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Null players, solidarity, and the egalitarian Shapley values

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Abstract

The Shapley value certainly is the most eminent single-point solution concept for TU-games. In its standard characterization, the null player property indicates the absence of solidarity among the players. First, we replace the null player property by a new axiom that guarantees null players non-negative payoffs whenever the grand coalition's worth is non-negative. Second, the equal treatment property is strengthened into desirability. This way, we obtain a new characterization of the class of egalitarian Shapley values, i.e., of convex combinations of the Shapley value and the equal division solution. We complement this result by characterizations of the class of generalized consensus values, i.e., of convex combinations of the Shapley value and the equal surplus division solution.

Key Words: Solidarity; egalitarian Shapley value; equal division value; desirability; generalized consensus value

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1. Introduction

A cooperative game with transferable utility (TU-game) consists of a non-empty and finite set of players, N , and a coalition function, $v \in \{f \mid f : 2^N \rightarrow \mathbb{R}, f(\emptyset) = 0\}$, which describes the worths $v(S)$ that can be generated by individuals that are willing to cooperate within coalitions $S \subseteq N$. Assuming that the grand coalition eventually forms, the question arises how to distribute the grand coalition's worth, $v(N)$. The most prominent single-point solution concept that answers this question certainly is the Shapley value (Shapley, 1953).

The Shapley value obeys marginality (Young, 1985), i.e., a player's payoff depends only on his *own* productivity measured by marginal contributions. Hence, the Shapley value does not allow for solidarity between the players. This can also be seen from the fact that unproductive players (null players) are assigned zero payoffs. Therefore, we depart from the standard characterization of the Shapley value by dropping the null player property¹ in order to create space for solidarity (as e.g. Nowak and Radzik (1994), van den Brink (2007), and van den Brink, Funaki and Ju (2011)).

Given that we no longer require the payoffs of null players to be zero, the nature of solidarity manifests itself in the way null players are treated. Since null players are completely unproductive, their “selfish” payoffs, i.e., their Shapley payoffs are zero. Consequently, any non-zero payoff must be due to solidarity among the players. Note that Nowak and Radzik (1994), van den Brink (2007), and Chameni-Nembua (2012) pursue another approach. Instead of dealing with the treatment of null players, they consider other notions of “null players”. In fact, they identify certain types of players that are supposed to obtain a zero payoff.

We suggest the *null player in a productive environment property* that requires a null player to obtain a non-negative payoff whenever the worth generated by the grand coalition is non-negative. This property rules out that one could find oneself in a wealthy society that drains out the unproductive players in order to make other players better off. We feel that this would stretch our sense of solidarity too far. Obviously, this property is implied by the null player property and therefore it is satisfied by the Shapley value.

Our main result states that the null player in a productive environment property together with additivity, efficiency, and desirability already characterizes the class of egalitarian Shapley values introduced by Joosten (1996). Note that desirability

¹Ruiz, Valenciano and Zarzuelo (1998), Driessen and Radzik (2003), Hernandez-Lamonedea, Juarez and Sanchez-Sanchez (2008), Chameni-Nembua and Andjiga (2008), and Chameni-Nembua (2012) provide formulae for the class of efficient, linear, and symmetric values.

demands more productive players to obtain no lower payoffs than less productive ones (Maschler and Peleg, 1966). Other characterizations of this class have been given by Joosten (1996), Malawski (2004), and van den Brink et al. (2011). We complement this result by characterizations of the class of generalized consensus values (Ju, Borm and Ruys, 2007).

This paper is organized as follows.

Section 2 provides further definitions and notations.

Section 3 provides our characterization of the egalitarian Shapley values.

Section 4 provides our characterizations of the generalized consensus values.

The appendix provides all the proofs.

2. Basic definitions and notation

The set of all coalition functions on N is denoted by $\mathbb{V}(N)$. Since we work within a fixed player set, it is dropped as an argument. In particular, we address $v \in \mathbb{V}$ as a game. Subsets of N are called **coalitions**, and $v(S)$ is called the worth of coalition S . For $v, w \in \mathbb{V}$, $\alpha \in \mathbb{R}$, the coalition functions $v + w \in \mathbb{V}$ and $\alpha \cdot v \in \mathbb{V}$ are given by $(v + w)(S) = v(S) + w(S)$ and $(\alpha \cdot v)(S) = \alpha \cdot v(S)$ for all $S \subseteq N$. For $T \subseteq N$, $T \neq \emptyset$, the game $u_T \in \mathbb{V}$, $u_T(S) = 1$ if $T \subseteq S$ and $u_T(S) = 0$ for $T \not\subseteq S$, is called a **unanimity game**. For $T \subseteq N$, $T \neq \emptyset$, the game $e_T \in \mathbb{V}$, $e_T(S) = 1$ if $T = S$ and $e_T(S) = 0$ for $T \neq S$, is called a **standard game**. Any $v \in \mathbb{V}$ can be uniquely represented by unanimity games,

