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Fairness Considerations in Labor Union Wage Setting: A Theoretical Analysis

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ABSTRACT

Fairness Considerations in Labor Union Wage Setting: A Theoretical Analysis

We consider a theoretical model in which unions not only take the outside option into account, but also base their wage-setting decisions on an internal reference, called the fairness reference. Wage and employment outcomes and the shape of the aggregate wage-setting curve depend on the weight and the size of the fairness reference relative to the outside option. If the fairness reference is relatively high compared to the outside option, higher wages and lower employment than in the standard model will prevail. If hit by an adverse technology shock, the economy will then react with a stronger downward adjustment in employment, whereas real wages are more rigid than in the standard model. With a low fairness reference the opposite results are obtained. An increase in the fairness weight amplifies the deviations of wages and employment from those of the standard model. It also leads to an increase in the degree of real wage rigidity if the fairness reference is high and an increase in the degree of real wage flexibility if the fairness reference is low. Thus, higher wages go hand in hand with more pronounced wage stickiness.

JEL Classification: J51, J64, E24

Keywords: labor unions, fairness, wage rigidity, wage flexibility, wage stickiness, wage-setting curve, wage-setting process, unemployment

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1 Introduction

Up to now, most models of unionized labor markets are strictly bound to the assumptions of classical homo economicus and neglect issues of fairness totally, see Pencavel (1991), Flanagan (1993), and Booth (1995). This is in line with the pioneering work of Dunlop (1944) who was the first to model union behavior using the neoclassical framework. Dunlop’s approach got heavily criticized by Ross (1948), who, among other things, considered equity comparisons and fairness to be major issues in union wage determination.

In the light of the available empirical evidence, it is highly likely that fairness in the form of relative comparisons indeed plays an important role for the wage-setting process when unions bargain with firms over wages. Clark and Oswald (1996), using data collected from 5,000 British workers, found evidence that utility does not only depend on absolute income, but also on income relative to a reference level. Based on a sample of 16,000 British workers, Brown et al. (2008) found further evidence for relative pay considerations and, more specifically, for the importance of the ordinal rank of an individual’s wage. Bewley (1999), having interviewed over 200 business executives in the United States, points out the importance of within firm comparisons which matter to workers when assessing the wage paid to be fair or not. Strom (1995) found empirical evidence that Norwegian unions compare wages to an internal reference. Agell and Bennmarker (2003) and (2007) did a representative survey on wage setters in Sweden. According to their results wages are compared to a within firm reference level as well as to the outside option.\(^1\)

Results from experimental economics and psychology also make it evident that it is a too narrow conception of utility if workers, or agents in general, are restricted to care only about material gains. Especially in settings with incomplete contracts, such as labor contracts, the phenomenon of reciprocal behavior based on the notion of fairness plays an important role, see, for example, Fehr et al. (1998), Fehr et al. (2002), Charness (2004), and Falk and Fischbacher (2006).

\(^1\)See Brown et al. (2008) for a more extensive overview.
Up to now, fairness did not enter the labor union literature except some works dis-
cussing union rivalry, see Oswald (1979) and Gylfason and Lindbeck (1984).2 Sessions
(1993) is one of the few exceptions incorporating behavioral assumptions (“status”) in
a wage–setting model. “Few models of unions [...] consider the role of equity concerns
on wage determination. This seems like a particularly egregious omission, however, for
even the most casual acquaintance with collective bargaining teaches one that equity
comparisons are both rife and important” (Kaufman 2002, p. 147).

In the light of the Dunlop-Ross controversy as well as the above mentioned evidence,
it is the key idea of our paper to include fairness considerations into a labor union model
to bring theory closer to real–world wage setting. Workers care about fairness and la-
bor unions have the necessary market power to transform these fairness considerations
into actual market outcomes. By trying to make up for the “egregious omission” we
contribute, over half a century later, to bridge the gap between Dunlop and Ross. We
demonstrate that the inclusion of fairness considerations into a union’s utility function
profoundly changes the workings of the wage–setting process and the reaction of the ag-
gregate economy to macroeconomic shocks. We proceed as follows. The next section
explains how we include fairness into the union’s utility function and how this affects the
labor union’s marginal rate of substitution between wages and employment. Section 3
presents the theoretical model. We first analyze the wage-setting behavior of the union
on the micro level and then discuss the implications for the wage-setting curve and for
labor-market outcomes on the aggregate level. In Section 4 it is analyzed how fairness
modifies the reaction of the economy if hit by an adverse technology shock. Section 5
concludes.

2There exists some literature which pursued the wage reference perspective to explain wage rigidity,
see Pehkonen (1990).
2 Fairness in the Union Utility Function

We consider firm–level labor unions and assume that all employed workers are members of the union. Workers who are dismissed or who voluntarily leave the firm also leave the labor union. Each union member in firm $i$ obtains a rent $\Omega_i$ (measured in terms of utility) that is generated by this employment relationship. Total utility $U_i$ of the labor union is this rent times the number of workers $N_i$ employed at firm $i$:  

$$U_i = N_i \cdot \Omega_i$$  \hspace{1cm} (1)

In traditional union models the rent is equal to the utility differential $\Omega_i^s = u(w_i) - u(\overline{w})$, where the superscript $s$ denotes the “standard model”, $w_i$ denotes the real wage in firm $i$ and $\overline{w}$ some expected alternative income the worker would earn when he or she is not employed at firm $i$. Hence, $\overline{w}$ serves as an external reference wage that is also called the outside option or simply the outside wage. If the earned wage does not exceed this reference, no rent is obtained. In this case the utility of being a union member in the firm under consideration equals zero. In traditional labor union models it is argued that the external reference is all that should matter for workers.

