r)$  is a standard Brownian motion. Define  $V_{\bar{c}}(r) = V_0(r, \bar{c})$ . Note that for the case of demeaning the limiting representation of the residual series is the same as the limiting representation of a random walk with no drift under the null ( $c = 0$ ). The value of the assumed local alternative  $\bar{c}$  does not enter that representation and therefore the critical values of any test based on the resid-



ual series will not depend on  $\bar{c}$ . This is not the case for the detrended series where the critical values will depend on  $\bar{c}$ .

### 3.3 STAR model

For the STAR model in (2)-(3) we now briefly describe the unit root test. Once the deterministic components have been removed as described above the test for the STAR case is constructed as follows. The null hypothesis is  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta > 0$ . Testing this null hypothesis directly is not feasible, since  $\gamma$  is not identified under the null. (See Davies, 1987). To overcome this problem Kapetanios, Snell, and Shin (2002) follow Luukkonen, Saikkonen and Teräsvirta (1988), and derive a t-type test statistic. The following auxiliary regression

$$\Delta y_t^d = \delta y_{t-1}^d + error. \quad (9)$$

obtained through a Taylor expansion, is used, where the significance of  $\hat{\delta}$  is tested using a  $t$ -test. Kapetanios, Snell, and Shin (2002) show that in the case of no deterministic components the asymptotic distribution of the test statistic, denoted by  $NLDF$  is given by

$$NLDF \Rightarrow \frac{\left\{ \frac{1}{4} W(1)^4 - \frac{3}{2} \int_0^1 W(r)^2 dr \right\}}{\sqrt{\int_0^1 W(r)^6 dr}} \quad (10)$$

As the GLS demeaned series has the same asymptotic representation under the null as a random walk with no drift the asymptotic distribution of the test using GLS demeaning and denoted by  $NLGLS^\mu$ , is given by (10). For the detrended case the arguments of Kapetanios, Snell, and Shin (2002) can be straightforwardly modified to show that the asymptotic distribution of

the test using GLS detrending, and denoted by  $NLGLS^\tau$  is given by

$$NLGLS^\tau \Rightarrow \frac{\left\{ \frac{1}{4} V_{\bar{c}}(1)^4 - \frac{3}{2} \int_0^1 V_{\bar{c}}(r)^2 dr \right\}}{\sqrt{\int V_{\bar{c}}(r)^6 dr}} \quad (11)$$

The 95% critical value for  $NLGLS^\tau$  for  $\bar{c} = -17.5$  which gives asymptotic local power<sup>1</sup> of 0.5, is -2.93. For the  $NLDF^\mu$  test the relevant value of  $\bar{c}$  is -9.

### 3.4 SETAR Model

Following the detrending, the model for testing for nonlinear SETAR stationarity can be compactly written as

$$\Delta y_t^d = \beta_1 y_{t-1}^d 1_{\{y_{t-1}^d \leq r_1\}} + \beta_2 y_{t-1}^d 1_{\{y_{t-1}^d > r_2\}} + u_t, \quad (12)$$

where  $y_{t-1}^d 1_{\{y_{t-1}^d \leq r_1\}}$  and  $y_{t-1}^d 1_{\{y_{t-1}^d > r_2\}}$  are orthogonal to each other by construction. Kapetanios and Shin (2002) consider the (joint) null hypothesis of unit root as

$$H_0 : \beta_1 = \beta_2 = 0, \quad (13)$$

against the alternative hypothesis of threshold stationarity. Writing (12) in matrix notation gives

$$\Delta \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{u}, \quad (14)$$

where  $\boldsymbol{\beta} = (\beta_1, \beta_2)'$ , and

$$\Delta \mathbf{y} = \begin{pmatrix} \Delta y_1^d \\ \Delta y_2^d \\ \vdots \\ \Delta y_T^d \end{pmatrix}; \quad \mathbf{X} = \begin{pmatrix} y_0^d 1_{\{y_0^d \leq r_1\}} & y_0^d 1_{\{y_0^d > r_2\}} \\ y_1^d 1_{\{y_1^d \leq r_1\}} & y_1^d 1_{\{y_1^d > r_2\}} \\ \vdots & \vdots \\ y_{T-1}^d 1_{\{y_{T-1}^d \leq r_1\}} & y_{T-1}^d 1_{\{y_{T-1}^d > r_2\}} \end{pmatrix}; \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_T \end{pmatrix}.$$

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<sup>1</sup>The local power was obtained via simulations using 5000 replications and processes of 1000 observations to discretely approximate the functionals of Brownian motions

Then, the joint null hypothesis of linear unit root against the nonlinear threshold stationarity can be tested using the Wald statistic given by

$$\mathcal{W}_{(r_1, r_2)} = \hat{\beta}' \left[ \text{Var} \left( \hat{\beta} \right) \right]^{-1} \hat{\beta} = \frac{\hat{\beta}' (\mathbf{X}' \mathbf{X}) \hat{\beta}}{\hat{\sigma}_u^2}, \quad (15)$$

where  $\hat{\beta}$  is the OLS estimator of  $\beta$ ,  $\hat{\sigma}_u^2 \equiv \frac{1}{T-2} \sum_{t=1}^T \hat{u}_t^2$ , and  $\hat{u}_t$  are the residuals obtained from (12).

