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Information cascades on the labor market*

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Abstract

A model of herding behavior on the labor market is discussed where employers only receive signals with limited precision about the workers' types, but can observe previous employers' decisions. In particular, we study a situation where the employer and the worker can influence the signal probabilities, in the sense that the employer tries to increase the precision of the signal about the worker's type whereas the worker tries to get a good signal, independent of her type. In a two-period model, we derive conditions for an equilibrium in which only down-cascades occur, i.e., the second employer does not hire a worker with a bad history even if he receives a favorable private signal about the worker's type, but he does follow his own signal if the worker's history is good.

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1 Introduction

One of the leading economic examples for rational herd behavior is the labor market.¹ When a worker is applying for jobs at different employers sequentially, current employers can infer something about the worker's abilities or "type" by observing previous employers' decisions. These decisions are summarized in the CV as spells of employment with particular employers or spells of unemployment. While good jobs in the past imply that previous employers received favorable signals about the abilities of the worker, unemployment spells are attributed to the fact that applications failed, i.e., potential employers chose not to hire the worker. Thus, an applicant who receives good offers in the beginning of her career can become a "star" whereas a bad start without good job offers can make subsequent employers unwilling to hire a worker. In this sense, information cascades may dominate a worker's career.

When comparing the labor market interpretation of herding theories with most other applications of information cascade games, such as investment and lending decisions, one difference seems particularly striking. In contrast to investment projects or loans, workers who can become the object of cascades are able to react and adjust to this phenomenon. For example, a worker who knows that the beginning of her career is decisive for her future success will send out many applications early in her life, and put in a lot of effort to prepare for interviews, assessment centers etc., or to obtain helpful letters of recommendation. In this paper, we incorporate such efforts by allowing the worker to increase the probability that the employer gets a favorable impression of her. Also, we endogenize the employer's choice of the precision of the test for a new applicant. Both of these possible manipulations of the signal probabilities, by the worker and by the employer, are assumed to create some cost to the agents, so they will only

¹See, e.g., Bikhchandani, Hirshleifer, and Welch (1992) and the experimental study by Anderson and Holt (1997). For a survey see Bikhchandani, Hirshleifer, and Welch (1998).

be used to a limited degree (which will depend on the employment history, and on the expectations about others' behavior). We investigate the effect of these choices on the occurrence of information cascades in the equilibrium of the game.

Moreover, we introduce the possibility that the required abilities of a worker differ between jobs. If the employer knows, for example, that the worker's previous job required completely different skills than the job he wants to fill, he cannot learn much from the previous employer's decision to hire the worker. On the other hand, the more similar the jobs are, the more employers can learn from the CV of an applicant.

In the two-period model specification that is studied below, the introduction of the signal manipulations by the worker and the employer causes a strict asymmetry between good and bad employment histories. While up-cascades (where the second employer follows the first employer after the worker has been employed) and down-cascades (reversely, with two periods of unemployment) are both possible in equilibrium, it is shown that only down-cascades can occur alone. I.e., for certain parameter ranges, early spells of unemployment are decisive for the worker's subsequent job search, but spells of employment are not. In this sense, having the possibility to improve one's chances of employment make unemployment a strong negative stigma in equilibrium.

In Section 2, the model and the main result are presented. The section also contains a numeric example which illustrates the size and ordering of the parameter ranges for which the different possible equilibria exist. Section 3 discusses the result in the context of related literature.

2 The model

2.1 Assumptions

Suppose there is one worker who can apply for a job in every period, and in every period there is one employer with an open post. The game has two periods, $t = 1, 2$, but jobs

last only one period and the worker cannot be reemployed by the same employer. When the firm hires a worker, it receives a return $V = 1$ if the worker is a good type (type G) and a return $V = 0$ if the worker is a bad type (type B). The wage payment to a worker, irrespective of her type, is 0.5. The prior probability of a good or a bad worker type is $\beta_1 = 0.5$.

The employer does not know the worker's type, but receives a signal about her abilities, which can be either high ($S_t = H$) or low ($S_t = L$).² In each period t , signal $S_t = H$ and signal $S_t = L$ occur with the following probabilities, given the worker is a good ($V = 1$) or a bad ($V = 0$) type:

	$V = 1$	$V = 0$
$S_t = H$	$0.5 + p_t + q_t^G$	$0.5 - p_t + q_t^B$
$S_t = L$	$0.5 - p_t - q_t^G$	$0.5 + p_t - q_t^B$

Table 1: Signal probabilities of the two types.