We integrate fairness considerations into this setup by assuming that workers also obtain a utility gain when they perceive the wage paid to be equitable. Noticing that “fairness always seemed to be judged by making some kind of wage comparison” (Rees, 1993, p. 244) and that “comparisons play a large and often dominant role as a standard of equity in the determination of wages under collective bargaining” (Ross, 1948, p. 50), we assume that employees compare their wage with the firm’s output per worker $Y_i/N_i$ to assess whether the firm pays a fair wage. This assumption is in line with Danthine and Kurmann (2007) and Koskela and Schöb (2009) who made a similar assumption within efficiency–wage models.\footnote{In our model all workers and workplaces are identical, so in each firm only one wage is paid. We therefore do not analyze the consequences of a fair or unfair wage structure within a firm, though this...}

\footnote{We do not take the heterogeneity of agents into account, thus neglecting the question of preference aggregation and principal-agent problems within the union.}
To simplify the general equilibrium analysis, union members are assumed to be risk-neutral, implying \( u(w) = w \). However, the qualitative results would not change if preferences were instead represented by a CRRA utility function of the form \( u(w) = \frac{w^{1-\beta}}{1-\beta} \). More specifically, \( \Omega_i \) is defined as

\[
\Omega_i = \Omega_i(w_i, N_i) \equiv \rho \left[ w_i - \nu \cdot \frac{Y_i}{N_i} \right] + (1 - \rho) \left[ w_i - \overline{w} \right],
\]

with \( 0 \leq \rho < 1 \) and \( 0 < \nu < 1 \). Labor is the only variable input of production, hence \( Y_i = Y_i(N_i) \). Notice that with \( \rho = 0 \) the standard model of a rent-maximizing union is obtained, whereas with \( \rho \to 1 \) only the fairness reference matters. With \( 0 < \rho < 1 \) the rent \( \Omega_i \) is a weighted average of a “fairness (or psychological) rent” (the term in the first bracket) and a “material rent” (the term in the second bracket) with \( \rho \) and \( 1 - \rho \) being the respective weights of these utility components. Having workers caring about both, we capture the insight of Agell and Bennmarker (2003, p. 25) that “both internal and external wages are important considerations in the local wage bargain.” Notice that the marginal rate of substitution between material rent and fairness rent depends on the parameter \( \rho \) which is called the “fairness parameter”.

The first term on the right-hand side of eq. (2), according to which workers compare their wage with their contribution to the firm’s output, is in line with Bewley (1999) and Rees (1993) who consider fairness to be a local phenomenon, meaning that wage comparisons are based on a reference which is close by. To ensure non-negative profits, wages can not be higher than average productivity. Employees know that and act rationally in not setting the fairness reference level too high. This is captured by the factor \( 0 < \nu < 1 \) in the definition of the fairness reference in eq. (2). In the following we interpret the parameter \( \nu \) as describing the size of the fairness reference. The fairness reference mirrors the principle of dual entitlement, see Kahneman et al. (1986b). Workers (and firms) behave as if they have an entitlement to the terms of the reference level. If the earned wage is higher than the fairness reference workers derive psychological utility whether or not the wage is low compared to the outside wage. With regard to the second term in eq. (2), also is a relevant aspect of the wage formation process.
Koskela and Schöb (2009, p. 81) ascribe unionization to play a “leading role here since it increases workers’ knowledge about external wages”.

To see the implications of the labor union’s utility function more clearly, the marginal rate of substitution (MRS) between employment and wages is computed:

\[
MRS = \frac{\partial U_i / \partial N_i}{\partial U_i / \partial w_i} = \frac{w_i - \bar{w} - \rho [\varepsilon_{YN} \cdot v \cdot (Y_i/N_i) - \bar{w}]}{N_i},
\]

(3)

where \(\varepsilon_{YN}\) denotes the elasticity of \(Y_i\) with respect to \(N_i\). We assume that the production function is subject to diminishing marginal returns to labor which are important for the workings of the model later on. Because of this assumption the elasticity of output with respect to employment is smaller than one.

The expression in eq. (3) can be easily compared with the marginal rate of substitution in the standard model by setting the fairness parameter \(\rho\) equal to zero. In this case results:

\[
MRS^* = \frac{\partial U_i^* / \partial N_i}{\partial U_i^* / \partial w_i} = \frac{w_i - \bar{w}}{N_i}
\]

(4)

Obviously, the difference in the marginal rate of substitution, and therefore in the slope of the indifference curve, is determined by the marginal utility of employment. In the standard case \(\partial U_i^* / \partial N_i\) denotes the rent \(\Omega_i^*\) which the marginal worker receives. In the general setting it holds that \(\partial U_i / \partial N_i = \Omega_i + N_i \partial \Omega_i / \partial N_i\). Notice that the difference in the rents obtained depends on whether the fairness reference is higher, equal or lower than the outside wage, or in more formal terms \(\Omega_i \leq \Omega_i^*\) if \(v(Y_i/N_i) - \bar{w} \geq 0\). Because the rent \(\Omega_i\) is a positive function of the firm’s employment level, \(\partial U_i / \partial N_i\) denotes not only the rent \(\Omega_i\) which the marginal worker receives, but additionally the change of the rent for all workers already employed. The latter effect arises because an increase in employment leads to a decline in output per worker, thereby lowering the fairness reference level. This leads to an increased differential to the wage paid, thus increasing fairness utility for all workers taken together by \(v \cdot (1 - \varepsilon_{YN}) \cdot (Y_i/N_i)\), where \((1 - \varepsilon_{YN})\) is the elasticity

\footnote{However, they disregard their insight by discussing the importance of the external wage within an efficiency wage model instead of a labor union model.}
of labor productivity with respect to employment in absolute values. Summing up, the difference in the marginal utility of employment does not only depend on the different rents of the marginal worker but also on the effect of a change in employment on the rent of all non-marginal workers. The net effect of the fairness reference on marginal utility of employment is equal to $v \cdot \varepsilon_{YN} \cdot (Y_i/N_i)$. This expression must be compared to the outside option in order to determine whether marginal utility of employment (and therefore the MRS) is higher or lower than the one in the standard model.

For the trade-off between wages and employment therefore the following three cases can be distinguished:

\[ \begin{align*}
MRS &< MRS^* \quad \text{for} \quad v \cdot \varepsilon_{YN} \cdot \frac{Y_i}{N_i} > \bar{w} \quad \text{case 1} \\
MRS &= MRS^* \quad \text{for} \quad v \cdot \varepsilon_{YN} \cdot \frac{Y_i}{N_i} = \bar{w} \quad \text{case 2} \\
MRS &> MRS^* \quad \text{for} \quad v \cdot \varepsilon_{YN} \cdot \frac{Y_i}{N_i} < \bar{w} \quad \text{case 3}
\end{align*} \]

In case 1 $\partial U_i/\partial N_i$ is smaller than in the standard case, which leads the union to be willing to give up more employment for an increase in wages. Thus, the indifference curve runs flatter in $w_i - N_i$ space than in the standard case. This case occurs when the fairness reference is of such a size that the rent of the marginal worker $\Omega_i$ plus the change in the fairness utility of all workers already employed is below the standard rent. Cases 2 and 3 can be interpreted analogously. Note that these cases are independent of the fairness weight $\rho$. What matters is the relative size of the fairness reference.

