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# **Common Factors Governing VDAX Movements and the Maximum Loss**

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## **Abstract**

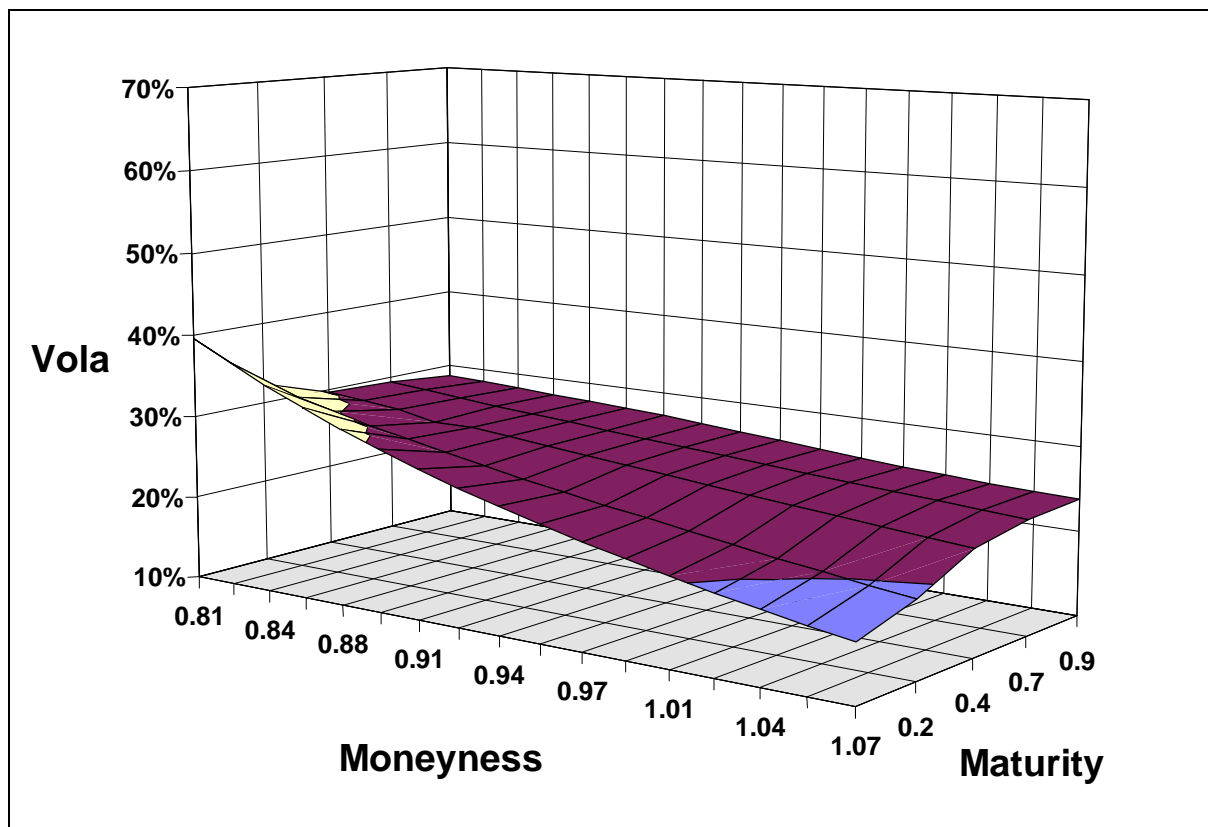
**Based on daily VDAX data this paper analyzes the factors governing the movements of implied volatilities of options on the German stock index DAX. Using Principal Components Analysis over the sample period from 1996 to 1997, we derive common factors representing „shift“ and „curvature“ of the term structure of „at the money“ DAX options. We present a risk management tool for options portfolios using the „Maximum Loss“ methodology based on Principal Components.**

*Keywords:* Implied Volatility, DAX Options, Term Structure Movements, PCA, Maximum Loss

## 1. Introduction

Pricing options and the estimation of real volatility of the underlying asset have an intimate relationship. Classical approaches based on continuous time Brownian motion yield exact analytical formulas, e.g. the *Black & Scholes* (B&S) option pricing formula. Indeed, given maturity  $T$ , strike price  $X$  and interest rate  $r$  the B&S formula is a one-to-one function of the volatility parameter. However, using quoted options prices on the basis of the inverted B&S formula one can find a stylized U-shape of so called implied volatilities across different strikes and maturities. The typical shape of the implied volatility surface for a certain point in time  $t$  is plotted in Figure 1, where “Moneyness” denotes the quotient of the underlyers price  $S$  and the strike  $X$  of the respective option. The volatility surface is a function of time to maturity  $T$  as well, and changes its shape and characteristics as time goes on.

Note that liquid options have discrete strikes and expirations, and so we have interpolated between them to create a continuous surface. For a large number of financial applications the monitoring of this surface is an important element of analysis and prediction.



**Figure 1:** Implied Volatility Surface of the DAX-Option on 18 July 1998

The objective of this paper is to identify common factors affecting implied volatility movements of “at the money” options on the German stock index DAX. We

also provide an intuitive meaning of the relevant risk factors observed in this market and give tests on the stability of these factors over time.