The test suffers from the Davies (1987) problem since unknown threshold parameters are not identified under the null. Most solutions to this problem involve some sort of integrating out unidentified parameters from the test statistics. This is usually achieved by calculating test statistics for a grid of possible values of threshold parameters,  $r_1$  and  $r_2$ , and then constructing the summary statistics. For stationary TAR models this problem has been studied in Tong (1990) and Hansen (1996). Following Andrews and Ploberger (1994), Kapetanios and Shin (2002) consider the three most commonly used statistics such as the supremum, the average and the exponential average of the Wald statistic defined respectively by

$$\mathcal{W}_{(r_1, r_2)}^{\text{sup}} = \sup_{i \in \# \Gamma} \mathcal{W}_{(r_1, r_2)}^{(i)}, \quad \mathcal{W}_{(r_1, r_2)}^{\text{avg}} = \frac{1}{\# \Gamma} \sum_{i=1}^{\# \Gamma} \mathcal{W}_{(r_1, r_2)}^{(i)}, \quad \mathcal{W}_{(r_1, r_2)}^{\text{exp}} = \frac{1}{\# \Gamma} \sum_{i=1}^{\# \Gamma} \exp \left( \frac{\mathcal{W}_{(r_1, r_2)}^{(i)}}{2} \right), \quad (16)$$

where  $\mathcal{W}_{(r_1, r_2)}^{(i)}$  is the Wald statistic obtained from the  $i$ -th point of the nuisance parameter grid,  $\Gamma$  and  $\# \Gamma$  is the number of elements of  $\Gamma$ . For more details on the the selection of the grid of threshold parameters see Kapetanios and Shin (2002).

The asymptotic distributions of  $\mathcal{W}_{(r_1, r_2)}^{\text{sup}}$  and  $\mathcal{W}_{(r_1, r_2)}^{\text{avg}}$  are the same and are given by the distribution of

$$\mathcal{W} \equiv \frac{\left\{ \int_0^1 1_{\{W(s) \leq 0\}} W(s) dW(s) \right\}^2}{\int_0^1 1_{\{W(s) \leq 0\}} W(s)^2 ds} + \frac{\left\{ \int_0^1 1_{\{W(s) > 0\}} W(s) dW(s) \right\}^2}{\int_0^1 1_{\{W(s) > 0\}} W(s)^2 ds},$$

when no deterministic components have been considered. The asymptotic distribution of  $\mathcal{W}_{(r_1, r_2)}^{exp}$  is given by the distribution of  $exp(\mathcal{W}/2)$ . As in the STAR case GLS demeaning prior to the application of the test does not change the asymptotic distributions given above. Kapetanios and Shin (2002) find that the supremum tests overrejects very significantly for reasonable sample sizes and so that test will not be considered in this paper. We denote the GLS demeaned tests by  $\mathcal{W}_{(r_1, r_2)}^{avg, GLS, \mu}$  and  $\mathcal{W}_{(r_1, r_2)}^{exp, GLS, \mu}$  and the GLS detrended tests by  $\mathcal{W}_{(r_1, r_2)}^{avg, GLS, \tau}$  and  $\mathcal{W}_{(r_1, r_2)}^{exp, GLS, \tau}$ . For the detrended case by tracing the steps in appendix A of Kapetanios and Shin (2002) we get that the asymptotic distribution of  $\mathcal{W}_{(r_1, r_2)}^{avg, GLS, \tau}$  is the same as that of

$$\mathcal{W}^\tau \equiv \frac{\left\{ \int_0^1 1_{\{V_{\bar{c}}(s) \leq 0\}} V_{\bar{c}}(s) dV_{\bar{c}}(s) \right\}^2}{\int_0^1 1_{\{V_{\bar{c}}(s) \leq 0\}} V_{\bar{c}}(s)^2 ds} + \frac{\left\{ \int_0^1 1_{\{V_{\bar{c}}(s) > 0\}} V_{\bar{c}}(s) dV_{\bar{c}}(s) \right\}^2}{\int_0^1 1_{\{V_{\bar{c}}(s) > 0\}} V_{\bar{c}}(s)^2 ds},$$

and that of  $\mathcal{W}_{(r_1, r_2)}^{exp, GLS, \tau}$  is given by the distribution of  $exp(\mathcal{W}^\tau/2)$ . The 95% critical value of  $\mathcal{W}_{(r_1, r_2)}^{avg, GLS, \tau}$  for  $\bar{c} = -13$ , which gives asymptotic local power equal to 0.5, is 8.81. The 95% critical value of  $\mathcal{W}_{(r_1, r_2)}^{exp, GLS, \tau}$  is 81.86.

## 4 Monte Carlo

### 4.1 STAR Model

In the first set of experiments we focus mainly on the size of the tests and thus construct the null model by

$$y_t = y_{t-1} + \varepsilon_t, \tag{17}$$

where  $\varepsilon_t$  is drawn from the standard normal distribution. This is experiment A1.

Secondly, in order to evaluate the power of alternative tests against globally stationary processes, we generate the DGP as follows:

$$\Delta y_t = \gamma y_{t-1} [1 - \exp(-\theta y_{t-1}^2)] + \varepsilon_t, \quad (18)$$

where  $\varepsilon_t \sim N(0, 1)$ . In particular, we choose a broad range of parameter values for  $\gamma = \{-1.5, -1, -0.5, -0.1\}$  and  $\theta = \{0.01, 0.05\}$  for a general power comparison. For  $\theta = 0.01$  we have experiments B1-B4, and for  $\theta = 0.05$  we have experiments B5-B8.