This discussion shows that the trade-off between wages and employment depends on whether and how social norms are included into the analysis. In line with this notion fairness considerations already found their way into the labor market literature, especially in efficiency wage theory, see, for example, Akerlof (1982) and Akerlof and Yellen (1990). More recently, fairness considerations are also taken into account in the international trade literature, see, e.g., Egger and Kreickemeier (2012).
wage model. Koskela and Schöb (2009) expand this model by considering as relevant reference a weighted average of the internal and external perspective. Because of the linear specification of $\Omega_i$ in eq. (2), this is equivalent to our formulation. Nevertheless, we would like to point out that here the references gain a different interpretation, more in line with findings from psychology. References are crucial to perform judgments of fairness, see Kahneman et al. (1986a). The choice of these reference transactions are subject to framing effects (Tversky and Kahneman, 1986 and Kubon-Gilke, 1990) which makes it rather implausible to determine a reference as weighted average of two references. The definition of $\Omega_i$ stresses the assumption that workers derive utility from fairness considerations as well as consumption possibilities with each having a single reference level.\footnote{Of course, the fairness reference influences wages set and therefore has an effect on consumption.} The notion to incorporate material and fairness utility can already be found in a paper of Rabin (1993). Considering the importance ascribed to unions in this context by Agell and Bennmarker (2003) and (2007) or by Koskela and Schöb (2009) this paper, to the best of our knowledge, is the first to explicitly introduce fairness in the union wage–setting process. Furthermore we do not only analyze the implications of the weights of the references, but also consider the impact of the relative size of the fairness reference on the level of wages and employment, and the degree of wage rigidity.

\section{The Model}

\subsection{Wages and employment at the firm level}

The goods market is described by the standard monopolistic competition framework. In the economy is a continuum of firms, indexed by $i \in [0,1]$, each of which has a labor demand function $N_i = N_i(w_i)$ with $\partial N_i / \partial w_i < 0$. Labor unions unilaterally determine wages at the firm level.\footnote{We consider a monopoly union model instead of a bargaining model in order to keep the analysis as simple as possible. A Nash bargaining model would lead to the same qualitative results.} The maximization of union utility in eq. (1) and eq. (2) subject
to the labor demand function leads to:

$$N_i \left[ \frac{\partial \Omega_i}{\partial w_i} + \frac{\partial \Omega_i}{\partial N_i} \frac{\partial N_i}{\partial w_i} \right] = -\frac{\partial N_i}{\partial w_i} \Omega_i$$  \hspace{1cm} (6)

In the utility maximum the marginal utility of wages (on the left-hand side) equals marginal costs (on the right-hand side). Marginal costs reflect the fact that the dismissed employees loose the rent related to the employment relationship. Marginal utility comprises both a direct and an indirect effect. The direct effect is the increase in the rent $\Omega_i$ for all employees because of the increase in the wage rate. The indirect effect emerges because the resulting decrease in employment increases labor productivity and therefore the fairness reference. As a consequence, the fairness rent decreases for all employees. The indirect effect only appears because of the inclusion of fairness considerations into the analysis. This effect lowers marginal utility of wages and *cet. par.* leads to lower wage pressure in comparison to the standard model. However, the rent $\Omega_i$ that is lost in case of dismissal also differs from the traditional model. For example, if the fairness reference is higher than the outside wage, then $\Omega_i < \Omega^S$, hence marginal cost is lower than in the standard model. In order to see whether the real wage is higher or lower than in the standard model, we rewrite eq. (6) as

$$w_i = \frac{\varepsilon_{NW}}{\varepsilon_{NW} - 1} \left\{ \bar{w} + \rho \left( v \cdot \varepsilon_{YN} \cdot \frac{Y_i}{N_i} - \bar{w} \right) \right\}$$ \hspace{1cm} (7)

where $\varepsilon_{NW}$ denotes the elasticity of labor demand with respect to the real wage (in absolute values). To facilitate the comparison with the standard model (in which $\rho = 0$), we now consider a more specific version of the model in which the labor demand elasticity is constant and therefore does not depend on the real wage. To derive a labor demand function with this property, it is assumed that each firm faces a goods demand function of the form $Y_i = p_i^{-\eta} Y$ with $\eta > 1$, where $p_i$ is the price of the firm’s product relative to the aggregate price level. The elasticity of the demand for goods is constant and equals $\eta$ (in absolute values). The variable $Y$ denotes an index of aggregate output which from the firm’s point of view is taken to be exogenous because of the assumed large number of firms. The production function is $Y_i = A N_i^\alpha$ with $0 < \alpha < 1$, where $A$ describes the
state of technology. The elasticity of output with respect to employment, $\varepsilon_{YN}$, equals the parameter $\alpha$. Profit maximization of the firm leads to the following labor demand ($LD$) function:

$$N_i = N_i(w_i) = \left[\alpha \kappa A^K Y^{1-\kappa} w_i^{-\kappa} \right]^{1/(1-\alpha\kappa)}$$

(8)

where $\kappa \equiv (\eta - 1)/\eta$. As a consequence, the elasticity of labor demand with respect to the real wage is constant and equals $\varepsilon_{NW} = 1/(1 - \alpha\kappa)$ in absolute values. In this case, it only depends on the terms in square brackets in eq. (7) whether the inclusion of fairness considerations lowers or increases wage pressure in comparison to the standard model. This leads to the same case distinctions that have already been derived for the differences in the marginal rate of substitution in eq. (5). It holds that

$$w_i > w_i^s \text{ and } N_i < N_i^s \text{ in case 1: } v\alpha AN_i^{\alpha-1} > \bar{w}$$

$$w_i = w_i^s \text{ and } N_i = N_i^s \text{ in case 2: } v\alpha AN_i^{\alpha-1} = \bar{w}$$

$$w_i < w_i^s \text{ and } N_i > N_i^s \text{ in case 3: } v\alpha AN_i^{\alpha-1} < \bar{w}$$

In Appendix A.1 it is shown that the optimum wage at the firm level is a strictly increasing function of the size of the fairness reference $\nu$, whereas optimum employment is strictly decreasing in that parameter. Moreover, there exists a specific value $\tilde{\nu} \in (0, 1)$ that generates case 2. It can therefore be concluded that, depending on the parameter $\nu$, all three cases represent possible outcomes at the firm level. Note that if wages and employment deviate from the respective levels of the standard model, i.e. if the term in square brackets in eq. (7) is not equal to zero, the fairness parameter $\rho$ amplifies the deviations of wages and employment from those of the standard model. The direction of the change depends on which of the above cases prevails.