$$v = \sum_{T \subseteq N: T \neq \emptyset} \lambda_T(v) \cdot u_T, \quad \lambda_T(v) := \sum_{S \subseteq T: S \neq \emptyset} (-1)^{|T|-|S|} \cdot v(S).$$

A game $v \in \mathbb{V}$ is called **zero-normalized** if $v(\{i\}) = 0$ for all $i \in N$; for $v \in \mathbb{V}$, the **associated zero-normalized game** $v^0 \in \mathbb{V}$ is given by

$$v^0 := v - \sum_{i \in N} \lambda_{\{i\}} \cdot u_{\{i\}} \quad \text{for all } S \subseteq N. \quad (1)$$

Player $i \in N$ is called a **null player** in $v \in \mathbb{V}$ if $v(S \cup \{i\}) - v(S) = 0$ for all $S \subseteq N \setminus \{i\}$; players $i, j \in N$ are called **symmetric** in $v \in \mathbb{V}$ if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$.

A **value** on N is an operator φ that assigns a payoff vector $\varphi(v) \in \mathbb{R}^N$ to any $v \in \mathbb{V}$. The **equal division value**, ED, is given by

$$\text{ED}_i(v) := \frac{v(N)}{|N|}, \quad \text{for all } i \in N.$$

The **equal surplus division value** (Driessen and Funaki, 1991), ES, is given by

$$\text{ES}_i(v) := v(\{i\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{|N|}, \quad \text{for all } i \in N.$$

The **Shapley value** (Shapley, 1953), Sh , given by

$$\text{Sh}_i(v) := \sum_{T \subseteq N: i \in T} \frac{\lambda_T(v)}{|T|}, \quad \text{for all } i \in N,$$

is characterized by the axioms **E**, **N**, [**A** or **L**], and [**ET** or **S**] below.

Efficiency, E. For all $v \in \mathbb{V}$, $\sum_{i \in N} \varphi_i(v) = v(N)$.

Null player, N. For all $v \in \mathbb{V}$ and every $i \in N$, who is a null player in v , $\varphi_i(v) = 0$.

Additivity, A. For all $v, w \in \mathbb{V}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$.

Linearity, L. For all $v, w \in \mathbb{V}$ and $\lambda \in \mathbb{R}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$ and $\varphi(\lambda \cdot v) = \lambda \cdot \varphi(v)$.

Equal treatment, ET. For all $v \in \mathbb{V}$ and $i, j \in N$, who are symmetric in $v \in \mathbb{V}$, $\varphi_i(v) = \varphi_j(v)$.

Symmetry, S. For all $v \in \mathbb{V}$, $i \in N$, and all bijections $\pi : N \rightarrow N$, $\varphi_{\pi(i)}(v \circ \pi^{-1}) = \varphi_i(v)$.

3. Treatment of null players in a productive environment

The “mixture” of the Shapley value and the equal division value, Sh^α , $\alpha \in \mathbb{R}$, is given by

$$\text{Sh}_i^\alpha(v) = \alpha \cdot \text{Sh}_i(v) + (1 - \alpha) \cdot \text{ED}_i(v), \quad \text{for all } i \in N \text{ and all } v \in \mathbb{V}. \quad (2)$$

For $0 \leq \alpha \leq 1$, the value Sh^α is said to be an egalitarian Shapley value (Joosten, 1996). The egalitarian Shapley values obey efficiency, symmetry, and additivity. For $\alpha \neq 1$, they do not satisfy the null player property.² But, they satisfy the following weaker one.

Null player in a productive environment, NPE. For all $v \in \mathbb{V}$ and $i \in N$ such that i is a null player in v and $v(N) \geq 0$, we have $\varphi_i(v) \geq 0$.