Identifying common factors is important from the perspective of risk management, vega-hedging and volatility trading. As a common practice option traders use delta-gamma neutral hedging strategies in order to establish direct trades on the vega exposure of their option portfolios, see Franke, Härdle and Stahl (2000). The natural technique to identify the number of stochastic shocks that move the implied volatility surface is Principal Components Analysis (PCA), see Skiadopoulos, Hodges and Clewlow (1998). With respect to risk management, PCA has the advantage that the complete term structure can be represented by a small number of variables, i.e. the dimensions of the risk factor space can be drastically reduced.

The options on the DAX are the most actively traded contracts at the German-Swiss derivatives exchange EUREX. Contracts of various strikes and maturities constitute a liquid market at any specific time. This liquidity yields a rich basket of implied volatilities for many pairs  $(X, T)$ . Estimating the first and second order sensitivities of a portfolio with respect to each implied volatility though is almost impossible from a practitioners point of view, given the constraints on accessible computer power that most market participants face.

Even if computers were not a constraint, treating each option implied volatility independent of others in order to estimate the sensitivity of the portfolio to that particular source of risk will result in unstructured non-informative volatility surfaces. This may result in miscalibration of option pricing models, leading to poor valuations and thus inaccurate estimates of sensitivities which may in fact deteriorate the performance of investment strategies. Principal Components Analysis (PCA) will generate smooth volatility structures, resulting in better volatility calibrations and more accurate estimates of portfolio sensitivities, see Fengler, Härdle and Villa (2000).

Our analysis indicates that two factors explain a large proportion of the total variance in the volatility matrix of “at the money” DAX options. As a result of our PCA, we present a risk management tool using the Maximum Loss (ML) methodology based on Principal Components.

The paper is organized as follows. In the next section we present the data and necessary cleaning and correction algorithms. In section 3 we perform the PCA procedure and identify the dominant factor components. In section 4 we discuss the stability of our analysis over time and in section 5 the ideas are applied to ML methodology. The last section presents the conclusions of our research.

## 2. Data Description

The subject of investigation here is implied volatility as measured by the German VDAX-Subindices available from Deutsche Börse AG. These indices, representing different options maturities from one to 24 month, measure volatility implied in European-style calls and puts with strikes equal to the current DAX level, i.e. those options are “at the money” (ATM). The index calculations are based on the assumption that the *Black & Scholes* options pricing formula is a suitable model for the formation of options prices. The B&S formula for a European call at time  $t$  is

$$C_t = S_t N(d_1) - X e^{-rT} N(d_2)$$

where  $N$  denotes the cumulative distribution function of a standard normal random variable and the coefficients are given by

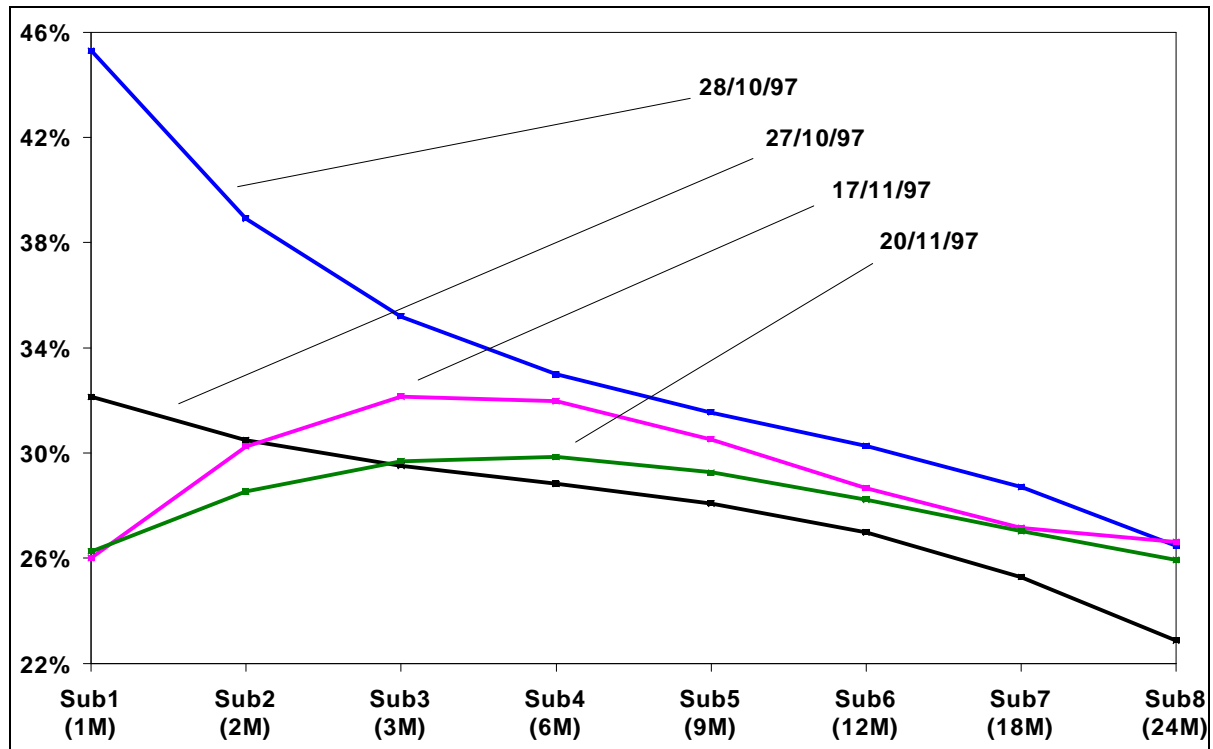
$$d_1 = \frac{\left[ \ln\left(\frac{S_t}{X}\right) + (r + 0,5s^2)T \right]}{s\sqrt{T}}; \quad d_2 = d_1 - s\sqrt{T}$$

Here  $r$  denotes the risk-free interest rate,  $S$  the price of the underlying,  $T = t + t$  the maturity and  $X$  the strike price. For ATM options the strike is  $X=S$ .