3.2 The aggregate wage-setting curve

In equilibrium all prices and wages are identical, thus $p_i = 1$ and $w_i = w$. Workers are homogenous and given by a [0-1] continuum such that $N_i = n$, where $n$ denotes
the employment rate. In order to derive the wage-setting equation, the outside option must be specified more precisely. It is assumed that with probability $n$ workers get a job elsewhere in the economy and earn $w$, whereas with probability $(1 - n)$ workers become unemployed and receive unemployment benefits $b$. Utility related to the outside option then is $\bar{w} = nw + (1 - n)b$. Taking account of eq. (7), the labor demand elasticity and the production function introduced in the preceding section, the following equation for the wage-setting curve (WS) can be derived:

$$w = \frac{\rho \alpha A n^{\alpha - 1} + (1 - \rho)(1 - n)b}{\alpha \kappa - (1 - \rho)n}$$

Wages are set as markup on the fairness reference and unemployment benefits. In contrast, in the standard model (with $\rho = 0$) wages are set as markup on unemployment benefits only:

$$w_s = \frac{1 - n}{\alpha \kappa - n}$$

In the standard model the wage-setting curve approaches an asymptote at $n_{\text{max}}^s = \alpha \kappa < 1$ that constitutes an upper bound for employment. In contrast, in the fairness model the WS curve reaches an asymptote at $n = \frac{\alpha \kappa}{(1 - \rho)}$. If $0 < \rho < 1 - \alpha \kappa$, this asymptote defines the upper bound for the employment rate, denoted $n_{\text{max}}$. In the complementary case, $1 - \alpha \kappa \leq \rho < 1$, the asymptote of the WS curve is not a binding constraint for the employment rate, because it must hold that $n \leq 1$. In this case $n_{\text{max}} = 1$.

Computing the slope of the wage-setting curve defined in eq. (9), one obtains:

$$\frac{\partial w}{\partial n} = \frac{\left[(2 - \alpha)(1 - \rho) - \alpha \kappa (1 - \alpha) \frac{1}{n}\right] \rho \alpha A n^{\alpha - 1} + [(1 - \rho) - \alpha \kappa](1 - \rho)b}{[\alpha \kappa - (1 - \rho)n]^2}$$

It is easy to see that in the standard model the wage-setting curve is positively sloped over the whole range of admissible employment rates $0 < n < n_{\text{max}}^s$. In contrast, in the fairness

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9The assumption that the employment probability equals the employment rate is made for simplicity. Beissinger and Egger (2004) consider an intertemporal model and show for various benefit systems that the employment probability in the steady state is a more complicated function of the (un)employment rate.
model the wage-setting curve follows a U–shaped pattern if the fairness parameter \( \rho \) is not too high, see Appendix A.2 for details. For very high values of the fairness parameter it is even possible that the wage-setting curve is negatively sloped over the whole range of admissible employment rates.\(^{10}\) In Appendix A.2 it is also shown that the employment rate, at which the U-shaped wage-setting curve reaches its minimum, rises with higher values of the fairness parameter \( \rho \) and the size of the fairness reference \( v \). The intuition behind the U–shape is the following. We have two wage references in the model: the outside option and the fairness reference. At low levels of employment average productivity is high and so is the fairness reference. This leads unions to demand high wages. As employment increases, average productivity declines and so does the fairness reference which leads to more moderate wage demands. This explains the downward sloping section of the WS–curve. However, the higher employment the higher is the outside option since the chance to find a job elsewhere in the economy increases. This effect leads unions to demand higher wages again. This explains the upward sloping section of the WS–curve.\(^{11}\)

Given a not too high value of \( \rho \), an increase in \( v \) leads to a more pronounced U–shape of the wage-setting curve. This is due to the fact that the mechanism just described is stronger the higher the value of the fairness reference is.

Turning to the location of the wage-setting curve, the impact of a change in \( v \) is computed as:

\[
\frac{\partial w}{\partial v} = \frac{\rho \alpha A n^{\alpha - 1}}{\alpha \kappa - (1 - \rho) n} > 0.
\]

Hence an increase in the size of the fairness reference, \( v \), leads to an upward shift of the wage-setting curve. For an increase in the fairness parameter \( \rho \) one obtains

\[
\frac{\partial w}{\partial \rho} = \frac{v \alpha A n^{\alpha - 1} - \overline{w}}{\alpha \kappa - (1 - \rho) n}.
\]

\(^{10}\)It can be shown that the second derivative of the wage-setting equation is positive for all admissible employment rates. As a consequence the WS curve is convex. The proof is available on request.

\(^{11}\)A non–standard shape for the wage-setting curve also results in the model of Sessions (1993). In that model, the relative social pressures associated with unemployment and employment lead to a local U–shape of the WS–curve. Koskela and Schöb (2009) derive a U–shaped WS–curve in their efficiency wage model, too.
where $\bar{w} \equiv nw + (1 - n)b$. The sign of the numerator depends on the three cases that have already been identified at the firm level, see Section 3.1. Taking into account that $w > b$, the numerator is positive for relatively low employment rates, whereas for higher employment rates the numerator is negative. This implies that an increase in $\rho$ causes a clockwise rotation of the wage-setting curve, i.e. for low $n$ the WS curve moves upwards, whereas for high $n$ the WS curve moves downwards. Note that the sign of the numerator also depends on the size of the fairness reference. A higher $\nu$ cet. par. increases the numerator.

### 3.3 General Equilibrium

In the general equilibrium the inverse labor demand function is given by:

$$w = \alpha \kappa An^{\alpha - 1}$$  \hspace{1cm} (14)

Using this equation and eq. (9) for the wage-setting curve, the equilibrium employment rate $n^*$ and equilibrium wages $w^*$ can be determined. In Appendix A.3 some restrictions for the size of the fairness reference $\nu$ are introduced to guarantee that $0 < n < n_{\text{max}}$ and that the fairness reference exceeds unemployment benefits.\(^{12}\) A variation in the size of the fairness reference $\nu$ leads to

$$\frac{\partial n^*}{\partial \nu} = -\frac{\rho \alpha An^{*(\alpha - 1)}}{(1 - \rho)[w^* - b + (1 - \alpha)\frac{(1-n^*)}{n^*}b]} < 0 \hspace{1cm} (15)$$

$$\frac{\partial w^*}{\partial \nu} = -(1 - \alpha)\frac{w^*}{n^*} \frac{\partial n^*}{\partial \nu} > 0 \hspace{1cm} (16)$$