This property guarantees null players to obtain a non-negative payoff whenever the worth generated by the grand coalition is non-negative. This is quite plausible. For we consider deviations from the Shapley payoffs as an expression of a certain degree of solidarity among players. When the whole society is productive, $v(N) \geq 0$, then it is not *necessary* that any player ends up with a negative payoff. In particular, null players need not receive a negative payoff. Since they do not do any harm to society, they actually should not obtain negative payoffs. It turns out that this

²Instead, each Sh^α , $\alpha \in \mathbb{R}$ satisfies the α -egalitarian null player property, stating that $\varphi_i(v) = (1 - \alpha) \cdot v(N) / |N|$ for all null players i in v (Joosten, 1996). In a sense, the parameter α determines the extent of solidarity among the players.

axiom has strong implication on the nature of solidarity among the players within the class of efficient, linear, and symmetric values; one is almost down to the class of egalitarian Shapley values.

Proposition 1. *A TU-value φ satisfies efficiency, linearity, the equal treatment property, and the null player in a productive environment property if and only if there is an $\alpha \leq 1$ such that $\varphi = \text{Sh}^\alpha$.*

The null player in a productive environment property in combination with standard axioms does not rule out that a higher productivity translates into lower pay-offs. For example, the TU-value $\text{Sh}^{-1} = 2 \cdot \text{ED} - \text{Sh}$ satisfies this weak requirement on null players as well as additivity, efficiency and the equal treatment property. Yet, for $1, 2 \in N$, we have $\text{Sh}_1^{-1}(u_{\{1\}}) = \frac{2}{|N|} - 1 < \frac{2}{|N|} = \text{Sh}_2^{-1}(u_{\{1\}})$ even though player 1 is more productive than player 2 in $u_{\{1\}}$. In a sense, the transfers from the productive player, 1, to the unproductive one, 2, are too high. We feel that this is also not in line with sound solidarity considerations. In order to rule out such awkward transfers, we invoke the following axiom.

Desirability³, D. For all $v \in \mathbb{V}$ and all $i, j \in N$, if $v(S \cup \{i\}) \geq v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$, then $\varphi_i(v) \geq \varphi_j(v)$.

Desirability compares two players in a game and ensures that their payoffs are not opposite to their productivities measured by marginal contributions. Replacing the equal treatment property by desirability does not only rule out adverse incentives, but also allows to weaken linearity into additivity.⁴ Next, we state the main result of this paper, a characterization of the class of egalitarian Shapley values.

Theorem 2. *A TU-value φ satisfies additivity, efficiency, desirability, and the null player in a productive environment property if and only if there exists an $\alpha \in [0, 1]$ such that $\varphi = \text{Sh}^\alpha$.*

Our characterization is non-redundant. The solution φ^{A} , given by $\varphi^{\text{A}}(v) = \text{Sh}(v)$ if $v(N) < 1$ and $\varphi^{\text{A}}(v) = \text{ED}(v)$ if $v(N) \geq 1$, satisfies all axioms but additivity. The solution given by $\varphi_i^{\text{E}}(v) = 0$ for all $i \in N$ and $v \in \mathbb{V}$ satisfies all axioms but efficiency. The solution given by $\varphi^{\text{D}} = \text{Sh}^{-1}$ satisfies all axioms but desirability. The solution given by $\varphi^{\text{NPE}} = \text{Sh}^2$ satisfies all axioms but the null player in a productive environment property.

³Desirability is also known as local monotonicity (e.g. van den Brink et al., 2011).

⁴Employing that fact that there are functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ but $f(x \cdot y) \neq x \cdot f(y)$ for some $x, y \in \mathbb{R}$ (Macho-Stadler, Pérez-Castrillo and Wettstein, 2007), it is easy to show that there are solutions that obey additivity, efficiency, and symmetry but not linearity.

Recently, van den Brink et al. (2011) characterize the class of egalitarian Shapley values employing efficiency, linearity, symmetry, and weak monotonicity below.

Weak monotonicity, M^- . For all $v, w \in \mathbb{V}$ and all $i \in N$ such that $v(N) \geq w(N)$ and $v(S \cup \{i\}) - v(S) \geq w(S \cup \{i\}) - w(S)$ for all $S \subseteq N \setminus \{i\}$, we have $\varphi_i(v) \geq \varphi_i(w)$.