The employer can influence the probability of receiving a good signal from a good type and a bad signal from a bad type (i.e., the precision of the signal) by choosing $p_t \in [0, \bar{p}]$, with some upper limit $\bar{p} < 0.5$. This costs him $K(p_t)$ with $K' > 0$, $K'' > 0$, $K(0) = 0$, $\lim_{p_t \rightarrow \bar{p}} K(p_t) = \infty$, and $K'(0) = 0$. Without loss of generality we write $K(p_t) = k \cdot \xi(p_t)$, in order to use k as a scaling parameter. The worker can influence the signal probabilities as well. She can choose $q_t, q_t \in [0, \bar{q}]$, to increase the probability of a good signal, independently of her true type. However, different types may choose different effort levels when applying for a job, denoted by q_t^G and q_t^B . The cost function $C(q_t)$ is identical for both types, monotonically increasing and strictly convex, $C' > 0$, $C'' > 0$, $C''' > 0$, $C(0) = 0$, $\lim_{q_t \rightarrow \bar{q}} C(q_t) = \infty$, and $C'(0) = 0$.³ Furthermore, assume that $\bar{p} + \bar{q} \leq 0.5$, to guarantee interior solutions.

²The assumption of a binary signal is not without consequences. If the signal were continuous, i.e., more precise, the employer could set different cutoff values, depending on the employment history of the worker. However, herding could still occur in this case, as long as the employment decision is discrete.

³The assumption that $C''' > 0$, which may be seen as rather restrictive, is only needed for one part of the main result, as will be specified below.

To allow for imperfectly correlated job profiles (e.g., workers switching to completely different jobs), we assume that a worker who was a good type in the first period may become a bad type in the second period, and vice versa. The probability of a good [bad] worker in period 1 of remaining a good [bad] worker in period 2 has a value of $\alpha \in [0.5, 1]$, which is common knowledge. E.g., if there is no correlation between the abilities required in period 1 and period 2, a good worker in period 1 is a good worker in period 2 with probability one half, i.e., $\alpha = 0.5$. Suppose further that before the second period starts neither the worker nor the employer know whether the worker's type changes, but in $t = 2$ the worker finds out whether she is a good or a bad type for the new job. The timing of the two-period game is as follows:

Period $t = 1$:

- The employer chooses p_t . Simultaneously, the worker learns her type, G or B , and chooses q_t .
- Firm 1 receives a signal S_t and either employs the worker or does not employ her. If employed, the worker receives the wage of 0.5 from the firm, and the firm gets the return V .

Period $t = 2$:

- The same as in $t = 1$, except that firm 2 learns the employment history of the worker, h_1 , as well as p_1 , before the period starts.

If the worker was employed in the first period, the history is denoted as $h_1 = 1$, and if she was not employed as $h_1 = 0$.

2.2 Optimal choices of the worker and the employer

To solve this game, we use the concept of Perfect Bayesian equilibrium. The worker determines her first-period effort by considering not only firm 1's behavior, but also

what will happen in the second period after being employed or unemployed in the first period. In equilibrium, the worker knows whether the second employer will hire her after a good or a bad signal S_2 , given her history. We say that a worker is in a cascade if in $t = 2$ she will be employed (in an up-cascade) or not employed (in a down-cascade) *independent* of the signal that the new employer receives about her type. The following lemma characterizes the worker's effort choice.⁴ (All proofs are relegated to the appendix.)

Lemma 1 *If the worker is in a cascade, she will always choose $q_t = 0$. Otherwise, the optimal q_t^A , $A = G, B$, is given by*

$$C'(q_t^A) = 0.5 + U_{t+1}^A(H) - U_{t+1}^A(L) \quad (1)$$

*with $U_{t+1}^A(S_t)$ denoting the equilibrium continuation payoff of type A after signal S_t in period t .*⁵

From Lemma 1 it follows directly that when there is no cascade in period 2, the optimal effort in this period satisfies $C'(q_2^A) = 0.5$. This effort level will be denoted by q^* . In period 1, it holds that the higher the continuation payoff after a good signal and the lower the continuation payoff after a bad signal are, the more effort the worker exerts in the current period.⁶

⁴Both Lemma 1 and Lemma 2 hold for any finite number of periods t , although we will only consider two periods in the analysis.

⁵For this notation to apply to period $t = 2$, define $U_3^A(H) = U_3^A(L) = 0$.

⁶For example, suppose the employer receives a good signal about the worker's type in period 1, he employs the worker, and no up-cascade starts. Then the continuation payoff for type G is given by

$$U_2^G(H) = \alpha[(0.5 + p_{2,(1)} + q^*)0.5 - C(q^*)] + (1 - \alpha)[(0.5 - p_{2,(1)} + q^*)0.5 - C(q^*)]$$

where $p_{2,(1)}$ stands for the employer's choice in the second period after the worker was employed in the first period. The continuation payoff $U_2^G(L)$ is equal to $U_2^G(H)$ if there is no cascade after signal L (because without cascades the employer's choice of signal precision in the second period is independent of the worker's employment history, i.e., $p_{2,(1)} = p_{2,(0)}$, which will be shown below). Analogously, for type B , if no cascade starts after signal H ,

$$U_2^B(H) = \alpha[(0.5 - p_{2,(1)} + q^*)0.5 - C(q^*)] + (1 - \alpha)[(0.5 + p_{2,(1)} + q^*)0.5 - C(q^*)]$$

Now consider the optimal choices of the firms. In the second period, the employer updates his beliefs about the worker's type based on whether she was employed in the first period or not. He chooses the precision of the signal, p_2 , given his beliefs about the worker's type and his beliefs about the effort q_2 chosen by the worker.