Equilibrium employment falls and wages increase with $\nu$. As has been explained in Section 2, an increase in $\nu$ leads to a smaller marginal utility of employment. This causes rising wage pressure at the firm level, as has been made evident in Section 3.1. Since employment and wages are monotonous functions of $\nu$, there also exists a threshold value $\tilde{\nu}$ that leads to the same wage and employment levels as in the standard model. In Appendix A.4 it is shown that in this model $\tilde{\nu} = \alpha \kappa^2$. There it is also demonstrated that, in

\(^{12}\)It can also be shown that multiple equilibria can be ruled out in this model. The proof is available on request.
line with the findings on the firm level, the following three cases can be distinguished:

\[ w^* > w_s^* \quad \text{and} \quad n^* < n_s^* \quad \text{for case 1} \]
\[ w^* = w_s^* \quad \text{and} \quad n^* = n_s^* \quad \text{for case 2} \]
\[ w^* < w_s^* \quad \text{and} \quad n^* > n_s^* \quad \text{for case 3} \]

Figure 1 depicts the equilibrium using values for \( \nu \) that lead to the three different cases.\(^\text{13}\) Equilibrium wages and employment are given by the intersection of the labor demand curve and the wage-setting curve. It can be seen that an increase in \( \nu \) leads to an upward shift of the wage-setting curve, thereby increasing wages and lowering employment. Moreover it makes the U–shape more pronounced. Note, that figure 1(b), representing case 2, produces the same equilibrium wage employment combination as the standard model. It constitutes the threshold between case 1 and 3.

\[ \text{Figure 1: Variation of the fairness reference on the macro level (}\rho = 0.5) \]

(a) Case 1 (\( \nu = 0.8 \)) (b) Case 2 (\( \nu = 0.648 \)) (c) Case 3 (\( \nu = 0.55 \))

\(^{13}\)We calibrated the model with \( \alpha = 0.8, \kappa = 0.9, A = 1, b = 0.2 \). The calibration results for wages and employment are not meant to describe actual economies, rather to demonstrate in a qualitative way how equilibrium outcomes depend on the three cases identified in this paper. It would certainly require a more complicated model to match the theoretical outcomes with the data.
Turning to a change in the fairness parameter \( \rho \), one obtains

\[
\frac{\partial n^*}{\partial \rho} = \frac{(\alpha k - n^*)v A n^*^{(\alpha - 1)} - (1 - n^*)kb}{-\frac{1}{\alpha}(1 - \rho)[\alpha k - (1 - \rho)n^*][w^* - b + (1 - \alpha)\frac{(1 - n^*)}{n^*} b]} \quad (17)
\]

\[
\frac{\partial w^*}{\partial \rho} = -(1 - \alpha) w^* \frac{\partial n^*}{n^*} \frac{\partial \rho}{\rho} \quad (18)
\]

Our results on the firm level suggest that the fairness parameter \( \rho \) again acts as an amplifier for the deviations of employment and wages from the corresponding values of the standard model. This result is passed on to the aggregate level. It is the numerator in eq. (17) which determines the sign of the partial derivatives. As has been shown in Appendix A.4, the numerator in eq. (17) corresponds to the three cases that have already been identified on the firm level. Hence, it ultimately rests upon \( v \) and the implied cases how \( \rho \) affects employment and wages.

- \( \frac{\partial w^*}{\partial \rho} > 0 \) \( \frac{\partial n^*}{\partial \rho} < 0 \) in case 1
- \( \frac{\partial w^*}{\partial \rho} = 0 \) \( \frac{\partial n^*}{\partial \rho} = 0 \) in case 2
- \( \frac{\partial w^*}{\partial \rho} < 0 \) \( \frac{\partial n^*}{\partial \rho} > 0 \) in case 3

Figure 2 demonstrates how an increase in the fairness parameter increases wages and lowers employment in case 1 (that prevails because of a relatively high fairness reference). In contrast, as shown in Figure 3, in case 3 (i.e. with a relatively low fairness reference) an increase in the fairness parameter has the opposite implications: it leads to lower wages and higher employment.

To summarize, the fairness reference as well as the fairness parameter have a major impact on aggregate outcomes. The size of the fairness reference determines the different cases which decide in which direction equilibrium wages and employment move when the fairness weight increases. The fairness parameter, measuring the weight which unions put on fairness, takes the working mode of an amplifier. The higher the fairness parameter, the more sensitive the WS curve reacts to changes in the size of the reference. As a conclusion, it is not only about \textit{whether}, but also \textit{how} fairness is included into union’s preferences.
4 Macroeconomic implications of technology shocks

This section analyzes how the inclusion of fairness considerations affects the response of wages and employment to macroeconomic shocks. The focus is on adverse technology
shocks that are modeled as a reduction in the parameter $A$. As a consequence, the labor demand curve (LD) shifts downwards, since
\[
\frac{\partial w}{\partial A} \bigg|_{\text{LD}} = \alpha \kappa \eta \alpha^{-1} > 0 \tag{19}
\]
In the standard model this shift in the LD curve is all that happens, because a change in the technology parameter $A$ does not affect the WS curve. The downward-shifting LD curve then leads to a decline in both real wages and employment as shown in Figure 4.\textsuperscript{14}

\textit{Figure 4: Adverse technology shock in the standard model}

Contrary to the standard model, in the fairness model the decline in $A$ also shifts the WS curve downwards, because the technology parameter is part of the fairness reference:
\[
\frac{\partial w}{\partial A} \bigg|_{\text{WS}} = \frac{\rho \upsilon \alpha n \alpha^{-1}}{\alpha \kappa - (1 - \rho) n} > 0 \tag{20}
\]
As is evident from eq. (20), the extent of the WS shift depends on the magnitude of $\rho$ and $\upsilon$, i.e. how much the union cares about fairness and of which size the fairness reference is. Comparing equations (19) and (20) reveals that the shift of the WS curve is less pronounced than that of the LD curve.\textsuperscript{15} It therefore matters for the equilibrium reaction

\textsuperscript{14}Throughout this section the shock is parameterized to $\Delta A = -0.1$.

\textsuperscript{15}The maximal shift of the WS curve is obtained for $\upsilon = \upsilon_{\text{max}}$ (which implies $n \to 0$) and with $\upsilon = \upsilon_{\text{min}}$ (which implies $n \to 1$). This maximal shift equals that of the LD curve. For the definition of $\upsilon_{\text{max}}$ and $\upsilon_{\text{min}}$ see Appendix A.3.
of wages and employment which shape the WS–curve has in or around the equilibrium.

