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## Unit and fractional roots in the presence of abrupt changes with an application to the Brazilian inflation rate

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**UNIT AND FRACTIONAL ROOTS IN THE PRESENCE OF ABRUPT CHANGES  
WITH AN APPLICATION TO THE BRAZILIAN INFLATION RATE**

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**ABSTRACT**

We analyse in this article the monthly structure of the Brazilian inflation rate by means of fractionally integrated techniques. This series is characterized by strong government interventions to bring inflation to a low level. We use a testing procedure due to Robinson (1994) which allow us to model the underlying dynamic of the series in terms of I(d) statistical models, while the government interventions are specified in terms of dummy variables. The results show that the series can be described in terms of an I(0.75) process with some of the interventions having little impact on the series.

**Keywords:** Long memory; Fractional integration.

**JEL Classification:** C22

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## 1. Introduction

Modelling the nonstationarity in macroeconomic data is a matter that still remains controversial. Deterministic models based on linear (or quadratic) functions of time were shown to be inappropriate in many cases and, stochastic models based on first (or second) differences of the data were then proposed, especially after the seminal paper by Nelson and Plosser (1982). In that paper, using tests of Fuller (1976) and Dickey and Fuller (1979), they showed that many US macroeconomic series may be well described in terms of unit root processes. Following this work, a battery of test statistics were developed for testing unit roots (eg, Said and Dickey, 1984; Phillips and Perron, 1988; Kwiatkowski et. al., 1992; etc.). Robinson (1994) also proposed tests of unit roots but, unlike the previous ones which are embedded in autoregressive (AR) alternatives, they are nested in a fractional model of form

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots,$$

where  $u_t$  is  $I(0)$  (properly defined in Section 2) and where the unit root null corresponds to  $d = 1$ .

On the other hand, Perron (1989, 1993) found that the 1929 crash shock and the 1973 oil price shock were a cause of nonrejection of the unit-root hypothesis, and that when these were taken into account, deterministic models were preferable. This question has been pursued by authors such as Christiano (1992), Demery and Duck (1992), Krol (1992), Serletis (1992), Ben-David and Papell (1994) and Mills (1994), the first author arguing that the date of the break should be treated as unknown. Similarly, Zivot and Andrews (1992) also allowed the structural break to be unknown.

This article try to connect both issues, testing unit and fractional roots in the presence of abrupt changes in the data. Fractional integration and structural breaks is a topic that has been scarcely investigated. Diebold and Inoue (1999) provide both theoretical and Monte Carlo evidence that structural breaks-based models and long memory processes are easily confused. Similarly, Granger and Hyung (1999) also developed a theory relating both types of

models and, in a recent article, Gil-Alana (2001a) shows that the order of integration of some series is reduced by the inclusion of dummy variables for structural breaks in the regression model.

The analysis in this article is directly motivated by the time series properties of the Brazilian inflation rate. This series is characterized by a period of hyperinflation, starting by the end of the 1980s and followed by government interventions to bring inflation to a low level for a short period of time. We use the same dataset as in Cati et al. (1999), i.e., the Brazilian monthly inflation rate (1974:1 – 1993:6). In that article, they show that the presence of outliers leads to a bias in the standard unit root tests in favour of stationarity where in fact the series is integrated of order one. In this article, we show, however, that the series is nonstationary but mean-reverting, with an order of integration smaller than one. The outline of this paper is as *follows*: Section 2 briefly describes the tests of Robinson (1994) for testing I(d) statistical models which allows us to include dummy variables to incorporate the government interventions. Section 3 applies different versions of Robinson's (1994) tests to the Brazilian inflation rate while Section 4 contains some concluding comments.

## 2. Testing of I(d) models with the tests of Robinson (1994)

For the purpose of the present article, we define an I(0) process  $\{u_t, t = 0, \pm 1, \dots\}$  as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that  $x_t$  is I(d) if

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

$$x_t = 0, \quad t \leq 0, \quad (2)$$

where the polynomial in (1) can be expanded in terms of its Binomial expansion such that for all real d,

$$(1 - L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(d+1)(-1)^j}{\Gamma(d-j+1)\Gamma(j+1)} L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \dots,$$

where  $\Gamma(x)$  means the gamma function. Clearly, if  $d = 0$  in (1),  $x_t = u_t$ , and a ‘weakly autocorrelated’  $x_t$  is allowed for. However, if  $d > 0$ ,  $x_t$  is said to be long memory, so-called because of the strong association between observations widely separated in time. This type of models was introduced by Granger (1980, 1981) and Hosking (1981), (though earlier work by Adenstedt, 1974 and Taqqu, 1975 shows an awareness of its representation) and were theoretically justified in terms of aggregation by Robinson (1978), Granger (1980) and more recently, in terms of the duration of shocks by Parke (1999).

Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o \quad (3)$$

for any real value  $d_o$ , in a model given by

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, \dots \quad (4)$$

and (1), where  $y_t$  is the time series we observe;  $\beta$  is a  $(k \times 1)$  vector of unknown parameters; and  $z_t$  is a  $(k \times 1)$  vector of deterministic regressors that may include, for example, an intercept ( $z_t \equiv 1$ ); a linear time trend ( $z_t = (1, t)'$ ) or dummy variables). Specifically, the test statistic is given by:

$$\hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \quad (5)$$

where  $T$  is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right),$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}.$$

$I(\lambda_j)$  is the periodogram of  $\hat{u}_t$ , where

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\beta}' w_t; \quad w_t = (1 - L)^{d_o} z_t; \quad \hat{\beta} = \left( \sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t (1 - L)^{d_o} y_t.$$

The function  $g$  above is a known function coming from the spectral density function of  $u_t$ ,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi,$$

evaluated at  $\hat{\tau} = \arg \min \sigma^2(\tau)$ . Note that the functional form of the test statistic will be affected by the specification we adopt for the I(0) disturbances  $u_t$  in (1). Thus, for example, if  $u_t$  is white noise, the test statistic greatly simplifies since  $g \equiv 1$  and  $\hat{A}$  below (5) becomes

$$\frac{2}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)^2,$$

which can be asymptotically approximated by  $\pi^2/6$ . However, the I(0) disturbances can also be weakly autocorrelated. If  $u_t$  is an AR(p) process of form:  $\phi_p(L)u_t = \varepsilon_t$ , with white noise  $\varepsilon_t$ ,

$$g = \left| \phi_p(e^{i\lambda_j}) \right|^{-2},$$

and  $\varepsilon(\lambda)$  is now a (px1) vector with 1<sup>th</sup> element given by:

$$2 \left\{ \cos l \lambda - \sum_{r=1}^p \tau_r \cos(l-r) \lambda \right\} g(\lambda; \tau).$$

In this article, we will also make use of other less conventional forms of I(0) processes. In particular, we will also employ the Bloomfield (1973) exponential spectral model. This is a non-parametric approach of modelling the disturbances, where  $u_t$  is exclusively specified in terms of its spectral density function, which is given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \exp \left\{ \sum_{r=1}^m \tau_r \cos \lambda r \right\}. \quad (6)$$

The intuition behind this model is the following. Let's suppose that  $u_t$  is an ARMA(p, q) process of form  $\phi_p(L) u_t = \theta_q(L) \varepsilon_t$ . Clearly, the spectral density function of this process is then

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \frac{|\theta_q(e^{i\lambda})|^2}{|\phi_p(e^{i\lambda})|^2}, \quad (7)$$

where  $\tau$  corresponds to all AR and MA coefficients and  $\sigma^2$  is the variance of  $\varepsilon_t$ . Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (6) approximates (7) well when  $p$  and  $q$  are small values, which usually happens in economics. Like the stationary AR( $p$ ) case, this model has exponentially decaying autocorrelations and thus, using this specification, we do not need to rely on so many parameters as in the ARMA processes, which always results tedious in terms of estimation, testing and model specification.

Based on (3), Robinson (1994) established under regularity conditions that

$$\hat{r} \rightarrow_d N(0, 1) \quad \text{as } T \rightarrow \infty. \quad (8)$$

The conditions on  $u_t$  in (8) are far more general than Gaussianity, with a moment condition only of order 2 required. Thus, an approximate one-sided  $100\alpha\%$ -level test of (3) against alternatives  $H_1: d > d_0$  is given by the rule: “Reject  $H_0$  (3) if  $\hat{r} > z_\alpha$ ”, where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . Conversely, an approximate one-sided  $100\alpha\%$  level test of (3) against alternatives:  $H_1: d < d_0$  is given by the rule: “Reject  $H_0$  (3) if  $\hat{r} < -z_\alpha$ ”. As these rules indicate, we are in a classical large sample testing situation for reasons described by Robinson (1994), who also showed that the above tests are efficient in the Pitman sense that against local alternatives  $H_1: d = d_0 + \delta T^{-1/2}$  for  $\delta \neq 0$ , the test has an asymptotic normal distribution with variance 1 and mean which cannot (when  $u_t$  is Gaussian) be exceeded in absolute value by that of any rival regular statistic. This version of the tests of Robinson (1994) was used in empirical applications in Gil-Alana and Robinson (1997) and Gil-Alana (2000) and, other versions of his tests, based on seasonal (quarterly and monthly) and cyclical models can be found respectively in Gil-Alana and Robinson (2001) and Gil-

Alana (1999, 2001b). In the following section, the tests are applied to the Brazilian inflation rate.

### 3. The Brazilian inflation rate

The series used in this paper is the monthly Brazilian inflation rate for the time period 1974:1 to 1993:6, and we use the same dataset as in Cati et al. (1999). Figures 1 and 2 show plots of the original series and its first differences respectively. Looking at the original data, we observe that the series is characterized in the 80's by several sudden drops that are important in magnitude. These are the outcome of the various shock plans instituted by the government in an attempt to stop the process of high and increasing inflation. They correspond to the time periods 86:3, 87:7, 89:2, 90:3 and 91:2. As in Cati et al. (1999), we stop the sample in 93:6 in order to avoid incorporating the Real Plan which is still in effect. The plot of the first differences clearly shows the importance of the outliers corresponding to these shock plans.

**(Figures 1 and 2 about here)**

Denoting the inflation series  $y_t$ , we initially employ throughout the model (1), (2) and (4) with  $z_t = (1, t)'$ ,  $t \geq 1$ ,  $z_t = (0, 0)'$  otherwise, so

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \dots, \quad (9)$$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (10)$$

treating separately the cases  $\alpha = \beta = 0$  a priori,  $\alpha$  unknown and  $\beta = 0$  a priori and  $(\alpha, \beta)$  unknown, i.e., we consider the cases of no regressors, an intercept, and an intercept and a linear time trend. We will model the  $I(0)$   $u_t$  to be both white noise and to have weak parametric autocorrelation, in the latter case assuming AR(1), MA(1) and Bloomfield(1) disturbances.