The only parameter in the B&S formula that cannot be observed directly is the actual volatility  $s$  of the underlying price process. One may substitute for  $s$  an estimate based on prior observed returns of the underlying. As mentioned before the B&S approach does not reflect option market prices, not even for ATM options. An alternative approach uses *implied volatilities*. The implied volatility is defined as the parameter  $s$  that yields the actually observed market price of a particular option when substituted into the B&S formula. Quoting option prices in terms of implied volatilities does not mean that market participants assume that the B&S formula with all its imperfections applies to the actual market. Instead they just use this formula as a convenient way of describing and quoting call option prices via implied volatilities. The implied volatility of a European put with the same strike and maturity can be deduced from  $P_t = C_t + X e^{-rT} - S_t$ . This relationship is known as “put-call parity”. Any violation of put-call parity gives rise to a pure arbitrage opportunity.

Implied volatilities can be calculated by iterative numerical techniques and may be used to monitor the market’s opinion about the volatility of a particular price process. The implied B&S volatility is not equal to the actual volatility  $s$  but may reflect in some way the expectations of market participants with respect to the future volatility of the underlying price process. Nevertheless, the links between actual and implied volatilities depend on certain theoretical assumptions and are of complex nature. For theoretical approaches of that kind we refer to Schönbucher (1998), Härdle and Hafner (1999).

Implied volatility estimates may be obtained by Deutsche Börse AG, which has been delivering daily closing prices of VDAX-subindices for maturities of 1, 2, 3, 6, 9, 12, 18 and 24 month since 18 March 1996. At that date long term options started trading at EUREX. The term structure for ATM DAX options can be derived from VDAX-subindices for any given trading day. Typical shapes are plotted in Figure 2, with fitted interpolation polynomials for illustration. Figure 2 clearly shows that the term structure at different times shifts and changes its shape.



**Figure 2:** Term Structure of ATM DAX Implied Volatilities

VDAX calculations are based on the assumption that the B&S-formula is a suitable model for the formation of option prices. For a given subindex, implied volatility is estimated from a subset of liquid “near the money” options. The contribution of each implied volatility estimate is not subject to an explicit weighting scheme. Instead, weights are determined implicitly by an ordinary least squares regression yielding an estimate of ATM implied volatility. For more information on VDAX calculation, see Redelberger (1994).

We did not exclusively confine our analysis to the highly liquid short term option contracts which are represented by the subindices 1 to 4. This has been done for two reasons: First, limited trading occurs on certain days in contracts near the liquid ones, thus there might be information in these data entries on those days. Ignoring them completely bears the risk of losing this information. VDAX subindices for long term options on the DAX are available since 18 March 1996. After the expiration day on 19 December 1997 the quality of VDAX data provided by Deutsche Börse AG unfortunately deteriorated in terms of missing data items and frequently

unchanged longer term subindices. For that reason we decided to limit our data base until 19 December 1997.

Second, for our analysis a constant option maturity is needed, instead of floating targets based on EUREX expiration dates. This is vital because changes in volatility arising from changing option maturities (due to the passage of time) will affect any statistical analysis. This observation forces us to use interpolated volatility values, which are most important for short-dated options. To accomplish constant time to maturity we linearly interpolated between neighbouring VDAX subindices. A similar weighting scheme is used by Deutsche Börse AG for the calculation of the “Volatility DAX” (VDAX). For a more detailed discussion on that topic see Redelberger (1994).

With  $\hat{s}_t(T_-)$  and  $\hat{s}_t(T_+)$ , the respective nearby and second nearby implied volatility subindices, we calculated indices  $\hat{s}_t(T_j^*)$  with fixed maturities of  $T_j^* = 30, 60, 90, 180, 270, 360, 540, 720$  calendar days by

$$\hat{s}_t(T^*) = \hat{s}_t(T_-) \left[ 1 - \frac{T^* - T_-}{T_+ - T_-} \right] + \hat{s}_t(T_+) \left[ \frac{T^* - T_-}{T_+ - T_-} \right]$$

Proceeding this way we obtained  $j = 8$  time series of fixed maturity. Each time series is a weighted average of two neighbouring maturities and contains  $n = 441$  data points of implied volatilities. From now on we will refer to fixed maturity when dealing with VDAX-indices.

### **3. Principal Components Analysis of Implied Volatility Dynamics**

In the following section we outline a procedure to extract common factors from historical term structure movements that govern the actual dynamics of implied volatilities. The basic data set for analysis of variations of implied volatilities is a collection of curves like in Figure 2. In order to identify common factors we used Principal Components Analysis (PCA). Changes in the term structure can be decomposed by PCA into a set of factors constituting an orthogonal base.

We computed Augmented Dickey-Fuller (ADF) tests on daily implied volatility indices  $\hat{s}_t(T_j^*)$ , see XploRe (2000). The ADF tests showed that the null hypothesis of instationarity could not be rejected for any of the VDAX-subindices even at the 90% confidence level. Our results suggested to perform Principal Components Analysis on first differences  $x_{jt} = \Delta[\hat{s}_t(T_j^*)]$  of implied volatility indices. First differences seemed to be stationary as an additional ADF analysis indicated.

Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Sub 6	Sub 7	Sub 8
2,08E-04	9,06E-05	6,66E-05	6,84E-05	4,29E-05	2,48E-05	2,11E-05	1,38E-05
9,06E-05	9,86E-05	6,67E-05	4,44E-05	3,21E-05	1,72E-05	1,11E-05	9,25E-06
6,66E-05	6,67E-05	6,43E-05	3,87E-05	2,63E-05	1,49E-05	1,01E-05	5,35E-06
6,84E-05	4,44E-05	3,87E-05	4,23E-05	2,66E-05	1,39E-05	1,38E-05	6,81E-06
4,29E-05	3,21E-05	2,63E-05	2,66E-05	2,62E-05	1,03E-05	1,02E-05	5,15E-06
2,48E-05	1,72E-05	1,49E-05	1,39E-05	1,03E-05	2,19E-05	6,36E-06	3,30E-06
2,11E-05	1,11E-05	1,01E-05	1,38E-05	1,02E-05	6,36E-06	1,76E-05	4,34E-06
1,38E-05	9,25E-06	5,35E-06	6,81E-06	5,15E-06	3,30E-06	4,34E-06	1,52E-05

**Table I:** Empirical Covariance Matrix  $\Omega$  of centered first differences

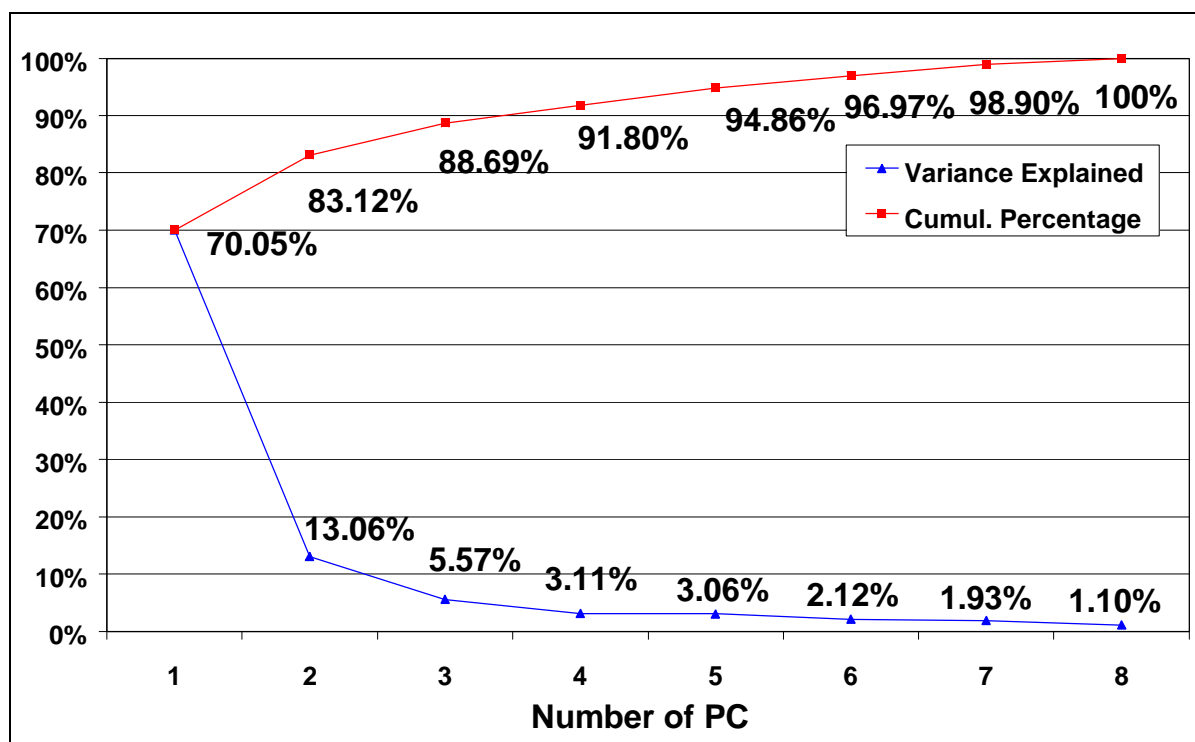
Let  $\bar{x}_j$  denote the sample means of first differences  $x_{jt}$ . Table I shows the empirical covariance matrix  $\Omega$  of centered individuals  $x_{jt}^c = x_{jt} - \bar{x}_j$ . Starting from  $\Omega$  we get the transformation  $\Omega = \Gamma^{Tr} \Lambda \Gamma$  by matrix rotation.  $\Gamma^{Tr} \Lambda \Gamma$  is the *Jordan Canonical Form* with an 8 x 8 diagonal matrix  $\Lambda$  of eigenvalues  $\mathbf{I}_k$ ,  $k = 1, 2, \dots, 8$  and an 8 x 8 matrix of eigenvectors  $\Gamma$ . Time series of principal components are obtained by  $Y = X_c \Gamma$ ;  $X_c$  denotes the 441 x 8 matrix of centered individuals.  $Y$  is the 441 x 8 matrix of principal components. A measure of how well the first  $l$  PCs explain variation of the underlying data is given by the relative proportion of eigenvalues

$$j_l = \frac{\sum_{k=1}^{p \leq 8} \mathbf{I}_k}{\sum_{k=1}^8 \mathbf{I}_k} = \frac{\sum_{l=1}^{p \leq 8} Var(y_l)}{\sum_{l=1}^8 Var(y_l)}$$