Define β_t as the employer's prior probability of a good worker type in period t . For example, before period 1 we have $\beta_1 = 0.5$, which is then updated by the first employer after he receives the signal.

Lemma 2 *Firm t sets $p_t = p^*$ such that $K'(p^*) = 0.5$ and employs the worker after observing $S_t = H$ (and does not employ her after observing $S_t = L$) iff the following three conditions are satisfied:⁷*

$$\beta_t(0.5 - p^* - q_t^G) \leq (1 - \beta_t)(0.5 + p^* - q_t^B) \quad (\text{non } U)$$

$$\beta_t(0.5 + p^* + q_t^G) \geq (1 - \beta_t)(0.5 - p^* + q_t^B) \quad (\text{non } D)$$

$$p^* + \beta_t q_t^G - (1 - \beta_t) q_t^B - 2K(p^*) \geq 0.5 |\beta_t - (1 - \beta_t)| \quad (C)$$

Otherwise, the firm chooses $p_t = 0$, employs the worker if $\beta_t \geq 0.5$, and does not employ the worker if $\beta_t < 0.5$.

Conditions (non U) and (non D) are no-cascade conditions, ensuring that the employer prefers to follow his own signal, given p^* . The third condition, (C), requires the cost $K(p^*)$ to be sufficiently small to make investing into the signal precision worthwhile. Notice that the optimal p^* does not depend on β_t, q_t^G, q_t^B , nor on α .

which, again, is equal to $U_2^B(L)$ when there is no cascade after signal L .

⁷The worker types' respective choices used in the three conditions, q_t^G and q_t^B , are the workers' equilibrium choices, given the history in period t . It follows from Lemma 1 (and from the convexity of $C(\cdot)$) that each worker type's optimal choice is unique in a given equilibrium, so the employer knows q_t^G and q_t^B with certainty.

This simplifies the analysis considerably. Regarding the employer's choice in the first period, it holds that $\beta_1 = 0.5$, and hence the employer always chooses $p = p^*$ as all three conditions of Lemma 2 are satisfied. In particular, condition (C) reduces to $p^* - 2K(p^*) \geq 0.5(q_t^B - q_t^G)$, which is always satisfied as $K(p)$ is convex and the right-hand side is smaller or equal to zero for all equilibrium values of q_t^A .⁸ For the same reason, conditions (*non U*) and (*non D*) are also satisfied as they are both equivalent to $2p^* \geq q_t^B - q_t^G$.

2.3 Equilibria of the game

An equilibrium of this game specifies the agents' behavior after any employment history. In particular, it is possible that the second employer is prescribed to herd behind the first employer's decision only after one employment history, but not after the other. E.g., we use the term "equilibrium with up-cascades only" if firm 2 hires the worker after a history $h_1 = 1$, regardless of his own signal, but follows his own signal if $h_1 = 0$. By analogy, there are three more possible pure-strategy equilibria, characterized by firm 2's behavior: equilibria with down-cascades only, equilibria with up-cascades and down-cascades, and equilibria with no cascades. Of course, there is also the possibility of mixed-strategy equilibria, where firm 2 follows firm 1 only with some probability. Also, a multiplicity of equilibria can arise.

The following proposition shows that there is a general asymmetry between up-cascades and down-cascades in the set of equilibria of the game. (The proposition takes as given the cost functions $C(q_t)$ and $\xi(p_t)$, as well as \bar{q} and \bar{p} , and views α and k as parameters.)

Proposition 1 (a) *There does not exist an equilibrium with up-cascades only, for any parameter constellation.*

⁸In all possible equilibria, $q_t^G \geq q_t^B$, which will be shown below.

(b) For k sufficiently small and α sufficiently close to 1, there is a unique equilibrium with down-cascades only.⁹

The workers' ability to influence the signal probabilities causes this asymmetry between up-cascades and down-cascades, as good signals become less informative than bad signals. In other words, if both workers strive to leave a good impression, the employer's likelihood of facing a good worker after receiving a good signal is lower than the likelihood of facing a bad worker after receiving a bad signal. This effect is reinforced when cascades are possible in equilibrium, because then the signal in the first period becomes more important and the workers increase their efforts further.

A second driving force for the result is the fact that good and bad worker types choose different effort levels when they expect cascades to occur (see Lemma 1). In particular, a good worker loses more from being in a down-cascade than a bad worker does, because without a cascade, the good worker would be more likely to get a good signal in the second period and to be employed. Therefore, a good type chooses a higher effort than a bad type in the first period, $q_1^G > q_1^B$, in the equilibrium with down-cascades only. This implies that a bad signal must come from a bad type with an even higher probability. Conversely, equilibria with up-cascades only are destabilized by the analogous logic. In any such proposed equilibrium with up-cascades only, the bad worker would have more to gain from a good signal in $t = 1$ than the good worker would. Hence, $q_1^B > q_1^G$ would hold, making a good signal even less informative, and the equilibrium breaks down.