The test statistic reported in Table 1 (and also in Tables 2 and 3) is the one-sided one given by (5), so that significantly positive values of this are consistent with alternatives with



higher orders of integration, whereas significantly negative ones are consistent with smaller values of  $d$ . We apply the tests for the time periods: 74:1 – 86:3, 86:4 – 93:6, and for the whole sample 74:1 – 93:6. Starting with the first subsample, we observe that practically all the non-rejection values of  $d$  oscillate between 0.25 and 0.75. We also observe a non-rejection value if  $d = 1$  for the case of no regressors and white noise disturbances. In general, the results seem quite robust to the different specifications of  $z_t$ , however, they substantially vary depending on how we specify the  $I(0)$  disturbances. Thus, if  $u_t$  is white noise, the non-rejected values are 0.75 and 1. If  $u_t$  is AR(1), we observe several other non-rejections. However, in this case, we also observe a lack of monotonic decrease in the value of the test statistic with respect to  $d$ , for small values of  $d$ . Such monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size, because, for example, we would wish that if  $d = 1$  is rejected against  $d > 1$ , an even more significant result in this direction should be expected when  $d = 0.75$  or  $d = 0.50$  are tested. However, in the event of misspecification, monotonicity is not necessarily to be expected: frequently misspecification inflates both numerator and denominator of  $\hat{\rho}$ , to varying degrees, and thus affects  $\hat{\rho}$  in a complicated way. Modelling  $u_t$  in terms of a MA(1) process or with the Bloomfield(1) exponential model, monotonicity is always obtained and the non-rejection values of  $d$  are 0.50 (in case of MA disturbances) and 0.50 and 0.25 with the Bloomfield model.

**(Table 1 about here)**

The results for the second subsample are a bit more ambiguous, given the higher proportion of non-rejection values, which could be largely due to the smaller sample size. These values range now widely between 0 and 1.25, again observing higher values of  $d$  if  $u_t$  is white noise rather than autocorrelated. We observe that the non-fractional cases ( $d = 0$  and  $d = 1$ ) cannot be rejected in some cases. Thus, for example, the unit root null cannot be rejected if  $u_t$  is white noise or AR(1). On the contrary,  $d = 0$  results non-rejected if  $u_t$  is also AR(1) or if follows the Bloomfield (1) exponential model.

Finally, we also present the results for the whole sample period. Definitely, they are less ambiguous than in the previous cases. Thus, if  $u_t$  is white noise,  $d = 1$  appears as the only non-rejection value for the three specifications in  $z_t$ . This may contradict the results in Cati et al. (1999), where they found strong evidence against unit roots using tests of Dickey and Fuller (1979), Phillips and Perron (1988) and Stock (1990). However, as mentioned in Section 1, these tests are based on AR models and do not consider fractionally integrated alternatives. Furthermore, we also observe in this table that imposing weakly autocorrelated disturbances, the unit root null hypothesis results rejected in favour of less nonstationary (or even stationary) alternatives, and the non-rejection values of  $d$  are now 0.25 and 0.50.

The results presented in Table 1 may be affected by the presence of outliers due to the government interventions during the 80's and early 90's. Thus, in Table 2, we recalculate the tests of Robinson (1994), but this time including in  $z_t$ , five dummy variables ( $D_{it}$ ) to incorporate these outliers. Thus, instead of (9), we have

$$y_t = \alpha + \beta t + \sum_{i=1}^5 \gamma_i D_{it} + x_t, \quad t = 1, 2, \dots, \quad (11)$$

where  $D_{it} = 1$   $I(t = T_i)$ , and  $T_i$  corresponding to the time periods of each of the government interventions. Other types of dummy (step and slope) variables were also considered but the coefficients were insignificant in practically all cases. Furthermore, the inclusion of impulse dummies also permit us to incorporate lag effects throughout the autocorrelated disturbances. Again, we perform  $\hat{r}$  given by (5), testing  $H_0$  (3) in model (10) and (11), for values  $d_0 = 0$ , (0.25), 2, and white noise and autocorrelated disturbances. The results are given in Table 2 and the non-rejection values of  $d$  correspond now to  $d = 1$  and 1.25 in case of white noise  $u_t$ ;  $d = 0.50$  and 0.75 for AR(1) and MA(1) disturbances; and  $d = 0.25$  and 0.50 when using the Bloomfield exponential spectral model. Higher orders for the AR, MA and Bloomfield models were also considered and the results were very similar to those reported across the tables, implying that higher autocorrelated orders were unnecessary to describe the short run

dynamics of the series. We also observe in this table that the significance of the coefficients in the regression model (11) also substantially vary depending on the degree of integration. Thus,  $D_1$  appears significant if  $d \geq 0.75$ ;  $D_2$  if  $d \geq 0.50$ ;  $D_3$  if  $d \geq 0.25$  and  $D_4$  if  $d = 0, 1.75$  and  $2$ .  $D_5$  always results insignificant across the different values of  $d$ . In view of this, we report in Table 3, the results for the same statistic as in Table 2, but taking into account only those regressors that were significant in Table 2.

**(Tables 2 and 3 about here)**

We see that the non-rejection values practically coincide with those obtained in Table 2. The only exception corresponds to the case of Bloomfield disturbances and  $d = 0.50$ . This model cannot be rejected in Table 2 but is rejected in Table 3 when including only the significant coefficients.