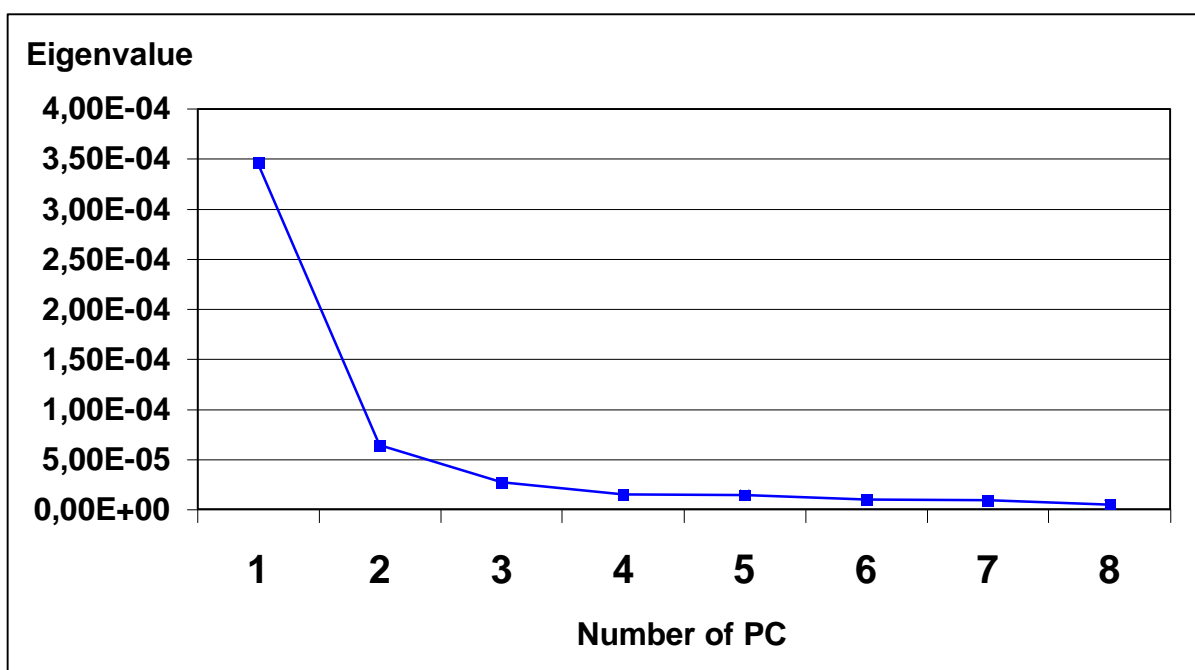
with variance  $j_l$  explained by the first  $l$  Principal Components. Figure 3 presents the proportion of variance and the cumulative proportion of variance explained by the respective principal components.

As is evident from Figure 3, the first PC captures 70% of the total data variability. The second PC captures an additional 13%. The third PC explains a considerably smaller amount of total variation in implied volatility. Thus the two dominant PCs cumulatively explain around 83% of total variance in implied ATM volatilities for DAX options.





**Figure 3:** In-Sample Variation Explained by  $l = 1, 2, \dots, 8$  Principal Components



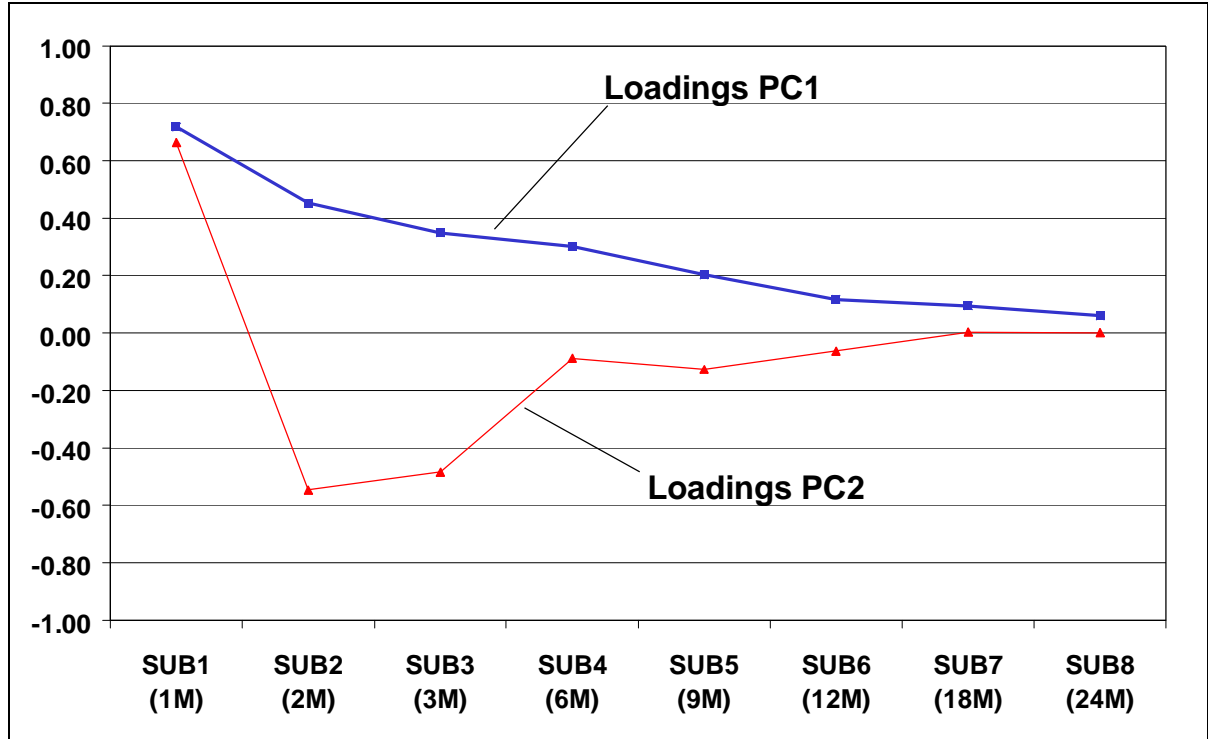
**Figure 4:** PC Eigenvalues, VDAX-Data from 18/03/1996 to 19/12/1997