Figure 5: Technology shock for different sizes of the fairness reference ($\rho = 0.5$)

![Figure 5](image)

(a) Case 1 ($\nu = 0.8$)  (b) Case 2 ($\nu = 0.648$)  (c) Case 3 ($\nu = 0.55$)

Figure 5 shows the old and new equilibrium in all three cases. The dashed lines mark the LD curve and the WS curve after the shock. In case 2 in Figure 5 b) the standard model WS–curve (dotted line) is depicted additionally. This shows that in case 2 the equilibrium reaction of wages and employment does not differ from the standard model. As in the previous analysis, case 2 constitutes the threshold between case 1 and 3. In case 1, depicted in Figure 5 a), the reaction in terms of employment is larger and in case 3, depicted in Figure 5 c), the employment response is smaller. For wages it is exactly the other way round. Thus in comparison to the standard model, wages are more rigid in case 1 and more volatile in case 3. This pattern is due to the U–shape of the WS–curve. In order to see this more clearly, in Figure 6 we consider small sections of Figure 5 a) and c).

In case 1 the WS curve is upward sloping, but relatively flat, at the intersection with the LD curve. Depending on the parameter values the WS curve could even be downward sloping. The new equilibrium will therefore lie relatively far to the left compared to the
old equilibrium. The less upward sloping or the more downward sloping the WS–curve in the old equilibrium, the stronger is the employment response and the weaker is the change in wages. Hence, case 1 is characterized by real wage rigidity.

**Figure 6: Shift of the WS–curve**

![Shift of the WS–curve](image)

(a) Case 1 ($\rho = 0.5$, $\nu = 0.8$)  
(b) Case 3 ($\rho = 0.5$, $\nu = 0.55$)

In case 3, in the old equilibrium point the WS–curve is upward sloping and steeper than in case 1. This leads to a weaker reaction in terms of employment and a stronger reaction in wages. In case 3 real wages are even more volatile as they already are in the standard model.

Since the fairness parameter $\rho$ has an impact on the shape of the WS–curve, it also evokes different equilibrium reactions. As depicted in figure 7, the higher the fairness parameter in case 1 the longer is the downward sloping part of the WS and therefore the more the accommodation takes place in terms of employment. Hence, in case 1 wage stickiness increases with the fairness parameter and employment gets more volatile. However, in case 3 wage rigidity decreases with the fairness parameter, see figure 8. If the fairness parameter is high enough as in figure 8 c), there is almost no reaction in employment at all.

The intuition for these results is as follows. An adverse technology shock leads *cet. par.*
Figure 7: Technology shock for different values of the fairness parameter in case 1 ($\nu = 0.8$)

(a) $\rho = 0.3$  
(b) $\rho = 0.5$  
(c) $\rho = 0.7$

Figure 8: Technology shock for different values of the fairness parameter in case 3 ($\nu = 0.55$)

(a) $\rho = 0.3$  
(b) $\rho = 0.5$  
(c) $\rho = 0.7$

to a decrease in the size of the fairness reference. The downward shift of the WS curve and the (stronger) downward shift of the LD curve lead to a decline in wages and employment.

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Table 1: Relative change of wages compared to employment

<table>
<thead>
<tr>
<th>Case distinctions</th>
<th>Case 1 ($v = 0.8$)</th>
<th>Case 2 ($v = 0.648$)</th>
<th>Case 3 ($v = 0.55$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 0.3$</td>
<td>$\rho = 0.5$</td>
<td>$\rho = 0.3$</td>
</tr>
<tr>
<td></td>
<td>2.05</td>
<td>4.30</td>
<td>3.08</td>
</tr>
<tr>
<td></td>
<td>$\rho = 0.5$</td>
<td></td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho = 0.5$</td>
<td></td>
<td>8.01</td>
</tr>
<tr>
<td></td>
<td>$\rho = 0.7$</td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>$\rho = 0.5$</td>
<td></td>
<td>61.00</td>
</tr>
<tr>
<td></td>
<td>$\rho = 0.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In case 1 the wage level is relatively high and the employment rate is relatively low in the initial equilibrium in comparison to the other cases. The reduction in employment then leads to a strong increase in average productivity and to a corresponding strong increase in the fairness reference thereby attenuating the decrease in wages. This effect is getting more pronounced with an increase in the fairness parameter. In case 3 the employment rate is higher than in case 1, because the wage level is lower. A change in the employment rate at this level has a much smaller effect on average productivity and therefore the fairness reference. The overall effect of an adverse technology shock then leads to a significant drop in wages. Again this effect is getting more pronounced if the fairness parameter increases. To summarize, it is evident that the inclusion of fairness considerations does not always lead to real wage rigidity. Depending on the size of the fairness reference it is even possible that wages are more volatile than in the standard
model. Moreover, it turns out that wage pressure and wage rigidity are closely related issues.

Table 1 summarizes the percentage change in wages relative to the percentage change in employment for the three cases. In the standard model as well as in case 2 the percentage change in wages is 4.3 times the percentage change in employment. This marks the threshold. In case 1 wages are more rigid, in case 3 they are more volatile. With an increasing preference for fairness these differences get amplified. As shown in Section 3.3 the same happens to the level of pay. Comparing the fairness model to the standard model results in either higher wages and more rigidity or lower wages and less rigidity depending on the size of the fairness reference and the fairness parameter.

5 Conclusions

The inclusion of fairness considerations leads to a different wage-setting behavior in unionized labor markets in comparison to that predicted by standard labor union models. In our theoretical model unions not only take the outside option of union members into account, but also base their wage-setting decisions on internal factors such as the output per worker produced by the respective firms. It turns out that two parameters are of special importance: first, the “fairness parameter” determining the relative weight of the fairness reference, and second, a parameter determining the size of the fairness reference relative to the size of the outside option.

The predictions of the theoretical model are driven by the fact that fairness considerations change the trade-off between wages and employment for each labor union. Using the standard model as benchmark three cases can be distinguished depending on the size of the fairness reference, i.e. the size of the threshold wage level above that the labor union will enjoy positive “fairness utility”. If the fairness reference is relatively high compared to the outside option, marginal utility of employment is relatively low. In that case, each union is willing to give up more employment for an increase in wages, leading to higher wages and lower employment compared to the standard model without fairness
considerations. If the size of the fairness reference is relatively low, the opposite results are obtained. There also exists a specific value of the fairness reference for which the same results as in the standard model are obtained. If wages and employment differ from the respective levels of the standard model, an increase in the fairness parameter (i.e. the fairness weight) amplifies the deviations of wages and employment from those of the standard model. These results hold for the single union as well as on the aggregate level.