Table 4 summarises the selected models according to the results in Table 3. That is, we write the estimated models based on (10) and (11), in which the null hypothesis  $H_0$  (3) was not rejected and all the coefficients were significantly different from zero. We see that all except model 7 are nonstationary ( $d \geq 0.5$ ), and the significant dummies appear to be  $D_1$ ,  $D_2$  and  $D_3$  in models 1, 2, 3, and 4; the time trend and  $D_2$  and  $D_3$  in models 5 and 6; while the time trend and  $D_3$  are the only significant regressors in model 7.

**(Table 4 about here)**

A more difficult task is to determine which is the correct model specification across the different models presented in that table. We display in the last column of Table 4 several diagnostic tests carried out on the residuals. We observe that if  $d = 1.25$  or  $1$  (Models 1 and 2), the models fail in relation to the homocedasticity property. Models 4, 5 and 6 (corresponding to  $d = 0.75$  and  $0.50$ ) fail in relation to the functional form, while Model 7 ( $d = 0.25$  and Bloomfield disturbances) cannot be evaluated because of its non-parametric autocorrelated structure. Thus, we see that Model 3 is the only one which passes all the diagnostics on the residuals, and it corresponds to

$$y_t = \gamma_1 D_{1t} + \gamma_2 D_{2t} + \gamma_3 D_{3t} + x_t, \quad t = 1, 2, \dots, \quad (12)$$

$$(1 - L)^{0.75} x_t = u_t; \quad u_t = \phi u_{t-1} + \varepsilon_t, \quad t = 1, 2, \dots, \quad (13)$$

giving the estimates:  $\gamma_1 = -7.469$ ;  $\gamma_2 = -9.040$ ;  $\gamma_3 = -16.457$  and  $\phi = -0.472$ . Thus, the impact of the government interventions appears especially relevant for the data in the cases of the first three plans, (i.e., 86:3, 87:7 and 89:2), while the fourth and fifth interventions, even being large in magnitude (in particular the fourth) have little impact on the series.

**(Figures 3 and 4 about here)**

To evaluate the responses of inflation to the plan shocks, we need to derive the impulse response functions. We do this next. Let  $(1 + 0.472L)(1 - L)^{0.75} = a(L)$ , and calling  $d_1(L) = -7.469a(L)$ ;  $d_2(L) = -9.040a(L)$ ;  $d_3(L) = -16.457a(L)$ , the model in (12) and (13) becomes

$$a(L)y_t = d_1(L)D_{1t} + d_2(L)D_{2t} + d_3(L)D_{3t} + \varepsilon_t, \quad t = 1, 2, \dots,$$

and using a power expansion of  $a(L)$ ,  $d_1(L)$ ,  $d_2(L)$  and  $d_3(L)$  in terms of its lags, with  $D_{1j} = D_{2j} = D_{3j} = 0$  for  $j \leq 0$ , we obtain

$$y_t = \sum_{j=1}^{T-1} a_j y_{t-j} + \sum_{j=1}^{t-1} d_{1j} D_{1t-j} + \sum_{j=1}^{t-1} d_{2j} D_{2t-j} + \sum_{j=1}^{t-1} d_{3j} D_{3t-j} + \varepsilon_t, \quad t = 1, 2, \dots, \quad (14)$$

where  $a_j$  are the coefficients of the impulse response functions, and  $d_{1j}$ ,  $d_{2j}$  and  $d_{3j}$  are the impacts of the shock plans on the inflation. Figure 3 summarizes these values for the impulse responses and Figure 4 for the impacts of the plans. We observe through the  $a_j$ 's that the effect of a shock on inflation tends to die away in the long run though it takes a very long period to disappear completely. In fact, we see that even 50 periods after the initial shock, 20% of its effect still remains on the series. The impact of the shocks is higher for the third plan (89:2) than for the others (86:3 and 87:7) and also they take a long time to disappear completely.

#### 4. Conclusions

The monthly structure of the Brazilian inflation rate (74:1 – 93:6) has been investigated in this article by means of fractionally integrated techniques. This series is characterized by a period of hyperinflation, starting by the end of the 80's and followed by government interventions to bring inflation to a low level for a short period of time. We have made use of a testing procedure due to Robinson (1994) that allows us to consider  $I(d)$  statistical models and at the same time to incorporate dummy variables for the government interventions, with no effect on the limit distribution of the tests. When the dummy variables are not included in the regression model, the results of Robinson's (1994) tests indicate that the series may be  $I(1)$  if the underlying disturbances are white noise. However, if they are autocorrelated, the order of integration seems to be smaller, oscillating between 0.25 and 0.50. This contradicts the results in Cati et al. (1994) which found that the series is  $I(1)$  but, in that paper, they do not consider fractionally integrated alternatives. Incorporating the dummies, the results also substantially vary depending on how we specify the disturbances. Thus, if they are white noise, the orders of integration are 1 and 1.25. If  $u_t$  is  $AR(1)$  or  $MA(1)$ ,  $d$  appears to be 0.50 and 0.75, while using the Bloomfield (1973) exponential model, the series seems to be stationary with  $d = 0.25$ . In order to choose which might be the best model specification across the potential models selected with the tests of Robinson (1994), we look at several diagnostic tests carried out on the residuals of these selected models, the results indicating that the series may be described in terms of an  $I(0.75)$  process with three dummy variables for the first three interventions (86:3, 87:7 and 89:2). Thus, the fourth and fifth interventions (even being large in magnitude) have little impact in the underlying structure of the series.

It would be worthwhile proceeding to get point estimates of  $d$ , perhaps especially in the Bloomfield case, where the results indicate that the series may be stationary as opposed to the nonstationary results obtained for the remaining cases. However, not only would this be

computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed.