This axiom is a relaxation of the well-known strong monotonicity axiom (Young, 1985) in the sense that its implication only is supposed to hold if $v(N) \geq w(N)$. Their characterization and ours are related as follows: van den Brink et al. (2011, Lemma 4.2) show that symmetry and weak monotonicity imply desirability. Moreover, since linearity determines the payoffs in a null game to be zero, it is immediate that linearity and weak monotonicity entail the null player in a productive environment property. Hence, we also provide an alternative proof of their characterization (van den Brink et al., 2011, Theorem 4.3).

4. Equal surplus value and the generalized consensus value

The “mixture” of the Shapley value and the equal surplus division value, Con^α , $\alpha \in \mathbb{R}$, is given by

$$\text{Con}_i^\alpha(v) = \alpha \cdot \text{Sh}_i(v) + (1 - \alpha) \cdot \text{ES}_i(v), \quad \text{for all } i \in N \text{ and all } v \in \mathbb{V}.$$

For $0 \leq \alpha \leq 1$, the values Con^α are called **generalized consensus values** (Ju et al., 2007).

Inspired by the previous section and Casajus and Huettner (2012), we provide a new characterization of the class of generalized consensus values. To this end, we suggest two new axioms, weak surplus monotonicity and null player in a surplus environment property.

Weak surplus monotonicity, SM^- . For all $v, w \in \mathbb{V}$ and all $i \in N$ such that $v^0(N) \geq w^0(N)$ and $v(S \cup \{i\}) - v(S) \geq w(S \cup \{i\}) - w(S)$ for all $S \subseteq N \setminus \{i\}$, we have $\varphi_i(v) \geq \varphi_i(w)$.

This axiom is a cousin of weak monotonicity and a relaxation of the well-known strong monotonicity axiom (Young, 1985). While weak monotonicity restricts strong monotonicity to situations where the overall worth generated does not decrease, weak surplus monotonicity refers to situations where the overall surplus does not decrease. Note that the equal division value violates weak surplus monotonicity for $|N| > 1$. For $1, 2 \in N$, $\text{ED}_1(\mathbf{0}) = 0$ but $\text{ED}_1(-u_{\{2\}}) = -1/|N|$ although $\mathbf{0}^0(N) = -u_{\{2\}}^0(N)$ and 1 is a null player in both in $\mathbf{0}$ and $-u_{\{2\}}$. This counterexample ($-u_{\{2\}}$) also shows that the equal division value fails the next property, which guarantees null

players a non-negative payoff whenever the surplus generated by the grand coalition is non-negative.

Null player in a surplus environment, NSE. For all $v \in \mathbb{V}$ and $i \in N$ such that i is a null player in v and $v^0(N) \geq 0$, we have $\varphi_i(v) \geq 0$.

Replacing null player in a productive environment property with the null player in a surplus environment property or with weak surplus monotonicity in Theorem 2 yields a characterization of the class of generalized consensus values.

Theorem 3. *A TU-value φ satisfies additivity, efficiency, desirability, and either (i) the null player in a surplus environment property or (ii) weak surplus monotonicity if and only if there exists an $\alpha \in [0, 1]$ such that $\varphi = \text{Con}^\alpha$.*

5. Appendix

We prepare the proof of Proposition 1 by a lemma which states that every efficient, linear, and symmetric value can be interpreted as follows: First, the Shapley payoffs for unanimity games u_T are taxed at a rate $\tau_{|T|}$ depending on the cardinality of T . Second, the tax revenue is distributed equally among all players.

Lemma 4. *A TU-value φ on N satisfies efficiency, linearity, and symmetry if and only if there exists an $\tau = (\tau_1, \dots, \tau_{|N|-1}) \in \mathbb{R}^{|N|-1}$ such that $\varphi = \zeta^\tau$ where*

$$\zeta_i^\tau(v) := \sum_{T \subseteq N, T \ni i} \left(\frac{1 - \tau_{|T|}}{|T|} \right) \cdot \lambda_T(v) + \sum_{T \subseteq N, T \neq \emptyset} \frac{\tau_{|T|}}{|N|} \cdot \lambda_T(v) + \frac{\lambda_N(v)}{|N|} \quad (3)$$

for all $i \in N$ and $v \in \mathbb{V}$.

Proof. It is clear that ζ^τ satisfies **E**, **L**, and **S** for all $\tau \in \mathbb{R}^{|N|-1}$. Now, let the TU-value φ satisfy **E**, **L**, and **S**. By **L**,

$$\varphi(v) = \sum_{T \subseteq N, T \neq \emptyset} \lambda_T \cdot \varphi(u_T), \quad \text{for all } v \in \mathbb{V}. \quad (4)$$

Note that by **E** and **S** we have $\varphi_i(u_N) = 1/|N|$ for all $i \in N$.