Fairness considerations also change the shape of the aggregate wage-setting curve. In the standard model the wage-setting curve is upward sloping in real wage–employment space. In contrast, in the fairness model the wage-setting curve follows a U–shaped pattern if the fairness parameter is not too high. For very high values of the fairness parameter it is even possible that the wage-setting curve is negatively sloped over the whole range of admissible employment rates. It can also be shown that the employment rate, at which the U–shaped wage–setting curve reaches its minimum, rises with higher values of the fairness parameter and the size of the fairness reference. An increase in the fairness reference leads to an upward shift of the wage-setting curve. In contrast, an increase in the fairness parameter leads to a clockwise rotation of the wage-setting curve.

Given the different possible shapes of the wage-setting curve it can be shown that the economy reacts in different ways to technology shocks. In the standard model an adverse technology shock only shifts the labor demand curve downwards, leading to a reduction in wages and in employment. If fairness considerations play a role, the wage–setting curve shifts downwards as well. The reaction of employment and wages depends on the three cases identified in this paper. If the fairness reference is relatively high, the decline in employment is stronger and the reduction in wages is weaker than in the standard model. In this scenario the inclusion of fairness considerations therefore leads to real wage rigidity that is the more pronounced the higher the fairness weight is. However, fairness considerations do not necessarily imply rigid wages. If the size of the fairness reference is relatively low, a stronger wage reduction and a less pronounced decline in employment compared to the standard model is observed. Again this effect is the stronger the higher the fairness weight is.
To summarize, it does not only matter if labor unions care about fairness, but also how they do. Depending on the size and the weight of the fairness reference either higher or lower real wage rigidity in comparison to the prediction of standard labor union models can be observed. A high degree of wage rigidity and a high level of pay appear to go hand in hand.
A Appendix

A.1 Optimum employment and wages at the firm level

Using eqs. (7) and (8) and taking into account that $\frac{Y_i}{N_i} = AN_i^{\alpha-1}$, $\varepsilon_{NW} = 1/(1 - \alpha \kappa)$ and $\varepsilon_{YN} = \alpha$, optimum employment and wages are given by

\[
(\alpha \kappa)^2 A^\kappa Y^{1-\kappa} N_i^{\alpha-1} = \rho \alpha v AN_i^{\alpha-1} + (1 - \rho) \bar{w}
\]

\[
w_i = \frac{\bar{w} + \rho [\alpha v AN_i^{\alpha-1} - \bar{w}]}{\alpha \kappa}
\]

(21)

(22)

It is evident that the wage is set as markup on the fairness reference and the outside option. In contrast, in the standard model $\rho = 0$, hence the wage is set as markup on the outside option only. Changing the size of the fairness reference $\upsilon$ yields:

\[
\frac{\partial N_i}{\partial \upsilon} = \frac{\rho \alpha AN_i^\alpha}{(1 - \alpha) \rho \alpha v AN_i^{\alpha-1} - (1 - \alpha \kappa)(\alpha \kappa)^2 A^\kappa Y^{1-\kappa} N_i^{\alpha-1}}
\]

\[
\frac{\partial w_i}{\partial \upsilon} = \frac{\rho}{\alpha \kappa} \left[ \alpha AN_i^{\alpha-1} - (1 - \alpha) \alpha v AN_i^{\alpha-2} \frac{\partial N_i}{\partial \upsilon} \right]
\]

The sign of the denominator of $\partial N_i/\partial \upsilon$ is easily determined, since it follows from eq. (21) that $(\alpha \kappa)^2 A^\kappa Y^{1-\kappa} N_i^{\alpha-1} > \rho \alpha v AN_i^{\alpha-1}$. Noticing that $(1 - \alpha \kappa) > (1 - \alpha)$ because $0 < \alpha < 1$ and $0 < \kappa < 1$, the following inequality is also true:

\[
(1 - \alpha \kappa)(\alpha \kappa)^2 A^\kappa Y^{1-\kappa} N_i^{\alpha-1} > (1 - \alpha) \rho \alpha v AN_i^{\alpha-1}
\]

Hence the denominator in $\partial N_i/\partial \upsilon$ is negative, leading to $\partial N_i/\partial \upsilon < 0$ and $\partial w_i/\partial \upsilon > 0$. The threshold value of $\upsilon$ generating case 2 is positive and given by

\[
\tilde{\upsilon} = \frac{\bar{w}}{\alpha AN_i^{\alpha-1}}
\]

Because of monopolistic competition, wages paid in each firm are lower than the marginal product of labor. In Section 3.3 it will be argued that $\bar{w}$ is a weighted average of wages elsewhere in the economy and unemployment benefits. Hence, $\bar{w}$ is lower than average wages and is certainly lower than the marginal product of labor. As a consequence, $\tilde{\upsilon} \in (0, 1)$. 

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A.2 The slope of the aggregate wage-setting curve

The slope of the wage-setting curve in the fairness model with $0 < \rho < 1$ is taken from eq. (11) and rewritten as

$$\frac{\partial w}{\partial n} = \Psi_A \rho v A n^{\alpha-1} + \Psi_b b$$

with $\Psi_A \equiv \alpha \left[ (2 - \alpha)(1 - \rho) - \alpha \kappa (1 - \alpha) \frac{1}{n} \right]$ and $\Psi_b \equiv [(1 - \rho) - \alpha \kappa] (1 - \rho)$. The sign of the slope depends on the sign of the numerator. To determine the sign, two cases must be distinguished.

Case (i): $0 < \rho < 1 - \alpha \kappa$

In this case $\Psi_b > 0$. As has been outlined in Section 3.3, the employment rate is restricted to the interval $0 < n < \alpha \kappa / (1 - \rho)$. Taking the upper limit for $n$ into account, $\Psi_A \geq 0$ for

$$\alpha \kappa (1 - \alpha) \frac{1 - \alpha}{1 - \rho} \leq n < \frac{\alpha \kappa}{1 - \rho}$$

which is sufficient (but not necessary) for $\partial w/\partial n > 0$. Hence in the range defined by eq. (24) the WS curve certainly is upward sloping. If

$$0 < n < \frac{\alpha \kappa}{1 - \rho} \frac{1 - \alpha}{2 - \alpha}$$

then $\Psi_A < 0$, but it may still hold that the slope of the wage-setting curve is positive. However, if $n \to 0$ then $\Psi_A \to (-\infty)$. As a consequence, for low $n$ the negative value of the first term in the numerator dominates implying $\partial w/\partial n < 0$. It can be concluded that the U-shaped wage-setting curve reaches an interior minimum at an employment rate $\tilde{n}$, with $0 < \tilde{n} < \alpha \kappa (1 - \alpha) / [(1 - \rho)(2 - \alpha)]$. Setting the numerator in eq. (23) to zero, implicit differentiation reveals that $\partial \tilde{n} / \partial v > 0$ and $\partial \tilde{n} / \partial \rho > 0$. The higher the fairness parameter $\rho$ and/or the higher the fairness reference $v$, the higher is the employment rate $\tilde{n}$ at which the wage-setting curve reaches its minimum.