The approach used in this article generates simply computed diagnostics for departures from any real  $d$ . It is thus not at all surprising that, when fractional hypotheses are entertained, some evidence supporting them appears, because this might happen even when the unit-root model is highly suitable. However, even though our practise of computing test statistics for a wide range of null hypotheses lead to ambiguous conclusions, often the bulk of these hypotheses are rejected, suggesting that the optimal local power properties of the tests may be supported by reasonable performance against non-local alternatives. Furthermore, the diagnostic tests carried out on the residuals of the selected models give further support for fractional models when describing the Brazilian inflation rate.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic data. Extensions of the tests of Robinson (1994) to include structural breaks at unknown periods of time are being developed. Also, the estimation of fractional models in the context of Bloomfield (1973) exponential spectral disturbances is being examined. Work is also proceeding on multivariate extensions of Robinson's (1994) tests, and this would lead to an alternative approach to the study of cointegration. How these approaches may affect to the conclusions obtained in this article still remains to be investigated.

## References

- Adenstedt, R.K., 1974, On large-sample estimation for the mean of a stationary random sequence, *Annals of Statistics* 2, 1095-1107.
- Ben-David, D. and D.H. Papell, 1994, The great wars, the great crash, and the unit root hypothesis: Some new evidence about an old stylised fact, National Bureau of Economic Research, Working Paper No. 4752.
- Bloomfield, P.J., 1973, An exponential model for the spectrum of a scalar time series, *Biometrika* 60, 217-226.
- Cati, R.C., M.G.P. Garcia and P. Perron, 1999, Unit roots in the presence of abrupt governmental interventions with an application to Brazilian data, *Journal of Applied Econometrics* 14, 27-56.
- Christiano, I.J., 1992, Searching for a break in GNP, *Journal of Business and Economic Statistics* 10, 237-250.
- Demery, D. and N.W. Duck, 1992, Are economic fluctuations really persistent? A reinterpretation of some international evidence, *The Economic Journal* 102, 1094-1101.
- Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association* 74, 427-431.
- Diebold, F.X. and A. Inoue, 1999, Long memory and structural change, Preprint.
- Fuller, W.A., 1976, *Introduction to statistical time series*, Willey, New York.
- Gil-Alana, L.A., 1999, Fractional integration with monthly data, *Economic Modelling* 16, 613-629.
- Gil-Alana, L.A., 2000, Mean reversion in the real exchange rates, *Economics Letters* 69, 285-288.
- Gil-Alana, L.A., 2001a, A fractionally integrated model with a mean shift for the US and the UK real oil prices, forthcoming in *Economic Modelling*.

Gil-Alana, L.A., 2001b, Testing stochastic cycles in macroeconomic time series, forthcoming in *Journal of Time Series Analysis*.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, *Journal of Econometrics* 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson, 2001, Testing of seasonal fractional integration in the UK and Japanese consumption and income, forthcoming in *Journal of Applied Econometrics*.

Granger, C.W.J., 1980, Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics* 14, 227-238.

Granger, C.W.J., 1981, Some properties of time series models and their use in econometric model specification, *Journal of Econometrics* 16, 121-130.

Granger, C.W.J. and Hyung, 1999, Occasional structural breaks and long memory, Discussion Paper 99-14, University of California, San Diego.

Hosking, J.R.M., 1981, Modelling persistence in hydrological time series using fractional differencing, *Water Resources Research* 20, 1898-1908.

Krol, R., 1992, Trends, random walks and persistence: an empirical study of disaggregated US industrial production, *The Review of Economics and Statistics* 74, 154-166.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin, 1992, Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics* 54, 159-178.

Mills, T.C., 1994, Infrequent permanent shocks and the unit root in quarterly UK output, *Bulletin of Economic Research* 46, 91-94

Nelson, C.R. and C.I. Plosser, 1982, Trends and random walks in macroeconomic time series, *Journal of Monetary Economics* 10, 139-162.

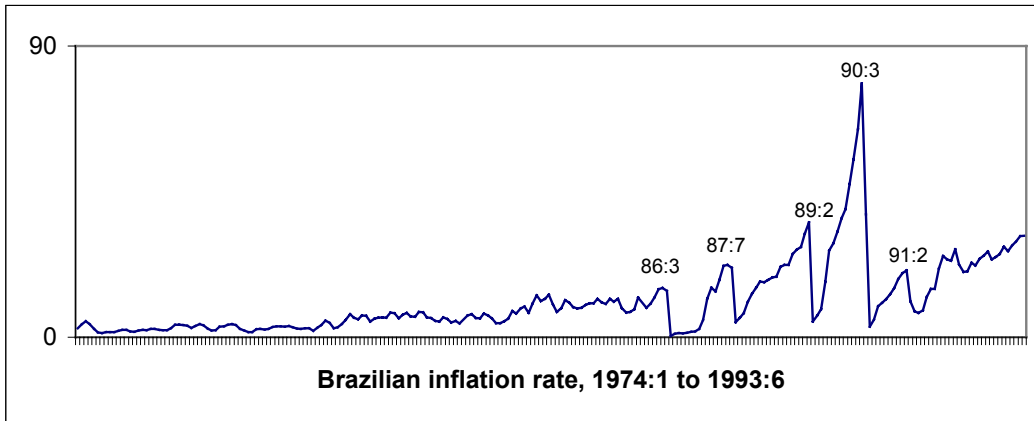
Parke, W.R., 1999, What is fractional integration?, *The Review of Economics and Statistics* 81, 632-638.

Perron, P., 1989, The great crash, the oil price shock and the unit root hypothesis, *Econometrica* 57, 1361-1401.