For $t \in \{1, \dots, |N| - 1\}$, set $\tau_t := |N| \cdot \varphi_i(u_T)$ for some $T \subseteq N$ such that $|T| = t$ and $i \in N \setminus T$. By **S**, $\varphi_i(u_T) = \varphi_j(u_S)$ for all $S, T \subseteq N$ such that $|S| = |T| > 0$ and $i \in N \setminus T$ and $j \in N \setminus S$. Therefore, τ_t do neither depend on the choice of $T \subseteq N$ nor on the choice of $i \in N \setminus T$.

Given $\varphi_j(u_T) = \tau_{|T|}/|N|$ for all $j \in N \setminus T$ and all $T \neq N$, we may infer from **E** and **S** that

$$\varphi_i(u_T) = \frac{1 - \sum_{j \in N \setminus T} \varphi_j(u_T)}{|T|} = \frac{\tau_{|T|}}{|N|} + \frac{1 - \tau_{|T|}}{|T|} \quad \text{for all } i \in T.$$

In view of (4), we have $\varphi = \zeta^\tau$. \square

Proof of Proposition 1. It is clear that Sh^α , $\alpha \leq 1$, inherits **L**, **E**, **ET**, and **NPE** from Sh and **ED**. Now, let the TU-value φ satisfy **L**, **E**, **ET**, and **NPE**. By Malawski (2008, Theorem 2), φ meets **S**. By Lemma 4, there is some $\tau \in \mathbb{R}^{|N|-1}$ such that $\varphi = \zeta^\tau$ given by (3).

Note that we have $\zeta^\tau = \text{Sh}$ for $\tau = (0, \dots, 0)$ and $\zeta^\tau = \text{ED}$ for $\tau = (1, \dots, 1)$. Since ζ^τ is linear in τ , we have $\text{Sh}^\alpha = \zeta^\tau$ for $\tau = (1 - \alpha, \dots, 1 - \alpha)$. Therefore, we have to show that $\tau = (c, \dots, c)$ for some $c \geq 0$. For $|N| = 1$, nothing is to show.

Suppose, $\tau_t < 0$ for some $t \in \{1, \dots, |N| - 1\}$. Consider $T \subsetneq N$, $|T| = t$. Then, $i \in N \setminus T$ is a null player in u_T and $u_T(N) = 1 \geq 0$. Yet, $\zeta_i^\tau(u_T) = \frac{\tau_t}{|N|} < 0$. Hence, $\tau_t \geq 0$ for all $t \in \{1, \dots, |N| - 1\}$.

Suppose now, $\tau_t > \tau_{t-1}$ for some $t \in \{2, \dots, |N| - 1\}$. Consider $T \subsetneq N$, $|T| = t$ and

$$v = -u_T + \frac{1}{|T|} \sum_{j \in T} u_{T \setminus \{j\}}.$$

Then, $i \in N \setminus T$ is a null player in v and $v(N) = 0$. Yet,

$$\zeta_i^\tau(v) = \frac{-\tau_t + \frac{1}{|T|} \sum_{j \in T} \tau_{t-1}}{|N|} = \frac{\tau_{t-1} - \tau_t}{|N|} < 0,$$

a contradiction to **NPE**. Hence, $\tau_t \leq \tau_{t-1}$ for all $t = 2, \dots, |N| - 1$. Analogously, one shows $\tau_t \geq \tau_{t-1}$. This concludes the proof. \square

We prepare the proof of Theorem 2 by a lemma showing that **E**, **A**, and **D** imply **L**.

Lemma 5. *Every TU-value φ that satisfies efficiency, additivity, and desirability is linear.*

Proof. Let φ meet **E**, **A**, and **D**. It suffices to show $\varphi(\lambda \cdot v) = \lambda \cdot \varphi(v)$ for all $\lambda \in \mathbb{R}$, $v \in \mathbb{V}$. First, we establish that for all $v, w \in \mathbb{V}$ and $i, j \in N$ such that

$$v(K \cup \{i\}) - v(K \cup \{j\}) \geq w(K \cup \{i\}) - w(K \cup \{j\}) \quad \text{for all } K \subseteq N \setminus \{i, j\}, \quad (5)$$

we have $\varphi_i(v) - \varphi_j(v) \geq \varphi_i(w) - \varphi_j(w)$.