For each of experiments we have computed the rejection probability of the null hypothesis. The nominal size of each of the tests is set at 0.05, the number of replications at 1000 and the sample size is considered for  $T = 50, 100, 200$ . In the comparisons we include the nonlinear STAR test proposed by Chortareas, Kapetanios, and Shin (2002) which uses the Schmidt and Phillips (1992) detrending. This test is denoted by *NLSP*. Its linear version is denoted by *SP*. We also include the linear DF test and the linear GLS DF test (denoted by GLS) suggested by Elliott, Rothenberg, and Stock (1996). Results are presented in Table 1 and 2.

It is clear that the nonlinear GLS detrending procedure improves upon the performance of the existing tests in a majority of cases and in those cases where no improvement is achieved the loss in power, compared to the best performing test, is minimal.

## 4.2 SETAR Models

In the first set of experiments we examine the size performance of the tests. Experiment A1 considers the random walk process:

$$y_t = y_{t-1} + u_t, \quad (19)$$

[11]

where the error term  $u_t$  is drawn from the independent standard normal distribution.

The next set of experiments examines the power performance of the tests, where the data is generated by

$$y_t = \begin{cases} \phi_1 y_{t-1} + u_t & \text{if } y_{t-1} \leq r_1 \\ \phi_0 y_{t-1} + u_t & \text{if } r_1 < y_{t-1} \leq r_2 \\ \phi_2 y_{t-1} + u_t & \text{if } y_{t-1} > r_2 \end{cases}, \quad t = 1, 2, \dots, T, \quad (20)$$

where  $u_t \sim N(0, 1)$ . Experiments B and C set  $\phi_0 = 1$ . Experiments B1-B5 consider the symmetric adjustment with  $\phi_1 = \phi_2 = 0.9$ , whereas we examine asymmetric adjustments in Experiment C1-C5 with  $\phi_1 = 0.85$  and  $\phi_2 = 0.95$ . Experiments D and E are as experiments B and C but with  $\phi_0 = 1.3$ , i.e. they assume an explosive corridor regime. Within each set of power experiments, we select five different sets of threshold parameter values from 0.90 to 3.90 and -0.90 to -3.90, at steps of 0.75 and -0.75, respectively.<sup>2</sup> For each sample the grid of either lower or upper threshold parameter comprises of eight equally spaced points between the 10% quantile (lower threshold) or the 90% quantile (upper threshold) of the sample and the mean of the sample.

All experiments are carried out using the following statistics: the two version of summary Wald statistics,  $\mathcal{W}_{(r_1, r_2)}^{avg}$  and  $\mathcal{W}_{(r_1, r_2)}^{exp}$ , defined by (16) proposed by Kapetanios and Shin (2002), their GLS detrended counterparts, the DF and the linear GLS detrending test suggested by Elliott, Rothenberg, and Stock (1996). For all power experiments, 200 initial observations are discarded to minimise the effect of initial conditions. All experiments are based on 1,000 replications, and samples of 100 and 200 are considered. Empirical size and power of the tests are evaluated at the 5% nominal level. As the GLS tests overreject we correct for that using empirical critical values

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<sup>2</sup>We also find via simulation that the processes have spent at least 10% of the time in each of the outer regimes even for the largest threshold parameter values considered.

obtained from the size experiment, A1. We also correct the nonlinear tests by Kapetanios and Shin (2002) to enable valid comparisons. Results are presented in Table 3 and 4. We reach the same conclusion as in the case of STAR models concerning the performance of the nonlinear detrending procedures on the existing tests for SETAR models. In particular the GLS nonlinear tests always outperform the existing nonlinear tests. They also outperform in the majority of cases the linear unit root tests. Although, there are some cases where the linear tests perform better, these are not the majority and in the cases where the nonlinear tests perform better the difference in performance is much more pronounced.

## 5 Stationarity of real exchange rates

In this section we apply the new tests to investigate the of stationarity properties of the Yen and Deutch Mark real exchange rates. Our choice for one of the data sets reflects previous work in this area by Chortareas, Kapetanios, and Shin (2002) on Yen real exchange rates using STAR based nonlinear unit root tests. That paper used nonlinear unit root tests to help explain the inability of standard unit root tests to reject the null hypothesis of non-stationarity in accordance with economic theory and the purchasing power parity hypothesis. We apply the STAR based tests on Yen real exchange rates and the SETAR based tests on the DM real exchange rates.

We construct bilateral real exchange rates against the  $i$ -th currency at time  $t$  ( $q_{i,t}$ ) as  $q_{i,t} = s_{i,t} + p_{J,t} - p_{i,t}^*$ , where  $s_{i,t}$  is the corresponding nominal exchange rate ( $i$ -th currency per numeraire currency),  $p_{J,t}$  the price level in the home country, and  $p_{i,t}^*$  the price level of the  $i$ -th country. Thus, a rise in  $q_{i,t}$  implies a real appreciation against the  $i$ -th currency. The price levels are

consumer price indices for Yen and wholesale price indices for the DM. All variables are in logs. All data are from the International Monetary Fund's *International Financial Statistics* in CD-ROM. The data are not seasonally adjusted. All data are quarterly, spanning from 1960Q1 to 2000Q4 and the bilateral nominal exchange rates against the currencies other than the US dollar are cross-rates computed using the US dollar rates. We consider a very large sample of countries in an attempt to make the empirical analysis more comprehensive. Results are presented for the STAR based tests in Tables 5-6 and for the SETAR based tests in Tables 7-8. Tables 5 and 7 present results for tests with no augmentations to take into account possible serial correlation, whereas Tables 6 and 8 present results for tests with 4 lags to take into account serial correlation in the series. All tests assume the presence of a trend under the alternative hypothesis. The empirical critical values obtained from the Monte Carlo experiments in the previous section are used for the empirical analysis to minimise the degree of overrejection under the null hypothesis for the GLS tests.