If the correlation between jobs is weak, i.e., α is close to 0.5, information cascades can disappear completely. And if the employer's cost of increasing the signal precision is high (large k), equilibria with up-cascades and down-cascades are likely to exist.

⁹For part (b) of the proposition, the assumption $C''' > 0$ is used.

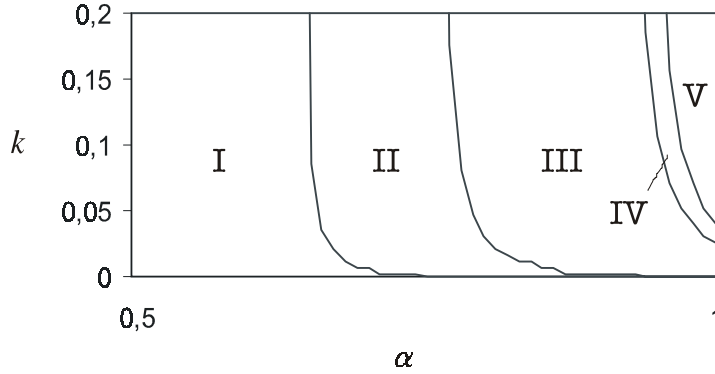


Figure 1: Parameter ranges of equilibria. Region I: Unique equilibrium with no cascades. II: Three equilibria exist; one with no cascades, one with down-cascades only, one mixed equilibrium. III: Unique equilibrium with down-cascades only. IV: Unique mixed equilibrium with a down-cascade after $h_1 = 0$, and a possible up-cascade after $h_1 = 1$. V: Unique equilibrium with up-and-down-cascades.

This is demonstrated in the example below.

2.4 An example

In this subsection, an example is presented for which the equilibria over the whole parameter range are characterized.¹⁰ For this example we suppose $C(q_t) = c[\frac{1}{\bar{q}-q_t} - (\frac{1}{\bar{q}})^2 q_t - \frac{1}{\bar{q}}]$ and similarly $K(p_t) = k[\frac{1}{\bar{p}-p_t} - (\frac{1}{\bar{p}})^2 p_t - \frac{1}{\bar{p}}]$, which are cost functions satisfying the above assumptions. Also, we set $\bar{q} = \bar{p} = 0.25$ and $c = 0.01$. Then, the equilibrium ranges depending on α and k are as illustrated in Figure 1.

Consider Figure 1 and first focus on the case where $\alpha = 1$, i.e., the same abilities are required in the first and in the second period. In this case, there are only two possible equilibria in pure strategies. If the employer's cost parameter k is below a critical level, the equilibrium is characterized by down-cascades only (region III), whereas if the employer's costs are high, both up-cascades and down-cascades occur (region V). Between these two areas, there is a unique mixed equilibrium in which the

¹⁰All calculations are in the appendix.

second period employer randomizes between $p_2 = p^*$ and $p_2 = 0$ with probability s and $(1 - s)$, respectively, if the worker was employed in the first period.¹¹

Moving from $\alpha = 1$ to the left first makes the equilibrium with up-cascades and down-cascades disappear: If the job requirements are less strongly correlated, the second employer does not want to follow all of his predecessor's decisions even if the interview costs are high. The same argument also eliminates the equilibrium with down-cascades for low α , such that only an equilibrium with no cascades exists when job requirements are barely correlated over time. Between the two ranges of unique equilibria, there are parameter values (region II) for which both an equilibrium with down-cascades only and an equilibrium with no cascades exists, as well as a mixed equilibrium in which a bad history induces a down-cascade with some positive probability.

3 Discussion

The paper analyzes the effect of endogenous signal probabilities on information cascades in a labor market setting. In contrast to the benchmark model developed by Bikhchandani, Hirshleifer, and Welch (1992), which corresponds to the limit case $\{\alpha = 1, \bar{q} = 0, \text{ and } k = 0\}$, cascades can start already in the second period, and down-cascades (i.e., cascades in which a worker is not employed) are more important than up-cascades (i.e., cascades in which a worker is employed).

There is a small literature on social learning, wages, and hiring decisions. Within the framework of matching models, Stern (1990) and Lockwood (1991) analyze how firms use the information conveyed by other firms' hiring decisions and condition their

¹¹The worker types choose effort levels contingent on s in this equilibrium. For the sake of a clearer exposition, the formulations of Lemmas 1 and 2 in Subsection 2.2 only apply to pure-strategy equilibria. Optimal behavior in the existing mixed-strategy equilibria, which is analogous, is addressed in the appendix.

decision on the applicant's employment history. Stern shows that wage offers fall with the length of an applicant's unemployment spell. Lockwood derives acceptable periods of unemployment, as the firms in his model choose a cutoff time after which they ignore their private information and never hire a worker. Also, Gibbons and Murphy (1992) and Farber and Gibbons (1996) model how Bayesian learning influences wages as unobservable abilities of workers are revealed over time. Our model differs from all of these models in that we restrict wages to be fixed, but allow workers to optimize, not only firms.