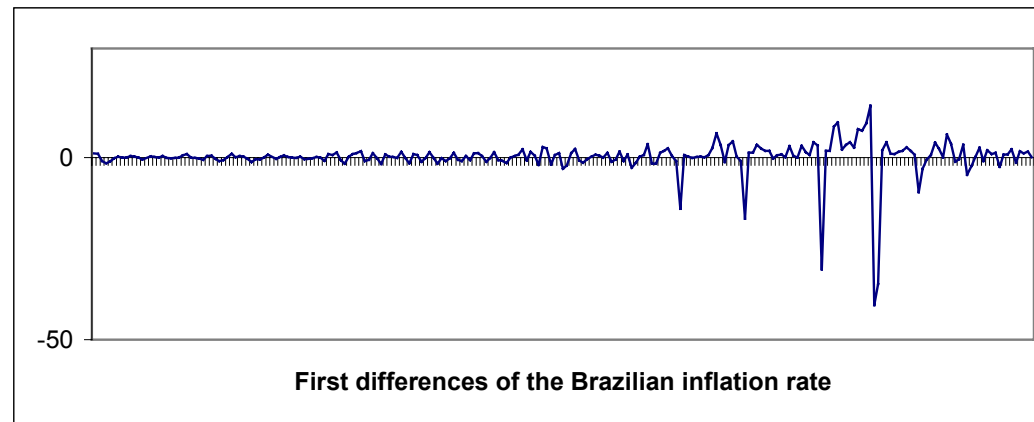


- Perron, P., 1993, Trends, unit roots and structural change in macroeconomic time series, Unpublished manuscript, University of Montreal, Montreal.
- Phillips, P.C.B. and P. Perron, 1988, Testing for a unit root in time series regression, *Biometrika* 75, 335-346.
- Robinson, P.M., 1978, Statistical inference for a random coefficient autoregressive model, *Scandinavian Journal of Statistics* 5, 163-168.
- Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, *Journal of the American Statistical Association* 89, 1420-1437.
- Said, S.E. and D.A. Dickey, 1984, Testing for unit roots in autoregressive moving average models of unknown order, *Biometrika* 71, 599-607.
- Serletis, A., 1992, The random walk in the Canadian output, *Canadian Journal of Economics* 2, 392-406.
- Stock, J.H., 1990, A class of tests for integration and cointegration, mimeo, Kennedy School of Government, Harvard University.
- Taqqu, M.S., 1975, Weak convergence to fractional Brownian motion and the Rosenblatt process, *Z. Wahrscheinlichkeitstheorie verw. Geb.* 31, 287-302.
- Zivot, E. and D.W.K. Andrews, 1992, Further evidence of the great crash, the oil price shock and the unit root hypothesis, *Journal of Business and Economics Statistics* 10, 251-270.