The results make interesting reading. For the Yen real exchange rates we see that the nonlinear GLS test based on the ESTAR model rejects the null hypothesis more often than any other test both for augmented and non-augmented test regression equations. When we carry out augmentation the test rejects for 26 out of 39 countries considered. The next best performing test is the *NLDF* test with 22 rejections. When there is no augmentation the nonlinear GLS test rejects less often but still produces twice the number of rejections compared to any other test examined.

Moving on to the DM real exchange rate and the SETAR model based tests we see that the nonlinear GLS tests rejects for 24 out of the 31 countries



considered. This is double the number of rejections of any other test. We note that we have investigated the Yen real exchange dataset using the SETAR based tests and the DM real exchange rate dataset using the STAR based tests and we could not find substantially different performance between the linear and nonlinear unit root tests. This may be taken to signify the possible presence of particular forms of nonlinearity which are different between the two datasets and which can perhaps be picked up more accurately by one or the other of the two classes of tests. In particular sudden step changes in the dynamic evolution of the real exchange rate processes may be better picked up by the SETAR model based tests whereas smoother adjustments may be more amenable to investigation through the STAR model based tests. Further this indicates that the tests may be used complementarily in empirical analysis.

## 6 Conclusion

In this paper we extend recent work on testing for unit roots against particular nonlinear alternatives by Kapetanios, Snell, and Shin (2002) and Kapetanios and Shin (2002). We focus on a particular aspect of their analysis which is the detrending procedure they use. It is well known in the literature on linear unit root testing that inefficient detrending can reduce the power of the tests significantly and therefore render the tests less useful. We investigate the ability of efficient detrending procedures used in linear unit root tests to improve the power performance on nonlinear unit root tests.

We find that the GLS detrending procedure can indeed improve the performance of the existing testing procedures in a majority of cases. This conclusion is supported both by extensive Monte Carlo experimentation and a comprehensive empirical analysis of Yen and DM real exchange rates.

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Table 1: Demeaned Tests for STAR Models ( $z_t = 1$ ).

Obs	Exp	$DF^\mu$	$NLDF^\mu$	$SP^\mu$	$NLSP^\mu$	$GLS^\mu$	$NLGLS^\mu$
50	A1	0.033	0.061	0.042	0.061	0.121	0.118
100	A1	0.044	0.066	0.049	0.066	0.095	0.101
200	A1	0.038	0.064	0.057	0.064	0.094	0.089
50	B1	0.193	0.264	0.193	0.305	0.283	0.387
	B2	0.170	0.170	0.170	0.197	0.195	0.281
	B3	0.112	0.110	0.112	0.119	0.130	0.159
	B4	0.076	0.076	0.076	0.055	0.074	0.094
	B5	0.788	0.795	0.788	0.839	0.871	0.853
	B6	0.524	0.603	0.524	0.661	0.672	0.715
	B7	0.253	0.277	0.253	0.332	0.329	0.443
	B8	0.105	0.090	0.105	0.082	0.126	0.118
100	B1	0.509	0.684	0.509	0.807	0.635	0.709
	B2	0.336	0.466	0.336	0.579	0.461	0.543
	B3	0.190	0.238	0.190	0.274	0.265	0.303
	B4	0.105	0.106	0.105	0.090	0.114	0.115
	B5	1.000	1.000	1.000	1.000	0.980	0.994
	B6	0.993	0.987	0.993	0.992	0.935	0.957
	B7	0.729	0.751	0.729	0.825	0.775	0.757
	B8	0.160	0.157	0.160	0.145	0.177	0.176
200	B1	1.000	0.988	1.000	0.999	0.942	0.974
	B2	0.967	0.952	0.967	0.988	0.885	0.933
	B3	0.618	0.720	0.618	0.858	0.724	0.749
	B4	0.174	0.182	0.174	0.195	0.243	0.261
	B5	1.000	1.000	1.000	1.000	1.000	1.000
	B6	1.000	1.000	1.000	1.000	0.994	0.999
	B7	1.000	0.997	1.000	1.000	0.967	0.994
	B8	0.394	0.429	0.394	0.483	0.557	0.532

Table 2: Detrended Tests for STAR Models ( $z_t = (1, t)'$ ).