Oberholzer-Gee (2000) focuses on the effect of unemployment spells on the re-employment probability before an information cascade begins. His results from a field experiment indicate that in Switzerland, a person who was unemployed for two and a half years is 47 % less likely to be hired than an employed person, while in the U.S. no unemployment stigma can be observed. Within our model, this can be explained with differences concerning the similarity of the jobs that a person holds during her lifetime in Switzerland, as compared to the United States. In Switzerland (just as in other Western European countries) employees rarely switch to jobs requiring a completely different set of abilities, which is not true for the U.S. Therefore, social learning may be much more important in the Swiss case than in the U.S., and spells of unemployment are more informative in Switzerland than in the U.S.¹²

All of the models mentioned above offer an information-based rationale for the finding that the re-employment probability depends negatively on the duration of unemployment.¹³ The main new feature of this paper is the introduction of endogenous signal qualities, leading to an asymmetry in the occurrence of successful and unsuccessful employment histories. In equilibrium it may well hold that employers do not

¹²Oberholzer-Gee explains his finding by wage subsidies and other measures for the unemployed in Switzerland which create an even stronger stigma for those who still cannot find a job.

¹³Other explanations are that the search intensity of unemployed persons declines over time or that human capital becomes obsolete.

hire a worker who was unemployed regardless of their own signal, but follow their own signal if the worker was employed previously. In this case, previous employment is less of a bonus than previous unemployment is a stigma.

Appendix

Proof of Lemma 1: First, when the worker is in a cascade, she will not exert any effort as the employer will not follow his own signal. Second, the expected utility of a good worker type is

$$U_t^G = (0.5 + p_t + q_t^G)(0.5 + U_{t+1}^G(H)) + (0.5 - p_t - q_t^G)U_{t+1}^G(L) - C(q_t^G)$$

(and similarly of a bad type after switching the sign in front of p_t). Taking the first derivative with respect to q_t yields equation (1). ■

Proof of Lemma 2: The optimal p_t is found by maximizing the employer's expected profit,

$$\Pi(p_t) = \beta_t(0.5 + p_t + q_t^G)0.5 + (1 - \beta_t)(0.5 - p_t + q_t^B)(-0.5) - K(p_t),$$

with respect to p_t , yielding p^* . The first two conditions, (*non U*) and (*non D*), ensure that the employer neither follows his prior belief if the worker was employed, nor if the worker was unemployed, but follows his own signal, given that p^* was chosen. The third condition, (*C*), is derived by comparing the employer's profit from setting $p_t = 0$ or $p_t = p^*$, respectively. If $\beta_t \geq 0.5$, $\Pi(p^*) \geq \Pi(0)$ iff

$$0.5(p^* + \beta_t q_t^G - (1 - \beta_t)q_t^B) - K(p^*) \geq 0.25(\beta_t - (1 - \beta_t)).$$

If $\beta_t < 0.5$, $\Pi(p^*) \geq \Pi(0)$ iff

$$0.5(\beta_t - (1 - \beta_t)) + p^* + \beta_t q_t^G - (1 - \beta_t)q_t^B - 2K(p^*) \geq 0.$$

Combining these two inequalities yields condition (C).

Proof of the Proposition: (a) In the equilibrium with up-cascades only, the worker chooses q_1^A such that $C'(q_1^A) = 1 - U_2^A(L)$. As a good worker's continuation payoff after a bad signal in period 1 is higher than that of a bad worker, $U_2^G(L) > U_2^B(L)$, it follows that $q_1^B > q_1^G$ (and both exceed q^*). In the second period, the worker exerts no effort after being employed in the first period, $q_{2,(1)}^G = q_{2,(1)}^B = 0$, but chooses some effort after not being employed, $q_{2,(0)}^G = q_{2,(0)}^B = q^*$. The employer sets $p_1 = p_{2,(0)} = p^*$ and $p_{2,(1)} = 0$.