By (5), we have $(v - w)(K \cup \{i\}) \geq (v - w)(K \cup \{j\})$ for all $K \subseteq N \setminus \{i, j\}$, i.e., $v - w$, i , and j meet the hypothesis of **D**. This entails

$$\begin{aligned} \varphi_i(v) - \varphi_j(v) &= \varphi_i(w + (v - w)) - \varphi_j(w + (v - w)) \\ &\stackrel{\mathbf{A}}{=} \varphi_i(w) - \varphi_j(w) + \varphi_i(v - w) - \varphi_j(v - w) \stackrel{\mathbf{D}}{\geq} \varphi_i(w) - \varphi_j(w) \end{aligned}$$

as claimed above.

It is well known that **A** implies homogeneity for rational scalars, i.e., $\varphi(\varrho \cdot v) = \varrho \cdot \varphi(v)$ for all $\varrho \in \mathbb{Q}$. By **A**, it suffices to consider coalition functions of the form $\lambda \cdot u_T$, $T \subseteq N$, $T \neq \emptyset$, $\lambda \in \mathbb{R} \setminus \mathbb{Q}$.

Let $\lambda > 0$ and $T \subseteq N$, $T \neq \emptyset$. Since the rational numbers are dense in the reals, there are rational sequences $(\lambda_k^-)_{k \in \mathbb{N}}$ and $(\lambda_k^+)_{k \in \mathbb{N}}$ such that $0 < \lambda_k^- \leq \lambda \leq \lambda_k^+$ for all $k \in \mathbb{N}$ and $\lim_{k \rightarrow \infty} \lambda_k^- = \lim_{k \rightarrow \infty} \lambda_k^+ = \lambda$. For all $i \in T$, it is clear that

$$\begin{aligned} \lambda_k^- \cdot (u_T(K \cup \{i\}) - u_T(K \cup \{j\})) &\leq \lambda \cdot (u_T(K \cup \{i\}) - u_T(K \cup \{j\})) \\ &\leq \lambda_k^+ \cdot (u_T(K \cup \{i\}) - u_T(K \cup \{j\})) \end{aligned}$$

for all $K \subseteq N \setminus \{i, j\}$ and $j \in N$. Hence, i , j , $\lambda_k^- \cdot u_T$, and $\lambda \cdot u_T$ (or $\lambda \cdot u_T$ and $\lambda_k^+ \cdot u_T$) satisfy the condition stated in (5). Thus, we have

$$\begin{aligned} (\varphi_i(\lambda_k^- \cdot u_T) - \varphi_j(\lambda_k^- \cdot u_T)) &\leq \varphi_i(\lambda \cdot u_T) - \varphi_j(\lambda \cdot u_T) \\ &\leq (\varphi_i(\lambda_k^+ \cdot u_T) - \varphi_j(\lambda_k^+ \cdot u_T)). \end{aligned}$$

Since **A** implies homogeneity for rational scalars, we obtain

$$\begin{aligned} \lambda_k^- \cdot (\varphi_i(u_T) - \varphi_j(u_T)) &\leq \varphi_i(\lambda \cdot u_T) - \varphi_j(\lambda \cdot u_T) \\ &\leq \lambda_k^+ \cdot (\varphi_i(u_T) - \varphi_j(u_T)). \end{aligned}$$

Taking the limit and by assumption, we thus have

$$\varphi_i(\lambda \cdot u_T) - \varphi_j(\lambda \cdot u_T) = \lambda \cdot (\varphi_i(u_T) - \varphi_j(u_T)).$$

Summing up over $j \in N$ gives

$$|N| \cdot \varphi_i(\lambda \cdot u_T) - \sum_{\ell \in N} \varphi_\ell(\lambda \cdot u_T) = |N| \cdot \lambda \cdot \varphi_i(u_T) - \lambda \cdot \sum_{\ell \in N} \varphi_\ell(u_T).$$

Using **E**, one obtains $\varphi_i(\lambda \cdot u_T) = \lambda \cdot \varphi_i(u_T)$. Analogously, this can be seen for $i \in N \setminus T$ or $\lambda < 0$. \square