**FIGURE 1**

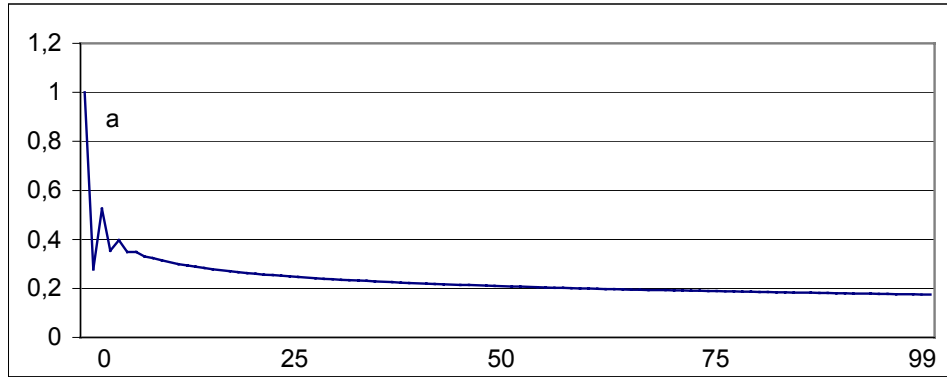


**FIGURE 2**



**FIGURE 3**

Impulse response function of the Brazilian inflation rate



**FIGURE 4**

Impacts of government interventions on the Brazilian inflation rate

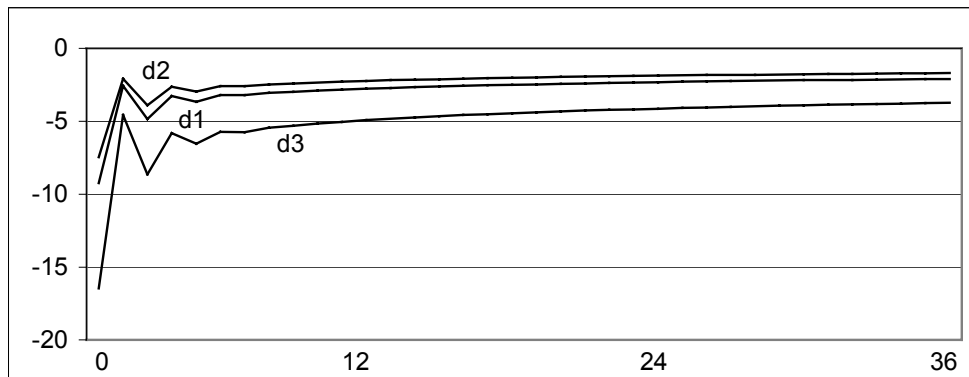


TABLE 1											
Testing the order of integration in the Brazilian inflation rate with the tests of Robinson (1994)											
Sample	$u_t$	$z_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
74.1 – 86.3	White noise	---	24.75	15.27	5.26	<b>0.78'</b>	<b>-1.49'</b>	-2.82	-3.62	-4.14	-4.51
		1	24.75	16.80	5.94	<b>0.42'</b>	-1.70	-2.86	-3.58	-4.08	-4.45
		(1, t)'	14.80	8.63	3.59	<b>0.23'</b>	-1.70	-2.85	-3.56	-4.03	-4.40
	AR (1)	---	-1.26'	-0.28'	<b>0.02'</b>	<b>-1.45'</b>	-2.51	-3.26	-3.73	-4.01	-4.20
		1	-1.26'	-1.21'	<b>-0.08'</b>	<b>-1.57'</b>	-2.61	-3.28	-3.71	-3.99	-4.18
		(1, t)'	-0.62'	<b>0.32'</b>	<b>-0.57'</b>	-1.70	-2.61	-3.26	-3.67	-3.89	-4.06
	MA (1)	---	18.95	7.52	<b>-0.49'</b>	-2.73	-3.51	-5.01	-6.39	-7.29	-7.93
		1	18.95	9.28	<b>-0.009'</b>	-3.12	-3.89	-5.07	-6.32	-7.18	-7.82
		(1, t)'	8.58	2.22	<b>-1.57'</b>	-3.25	-3.88	-5.05	-6.29	-7.09	-7.73
	Bloomfield (1)	---	10.22	3.65	<b>-0.54'</b>	-2.45	-3.35	-4.03	-4.42	-4.65	-4.79
		1	10.22	4.98	<b>-0.19'</b>	-2.77	-3.74	-4.24	-4.57	-4.74	-4.86
		(1, t)'	2.75	<b>0.34'</b>	<b>-1.61'</b>	-2.89	-3.73	-4.21	-4.50	-4.63	-4.70
Sample	$u_t$	$z_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
86.4 – 93.6	White noise	---	9.55	5.90	3.60	1.67	<b>0.10'</b>	<b>-1.09'</b>	-2.00	-2.69	-3.21
		1	9.55	6.24	3.56	<b>1.64'</b>	<b>0.10'</b>	<b>-1.09'</b>	-2.00	-2.69	-3.21
		(1, t)'	9.53	6.44	3.83	1.72	<b>0.10'</b>	<b>-1.10'</b>	-2.00	-2.69	-3.21
	AR (1)	---	<b>0.90'</b>	<b>0.29'</b>	<b>-0.47'</b>	<b>-1.07'</b>	<b>-1.53'</b>	-1.90	-2.21	-2.47	-2.71
		1	<b>0.90'</b>	<b>-0.01'</b>	<b>-0.61'</b>	<b>-1.09'</b>	<b>-1.53'</b>	-1.90	-2.21	-2.47	-2.71
		(1, t)'	<b>1.37'</b>	<b>0.38'</b>	<b>-0.41'</b>	<b>-1.05'</b>	<b>-1.53'</b>	-1.90	-2.21	-2.47	-2.71
	MA (1)	---	5.88	2.36	<b>0.30'</b>	<b>-1.12'</b>	-2.19	-2.88	-3.71	-4.94	-5.88
		1	5.88	2.56	<b>0.29'</b>	<b>-1.14'</b>	-2.20	-2.88	-3.71	-4.94	-5.89
		(1, t)'	5.87	2.79	<b>0.47'</b>	<b>-1.09'</b>	-2.19	-2.89	-3.71	-4.94	-5.88
	Bloomfield (1)	---	<b>1.33'</b>	<b>0.63'</b>	<b>-1.54'</b>	-2.17	-2.65	-3.01	-3.30	-3.56	-3.74
		1	<b>1.33'</b>	<b>-0.33'</b>	<b>-1.53'</b>	-2.20	-2.65	-3.01	-3.30	-3.56	-3.74
		(1, t)'	<b>1.21'</b>	<b>-0.39'</b>	<b>-1.36'</b>	-2.13	-2.65	-3.02	-3.31	-3.56	-3.73
Sample	$u_t$	$z_t / d_0$	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
74.1 – 93.6	White noise	---	27.75	13.49	6.12	2.49	<b>0.04'</b>	-1.76	-3.12	-4.14	-4.92
		1	27.75	14.85	6.35	2.48	<b>0.03'</b>	-1.76	-3.12	-4.14	-4.91
		(1, t)'	16.13	10.15	5.20	2.44	<b>0.03'</b>	-1.76	-3.12	-4.14	-4.91
	AR (1)	---	1.66	<b>1.44'</b>	<b>-0.69'</b>	-2.01	-2.78	-3.34	-3.81	-4.22	-4.58
		1	1.66	<b>1.18'</b>	<b>-0.69'</b>	-2.03	-2.78	-3.34	-3.81	-4.22	-4.58
		(1, t)'	3.01	<b>0.96'</b>	<b>-0.86'</b>	-2.04	-2.78	-3.34	-3.81	-4.22	-4.58
	MA (1)	---	20.51	7.04	<b>0.96'</b>	-1.69	-3.23	-4.19	-5.41	-7.14	-8.45
		1	20.51	8.11	<b>1.13'</b>	-1.69	-3.24	-4.20	-5.41	-7.14	-8.45
		(1, t)'	10.54	4.56	<b>0.64'</b>	-1.72	-3.24	-4.19	-5.41	-7.14	-8.45
	Bloomfield (1)	---	9.97	1.75	<b>-1.64'</b>	-2.92	-3.68	-4.21	-4.63	-5.00	-5.25
		1	9.97	2.41	<b>-1.49'</b>	-2.93	-3.69	-4.22	-4.63	-5.00	-5.25
		(1, t)'	2.53	<b>-0.15'</b>	<b>-1.79'</b>	-2.96	-3.69	-4.21	-4.63	-4.99	-5.24