Obs	Exp	$DF^\tau$	$NLDF^\tau$	$SP^\tau$	$NLSP^\tau$	$GLS^\tau$	$NLGLS^\tau$
50	A1	0.048	0.066	0.047	0.059	0.092	0.097
100	A1	0.053	0.064	0.049	0.051	0.076	0.074
200	A1	0.060	0.048	0.061	0.054	0.067	0.052
50	B1	0.154	0.169	0.177	0.203	0.151	0.208
	B2	0.119	0.134	0.132	0.158	0.115	0.151
	B3	0.082	0.088	0.097	0.089	0.073	0.093
	B4	0.073	0.062	0.066	0.066	0.057	0.069
	B5	0.555	0.654	0.619	0.608	0.600	0.703
	B6	0.318	0.434	0.399	0.418	0.349	0.481
	B7	0.155	0.173	0.183	0.190	0.144	0.200
	B8	0.098	0.073	0.106	0.082	0.087	0.079
100	B1	0.323	0.461	0.370	0.459	0.354	0.552
	B2	0.237	0.286	0.280	0.310	0.252	0.348
	B3	0.144	0.149	0.152	0.192	0.140	0.205
	B4	0.083	0.082	0.094	0.091	0.071	0.088
	B5	0.999	0.987	0.975	0.950	0.991	0.980
	B6	0.929	0.913	0.917	0.834	0.936	0.917
	B7	0.432	0.530	0.545	0.515	0.525	0.614
	B8	0.108	0.101	0.130	0.121	0.110	0.120
200	B1	0.955	0.959	0.936	0.880	0.960	0.922
	B2	0.783	0.843	0.817	0.764	0.857	0.833
	B3	0.349	0.461	0.451	0.493	0.505	0.525
	B4	0.125	0.125	0.127	0.141	0.143	0.132
	B5	1.000	1.000	1.000	0.999	1.000	1.000
	B6	1.000	1.000	0.999	0.994	1.000	0.998
	B7	0.997	0.978	0.973	0.910	0.992	0.957
	B8	0.228	0.237	0.287	0.281	0.316	0.289

Table 3: Demeaned Tests for SETAR Models ( $z_t = 1$ ).

Obs	Exp	$W^{ave}$	$W^{exp}$	$DF$	$W_{GLS}^{ave}$	$W_{GLS}^{exp}$	$GLS$
100	A1	0.048	0.065	0.046	0.079	0.107	0.064
200	A1	0.040	0.060	0.041	0.045	0.079	0.051
100	B1	0.250	0.223	0.318	0.315	0.264	0.455
	B2	0.246	0.216	0.280	0.317	0.281	0.430
	B3	0.206	0.200	0.217	0.275	0.250	0.323
	B4	0.171	0.183	0.155	0.230	0.225	0.271
	B5	0.159	0.170	0.144	0.195	0.204	0.183
100	C1	0.219	0.204	0.271	0.260	0.236	0.361
	C2	0.209	0.196	0.225	0.256	0.241	0.296
	C3	0.189	0.185	0.175	0.230	0.224	0.274
	C4	0.179	0.204	0.136	0.222	0.230	0.211
	C5	0.136	0.150	0.106	0.165	0.174	0.147
100	D1	0.267	0.249	0.298	0.341	0.301	0.473
	D2	0.257	0.245	0.205	0.326	0.309	0.312
	D3	0.300	0.338	0.207	0.359	0.390	0.215
	D4	0.486	0.576	0.455	0.553	0.617	0.419
	D5	0.749	0.826	0.723	0.765	0.828	0.701
100	E1	0.208	0.198	0.233	0.265	0.241	0.344
	E2	0.231	0.237	0.185	0.285	0.282	0.256
	E3	0.318	0.370	0.244	0.358	0.400	0.275
	E4	0.526	0.587	0.485	0.569	0.615	0.499
	E5	0.759	0.810	0.713	0.779	0.827	0.715
200	B1	0.748	0.738	0.886	0.817	0.789	0.799
	B2	0.750	0.743	0.839	0.814	0.793	0.778
	B3	0.722	0.719	0.747	0.826	0.802	0.723
	B4	0.620	0.650	0.506	0.768	0.742	0.583
	B5	0.489	0.552	0.355	0.645	0.653	0.507
200	C1	0.604	0.586	0.663	0.703	0.670	0.676
	C2	0.564	0.569	0.619	0.682	0.650	0.639
	C3	0.529	0.548	0.546	0.634	0.622	0.586
	C4	0.512	0.552	0.433	0.635	0.624	0.512
	C5	0.355	0.419	0.279	0.512	0.524	0.386
200	D1	0.748	0.748	0.847	0.818	0.804	0.796
	D2	0.770	0.790	0.712	0.846	0.847	0.678
	D3	0.799	0.860	0.413	0.881	0.894	0.479
	D4	0.892	0.953	0.362	0.949	0.972	0.397
	D5	0.958	0.992	0.607	0.986	0.989	0.587
200	E1	0.631	0.632	0.681	0.728	0.698	0.652
	E2	0.596	0.639	0.530	0.678	0.710	0.613
	E3	0.576	0.696	0.341	0.669	0.743	0.414
	E4	0.730	0.834	0.480	0.797	0.853	0.467
	E5	0.889	0.949	0.701	0.924	0.952	0.657

Table 4: Detrended Tests for SETAR Models ( $z_t = (1, t)'$ ).