Proof of Theorem 2. It is clear that Sh^α , $\alpha \in [0, 1]$ inherits **A**, **E**, **D**, and **NPE** from Sh . Let the TU-value φ satisfy **A**, **E**, **D**, and **NPE**. By Lemma 5, φ meets **L**. Since **D** implies **ET**, Proposition 1 already entails $\varphi = \text{Sh}^\alpha$ for some $\alpha \leq 1$. Suppose $0 > \alpha$. This entails

$$\varphi_i^\alpha(u_{N \setminus \{i\}}) = \frac{1 - \alpha}{n} > \frac{1 - \alpha}{n} + \frac{\alpha}{n - 1} = \varphi_j^\alpha(u_{N \setminus \{i\}}) \quad \text{for } i \in N, j \in N \setminus \{i\}.$$

Yet, the marginal contributions of i are not greater than those of j in $u_{N \setminus \{i\}}$, contradicting **D**. Hence $0 \leq \alpha \leq 1$. \square

Proof of Theorem 3. It is clear that Con^α , $\alpha \in [0, 1]$ inherits **A**, **E**, **D**, **NSE** and **SM**⁻ from Sh and Con . Let the TU-value φ satisfy **A**, **E**, **D**, and **NSE**. By Lemma 5, φ meets **L**. Moreover, **D** implies **ET**. Thus, Malawski (2008, Theorem 2) implies that φ meets **S**.

For all $i \in N$ we have $(u_{\{i\}})^0 = \mathbf{0} = (-u_{\{i\}})^0$. Since $\mathbf{0}(N) = 0$, **NSE** implies $\varphi_j(u_{\{i\}}) \geq 0$ and $\varphi_j(-u_{\{i\}}) \geq 0$ for all $j \in N \setminus \{i\}$. By **L**, $\varphi_j(-u_{\{i\}}) = -\varphi_j(u_{\{i\}})$. Hence, $\varphi_j(u_{\{i\}}) = 0$. By **E**, $\varphi_i(u_{\{i\}}) = 1$. Therefore, $\varphi(u_{\{i\}}) = \text{Sh}(u_{\{i\}}) = \text{ED}(u_{\{i\}}) = \text{Con}^\alpha(u_{\{i\}})$ for all $i \in N$ and all $\alpha \in \mathbb{R}$. Moreover, from (1) and **L** follows

$$\varphi_i(v) = \varphi_i(v^0) + v(\{i\}) \quad \text{for } i \in N \text{ and } v \in \mathbb{V}. \quad (6)$$

Note that on the class of zero-normalized games **NSE** becomes **NPE**. Further, one easily checks that the proof of Proposition 1 runs through smoothly within the class of zero-normalized games. Hence, $\varphi(v^0) = \text{Sh}^\alpha(v^0)$ for some $\alpha \leq 1$ and all $v \in \mathbb{V}$. Using (6), one infers $\varphi_i(v) = \text{Sh}_i^\alpha(v^0) + v(\{i\}) = \text{Con}^\alpha(v)$ for some $\alpha \leq 1$.

For $|N| \leq 2$, $\text{Sh} = \text{ED} = \text{Con}^\alpha$ for all $\alpha \in \mathbb{R}$. Thus, it suffices to show that $0 > \alpha$ contradicts **D** for $|N| \geq 3$. Suppose $0 > \alpha$. Then,

$$\varphi_i^\alpha(u_{N \setminus \{i\}}) = \frac{1 - \alpha}{n} > \frac{1 - \alpha}{n} + \frac{\alpha}{n - 1} = \varphi_j^\alpha(u_{N \setminus \{i\}}) \quad \text{for } i \in N, j \in N \setminus \{i\}.$$

Yet, the marginal contributions of i are not greater than those of j in $u_{N \setminus \{i\}}$, contradicting **D**. Hence $0 \leq \alpha \leq 1$. This proves the first part of the theorem.

Concerning (ii), it suffices to show that **L** and **SM**[−] imply **NSE**. Consider $v \in \mathbb{V}$ and $i \in N$ such that i is a null player in v and $v^0(N) \geq 0$. This entails $v^0(N) \geq \mathbf{0}^0(N)$ and $v(S \cup \{i\}) - v(S) \geq \mathbf{0}(S \cup \{i\}) - \mathbf{0}(S)$. By **SM**[−], we have $\varphi_i(v) \geq \varphi_i(\mathbf{0})$. Finally, **L** implies $\varphi_i(\mathbf{0}) = 0$ and we are done. \square

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