TABLE 2					
Testing the order of integration with the tests of Robinson (1994) including dummy variables					
d	Significant regressors	White noise	AR(1)	MA(1)	Bloomfield (1)
0.00	1; t; D <sub>4</sub>	15.59	2.30	9.76	2.83
0.25	T; D <sub>3</sub>	10.53	1.86	5.44	<b>-0.75'</b>
0.50	T; D <sub>2</sub> ; D <sub>3</sub>	6.00	<b>-0.93'</b>	<b>1.19'</b>	<b>-1.44'</b>
0.75	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	3.43	<b>-1.29'</b>	<b>-1.10'</b>	-3.92
1.00	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	<b>1.19'</b>	-2.41	-2.92	-4.51
1.25	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	<b>-0.51'</b>	-3.22	-4.25	-4.98
1.50	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	-1.82	-3.84	-5.17	-5.29
1.75	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ; D <sub>4</sub>	-2.83	-4.33	-5.81	-5.56
2.00	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ; D <sub>4</sub>	-3.63	-4.73	-6.25	-5.76

<sup>a</sup> and in bold: Non-rejection values of the null hypothesis H<sub>0</sub> (3) in (10) and (11) at the 95% significant level.

TABLE 3					
Testing the order of integration with the tests of Robinson (1994) including only significant regressors					
d	Regressors	White noise	AR(1)	MA(1)	Bloomfield (1)
0.00	1; t; D <sub>4</sub>	15.33	2.66	9.18	3.24
0.25	T; D <sub>3</sub>	10.74	1.86	5.54	<b>-0.63'</b>
0.50	T; D <sub>2</sub> ; D <sub>3</sub>	6.40	<b>0.07'</b>	<b>1.56'</b>	-2.65
0.75	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	3.42	<b>-1.35'</b>	<b>-1.15'</b>	-3.72
1.00	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	<b>1.07'</b>	-2.49	-3.03	-4.37
1.25	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	<b>-0.67'</b>	-3.28	-4.27	-4.81
1.50	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ;	-2.01	-3.87	-5.12	-5.16
1.75	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ; D <sub>4</sub>	-2.89	-4.34	-5.76	-5.52
2.00	D <sub>1</sub> ; D <sub>2</sub> ; D <sub>3</sub> ; D <sub>4</sub>	-3.66	-4.67	-6.31	-5.65

<sup>a</sup> and in bold: Non-rejection values of the null hypothesis H<sub>0</sub> (3) at the 95% significant level.

TABLE 4		
Selected models for the Brazilian inflation rate according to Table 3		
Model		Diagnostics
1	$y_t = -7.346 D_{1t} - 8.887 D_{2t} - 16.222 D_{3t} + x_t$ <p style="text-align: center;">(2.80)            (2.80)            (2.80)</p> $(1 - L)^{1.25} x_t = \varepsilon_t.$	A; B; C
2	$y_t = -7.395 D_{1t} - 9.040 D_{2t} - 16.250 D_{3t} + x_t$ <p style="text-align: center;">(3.20)            (3.20)            (3.20)</p> $(1 - L) x_t = \varepsilon_t.$	A; B; C
3	$y_t = -7.469 D_{1t} - 9.244 D_{2t} - 16.457 D_{3t} + x_t$ <p style="text-align: center;">(3.78)            (3.78)            (3.78)</p> $(1 - L)^{0.75} x_t = u_t; \quad u_t = -0.472 u_{t-1} + \varepsilon_t.$ <p style="text-align: center;">(0.057)</p>	A; B; C; D
4	$y_t = -7.469 D_{1t} - 9.244 D_{2t} - 16.457 D_{3t} + x_t$ <p style="text-align: center;">(3.78)            (3.78)            (3.78)</p> $(1 - L)^{0.75} x_t = u_t; \quad u_t = \varepsilon_t + 0.701 \varepsilon_{t-1}.$ <p style="text-align: center;">(17.277)</p>	A; C; D
5	$y_t = 0.110t - 9.533 D_{2t} - 16.900 D_{3t} + x_t$ <p style="text-align: center;">(0.027)            (4.62)            (4.62)</p> $(1 - L)^{0.50} x_t = u_t; \quad u_t = 0.590 u_{t-1} + \varepsilon_t.$ <p style="text-align: center;">(0.052)</p>	A; C; D
6	$y_t = 0.110t - 9.533 D_{2t} - 16.900 D_{3t} + x_t$ <p style="text-align: center;">(0.027)            (4.62)            (4.62)</p> $(1 - L)^{0.50} x_t = u_t; \quad u_t = \varepsilon_t + 0.727 \varepsilon_{t-1}.$ <p style="text-align: center;">(19.872)</p>	A; C; D
7	$y_t = 0.104t - 17.004 D_{3t} + x_t$ <p style="text-align: center;">(0.009)            (5.93)</p> $(1 - L)^{0.25} x_t = u_t; \quad u_t \approx \text{Bloomfield (1)}.$	--- --- --- ---

\*: Non-rejections at the 99% significant level of A): No serial correlation; B): Functional form; C): Normality, and D): Homocedasticity. Standard errors in parenthesis.