The plot of eigenvalues in Figure 4 shows that there is an “elbow” like shape at PC No. 2. With respect to the eigenvalues the so called “elbow criterion” suggests to retain the first two components which account for more than 80% of total data variability. The remaining residual information may be interpreted as unsystematic noise.

Figure 5 presents the factor loadings for the first two PCs. Taking orthogonality of Principal Components into account these loadings can be estimated by least squares from the single equation factor model

$$x_{jt}^c = \sum_{l=1}^2 b_{jl} y_{lt} + e_t,$$

where  $e_t$  is an i.i.d. nuisance term.



**Figure 5:** Factor Loadings for First and Second Principal Component

Obviously, a shock on the first factor tends to affect all maturities in a similar manner, causing a non-parallel shift. A shock in the second factor has a strong positive impact on the front maturity but a negative impact on the longer ones, thus causing a change of curvature in the term structure of implied volatilities. These results are a little bit at odds with Sylla and Villa (1999) who applied PCA on the implied ATM volatilities derived from traded options on the CAC 40-index. The authors found a pronounced Z-shape in the factor loadings which they explain by a distinct market segmentation for long term and short term options. A similar effect was not readily observable in the DAX options market in 1996 and 1997.

#### **4. Stability Analysis**

In order to be able to use PCs for risk management, we need to investigate the stability of this procedure over time. If PCs change shape frequently, they will not be

able to explain much of the future variation in implied volatility, and would therefore be ineffective for risk management purposes.

Our stability test extends in two directions: First, daily data may be noisy because traders often consider different sections of the DAX options implied volatility matrix as distinct from each other, sometimes updating options quotes in the more liquid shorter maturities while ignoring the longer ones. Sections of the DAX options market with little or no order flow during the day thus may include stale data during, and possibly at the end of, the trading day. We estimated Principal Components using weekly data rather than daily. If PCs are stable, one would not expect to find substantially different results with daily versus weekly data.

Second, we divided the data set into two non-overlapping periods of equal length. Each period contains  $m = 220$  daily observations of centered first differences of implied VDAX indices. We then conducted pairwise testing of  $\mathbf{I}_{k,y}$  with  $k=1,2$  and  $y = 1,2$  to see if the eigenvalues changed appreciably over time.

PC No.	1	2	3	4	5	6	7	8
<b>Weekly: 18/03/96-19/12/97</b>	<b>73.85%</b>	<b>91.59%</b>	<b>95.09%</b>	<b>97.38%</b>	<b>98.80%</b>	<b>99.61%</b>	<b>99.88%</b>	<b>100%</b>
<b>Daily: 18/03/96-19/12/97</b>	<b>70.05%</b>	<b>83.12%</b>	<b>88.69%</b>	<b>91.80%</b>	<b>94.86%</b>	<b>96.97%</b>	<b>98.90%</b>	<b>100%</b>
<b>Sub 1: 18/03/96-05/02/97</b>	83.36%	91.84%	94.65%	96.39%	97.76%	98.78%	99.52%	100%
<b>Sub 2: 05/02/97-19/12/97</b>	68.22%	82.21%	87.99%	91.35%	94.64%	96.93%	98.86%	100%

**Table II:** Historical PC Analysis: Cumulative Percentages of Variance Explained

As mentioned before, daily data may be noisy compared to weekly data items. To reduce the potential non-synchronicity in the data due to low liquidity in the long maturities, we checked our analysis using weekly data. As is clear from Table II, the in-sample proportion of variance explained by the first two PCs in weekly data is higher than in daily data by around 8.5 percentage points. This is not surprising, since one would expect fewer unsystematic volatility movements in (higher quality) weekly data. In fact weekly results are qualitatively very similar to daily results, so we do not further discuss them here.