First consider a worker with a good history, $h_1 = 1$. The employer's updated prior is $\beta_2 = \frac{\alpha(0.5+p^*+q_1^G)+(1-\alpha)(0.5-p^*+q_1^B)}{1+q_1^G+q_1^B}$. For a proof by contradiction suppose that firm 2 deviates, i.e., sets $p_{2,(1)} = p^*$ and follows its own signal (as the best possible deviation). For the equilibrium to exist, this deviation must not be profitable, which implies that either (C) or (*non U*) or (*non D*) must be violated. Condition (*non U*) requires that $\beta_2(0.5 - p^* - q^*) \leq (1 - \beta_2)(0.5 + p^* - q^*)$. Note that β_2 is increasing in α . (This follows from $0.5 + p^* + q_1^G \geq 0.5 - p^* + q_1^B$, which must hold in equilibrium, because otherwise a good signal would indicate a bad worker, implying $q_1^G = q_1^B = 0$ as a best response.) Thus, if (*non U*) holds for $\alpha = 1$, it must always hold. Some manipulations together with $\alpha = 1$ yield $-q^*(2p^* + q_1^G + q_1^B) \leq (0.5 + p^*)q_1^B - (0.5 - p^*)q_1^G$, which is always true (because $q_1^G > q_1^B$). As (*non D*) holds also after $h_1 = 1$, it follows that the deviation is profitable iff (C) is true.¹⁴ Condition (C) demands that

$$p^* - 2K(p^*) \geq (0.5 - q^*) \frac{2\alpha(2p^* + q_1^G - q_1^B) - 2p^* - q_1^G + q_1^B}{1 + q_1^G + q_1^B}. \quad (2)$$

Suppose that this inequality is violated, which means that an up-cascade exists. But now consider whether this is consistent with no down-cascades after $h_1 = 0$. In partic-

¹⁴In all equilibria, condition (*non D*) [*(non U)*] is trivially satisfied after history $h_1 = 1$ [$h_1 = 0$].

ular, after $h_1 = 0$, condition (C) must be satisfied for down-cascades not to exist:

$$p^* - 2K(p^*) \geq (0.5 + q^*) \frac{2\alpha(2p^* + q_1^G - q_1^B) - 2p^* - q_1^G + q_1^B}{1 - q_1^G - q_1^B}. \quad (3)$$

If (2) is violated, then (3) must be violated, too. This implies that an equilibrium with up-cascades only does not exist.

(b) First, show that an equilibrium with no cascades does not exist for large α and small k . In such an equilibrium, there is no learning. Thus, the worker and the employers choose the same optimal effort level in each period, $q_t = q^*, p_t = p^*, t = 1, 2$. Suppose the worker was not employed in the first period, $h_1 = 0$. Then, the second employer's updated probability for a good type is $\beta_2 = \frac{0.5 - q^* + p^*(1 - 2\alpha)}{1 - 2q^*}$, after some reformulations. Condition (C) of Lemma 2 then requires $p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 + q^*)(2p^*)(1 - 2q^*)$. For $\alpha \rightarrow 1$ and $k \rightarrow 0$ (the latter implying that $p^* \rightarrow \bar{p}$), this condition can hold only if $\bar{p}(1 - 2q^*) > \bar{p}(1 + 2q^*)$, which is never satisfied. Thus, an equilibrium with no cascades does not exist.

Now consider whether there is an equilibrium with up-cascades and down-cascades. Both worker types set q_1^A such that $C'(q_1^A) = 1$, denoted by q^{**} , and $q_2^A = 0$. The employer chooses $p_1 = p^*$ and $p_2 = 0$. This equilibrium does not exist if the employer sets $p_2 = p^*$ after some history h_1 , i.e., if all three conditions of Lemma 2 are satisfied for either $h_1 = 1$ or $h_1 = 0$. For $h_1 = 1$, (*non U*) must hold, i.e., $(0.5 + q^{**} + (2\alpha - 1)p^*)(0.5 - p^*) \leq (0.5 + q^{**} - (2\alpha - 1)p^*)(0.5 + p^*)$, which is satisfied for all α . Also, (*non D*) holds, such that (C) is the only condition remaining to be checked. With $\beta_2 = (0.5 + q^{**} + p^*(2\alpha - 1))/(1 + 2q^{**})$, it is given by $p^* - 2K(p^*) \geq (2\alpha - 1)p^*/(1 + 2q^{**})$, which for $\alpha \rightarrow 1$ and $k \rightarrow 0$ becomes $\bar{p} \geq \bar{p}/(1 + 2q^{**})$. This is always satisfied implying that the equilibrium with up-cascades and down-cascades does not exist.

Finally, the equilibrium with down-cascades only requires that all three conditions of Lemma 2 are satisfied for $h_1 = 1$ and that at least one of them is violated for $h_1 = 0$.