Obs	Exp	$\mathcal{W}^{ave}$	$\mathcal{W}^{exp}$	$DF$	$\mathcal{W}_{GLS}^{ave}$	$\mathcal{W}_{GLS}^{exp}$	$GLS$
100	A1	0.044	0.068	0.045	0.074	0.099	0.066
200	A1	0.047	0.066	0.056	0.063	0.083	0.052
100	B1	0.155	0.165	0.201	0.195	0.195	0.209
	B2	0.145	0.156	0.172	0.218	0.223	0.199
	B3	0.097	0.107	0.120	0.143	0.146	0.125
	B4	0.103	0.112	0.119	0.160	0.172	0.128
	B5	0.088	0.106	0.101	0.123	0.138	0.106
100	C1	0.130	0.126	0.167	0.183	0.178	0.188
	C2	0.114	0.129	0.140	0.165	0.174	0.155
	C3	0.117	0.124	0.128	0.155	0.173	0.130
	C4	0.105	0.114	0.108	0.155	0.167	0.124
	C5	0.087	0.093	0.084	0.102	0.122	0.090
100	D1	0.158	0.171	0.205	0.251	0.247	0.226
	D2	0.136	0.155	0.163	0.208	0.228	0.171
	D3	0.205	0.231	0.240	0.269	0.290	0.229
	D4	0.377	0.411	0.440	0.441	0.457	0.433
	D5	0.638	0.684	0.715	0.701	0.713	0.715
100	E1	0.110	0.121	0.137	0.176	0.184	0.160
	E2	0.132	0.146	0.129	0.178	0.196	0.138
	E3	0.208	0.230	0.229	0.266	0.296	0.234
	E4	0.345	0.381	0.415	0.446	0.459	0.420
	E5	0.597	0.629	0.633	0.678	0.699	0.645
200	B1	0.524	0.544	0.649	0.641	0.617	0.704
	B2	0.495	0.505	0.565	0.591	0.566	0.618
	B3	0.419	0.448	0.464	0.539	0.527	0.531
	B4	0.302	0.343	0.335	0.402	0.416	0.334
	B5	0.228	0.283	0.231	0.335	0.347	0.271
200	C1	0.353	0.371	0.430	0.453	0.430	0.466
	C2	0.327	0.352	0.384	0.436	0.415	0.402
	C3	0.304	0.320	0.308	0.407	0.395	0.357
	C4	0.292	0.332	0.294	0.396	0.409	0.316
	C5	0.195	0.246	0.193	0.257	0.278	0.193
200	D1	0.504	0.530	0.617	0.612	0.593	0.650
	D2	0.437	0.473	0.438	0.562	0.548	0.503
	D3	0.429	0.498	0.402	0.559	0.580	0.410
	D4	0.593	0.644	0.576	0.720	0.744	0.539
	D5	0.762	0.797	0.747	0.854	0.866	0.750
200	E1	0.404	0.420	0.435	0.474	0.457	0.475
	E2	0.335	0.381	0.347	0.444	0.456	0.393
	E3	0.356	0.438	0.346	0.461	0.497	0.342
	E4	0.575	0.622	0.585	0.642	0.679	0.579
	E5	0.775	0.803	0.779	0.836	0.852	0.754

Table 5: Detrended<sup>a</sup> Tests using STAR Model ( $z_t = (1, t)'$ ).

Country	$DF^\tau$	$SP^\tau$	$GLS^\tau$	$NLDF^\tau$	$NLSP^\tau$	$NLGLS^\tau$
<i>US</i>	-1.666	-1.891	-1.626	-2.618	-2.203	-2.415
<i>Germany</i>	-2.669	-2.488	-2.728	-3.222	-2.065	-3.234*
<i>France</i>	-2.412	-2.409	-2.512	-2.939	-2.364	-3.245*
<i>Italy</i>	-2.015	-2.120	-2.090	-3.230	-2.587	-3.155*
<i>UK</i>	-2.112	-2.166	-2.038	-2.811	-2.287	-2.657
<i>Canada</i>	-2.309	-2.325	-2.098	-2.537	-2.093	-2.280
<i>Australia</i>	-2.634	-2.641	-2.648	-3.793*	-3.901*	-3.809*
<i>Austria</i>	-0.712	-1.222	-1.024	-1.600	-2.156	-2.274
<i>Belgium</i>	-2.379	-2.453	-2.425	-2.813	-1.821	-2.769
<i>CzechRep.</i>	-0.738	-1.186	-1.044	-1.623	-1.715	-1.843
<i>Denmark</i>	-2.710	-2.691	-2.751	-3.159	-2.417	-3.030
<i>Finland</i>	-2.617	-2.626	-2.646	-2.366	-2.006	-2.355
<i>Greece</i>	-2.134	-2.095	-2.053	-2.310	-2.480	-2.312
<i>Hungary</i>	-2.043	-1.693	-1.737	-1.983	-1.738	-1.834
<i>Iceland</i>	-2.008	-2.029	-2.027	-2.483	-2.889	-2.539
<i>Korea</i>	-2.931	-2.550	-2.289	-2.621	-2.214	-2.174
<i>Mexico</i>	-1.839	-2.142	-1.652	-3.911*	-3.541*	-3.978*
<i>Netherlands</i>	-2.622	-2.299	-2.513	-2.815	-2.321	-2.668
<i>NewZealand</i>	-2.515	-2.273	-2.347	-2.593	-2.061	-2.277
<i>Norway</i>	-2.916	-2.951	-2.896*	-3.566*	-3.616*	-3.855*
<i>Poland</i>	-2.426	-2.590	-2.469	-2.892	-1.593	-2.751
<i>Portugal</i>	-1.906	-2.055	-1.985	-2.523	-1.746	-2.375
<i>Spain</i>	-2.651	-2.553	-2.632	-3.237	-3.027*	-3.230*
<i>Sweden</i>	-2.847	-2.891	-2.792	-4.139*	-3.268*	-4.156*
<i>Switzerland</i>	-2.846	-2.504	-2.862	-2.684	-2.824	-2.910
<i>Turkey</i>	-2.878	-2.553	-1.843	-3.107	-3.090*	-3.150*
<i>HongKong</i>	-1.472	-1.448	-1.553	-2.853	-2.062	-2.753
<i>Singapore</i>	-1.944	-1.894	-1.696	-2.105	-1.815	-2.042
<i>Malaysia</i>	-2.373	-2.381	-2.382	-2.359	-2.534	-2.396
<i>Indonesia</i>	-2.338	-2.242	-2.125	-2.906	-1.694	-2.615
<i>Thailand</i>	-2.532	-2.541	-2.551	-2.975	-2.975*	-2.978
<i>Philippines</i>	-3.566*	-2.826	-2.360	-3.938*	-2.459	-3.237*
<i>SriLanka</i>	-0.782	-1.216	-0.966	-1.128	-1.423	-1.234
<i>Argentina</i>	-3.542*	-2.541	-3.136*	-2.448	-2.511	-2.336
<i>Bolivia</i>	-2.501	-1.651	-1.630	-3.201	-1.066	-3.245*
<i>Brazil</i>	-2.246	-1.876	-2.235	-2.510	-2.797	-2.674
<i>Chile</i>	-2.311	-1.940	-1.711	-2.122	-2.468	-2.500
<i>Colombia</i>	-2.037	-2.089	-2.027	-2.654	-1.566	-2.470
<i>Venezuela</i>	-1.703	-2.034	-2.023	-0.454	-3.856*	-1.275
Total No. of Rejections	2	0	2	5	8	11