As stands out in Table II, the proportion of explained daily data variability explained by the first two PCs diminished over the second subperiods. In light of this, we needed to test for stability of Principal Components over time: We computed eigenvalues over each subperiod. The estimated eigenvalues can be considered as independent. Hence, a two-sided confidence interval for some pair of eigenvalues is given by

$$\ln \mathbf{I}_{k,1} - 2q_a \sqrt{\frac{2}{m-1}} \leq \ln \mathbf{I}_{k,2} \leq \ln \mathbf{I}_{k,1} + 2q_a \sqrt{\frac{2}{m-1}},$$

where  $q_a$  denotes the  $a$ -quantil of a standard normal distribution, see Härdle and Simar (2001, Chap 10). We immediately have

$$|\ln \mathbf{I}_{k,1} - \ln \mathbf{I}_{k,2}| \geq 2q_a \sqrt{\frac{2}{m-1}}$$

as a two-sided test for  $H_0 : \mathbf{I}_{k,1} = \mathbf{I}_{k,2}$ . The null hypothesis implies equality of eigenvalues in both subperiods and would thus be rejected if the above expression holds at any statistically significant level.

P-values for the null hypothesis are 0,313 (90%), 0,373 (95%) and 0,490 (99%). The estimated differences for eigenvalues  $(\mathbf{I}_{1,1}; \mathbf{I}_{1,2})$  and  $(\mathbf{I}_{2,1}; \mathbf{I}_{2,2})$  are 0,667 and 1,183 respectively, and hence are both significant different from zero at the 99% level. Our analysis indicates that the common factors governing implied volatility movements measured by the German VDAX are not stable over time. With respect to this result we suggest some adaptive estimation technique as proposed by Härdle, Spokoiny and Teyssiere (2000) for risk management purposes. In the next section we outline a method to determine the maximum loss of option portfolios on a day by day basis.

## **5. Measuring Implied Volatility Risk using Principal Components Analysis**

One of the key issues of portfolio management is the measurement of inherent market risk. Monte-Carlo simulation techniques can be used to assess the risks of highly non-linear portfolios. Unfortunately these techniques are computationally expensive and time consuming. In the following we introduce an approach to approximate the maximum loss of Delta-Gamma hedged options portfolios analytically.

The market value  $P$  of a portfolio consisting of  $w$  single options on the same underlying is susceptible to changes in interest rates, the price of the underlying, time to maturity and implied volatilities. The Taylor series expansion of the change in market value from time  $t-1$  to time  $t$  is

$$\Delta P_t \approx \sum_{u=1}^w \left[ \frac{\partial O_u^{(t)}}{\partial \mathbf{s}_{impl}^{(t)}(T_u)} \Delta \mathbf{s}_{impl}^{(t)}(T_u) + \frac{\partial O_u^{(t)}}{\partial t} \Delta t + \frac{\partial O_u^{(t)}}{\partial r_t} \Delta r_{T_u}^{(t)} + \frac{\partial O_u^{(t)}}{\partial S_t} \Delta S_t + \frac{1}{2} \frac{\partial^2 O_u^{(t)}}{\partial S_t^2} (\Delta S_t)^2 \right],$$

where  $O_u^{(t)}$  denotes the price of a particular option with maturity  $T_u$  in  $t$ . Interest rates may be assumed to be constant over short investment periods.

As a common practice traders directly trade the so called ‘‘Vega’’ of their portfolios, which is the portfolio sensitivity to the implied volatility. For European-style options on an underlying asset paying no dividends Vega can be derived from the B&S formula. In order to establish Vega trades market professionals use Delta-

Gamma neutral hedging strategies which are insensitive to changes in the underlying and are insensitive to time decay on a regular basis, see Taleb (1997).

A popular strategy to exploit changes of implied volatility term structure is buying and selling of “straddles” of different option maturities at the same time. A straddle consists of the same number of “at the money” calls and puts. For instance, when a trader expects the short term implied volatilities to rise relatively to the longer ones, she will buy straddles in short term contracts and simultaneously sell straddles in longer maturities. The resulting portfolio will be Delta-Gamma neutral and nearly Theta-neutral over short periods of time (i.e. nearly insensitive to time decay on a day-by-day basis). Hence, the Taylor series simplifies to

$$\Delta P_t \approx \sum_{u=1}^w \left[ \frac{\partial O_u^{(t)}}{\partial \mathbf{s}_{impl}^{(t)}(T_u)} \Delta \mathbf{s}_{impl}^{(t)}(T_u) \right]$$

Principal Components Analysis allows us to write the differences of implied volatilities as a linear combination of PCs. Thus, taking nearby fixed maturity subindices  $\hat{\mathbf{s}}_t(T_j^*)$  as proxies for  $\mathbf{s}_{impl}^{(t)}(T_u)$ , we get

$$\Delta P_t \approx \sum_{u=1}^w \left[ \frac{\partial O_u^{(t)}}{\partial \mathbf{s}_{impl}^{(t)}(T_u)} \left( \sum_{l=1}^8 b_{jl} y_{lt} \right) \right]$$

In the following we present a risk management tool for options portfolios using the „Maximum Loss“ methodology based on Principal Components:

Maximum loss analysis is based on the probability distribution of a change in the value of a portfolio attributable to a change in fundamentals over a short period of time. Since implied volatilities make up the fundamentals that determine the prices of delta-gamma hedged derivatives portfolios, we require an accurate characterization of the future probability distribution of volatilities of various-maturity options portfolios.

Maximum loss is defined as the maximum possible loss

- over a given risk factor space  $A_t$ , where  $A_t$  will be assumed a closed set with confidence level  $\Pr(y|y \in A_t) = \mathbf{a}$
- for some holding period  $t$ .

In order to maintain Delta-Gamma neutrality, dynamic hedgers usually revise their derivatives portfolios after short periods of time.  $t = t - t(-1)$  may be a realistic assumption from a practitioners point of view, which implies a portfolio revision on every single day.