The worker chooses $q_1^G > q_1^B (> q^*)$ in the first period. In the second period she sets $q_{2,(1)}^G = q_{2,(1)}^B = q^*$ after being employed in the first period and $q_{2,(0)}^G = q_{2,(0)}^B = 0$ when she was not employed. The employer tests the worker in the first period and in the second period after a good history, but not after she was unemployed: $p_1 = p_{2,(1)} = p^*$ and $p_{2,(0)} = 0$. After a good history $h_1 = 1$, the second employer's updated prior for a good type is $\beta_2 = ((2\alpha - 1)p^* + 0.5 + \alpha q_1^G + (1 - \alpha)q_1^B)/(1 + q_1^G + q_1^B)$. Condition (*non U*) then requires $\beta_2(0.5 - p^* - q^*) \leq (1 - \beta_2)(0.5 + p^* - q^*)$. Using the fact that $\beta_2 \geq 0.5$, it is sufficient for this condition to hold that $\beta_2 \leq p^* + 0.5$, which can be reformulated, setting $\alpha = 1$ and $p^* = \bar{p}$, to

$$\frac{q_1^G}{q_1^G + q_1^B} \leq \bar{p} + 0.5. \quad (4)$$

Using Lemma 1, we have $C'(q_1^B) > 0.5$. Together with the fact that $C'(q_1^G) - C'(q_1^B) = p^*$ holds in this equilibrium (again from Lemma 1), this can be rewritten as $C'(q_1^B) > \frac{1}{2\bar{p}}(C'(q_1^G) - C'(q_1^B))$. Since C' is an increasing and convex function ($C'' > 0$ and $C''' > 0$), this implies

$$q_1^B > \frac{1}{2\bar{p}}(q_1^G - q_1^B). \quad (5)$$

Reformulating this to $q_1^B > q_1^G/(2\bar{p} + 1)$ and replacing q_1^B in (4) yields as a sufficient condition for (*non U*) that $2\bar{p} + 2 \geq 2$, which is always satisfied. Next, examine condition (*C*) after $h_1 = 1$, which must also hold in an equilibrium with down-cascades only,

$$p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 - q^*) \frac{2p^* + q_1^G - q_1^B}{1 + q_1^G + q_1^B}.$$

For $\alpha \rightarrow 1$ and $k \rightarrow 0$ this will hold if $\bar{p} \geq 0.5(2\bar{p} + q_1^G - q_1^B)/(1 + q_1^G + q_1^B)$, which is equivalent to $q_1^G + q_1^B \geq (q_1^G - q_1^B)/(2\bar{p})$. The latter is always satisfied, as it is implied by inequality (5).

After a bad history, $h_1 = 0$, either (*non D*) or (*C*) must be violated. Consider (*C*):

$$p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 + q^*) \frac{2p^* + q_1^G - q_1^B}{1 - q_1^G - q_1^B}$$

For $\alpha \rightarrow 1$ and $k \rightarrow 0$, this becomes $p \geq (0.5 + q^*)(2\bar{p} + q_1^G - q_1^B)/(1 - q_1^G - q_1^B)$. A simple rearrangement leads to $2\bar{p}(0.5 - 0.5q_1^G - 0.5q_1^B) \geq (2\bar{p} + q_1^G - q_1^B)(0.5 + q^*)$, which can never be satisfied. Thus, the equilibrium with down-cascades only exists. ■

Calculation of the example: In order to generate Figure 1, Lemmas 1 and 2 are used, in a way similar to the proof of the proposition. In particular, the four lines in the figure are given by, from left to right, condition (*C*) after $h_1 = 0$ for the equilibrium with down-cascades, condition (*C*) after $h_1 = 0$ for the equilibrium without cascades, condition (*C*) after $h_1 = 1$ for the equilibrium with down-cascades, and condition (*C*) after $h_1 = 1$ for the equilibrium with up- and down cascades. It can be shown that these are the binding conditions for the pure-strategy equilibria to exist, dividing the parameter space into different equilibrium ranges.

No equilibrium in pure strategies exists when

$$\frac{2p^*}{1 + 2q^{**}} \leq \frac{p^* - 2K(p^*)}{(0.5 - q^*)(2\alpha - 1)} < \frac{2p^* + q_1^G - q_1^B}{1 + q_1^G + q_1^B} \quad (6)$$

(where q^* and q^{**} satisfy, as above, $C'(q^*) = 0.5$ and $C'(q^{**}) = 1$, respectively, and q_1^G and q_1^B are the worker types' effort levels in the equilibrium with down-cascades only, in $t = 1$). In an according equilibrium with mixed strategies (region IV), the firm has to be indifferent between setting $p = 0$ and $p = p^*$ if $h_1 = 1$. This is the case if condition (*C*) holds with equality,

$$\frac{p^* - 2K(p^*)}{(0.5 - \tilde{q}_{2,(1)}(s))(2\alpha - 1)} = \frac{2p^* + \tilde{q}_1^G(s) - \tilde{q}_1^B(s)}{1 + \tilde{q}_1^G(s) + \tilde{q}_1^B(s)} \quad (7)$$

where s is the probability of the employer choosing $p = p^*$, $\tilde{q}_1^G(s)$ and $\tilde{q}_1^B(s)$ are the worker types' optimal effort levels, given s , in $t = 1$, and $\tilde{q}_{2,(1)}(s)$ is the effort of both