<sup>a</sup>Starred entries indicate significance at the 5% significance level



Table 6: Augmented Detrended Tests using STAR Model ( $z_t = (1, t)'$ ).

Country	$DF^\tau$	$SP^\tau$	$GLS^\tau$	$NLDF^\tau$	$NLSP^\tau$	$NLGLS^\tau$
<i>US</i>	-2.768	-2.875	-2.674	-3.928*	-3.353*	-3.593*
<i>Germany</i>	-2.960	-2.540	-2.953*	-3.728*	-2.029	-3.613*
<i>France</i>	-3.115	-2.729	-3.123*	-3.947*	-2.737	-4.299*
<i>Italy</i>	-2.853	-2.729	-2.876	-4.606*	-3.461*	-4.402*
<i>UK</i>	-3.050	-3.099*	-2.938*	-3.986*	-3.237*	-3.739*
<i>Canada</i>	-3.748*	-3.766*	-3.435*	-3.848*	-3.318*	-3.457*
<i>Australia</i>	-3.587*	-3.600*	-3.611*	-5.106*	-5.301*	-5.127*
<i>Austria</i>	-1.551	-1.771	-1.805	-2.667	-3.266*	-3.575*
<i>Belgium</i>	-2.802	-2.701	-2.820	-3.311	-1.872	-3.177*
<i>CzechRep.</i>	-0.585	-1.648	-1.495	-2.636	-2.346	-2.752
<i>Denmark</i>	-3.397	-3.268*	-3.420*	-4.243*	-2.945	-3.974*
<i>Finland</i>	-3.945*	-3.851*	-3.952*	-3.456*	-2.956	-3.478*
<i>Greece</i>	-2.616	-2.565	-2.489	-2.776	-3.094*	-2.790
<i>Hungary</i>	-2.769	-2.444	-2.484	-2.554	-2.262	-2.386
<i>Iceland</i>	-2.534	-2.557	-2.573	-3.079	-3.699*	-3.163*
<i>Korea</i>	-3.880*	-3.143*	-2.675	-3.331	-2.608	-2.477
<i>Mexico</i>	-1.680	-2.017	-1.565	-4.010*	-3.664*	-4.108*
<i>Netherlands</i>	-2.867	-2.295	-2.617	-2.916	-2.294	-2.671
<i>NewZealand</i>	-2.877	-2.525	-2.626	-3.073	-2.308	-2.616
<i>Norway</i>	-3.803*	-3.715*	-3.695*	-4.713*	-4.640*	-5.281*
<i>Poland</i>	-2.039	-2.180	-2.171	-2.664	-1.409	-2.657
<i>Portugal</i>	-2.768	-2.658	-2.800	-3.590*	-2.165	-3.233*
<i>Spain</i>	-3.941*	-3.606*	-3.819*	-4.753*	-4.303*	-4.755*
<i>Sweden</i>	-4.025*	-4.051*	-3.893*	-5.960*	-4.337*	-5.989*
<i>Switzerland</i>	-3.809*	-2.911	-3.635*	-3.394*	-3.468*	-3.677*
<i>Turkey</i>	-3.318	-2.824	-1.879	-3.955*	-3.218*	-3.155*
<i>HongKong</i>	-2.030	-1.844	-2.105	-4.228*	-2.901	-3.993*
<i>Singapore</i>	-3.435*	-3.406*	-3.122*	-3.612*	-3.216*	-3.544*
<i>Malaysia</i>	-3.565*	-3.576*	-3.578*	-3.328	-3.615*	-3.389*
<i>Indonesia</i>	-2.756	-2.634	-2.473	-4.624*	-2.074	-3.899*
<i>Thailand</i>	-3.281	-3.293*	-3.307*	-3.777*	-3.761*	-3.779*
<i>Philippines</i>	-4.216*	-2.947	-2.287	-5.036*	-2.370	-3.295*
<i>SriLanka</i>	-1.282	-1.578	-1.462	-1.761	-2.044	-1.867
<i>Argentina</i>	-4.255*	-4.368*	-4.360*	-3.242	-3.156*	-2.867
<i>Bolivia</i>	-5.665*	-1.275	-1.279	-11.262*	-0.704	-3.339*
<i>Brazil</i>	-1.734	-1.706	-1.911	-2.034	-1.623	-1.909
<i>Chile</i>	-2.004	-1.700	-1.535	-1.988	-1.539	-1.702
<i>Colombia</i>	-2.342	-2.375	-2.342	-2.942	-1.763	-2.762
<i>Venezuela</i>	-2.023	-2.044	-2.326	-0.502	-3.808*	-1.842
Total No. of Rejections	13	13	15	22	19	26

Table 7: Detrended<sup>a</sup> Tests using SETAR Model ( $z_t = (1, t)'$ ).