Case (ii): $1 - \alpha \kappa \leq \rho < 1$

In this case $\Psi_b \leq 0$. The wage-setting curve is now defined over the whole range $0 < n \leq 1$. If the employment rate is in the range defined by eq. (25) it holds that $\Psi_A < 0$. Notice that
in Case (ii) $\alpha\kappa/(1-\rho) \geq 1$. If $\rho > 1 - [\alpha\kappa(1-\alpha)/(2-\alpha)]$, the upper limit for $n$ in eq. (25) would be greater than one and must therefore be replaced by one. For such high values of the fairness parameter the wage-setting curve would therefore be downward-sloping over all employment rates in the interval $(0,1]$. If $1 - \alpha\kappa < \rho < 1 - [\alpha\kappa(1-\alpha)/(2-\alpha)]$, there is a range of employment rates, namely

$$
\frac{\alpha\kappa}{(1-\rho)} \frac{(1-\alpha)}{(2-\alpha)} \leq n \leq 1,
$$

(26)

where $\Psi_A > 0$ holds. This is a necessary, but not a sufficient condition for a positively sloped wage-setting curve. However, if $n \to 1$ then $\partial w/\partial n > 0$. In this case there is again an interior minimum at an employment rate $\hat{n}$ implying a U-shaped wage-setting curve.

A.3 Parameter restrictions for the general equilibrium

There are two reasons, why the interval for the parameter $\nu$ determining the size of the fairness reference has to be further restricted. First, it would be rather implausible if the fairness reference would be lower than unemployment benefits. Second, it must be guaranteed that the equilibrium employment rate $n$ lies in the interval $(0, 1]$.

With regard to the first reason, we assume that $\nu \alpha n^{\alpha-1} > b$. With that we are in line with the common notion that “comparison [is] always made upward rather than downward” (Rees, 1993, p. 244). Since we get the highest lower bound of the fairness reference for $\lim_{n \to n_{\text{max}}}$, this “Rees Assumption” (RA) leads to $\nu > \nu_{\text{RA}} = (n_{\text{max}})^{1-\alpha} \cdot b/A$. For $0 < \rho < 1 - \alpha\kappa$ the upper bound for employment is given by $n_{\text{max}} = \alpha\kappa/(1-\rho)$. For $1 - \alpha\kappa \leq \rho < 1$ the upper employment threshold is $n_{\text{max}} = 1$.

With regard to the second reason, only in the case $1 - \alpha\kappa \leq \rho < 1$ a restriction for $\nu$ is necessary to guarantee that $n \leq 1$. Since $\partial n/\partial \nu < 0$ (compare Appendix A.1), a decrease in $\nu$ leads to an increase in $n$. There must therefore be a lower bound $\nu_{\text{min}}$ leading to $n = 1$. This lower bound is derived from the macroeconomic equilibrium. Using the labor demand equation (14) and the wage-setting equation (9), in equilibrium it holds that

$$
\alpha\kappa An^{\alpha-1} = \frac{\rho \nu \alpha An^{\alpha-1} + (1-\rho)(1-n)b}{\alpha\kappa - (1-\rho)n}
$$
By solving the implicit equilibrium employment rate at \( \lim_{n \to 1} \) for \( \upsilon \), we get the restriction \( \upsilon > \upsilon^{\text{min}} = (\alpha \kappa - (1 - \rho))\kappa/\rho \) not only assuring that \( n^* < 1 \) but also \( w^* > b \). Notice that in the case \( 1 - \alpha \kappa \leq \rho < 1 \) we therefore have two lower bounds \( \upsilon^{\text{RA}} \) and \( \upsilon^{\text{min}} \). Always the larger of the two is binding.

In addition to the lower bound an upper bound for \( \upsilon < \upsilon^{\text{max}} = \alpha \kappa^2/\rho \) exists for \( \lim_{n \to 0} \). Again this is derived from the equilibrium employment rate. Moreover, we assume, that \( \upsilon \) can not be higher than 1. In the sense of fairness it would be contradictory to demand something higher than the own productivity. Always the lower of the two upper bounds for \( \upsilon \) is binding.

Taken together these limits denote the range of all possible \( \rho - \upsilon \) combinations generating an employment rate between zero and one \( (0 < n^* < 1) \).

### A.4 Case distinctions on the aggregate level

Aggregating the case distinctions of section 3.1 yields the following case distinctions in the general equilibrium.

\[
(\alpha \kappa - n)\upsilon^\alpha An^{-1} > (1 - n)\kappa b \quad \text{case 1}
\]

\[
(\alpha \kappa - n)\upsilon^\alpha An^{-1} = (1 - n)\kappa b \quad \text{case 2}
\]

\[
(\alpha \kappa - n)\upsilon^\alpha An^{-1} < (1 - n)\kappa b \quad \text{case 3}
\]

By rewriting the WS curve from eq. (9) as

\[
w = \frac{1 - n}{\alpha \kappa - n} b + \frac{\rho \alpha [(\alpha \kappa - n)\upsilon^\alpha An^{-1} - (1 - n)\kappa b]}{(\alpha \kappa - n)(\alpha \kappa - (1 - \rho)n)},
\]

the case distinctions can be easily obtained, too. The first part of the equation equals the standard WS curve from eq. (10). The second part already contains the case distinctions in the numerator. Since employment and wages are monotonous functions of \( \upsilon \), it remains to proof that there exists a \( \tilde{\upsilon} \) with \( \max\{\upsilon^{\text{min}}, \upsilon^{\text{RA}}\} < \tilde{\upsilon} < \min\{\upsilon^{\text{max}}, 1\} \) which sets the equation in brackets in equation (27) equal to 0. This is true for all parameter ranges since \( \tilde{\upsilon} = \alpha \kappa^2 \). All three cases prevail on the aggregate level. They can be rewritten in
the following short notation:

\[
\begin{align*}
\min\{v^{\text{max}}, 1\} & > v > \tilde{v} \quad \text{case 1} \\
\tilde{v} & = v \quad \text{case 2} \\
\tilde{v} & > v > \max\{v^{\text{min}}, v^{\text{RA}}\} \quad \text{case 3}
\end{align*}
\]
References