worker types in $t = 2$, after $h_1 = 1$. Is there always a unique probability s such that the optimal $\tilde{q}_{2,(1)}(s)$, $\tilde{q}_1^G(s)$, and $\tilde{q}_1^B(s)$ fulfill this equality? To answer this, first define the LHS of (7) as $L(\tilde{q}_2) := \frac{p^* - 2K(p^*)}{(0.5 - \tilde{q}_2)(2\alpha - 1)}$, and the RHS of (7) as $R(\tilde{q}_1^G, \tilde{q}_1^B) := \frac{2p^* + \tilde{q}_1^G - \tilde{q}_1^B}{1 + \tilde{q}_1^G + \tilde{q}_1^B}$. Also, notice that $\tilde{q}_{2,(1)}(0) = 0$, $\tilde{q}_{2,(1)}(1) = q^*$, $\tilde{q}_1^G(0) = \tilde{q}_1^B(0) = q^{**}$, $\tilde{q}_1^G(1) = q_1^G$, and $\tilde{q}_1^B(1) = q_1^B$ (all from Lemma 1), and that $\tilde{q}_{2,(1)}(s)$, $\tilde{q}_1^G(s)$, and $\tilde{q}_1^B(s)$ are continuous in $s \in [0, 1]$, $L(\tilde{q}_2)$ is continuous in \tilde{q}_2 , and $R(\tilde{q}_1^G, \tilde{q}_1^B)$ is continuous in \tilde{q}_1^G and \tilde{q}_1^B . Then, the wanted probability always exists, as the expression $R(\tilde{q}_1^G, \tilde{q}_1^B)$ – the arguments of which move from (q^{**}, q^{**}) to (q_1^G, q_1^B) as s moves from 0 to 1 – intersects the point $(p^* - 2K(p^*)) / [(0.5 - q^*)(2\alpha - 1)]$, according to (6). Hence, L and R need to be equal for some s . To show that s is unique, first note that – analogous to Lemma 1 – the optimal effort levels of the worker types are determined by

$$\begin{aligned} C'(\tilde{q}_1^G(s)) &= 0.5 + s(0.5 + p^* + \tilde{q}_2(s))0.5 + (1 - s)0.5 - C(\tilde{q}_2(s)), \\ C'(\tilde{q}_1^B(s)) &= 0.5 + s(0.5 - p^* + \tilde{q}_2(s))0.5 + (1 - s)0.5 - C(\tilde{q}_2(s)), \\ \text{and } C'(\tilde{q}_{2,(1)}) &= s \cdot 0.5. \end{aligned}$$

Thus, $d(\tilde{q}_1^G(s) - \tilde{q}_1^B(s))/ds > 0$, as $C''' > 0$ and $d\tilde{q}_1^B/ds < 0$. In addition, it holds with $R(\tilde{q}_1^G, \tilde{q}_1^B) = \frac{2p^* + (\tilde{q}_1^G - \tilde{q}_1^B)}{1 + (\tilde{q}_1^G - \tilde{q}_1^B) + 2\tilde{q}_1^B}$ that $\partial R / \partial(\tilde{q}_1^G - \tilde{q}_1^B) > 0$, and therefore $\frac{dR}{ds} > 0$. Similarly, one can check that $\frac{d^2 R}{ds^2} < 0$, $\frac{dL}{ds} > 0$, and $\frac{d^2 L}{ds^2} > 0$ hold, which completes the proof of uniqueness as R and L can therefore be equal for at most one s . Given $\tilde{q}_1^G(s), \tilde{q}_1^B(s), p = p^*$, do conditions (*non U*) and (*non D*) hold? This is equivalent to checking these conditions in the equilibrium with down-cascades after replacing (q_1^G, q_1^B, q^*) by $(\tilde{q}_1^G(s), \tilde{q}_1^B(s), \tilde{q}_2(s))$. Thus, the mixed equilibrium exists and is unique for $C''' > 0$. The mixed equilibrium in region II is derived analogously.

References

- [1] Anderson, L., and C. Holt, 1997. Information cascades in the laboratory. *American Economic Review*, 87, 847-862.
- [2] Banerjee, A., 1992. A simple model of herd behavior. *Quarterly Journal of Economics*, 107, 797-817.
- [3] Bikhchandani, S., D. Hirshleifer, and I. Welch, 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, 100, 992-1026.
- [4] Bikhchandani, S., D. Hirshleifer, and I. Welch, 1998. Learning from the behavior of others. *Journal of Economic Perspectives*, 12, 151-170.
- [5] Farber, H., and R. Gibbons, 1996. Learning and Wage Dynamics. *Quarterly Journal of Economics*, 111, 1007-1047.
- [6] Gibbons, R., and K.J. Murphy, 1992. Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy* 100, 468-505.
- [7] Lockwood, B., 1991. Information externalities in the labour market and the duration of unemployment. *Review of Economic Studies*, 58, 733-753.
- [8] Oberholzer-Gee, F., 2000. Unemployment stigma as rational herding: Hiring practices in Europe and in the United States. *Mimeo*.
- [9] Stern, S., 1990, The effects of firm optimizing behavior in matching models. *Review of Economic Studies*, 57, 647-660.