The ML-definition looks similar to the well known Value-at-risk (VaR) definition at first glance. However, there is one important difference. Whereas VaR calculation requires the return distribution to be known, ML is defined directly in the risk factor space and thus has an additional degree of freedom called “trust region”. For a detailed discussion of maximum loss, see Studer (1995).

In our analysis we decomposed the term structure of implied volatilities into a set of two principal components that explain an essential part of term structure variance. Hence, the first two PCs represent the risk factors we use for maximum loss modelling. The portfolio profit and loss (P&L) surface is constructed with the help of a suitable valuation model (B&S in this case). To accomplish this, the whole portfolio has to be re-valuated for every point in the factor space. This is approximated by marking the portfolio to the market at discrete points  $y_l^{z_l}$  ( $z_l = 1, \dots, N_l$ ) while keeping the other risk factors constant. Because of the orthogonality of PCs the P&L function is strictly additive with  $PL(y_1^{z_1}, y_2^{z_2}) = PL(y_1^{z_1}) + PL(y_2^{z_2})$ . Thus, the one-dimensional grid-values can be extrapolated onto a two-dimensional grid. The P&L of points not lying on the grid may be obtained by interpolation procedures.

Assuming multinormal distributed PCs, trust regions can be constructed using the joint density function

$$f_t(y) = \frac{1}{(2\pi)^{1/2} \sqrt{\det \Lambda_2}} \exp(-0.5 y^{Tr} \Lambda_2^{-1} y),$$

where matrix  $\Lambda_2$  is a 2 x 2 diagonal matrix of eigenvalues  $\mathbf{I}_k$ ,  $k = 1, 2$  and  $y = (y_1, y_2)^{Tr}$ . It can be shown that  $y^{Tr} \Lambda_2^{-1} y = X_1^2 + X_2^2$  results in a random variable which is Chi-square distributed with 2 degrees of freedom, see Studer (1995). Hence, a valid trust region for our portfolio is obtained by the equation for a centered ellipse  $A_t = \{y | y^{Tr} \Lambda_2^{-1} y \leq c_a\}$ , where  $c_a$  denotes the  $a$ -quantil of a Chi-squared distribution with 2 degrees of freedom.

## **6. Summary**

In this paper we outlined a procedure for using principal components analysis to determine the maximum loss of option portfolios bearing Vega exposure. The term structure of implied volatilities “at the money” is decomposed into two orthogonal factors that are then used to determine the price sensitivity of ATM DAX options. In the last section of our paper we proposed a parsimonious way of determining the maximum loss of a derivatives portfolio whose primary source of risk is that associated with the term structure of implied volatilities. Financial institutions may find our maximum loss approach useful in monitoring the Vega exposure of their investment positions and in setting margin requirements for clients who trade with them.

Stability analysis indicated that the common factors governing implied volatility movements of DAX options may vary over time. With respect to this, an adaptive approach to PC estimation should be applied for risk management purposes. We leave this topic to further research.

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## **References**

- Fengler, M., Härdle, W. and Villa, Ch. (2000) Common Principal Components Analysis and the Dynamics of Implied Volatilities, *Discussion Paper, Sonderforschungsbereich 373, Humboldt University Berlin*.
- Franke, J. / Härdle, W. and Stahl, G. (eds) (2000) Measuring Risk in Complex Stochastic Systems, *LNS 147. Springer Verlag, New York*.
- Härdle, W. and Ch. Hafner (1997) Discrete Time Option Pricing with Flexible Volatility Estimation, *Discussion Paper, Sonderforschungsbereich 373, Humboldt University Berlin*.
- Härdle, W. and Simar, L. (2001) Applied Multivariate Statistical Analysis, *Springer Verlag Heidelberg, Heidelberg*.
- Härdle, W., Spokoiny, V. and Teyssiere, G. (2000) Adaptive Estimation for a Time Inhomogeneous Stochastic-Volatility Model, *Discussion Paper No. 6, Sonderforschungsbereich 373, Humboldt University Berlin*.
- Redelberger, T. (1994) Grundlagen und Konstruktion des VDAX-Volatilitätsindex der Deutsche Börse AG. *Deutsche Börse AG*.
- Schönbucher, J. (1998) A Market Model for Stochastic Implied Volatility, *Discussion Paper, Department of Statistics, Bonn University*.
- Skiadopoulos, G., Hodges, S. and Clewlow, L. (1998) The Dynamics of Implied Volatility Surfaces, *Working Paper, Financial Options Research Centre, Warwick Business School University of Warwick, Coventry*.
- Studer, G. (1995) Value at Risk and Maximum Loss Optimization, *Discussion Paper, ETHZ, RiskLab: Technical Report*.
- Sylla, A. and Villa, Ch. (1999) Measuring Implied Surface Risk using PCA, *Franke, J., Härdle, W. and Stahl, G. (eds) (2000) Measuring Risk in Complex Stochastic Systems, LNS 147, Springer Verlag, New York, p 131- 147*.
- Taleb, N. (1997) Dynamic Hedging: Managing Vanilla and Exotic Options, *John Wiley, New York*.
- XploRe Application Guide ([www.i-xplore.de](http://www.i-xplore.de)) (2000) Härdle, W., Hlavka, Z. and S. Klinke (eds), *Springer Verlag Heidelberg, Heidelberg*.