Country	$DF^\tau$	$GLS^\tau$	$W^{exp}$	$W_{GLS}^{exp}$
<i>US</i>	-2.018	-2.001	30.417	31.885
<i>Germany</i>	-3.029	-2.493	21.617	371.184*
<i>Italy</i>	-1.550	-1.600	4.569	5.281
<i>UK</i>	-1.368	-1.376	2.611	2.583
<i>Canada</i>	-2.309	-2.109	9.049	15.332
<i>Australia</i>	-2.718	-2.758	26.749	27.129
<i>Austria</i>	-3.220	-3.132*	129.876	182.792*
<i>Belgium</i>	-1.158	-1.289	3.130	3.359
<i>Denmark</i>	-2.884	-2.882	193.689	179.178*
<i>Finland</i>	-1.976	-2.000	21.031	20.314
<i>Greece</i>	-3.109	-3.152*	483.571	648.997*
<i>Hungary</i>	-1.392	-1.461	27.197	25.501
<i>Ireland</i>	-2.464	-2.484	71.521	84.536
<i>Korea</i>	-3.913*	-1.792	5.764	7196.804*
<i>Mexico</i>	-3.112	-3.139*	150.811	140.496
<i>Netherlands</i>	-1.643	-1.663	16.911	15.533
<i>Norway</i>	-1.805	-1.765	5.585	5.105
<i>Spain</i>	-2.721	-2.579	279.802	656.358*
<i>Sweden</i>	-2.545	-2.551	31.005	36.293
<i>Switzerland</i>	-3.040	-2.975*	255.250	380.021*
<i>Turkey</i>	-2.389	-2.212	15.109	26.186
<i>Singapore</i>	-1.471	-1.493	4.498	5.483
<i>Malaysia</i>	-3.583*	-2.587	76.817	5761.784*
<i>Indonesia</i>	-1.848	-1.420	5.781	11.051
<i>Thailand</i>	-2.802	-2.677	79.602	160.068*
<i>Philippines</i>	-3.037	-1.685	20.890	144.733
<i>SriLanka</i>	-2.050	-1.687	7.618	22.242
<i>Argentina</i>	-5.165*	-2.407	33.843	4623153*
<i>Chile</i>	-2.909	-2.516	1058.658*	16315.06*
<i>Colombia</i>	-2.140	-2.149	16.778	16.885
<i>Venezuela</i>	-2.022	-2.071	13.117	10.126
Total No. of Rejections	4	5	2	12

<sup>a</sup>Starred entries indicate significance at the 5% significance level

Table 8: Augmented Detrended Tests using SETAR Model ( $z_t = (1, t)'$ ).

Country	$DF^\tau$	$GLS^\tau$	$W^{exp}$	$W_{GLS}^{exp}$
<i>US</i>	-2.995	-2.982*	631.439*	702.101*
<i>Germany</i>	-3.249	-2.491	27.803	2943.871*
<i>Italy</i>	-2.238	-2.247	22.700	29.338
<i>UK</i>	-2.082	-2.079	13.153	8.673
<i>Canada</i>	-3.289	-3.085*	122.689	282.461*
<i>Australia</i>	-3.690*	-3.707*	302.119	337.048*
<i>Austria</i>	-3.429*	-3.143*	217.976	422.499*
<i>Belgium</i>	-2.530	-2.752	155.902	124.441
<i>Denmark</i>	-3.528*	-3.545*	3011.942*	2492.441*
<i>Finland</i>	-3.311	-3.332*	1866.803*	1737.248*
<i>Greece</i>	-2.679	-2.744	171.794	246.429*
<i>Hungary</i>	-1.735	-1.790	40.063	42.969
<i>Ireland</i>	-2.633	-2.647	180.433	211.145*
<i>Korea</i>	-3.059	-1.862	6.843	225.669*
<i>Mexico</i>	-4.747*	-4.708*	315031.4*	275743.3*
<i>Netherlands</i>	-2.566	-2.432	371.038	230.185*
<i>Norway</i>	-3.552*	-3.645*	1159.313*	878.808*
<i>Spain</i>	-2.825	-2.659	759.037*	1615.200*
<i>Sweden</i>	-3.884*	-3.840*	1360.870*	1587.833*
<i>Switzerland</i>	-3.063	-2.937*	682.176*	1605.988*
<i>Turkey</i>	-2.257	-2.302	30.638	37.589
<i>Singapore</i>	-2.428	-2.462	170.091	255.528*
<i>Malaysia</i>	-3.147	-2.346	41.731	748.718*
<i>Indonesia</i>	-2.290	-1.642	63.976	229.395*
<i>Thailand</i>	-3.416*	-3.232*	848.976*	1688.448*
<i>Philippines</i>	-3.833*	-1.852	59.831	5858.578*
<i>SriLanka</i>	-3.499*	-2.772	250.551	4455.678*
<i>Argentina</i>	-5.701*	-1.693	3.985	10064985*
<i>Chile</i>	-1.989	-1.831	97.113	9551.004*
<i>Colombia</i>	-2.269	-2.287	27.936	27.388
<i>Venezuela</i>	-2.218	-2.281	28.026	21.126
Total No. of Rejections	10	11	9	24

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